



Research article**Dynamics of a periodically switched predator-prey model with seasonal birth, predation-induced fear, intraspecific cooperative hunting and predator migrating for chasing foods****Hui Jiao^{1,*}, Akanat Wetayawanich¹ and Jianjun Jiao^{2,*}**¹ School of Mathematics and Geoinformations, Institute of Science, Suranaree University of Technology, Nakhon, Ratchasima 30000, Thailand² School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang 550004, China*** Correspondence:** Email: jiaojianjun05@126.com; jiaohui_math2025@126.com.

Abstract: In view of seasonal birth, predation-induced fear, intraspecific cooperative hunting, and predator migrating for chasing foods, we constructed a new periodically switched predator-prey model for ecological protection. Exploiting theories of impulsive differential equations, we obtained the sufficient conditions for system persistence and the prey-eradication boundary periodic solution being globally asymptotically stable. Bifurcation analysis was used to reveal rich dynamical properties of system (2.1) with numerical simulations, including the periodic solution, period-doubling solutions, and chaos. We also found that the fear effect, intraspecific cooperative hunting, and predator seasonal migration have significant impacts on the persistence of system (2.1).

Keywords: periodically switched predator-prey model; seasonally migrating for chasing food; intraspecific cooperative hunting; fear effect; persistence

Mathematics Subject Classification: 34A37, 92D25

1. Introduction

Population migration is a ubiquitous global phenomenon. Every year, seasonal changes trigger large-scale animal migrations. This cyclical journey allows species to cope with shifts in weather and food supply, and to breed successfully. During these movements, whole populations travel from their home grounds to a new area, only to return again later in the year [1]. According to ecological traits affecting the seasonal migration patterns of breeding birds in the Hengduan Mountains of China, the research results of reference [2] indicated that most species ($n = 30$) bred at a higher altitude in breeding season, and migrated downward in non-breeding season, while some species ($n = 13$) tended

to migrate to a higher altitude in non-breeding seasons. Liao et al. [3] presented the virus migration and meteorological elements shaping the seasonality of influenza a/H3N2. Zhang et al. [4] presented an ecosystem-based analysis characterized by the dynamics of salmon migration and aquaculture. Jiao et al. [5] introduced impulsive vaccination and dispersal into a susceptible-infected-recovered (SIR) epidemic model. Jiao et al. [6] also proposed the periodic migration described by time delay into a pest-natural enemy model. Especially, Jiao et al. [7] introduced the predator-population seasonally large-scale migration in an impulsive predator-prey model as

$$\left\{ \begin{array}{l} \frac{dX_1(t)}{dt} = X_1(t)(a_{11} - b_{11}X_1(t)), \\ \frac{dX_2(t)}{dt} = X_2(t)(a_{12} - b_{12}X_2(t)), \\ \Delta X_1(t) = -hX_1(t), \\ \Delta X_2(t) = -\varepsilon_1 X_2(t), \end{array} \right\} t \in (n\tau, (n+l)\tau],$$

$$\left\{ \begin{array}{l} \frac{dX_1(t)}{dt} = X_1(t)(a_{21} - b_{21}X_1(t)) - \beta X_1(t)X_2(t), \\ \frac{dX_2(t)}{dt} = X_2(t)(a_{22} - b_{22}X_2(t)) + k\beta X_1(t)X_2(t), \\ \Delta X_1(t) = 0, \\ \Delta X_2(t) = -\varepsilon_2 X_2(t) \end{array} \right\} t \in ((n+l)\tau, (n+1)\tau],$$

$$\left\{ \begin{array}{l} \Delta X_1(t) = 0, \\ \Delta X_2(t) = -\varepsilon_2 X_2(t) \end{array} \right\} t = (n+1)\tau, \quad (1.1)$$

where system (1.1) is constructed by two patches and the predator is assumed to migrate seasonally between these two patches. $X_1(t)$ represents the density of the prey population, and prey population X_1 lives in patch 2. $X_2(t)$ represents the density of the predator population. The biological meanings of the parameters in system (1.1) can be found in [7].

Cooperative hunting [8] can promote the evolution of group living by enhancing foraging efficiency and success through multiple mechanisms: it increases the likelihood of a successful kill, enables the acquisition of larger prey, improves the chances of securing multiple prey in a single event, and reduces both travel distance and individual energy expenditure. Understanding these effects is increasingly important for the conservation of endangered species. Intraspecific cooperative hunting [9–11], Communal hunting [12], and interspecific cooperative hunting [13, 14] have been studied extensively. Further, Banerjee et al. [15] presented a model that developed a scenario where a pack hunter predate on two prey species in mutualistic relationship:

$$\left\{ \begin{array}{l} \frac{dX(t)}{dt} = r_x X(t) \left(1 - \frac{X(t)}{K_x}\right) - (p_x + \alpha_x Y(t))X(t)Y(t) + \frac{\mu_1 Z(t)X(t)}{H_z + Z(t)}, \\ \frac{dY(t)}{dt} = -dY(t) + e_x(p_x + \alpha_x Y(t))X(t)Y(t) + e_z(p_z + \alpha_z Y(t))Z(t)Y(t), \\ \frac{dZ(t)}{dt} = r_z Z(t) \left(1 - \frac{Z(t)}{K_z}\right) - (p_z + \alpha_z Y(t))Z(t)Y(t) + \frac{\mu_2 X(t)Z(t)}{H_z + X(t)}, \end{array} \right. \quad (1.2)$$

where $X(t)$ and $Z(t)$ are the densities of two kinds of prey. $Y(t)$ is the density of the predator. The biological meanings of the parameters in system (1.2) can also be seen in [15].

Generally, the predator influences prey in two ways in predator-prey systems: one is through direct killing and the other is called the fear effect of predator. Some experiments showed that the fear effect

had greater influence on prey than direct killing. The fear effect was introduced into a predator-prey system in [16]. Many results have shown that fear effect [17, 18] greatly affected the dynamical behaviors of predator-prey systems, and the fear effect is even more powerful than predation [19, 20]. All in all, the fear effect of the predator plays a negative role on the prey population. Many references [21–23] modeled the fear effect in predator-prey interactions. Furthermore, Sun et al. [24] established a periodic switched pest-management model under the influence of the fear effect:

$$\left\{ \begin{array}{l} \frac{dP_1(t)}{dt} = \frac{rP_1(t)}{1 + m\alpha P_2(t)} - \alpha_1 P_1(t) - \beta_1 P_1^2(t) - \frac{\beta_{11}P_1(t)P_2(t)}{1 + \alpha_{11}P_1(t)}, \\ \frac{dP_2(t)}{dt} = -c_{12}P_2(t) + \frac{k_1\beta_{11}P_1(t)P_2(t)}{1 + \alpha_{11}P_1(t)}, \\ \Delta P_1(t) = -h_1 P_1(t), \\ \Delta P_2(t) = -h_2 P_2(t) + \frac{\tau_1}{1 + \alpha_{12}P_2(t)}, \end{array} \right\} t \in (n\tau, (n+l)\tau], \quad (1.3)$$

$$\left\{ \begin{array}{l} \frac{dP_1(t)}{dt} = -\alpha_2 P_1(t) - P_2(t)(1 - e^{-\gamma P_1(t)}), \\ \frac{dP_2(t)}{dt} = k_2 P_2(t)(1 - e^{-\gamma P_1(t)}) - c_2 P_2(t), \\ \Delta P_1(t) = 0, \\ \Delta P_2(t) = \tau_2 \end{array} \right\} t \in ((n+l)\tau, (n+1)\tau],$$

$$\left\{ \begin{array}{l} \Delta P_1(t) = 0, \\ \Delta P_2(t) = \tau_2 \end{array} \right\} t = (n+1)\tau,$$

where we presume that $P_1(t)$ represents the pest population at time t . $P_2(t)$ represents the number of the natural enemy population at time t . The biological meanings of the parameters in system (1.3) can also be seen in [24].

Though ordinary differential equations may exquisitely capture natural scenes, such as dispersal [25], tumor and epidemic [26–28], theories of impulsive differential equations have recently been introduced into population dynamics [29–31]. Impulsive equations are found in almost every domain of applied science and have been studied in many investigations [32–34], as they generally describe phenomena which are subject to steep or instantaneous changes.

As can be seen from the above discussion, differential equations are employed to study population dynamics [35–37]. Many papers have paid close attention to single-mechanism coupled predator-prey models for ecological conservation scenarios. In this paper, we combine migration, fear effects, and cooperative hunting into population dynamic models.

2. The model

Integrating the above discussion with the dynamic evolution process in Figure 1, we establish a periodically switched predator-prey model with seasonal birth, predation-induced fear, intraspecific cooperative hunting, and predator migration for chasing food:

$$\left\{ \begin{array}{l} \frac{dP_1(t)}{dt} = -d_{11}P_1(t), \\ \frac{dP_2(t)}{dt} = -d_{12}P_2(t), \\ \frac{dP(t)}{dt} = P(t)(a_{13} - b_{13}P(t)), \\ \Delta P_1(t) = P_1(t)(a_1 - b_1P_1(t)), \\ \Delta P_2(t) = 0, \\ \Delta P(t) = -\epsilon_1P(t), \end{array} \right\} t \in (nT, (n + \gamma_1)T], \\
 \left\{ \begin{array}{l} \frac{dP_1(t)}{dt} = P_1(t)\left(\frac{a_{21}}{1 + \delta_1P(t)} - b_{21}P_1(t) - \beta_1(1 + \beta_{11}P(t))P_1(t)P(t), \right. \\ \frac{dP_2(t)}{dt} = -d_{22}P_2(t), \\ \left. \frac{dP(t)}{dt} = -d_{23}P(t) + c_1\beta_1(1 + \beta_{11}P(t))P_1(t)P(t), \right. \\ \Delta P_1(t) = 0, \\ \Delta P_2(t) = P_2(t)(a_2 - b_2P_2(t)), \\ \Delta P(t) = -\epsilon_2P(t), \end{array} \right\} t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\
 \left\{ \begin{array}{l} \frac{dP_1(t)}{dt} = -d_{31}P_1(t), \\ \frac{dP_2(t)}{dt} = P_2(t)\left(\frac{a_{32}}{1 + \delta_2P(t)} - b_{32}P_2(t) - \beta_2(1 + \beta_{22}P(t))P_2(t)P(t), \right. \\ \frac{dP(t)}{dt} = -d_{33}P(t) + c_2\beta_2(1 + \beta_{22}P(t))P_2(t)P(t), \\ \Delta P_1(t) = 0, \\ \Delta P_2(t) = 0, \\ \Delta P(t) = -\epsilon_3P(t) \end{array} \right\} t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\
 \left\{ \begin{array}{l} \Delta P_1(t) = 0, \\ \Delta P_2(t) = 0, \\ \Delta P(t) = -\epsilon_3P(t) \end{array} \right\} t = (n + 1)T,
 \end{array} \quad (2.1)$$

where the habits of two preys and a predator in system (2.1) are distributed across three patches. $P_1(t)$ represents the density of one prey at time t , where prey P_1 lives in patch 1. $P_2(t)$ represents the density of the other prey at time t , where prey P_2 lives in patch 2. $P(t)$ represents the density of predator at time t . It is also assumed that prey P_1 's seasonally birth at time $t = (n + \gamma_1)T$, prey P_2 seasonally birth at time $t = (n + \gamma_1)T$, and predator P migrates for chasing food among these three patches. The biological meanings of the parameters in system (2.1) are explained as follows.

