



*Research article***Optimal dissimulation control of insider trading under mandatory disclosure and quadratic penalty****Ji Wu^{1,2}, Qihong Nie¹ and Yonghui Zhou^{3,*}**¹ School of Mathematical Sciences, Guizhou Normal University, Guiyang, 550025, China² Guizhou Industry Polytechnic University, Guiyang, 550025, China³ School of Big Data and Computer Sciences, Guizhou Normal University, Guiyang, 550001, China*** Correspondence:** Email: yonghuizhou@gznu.edu.cn; Tel: +86-18785009082.

Abstract: Within the continuous-time Kyle framework, this paper develops an equilibrium model of insider trading under dual regulatory constraints: Mandatory public disclosure and quadratic economic penalties. By applying the Pontryagin maximum principle, we derive explicit closed-form solutions for all market equilibrium parameters. Three core findings are obtained. First, regulatory penalties curb insiders' aggressive terminal trading, reducing trading intensity and profits while preventing full private information incorporation into prices, leading to non-zero terminal residual information. Second, penalty constraints endogenously distort insiders' dissimulation strategies. Without penalties, full dissimulation $\sigma_x = \sigma_z$ (where σ_x denotes the insider's dissimulation noise volatility and σ_z denotes the market noise volatility) is optimal; with penalties, the optimal dissimulation volatility decreases with the growth of penalty intensity, and insiders abandon dissimulation entirely ($\sigma_x = 0$) as the penalty tends to infinity. Third, there exists a unique critical dissimulation ratio $k^* \approx 0.6823$ for terminal residual information. When the dissimulation ratio is below this critical threshold, terminal residual information in the no-disclosure regime is larger than that under mandatory disclosure, and the opposite holds when the ratio exceeds the threshold. Moreover, insiders' equilibrium profit without mandatory disclosure is always higher than that under disclosure for any admissible dissimulation ratio. We also briefly examine the optimal penalty design within this framework. These results revise the oversimplified assumption widely adopted by existing analogous models that fixes dissimulation volatility equal to market noise volatility, and provide theoretical guidance for the design of insider trading regulatory policies.

Keywords: mandatory public disclosure; quadratic penalty; insider trading; optimal dissimulation; Kyle model; stochastic control

Mathematics Subject Classification: Primary 60H10, 91G80; Secondary 60G35, 93E11

Nomenclature

Symbol	Definition
T	Trading horizon
Σ_0	Initial variance of the asset payoff V
σ_z	Volatility of noise traders' aggregate order flow
σ_x	Volatility of the insider's dissimulation noise component
R	Penalty intensity coefficient (dimensionless)
β_t	Insider's trading intensity coefficient
$\widetilde{\lambda}_t$	Price impact coefficient in the trading phase
$\widetilde{\mu}_t$	Price impact coefficient in the quote updating phase
θ_t	Insider's trading rate
P_t	Transaction price in the trading phase
P_t^*	Post-disclosure price in the quote updating phase
y_t	Aggregate order flow, $y_t = x_t + z_t$
$\widetilde{\Sigma}_t$	Conditional variance of V given public information under mandatory disclosure
$\widetilde{\Pi}$	Insider's equilibrium expected profit under mandatory disclosure

Note: All variables are dimensionless under the normalized model setting.

1. Introduction

Insider trading has long been a core challenge for financial market regulation. By exploiting undisclosed private information to capture excess profits, insider traders impair market fairness and the efficiency of price discovery. To curb such conduct, regulatory authorities worldwide generally adopt two types of regulatory instruments. The first is mandatory public disclosure, which requires insider traders to timely release order information to the market upon transaction completion so as to improve market transparency. The second is ex post economic penalty, whereby fines are imposed on verified insider trading violations. Nevertheless, the concurrent implementation of the two regulatory tools generates non-negligible interaction effects: Mandatory disclosure forces insiders to inject dissimulation noise component into their trades to conceal private information advantages, while ex post penalty schemes alter the optimal selection of their dissimulation strategies. This paper centers on theoretically characterizing the market equilibrium of insider trading subject to dual-layer joint regulation, which constitutes its core research question.

From the perspective of theoretical research progress, Kyle [1] first proposed a dynamic insider trading model under the discrete auction framework. Back [2] further extended this model to a continuous-time setup, establishing the classic Kyle-Back continuous auction model, which has been widely applied to quantitative analysis of market liquidity, price discovery efficiency and the impacts of regulatory policies. For extensions to transparent markets with mandatory public disclosure, Huddart et al. [3] were the first to identify that informed insiders superimpose noise terms on their trading

demand to prevent market makers from inferring private fundamental information from order flows, and such hidden trading behavior is defined as “trade dissimulation” in academia. Han et al. [4] subsequently derived the transparent market equilibrium within an environment with long-memory stochastic liquidity, with an exogenous assumption that the volatility of insider’s dissimulation noise component equals the volatility of market noise, namely the perfect imitation strategy. Nie and Zhou [5] constructed a closed-form σ_z -equilibrium under the premise of perfect imitation within an insider trading model with trading constraints in transparent markets. In terms of studies on ex post penalty mechanisms, Shin [6] categorizes informed market agents into professional speculators and corporate insiders. He demonstrates that strengthening competition between the two groups of informed agents can achieve regulatory objectives, and verifies that moderate tolerance of insider trading may become the optimal regulatory scheme under specific parametric conditions. Carré [7] proves that a stable market equilibrium exists regardless of the penalty rules formulated by regulators when market noise follows a uniform distribution. Within a specific model framework, he designs a differentiated penalty rule featuring “heavy penalties for minor violations and lenient penalties for severe violations”, balancing the dual regulatory goals of restraining insider trading and sustaining market liquidity. Çetin [8] investigates insider trading regulation in the continuous-time Kyle model via quadratic backward stochastic differential equations (BSDEs) and entropy penalty techniques. He constructs market equilibria for general asset payoff distributions and derives the optimal regulatory mechanism balancing market operational efficiency and trading fairness. Apart from the above representative studies, extended research on transparent markets and mandatory disclosure can be found in references [9–16], which respectively examine competitive trading and opaque strategies [9, 11], behavioral biases and heterogeneous beliefs [10, 14], as well as risk aversion, information sharing, and alternative disclosure formats [12, 13, 15, 16]. Meanwhile, theoretical contributions focusing on penalty constraints on trades are summarized in references [17–23], covering optimal enforcement and legal risk [17, 18, 23], behavioral and heterogeneous-belief extensions [19, 21], and alternative price impact specifications or information structures [20, 22].

A review of existing literature reveals two primary limitations of prior research: Existing studies either examine penalty mechanisms or mandatory disclosure as a single regulatory channel in isolation, and the perfect imitation assumption in transparent market equilibrium models such as Han [4], and Nie and Zhou [5] is imposed exogenously without rigorous justification. To address these gaps, this paper makes three main contributions. First, we develop a unified continuous-time Kyle framework that incorporates both mandatory disclosure and quadratic penalties. Second, by applying the Pontryagin maximum principle, we endogenously derive the optimal dissimulation strategy: We prove that perfect imitation is globally optimal in the absence of penalties (thereby justifying the exogenous assumption in prior literature), and reveal that the optimal dissimulation volatility is strictly smaller than the market noise volatility under positive penalties, decreasing with the penalty intensity. Third, we identify a critical dissimulation ratio that determines the relative informational efficiency of mandatory disclosure versus no disclosure, and we examine the optimal penalty design from the regulator’s perspective. Methodologically, the Pontryagin maximum principle is more intuitive and analytically tractable than the quadratic BSDE approach in Çetin [8] (for comprehensive treatments of the maximum principle, see [24–26]), yielding explicit closed-form solutions for all equilibrium parameters. The remainder of this paper is organized as follows. Section 2 lays out the fundamental model setup. Section 3 derives necessary conditions for market equilibrium and solves the explicit equilibrium solutions using the

Pontryagin maximum principle. Section 4 conducts numerical simulations and parametric sensitivity analysis. Section 5 concludes the paper and proposes directions for future research extensions. For the convenience of readers, a Nomenclature summarizing the key symbols and acronyms is provided at the end of the paper.

2. Model setup

In the following, all random variables or processes are defined on a filtered probability space satisfying the usual conditions [27, 28].

