



Research article

Applications to Kirchoff's law and Wheatstone bridge model through fixed point approach in the setting of interpolative metric spaces

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Abstract: The notion of Jaggi-Suzuki-type hybrid contraction in interpolative metric space was presented in this study and some fixed point results were proved for such mappings. An example was also given to confirm the main result's validity. The theory that was presented expanded a number of previous findings. Additionally, a Jaggi-Suzuki type hybrid interpolative fixed point theorem was used to illustrate applications to an integral equation and electrical models, such as the Wheatstone bridge and an Inductor-Capacitor-Resistor (LCR) circuit via Kirchhoff's law.

Keywords: fixed point; interpolative metric space; hybrid contraction; metric space

Mathematics Subject Classification: 47H10, 54H25

1. Introduction

Metric fixed point theory has become an important subject of differential equations, functional analysis, and related fields throughout the past few decades. A fundamental tool in both pure and applied mathematics, the Banach Contraction Principle was first presented by Stefan Banach [1] in 1922 and ensures the presence and uniqueness of fixed points in full metric spaces. Later on, Caccioppoli [2] expanded it to include more classes of mappings.

In 1968, Kannan [3] presented generalised version of Banach Fixed Point Theorem. Later, in 1974, L. B. Ćirić [4] further generalised the result. Later, in 1993 [5] presented fixed point results in b-metric spaces and in 2018, Karapinar [6] revisited Kannan fixed point results.

In 1969, [7] introduced Meir-Keeler contraction and studied fixed point results in the setting of metric spaces. In 2015, [8] presented fixed point results using simulation functions. Recently, in 2026, Kajal et al., presented fixed point results in the setting of Interpolative metric spaces [9–11]. Later

new types of contractive conditions were introduced by Suzuki [12] and Wardowski [13]. Recently researchers have applied fixed point results to physical problems involving electric circuits, see [14–16].

In 2021, Noorwali and Yesilkaya [17] proposed a hybrid contraction combining Jaggi- and Suzuki-type contractions and established corresponding fixed point results in interpolative metric spaces.

Throughout the paper, $\mathcal{W}$ denotes an arbitrary nonempty set and $\mathbb{N}$ denotes the set of nonnegative integers.

Definition 1.1 ([17]). Let $(\mathcal{W}, \delta)$ be a interpolative metric space. A self mapping $\varpi : \mathcal{W} \rightarrow \mathcal{W}$ is said to be a Jaggi-Suzuki type hybrid contraction if there exist $\psi \in \Psi$ and $\wp : \mathcal{W} \times \mathcal{W} \rightarrow [0, +\infty)$ such that

$$\frac{1}{2}\delta(\xi, \varpi\xi) \leq \delta(\xi, \varrho) \implies \wp(\xi, \varrho)\delta(\varpi\xi, \varpi\varrho) \leq \psi(\mathcal{M}(\xi, \varrho)).$$

For each $\xi, \varrho \in \mathcal{W}$ where $\eta \geq 0$ and $\lambda_i \geq 0$, $i = 1, 2$, such that $\lambda_1 + \lambda_2 = 1$ with $\lambda_1 < \frac{1}{2}$,

$$\mathcal{M}(\xi, \varrho) = \begin{cases} \left[\lambda_1 \left(\left(\frac{\delta(\xi, \varpi\xi)\delta(\varrho, \varpi\varrho)}{\delta(\xi, \varrho)} \right)^\eta \right) + \lambda_2 (\delta(\xi, \varrho))^\eta \right]^{\frac{1}{\eta}}, & \eta > 0, \\ [\delta(\xi, \varpi\xi)]^{\lambda_1} [\delta(\varrho, \varpi\varrho)]^{\lambda_2}, & \eta = 0. \end{cases}$$

In 2024, Karapinar [18–20] introduced the concept of interpolative metric spaces by modifying the standard triangle inequality with the interpolative inequality.

Definition 1.2 ([18, 19]). Let $\mathcal{W}$ be a nonempty set. We say that $\delta : \mathcal{W} \times \mathcal{W} \rightarrow [0, +\infty)$ is $(\wp, \mathfrak{P})$ -interpolative metric if

- (1) $\delta(\xi, \varrho) = 0$, if and only if, $\xi = \varrho$, for all $\xi, \varrho \in \mathcal{W}$.
- (2) $\delta(\xi, \varrho) = \delta(\varrho, \xi)$, for all $\xi, \varrho \in \mathcal{W}$.
- (3) There exists $\wp \in (0, 1)$ and $\mathfrak{P} \geq 0$ such that

$$\delta(\xi, \varrho) \leq \delta(\xi, \varpi) + \delta((\varpi, \varrho)) + \mathfrak{P}[(\delta(\xi, \varpi))^\wp (\delta(\varpi, \varrho))^{1-\wp}]$$

for all $\xi, \varrho, \varpi \in \mathcal{W}$.

The pair $(\mathcal{W}, \delta)$ is called an $(\wp, \mathfrak{P})$ -interpolative metric space.

Example 1.1. Let $\mathcal{W} = [0, 1]$ and define a function $\delta : \mathcal{W} \times \mathcal{W} \rightarrow [0, \infty]$ as follows:

$$\delta(\xi, \varrho) = |\xi - \varrho|^7, \quad \text{for all } \xi, \varrho \in \mathcal{W}.$$

It is clear that conditions (1) and (2) are satisfied. Now, we verify condition (3) as follows:

$$\begin{aligned} \delta(\xi, \varrho) &= |\xi - \varrho|^7 = |\xi - \varpi + \varpi - \varrho|^7, \\ \delta(\xi, \varrho) &\leq |\xi - \varpi|^7 + 7|\xi - \varpi|^6|\varpi - \varrho| + 21|\xi - \varpi|^5|\varpi - \varrho|^2 + 35|\xi - \varpi|^4|\varpi - \varrho|^3 \\ &\quad + 35|\xi - \varpi|^3|\varpi - \varrho|^4 + 21|\xi - \varpi|^2|\varpi - \varrho|^5 + 7|\xi - \varpi||\varpi - \varrho|^6 + |\varpi - \varrho|^7 \\ &\leq \delta(\xi, \varpi) + 7[\delta(\xi, \varpi)]^{\frac{6}{7}}\delta(\varpi, \varrho)^{\frac{1}{7}} + 21[\delta(\xi, \varpi)]^{\frac{5}{7}}\delta(\varpi, \varrho)^{\frac{2}{7}} + 35[\delta(\xi, \varpi)]^{\frac{4}{7}}\delta(\varpi, \varrho)^{\frac{3}{7}} \\ &\quad + 35[\delta(\xi, \varpi)]^{\frac{3}{7}}\delta(\varpi, \varrho)^{\frac{4}{7}} + 21[\delta(\xi, \varpi)]^{\frac{2}{7}}\delta(\varpi, \varrho)^{\frac{5}{7}} + 7[\delta(\xi, \varpi)]^{\frac{1}{7}}\delta(\varpi, \varrho)^{\frac{6}{7}} + \delta(\varpi, \varrho), \end{aligned}$$