i) $d_{11} > 0$ represents the mortality rate of prey P_1 in patch 1 on $(nT, (n + \gamma_1)T]$. $d_{12} > 0$ represents the mortality rate of prey P_2 in patch 2 on $(nT, (n + \gamma_1)T]$. $a_{13} > 0$ represents the growth rate of predator P in patch 3 on $(nT, (n + \gamma_1)T]$. $b_{13} > 0$ represents the intraspecific competition coefficient of predator P in patch 3 on $(nT, (n + \gamma_1)T]$.

ii) $a_1 > 0$ represents the initial birth coefficient of prey P_1 in patch 1 at time $t = (n + \gamma_1)T$. $\frac{a_1}{b_1} > 0$ represents the initial birth maximum capacity of prey P_1 in patch 1 at $t = (n + \gamma_1)T$. $0 < \varepsilon_1 < 1$ represents the loss proportion of predator P during their migration for chasing food from patch 3 to patch 1 at $t = (n + \gamma_1)T$.

iii) $a_{21} > 0$ represents the growth rate of prey P_1 in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$, which is called the child-bearing period of prey P_1 . One factor contributing to a reduction in prey P_1 is the predation-induced fear of predator P migrating from patch 3, given by $\frac{1}{1+\delta_1 P(t)}$ ($\delta_1 > 0$ is the level of the predation-induced fear effect from the migrated predator P arriving in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$). $b_{21} > 0$ represents the intraspecific competition coefficient of prey P_1 in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$. $d_{22} > 0$ represents the mortality rate of prey P_2 in patch 2 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$. $d_{23} > 0$ represents the mortality rate of predator P in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$. When predator P arrives in patch 1, they may hunt prey P_1 cooperatively. Let predator P consume prey P_1 with predation coefficient $\beta_1 > 0$ in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$. $\beta_{11} \geq 0$ represents the hunting cooperative factor of predator P in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$, so $\beta_{11}P(t)$ represents the increase in the predation rate of predator P due to the assistance of intraspecific cooperative hunting of predator P in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$. Therefore, a new functional response with intraspecific cooperative hunting is described as:

$$\underbrace{\beta_1(1 + \beta_{11}P(t))P_1(t)}_{\text{The predation functional response of predator } P \text{ with intraspecific cooperative hunting on prey } P_1}. \quad (2.2)$$

The predation functional response of predator P with intraspecific cooperative hunting on prey P_1 .

$c_1 > 0$ is the nutrient conversion rate from prey P_1 into predator P in patch 1 on $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$.

iv) $a_2 > 0$ represents the initial birth coefficient of prey P_2 in patch 2 at time $t = (n + \gamma_1 + \gamma_2)T$. $\frac{a_2}{b_2} > 0$ represents the initial birth maximum capacity of prey P_2 in patch 2 at time $t = (n + \gamma_1 + \gamma_2)T$. $0 < \varepsilon_2 < 1$ represents the loss proportion of predator P during their migrating for chasing foods from patch 1 to patch 2 at $t = (n + \gamma_1 + \gamma_2)T$.

v) $d_{31} > 0$ represents the mortality rate of prey P_1 in patch 1 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$, which is also called the child-bearing period of prey P_2 . $a_{32} > 0$ represents the growth rate of prey P_2 in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$. One factor contributing to a reduction in prey P_2 is the predation-induced factor of migrated predator P from patch 1, given by $\frac{1}{1+\delta_2 P(t)}$ (where $\delta_2 > 0$ is the level of the predation-induced fear from the migrated predator P arriving in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$). $b_{32} > 0$ represents the intraspecific competition coefficient of prey P_2 in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$. $d_{33} > 0$ represents the mortality rate of predator P in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$. When predator P arrives in patch 2, predator P may hunt prey P_2 cooperatively. Let predator P consume prey P_2 with predation coefficient $\beta_2 > 0$ in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$. $\beta_{22} \geq 0$ represents the hunting cooperative factor of predator P in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$, and $\beta_{22}P(t)$ represents the increase in the predation rate of predator P due to the assistance of intraspecific cooperative hunting of predator P in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$. Therefore, a new functional response with intraspecific cooperative hunting is also described as:

$$\underbrace{\beta_2(1 + \beta_{22}P(t))P_2(t)}_{\text{The predation functional response of } P_2 \text{ of predator } P \text{ with intraspecific cooperative hunting prey } P_2}. \quad (2.3)$$

The predation functional response of P_2 of predator P with intraspecific cooperative hunting prey P_2 .

$c_2 > 0$ is the nutrient conversion rate from prey P_2 into predator P in patch 2 on $((n + \gamma_1 + \gamma_2)T, (n + 1)T]$.

vi) $0 < \epsilon_3 < 1$ represents the loss proportion of predator P during their migration for chasing food from patch 2 to patch 3 at $t = (n + 1)T$.

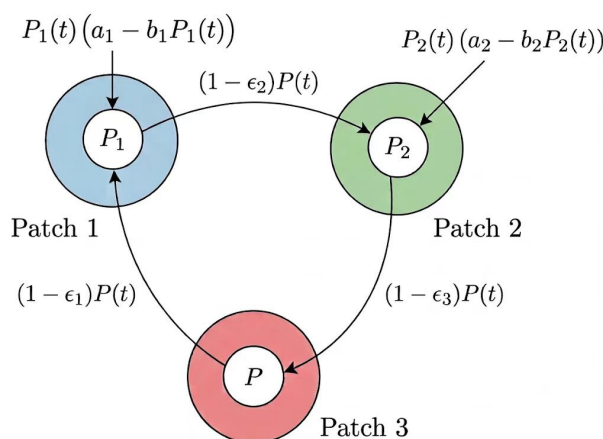


Figure 1. Dynamic evolution process for a periodically switched predator-prey model with seasonal birth, predation-induced fear, intraspecific cooperative hunting, and predator migration for chasing food.

The seasonal migration of predator P occurs every T period (T is a positive constant) or the prey P_i ($i = 1, 2$) gives birth every T period, $0 < \gamma_i \leq 1$ ($i = 1, 2$) and $0 < \gamma_1 + \gamma_2 \leq 1$.

3. The lemmas

The solution $X(t) = (P_1(t), P_2(t), P(t))'$ of system (2.1) is a piecewise continuous function $X: R_+ \rightarrow R_+^3$, where $R_+ = [0, \infty)$, $R_+^3 = \{X \in R^3 : X > 0\}$. $X(t)$ is continuous on $(nT, (n + \gamma_1)T] \times R_+^3$, $((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T] \times R_+^3$ and $((n + \gamma_1 + \gamma_2)T, (n + 1)T] \times R_+^3$ ($n \in Z_+$, $0 \leq \gamma_i \leq 1$ ($i = 1, 2$), $0 \leq \gamma_1 + \gamma_2 \leq 1$). According to reference [38], the global existence and uniqueness of solutions of system (2.1) is guaranteed by the smoothness properties of f , which denote the mapping defined by the right-side of system (2.1).

If $P_i(t) = 0$ ($i = 1, 2$), there exists a subsystem of system (2.1) as

$$\left\{ \begin{array}{l} \frac{dP(t)}{dt} = P(t)(a_{13} - b_{13}P(t)), \quad t \in (nT, (n + \gamma_1)T], \\ \Delta P(t) = -\epsilon_1 P(t), \quad t = (n + \gamma_1)T, \\ \frac{dP(t)}{dt} = -d_{23}P(t), \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta P(t) = -\epsilon_2 P(t), \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dP(t)}{dt} = -d_{33}P(t), \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\ \Delta P(t) = -\epsilon_3 P(t), \quad t = (n + 1)T. \end{array} \right. \quad (3.1)$$

The analytic solution of system (3.1) is

$$P(t) = \begin{cases} \frac{a_{13}P(nT^+)e^{a_{13}(t-nT)}}{a_{13} + b_{13}P(nT^+)(e^{a_{13}(t-nT)} - 1)}, & t \in (nT, (n + \gamma_1)T], \\ \frac{P((n + \gamma_1)T^+)e^{-d_{23}(t-(n+\gamma_1)T)}}{P((n + \gamma_1)T^+)e^{-d_{23}(t-(n+\gamma_1)T)}}, & t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \frac{P((n + \gamma_1 + \gamma_2)T^+)e^{-d_{33}(t-(n+\gamma_1+\gamma_2)T)}}{P((n + \gamma_1 + \gamma_2)T^+)e^{-d_{33}(t-(n+\gamma_1+\gamma_2)T)}}, & t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T]. \end{cases} \quad (3.2)$$

Then, the stroboscopic mapping of system (3.1) is

$$P((n + 1)T^+) = \frac{\prod_{i=1}^3 (1 - \epsilon_i) \times a_{13} e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2)]T} P(nT^+)}{a_{13} + b_{13}(e^{a_{13}\gamma_1 T} - 1)P(nT^+)}. \quad (3.3)$$

Two fixed points such as 0 and P^* of (3.3) can be easily computed, where P^* is

$$P^* = \left. \begin{aligned} & \frac{a_{13}[\prod_{i=1}^3 (1 - \epsilon_i) e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T} - 1]}{b_{13}(e^{a_{13}\gamma_1 T} - 1)} > 0, \\ & \prod_{i=1}^3 (1 - \epsilon_i) e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2)]T} > 1. \end{aligned} \right\} \quad (3.4)$$

Lemma 3.1.

i) If $\prod_{i=1}^3 (1 - \epsilon_i) e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2)]T} < 1$ holds, the fixed point 0 of system (3.3) is globally asymptotically stable.

ii) If $\prod_{i=1}^3 (1 - \epsilon_i) e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2)]T} > 1$ holds, the positive fixed point P^* of system (3.3) is globally asymptotically stable.

