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Research article

Certain families of admissible functions defined via fuzzy differential subordination and a generalized Mittag-Leffler function

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Abstract: In the present paper, we employ the generalized Mittag-Leffler function to investigate several fuzzy differential subordination results associated with suitable families of admissible functions in the open unit disk. By utilizing a refined analytic framework, we derive new inclusion relationships and establish sufficient conditions for fuzzy subordinating functions defined via Mittag-Leffler-type operators. Furthermore, the obtained results unify and extend a number of earlier findings in the theory of fuzzy analytic functions. These developments provide a deeper insight into the interaction between generalized special functions and the structure of fuzzy differential subordinations, offering potential applications to broader subclasses of analytic and bi-univalent functions, as well as to various operator-defined families in geometric function theory.

Keywords: fuzzy set; fuzzy differential subordination; fuzzy best dominant; analytic functions; convolution; admissible functions
Mathematics Subject Classification: 30C20, 30C45, 30A20

1. Introduction and preliminaries

The notion of a fuzzy set was introduced by Zadeh in 1965 [28], and since then it has given rise to a rich and rapidly developing theory with a wide range of applications in science and engineering. Fuzzy set theory provides a flexible framework for modeling vagueness and uncertainty, and it has been successfully combined with many classical branches of analysis and applied mathematics.

In the context of geometric function theory, the method of differential subordination has proved to be a powerful tool for obtaining inclusion relationships and coefficient estimates for analytic functions. This concept, originally formulated by Miller and Mocanu [10], has motivated an extensive literature in which various authors have adapted and generalized their approach in different directions.

A significant extension of this concept to the fuzzy framework was proposed by Oros and Oros in [15], where they introduced a fuzzy analog of differential subordination. In a subsequent series of works [16–18], they developed the theory of fuzzy differential subordinations and related notions such as dominants and best dominants, as well as Briot–Bouquet-type fuzzy differential subordinations. These contributions established an important link between fuzzy set theory and geometric function theory.

An overview on the developments of this study is given in [7] and several other authors, have considered new classes of fuzzy differential subordinations, often in connection with different operators or subclasses of analytic functions. In this way, fuzzy differential subordination can be regarded as a natural extension of classical differential subordination, obtained by incorporating membership functions and fuzzy inclusion into the analytic framework. In [2], Altinkaya and Wanas presented some properties for fuzzy differential subordination associated with Wanas operator. Azzam et al. [5] discussed fuzzy spiral-like functions for linear operators. In 2023, Lupas et al. [9] combined quantum calculus applications with fuzzy set theory, while in 2024 Ali et al. [1] considered the fuzzy differential subordination for p-valent analytic functions related to a new generalized q-calculus operator. Also, some related findings are shown in [13, 14, 24–27].

We denote by $\mathcal{H}(\Omega)$ the class of all analytic functions in the open unit disk:

$$\Omega = \{\xi \in \mathbb{C} : |\xi| < 1\}.$$

We also write $\overline{\Omega} = \{\xi \in \mathbb{C} : |\xi| \leq 1\}$ for its closure.

For a positive integer n and $a \in \mathbb{C}$, we consider

$$\mathcal{H}[a, n] = \{\Xi \in \mathcal{H}(\Omega) : \Xi(\xi) = a + a_n\xi^n + a_{n+1}\xi^{n+1} + \ldots,\ \xi \in \Omega\}$$

and

$$\mathcal{A}_n = \{\Xi \in \mathcal{H}(\Omega) : \Xi(\xi) = a + a_n\xi^n + a_{n+1}\xi^{n+1} + a_{n+2}\xi^{n+2} + \ldots,\ \xi \in \Omega\},$$

with the usual notation $\mathcal{A}_1 = \mathcal{A}$ for the standard class of normalized analytic functions.

The classical Mittag-Leffler function $E_\alpha(\xi)$ ($\xi \in \mathbb{C}$) (see [11, 12]) is given by

$$E_\alpha(\xi) = \sum_{n=0}^{\infty} \frac{\xi^n}{\Gamma(\alpha n + 1)}, \qquad (\alpha \in \mathbb{C},\ \Re(\alpha) > 0).$$

The properties and generalizations of the Mittag-Leffler function have been extensively studied; see, for instance, [6, 8, 20, 21]. In particular, Srivastava and Tomovski [23] introduced the generalized function

$E^{\gamma,\lambda}_{\alpha,\beta}(\xi)$ ($\xi \in \mathbb{C}$) by

$$E^{\gamma,\lambda}_{\alpha,\beta}(\xi) = \sum_{n=0}^{\infty} \frac{(\gamma)_{n\lambda}\xi^n}{\Gamma(\alpha n + \beta)n!},$$

where $\alpha, \beta, \gamma \in \mathbb{C}$, $\Re(\alpha) > \max\{0, \Re(\lambda) - 1\}$, $\Re(\lambda) > 0$, and $(x)_n$ denotes the Pochhammer symbol defined by