$$\begin{aligned} \delta(\xi, \varrho) &\leq \delta(\xi, \varrho) + 7[\delta(\xi, \varrho)^{\frac{6}{7}}\delta(\varrho, \varrho)^{\frac{1}{7}}] + 7[\delta(\xi, \varrho)^{\frac{1}{7}}\delta(\varrho, \varrho)^{\frac{6}{7}}] \\ &\quad + 21[\delta(\xi, \varrho)^{\frac{5}{7}}\delta(\varrho, \varrho)^{\frac{2}{7}}] + 21[\delta(\xi, \varrho)^{\frac{2}{7}}\delta(\varrho, \varrho)^{\frac{5}{7}}] + 35[\delta(\xi, \varrho)^{\frac{4}{7}}\delta(\varrho, \varrho)^{\frac{3}{7}}] \\ &\quad + 35[\delta(\xi, \varrho)^{\frac{3}{7}}\delta(\varrho, \varrho)^{\frac{4}{7}}] + \delta(\varrho, \varrho). \end{aligned}$$

Without loss of generality, we assume $\delta(\xi, \varrho) \geq \delta(\varrho, \varrho)$,

$$\delta(\xi, \varrho) \leq \delta(\xi, \varrho) + \delta(\varrho, \varrho) + 126[\delta(\xi, \varrho)^{\frac{6}{7}}\delta(\varrho, \varrho)^{\frac{1}{7}}].$$

Thus, condition (3) is satisfied for $\mathfrak{P} \geq 126$ for any $\wp = \frac{1}{7} \in (0, 1)$.

Hence, $(\mathcal{W}, \delta)$ is a $(\frac{1}{7}, 126)$ -interpolative metric space.

Definition 1.3 ([18]). Let $(\mathcal{W}, \delta)$ be a $(\wp, \mathfrak{P})$ -interpolative metric space and $\{\xi_n\}$ be a sequence in $\mathcal{W}$.

- (1) $\{\xi_n\}$ converges to ξ in $\mathcal{W}$ if, and only if, $\delta(\xi_n, \xi) \rightarrow 0$, as $n \rightarrow \infty$.
- (2) $\{\xi_n\}$ is a Cauchy sequence in $\mathcal{W}$ if, and only if, $\limsup_{n \rightarrow \infty} \{\delta(\xi_n, \xi_m) : m > n\} = 0$.
- (3) If every Cauchy sequence converges in $\mathcal{W}$, then $(\mathcal{W}, \delta)$ is a complete $(\wp, \mathfrak{P})$ -interpolative metric space.

Definition 1.4 ([21]). Let $\sqsubset : \mathcal{W} \rightarrow \mathcal{W}$ be a mapping and $\wp : \mathcal{W} \times \mathcal{W} \rightarrow [0, +\infty)$. We say that $\sqsubset$ is $\wp$ -admissible if

$$\xi, \varrho \in \mathcal{W}, \wp(\xi, \varrho) \geq 1 \implies \wp(\sqsubset \xi, \sqsubset \varrho) \geq 1.$$

Definition 1.5 ([22]). Let $\sqsubset : \mathcal{W} \rightarrow \mathcal{W}$ be a mapping and $\wp : \mathcal{W} \times \mathcal{W} \rightarrow [0, +\infty)$. We say that $\sqsubset$ is $\wp$ -orbital admissible if

$$\xi, \varrho \in \mathcal{W}, \wp(\xi, \sqsubset \xi) \geq 1 \implies \wp(\sqsubset \xi, \sqsubset^2 \xi) \geq 1.$$

2. Main result

In this section, we shall introduce a notion of Jaggi-Suzuki-type hybrid contraction within the framework of interpolative metric space and provide an example also to illustrate the validity of the result.

Let Ψ denote the family of functions $\psi : [0, \infty) \rightarrow [0, \infty)$ satisfying the following conditions:

- (1) ψ is nondecreasing.
- (2) There exists $a \in (0, 1)$ and a convergent series $\sum_{n=1}^{\infty} r_n$ such that

$$\psi^{n+1}(t) \leq r_n \psi^n(t), \quad \text{for all } t > 0 \text{ and } n \in \mathbb{N}.$$

Here, these functions are known as ψ -comparison functions (see [23]).

For every $\psi \in \Psi$, the following properties hold:

- (1) $\psi^n(t)$ converges to 0 as $n \rightarrow \infty$ for all $t > 0$;

- (2) $\psi(t) < t$, for any $t > 0$;
- (3) ψ is continuous at 0;
- (4) The series $\sum_{n=1}^{\infty} \psi^n(t)$ is convergent for $t > 0$.

Lemma 2.1 ([24]). Let $\mathcal{W}$ be a nonempty set. Assume that $\wp : \mathcal{W} \times \mathcal{W} \rightarrow [0, +\infty)$ is a given function and $\beth : \mathcal{W} \rightarrow \mathcal{W}$ is an $\wp$ -orbital admissible mapping. If there exists an element $\mathfrak{H}_0 \in \mathcal{W}$ such that $\wp(\mathfrak{H}_0, \beth \mathfrak{H}_0) \geq 1$, and sequence $\{\mathfrak{H}_n\}$ is defined by

$$\mathfrak{H}_n = \beth \mathfrak{H}_{n-1}, \quad \text{for } n \geq 0,$$

then it follows that

$$\wp(t_0, \beth t_0) \geq 1, \quad \text{for all } n \geq 0.$$

Theorem 1. Let $(\mathcal{W}, \mathfrak{d})$ be an interpolative complete metric space and $\beth : \mathcal{W} \rightarrow \mathcal{W}$ be a Jaggi-Suzuki-type hybrid mapping. Assume also that $\beth$ is $\wp$ -orbital admissible mapping and

$$\wp(\mathfrak{H}_0, \beth \mathfrak{H}_0) \geq 1, \quad (2.1)$$

for some $\mathfrak{H}_0 \in \mathcal{W}$. Then $\beth$ has a fixed point in $\mathcal{W}$.

Proof. Define a sequence $\{\wp_n\}$ in $\mathcal{W}$ by $\mathfrak{H}_{n+1} = \beth \mathfrak{H}_n$, for all $n \geq 0$.

Thus, $\beth$ is $\wp$ -orbital admissible and $\wp(\mathfrak{H}_0, \beth \mathfrak{H}_0) \geq 1$ for some $\mathfrak{H}_0 \in \mathcal{W}$.