Proof. Making the notation

$$F(P(nT^+)) = \frac{\prod_{i=1}^3 (1 - \epsilon_i) \times a_{13} e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T} P(nT^+)}{a_{13} + b_{13}(e^{a_{13}\gamma_1 T} - 1)P(nT^+)}, \quad (3.5)$$

we derivate (3.5) as

$$\frac{\partial F(P(nT^+))}{\partial P(nT^+)} = \frac{a_{13} \prod_{i=1}^3 (1 - \epsilon_i) \times a_{13} e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2)]T}}{[a_{13} + b_{13}(e^{a_{13}\gamma_1 T} - 1)P(nT^+)]^2}. \quad (3.6)$$

i) If $e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T} < 1$, the uniquely fixed point of system (3.3) is 0. Then,

$$\begin{aligned} 0 < \frac{\partial F(P(nT^+))}{\partial P(nT^+)} \Big|_{P(nT^+)=0} &= \frac{a_{13}^2 \prod_{i=1}^3 (1 - \epsilon_i) \times e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T}}{(a_{13} + b_{13}(e^{a_{13}\gamma_1 T} - 1)P(nT^+))^2} \Big|_{P(nT^+)=0} \\ &= \prod_{i=1}^3 (1 - \epsilon_i) \times e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T} < 1. \end{aligned} \quad (3.7)$$

From reference [29], we can deduce that the unique fixed point 0 of system (3.3) is globally asymptotically stable.

ii) If $e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T} > 1$, there are two fixed points, 0 and P^* . Obviously, the fixed point 0 of system (3.3) is unstable, while

$$\begin{aligned} \frac{\partial F(P(nT^+))}{\partial P(nT^+)} \Big|_{P^*=0} &= \frac{a_{13}^2 \prod_{i=1}^3 (1 - \epsilon_i) \times e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T}}{(a_{13} + b_{13}(e^{a_{13}\gamma_1 T} - 1)P(nT^+))^2} \Big|_{P^*=0} \\ &= \frac{1}{\prod_{i=1}^3 (1 - \epsilon_i) \times e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T}} < 1. \end{aligned} \quad (3.8)$$

From reference [29], we can deduce that the fixed point P^* of system (3.3) is globally asymptotically stable. $\square$

Similar to reference [32], we have the following lemma.

Lemma 3.2.

i) If $\prod_{i=1}^3 (1 - \epsilon_i) e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2)]T} < 1$ holds, the normal periodic solution 0 of system (3.1) is globally asymptotically stable.

ii) If $\prod_{i=1}^3 (1 - \epsilon_i) e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2)]T} > 1$ holds, the positive periodic solutions $\overline{P(t)}$ of system (3.1) are globally asymptotically stable, where $\overline{P(t)}$ is defined as

$$\overline{P(t)} = \begin{cases} \frac{a_{13}P^*e^{a_{13}(t-nT)}}{a_{13} + b_{13}P^*(e^{a_{13}(t-nT)} - 1)}, & t \in (nT, (n + \gamma_1)T], \\ \frac{P^{**}e^{-d_{23}(t-(n+\gamma_1)T)}}{P^{**}e^{-d_{23}(t-(n+\gamma_1)T)}}, & t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ P^{***}e^{-d_{33}(t-(n+\gamma_1+\gamma_2)T)}, & t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \end{cases} \quad (3.9)$$

where

$$\begin{cases} P^* = \frac{a_{13}[\prod_{i=1}^3 (1 - \epsilon_i) e^{(a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1-\gamma_1-\gamma_2))T} - 1]}{b_{13}(e^{a_{13}\gamma_1 T} - 1)} > 0, \\ P^{**} = \frac{(1 - \epsilon_1)a_{13}P^*e^{a_{13}\gamma_1 T}}{a_{13} + b_{13}P^*(e^{a_{13}\gamma_1 T} - 1)} > 0, \\ P^{***} = (1 - \epsilon_2)P^{**}e^{-d_{23}\gamma_2 T} > 0. \end{cases} \quad (3.10)$$

Lemma 3.3. There exists a constant $M > 0$ such that $P_i(t) \leq M$ ($i = 1, 2$) and $P(t) \leq M$ with all t large enough.

Proof. Denoting $d = \min\{d_{11}, d_{12}, d_{22}, d_{23}, d_{31}, d_{33}\} > 0$ and $V(t) = cP_1(t) + cP_2(t) + P(t)$ with $c = \max\{c_1, c_2\} > 0$, we get

$$\frac{dV(t)}{dt} + dV(t) \leq \frac{(a_{13} + d)^2}{4b_{13}} = M_1, \quad t \in (nT, (n + \gamma_1)T],$$

with

$$\begin{aligned} V((n + \gamma_1)T^+) &= cP_1((n + \gamma_1)T) + cP_1((n + \gamma_1)T)(a_1 + b_1P_1((n + \gamma_1)T)) + cP_2((n + \gamma_1)T) \\ &\quad + (1 - \epsilon_1)P((n + \gamma_1)T) \leq V((n + \gamma_1)T) + \frac{ca_1^2}{4b_1}, \quad t = (n + \gamma_1)T, \end{aligned}$$

and

$$\frac{dV}{dt} + dV(t) \leq \frac{c(a_{21} + d)^2}{4b_{21}} = M_2, \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T],$$

with

$$\begin{aligned} V((n + \gamma_1 + \gamma_2)T^+) &= cP_1((n + \gamma_1 + \gamma_2)T) + cP_2((n + \gamma_1 + \gamma_2)T) + cP_2((n + \gamma_1 + \gamma_2)T)(a_2 + b_2P_2((n + \gamma_1 \\ &\quad + \gamma_2)T)) + (1 - \epsilon_1)P((n + \gamma_1 + \gamma_2)T) \leq V((n + \gamma_1 + \gamma_2)T) + \frac{ca_2^2}{4b_2}, \quad t = (n + \gamma_1 + \gamma_2)T, \end{aligned}$$

and

$$\frac{dV}{dt} + dV(t) \leq \frac{c(a_{32} + d)^2}{4b_{32}} = M_3, \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T],$$

with

$$V((n+1)T^+) \leq cP_1((n+1)T) + cP((n+1)T) + (1 - \epsilon_3)P((n+1)T) \leq V((n+1)T), \quad t = (n+1)T.$$

Then,

$$\begin{cases} \frac{dV}{dt} + dV \leq M_1, \quad t \in (nT, (n + \gamma_1)T], \\ \Delta V(t) \leq \frac{ca_1^2}{4b_1}, \quad t = (n + \gamma_1)T, \\ \frac{dV}{dt} + dV \leq M_2, \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta V(t) \leq \frac{ca_2^2}{4b_2}, \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dV}{dt} + dV \leq M_3, \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \end{cases} \quad (3.11)$$

where $A = \frac{M_1}{d}(e^{d\gamma_1 T} - 1)e^{-dT} + \frac{ca_1^2}{4b_1}e^{-d(1-\gamma_1)T} + \left[\frac{M_2}{d}(1 - e^{-d\gamma_2 T}) + \frac{ca_2^2}{4b_2} \right]e^{-d(1-\gamma_1-\gamma_2)T} + \frac{M_3}{d}(1 - e^{-d(1-\gamma_1-\gamma_2)T})$. According to the comparison theorem, we can get

$$\begin{aligned} V(t) &\leq (V(0^+) - V^*)e^{-dt} + \widetilde{V}(t) \\ &\leq (V(0^+) - V^*)e^{-dt} + \max \left\{ V^* + \frac{M_1}{d}, V^{**} + \frac{M_2}{d}, V^{***} + \frac{M_3}{d} \right\}, \\ &\rightarrow \max \left\{ V^* + \frac{M_1}{d}, V^{**} + \frac{M_2}{d}, V^{***} + \frac{M_3}{d} \right\} = M \quad (t \rightarrow \infty), \end{aligned} \quad (3.12)$$

where

$$\widetilde{V}(t) = \begin{cases} V^* e^{-d(t-nT)} + \frac{M_1}{d}(1 - e^{-d(t-nT)}), & t \in (nT, (n + \gamma_1)T], \\ V^{**} e^{-d(t-(n+\gamma_1)T)} + \frac{M_2}{d}(1 - e^{-d(t-(n+\gamma_1)T)}), & t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ V^{***} e^{-d(t-(n+\gamma_1+\gamma_2)T)} + \frac{M_3}{d}(1 - e^{-d(t-(n+\gamma_1+\gamma_2)T)}), & t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \end{cases} \quad (3.13)$$

and

$$\begin{cases} V^* = \frac{A}{1 - e^{-dT}}, \\ V^{**} = V^* e^{-d\gamma_1 T} + \frac{M_1}{d}(1 - e^{-d\gamma_1 T}) + \frac{ca_1^2}{4b_1}, \\ V^{***} = V^{**} e^{-d\gamma_2 T} + \frac{M_2}{d}(1 - e^{-d\gamma_2 T}) + \frac{ca_2^2}{4b_2}. \end{cases} \quad (3.14)$$

□

For convenience, we make the following notations:

$$R_1 = \ln \Pi_{i=1}^3 (1 - \epsilon_i) + a_{13}\gamma_1 T - 2 \ln \frac{a_{13} + b_{13}P^*(e^{a_{13}\gamma_1 T} - 1)}{a_{13}},$$

$$R_2 = \ln(1 + a_1) - d_{11}\gamma_1 T + \frac{a_{21}}{d_{23}} \ln \frac{\delta_1 P^{**} + e^{d_{23}(\gamma_1 + \gamma_2)T}}{\delta_1 P^{**} + e^{d_{23}\gamma_1 T}} \\ - \frac{\beta_1 P^{**}}{d_{31}} (e^{-d_{31}\gamma_1 T} - e^{-d_{23}(\gamma_1 + \gamma_2)T}) - \frac{\beta_1 \beta_{11} (P^{**})^2}{2d_{23}} (e^{-2d_{23}\gamma_1 T} - e^{-2d_{23}(\gamma_1 + \gamma_2)T}) - d_{23}(1 - \gamma_1 - \gamma_2)T,$$

and

$$R_3 = \ln(1 + a_1) - (d_{12}\gamma_1 + d_{22}\gamma_2)T + \frac{a_{32}}{d_{33}} \ln \frac{\delta_2 P^{***} + e^{d_{33}T}}{\delta_2 P^{***} + e^{d_{33}(\gamma_1 + \gamma_2)T}} \\ - \frac{\beta_2 P^{***}}{d_{33}} (e^{-d_{33}(\gamma_1 + \gamma_2)T} - e^{-d_{33}T}) - \frac{\beta_2 \beta_{22} (P^{***})^2}{2d_{33}} (e^{-2d_{33}(\gamma_1 + \gamma_2)T} - e^{-2d_{33}T}).$$

4. The dynamics

For $\prod_{i=1}^3 (1 - \epsilon_i) e^{[a_{13}\gamma_1 - d_{23}\gamma_2 - d_{33}(1 - \gamma_1 - \gamma_2)]T} > 1$, there exists a prey-eradication boundary periodic solution $(0, 0, \overline{P(t)})$ of system (2.1). Furthermore, it will be proved to be globally asymptotically stable.