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} 1, & (n = 0), \\ x(x+1)\ldots(x+n-1), & (n \in \mathbb{N}). \end{cases}$$

Let $\Xi_i \in \mathcal{A}$ ($i = 1, 2$) be defined by

$$\Xi_i(\xi) = \xi + \sum_{n=2}^{\infty} a_{n,i}\xi^n \quad (i = 1, 2).$$

The Hadamard product (or convolution) of Ξ_1 and Ξ_2 is given by

$$(\Xi_1 * \Xi_2)(\xi) = \xi + \sum_{n=2}^{\infty} a_{n,1}a_{n,2}\xi^n = (\Xi_2 * \Xi_1)(\xi).$$

In [4], Attiya introduced the operator $\mathcal{H}^{\gamma,\lambda}_{\alpha,\beta} : \mathcal{A} \to \mathcal{A}$ defined by

$$\mathcal{H}^{\gamma,\lambda}_{\alpha,\beta}\Xi(\xi) = Q^{\gamma,\lambda}_{\alpha,\beta}(\xi) * \Xi(\xi), \qquad \xi \in \Omega,$$

where

$$Q^{\gamma,\lambda}_{\alpha,\beta}(\xi) = \frac{\Gamma(\alpha+\beta)}{(\gamma)_\lambda}\left(E^{\gamma,\lambda}_{\alpha,\beta}(\xi) - \frac{1}{\Gamma(\beta)}\right),$$

and $\alpha, \beta, \gamma \in \mathbb{C}$, $\Re(\alpha) > \max\{0, \Re(\lambda) - 1\}$, $\Re(\lambda) > 0$.

A straightforward computation shows that

$$\mathcal{H}^{\gamma,\lambda}_{\alpha,\beta}\Xi(\xi) = \xi + \sum_{n=2}^{\infty} \frac{\Gamma(\gamma + n\lambda)\Gamma(\alpha+\beta)}{\Gamma(\gamma+\lambda)\Gamma(\beta+\alpha n)n!}a_n\xi^n. \tag{1.1}$$

From (1.1), it is easy to verify that

$$\xi\left(\mathcal{H}^{\gamma,\lambda}_{\alpha,\beta}\Xi(\xi)\right)' = \frac{\gamma+\lambda}{\lambda}\mathcal{H}^{\gamma+1,\lambda}_{\alpha,\beta}\Xi(\xi) - \frac{\gamma}{\lambda}\mathcal{H}^{\gamma,\lambda}_{\alpha,\beta}\Xi(\xi). \tag{1.2}$$

Classical subordination theory constitutes a fundamental tool in complex and geometric function theory, offering a systematic approach to relate analytic functions through dominance relations. Traditionally, subordination relies on a fixed analytic mapping that precisely controls the behavior of another function within a given domain. Although this framework is highly effective for idealized mathematical settings, it presumes exact functional dependencies and well-defined parameters. In contrast, many modern models, especially in applied contexts, involve uncertainty, vagueness, or imprecise information, which limits the descriptive capability of the classical subordination approach.

In practical systems, uncertainty often stems from incomplete data, observational inaccuracies, heterogeneous structures, or dynamics with memory effects. Owing to its deterministic and rigid nature, classical subordination theory is not sufficiently adaptable to describe such nonlocal or uncertain phenomena. These shortcomings have led to the introduction of generalized subordination frameworks and analytical methods capable of incorporating fractional-order behavior, operators of non-integer order, and fuzzy-type characteristics. Such extensions provide a more realistic representation of systems in which the governing behavior is influenced by distributed or hereditary mechanisms rather than being uniquely prescribed.

From an applied standpoint, generalized Mittag–Leffler functions arise as essential components in the analysis of fractional-order systems. As extensions of the classical exponential function, they serve a comparable role in fractional calculus, particularly in expressing solutions to fractional differential and integral equations. Owing to their ability to bridge power-law and exponential behaviors, these functions are well suited for describing anomalous transport phenomena, viscoelastic responses, memory-dependent electrical networks, and intricate relaxation dynamics.

We next recall several basic definitions related to the theory of fuzzy sets and fuzzy subordinations that will be needed in the subsequent section.

Definition 1.1. [10] Let $\mathbb{Q}$ denote the collection of functions q that are analytic and injective on the domain $\overline{\Omega} \setminus E(q)$, where

$$E(q) = \{\xi \in \partial\Omega : \lim_{z\to\xi} q(\xi) = \infty\},$$

and such that $q'(\xi) \neq 0$ for $\xi \in \partial\Omega \setminus E(q)$. The subclass of $\mathbb{Q}$ consisting of functions with prescribed value at the origin, $q(0) = \alpha$, will be denoted by $\mathbb{Q}(\alpha)$. In particular, we write $\mathbb{Q}(0) = \mathbb{Q}_0$ and $\mathbb{Q}(1) = \mathbb{Q}_1$.