From Lemma 2.1, we have

$$\mathfrak{d}(\mathfrak{H}_n, \mathfrak{H}_{n+1}) > 0, \quad \text{for every } n \geq 0. \quad (2.2)$$

Without loss of generality, we assume that $\mathfrak{H}_n \neq \mathfrak{H}_{n+1}$, for all $n \in \mathbb{N}$.

If, for some k , $\mathfrak{H}_k = \mathfrak{H}_{k+1}$, then $\mathfrak{H}_k$ is a fixed point of $\beth$, and the proof is complete.

By Definition 1.1, there exists a nondecreasing function $\psi \in \Psi$ such that

$$\begin{aligned} \mathfrak{d}(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}) &= \mathfrak{d}(\beth \mathfrak{H}_n, \beth \mathfrak{H}_{n+1}), \\ \mathfrak{d}(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}) &\leq \wp(\mathfrak{H}_n, \mathfrak{H}_{n+1}) \mathfrak{d}(\beth \mathfrak{H}_n, \beth \mathfrak{H}_{n+1}), \end{aligned} \quad (2.3)$$

$$\mathfrak{d}(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}) \leq \psi(\mathcal{M}(\mathfrak{H}_n, \mathfrak{H}_{n+1})). \quad (2.4)$$

From here, two cases arise:

Case 1: For $\mathfrak{v} > 0$,

$$\begin{aligned} \mathcal{M}(\mathfrak{H}_n, \mathfrak{H}_{n+1}) &= \left[\lambda_1 \left(\frac{\mathfrak{d}(\mathfrak{H}_n, \beth \mathfrak{H}_n) \mathfrak{d}(\mathfrak{H}_{n+1}, \beth \mathfrak{H}_{n+1})}{\mathfrak{d}(\mathfrak{H}_n, \mathfrak{H}_{n+1})} \right)^{\mathfrak{v}} + \lambda_2 (\mathfrak{d}(\mathfrak{H}_n, \mathfrak{H}_{n+1}))^{\mathfrak{v}} \right]^{\frac{1}{\mathfrak{v}}}, \\ \mathcal{M}(\mathfrak{H}_n, \mathfrak{H}_{n+1}) &= \left[\lambda_1 \left(\frac{\mathfrak{d}(\mathfrak{H}_n, \mathfrak{H}_{n+1}) \mathfrak{d}(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2})}{\mathfrak{d}(\mathfrak{H}_n, \mathfrak{H}_{n+1})} \right)^{\mathfrak{v}} + \lambda_2 (\mathfrak{d}(\mathfrak{H}_n, \mathfrak{H}_{n+1}))^{\mathfrak{v}} \right]^{\frac{1}{\mathfrak{v}}}. \end{aligned}$$

From (2.3), we have

$$\begin{aligned}
\delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}) &\leq \wp(\mathfrak{H}_n, \mathfrak{H}_{n+1}) \delta(\sqsupset \mathfrak{H}_n, \sqsupset \mathfrak{H}_{n+1}) \\
&\leq \psi((\lambda_1 + \lambda_2) \delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}))^{\frac{1}{\wp}} \\
&= \psi\left((\lambda_1 + \lambda_2)^{\frac{1}{\wp}} \delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2})\right) \\
&< (\lambda_1 + \lambda_2)^{\frac{1}{\wp}} \delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}), \\
\delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}) &< \delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}),
\end{aligned} \tag{2.5}$$

a contradiction. Then, we get

$$\delta(\mathfrak{H}_n, \mathfrak{H}_{n+1}) < \delta(\mathfrak{H}_{n-1}, \mathfrak{H}_n).$$

Moreover, inequality (2.4) implies that:

$$\delta(\mathfrak{H}_n, \mathfrak{H}_{n+1}) \leq \psi(\delta(\mathfrak{H}_{n-1}, \mathfrak{H}_n)) < \delta(\mathfrak{H}_{n-1}, \mathfrak{H}_n). \tag{2.6}$$

Applying the inequality successively, we obtain

$$\delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}) \leq \psi^n(\delta(\mathfrak{H}_1, \mathfrak{H}_0)), \quad \text{for all } n \geq 1. \tag{2.7}$$

Since $\psi \in \Psi$, we get

$$\lim_{n \rightarrow \infty} \delta(\mathfrak{H}_n, \mathfrak{H}_{n+1}) = 0. \tag{2.8}$$

Using elementary induction,

$$\delta(\mathfrak{H}_n, \mathfrak{H}_{n+2}) \leq \delta(\mathfrak{H}_n, \mathfrak{H}_{n+1}) + \delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}) + \wp \left[(\delta(\mathfrak{H}_n, \mathfrak{H}_{n+1}))^\wp (\delta(\mathfrak{H}_{n+1}, \mathfrak{H}_{n+2}))^{1-\wp} \right]. \tag{2.9}$$

Using (2.7) and the limit as $n \rightarrow \infty$, we obtain

$$\lim_{n \rightarrow \infty} \delta(\mathfrak{H}_n, \mathfrak{H}_{n+2}) = 0. \tag{2.10}$$

Additionally, we have

$$\delta(\mathfrak{H}_n, \mathfrak{H}_{n+3}) \leq \delta(\mathfrak{H}_n, \mathfrak{H}_{n+2}) + \delta(\mathfrak{H}_{n+2}, \mathfrak{H}_{n+3}) + \wp [(\delta(\mathfrak{H}_n, \mathfrak{H}_{n+2}))^\wp (\delta(\mathfrak{H}_{n+2}, \mathfrak{H}_{n+3}))^{1-\wp}]. \tag{2.11}$$

By (2.7) and (2.10), taking $n \rightarrow \infty$ in the above inequality, we deduce that

$$\lim_{n \rightarrow \infty} \delta(\mathfrak{H}_n, \mathfrak{H}_{n+3}) = 0. \tag{2.12}$$

We now assume that our claim is true in its general case; that is, we have

$$\lim_{n \rightarrow \infty} \delta(\mathfrak{H}_n, \mathfrak{H}_{n+r}) = 0, \quad \text{for some } r \in \mathbb{N}. \tag{2.13}$$

According to the theorem,

$$\delta(\mathfrak{H}_n, \mathfrak{H}_{n+r+1}) \leq \delta(\mathfrak{H}_n, \mathfrak{H}_{n+r}) + \delta(\mathfrak{H}_{n+r}, \mathfrak{H}_{n+r+1}) + \wp \left[(\delta(\mathfrak{H}_n, \mathfrak{H}_{n+r}))^\wp (\delta(\mathfrak{H}_{n+r}, \mathfrak{H}_{n+r+1}))^{1-\wp} \right]. \tag{2.14}$$

Using (2.7) and (2.13) and taking the limit as $n \rightarrow \infty$ in the above inequality, we conclude that

$$\lim_{n \rightarrow \infty} \delta(\xi_n, \xi_{n+r+1}) = 0. \quad (2.15)$$

Consequently, one can deduce that

$$\delta(\xi_{n+1}, \xi_m) < 1, \quad (2.16)$$

for $m > n > l$, for some $l \in \mathbb{N}$. We now prove that the generated sequence $\{\xi_n\}$ is Cauchy.