Theorem 4.1. *If*

$$R_1 < 0 \tag{4.1}$$

and

$$\max\{R_2, R_3\} < 0 \tag{4.2}$$

hold, the prey-eradication boundary periodic solution $(0, 0, \overline{P(t)})$ of system (2.1) is GAS (globally asymptotically stable).

Proof. Applying the transformation $x_i(t) = P_i(t)$ ($i = 1, 2$), $z(t) = P(t) - \overline{P(t)}$, we derive the linearly similar system of system (2.1) with $(0, 0, \overline{P(t)})$:

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dz(t)}{dt} \end{pmatrix} = \begin{pmatrix} -d_{11} & 0 & 0 \\ 0 & -d_{12} & 0 \\ 0 & 0 & a_{13} - 2b_{13}\overline{P(t)} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ z(t) \end{pmatrix}, \quad t \in (nT, (n + \gamma_1)T],$$

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dz(t)}{dt} \end{pmatrix} = \begin{pmatrix} \frac{a_{21}}{1 + \delta_1 \overline{P(t)}} - \beta_1(1 + \beta_{11}\overline{P(t)})\overline{P(t)} & 0 & 0 \\ 0 & -d_{22} & 0 \\ c_1\beta_1(1 + \beta_{11}\overline{P(t)})\overline{P(t)} & 0 & -d_{23} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ z(t) \end{pmatrix}, \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T],$$

and

$$\begin{pmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dz(t)}{dt} \end{pmatrix} = \begin{pmatrix} -d_{31} & 0 & 0 \\ 0 & \frac{a_{32}}{1 + \delta_2 \overline{P(t)}} - \beta_2(1 + \beta_{22}\overline{P(t)})\overline{P(t)} & 0 \\ 0 & c_2\beta_2(1 + \beta_{22}\overline{P(t)})\overline{P(t)} & -d_{23} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ z(t) \end{pmatrix}, \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T].$$

Their corresponding fundamental solution matrices are

$$\Phi_1(t) = \begin{pmatrix} e^{-d_{11}(t-nT)} & 0 & 0 \\ 0 & e^{-d_{12}(t-nT)} & 0 \\ 0 & 0 & e^{\int_{nT}^t a_{13}-2b_{13}\overline{P(s)}ds} \end{pmatrix}, \quad t \in (nT, (n+\gamma_1)T],$$

$$\Phi_2(t) = \begin{pmatrix} e^{\int_{(n+\gamma_1)T}^t (\frac{a_{21}}{1+\delta_1\overline{P(s)}} - \beta_1(1+\beta_{11}\overline{P(s)})\overline{P(s)})ds} & 0 & 0 \\ 0 & e^{-d_{22}(t-(n+\gamma_1)T)} & 0 \\ \ddagger_1 & 0 & e^{-d_{23}(t-(n+\gamma_1)T)} \end{pmatrix},$$

$$t \in ((n+\gamma_1)T, (n+\gamma_1+\gamma_2)T],$$

and

$$\Phi_3(t) = \begin{pmatrix} e^{-d_{31}(t-(n+\gamma_1+\gamma_2)T)} & 0 & 0 \\ 0 & e^{\int_{(n+\gamma_1+\gamma_2)T}^t (\frac{a_{32}}{1+\delta_2\overline{P(s)}} - \beta_2(1+\beta_{22}\overline{P(s)})\overline{P(s)})ds} & 0 \\ 0 & \ddagger_2 & e^{-d_{33}(t-(n+\gamma_1)T)} \end{pmatrix},$$

$$t \in ((n+\gamma_1+\gamma_2)T, (n+1)T].$$

There is no need to calculate the exact form of $\ddagger_i$ ($i = 1, 2$) as they are not required in the next analysis process. The linearizations of impulsive terms of system (2.1) are

$$\begin{pmatrix} x_1((n+\gamma_1)T^+) \\ x_2((n+\gamma_1)T^+) \\ z((n+\gamma_1)T^+) \end{pmatrix} = \begin{pmatrix} 1+a_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\epsilon_1 \end{pmatrix} \begin{pmatrix} x_1((n+\gamma_1)T) \\ x_2((n+\gamma_1)T) \\ z((n+\gamma_1)T) \end{pmatrix}, \quad t = (n+\gamma_1)T,$$

$$\begin{pmatrix} x_1((n+\gamma_1+\gamma_2)T^+) \\ x_2((n+\gamma_1+\gamma_2)T^+) \\ z((n+\gamma_1+\gamma_2)T^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1+a_2 & 0 \\ 0 & 0 & 1-\epsilon_2 \end{pmatrix} \begin{pmatrix} x_1((n+\gamma_1+\gamma_2)T) \\ x_2((n+\gamma_1+\gamma_2)T) \\ z((n+\gamma_1+\gamma_2)T) \end{pmatrix}, \quad t = (n+\gamma_1+\gamma_2)T,$$

and

$$\begin{pmatrix} x_1((n+1)T^+) \\ x_2((n+1)T^+) \\ z((n+1)T^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\epsilon_3 \end{pmatrix} \begin{pmatrix} x_1((n+1)T) \\ x_2((n+1)T) \\ z((n+1)T) \end{pmatrix}, \quad t = (n+1)T.$$

The local stability of the periodic solution $(0, 0, \overline{P(t)})$ is determined by the eigenvalues of

$$M = \Phi_1(\gamma_1 T) \begin{pmatrix} 1 + a_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \epsilon_1 \end{pmatrix} \Phi_2((\gamma_1 + \gamma_2)T) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 + a_2 & 0 \\ 0 & 0 & 1 - \epsilon_2 \end{pmatrix} \\ \times \Phi_3(T) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - \epsilon_3 \end{pmatrix},$$

which are

$$\lambda_1 = (1 + a_1)e^{-d_{11}\gamma_1 T + \int_{\gamma_1 T}^{(\gamma_1 + \gamma_2)T} \frac{a_{21}}{1 + \delta_1 \overline{P(s)}} - \beta_1(1 + \beta_{11}\overline{P(s)})\overline{P(s)}ds - d_{31}(1 - \gamma_1 - \gamma_2)T} > 0, \\ \lambda_2 = (1 + a_2)e^{-d_{12}\gamma_1 T - d_{22}\gamma_2 T + \int_{(\gamma_1 + \gamma_2)T}^T \frac{a_{32}}{1 + \delta_2 \overline{P(s)}} - \beta_2(1 + \beta_{22}\overline{P(s)})\overline{P(s)}ds} > 0,$$

and

$$\lambda_3 = \prod_{i=1}^3 (1 - \epsilon_i)e^{\int_0^{\gamma_1 T} a_{13} - 2b_{13}\overline{P(s)}ds - d_{23}\gamma_2 T - d_{33}(1 - \gamma_1 - \gamma_2)T} > 0.$$

From conditions (4.1)-(4.2), we have $|\lambda_i| < 1$ ($i = 1, 2, 3$). According to Floquet theory [38], the prey-eradication boundary periodic solution $(0, 0, \overline{P(t)})$ of system (2.1) is locally asymptotically stable.

The next piece is to prove the prey-eradication periodic solution $(0, 0, \overline{P(t)})$ to be globally attractive. According to conditions (4.1)-(4.2), we may choose $\varepsilon > 0$ such that

$$\rho_1 = (1 + a_1)e^{-d_{11}\gamma_1 T + \int_{\gamma_1 T}^{(\gamma_1 + \gamma_2)T} \frac{a_{21}}{1 + \delta_1(\overline{P(s)} - \varepsilon)} - \beta_1(1 + \beta_{11}(\overline{P(s)} - \varepsilon))(\overline{P(s)} - \varepsilon)ds - d_{31}(1 - \gamma_1 - \gamma_2)T} < 1$$

and

$$\rho_2 = (1 + a_2)e^{-d_{12}\gamma_1 T - d_{22}\gamma_2 T + \int_{(\gamma_1 + \gamma_2)T}^T \frac{a_{32}}{1 + \delta_2(\overline{P(s)} - \varepsilon)} - \beta_2(1 + \beta_{22}(\overline{P(s)} - \varepsilon))(\overline{P(s)} - \varepsilon)ds} < 1.$$

From the ninth and fifteenth equations of system (2.1), we get that $\frac{dP(t)}{dt} \geq -d_{23}P(t)$, $t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T]$, and $\frac{dP(t)}{dt} \geq -d_{33}P(t)$, $t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T]$. Therefore, the following impulsive differential equations are considered:

$$\begin{cases} \frac{dY(t)}{dt} = Y(t)(a_{13} - b_{13}Y(t)), & t \in (nT, (n + \gamma)T], \\ \Delta Y(t) = -\epsilon_1 Y(t), & t = (n + \gamma_1)T, \\ \frac{dY(t)}{dt} = -d_{23}Y(t), & t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta Y(t) = -\epsilon_2 Y(t), & t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dY(t)}{dt} = -d_{33}Y(t), & t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\ \Delta Y(t) = -\epsilon_3 Y(t), & t = (n + 1)T. \end{cases} \quad (4.3)$$

From Lemma 3.2 and the comparison theorem of impulsive differential equations [30], we have $P(t) \geq Y(t)$ and $Y(t) \rightarrow \overline{Y(t)}$ as $t \rightarrow \infty$. Then, $P(t) \geq Y(t) \geq \overline{Y(t)} - \varepsilon = \overline{P(t)} - \varepsilon$ for all t large enough. For

convenience, we assume that they hold for all $t \geq 0$. From system (2.1), we get

$$\left\{ \begin{array}{l} \frac{dP_1(t)}{dt} = -d_{11}P_1(t), \quad t \in (nT, (n + \gamma_1)T], \\ \Delta P_1(t) = P_1(t)(a_1 - b_1P_1(t)), \quad t = (n + \gamma_1)T, \\ \frac{dP_1(t)}{dt} \leq P_1(t) \left[\frac{a_{21}}{1 + \delta_1(\overline{P}(t) - \varepsilon)} - \beta_1(1 + \beta_{11}(\overline{P}(t) - \varepsilon))(\overline{P}(t) - \varepsilon) \right], \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta P_1(t) = 0, \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dP_1(t)}{dt} = -d_{31}P_1(t), \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\ \Delta P_1(t) = 0, \quad t = (n + 1)T, \end{array} \right. \quad (4.4)$$

and

$$\left\{ \begin{array}{l} \frac{dP_2(t)}{dt} = -d_{12}P_2(t), \quad t \in (nT, (n + \gamma_1)T], \\ \Delta P_2(t) = 0, \quad t = (n + \gamma_1)T, \\ \frac{dP_2(t)}{dt} = -d_{22}P_2(t), \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta P_2(t) = P_2(t)(a_2 - b_2P_2(t)), \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dP_2(t)}{dt} \leq P_2(t) \left[\frac{a_{32}}{1 + \delta_2(\overline{P}(t) - \varepsilon)} - \beta_2(1 + \beta_{22}(\overline{P}(t) - \varepsilon))(\overline{P}(t) - \varepsilon) \right], \\ \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\ \Delta P_2(t) = 0, \quad t = (n + 1)T. \end{array} \right. \quad (4.5)$$