Consider a financial market for a risky asset trading in a finite interval $[0, T]$, whose terminal payoff V follows a normal distribution $\mathcal{N}(p_0, \Sigma_0)$ and is publicly revealed at the terminal time T . As the classical modeling framework of insider trading in [8], the market consists of four types of participants: Insider traders, noise traders, market makers, and regulators. The detailed behavioral assumptions of each participant are specified as follows:

(i) *Noise traders*, who possess no private information on the asset value, and submit orders z_t at time $t \in [0, T]$ randomly satisfying a dynamics below

$$dz_t = \sigma_z dW_t \quad (2.1)$$

where $z_0 = 0$, $\sigma_z > 0$ is an exogenous market parameter and W denotes a standard Brownian motion independent of V .

(ii) *An insider*, who knows the asset value V and submits aggregate order flow x_t governed by [4]

$$dx_t = \theta_t dt + \sigma_x d\tilde{W}_t, \quad x_0 = 0 \quad (2.2)$$

where the process $\theta_t : [0, T] \rightarrow \mathbb{R}$ with $\mathbb{E} \left[\int_0^T \theta_s^2 ds \right] < \infty$ and constant $\sigma_x > 0$ stands for the volatility of the noise component the insider introduces for dissimulation, and it is an endogenous decision variable chosen by the insider at $t = 0$ to conceal private information. Respectively, here $\tilde{W}$ is a standard Brownian motion independent of V and W .

(iii) *Market makers*, who only observe the aggregate market order flow $y_t = x_t + z_t$ (not disentangling the individual components x_t and z_t). In particular, market makers do not directly observe the insider's dissimulation volatility σ_x , but know the insider's strategy space and the disclosure rule, and set pricing rule in a semi-strong efficient way in trading phrase and quote updating phrase respectively as in [1, 2],

$$P_t = \mathbb{E}[V | y^t, x_{t-}], \quad P_t^* = \mathbb{E}[V | x^t], \quad (2.3)$$

where $y^t = \sigma\{y_s, 0 \leq s \leq t\}$ and $x^t = \sigma\{x_s, 0 \leq s \leq t\}$.

(iv) *A regulator*, who implements ex post supervision over insider trading, as documented in references [6–8], with probability $p \in [0, 1)$ of successfully convicting the insider and penalty cost taking the quadratic form $c = \frac{1}{2}pa \int_0^T \theta_t^2 dt$ with the penalty intensity coefficient; in fact, $R = \frac{pa}{1-p}$ determines a regulation.

The timing of events over $[0, T]$ is as follows. In the trading stage, the insider submits x_t based on his private information, noise traders submit z_t , and market makers observe the aggregate order flow $y_t = x_t + z_t$ and set the market price P_t . Once the trade is completed, the insider's order flow x_t is publicly

disclosed, and market makers update the price to P_t^* . This timing follows the transparent-market setting of [4] (see Chapter 4 and Figure 1 therein).

Definition 1. An equilibrium of the market is a profile of $(x; P, P^*)$ for a given regulation R satisfying

(i) Given (P, P^*) , the insider's strategy x maximizes

$$\mathbb{E} \left[\int_0^T (V - P_{t+}) dx_t - \frac{1}{2} R \int_0^T \theta_t^2 dt \right]; \quad (2.4)$$

(ii) Given x , the pricing rule (P, P^*) satisfies a semi-strong efficient way as (2.3).

For solvability of equilibrium, we here focus on linear strategies as in [4]: For the insider,

$$\theta_t = \tilde{\beta}_t (V - P_t^*), \quad (2.5)$$

and for market makers,

$$dP_t = P_{t+dt} - P_t^* = \tilde{\lambda}_t dy_t, \quad dP_t^* = P_t^* - P_t = \tilde{\mu}_t dx_t, \quad (2.6)$$

where $\tilde{\beta}_t$, $\tilde{\lambda}_t$ and $\tilde{\mu}_t$ are all $\mathcal{F}_t^y$ -adapted.

Definition 2. A profile $(\tilde{\beta}, \sigma_x; \tilde{\lambda}, \tilde{\mu})$ is called a linear equilibrium if $(x; P, P^*)$ is an equilibrium given a regulation R .

Remark 1. As in (2.4), we adopt an ex post penalty scheme $-\frac{1}{2} R \int_0^T \theta_t^2 dt$, that is, monetary sanctions are only imposed on the informed trading component of the insider's order flow, while the dissimulation noise component is exempted from penalty calculation. The rationality of this modeling choice is elaborated from three perspectives:

(i) Modeling foundation. We follow the framework of [7] for ex post regulatory supervision. The regulator only levies fines when investigations yield sufficient evidence to confirm illegal insider trading; otherwise, no penalty is charged. Under this setup, the regulator never makes type 1 errors (never convicts a trader that didn't use insider information) but can make type 2 errors (not convicting a trader that did use insider information). The essential illegality of insider trading lies in capturing illicit gains via private information advantages, rather than injecting dissimulation noise to mask trading positions.

(ii) Stochastic control theory. The dissimulation noise component constitutes a martingale process whose conditional expectation equals zero under any information filtration, which exerts no impact on market pricing efficiency or the insider's expected profit. Restricting quadratic penalty constraints solely to the drift control variable aligns with standard modeling practices in continuous-time stochastic optimal control.

(iii) Real-world regulatory practice. U.S. SEC Rule 10b5-1(c) permits insiders to assert affirmative defenses for partial trades via pre-established written trading plans. Meanwhile, the Securities Law of the People's Republic of China (Revised) stipulates that fines for insider trading are calculated based on illegal gains. Since the martingale dissimulation component generates no illicit profit, excluding it from the penalty base is consistent with real-world regulatory rules.

3. Linear equilibrium

Based on the model constructed in Section 2, this section simplifies the optimization problem under linear trading strategies, solves the insiders optimal control problem (2.4) via the Pontryagin maximum principle [29] under two scenarios: Zero penalty cost ($R = 0$) and positive penalty cost ($R > 0$). In this section, we will study the uniqueness and existence of linear equilibrium for our insider trading model.

Throughout this paper, the tilde notation $\tilde{\cdot}$ represents the scenario with public disclosure, whereas variables without a tilde correspond to the setting without public disclosure. The superscript 0 ($\cdot^0$) indicates the penalty-free case, while those without denote the general case with penalties.

3.1. Case I: $R = 0$

Theorem 1. In the absence of regulatory penalties, there is a unique linear equilibrium $(\tilde{\beta}^0, \sigma_x; \tilde{\lambda}^0, \tilde{\mu}^0)$ satisfying

$$\tilde{\beta}_t^0 = \frac{\sigma_x}{T-t} \sqrt{\frac{T}{\Sigma_0}}, \quad \sigma_x = \sigma_z, \quad \tilde{\lambda}_t^0 = \frac{\sigma_x}{(\sigma_x^2 + \sigma_z^2)} \sqrt{\frac{\Sigma_0}{T}}, \quad \tilde{\mu}_t^0 = \sqrt{\frac{\Sigma_0}{\sigma_x^2 T}}. \quad (3.1)$$

Moreover, the maximum expected profit of the insider is

$$\tilde{\Pi}^0 = \frac{\sigma_z}{2} \sqrt{T \Sigma_0}, \quad (3.2)$$

and the residual private information variance after order disclosure, defined by $\tilde{\Sigma}_t^0 = \mathbb{E}(V - P_t^*)^2$, reads

$$\tilde{\Sigma}_t^0 = \Sigma_0 - \frac{t}{T} \Sigma_0. \quad (3.3)$$

Proof. Firstly, for market makers, from (2.2) and (2.5), there is a signal observation system with respect to (V, x) in the quote updating phrase governed by

$$\begin{cases} dV = 0, \\ dx_t = \tilde{\beta}_t(V - P_t^*)dt + \sigma_x d\tilde{W}_t. \end{cases} \quad (3.4)$$

Since the post-disclosure price P_t^* satisfies the semi-strong efficient pricing (2.3), it follows from Theorem 12.1 in [28] that

$$\tilde{\mu}_t = \frac{\tilde{\Sigma}_t \tilde{\beta}_t}{\sigma_x^2}, \quad \frac{d\tilde{\Sigma}_t}{dt} = -\frac{\tilde{\Sigma}_t^2 \tilde{\beta}_t^2}{\sigma_x^2}.$$

Secondly, similar to the proof of Lemma 4 in [4], and from (2.1) and (2.2), a heuristic derivation for the expression of λ is given below:

$$\begin{aligned} P_{t+dt} &= E[V | y^{t+dt}, x^t] = E[V | y^t, x^t, dy_t] = E[V | x^t, dy_t] \\ &= E[V | x^t] + \frac{\text{cov}(V, dy_t | x^t)}{\text{Var}(dy_t | x^t)} \times (dy_t - E[dy_t | x^t]) \\ &= P_t^* + \frac{\tilde{\Sigma}_t \tilde{\beta}_t}{\sigma_x^2 + \sigma_z^2} dy_t, \end{aligned} \quad (3.5)$$

where $dy_t = \tilde{\beta}_t(V - P_t^*)dt + \sigma_x d\tilde{W}_t + \sigma_z dW_t$. According to (2.6), we have $\tilde{\lambda}_t = \frac{\tilde{\Sigma}_t \tilde{\beta}_t}{\sigma_x^2 + \sigma_z^2}$.