For $m > n > k = \{M, l\}$, let

$$\begin{aligned} \delta(\xi_n, \xi_m) &\leq \delta(\xi_n, \xi_{n+1}) + \delta(\xi_{n+1}, \xi_m) + \mathfrak{P}[(\delta(\xi_n, \xi_{n+1}))^\varphi (\delta(\xi_{n+1}, \xi_m))^{1-\varphi}] \\ &\leq \psi^{n-k}(\delta(\xi_k, \xi_{k+1})) + \delta(\xi_{n+1}, \xi_m) + \mathfrak{P}[(\psi^{n-k}(\delta(\xi_k, \xi_{k+1})))^\varphi (\delta(\xi_{n+1}, \xi_m))^{1-\varphi}]. \end{aligned} \quad (2.17)$$

Using fact (2.34), we have $\delta(\xi_{n+1}, \xi_m) < 1$, and hence we get

$$(\delta(\xi_{n+1}, \xi_m))^{1-\varphi} < 1. \quad (2.18)$$

Hence, the righthand side of the inequality (2.17) becomes

$$\begin{aligned} &\leq \psi^{n-k}(\delta(\xi_k, \xi_{k+1})) + \delta(\xi_{n+1}, \xi_m) + [\mathfrak{P}\psi^{n-k}(\delta(\xi_k, \xi_{k+1}))^\varphi (\delta(\xi_{n+1}, \xi_m))^{1-\varphi}] \\ &\leq \psi^{n-k}(\delta(\xi_k, \xi_{k+1})) + [1 + \mathfrak{P}\psi^{n-k}(\delta(\xi_k, \xi_{k+1}))^\varphi (\delta(\xi_{n+1}, \xi_m))^{-\varphi}] \delta(\xi_{n+1}, \xi_m). \end{aligned} \quad (2.19)$$

So, we have

$$\delta(\xi_n, \xi_m) \leq \psi^{n-k}(\delta(\xi_k, \xi_{k+1})) + [1 + \mathfrak{P}(\psi^{n-k})^\varphi] \delta(\xi_{n+1}, \xi_m). \quad (2.20)$$

Additionally, we see that

$$\begin{aligned} \delta(\xi_{n+1}, \xi_m) &\leq \delta(\xi_{n+1}, \xi_{n+2}) + \delta(\xi_{n+2}, \xi_m) + \mathfrak{P}(\delta(\xi_k, \xi_{k+1}))^\varphi (\delta(\xi_{n+2}, \xi_m))^{1-\varphi} \\ &\leq \psi^{n-k+1}(\delta(\xi_k, \xi_{k+1})) + \delta(\xi_{n+2}, \xi_m) + [\mathfrak{P}\psi^{n-k+1}(\delta(\xi_k, \xi_{k+1}))^\varphi (\delta(\xi_{n+2}, \xi_m))^{-\varphi}] \delta(\xi_{n+2}, \xi_m) \\ &\leq \psi^{n-k+1}(\delta(\xi_k, \xi_{k+1})) + [1 + \mathfrak{P}\psi^{n-k+1}] \delta(\xi_{n+2}, \xi_m). \end{aligned} \quad (2.21)$$

Combining the inequalities (2.20) and (2.21), we get that

$$\begin{aligned} \delta(\xi_n, \xi_m) &\leq \psi^{n-k}(\delta(\xi_k, \xi_{k+1})) \psi^{n-k+1}(\delta(\xi_k, \xi_{k+1})) [1 + \mathfrak{P}(\psi^{n-k})^\varphi] \\ &\quad + [1 + \mathfrak{P}(\psi^{n-k})^\varphi] [(1 + \mathfrak{P}\psi^{n-k+1})^\varphi] \delta(\xi_{n+2}, \xi_m). \end{aligned} \quad (2.22)$$

Considering each of these findings, we concluded that

$$\begin{aligned} \delta(\xi_n, \xi_m) &\leq \psi^{n-k}(\delta(\xi_k, \xi_{k+1})) \sum_{i=0}^{m-n-1} \psi^i \prod_{j=0}^{i-1} (1 + \mathfrak{P}\psi^{n-k+j})^\varphi \\ &\leq \psi^{n-k}(\delta(\xi_k, \xi_{k+1})) \sum_{i=0}^{m-n-1} \psi^i \prod_{j=0}^{i-1} (1 + \mathfrak{P}\psi^j)^\varphi. \end{aligned} \quad (2.23)$$

The series $\sum_{i=0}^{\infty} S_j$, which is convergent when $n, m \rightarrow \infty$, dominates the righthand side of the abovementioned inequality.

$$S_j = \psi^i(\delta(\xi_k, \xi_{k+1})) \prod_{j=0}^{i-1} (1 + \mathfrak{P}\psi^j(\delta(\xi_k, \xi_{k+1})))^\varphi < \psi^i(\delta(\xi_k, \xi_{k+1})) \prod_{j=0}^{i-1} (1 + \mathfrak{P}\delta(\xi_k, \xi_{k+1}))^\varphi.$$

Thus $\{\xi_n\}$ is Cauchy sequence in $\mathcal{W}$.

As $(\mathcal{W}, \delta)$ is a complete $(\varphi, \mathfrak{P})$ -interpolative metric space, the sequence $\{\xi_n\}$ converges to $r \in \mathcal{W}$.

Assume, to the contrary that $\delta(r, \sqsupset r) > 0$.

Note that

$$\delta(\xi_{n+1}, \sqsupset r) = \delta(\sqsupset \xi_n, \sqsupset r) \leq \psi(\max\{\delta(\xi_n, r), \delta(\xi_n, \xi_{n+1}), \delta(\sqsupset r, r)\}). \quad (2.24)$$

Since $\xi_n \rightarrow r$ and $\xi_{n+1} \rightarrow r$, we have $\delta(\xi_n, r) \rightarrow 0$ and $\delta(\xi_n, \xi_{n+1}) \rightarrow 0$.

Therefore, by taking lim sup of both sides of the inequalities (2.24), we obtain

$$\begin{aligned} \delta(\sqsupset r, r) &\leq \psi(\delta(\sqsupset r, r)), \\ \delta(\sqsupset r, r) &< \delta(\sqsupset r, r) \quad \text{since we assume } \delta(r, \sqsupset r) > 0, \end{aligned} \quad (2.25)$$

which is a contradiction. As a result, $\sqsupset r = r$ is the fixed point of $\sqsupset$ in $\mathcal{W}$.