Integrating (4.4) and (4.5) on $(nT, (n + 1)T]$, we have

$$P_1((n + 1)T) \leq P_1(nT^+)(1 + a_1)e^{-d_{11}\gamma_1 T + \int_{\gamma_1 T}^{(\gamma_1 + \gamma_2)T} \frac{a_{21}}{1 + \delta_1(\overline{P}(s) - \varepsilon)} - \beta_1(1 + \beta_{12}(\overline{P}(s) - \varepsilon))(\overline{P}(s) - \varepsilon) ds - d_{31}(1 - \gamma_1 - \gamma_2)T}$$

and

$$P_2((n + 1)T) \leq P_2(nT^+)(1 + a_2)e^{-d_{12}\gamma_1 T - d_{22}\gamma_2 T + \int_{(\gamma_1 + \gamma_2)T}^T \frac{a_{32}}{1 + \delta_2(\overline{P}(s) - \varepsilon)} - \beta_2(1 + \beta_{22}(\overline{P}(s) - \varepsilon))(\overline{P}(s) - \varepsilon) ds}.$$

Hence $P_i(nT) \leq P_i(0^+)\rho_i^n$ ($i = 1, 2$) and $P_i(nT) \rightarrow 0$ ($i = 1, 2$) as $n \rightarrow \infty$. Therefore, $P_i(t) \rightarrow 0$ ($i = 1, 2$) as $t \rightarrow \infty$.

Finally, we will prove that $P(t) \rightarrow \overline{P}(t)$ as $t \rightarrow \infty$. For $\varepsilon > 0$ small enough, there must exist a $t_0 > 0$ such that $0 < P_i(t) < \varepsilon$ ($i = 1, 2$) for all $t \geq t_0$. Without loss of generality, we assume that $0 < P_i(t) < \varepsilon$ ($i = 1, 2$) for all $t \geq 0$, and then we have

$$-d_{23}P(t) \leq \frac{dP(t)}{dt} \leq -[d_{23} - \beta_1(1 + \beta_{11}M)\varepsilon]P(t), \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T],$$

and

$$-d_{33}P(t) \leq \frac{dP(t)}{dt} \leq -[d_{33} - \beta_2(1 + \beta_{22}M)\varepsilon]P(t), \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T].$$

Therefore, $Y(t) \leq P(t) \leq Y_1(t)$ and $Y_1(t) \rightarrow \overline{Y_1(t)}$ as $t \rightarrow \infty$, while $Y(t)$ and $Y_1(t)$ are, respectively, the solutions of system (4.3), and

$$\left\{ \begin{array}{l} \frac{dY_1(t)}{dt} = Y_1(t)(a_{13} - b_{13}Y_1(t)), \quad t \in (nT, (n + \gamma)T], \\ \Delta Y_1(t) = -\epsilon_1 Y_1(t), \quad t = (n + \gamma_1)T, \\ \frac{dY_1(t)}{dt} = -[d_{23} - c_1\beta_1(1 + \beta_{11}M)\epsilon]Y_1(t), \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta Y_1(t) = -\epsilon_2 Y_1(t), \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dY_1(t)}{dt} = -[d_{33} - c_2\beta_2(1 + \beta_{22}M)\epsilon]Y_1(t), \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\ \Delta Y_1(t) = -\epsilon_3 Y_1(t), \quad t = (n + 1)T. \end{array} \right. \quad (4.6)$$

From Lemma 3.2, we know that the positive periodic solutions $\overline{Y_1(t)}$ of system (4.6) are globally asymptotically stable, where

$$\overline{Y_1(t)} = \begin{cases} \frac{a_{13}Y_1^*e^{a_{13}(t-nT)}}{a_{13} + b_{13}Y_1^*(e^{a_{13}(t-nT)} - 1)}, & t \in (nT, (n + \gamma_1)T], \\ Y_1^{**}e^{-[d_{23}-c_1\beta_1(1+\beta_{11}M)\epsilon](t-(n+\gamma_1)T)}, & t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ Y_1^{***}e^{-[d_{33}-c_2\beta_2(1+\beta_{22}M)\epsilon](t-(n+\gamma_1+\gamma_2)T)}, & t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \end{cases} \quad (4.7)$$

with

$$\left\{ \begin{array}{l} Y_1^* = \frac{a_{13}(\prod_{i=1}^3(1 - \epsilon_i)e^{(a_{13}\gamma_1 - [d_{23}-c_1\beta_1(1+\beta_{11}M)\epsilon]\gamma_2 - [d_{33}-c_2\beta_2(1+\beta_{22}M)\epsilon](1-\gamma_1-\gamma_2)T} - 1)}}{b_{13}(e^{a_{13}\gamma_1 T} - 1)} > 0, \\ Y_1^{**} = \frac{(1 - \epsilon_1)a_{13}P^*e^{a_{13}\gamma_1 T}}{a_{13} + b_{13}P^*(e^{a_{13}\gamma_1 T} - 1)} > 0, \\ Y_1^{***} = (1 - \epsilon_2)P^{**}e^{-[d_{23}-c_1\beta_1(1+\beta_{11}M)\epsilon]\gamma_2 T} > 0. \end{array} \right. \quad (4.8)$$

Therefore, for any $\epsilon_1 > 0$ small enough, there exists a $t_1, t > t_1$ such that

$$\overline{Y(t)} - \epsilon_1 < P(t) < \overline{Y_1(t)} + \epsilon_1.$$

Let $\epsilon \rightarrow 0$ and we have

$$\overline{P(t)} - \epsilon_1 < P(t) < \overline{P(t)} + \epsilon_1,$$

for t large enough. This implies $P(t) \rightarrow \overline{P(t)}$ as $t \rightarrow \infty$. $\square$

Theorem 4.2. *If*

$$\min\{R_2, R_3\} > 0 \quad (4.9)$$

holds, system (2.1) is permanent.

Proof. From Lemma 3.3, we have proved that there exists a constant $M > 0$ such that $P_i(t) \leq M$ ($i = 1, 2$) and $P(t) \leq M$ for t large enough. We may assume $P_i(t) \leq M$ ($i = 1, 2$) and $P(t) \leq M$ for $t \geq 0$. From the ninth and fifteenth equations of system (2.1), we have $\frac{dP(t)}{dt} \geq -d_{23}P(t), t \in ((n + \gamma_1)T, (n +$

$\gamma_1 + \gamma_2)T]$, and $\frac{dP(t)}{dt} \geq -d_{33}P(t)$, $t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T]$. From the proof process of Theorem 4.1 and for any $\varepsilon_2 > 0$, there exists $T_0 > 0$, when $t > T_0$, and we have

$$P(t) \geq Y(t) \geq \overline{Y(t)} - \varepsilon_2 = \overline{P(t)} - \varepsilon_2 > P^* + P^{**}e^{-d_{23}\gamma_2 T} + P^{***}e^{-d_{33}(1-\gamma_1-\gamma_2)T} - \varepsilon_2 = m_0.$$

Thus, we only need to find $m_i > 0$ ($i = 1, 2$) such that $P_i(t) \geq m_i$ ($i = 1, 2$) for t large enough, which we will do in the next step.

By condition (4.9) we may select

$$0 < m_{11} < \frac{d_{23}}{c_1\beta_1(1 + \beta_{11}M)}$$

and

$$0 < m_{22} < \frac{d_{33}}{c_2\beta_2(1 + \beta_{22}M)},$$

and $\varepsilon > 0$ small enough such that

$$\begin{aligned} \xi_1 = & \ln(1 + a_1) - d_{11}\gamma_1 T \\ & + \frac{a_{21}}{(1 + \delta_1\varepsilon)[d_{23} - c_1\beta_1(1 + \beta_{11}M)m_{11}]} \ln \frac{\delta_1 Z_1^{**} + (1 + \delta_1\varepsilon)e^{[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}](\gamma_1+\gamma_2)T}}{\delta_1 Z_1^{**} + (1 + \delta_1\varepsilon)e^{[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}]\gamma_1 T}} \\ & - \frac{\beta_1(1 + \beta_{11}\varepsilon)Z_1^{**}}{d_{23} - c_1\beta_1(1 + \beta_{11}M)m_{11}} (e^{-[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}]\gamma_1 T} - e^{-[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}](\gamma_1+\gamma_2)T}) \\ & - \frac{\beta_1\beta_{11}(Z_1^{**})^2}{2[d_{23} - c_1\beta_1(1 + \beta_{11}M)m_{11}]} (e^{-2[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}]\gamma_1 T} - e^{-2[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}](\gamma_1+\gamma_2)T}) \\ & - \beta_{11}(1 + \beta_{11}\varepsilon)\varepsilon\gamma_2 T - d_{31}(1 - \gamma_1 - \gamma_2)T > 0 \end{aligned}$$

and

$$\begin{aligned} \xi_2 = & \ln(1 + a_2) - (d_{12}\gamma_1 + d_{22}\gamma_2)T \\ & + \frac{a_{32}}{(1 + \delta_2\varepsilon)[d_{33} - c_2\beta_2(1 + \beta_{22}M)m_{22}]} \ln \frac{\delta_2 Z_1^{***} + (1 + \delta_2\varepsilon)e^{[d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}]T}}{\delta_2 Z_1^{***} + (1 + \delta_2\varepsilon)e^{[d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}](\gamma_1+\gamma_2)T}} \\ & - \frac{\beta_2(1 + \beta_{22}\varepsilon)Z_1^{***}}{[d_{33} - c_2\beta_2(1 + \beta_{22}M)m_{22}]} (e^{-[d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}](\gamma_1+\gamma_2)T} - e^{-[d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}]T}) \\ & - \frac{\beta_2(1 + \beta_{22}\varepsilon)(Z_1^{***})^2}{2[d_{33} - c_2\beta_2(1 + \beta_{22}M)m_{22}]} (e^{-2[d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}](\gamma_1+\gamma_2)T} - e^{-2[d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}]T}) \\ & \times \beta_2(1 + \beta_{22}\varepsilon)\varepsilon(1 - \gamma_1 - \gamma_2)T > 0, \end{aligned}$$

with Z_1^* , Z_1^{**} , and Z_1^{***} defined as in (4.13).