The insider's optimization proceeds in two stages. Since σ_x is chosen at $t = 0$, while $\tilde{\beta}_t$ is an adapted process, we first fix σ_x and optimize $\tilde{\beta}_t$, and then optimize σ_x over the resulting value function.

Therefore, the problem of the insider (2.4) simplified to

$$\tilde{\Pi}^0 = \max_{\tilde{\beta}} \mathbb{E} \int_0^T (V - P_t^* - \tilde{\lambda}_t dy_t) dx_t = \max_{\tilde{\beta}} \mathbb{E} \int_0^T \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\Sigma}_t \tilde{\beta}_t dt. \quad (3.6)$$

Then, we construct the Hamiltonian function for the optimal control problem

$$H^0 = \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\beta}_t^0 \tilde{\Sigma}_t^0 + \gamma_t^0 \left(-\frac{(\tilde{\beta}_t^0)^2 (\tilde{\Sigma}_t^0)^2}{\sigma_x^2} \right). \quad (3.7)$$

By Pontryagin's maximum principle, the first-order necessary condition of (3.7) is:

$$\frac{\partial H^0}{\partial \tilde{\beta}_t^0} = \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\Sigma}_t^0 - \gamma_t^0 \frac{2\tilde{\beta}_t^0 (\tilde{\Sigma}_t^0)^2}{\sigma_x^2} = 0, \quad (3.8)$$

which implies

$$\tilde{\beta}_t^0 = \frac{\sigma_x^2 \sigma_z^2}{2\gamma_t^0 \tilde{\Sigma}_t^0 (\sigma_x^2 + \sigma_z^2)}, \quad (3.9)$$

and state equations should satisfy:

$$\begin{aligned} \frac{d\tilde{\Sigma}_t^0}{dt} &= \frac{\partial H^0}{\partial \gamma_t^0} = -\frac{(\tilde{\beta}_t^0)^2 (\tilde{\Sigma}_t^0)^2}{\sigma_x^2} = -\frac{\sigma_x^2 \sigma_z^4}{4(\gamma_t^0)^2 (\sigma_x^2 + \sigma_z^2)^2}, \\ \frac{d\gamma_t^0}{dt} &= -\frac{\partial H^0}{\partial \tilde{\Sigma}_t^0} = \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\beta}_t^0 - \frac{2\gamma_t^0 (\tilde{\beta}_t^0)^2 \tilde{\Sigma}_t^0}{\sigma_x^2} = 0. \end{aligned} \quad (3.10)$$

It follows that

$$\begin{aligned} \gamma_t^0 &= \sqrt{\frac{\sigma_x^2 \sigma_z^4 T}{4(\sigma_x^2 + \sigma_z^2)^2 (\Sigma_0 - \tilde{\Sigma}_{T-}^0)}}, \\ \tilde{\beta}_t^0 &= \frac{\sigma_x^2 \sigma_z^2}{\sqrt{\frac{\sigma_x^2 \sigma_z^4 T}{(\sigma_x^2 + \sigma_z^2)^2 (\Sigma_0 - \tilde{\Sigma}_{T-}^0)}} \left(\Sigma_0 - \frac{t(\Sigma_0 - \tilde{\Sigma}_{T-}^0)}{T} \right) (\sigma_x^2 + \sigma_z^2)}. \end{aligned} \quad (3.11)$$

So the profit of the insider is

$$\tilde{\Pi}^0 = \mathbb{E} \left[\int_0^T \left(\frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\Sigma}_t^0 \tilde{\beta}_t^0 \right) dt \right] = \frac{\sigma_x \sigma_z^2}{\sigma_x^2 + \sigma_z^2} \sqrt{T(\Sigma_0 - \tilde{\Sigma}_{T-}^0)}. \quad (3.12)$$

Since $\tilde{\Pi}^0$ in (3.12) is strictly decreasing in $\tilde{\Sigma}_{T-}^0$ and $\tilde{\Sigma}_{T-}^0 \geq 0$, then we can choose $\tilde{\beta}_t^0$ such that the terminal residual variance $\tilde{\Sigma}_{T-}^0 = 0$ to maximize the above profit for a given σ_x , which is

$$\tilde{\Pi}^0(\sigma_x) = \frac{\sigma_x \sigma_z^2}{\sigma_x^2 + \sigma_z^2} \sqrt{T \Sigma_0}; \quad (3.13)$$

and then choose an optimal σ_x to maximize the former profit by the first-order derivative condition

$$\frac{d\tilde{\Pi}^0(\sigma_x)}{d\sigma_x} = \frac{\sigma_z^2(\sigma_z^2 - \sigma_x^2)}{(\sigma_x^2 + \sigma_z^2)^2} \sqrt{T\Sigma_0} = 0, \quad (3.14)$$

which vanishes at $\sigma_x = \sigma_z$. Since $\tilde{\Pi}^0(0) = \tilde{\Pi}^0(\infty) = 0$ and $\tilde{\Pi}^0(\sigma_x) > 0$, this stationary point is the unique global maximum. Therefore the maximal profit is

$$\tilde{\Pi}^0 = \frac{\sigma_z}{2} \sqrt{T\Sigma_0}.$$

And we have that

$$\tilde{\beta}_t^0 = \frac{\sigma_z}{T-t} \sqrt{\frac{T}{\Sigma_0}}, \quad \tilde{\Sigma}_t^0 = \Sigma_0 - \frac{t}{T} \Sigma_0, \quad \tilde{\mu}_t^0 = \sqrt{\frac{\Sigma_0}{\sigma_x^2 T}}, \quad \tilde{\lambda}_t^0 = \frac{\sigma_x}{(\sigma_x^2 + \sigma_z^2)} \sqrt{\frac{\Sigma_0}{T}}. \quad (3.15)$$

□

3.2. Case II: $R > 0$

Theorem 2. With regulatory penalties imposed, there is a unique linear equilibrium $(\tilde{\beta}, \sigma_x; \tilde{\mu}, \tilde{\lambda})$ satisfying

$$\begin{aligned} \tilde{\beta}_t &= \frac{\sigma_z^2 \tilde{\Sigma}_{T-}}{(\sigma_x^2 + \sigma_z^2) R \left(\Sigma_0 - \frac{\sigma_z^4 \tilde{\Sigma}_{T-}^2}{R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2} t \right)}, \quad \sigma_x \in (0, \sigma_z), \\ \tilde{\mu}_t &= \frac{\sigma_z^2 \tilde{\Sigma}_{T-}}{\sigma_x^2 (\sigma_x^2 + \sigma_z^2) R}, \quad \tilde{\lambda}_t = \frac{\sigma_z^2 \tilde{\Sigma}_{T-}}{(\sigma_x^2 + \sigma_z^2)^2 R}. \end{aligned} \quad (3.16)$$

Moreover, the maximum expected profit of the insider is

$$\tilde{\Pi} = \frac{\sigma_z^4 \tilde{\Sigma}_{T-}}{R(\sigma_x^2 + \sigma_z^2)^2} T + \frac{1}{2} R \sigma_x^2 \left(\ln \frac{\tilde{\Sigma}_{T-}}{\Sigma_0} \right), \quad (3.17)$$

where

$$\tilde{\Sigma}_{T-} = \frac{-R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2 + \sqrt{(R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2)^2 + 4 \sigma_z^4 T R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2 \Sigma_0}}{2 \sigma_z^4 T}. \quad (3.18)$$

The residual private information variance after order disclosure is

$$\tilde{\Sigma}_t = \Sigma_0 - \frac{\sigma_z^4 \tilde{\Sigma}_{T-}^2}{R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2} t. \quad (3.19)$$

Proof. Since $R > 0$, the Hamiltonian function for the optimal control problem (2.4) can be constructed as

$$H = \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\beta}_t \tilde{\Sigma}_t - \frac{1}{2} R \tilde{\beta}_t^2 \tilde{\Sigma}_t + \gamma_t \left(-\frac{\tilde{\Sigma}_t^2 \tilde{\beta}_t^2}{\sigma_x^2} \right). \quad (3.20)$$