Case 2: For $\eta = 0$,

$$\mathcal{M}^\sim(\xi_n, \xi_{n+1}) = (\delta(\xi_n, \sqsupset \xi_n))^{\lambda_1} (\delta(\xi_{n+1}, \sqsupset \xi_{n+1}))^{\lambda_2}, \quad (2.26)$$

$$\mathcal{M}^\sim(\xi_n, \xi_{n+1}) = (\delta(\xi_n, \xi_{n-1}))^{\lambda_1} (\delta(\xi_{n+1}, \xi_n))^{\lambda_2}. \quad (2.27)$$

Now, if $\delta(\xi_n, \xi_{n-1}) \leq \delta(\xi_{n+1}, \xi_n)$, then

$$\begin{aligned} \mathcal{M}^\sim(\xi_n, \xi_{n+1}) &= (\delta(\xi_n, \xi_{n-1}))^{\lambda_1} ([\delta(\xi_{n+1}, \xi_n)]^{\lambda_2}) \\ &= (\delta(\xi_n, \xi_{n+1}))^{\lambda_1 + \lambda_2} \\ &\leq \delta(\xi_n, \xi_{n+1}). \end{aligned}$$

From (2.3), we have

$$\delta(\xi_{n+1}, \xi_{n+2}) < \delta(\sqsupset \xi_n, \sqsupset \xi_{n+1}), \quad (2.28)$$

which is contradiction. Therefore,

$$\begin{aligned} \delta(\xi_n, \xi_{n+1}) &< \psi(\delta(\xi_n, \xi_{n-1})), \\ \delta(\xi_n, \xi_{n+1}) &< \psi^n(\delta(\xi_0, \xi_1)). \end{aligned} \quad (2.29)$$

Since $\psi \in \Psi$, we get

$$\lim_{n \rightarrow \infty} \delta(\xi_n, \xi_{n+1}) = 0. \quad (2.30)$$

To reach our goal, we first consider the following inequality

$$\delta(\xi_n, \xi_{n+3}) \leq \delta(\xi_n, \xi_{n+2}) + \delta(\xi_{n+2}, \xi_{n+3}) + \mathfrak{P}[(\delta(\xi_n, \xi_{n+2}))^\varphi (\delta(\xi_{n+1}, \xi_{n+3}))^{1-\varphi}]. \quad (2.31)$$

Based on (2.30), as $n \rightarrow \infty$ in the inequality above, we have

$$\lim_{n \rightarrow \infty} \delta(\xi_n, \xi_{n+3}) = 0. \quad (2.32)$$

Iterative application of (3) in Definition 1.2 gives

$$\lim_{n \rightarrow \infty} \delta(\xi_n, \xi_{n+r+1}) = 0. \quad (2.33)$$

Based on this observation, it follows that

$$\delta(\xi_{n+1}, \xi_m) < 1. \quad (2.34)$$

Proceeding from (2.17) in a similar manner to the case $\eta > 0$, we can show that the sequence $\{\xi_n\}$ forms a cauchy sequence in $\mathcal{W}$. Hence, there exists $r \in \mathcal{W}$ such that $\lim_{n \rightarrow \infty} \delta(\xi_n, r) = 0$. Since $\mathcal{W}$ is complete and $\mathfrak{J}$ is continuous, it follows that r is a fixed point of $\mathfrak{J}$. $\square$

Example 2.1. Let $\mathcal{W} = \mathbb{R}$, with metric $\delta(\xi, \varrho) = |\xi - \varrho|^2$. Clearly, $(\mathcal{W}, \delta)$ is a complete interpolative metric space.

Define the mapping $\mathfrak{J} : \mathcal{W} \rightarrow \mathcal{W}$ by $\mathfrak{J}\xi = \frac{\xi}{2}$. Then, $\mathfrak{J}$ has a unique fixed point.

Solution. Let us choose the constants as follows:

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = \frac{1}{2}, \quad \eta = 1, \quad \wp(\xi, \varrho) = 1, \quad \psi(t) = \frac{t}{2}.$$

Now for any $\xi, \varrho \in \mathbb{R}$, we compute:

$$\begin{aligned} \delta(\xi, \mathfrak{J}\xi) &= \left| \xi - \frac{\xi}{2} \right|^2 = \frac{\xi^2}{4}, \\ \delta(\varrho, \mathfrak{J}\varrho) &= \left| \varrho - \frac{\varrho}{2} \right|^2 = \frac{\varrho^2}{4}, \\ \delta(\mathfrak{J}\xi, \mathfrak{J}\varrho) &= \left| \frac{\xi}{2} - \frac{\varrho}{2} \right|^2 = \frac{1}{4} |\xi - \varrho|^2. \end{aligned}$$

Then, by the formula in the theorem, we have

$$\mathcal{M}^\sim(\xi, \varrho) = \left[\lambda_1 \left(\left(\frac{\delta(\xi, \mathfrak{J}\xi)\delta(\varrho, \mathfrak{J}\varrho)}{\delta(\xi, \varrho)} \right)^\eta \right) + \lambda_2 (\delta(\xi, \varrho))^q \right]^{\frac{1}{\eta}}. \quad (2.35)$$

Here,

$$\begin{aligned} \frac{\delta(\xi, \mathfrak{J}\xi)\delta(\varrho, \mathfrak{J}\varrho)}{\delta(\xi, \varrho)} &= \frac{\xi^2 \varrho^2}{16 |\xi - \varrho|^2}, \\ \lambda_1 \left(\frac{\delta(\xi, \mathfrak{J}\xi)\delta(\varrho, \mathfrak{J}\varrho)}{\delta(\xi, \varrho)} \right) &= \frac{\xi^2 \varrho^2}{32 |\xi - \varrho|^2}, \\ \lambda_2 (\delta(\xi, \varrho))^1 &= \frac{|\xi - \varrho|^2}{2}. \end{aligned}$$

Putting all these values in Eq (2.35), we get

$$\widetilde{\mathcal{M}}(\mathfrak{H}, \mathfrak{Q}) = \frac{\mathfrak{H}^2 \mathfrak{Q}^2}{32|\mathfrak{H} - \mathfrak{Q}|^2} + \frac{1}{2}|\mathfrak{H} - \mathfrak{Q}|^2.$$