We will prove $P_i(t) < m_{ii}$ ($i = 1, 2$) cannot hold for $t \geq 0$. Otherwise,

$$\left\{ \begin{array}{l} \frac{dZ(t)}{dt} = Z(t)(a_{13} - b_{13}Z(t)), \quad t \in (nT, (n + \gamma)T], \\ \Delta Z(t) = -\epsilon_1 Z(t), \quad t = (n + \gamma_1)T, \\ \frac{dZ(t)}{dt} \leq -[d_{23} - c_1\beta_1(1 + \beta_{11}M)m_{11}]Z(t), \quad t \in ((n + \gamma_1)\tau, (n + \gamma_1 + \gamma_2)T], \\ \Delta Z(t) = -\epsilon_2 Z(t), \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dZ(t)}{dt} \leq -[d_{33} - c_2\beta_2(1 + \beta_{22}M)m_{22}]Z(t), \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)\tau], \\ \Delta Z(t) = -\epsilon_3 Z(t), \quad t = (n + 1)\tau. \end{array} \right. \quad (4.10)$$

Then, the comparative impulsive differential equations of (4.10) are

$$\left\{ \begin{array}{l} \frac{dZ_1(t)}{dt} = Z_1(t)(a_{13} - b_{13}Z_1(t)), \quad t \in (nT, (n + \gamma)T], \\ \Delta Z_1(t) = -\epsilon_1 Z_1(t), \quad t = (n + \gamma_1)T, \\ \frac{dZ_1(t)}{dt} = -[d_{23} - c_1\beta_1(1 + \beta_{11}M)m_{11}]Z_1(t), \quad t \in ((n + \gamma_1)\tau, (n + \gamma_1 + \gamma_2)T], \\ \Delta Z_1(t) = -\epsilon_2 Z_1(t), \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dZ_1(t)}{dt} = -[d_{33} - c_2\beta_2(1 + \beta_{22}M)m_{22}]Z_1(t), \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)\tau], \\ \Delta Z_1(t) = -\epsilon_3 Z_1(t), \quad t = (n + 1)\tau. \end{array} \right. \quad (4.11)$$

From Lemma 3.2, the positive periodic solutions $\overline{Z_1(t)}$ of system (4.11) are globally asymptotically stable, where

$$\overline{Z_1(t)} = \begin{cases} \frac{a_{13}Z_1^*e^{a_{13}(t-nT)}}{a_{13} + b_{13}Z_1^*(e^{a_{13}(t-nT)} - 1)}, & t \in (nT, (n + \gamma_1)T], \\ Z_1^{**}e^{-[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}](t-(n+\gamma_1)\tau)}, & t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ Z_1^{***}e^{-[d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}](t-(n+\gamma_1+\gamma_2)T)}, & t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \end{cases} \quad (4.12)$$

with

$$\left\{ \begin{array}{l} Z_1^* = \frac{a_{13}(\prod_{i=1}^3(1 - \epsilon_i)e^{(a_{13}\gamma_1 - [d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}]\gamma_2 - [d_{33}-c_2\beta_2(1+\beta_{22}M)m_{22}](1-\gamma_1-\gamma_2)T} - 1)}}{b_{13}(e^{a_{13}\gamma_1 T} - 1)}, \\ Z_1^{**} = \frac{(1 - \epsilon_1)a_{13}Z_1^*e^{a_{13}\gamma_1 T}}{a_{13} + b_{13}Z_1^*(e^{a_{13}\gamma_1 T} - 1)}, \\ Z_1^{***} = (1 - \epsilon_2)Z_1^{**}e^{-[d_{23}-c_1\beta_1(1+\beta_{11}M)m_{11}]\gamma_2 T}. \end{array} \right. \quad (4.13)$$

By the comparative theorem of impulsive differential equations and Lemma 3.2, we have $Z(t) < Z_1(t)$ and $Z_1(t) \rightarrow \overline{Z_1(t)}$, as $t \rightarrow \infty$. Substituting (4.12) into the seventh and fourteenth equations of

system (2.1), we easily have

$$\left\{ \begin{array}{l} \frac{dP_1(t)}{dt} = -d_{11}P_1(t), \quad t \in (nT, (n + \gamma_1)T], \\ \Delta P_1(t) = P_1(t)(a_1 - b_1P_1(t)), \quad t = (n + \gamma_1)T, \\ \frac{dP_1(t)}{dt} \geq P_1(t) \left[\frac{a_{21}}{1 + \delta_1(\overline{Z_1(t)} + \varepsilon)} - \beta_1(1 + \beta_{11}(\overline{Z_1(t)} + \varepsilon))(\overline{Z_1(t)} + \varepsilon) \right], \\ \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta P_1(t) = 0, \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dP_1(t)}{dt} = -d_{31}P_1(t), \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\ \Delta P_1(t) = 0, \quad t = (n + 1)T, \end{array} \right. \quad (4.14)$$

and

$$\left\{ \begin{array}{l} \frac{dP_2(t)}{dt} = -d_{12}P_2(t), \quad t \in (nT, (n + \gamma_1)T], \\ \Delta P_2(t) = 0, \quad t = (n + \gamma_1)T, \\ \frac{dP_2(t)}{dt} = -d_{22}P_2(t), \quad t \in ((n + \gamma_1)T, (n + \gamma_1 + \gamma_2)T], \\ \Delta P_2(t) = P_2(t)(a_2 - b_2P_2(t)), \quad t = (n + \gamma_1 + \gamma_2)T, \\ \frac{dP_2(t)}{dt} \geq P_2(t) \left[\frac{a_{32}}{1 + \delta_2(\overline{Z_1(t)} + \varepsilon)} - \beta_2(1 + \beta_{22}(\overline{Z_1(t)} + \varepsilon))(\overline{Z_1(t)} + \varepsilon) \right], \\ \quad t \in ((n + \gamma_1 + \gamma_2)T, (n + 1)T], \\ \Delta P_2(t) = 0, \quad t = (n + 1)T. \end{array} \right. \quad (4.15)$$

Integrating (4.14) and (4.15) on $(nT, (n + 1)T]$, we have

$$P_1((n + 1)T) \geq P_1(nT^+)(1 + a_1) \times e^{-d_{11}\gamma_1 T + \int_{\gamma_1 T}^{(\gamma_1 + \gamma_2)T} \frac{a_{21}}{1 + \delta_1(\overline{Z_1(s)} + \varepsilon)} - \beta_1(1 + \beta_{12}(\overline{Z_1(s)} + \varepsilon))(\overline{Z_1(s)} + \varepsilon) ds - d_{31}(1 - \gamma_1 - \gamma_2)T} > P_1(nT^+)e^{\xi_1}$$

and

$$P_2((n + 1)T) \geq P_2(nT^+)(1 + a_2) \times e^{-d_{12}\gamma_1 T - d_{22}\gamma_2 T + \int_{(\gamma_1 + \gamma_2)T}^T \frac{a_{32}}{1 + \delta_2(\overline{Z_1(s)} + \varepsilon)} - \beta_2(1 + \beta_{22}(\overline{Z_1(s)} + \varepsilon))(\overline{Z_1(s)} + \varepsilon) ds} > P_2(nT^+)e^{\xi_2}.$$

For $t \geq T_1$, let $N_1 \in \mathbb{N}$ and $N_1 T > T_1$, and then $P_i((N_1 + k)T) \geq P_i(N_1 T^+)e^{k\sigma} \rightarrow \infty$ ($i = 1, 2$), as $k \rightarrow \infty$, which is a contradiction to the boundedness of $P_i(t)$ ($i = 1, 2$). Hence there exists $t_1 > 0$ such that $P_i(t) \geq m_i$ ($i = 1, 2$). The proof is complete.

5. Numerical simulations

In this section, we conduct numerical simulations to verify our results and explore the effect of some vital parameters affecting system (2.1) by numerical analysis. If it is assumed that $d_{11} = 0.15$, $d_{12} = 0.45$, $a_{13} = 3$, $b_{13} = 2$, $a_{21} = 2.3$, $\delta_1 = 4$, $b_{21} = 0.5$, $\beta_1 = 3.5$, $\beta_{11} = 6$, $d_{22} = 0.9$,

$d_{23} = 0.65$, $c_1 = 1.5$, $d_{31} = 0.25$, $a_{32} = 0.15$, $\delta_2 = 6$, $b_{32} = 0.25$, $\beta_2 = 4.4$, $\beta_{22} = 2.5$, $d_{33} = 0.55$, $c_2 = 2$, $T = 7$, $\gamma_1 = 0.65$, $\gamma_2 = 0.3$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.1$, $a_1 = 8$, $b_1 = 0.1$, $a_2 = 1.5$, and $b_2 = 0.6$, then $R_1 = -10.1618 < 0$, $\max\{R_2, R_3\} = -0.3961 < 0$, and the prey-eradication boundary periodic solution $(0, 0, \overline{P(t)})$ is GAS as shown in Figure 2. If it is also assumed that $d_{11} = 0.15$, $d_{12} = 0.45$, $a_{13} = 3$, $b_{13} = 2$, $a_{21} = 2.3$, $\delta_1 = 0.5$, $b_{21} = 0.5$, $\beta_1 = 3$, $\beta_{11} = 0.5$, $d_{22} = 0.9$, $d_{23} = 0.65$, $c_1 = 1.5$, $d_{31} = 0.25$, $a_{32} = 8$, $\delta_2 = 0.1$, $b_{32} = 0.05$, $\beta_2 = 3$, $\beta_{22} = 0.5$, $d_{33} = 0.55$, $c_2 = 0.2$, $T = 2$, $\gamma_1 = 0.65$, $\gamma_2 = 0.3$, $\epsilon_1 = 0.6$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.1$, $a_1 = 8$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.6$, and $P_1(0) = P_2(0) = P(0) = 0.5$, then $\min\{R_2, R_3\} = 0.3149 > 0$ and system (2.1) is permanent as shown in Figure 3.