By Pontryagin's maximum principle, the first-order necessary condition of (3.20) is

$$\frac{\partial H}{\partial \tilde{\beta}_t} = \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\Sigma}_t - R \tilde{\beta}_t \tilde{\Sigma}_t - 2\gamma_t \frac{\tilde{\Sigma}_t^2 \tilde{\beta}_t}{\sigma_x^2} = 0, \quad (3.21)$$

which implies

$$\tilde{\beta}_t = \frac{\sigma_z^2 \sigma_x^2}{(\sigma_x^2 + \sigma_z^2)(R\sigma_x^2 + 2\gamma_t \tilde{\Sigma}_t)}, \quad (3.22)$$

and state equations should satisfy

$$\frac{d\gamma_t}{dt} = -\frac{\partial H}{\partial \tilde{\Sigma}_t} = -\frac{R\sigma_z^4 \sigma_x^4}{2(\sigma_x^2 + \sigma_z^2)^2 (R\sigma_x^2 + 2\gamma_t \tilde{\Sigma}_t)^2}, \quad (3.23)$$

$$\frac{d\tilde{\Sigma}_t}{dt} = -\frac{\tilde{\Sigma}_t^2 \tilde{\beta}_t^2}{\sigma_x^2} = -\frac{\tilde{\Sigma}_t^2 \sigma_z^4 \sigma_x^2}{(\sigma_x^2 + \sigma_z^2)^2 (R\sigma_x^2 + 2\gamma_t \tilde{\Sigma}_t)^2}. \quad (3.24)$$

We conjecture that $\tilde{\Sigma}_{T-} > 0$ (whereas $\tilde{\Sigma}_{T-} = 0$ when $R = 0$).

In fact, for $R > 0$, if we impose $\tilde{\Sigma}_{T-} = 0$, we obtain

$$\frac{d\tilde{\Sigma}_t}{d\gamma_t} = \frac{2\tilde{\Sigma}_t^2}{R\sigma_x^2}.$$

Then together with (3.24), it gives

$$\gamma_t = \left(\frac{1}{\tilde{\Sigma}_0} - \frac{1}{\tilde{\Sigma}_t} \right) \frac{R\sigma_x^2}{2} + \gamma_0,$$

$$\tilde{\Sigma}_t = \tilde{\Sigma}_0 \left(\frac{T-t}{T} \right).$$

Then we have

$$\tilde{\beta}_t = \frac{\sigma_x}{T-t} \sqrt{\frac{T}{\tilde{\Sigma}_0}}.$$

So the corresponding profit functional reads

$$\begin{aligned} \tilde{\Pi} &= \mathbb{E} \left[\int_0^T \left(\frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} \tilde{\beta}_t \tilde{\Sigma}_t - \frac{1}{2} R \tilde{\beta}_t^2 \tilde{\Sigma}_t \right) dt \right] \\ &= \frac{\sigma_z^2 \sigma_x}{\sigma_x^2 + \sigma_z^2} \sqrt{\tilde{\Sigma}_0 T} + \frac{1}{2} R \sigma_x^2 \ln(T-t)|_0^{T-} = -\infty, \end{aligned}$$

which means that $\tilde{\Sigma}_{T-} > 0$.

By the Pontryagin maximum principle, the transversality condition must hold at terminal time: $\gamma_{T-} \delta \tilde{\Sigma}_{T-} = 0$, since $\tilde{\Sigma}_{T-} > 0$, the variation $\delta \tilde{\Sigma}_{T-}$ is not identically zero. Hence we must have $\gamma_{T-} = 0$.

We then re-solve the system consisting of (3.23) and (3.24):

$$\gamma_t = \frac{R\sigma_x^2}{2} \left(\frac{1}{\tilde{\Sigma}_{T-}} - \frac{1}{\tilde{\Sigma}_t} \right), \quad (3.25)$$

$$\frac{d\widetilde{\Sigma}_t}{dt} = -\frac{\sigma_z^4 \widetilde{\Sigma}_{T-}^2}{R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2}. \quad (3.26)$$

Hence we obtain

$$\widetilde{\Sigma}_t = \Sigma_0 - \frac{\sigma_z^4 \widetilde{\Sigma}_{T-}^2}{R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2} t, \quad (3.27)$$

$$\widetilde{\beta}_t = \frac{\sigma_z^2 \widetilde{\Sigma}_{T-}}{(\sigma_x^2 + \sigma_z^2) R \widetilde{\Sigma}_t} = \frac{\sigma_z^2 \widetilde{\Sigma}_{T-}}{(\sigma_x^2 + \sigma_z^2) R \left(\Sigma_0 - \frac{\sigma_z^4 \widetilde{\Sigma}_{T-}^2}{R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2} t \right)}, \quad (3.28)$$

$$\widetilde{\mu}_t = \frac{\sigma_z^2 \widetilde{\Sigma}_{T-}}{\sigma_x^2 (\sigma_x^2 + \sigma_z^2) R}, \quad \widetilde{\lambda}_t = \frac{\sigma_z^2 \widetilde{\Sigma}_{T-}}{(\sigma_x^2 + \sigma_z^2)^2 R} \quad (3.29)$$

where

$$\widetilde{\Sigma}_{T-} = \frac{-R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2 + \sqrt{(R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2)^2 + 4 \sigma_z^4 T R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2 \Sigma_0}}{2 \sigma_z^4 T}. \quad (3.30)$$

The maximum expected profit is

$$\widetilde{\Pi} = \frac{\sigma_z^4 \widetilde{\Sigma}_{T-}}{R (\sigma_x^2 + \sigma_z^2)^2} T + \frac{1}{2} R \sigma_x^2 \left(\ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0} \right). \quad (3.31)$$

Therefore,

$$\frac{d\widetilde{\Sigma}_{T-}}{d\sigma_x} = \frac{2 (3\sigma_x^2 + \sigma_z^2) \sigma_z^4 T \widetilde{\Sigma}_{T-}^2}{\sigma_x (\sigma_x^2 + \sigma_z^2) \sqrt{(R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2)^2 + 4 \sigma_z^4 T R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2 \Sigma_0}}, \quad (3.32)$$

$$\frac{d\widetilde{\Pi}}{d\sigma_x} = \frac{\sigma_z^4 T \widetilde{\Sigma}_{T-}}{R \sigma_x (\sigma_x^2 + \sigma_z^2)^3} (\sigma_z^2 - \sigma_x^2) + R \sigma_x \ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0}. \quad (3.33)$$

Since $\widetilde{\Sigma}_{T-} < \Sigma_0$, the second term on the right-hand side of (3.33) is uniformly negative. Consequently, whenever $\sigma_x^* \in [\sigma_z, \infty)$, it follows that $\frac{d\widetilde{\Pi}}{d\sigma_x} < 0$. And since

$$\lim_{\sigma_x \rightarrow 0} \frac{d\widetilde{\Pi}}{d\sigma_x} = \sqrt{T \Sigma_0} > 0, \quad (3.34)$$

there exists a $\sigma_x^* \in (0, \sigma_z)$ such that $\frac{d\widetilde{\Pi}}{d\sigma_x} = 0$.

We now proceed to prove the uniqueness of this solution. Let

$$f(\sigma_x) = \frac{\sigma_z^4 T \widetilde{\Sigma}_{T-}}{R \sigma_x (\sigma_x^2 + \sigma_z^2)^3} (\sigma_z^2 - \sigma_x^2) + R \sigma_x \ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0} = 0. \quad (3.35)$$

Rearranging and simplifying the above equation yields

$$-\frac{\widetilde{\Sigma}_{T-}}{\Sigma_0 - \widetilde{\Sigma}_{T-}} \ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0} = \frac{\sigma_z^2 - \sigma_x^2}{\sigma_x^2 + \sigma_z^2}. \quad (3.36)$$

Define $\Sigma = \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0} \in (0, 1)$, and set $g(\Sigma) = -\frac{\widetilde{\Sigma}_{T-}}{\Sigma_0 - \widetilde{\Sigma}_{T-}} \ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0} = -\frac{\Sigma}{1 - \Sigma} \ln \Sigma$. Then direct differentiation gives

$$g'(\Sigma) = \frac{-\ln \Sigma - 1 + \Sigma}{(1 - \Sigma)^2} > 0. \quad (3.37)$$

Together with Eq (3.32), the chain rule implies

$$\frac{dg}{d\sigma_x} = \frac{dg}{d\Sigma} \frac{d\Sigma}{d\sigma_x} > 0. \quad (3.38)$$