Now, we have

$$\begin{aligned} \wp(\mathfrak{H}, \mathfrak{Q})\mathfrak{d}(\mathfrak{Q}\mathfrak{H}, \mathfrak{Q}\mathfrak{Q}) &\leq \psi(\mathcal{M}^{\sim}(\mathfrak{H}, \mathfrak{Q})), \\ \frac{|\mathfrak{H} - \mathfrak{Q}|^2}{4} &\leq \frac{1}{2} \left(\frac{\mathfrak{H}^2 \mathfrak{Q}^2}{32|\mathfrak{H} - \mathfrak{Q}|^2} + \frac{|\mathfrak{H} - \mathfrak{Q}|^2}{2} \right), \\ \frac{|\mathfrak{H} - \mathfrak{Q}|^2}{4} &\leq \left(\frac{\mathfrak{H}^2 \mathfrak{Q}^2}{64|\mathfrak{H} - \mathfrak{Q}|^2} + \frac{|\mathfrak{H} - \mathfrak{Q}|^2}{4} \right). \end{aligned}$$

So,

$$\begin{aligned} 0 &\leq \frac{\mathfrak{H}^2 \mathfrak{Q}^2}{64|\mathfrak{H} - \mathfrak{Q}|^2}, \\ 0 &\leq \mathfrak{H}^2 \mathfrak{Q}^2. \end{aligned}$$

Now, all the assumptions of Theorem 1 are satisfied. Consequently $\mathfrak{Q}$, has a fixed point.

Now,

$$\begin{aligned} \frac{\mathfrak{H}}{2} &= \mathfrak{H} \\ \Rightarrow \mathfrak{H} &= 2\mathfrak{H} \\ \Rightarrow \mathfrak{H} &= 0. \end{aligned}$$

So, the unique fixed point is $\mathfrak{H} = 0$.

3. Consequences

Corollary 3.1. Let $(\mathcal{W}, \mathfrak{d})$ be an interpolative complete metric space and $\mathfrak{Q} : \mathcal{W} \rightarrow \mathcal{W}$ a continuous mapping. If there exists $\psi \in \Psi$ such that

$$\frac{1}{2}\mathfrak{d}(\mathfrak{H}, \mathfrak{Q}\mathfrak{H}) \leq \mathfrak{d}(\mathfrak{H}, \mathfrak{Q}) \implies \mathfrak{d}(\mathfrak{Q}\mathfrak{H}, \mathfrak{Q}\mathfrak{Q}) \leq \psi(\mathcal{M}^{\sim}(\mathfrak{H}, \mathfrak{Q})),$$

for $\mathfrak{H}, \mathfrak{Q} \in \mathcal{W}$ where $\mathfrak{q} \geq 0$; $\lambda_i \geq 0$, $i = 1, 2$; and $\lambda_1 + \lambda_2 = 1$ with $\lambda_1 < 1/2$ and $\wp \in (0, 1)$.

$$\mathcal{M}^{\sim}(\mathfrak{H}, \mathfrak{Q}) = \begin{cases} \left[\lambda_1 \left(\left(\frac{\mathfrak{d}(\mathfrak{H}, \mathfrak{Q}\mathfrak{H})\mathfrak{d}(\mathfrak{Q}, \mathfrak{Q}\mathfrak{Q})}{\mathfrak{d}(\mathfrak{H}, \mathfrak{Q})} \right)^{\mathfrak{q}} \right) + \lambda_2 (\mathfrak{d}(\mathfrak{H}, \mathfrak{Q}))^{\mathfrak{q}} \right]^{\frac{1}{\mathfrak{q}}}, & \mathfrak{q} > 0, \\ \mathfrak{d}(\mathfrak{H}, \mathfrak{Q}\mathfrak{H})^{\wp} [\mathfrak{d}(\mathfrak{Q}, \mathfrak{Q}\mathfrak{Q})]^{1-\wp}, & \mathfrak{q} = 0, \end{cases}$$

therefore, $\mathfrak{Q}$ has a fixed point in $\mathcal{W}$.

Proof. Case 1: Letting $\wp(\mathfrak{H}, \mathfrak{Q}) = 1$ for $\mathfrak{H}, \mathfrak{Q} \in \mathcal{W}$, it follows from Theorem 1 for $\mathfrak{q} > 0$.

Case 2: Letting $\wp(\mathfrak{H}, \mathfrak{Q}) = 1$ for $\mathfrak{H}, \mathfrak{Q} \in \mathcal{W}$ and $\lambda_1 = \wp$, $\lambda_2 = 1 - \wp$, it follows from Theorem 1 for $\mathfrak{q} = 0$. $\square$

Corollary 3.2. Let $(\mathcal{W}, \delta)$ be an interpolative complete metric space and $\mathfrak{N} : \mathcal{W} \rightarrow \mathcal{W}$ a continuous mapping. If there exists $s \in (0, 1)$ such that

$$\frac{1}{2}\delta(\mathfrak{H}, \mathfrak{N}\mathfrak{H}) \leq \delta(\mathfrak{H}, \mathfrak{L}) \implies \delta(\mathfrak{N}\mathfrak{H}, \mathfrak{N}\mathfrak{L}) \leq s(\mathcal{M}^\sim(\mathfrak{H}, \mathfrak{L})).$$

For $\mathfrak{H}, \mathfrak{L} \in \mathcal{W}$ where $\eta \geq 0$; $\lambda_i \geq 0$, $i = 1, 2$; and $\lambda_1 + \lambda_2 = 1$ with $\lambda_1 < 1/2$ and $\wp \in (0, 1)$.

$$\mathcal{M}^\sim(\mathfrak{H}, \mathfrak{L}) = \begin{cases} \left[\lambda_1 \left(\left(\frac{\delta(\mathfrak{H}, \mathfrak{N}\mathfrak{H})\delta(\mathfrak{L}, \mathfrak{N}\mathfrak{L})}{\delta(\mathfrak{H}, \mathfrak{L})} \right)^\eta \right) + \lambda_2 (\delta(\mathfrak{H}, \mathfrak{L}))^\eta \right]^{\frac{1}{\eta}}, & \eta > 0, \\ [\delta(\mathfrak{H}, \mathfrak{N}\mathfrak{H})]^\wp [\delta(\mathfrak{L}, \mathfrak{N}\mathfrak{L})]^{1-\wp}, & \eta = 0, \end{cases}$$

therefore, $\mathfrak{N}$ has a fixed point in $\mathcal{W}$.

Proof. Case 1: Letting $\psi(r) = sr$ for any $r \geq 0$, it follows from Corollary 3.1 for $\eta > 0$.