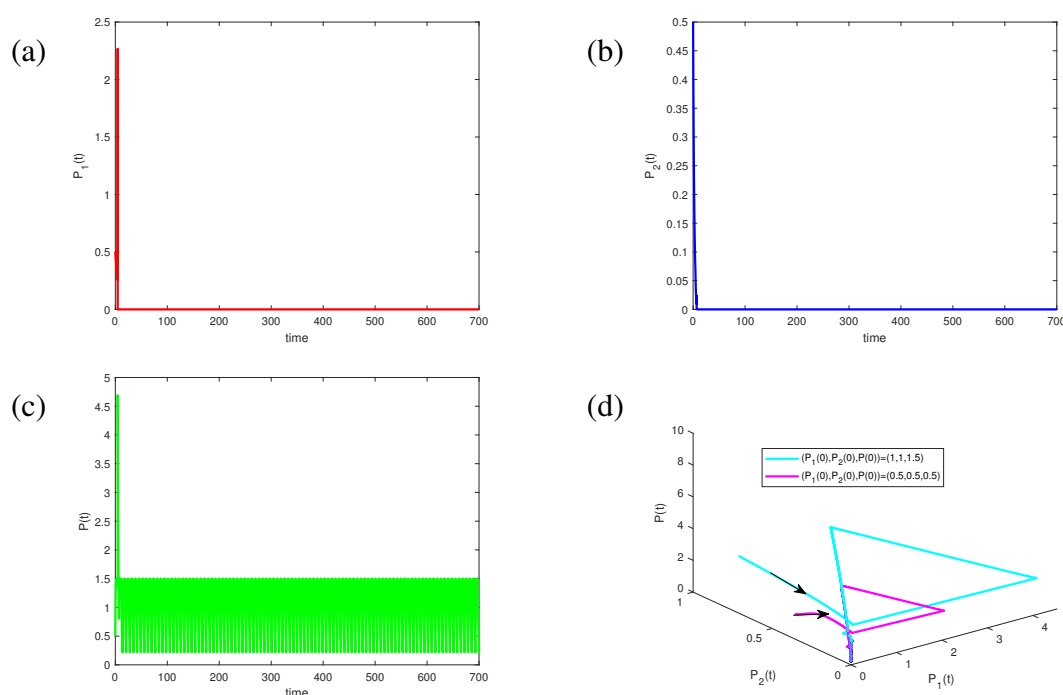


Figure 2. The prey-eradication boundary periodic solution $(0, 0, \overline{P(t)})$ of system (2.1) is GAS with $d_{11} = 0.15$, $d_{12} = 0.45$, $a_{13} = 3$, $b_{13} = 2$, $a_{21} = 2.3$, $\delta_1 = 4$, $b_{21} = 0.5$, $\beta_1 = 3.5$, $\beta_{11} = 6$, $d_{22} = 0.9$, $d_{23} = 0.65$, $c_1 = 1.5$, $d_{31} = 0.25$, $a_{32} = 0.15$, $\delta_2 = 6$, $b_{32} = 0.25$, $\beta_2 = 4.4$, $\beta_{22} = 2.5$, $d_{33} = 0.55$, $c_2 = 2$, $T = 7$, $\gamma_1 = 0.65$, $\gamma_2 = 0.3$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.1$, $a_1 = 8$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.6$, and $P_1(0) = P_2(0) = P(0) = 0.5$. (a) The time-series of $P_1(t)$. (b) The time-series of $P_2(t)$. (c) The time-series of $P(t)$. (d) The corresponding phase portrait of system (2.1).

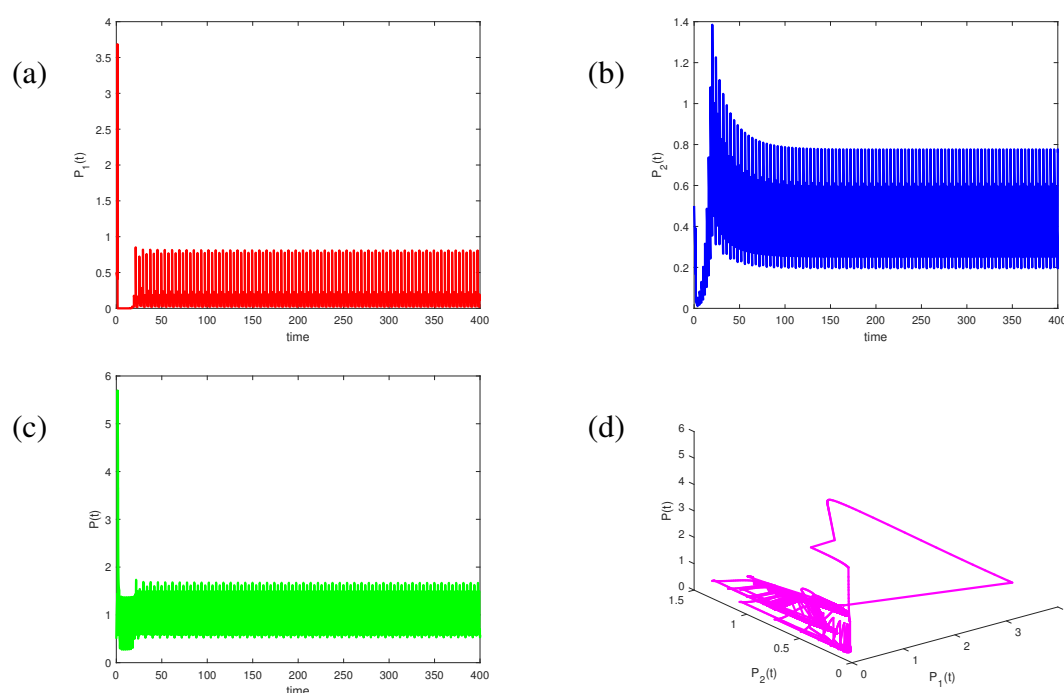


Figure 3. System (2.1) is permanent with $d_{11} = 0.15$, $d_{12} = 0.45$, $a_{13} = 3$, $b_{13} = 2$, $a_{21} = 2.3$, $\delta_1 = 0.5$, $b_{21} = 0.5$, $\beta_1 = 3$, $\beta_{11} = 0.5$, $d_{22} = 0.9$, $d_{23} = 0.65$, $c_1 = 1.5$, $d_{31} = 0.25$, $a_{32} = 8$, $\delta_2 = 0.1$, $b_{32} = 0.05$, $\beta_2 = 3$, $\beta_{22} = 0.5$, $d_{33} = 0.55$, $c_2 = 0.2$, $T = 2$, $\gamma_1 = 0.65$, $\gamma_2 = 0.3$, $\epsilon_1 = 0.6$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.1$, $a_1 = 8$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.6$, and $P_1(0) = P_2(0) = P(0) = 0.5$. (a) The time-series graph of $P_1(t)$. (b) The time-series of $P_2(t)$. (c) The time-series of $P(t)$. (d) The corresponding phase portrait of system (2.1).

To investigate the effects of fear effects, intraspecific cooperative hunting, and predator migration on system (2.1), and to explore its rich dynamical behaviors, we plot the bifurcation parameter diagram. First, we select a series of parameters: $d_{11} = 0.1$, $d_{12} = 0.2$, $a_{13} = 3$, $b_{13} = 1$, $a_{21} = 9$, $b_{21} = 0.25$, $\beta_1 = 3$, $\beta_{11} = 0.5$, $d_{22} = 0.3$, $d_{23} = 0.25$, $c_1 = 0.75$, $d_{31} = 0.25$, $a_{32} = 4$, $\delta_2 = 0.1$, $b_{32} = 0.05$, $\beta_2 = 3$, $\beta_{22} = 0.1$, $d_{33} = 0.55$, $c_2 = 0.2$, $T = 1.2$, $\gamma_1 = 0.65$, $\gamma_2 = 0.3$, $\epsilon_1 = 0.6$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.1$, $a_1 = 4$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.6$, and $P_1(0) = P_2(0) = P(0) = 0.5$. Then, the bifurcation parameter graphs with δ_1 , as shown in Figure 4, are also presented as the phase portraits corresponding to different values of δ_1 in Figure 5. Based on Figure 5, we can observe that as the fear effect intensifies, the system undergoes a series of dynamical transitions: from chaos to period-4 solutions, then to period-2 solutions, and finally to period-1 solutions. These justly explain that low levels of fear may drive prey population extinction. As fear intensity rises, prey can evolve adaptive anti-predator traits, which promote population stability. This demonstrates the dual influence of fear-mediated non-consumptive effects. Mild predation risk suppresses prey survival and reproduction, leading to population collapse. By contrast, elevated fear pressure triggers evolutionary adjustments in prey, buffering the negative consequences of predation risk and sustaining population persistence.

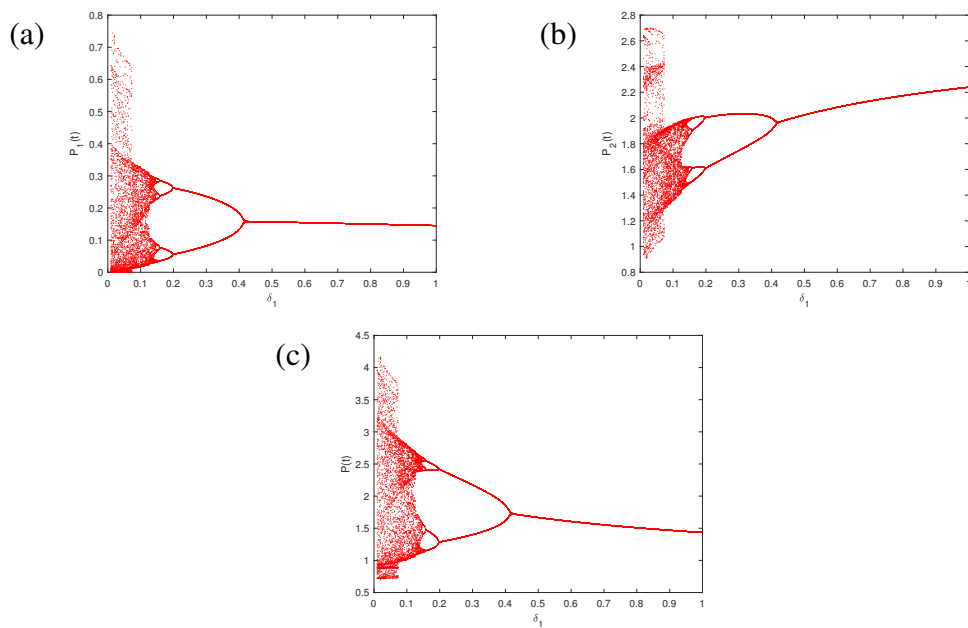


Figure 4. The bifurcation graphs of system (2.1) with $d_{11} = 0.1$, $d_{12} = 0.2$, $a_{13} = 3$, $b_{13} = 1$, $a_{21} = 9$, $b_{21} = 0.25$, $\beta_1 = 3$, $\beta_{11} = 0.5$, $d_{22} = 0.3$, $d_{23} = 0.25$, $c_1 = 0.75$, $d_{31} = 0.25$, $a_{32} = 4$, $\delta_2 = 0.1$, $b_{32} = 0.05$, $\beta_2 = 3$, $\beta_{22} = 0.1$, $d_{33} = 0.55$, $c_2 = 0.2$, $T = 1.2$, $\gamma_1 = 0.65$, $\gamma_2 = 0.3$, $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.15$, $\varepsilon_3 = 0.1$, $a_1 = 4$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.6$, $P_1(0) = P_2(0) = P(0) = 0.5$, and δ_1 varying from 0 to 1. (a) Bifurcation portrait of $P_1(t)$. (b) Bifurcation portrait of $P_2(t)$. (c) Bifurcation portrait of $P(t)$.