Let $h(\sigma_x) = \frac{\sigma_z^2 - \sigma_x^2}{\sigma_x^2 + \sigma_z^2}$. Then,

$$h'(\sigma_x) = \frac{-4\sigma_x\sigma_z^2}{(\sigma_x^2 + \sigma_z^2)^2} < 0. \quad (3.39)$$

Now define the auxiliary function

$$H(\sigma_x) = \frac{\sigma_z^2 - \sigma_x^2}{\sigma_x^2 + \sigma_z^2} + \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0 - \widetilde{\Sigma}_{T-}} \ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0}, \quad (3.40)$$

whose first derivative satisfies

$$H'(\sigma_x) = h'(\sigma_x) - g'(\sigma_x) < 0. \quad (3.41)$$

Moreover, we have the following limit results:

$$\lim_{\sigma_x \rightarrow 0} H(\sigma_x) = 1, \quad \lim_{\sigma_x \rightarrow \sigma_z} H(\sigma_x) < 0. \quad (3.42)$$

Combining the Intermediate Value Theorem and the strict monotonicity of $H(\sigma_x)$, we conclude that there exists a unique threshold $\sigma_x^* \in (0, \sigma_z)$ such that $H(\sigma_x) = 0$. Therefore, $\frac{d\bar{\Pi}}{d\sigma_x} = 0$ has a unique solution. At this point, the profit attains its maximum, and the corresponding maximized profit is given by (3.31). $\square$

4. Properties and trading regulation

4.1. Properties of linear equilibrium

Let the optimal dissimulation ratio be $k = \frac{\sigma_x}{\sigma_z} \in (0, 1]$ in Theorem 1 and Theorem 2, and define k^* the solution to the equation $k^3 + k - 1 = 0$ approximately 0.6823.

Property 1 (Comparative static properties with respect to penalty coefficient).

$$(1) \frac{d\bar{\beta}_t}{dR} < 0; \quad \frac{d\bar{\Pi}}{dR} < 0; \quad \frac{d\widetilde{\Sigma}_{T-}}{dR} > 0; \quad \frac{dk}{dR} < 0.$$

$$(2) \lim_{R \rightarrow 0} \tilde{\beta}_t = \tilde{\beta}_t^0; \quad \lim_{R \rightarrow 0} \tilde{\Sigma}_{T-} = \tilde{\Sigma}_{T-}^0; \quad \lim_{R \rightarrow 0} \tilde{\Pi} = \tilde{\Pi}^0; \quad \lim_{R \rightarrow 0} k = 1.$$

$$(3) \lim_{R \rightarrow \infty} \tilde{\beta}_t = 0; \quad \lim_{R \rightarrow \infty} \tilde{\Sigma}_{T-} = \Sigma_0; \quad \lim_{R \rightarrow \infty} \tilde{\Pi} = 0; \quad \lim_{R \rightarrow \infty} k = 0.$$

Proof. From (3.28), (3.30), (3.31) and (3.15), (3.13) we can get $\frac{d\tilde{\beta}_t}{dR} < 0$, $\frac{d\tilde{\Pi}}{dR} < 0$, $\frac{d\tilde{\Sigma}_{T-}}{dR} > 0$. And from (3.36), we obtain $-\frac{\tilde{\Sigma}_{T-}}{\Sigma_0 - \tilde{\Sigma}_{T-}} \ln \frac{\tilde{\Sigma}_{T-}}{\Sigma_0} = \frac{1-k^2}{1+k^2}$. Then differentiating both sides with respect to $\tilde{\Sigma}_{T-}$ yields $-\frac{4k}{(1+k^2)^2} \frac{dk}{d\tilde{\Sigma}_{T-}} > 0$, which implies $\frac{dk}{d\tilde{\Sigma}_{T-}} < 0$. So $\frac{dk}{dR} = \frac{dk}{d\tilde{\Sigma}_{T-}} \frac{d\tilde{\Sigma}_{T-}}{dR} < 0$.

The remaining results can be calculated according to the above formula.

Property 2 (Market characteristics varying with time).

$$(1) \lim_{t \rightarrow T-} \tilde{\beta}_t = \frac{\sigma_z^2}{(\sigma_x^2 + \sigma_z^2)R}; \quad \lim_{t \rightarrow T-} \tilde{\beta}_t^0 = +\infty.$$

$$(2) \lim_{t \rightarrow T-} \tilde{\Sigma}_t = \frac{-L + \sqrt{L^2 + 4\sigma_z^4 T L \Sigma_0}}{2\sigma_z^4 T} > 0 \text{ with } L = R^2 \sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2; \quad \lim_{t \rightarrow T-} \tilde{\Sigma}_t^0 = 0.$$

$$(3) \frac{d\tilde{\beta}_t}{dt} > 0; \quad \frac{d\tilde{\beta}_t^0}{dt} > 0; \quad \frac{d\tilde{\Sigma}_t}{dt} < 0; \quad \frac{d\tilde{\Sigma}_t^0}{dt} < 0.$$

Proof. It follows from (3.28), (3.30) and (3.15).

Property 3 (Parameter comparison between public disclosure and no public disclosure).

$$(1) \tilde{\Sigma}_{T-} \geq \Sigma_{T-} \text{ if } k \geq k^*, \text{ or } \tilde{\Sigma}_{T-} < \Sigma_{T-} \text{ if } k < k^*; \quad \tilde{\Sigma}_{T-}^0 = \Sigma_{T-}^0 = 0, \text{ and } k = 1.$$

$$(2) \Pi > \tilde{\Pi}; \quad \Pi^0 > \tilde{\Pi}^0.$$

Proof. In the no-disclosure regime, market makers observe only the aggregate order flow $y_t = x_t + z_t$, and the filtering step uses the observation equation dy_t instead of (dx_t, dy_t) as in the disclosure regime. The conditional mean and variance can be obtained either by following the standard Kyle-Back derivation [1, 2] or by applying the same maximum-principle argument as in Section 3; both approaches yield identical results. We can separately derive the expressions of Σ_{T-} and Π under the no-public-disclosure scenario. By rearranging $\tilde{\Sigma}_{T-}$ and $\tilde{\Pi}$ via Eqs (3.30) and (3.31), we obtain

$$\Sigma_{T-} = \frac{-R^2 \sigma_z^2 + \sqrt{(R^2 \sigma_z^2)^2 + 4R^2 \sigma_z^2 \Sigma_0 T}}{2T}, \quad (4.1)$$

$$\tilde{\Sigma}_{T-} = \frac{-R^2 \frac{\sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2}{\sigma_z^4} + \sqrt{\left(R^2 \frac{\sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2}{\sigma_z^4}\right)^2 + 4R^2 \Sigma_0 T \frac{\sigma_x^2 (\sigma_x^2 + \sigma_z^2)^2}{\sigma_z^4}}}{2T}, \quad (4.2)$$

$$\Pi = R\sigma_z^2 \left(\frac{\Sigma_0 - \Sigma_{T-}}{\Sigma_{T-}} + \frac{1}{2} \ln \frac{\Sigma_{T-}}{\Sigma_0} \right), \quad (4.3)$$

$$\tilde{\Pi} = Rk^2 \sigma_z^2 \left(\frac{\Sigma_0 - \tilde{\Sigma}_{T-}}{\tilde{\Sigma}_{T-}} + \frac{1}{2} \ln \frac{\tilde{\Sigma}_{T-}}{\Sigma_0} \right). \quad (4.4)$$

Define

$$f(\Sigma) = \frac{-R^2 \Sigma + \sqrt{R^4 \Sigma^2 + 4R^2 \Sigma_0 T \Sigma}}{2T}, \quad (4.5)$$

then

$$f'(\Sigma) = \frac{R}{2T} \cdot \frac{\Sigma R^2 + 2\Sigma_0 T - \sqrt{R^4 \Sigma^2 + 4\Sigma_0 T \Sigma R^2}}{\sqrt{R^4 \Sigma^2 + 4\Sigma_0 T \Sigma}} > 0. \quad (4.6)$$

Set $\frac{\sigma_x^2(\sigma_x^2 + \sigma_z^2)^2}{\sigma_z^4} - \sigma_z^2 = \sigma_z^2(k^3 + k + 1)(k^3 + k - 1) = 0$. So solving this equation yields the critical value $k^* \approx 0.6823$.

Therefore, if $k \geq k^*$, we have $\widetilde{\Sigma}_{T-} \geq \Sigma_{T-}$; if $k < k^*$, we have $\widetilde{\Sigma}_{T-} < \Sigma_{T-}$.