Case 2: Letting $\psi(v) = sr$ for any $r \geq 0$ and $\lambda_1 = \wp$, $\lambda_2 = 1 - \wp$, it follows from Corollary 3.1 for $\eta = 0$. $\square$

Corollary 3.3. Let $(\mathcal{W}, \delta)$ be an interpolative complete metric space and $\mathfrak{N} : \mathcal{W} \rightarrow \mathcal{W}$ a continuous mapping. If there exists $s \in (0, 1)$ such that

$$\begin{aligned} \frac{1}{2}\delta(\mathfrak{H}, \mathfrak{N}\mathfrak{H}) \leq \delta(\mathfrak{H}, \mathfrak{L}) &\implies \delta(\mathfrak{N}\mathfrak{H}, \mathfrak{N}\mathfrak{L}) \\ &\leq s(\sqrt{\delta(\mathfrak{H}, \mathfrak{N}\mathfrak{H})\delta(\mathfrak{L}, \mathfrak{N}\mathfrak{L})}), \end{aligned}$$

for $\mathfrak{H}, \mathfrak{L} \in \mathcal{W}$; therefore, $\mathfrak{N}$ has a fixed point in $\mathcal{W}$.

Proof. Letting $\alpha = 1/2$, it follows from Corollary 3.1 for $\eta = 0$. $\square$

Corollary 3.4. Let $(\mathcal{W}, \delta)$ be an interpolative complete metric space and $\mathfrak{N} : \mathcal{W} \rightarrow \mathcal{W}$ a continuous mapping such that

$$\begin{aligned} \frac{1}{2}\delta(\mathfrak{H}, \mathfrak{N}\mathfrak{H}) \leq \delta(\mathfrak{H}, \mathfrak{L}) &\implies \delta(\mathfrak{N}\mathfrak{H}, \mathfrak{N}\mathfrak{L}) \\ &\leq \wp \frac{\delta(\mathfrak{H}, \mathfrak{N}\mathfrak{H})\delta(\mathfrak{L}, \mathfrak{N}\mathfrak{L})}{\delta(\mathfrak{H}, \mathfrak{L})} + \beta \delta(\mathfrak{H}, \mathfrak{L}), \quad \mathfrak{H} \neq \mathfrak{L}, \end{aligned}$$

for $\mathfrak{H}, \mathfrak{L} \in \mathcal{W}$, where $\alpha = s\lambda_1$ and $\beta = s\lambda_2$ for some $s \in (0, 1)$. Hence $\wp, \beta \in (0, 1)$, $\wp + \beta < 1$. Thus, $\mathfrak{N}$ has a fixed point in $\mathcal{W}$.

Proof. Letting $\alpha = s\lambda_1$ and $\beta = s\lambda_2$ with $s \in (0, 1)$ for $\mathfrak{H}, \mathfrak{L} \in \mathcal{W}$, it follows from Corollary 3.3 for $\eta = 1$. $\square$

4. Application to Kirchoff's law in an LCR circuit via Jaggi-Suzuki type hybrid interpolative fixed point theorem

As an application of Theorem 1, we consider a nonlinear electrical network governed by Kirchoff's voltage law (KVL) to demonstrate how the hybrid interpolative fixed point framework can be used to analyze the stability of current in the Inductor-Capacitor-Resistor (LCR) circuit. Let the circuit consist of an inductor L , a capacitor C , and a resistor R connected in series with an external voltage source $E(t)$. According to KVL, the sum of voltage drop around the loop is zero.

Thus, the system can be represented as

$$\begin{aligned} L \frac{di}{dt} + Ri + \frac{1}{c} \int_0^t i(s) ds &= E(t), \\ L \frac{di}{dt} &= E(t) - Ri - \frac{1}{c} \int_0^t i(s) ds. \end{aligned} \quad (4.1)$$

Dividing both sides by ' L ', we get

$$\begin{aligned} \frac{di}{dt} &= \frac{1}{L} \left[E(t) - Ri - \frac{1}{c} \int_0^t i(s) ds \right], \\ \frac{di}{dt} &= \frac{1}{L} \left[E(t) - Ri(t) - \frac{1}{C} \int_0^t i(s) ds \right]. \end{aligned} \quad (4.2)$$

Integrating both sides with respect to t from 0 to t ,

$$\int_0^t \frac{di}{du} du = \frac{1}{L} \left[\int_0^t E(u) du - R \int_0^t i(u) du - \frac{1}{C} \int_0^t \left(\int_0^u i(s) ds \right) du \right].$$

Using the Fundamental Theorem of Calculus,

$$\begin{aligned} i(t) - i(0) &= \frac{1}{L} \left[\int_0^t E(u) du - R \int_0^t i(u) du - \frac{1}{C} \int_0^t \left(\int_0^u i(s) ds \right) du \right], \\ i(t) &= \frac{1}{L} \left[E(t) - Ri(t) - \frac{1}{c} \int_0^t i(s) ds \right]. \end{aligned} \quad (4.3)$$

Let $W = L^2([0, 1], R)$ be the space of square integrable real valued function on $[0, 1]$.

$$(\mathfrak{N}i)(t) = \frac{1}{L} \left[E(t) - Ri(t) - \frac{1}{c} \int_0^t i(s) ds \right]. \quad (4.4)$$

Let $\mathfrak{N}$ be a Jaggi-Suzuki hybrid contraction satisfying

$$\frac{1}{2} \delta(i, \mathfrak{N}i) \leq \delta(i, \mathcal{U}) \implies \wp(i, \mathcal{U}) \delta(\mathfrak{N}i, \mathfrak{N}\mathcal{U}) \leq \psi(\mathcal{M}(i, \mathcal{U})).$$

Choose $\delta(i, \mathcal{U}) = |i - \mathcal{U}|^2$, $\psi(t) = \mathfrak{A}t$ with $0 < \mathfrak{A} < 1$, $\wp(i, \mathcal{U}) = 1$,

$$\begin{aligned}\delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) &= |\mathfrak{A}i - \mathfrak{A}\mathcal{U}|^2, \\ \delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) &= \frac{1}{L^2} \left[-R(i(t) - \mathcal{U}(t)) - \frac{1}{c} \int_0^t (i(s) - \mathcal{U}(s)) ds \right].\end{aligned}\quad (4.5)$$

So,

$$\begin{aligned}\delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) &= \int_0^1 |\mathfrak{A}i(t) - \mathfrak{A}\mathcal{U}(t)|^2 dt, \\ \delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) &= \int_0^1 \left| \frac{1}{L} \left[-R(i(t) - \mathcal{U}(t)) - \frac{1}{c} \int_0^t (i(s) - \mathcal{U}(s)) ds \right] \right|^2 dt, \\ \delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) &= \frac{1}{L^2} \int_0^1 \left| R(i(t) - \mathcal{U}(t)) + \frac{1}{c} \int_0^t (i(s) - \mathcal{U}(s)) ds \right|^2 dt.\end{aligned}\quad (4.6)$$

Applying the triangle and Cauchy-Schwarz inequality:

$$\delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) = \frac{1}{L^2} \left(R^2 + \frac{1}{C^2} \right) |i - \mathcal{U}|^2. \quad (4.7)$$

Thus, for suitable $\mathfrak{A} = \frac{1}{L^2} (R^2 + \frac{1}{C^2}) < 1$, we get

$$\begin{aligned}\delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) &\leq \mathfrak{A} |i - \mathcal{U}|^2, \\ \delta(\mathfrak{A}i, \mathfrak{A}\mathcal{U}) &= \psi(\mathcal{M}(i, \mathcal{U})).\end{aligned}$$

Hence, all the conditions of Theorem 1 are satisfied. It follows that $\mathfrak{A}$ has a fixed point $i^*(t) \in \mathcal{W}$, which corresponds to a solution to the integral Eq (4.3). Figure 1 shows the input voltage $E(t)$ and the current response $i(t)$ of the LCR circuit corresponding to the fixed point obtained by Theorem 1.