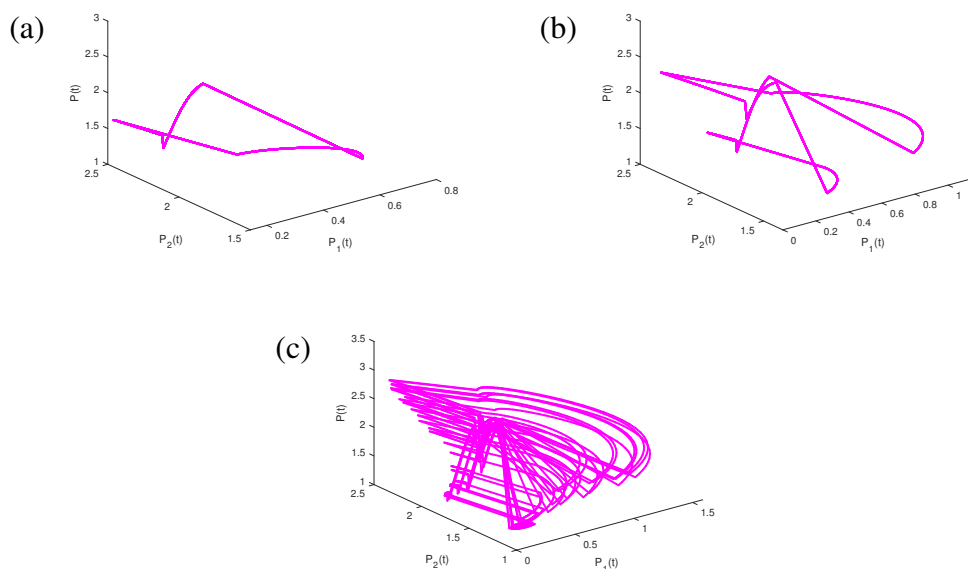


Figure 5. The phase portraits corresponding to Figure 4. (a) Period-1 solutions with $\delta_1 = 0.7$. (b) Period-2 solutions with $\delta_1 = 0.3$. (c) The chaotic attractor with $\delta_1 = 0.1$.

If we take $\delta_1 = 0.35$, varying ϵ_1 from 0.01 to 0.75, and keep the other parameters unchanged, then the bifurcation parameter graph versus ϵ_1 is shown in Figure 6. It can be seen from the figure that the smaller the loss of predators during migration, the more likely the prey in both patches will go extinct. Conversely, the greater the loss, the more likely the prey in both patches will persist. When $\epsilon_1 > 0.25$, the prey in the second patch persists, and when $\epsilon_1 > 0.43$, the prey in the first patch also persists. Figure 7 presents the phase portraits corresponding to different values of ϵ_1 . Figure 7 illustrates period-1 solutions, period-2 solutions, and chaos. These graphs also show that as predator loss intensifies during seasonal migration, prey populations gradually transition from extinction to stability, and eventually collapse into extinction once more. From an ecological conservation perspective, this indicates that we should provide appropriate protection methods for migrating predators in their migration process.

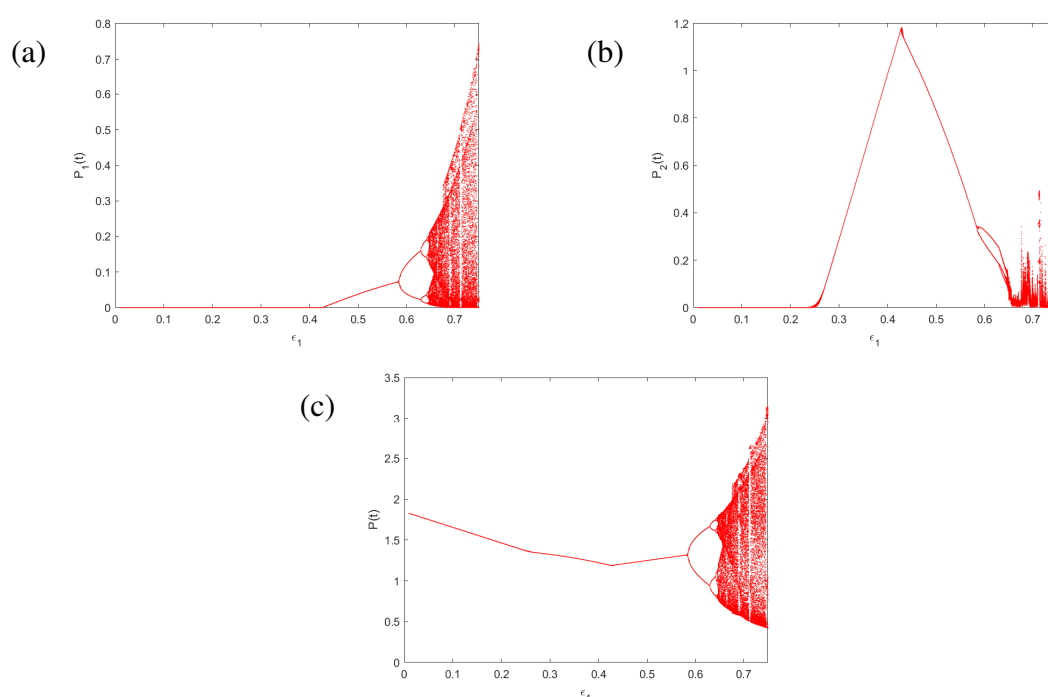


Figure 6. The bifurcation portraits of system (2.1) with $d_{11} = 0.1$, $d_{12} = 0.2$, $a_{13} = 3$, $b_{13} = 1$, $a_{21} = 9$, $b_{21} = 0.25$, $\beta_1 = 3$, $\beta_{11} = 0.5$, $d_{22} = 0.3$, $d_{23} = 0.25$, $c_1 = 0.75$, $d_{31} = 0.25$, $a_{32} = 4$, $\delta_1 = 0.35$, $\delta_2 = 0.1$, $b_{32} = 0.05$, $\beta_2 = 3$, $\beta_{22} = 0.5$, $d_{33} = 0.55$, $c_2 = 0.2$, $T = 2$, $\gamma_1 = 0.65$, $\gamma_2 = 0.3$, $\epsilon_2 = 0.15$, $\epsilon_3 = 0.1$, $a_1 = 4$, $b_1 = 0.1$, $a_2 = 1.5$, $b_2 = 0.6$, $P_1(0) = P_2(0) = P(0) = 0.5$, and ϵ_1 varying from 0.01 to 0.75. (a) Bifurcation portrait of $P_1(t)$. (b) Bifurcation portrait of $P_2(t)$. (c) Bifurcation portrait of $P(t)$.

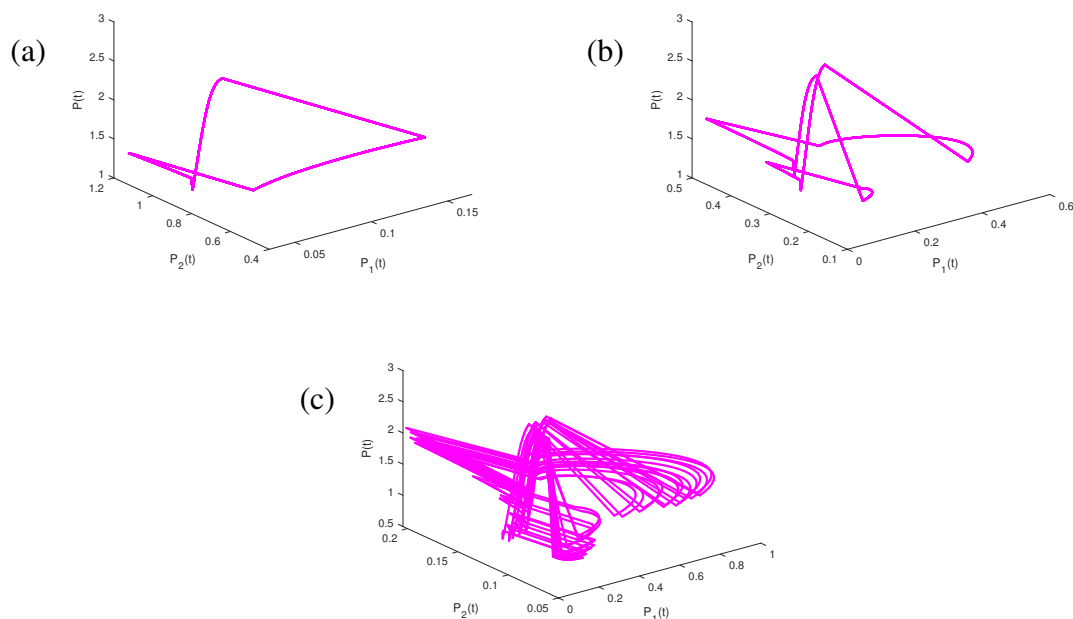


Figure 7. The phase graphs corresponding to Figure 6. (a) Period-1 solutions with $\epsilon_1 = 0.5$. (b) Period-2 solutions with $\epsilon_1 = 0.6$. (c) The chaotic attractor with $\epsilon_1 = 0.65$.

6. Conclusions

In this paper, we studied a periodically switched predator-prey model with seasonal birth, predation-induced fear, intraspecific cooperative hunting, and predator migrating for chasing food. Using impulsive theories of impulsive differential equations, we obtained the sufficient conditions for system persistence and the prey-eradication boundary periodic solution of (2.1) being globally asymptotically stable. Our results were verified by numerical simulations, and bifurcation diagrams revealed rich dynamical properties of system (2.1), including period-doubling bifurcation and chaos. Finally, through numerical analysis, the main results were verified as follows:

(i) If $R_1 < 0$ and $\max\{R_2, R_3\} < 0$ hold, the prey-eradication period solution $(0, 0, \overline{P(t)})$ is GAS (globally asymptotically stable).

(ii) If $\min\{R_2, R_3\} > 0$ holds, system (2.1) is permanent.

In future work, we can look for some strategies for the ecological conservation perspective via bifurcation graphs. Through numerical analysis, we can provide appropriate controlling of invaded migratory species for protecting local species. Additionally, it is essential to protect predators during their migration to safeguard the biodiversity of ecological systems. Then, the optimally conservational strategies under different situations should be explored.

Author contributions

Hui Jiao: Writing—original draft, Methodology, and Editing. Akanat Wetayawanich: Supervision and Simulations. Jianjun jiao: Writing—review. All authors have read and approved the final version of

the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that no Artificial Intelligence (AI) tools were used in the composition of this article.

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Conflict of interest

The authors declare no competing interests.

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