Similarly, define

$$g(\Sigma) = \frac{\Sigma_0 - \Sigma}{\Sigma} + \frac{1}{2} \ln \frac{\Sigma}{\Sigma_0} > 0, \quad (4.7)$$

and its derivative reads

$$g'(\Sigma) = \frac{-2\Sigma_0 + \Sigma}{2\Sigma^2} < 0. \quad (4.8)$$

When $k \geq k^*$, which implies $\widetilde{\Sigma}_{T-} \geq \Sigma_{T-}$, $g(\Sigma_{T-}) \geq g(\widetilde{\Sigma}_{T-})$.

Therefore

$$\Pi - \widetilde{\Pi} = R\sigma_z^2 g(\Sigma_{T-}) - Rk^2 \sigma_z^2 g(\widetilde{\Sigma}_{T-}) \geq R\sigma_z^2 g(\widetilde{\Sigma}_{T-}) - Rk^2 \sigma_z^2 g(\widetilde{\Sigma}_{T-}) > 0. \quad (4.9)$$

When $k < k^*$, there is $\widetilde{\Sigma}_{T-} < \Sigma_{T-}$. And Eq (3.36) can be rewritten as

$$\ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0} = \frac{1 - k^2}{1 + k^2} \left(-\frac{\Sigma_0 - \widetilde{\Sigma}_{T-}}{\widetilde{\Sigma}_{T-}} \right). \quad (4.10)$$

From Eqs (4.1) and (4.2), we derive

$$\frac{\Sigma_0 - \widetilde{\Sigma}_{T-}}{\widetilde{\Sigma}_{T-}^2} = \frac{\Sigma_0 - \Sigma_{T-}}{\Sigma_{T-}^2 k^2 (1 + k^2)^2}. \quad (4.11)$$

Therefore, from Eqs (4.10) and (4.11), we derive

$$\begin{aligned} \Pi - \widetilde{\Pi} &= R\sigma_z^2 \left[\frac{\Sigma_0 - \Sigma_{T-}}{\Sigma_{T-}} + \frac{1}{2} \ln \frac{\Sigma_{T-}}{\Sigma_0} - \frac{(1 + 3k^2)(\Sigma_0 - \Sigma_{T-})\widetilde{\Sigma}_{T-}}{2\Sigma_{T-}^2(1 + k^2)^3} \right] \\ &> \frac{1}{2} R\sigma_z^2 \left[\ln \frac{\Sigma_{T-}}{\Sigma_0} + \frac{\Sigma_0 - \Sigma_{T-}}{\Sigma_{T-}} \right] \geq 0, \end{aligned} \quad (4.12)$$

and $\Pi^0 = \sigma_z \sqrt{T\Sigma_0}$, which immediately implies $\Pi^0 > \widetilde{\Pi}^0$.

Remark 2. (1) From Property 1, as the penalty coefficient rises, both the trading intensity β_t and total expected profit Π decline, whereas the terminal residual information $\widetilde{\Sigma}_{T-}$ increases. This result is consistent with economic intuition.

(2) From Property 2, in the absence of regulatory penalties, the trading intensity β_t diverges to infinity as time approaches terminal date, and the residual private information $\widetilde{\Sigma}_t$ decays to zero upon public disclosure. When penalties are present, β_t still rises over time but converges to a finite limit, while the residual information $\widetilde{\Sigma}_t$ never declines to zero.

(3) The parameter results under no public disclosure in Property 1 and Property 3 are consistent with the Gaussian framework in Reference [8]. Here, $k = \frac{\sigma_x}{\sigma_z} \in (0, 1)$ measures the degree of matching between the insider's dissimulation noise and the noise traders' order flow volatility, with $k = 1$ corresponding to perfect dissimulation. Under positive penalties, the penalty constraint endogenously

distorts the optimal dissimulation ratio: The optimal dissimulation ratio is no longer equal to unity, but decreases monotonically with the penalty intensity R . Specifically, the smaller the penalty coefficient, the closer the optimal dissimulation ratio approaches unity; as R tends to infinity, the optimal strategy degenerates to zero dissimulation. Furthermore, in the no-disclosure regime, information is transmitted solely through the order flow. In contrast, under mandatory disclosure, information is conveyed through a dual channel: The order flow plus the public disclosure itself. When k is relatively large ($k > k^*$), the order flow is heavily diluted by dissimulation noise, and although mandatory disclosure publicly releases additional information, the amount disclosed is insufficient to compensate for the information lost due to dilution; consequently, more private information remains unabsorbed at the terminal date, i.e., $\tilde{\Sigma}_{T-} \geq \Sigma_{T-}$. Conversely, when k is relatively small ($k < k^*$), the order flow itself already carries a substantial amount of genuine private information. In this case, mandatory disclosure serves as a second channel that plays a “signal verification” and “information amplification” role: Market makers can not only extract information from the highly informative order flow, but also corroborate its reliability through public disclosure. The combined effect of these dual channels enables the price to absorb more total information than under the single-channel no-disclosure regime. Hence, the terminal residual information under mandatory disclosure is smaller, i.e., $\tilde{\Sigma}_{T-} < \Sigma_{T-}$. The critical threshold k^* is determined by the equation $k^3 + k - 1 = 0$.

(4) A structured comparison of the key equilibrium outcomes across different regulatory regimes is provided in Table 1.

Table 1. Summary of equilibrium outcomes under different regulatory regimes

Scenario	Optimal dissimulation ratio	Terminal information residual	Expected profit	Terminal trading intensity
No penalty ($R = 0$), no disclosure		0	$\sigma_z \sqrt{T\Sigma_0}$	$+\infty$
No penalty 1 ($R = 0$), mandatory disclosure		0	$\frac{\sigma_z}{2} \sqrt{T\Sigma_0}$	$+\infty$
Penalty ($R > 0$), no disclosure		> 0 , see (4.1)	Π , see (4.3)	Converges to $1/R$
Penalty ($R > 0$), mandatory disclosure	< 1 , decreasing in R	> 0 , see (4.2)	$\tilde{\Pi}$, see (4.4)	Converges to $\frac{\sigma_z^2}{(\sigma_x^2 + \sigma_z^2)R}$

4.2. Optimal penalty design under mandatory disclosure

From the regulators perspective, this section analyzes the optimal choice of penalty coefficient R . Following the modeling framework of [8], the regulator balances the expected loss of noise traders and the inefficiency of terminal prices. Define $\zeta(R) := \frac{\sigma_z^4 \tilde{\Sigma}_{T-T}}{R(\sigma_x^2 + \sigma_z^2)^2}$ as the expected loss of noise traders, and

define the price inefficiency metric $I(R) := \widetilde{\Sigma}_{T-}$. The regulators objective is formulated as

$$\min_{R>0} J(R) := \zeta(R) + rI(R), \quad (4.13)$$

where $r > 0$ measures the regulators policy sensitivity to price efficiency.

The deterrence constraint is imposed as

$$P(R) := -\frac{1}{2}R\sigma_x^2 \ln \frac{\widetilde{\Sigma}_{T-}}{\Sigma_0} \geq b, \quad (4.14)$$

where $b > 0$ denotes the minimum required expected penalty (investigation cost).

Following [8], set $R = \kappa \sqrt{\frac{\Sigma_0}{\sigma_x^2 T}}$, and $\tilde{\mu}_t = \wedge(\kappa) \sqrt{\frac{\Sigma_0}{\sigma_x^2 T}}$, where the mapping $\wedge : (0, \infty) \rightarrow (0, 1)$ takes the form

$$\wedge(\kappa) = \frac{-\kappa \frac{k^2 + 1}{T} + \sqrt{\left(\frac{k^2 + 1}{T}\right)^2 \kappa^2 + 4}}{2}. \quad (4.15)$$

The core distinction from [8] is that mandatory disclosure makes $\wedge(\kappa)$ dependent not only on κ but also coupled with the dissimulation parameter k .

In particular, $\wedge(0^+) = 1 = 1 - \wedge(\infty)$.