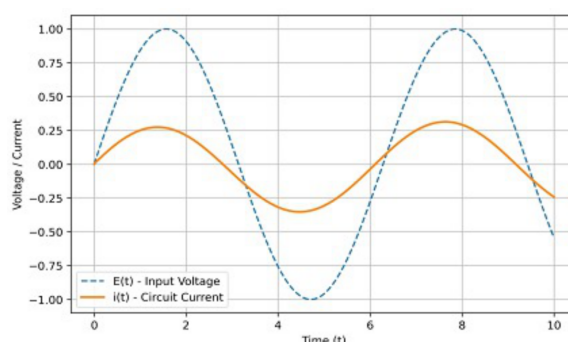


Figure 1. LCR Circuit: voltage and current behavior.

5. Electrical interpretation via Wheatstone bridge model

This application illustrates the validity of Theorem 1 through an electrical interpretation based on the Wheatstone bridge. The bridge circuit consists of four resistors R_1, R_2, R_3, R_4 arranged in a network to determine an unknown resistance.

When the bridge satisfies the condition:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4},$$

the equilibrium condition can be viewed as an iterative self-adjustment of the variable resistance until a steady configuration (fixed point) is obtained.

Let $\mathcal{W} = \mathbb{R}$, with standard metric $\delta(\mathfrak{H}, \mathfrak{L}) = |\mathfrak{H} - \mathfrak{L}|^5$.

Define the mapping $\mathcal{T} : \mathcal{W} \rightarrow \mathcal{W}$ by

$$\mathcal{T}\mathfrak{H} = \frac{\mathfrak{H} + 2}{2}.$$

By solving the fixed point equation $\mathcal{T}\mathfrak{H} = \mathfrak{H}$, we obtain $\mathfrak{H} = 2$, which represents the equilibrium value of the iterative balancing process.

Here, $\mathcal{T}$ represents the successive adjustment of resistance ratios in the Wheatstone Bridge, which is an electrical network used to measure an unknown resistance by balancing two arms of a bridge circuit.

The bridge attains equilibrium when the ratio condition is

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

This balancing process can be viewed as an iterative approach to a fixed point, corresponding to a steady-state current.

Although $\mathfrak{H}$ can be obtained directly from $\mathcal{T}\mathfrak{H} = \mathfrak{H}$, this formulation demonstrates the applicability of Theorem 1 and verifies the contractive condition.

Now, for all $\mathfrak{H}, \mathfrak{L} \in \mathcal{W}$, we have

$$\begin{aligned}\delta(\mathfrak{H}, \mathcal{T}\mathfrak{H}) &= \left| \mathfrak{H} - \frac{\mathfrak{H} + 2}{2} \right|^5 = \frac{|\mathfrak{H} - 2|^5}{32}, \\ \delta(\mathfrak{L}, \mathcal{T}\mathfrak{L}) &= \left| \mathfrak{L} - \frac{\mathfrak{L} + 2}{2} \right|^5 = \frac{|\mathfrak{L} - 2|^5}{32}, \\ \delta(\mathcal{T}\mathfrak{H}, \mathcal{T}\mathfrak{L}) &= \left| \frac{\mathfrak{H} + 2}{2} - \frac{\mathfrak{L} + 2}{2} \right|^5 = \frac{1}{32} |\mathfrak{H} - \mathfrak{L}|^5.\end{aligned}$$

Define the auxillary function by $\mathfrak{H} : \mathcal{W} \times \mathcal{W} \rightarrow [0, 1)$ by $\wp(\mathfrak{H}, \mathfrak{L}) = 1$, for all $\mathfrak{H}, \mathfrak{L} \in \mathcal{W}$.

Let $\lambda_1 = \lambda_2 = \frac{1}{2}$, $\eta = 1$.

Then, according to the Jaggi-Suzuki-Type contraction:

$$\begin{aligned}\mathcal{M}^{\sim}(\mathfrak{H}, \mathfrak{L}) &= \left[\frac{1}{2} \left(\left(\frac{\delta(\mathfrak{H}, \mathcal{T}\mathfrak{H})\delta(\mathfrak{L}, \mathcal{T}\mathfrak{L})}{\delta(\mathfrak{H}, \mathfrak{L})} \right)^{\eta} \right) + \frac{1}{2} (\delta(\mathfrak{H}, \mathfrak{L}))^{\eta} \right]^{\frac{1}{\eta}}, \\ \mathcal{M}^{\sim}(\mathfrak{H}, \mathfrak{L}) &= \frac{1}{2} \left[\frac{|\mathfrak{H} - 2|^5 |\mathfrak{L} - 2|^5}{1024 |\mathfrak{H} - \mathfrak{L}|^5} \right].\end{aligned}$$

So,

$$\frac{1}{32} |\mathfrak{H} - \mathfrak{L}|^5 \leq \frac{1}{2} \left[\frac{|\mathfrak{H} - 2|^5 |\mathfrak{L} - 2|^5}{1024 |\mathfrak{H} - \mathfrak{L}|^5} \right].$$

Thus, all the conditions of Theorem 1 are satisfied. Consequently, $\mathcal{T}$ has a fixed point.

Clearly, the unique fixed point of $\mathcal{T}$ is $\mathfrak{H} = 2$, which represents the equilibrium value of the iterative balancing process.

6. Conclusions

In this work, we introduce and investigated the notion of Jaggi-Suzuki-Type hybrid contraction within the setting of interpolative metric spaces, and established corresponding fixed point results. The validity of the proposed theorem is demonstrated through illustrative examples. Furthermore, its applicability is verified by employing it to the Wheatstone bridge model and Kirchoff's law in an LCR circuit, confirming both its theoretical relevance and practical significance.

Author contributions

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All authors have read and agreed to the final version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Funding

The authors extend their appreciation to Prince Sattam bin Abdulaziz University for funding this research work through the Project Number (2026/01/45669).

Conflict of interest

The authors declare no conflicts of interest.

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