The mapping $\wedge(\kappa)$ satisfies the quadratic identity

$$\wedge(\kappa)^2 + \frac{k^2 + 1}{T} \wedge(\kappa) - 1 = 0. \quad (4.16)$$

The regulators optimization problem is then rewritten as

$$\min_{\kappa>0} J(\kappa) := \frac{\wedge(\kappa)\sigma_x \sqrt{\Sigma_0 T}}{k^2 + 1} + r\kappa\Sigma_0(1 - \wedge(\kappa)^2), \quad (4.17)$$

$$\text{s.t. } P(\kappa) := -\frac{1}{2}\kappa \sqrt{\frac{\sigma_x^2 \Sigma_0}{T}} \ln((k^2 + 1)\kappa \wedge(\kappa)) \geq b. \quad (4.18)$$

Define the supremum standardized penalty

$$\overline{P} := \sup_{\kappa} P(\kappa) \sqrt{\frac{T}{\Sigma_0 \sigma_x^2}} < \infty. \quad (4.19)$$

This motivates the introduction of standardized investigation cost $b_0 := b \sqrt{\frac{T}{\Sigma_0 \sigma_x^2}}$.

If $b_0 > \overline{P}$, the deterrence constraint is infeasible. If $b_0 \leq \overline{P}$, the optimal κ^* is determined by the relative position of the unconstrained minimizer and the feasible interval $[\kappa_1, \kappa_2]$, where $P(\kappa_1) = P(\kappa_2) = b_0$.

5. Numerical simulations

This section conducts numerical verification on the theoretical results derived from Theorem 1 and Theorem 2. The analysis is carried out from three dimensions: The time dynamic evolution of equilibrium variables, comparative statics of penalty intensity, and optimal dissimulation strategy. It intuitively reveals the differences in market equilibrium under scenarios with and without regulatory penalties, with a focus on verifying the heterogeneous conclusion regarding the insider's optimal dissimulation volatility under the mandatory disclosure framework. For comparability, the initial residual information, total trading horizon, and noise volatility parameters are fixed in Figures 1 and 2, with parameter settings as follows: $\Sigma_0 = 1$, $T = 1$, $\sigma_z = 0.5$.

On the one hand, with the penalty coefficient R and $\sigma_x = 0.5$ fixed, Figure 1 illustrates that the trading intensity β_t rises over trading time. In the absence of penalties, the trading intensity diverges as maturity approaches, and insiders adopt aggressive trading strategies near the terminal date. When penalties are introduced, the trading intensity converges within a finite interval throughout the whole trading window; moreover, a higher penalty corresponds to a lower trading intensity at any given time. This indicates that penalties effectively restrain insiders' aggressive trading behaviors. The residual information after disclosure monotonically declines over time. Under the no-penalty scenario, private information is fully incorporated into prices, with terminal residual information variance equal to zero. In contrast, a positive amount of private information remains unincorporated into prices at maturity when penalties exist, and stronger penalties lead to greater terminal residual information. This implies that penalties slow down the speed at which private information is impounded into market prices, such that information cannot be fully reflected in prices by the end of trading.

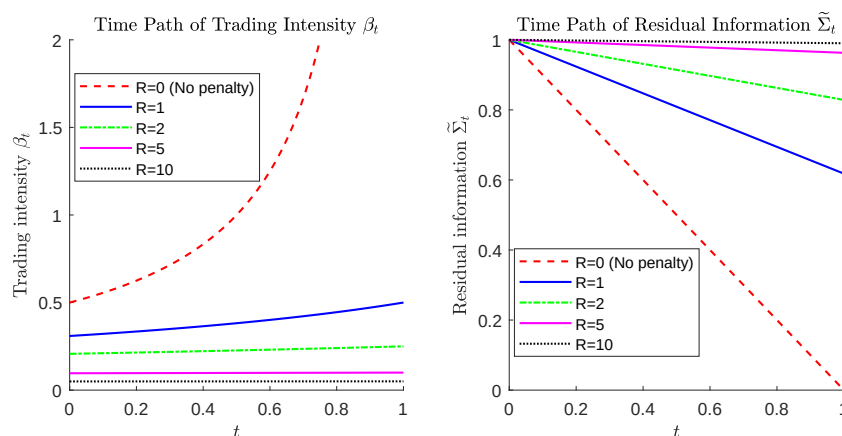


Figure 1. Time path of trading intensity β_t and residual information $\tilde{\Sigma}_t$ under different penalty intensities.

On the other hand, we fix time t and $\sigma_x = 0.5$. To avoid meaningless observations caused by zero trading intensity at $t = 0$ and divergent trading intensity at $t = 1$, the mid-term trading intensity $\beta_{0.5}$ at $t = 0.5$ is selected as the core indicator, together with the global total expected profit Π and terminal residual variance $\tilde{\Sigma}_{T-}$ for joint analysis. Numerical results show that both the mid-term trading intensity $\beta_{0.5}$ and total expected profit Π monotonically decrease as the penalty coefficient rises. This suggests that regulation effectively constrains insiders' trading activities, reduces their trading

intensity and arbitrage profits, and thus better protects noise traders, consistent with economic intuition. Conversely, the terminal residual information monotonically increases with R . This demonstrates that heavier penalties incentivize insiders to withhold more private information and weaken their motives to manipulate the market via private information.

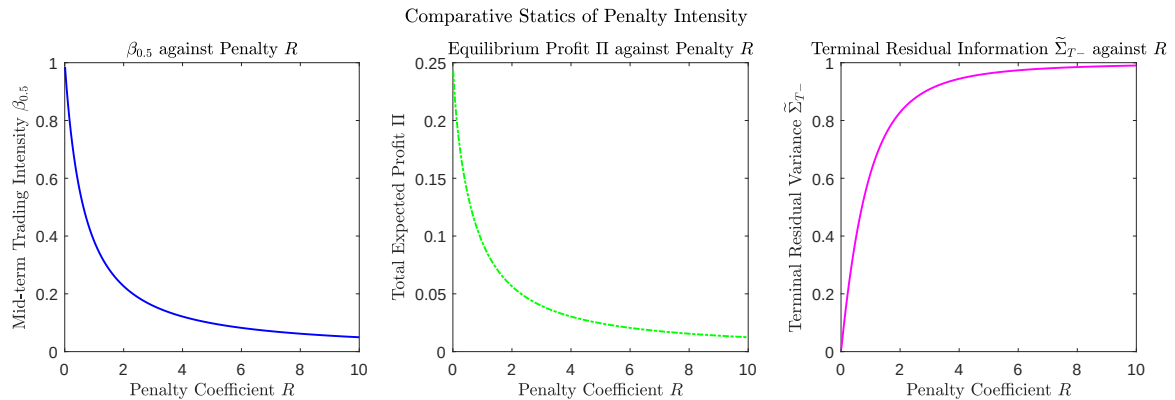


Figure 2. Comparative statics of penalty intensity on mid-term trading intensity, equilibrium profit, and terminal residual information.

Figure 3 plots the insiders expected profit $\tilde{\Pi}$ against dissimulation volatility σ_x under multiple penalty intensities. All curves exhibit a single-peaked pattern that first rises and then falls. Without penalties, profit reaches its global maximum at $\sigma_x = \sigma_z = 0.5$, which verifies that the exogenous setting $\sigma_x = \sigma_z$ adopted in existing literature is only valid in unregulated environments. After introducing quadratic penalties, the optimal dissimulation volatility corresponding to the profit peak satisfies $\sigma_x^* < \sigma_z$, and the optimal σ_x^* decreases as the penalty intensity R increases. This conclusion addresses the limitation of fixed $\sigma_x = \sigma_z$ embedded in prior mandatory disclosure models. It reveals that regulatory penalties distort insiders' dissimulation strategies: Insiders opt for a dissimulation intensity lower than the market noise level to maximize their net profit.

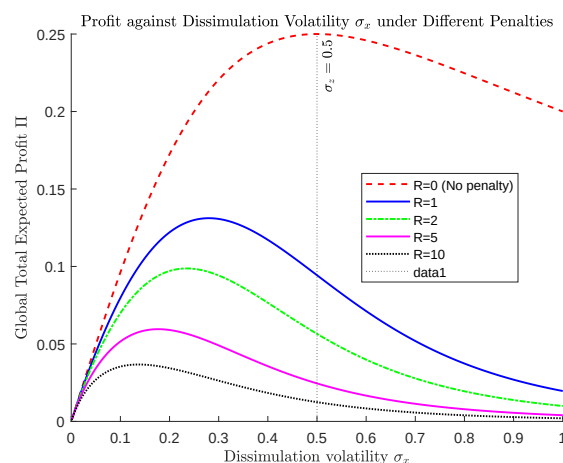


Figure 3. Insider's expected profit against dissimulation volatility σ_x under different penalties.

Figure 4 illustrates the evolutionary law of the optimal dissimulation ratio k . Subplot (a) demonstrates that each pair of curves of residual information variance under the no-disclosure regime and the mandatory disclosure regime intersects at a single point, which corresponds to the vertical dashed line at approximately 0.6823. When $k < 0.6823$, the residual information variance without mandatory disclosure is larger; conversely, the residual information variance under mandatory disclosure becomes larger when $k > 0.6823$. Subplot (b) compares the insiders equilibrium expected profits with and without mandatory disclosure under different penalty intensities. The numerical results indicate that the profit in the absence of mandatory disclosure is always higher than that under mandatory disclosure, regardless of the value of the optimal dissimulation ratio k . Finally, Subplot (c) reveals that the optimal dissimulation ratio decreases monotonically with the penalty coefficient R , and the curve exhibits a distinct concave shape. As $R \rightarrow 0$, the optimal dissimulation ratio k converges to 1; as $R \rightarrow +\infty$, the optimal dissimulation ratio k asymptotically approaches 0. The above numerical findings fully verify the distortion effect of the penalty mechanism on insiders dissimulated trading strategies.

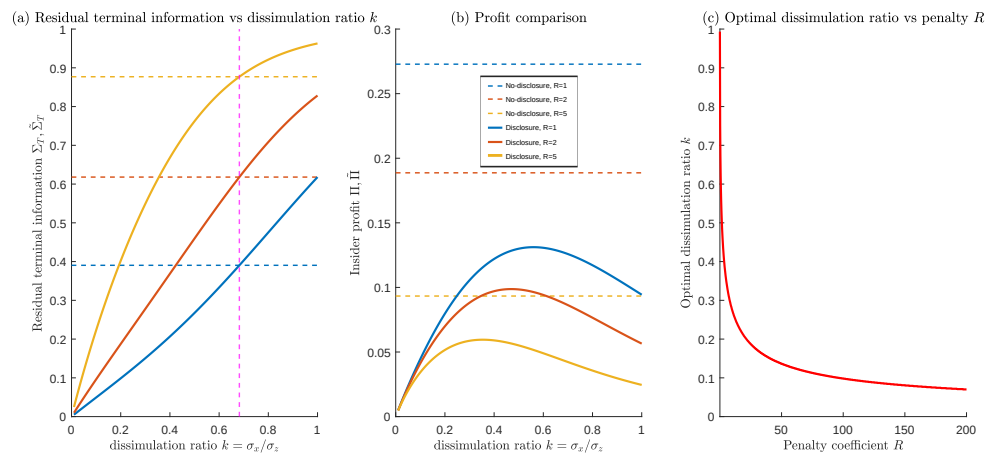


Figure 4. Residual information, profit comparison, and optimal dissimulation ratio under different regulatory regimes.

Figure 5 plots the expected penalty paid by insiders in equilibrium as a function of κ . The vertical axis is standardized by the scaling factor $\sqrt{\frac{T}{\Sigma_0 \sigma_x^2}}$. The curve of expected penalty with respect to κ displays pronounced left skewness. Under mandatory public disclosure, relative to the benchmark model proposed by [8], incorporating the dissimulation parameter k significantly reduces the overall peak value of the curve, shifts the peak to a smaller value of κ , and substantially narrows the feasible interval. Moreover, a larger dissimulation ratio k yields a lower peak and a narrower feasible region. This result indicates that insiders dissimulation behavior depresses the expected penalty for a fixed penalty coefficient, contracts the feasible range available for regulatory penalty implementation, and undermines the regulatory deterrence effect.

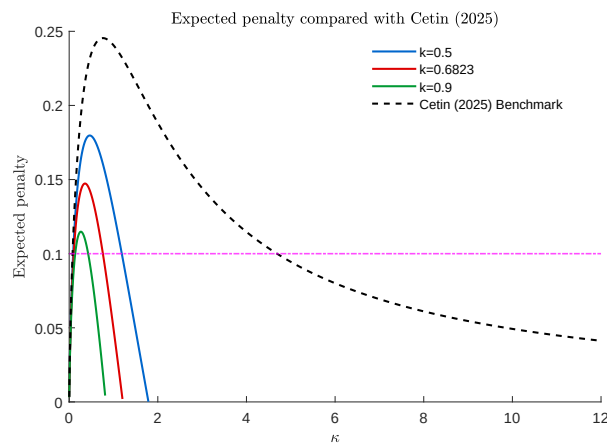


Figure 5. Expected penalty compared with Çetin (2025) under different dissimulation ratios.

6. Discussion

Our findings offer several insights for the regulatory design of insider trading. The endogenous distortion of the dissimulation strategy—specifically, the monotonic decline of $k = \sigma_x/\sigma_z$ with R reveals a subtle trade-off: While heavier penalties effectively deter informed trading, they simultaneously reduce the insider's incentive to camouflage, thereby making the order flow more informative. This implies that regulators cannot treat deterrence and market transparency as independent objectives; an optimal penalty coefficient must balance the two, as examined in our optimal penalty design framework in Section 4.2.

The critical threshold $k^* \approx 0.6823$ further qualifies the conventional presumption that mandatory disclosure uniformly enhances market transparency. Its effectiveness is contingent upon the insider's dissimulation intensity—a factor endogenously shaped by the penalty regime itself. This suggests that a one-size-fits-all disclosure policy may not be optimal across different market microstructures.

From a practical standpoint, the closed-form nature of our solutions enables transparent policy simulations, allowing regulators to evaluate the marginal impact of penalty adjustments on market outcomes without relying on black-box numerical procedures. This analytical tractability is a key advantage over more complex approaches that obscure the economic mechanisms at play.

7. Conclusions

This paper constructs a continuous-time linear equilibrium model of insider trading with quadratic trading penalties, and derives closed-form equilibrium solutions under the mandatory disclosure framework. We characterize the impacts of penalties on trading behaviors, residual private information and dissimulation strategies via formal theorems, and verify all theoretical findings through numerical simulations. The main results are summarized as follows.

From the time dimension, without penalty, the insider trading intensity rises over time and diverges at the terminal date, and private information is fully incorporated into market prices by maturity. After introducing quadratic penalties, the trading intensity converges globally. Higher penalty coefficients correspond to lower trading intensities, while unabsorbed private information remains at the terminal

date, which reduces market information efficiency.

In terms of the penalty coefficient, as the penalty coefficient rises, the insiders equilibrium trading intensity and total expected profit decrease monotonically, whereas the terminal residual private information increases monotonically. Moreover, the optimal dissimulation ratio declines with the growth of the penalty coefficient. When the penalty coefficient approaches zero, the optimal dissimulation strategy corresponds to full dissimulation; when the penalty coefficient tends to infinity, the optimal strategy degenerates to no dissimulation at all. This result remedies the limitation of fixed full dissimulation embedded in existing models.

From the perspective of dissimulation ratio, there exists a critical dissimulation threshold $k^* \approx 0.6823$ for terminal residual information. If $k < 0.6823$, the terminal residual information under mandatory disclosure is smaller; if $k > 0.6823$, the residual information is lower in the no-disclosure regime. In addition, the insiders equilibrium expected profit without mandatory disclosure is always higher than that under mandatory disclosure for any admissible dissimulation ratio. We also extend the benchmark analysis of Cetin (2025) to examine the optimal penalty design under mandatory disclosure, finding that the dissimulation behavior reduces the expected penalty for a given penalty coefficient and narrows the feasible range for regulatory implementation.

Overall, this paper extends and improves the analytical framework for continuous-time insider trading equilibrium subject to dual regulatory constraints of mandatory disclosure and quadratic trading penalties. We uncover an inherent trade-off embedded in the regulatory mechanism: Administrative penalties effectively restrain insiders' arbitrage incentives yet endogenously distort their dissimulation strategies. Admittedly, our analysis is subject to several simplifying assumptions, including linear trading strategies, Gaussian payoff distributions, and the single-insider setting; these limitations do not compromise the qualitative validity of our main conclusions but suggest fruitful avenues for future work. Future research can be extended to settings with competing insiders, dynamic penalty schemes, non-Gaussian information structures, and other related directions.

Author contributions

In this paper, all authors have equal contributions.

Use of Generative-AI tools declaration

The authors used Deepseek for English language polishing. The AI tool was used solely for language improvement; all content, analyses, and conclusions are the authors' own.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant Nos. 11861025, 12661097), the Guizhou Science and Technology Plan Project (Qian Ke Zhong Yin Di [2022] No. 4055), and the Natural Science Research Project of Guizhou Provincial Department of Education (Qian Jiao Ji [2023] No. 011).

Conflict of interest

All authors declare no conflicts of interest in this paper.

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