Definition 1.2. [28] Let X be a non-empty set. A mapping $F : X \to [0, 1]$ is called a fuzzy subset of X. More precisely, a pair (A, F_A), where $F_A : X \to [0, 1]$ and

$$A = \{x \in X : 0 < F_A(x) \leq 1\} = \sup(A, F_A),$$

is referred to as a fuzzy subset of X. The function F_A is called the membership function associated with the fuzzy subset (A, F_A).

Definition 1.3. [15] Let (M, F_M) and (N, F_N) be two fuzzy subsets of X. We say that the fuzzy subsets M and N are equal if and only if $F_M(x) = F_N(x)$ for all $x \in X$, in which case we write $(M, F_M) = (N, F_N)$. We say that the fuzzy subset (M, F_M) is contained in the fuzzy subset (N, F_N) if and only if

$$F_M(x) \leq F_N(x), \quad x \in X,$$

and we denote this inclusion by $(M, F_M) \subseteq (N, F_N)$.

Definition 1.4. [15] Let $\mathbb{D} \subseteq \mathbb{C}$, let $\xi_0 \in \mathbb{D}$ be a fixed point, and let $\Xi, \Theta : \mathbb{D} \to \mathbb{C}$ be two functions. We say that Ξ is fuzzy subordinate to Θ, written $\Xi \prec_F \Theta$ or $\Xi(\xi) \prec_F \Theta(\xi)$, if the following conditions hold:

(1) $\Xi(\xi_0) = \Theta(\xi_0)$;
(2) $F_{\Xi(\mathbb{D})}(\Xi(\xi)) \leq F_{\Theta(\mathbb{D})}(\Theta(\xi))$ for all $\xi \in \mathbb{D}$,

where

$$\Xi(\mathbb{D}) = \sup(\Xi(\mathbb{D}), F_{\Xi(\mathbb{D})}) = \{\Xi(\xi) : 0 < F_{\Xi(\mathbb{D})}(\Xi(\xi)) \leq 1,\ \xi \in \mathbb{D}\}$$

and

$$\Theta(\mathbb{D}) = \sup(\Theta(\mathbb{D}), F_{\Theta(\mathbb{D})}) = \{\Theta(\xi) : 0 < F_{\Theta(\mathbb{D})}(\Theta(\xi)) \leq 1, \ \xi \in \mathbb{D}\}.$$

We note that the fuzzy subordinate relation $\prec_F$ is transitive, i.e., if $\Xi \prec_F \Theta$ and $\Theta \prec_F \Upsilon$, then we have $\Xi \prec_F \Upsilon$. Also, the fuzzy subordinate relation $\prec_F$ is antisymmetric, i.e., if $\Xi \prec_F \Theta$ and $\Theta \prec_F \Xi$, then we have $\Xi = \Upsilon$.

Definition 1.5. [16] Let $\Delta \subseteq \mathbb{C}$, and let $\psi : \mathbb{C}^3 \times \Omega \to \mathbb{C}$ be a given function. Assume that h is univalent in the domain Ω. Suppose that p is an analytic function on Ω which satisfies the (second-order) fuzzy differential subordination

$$F_{\psi(\mathbb{C}^3\times\Omega)}(\psi(p(\xi), \xi p'^2(\xi), \xi^2 p''(\xi) : \xi)) \leq F_{h(\Omega)}(h(\xi)), \tag{1.3}$$

that is,

$$\psi(p(\xi), \xi p'^2(\xi), \xi^2 p''(\xi) : \xi) \prec_F h(\xi), \quad \xi \in \Omega.$$

In this case, p is called a fuzzy solution of the fuzzy differential subordination. A univalent function q is said to be a fuzzy dominant of the fuzzy solutions of the fuzzy differential subordination (or simply a fuzzy dominant) if

$$p(\xi) \prec_F q(\xi), \quad \xi \in \Omega,$$

for every p that satisfies (1.3). A fuzzy dominant $\bar{q}$ is called the fuzzy best dominant of (1.3) if

$$\bar{q}(\xi) \prec_F q(\xi), \quad \xi \in \Omega,$$

for all fuzzy dominants q associated with (1.3).

Definition 1.6. [16] Let Δ be a subset of $\mathbb{C}$, let $q \in \mathbb{Q}$, and let n be a positive integer. The class of admissible functions $\Psi_n[\Delta, q]$ consists of all functions $\Psi : \mathbb{C}^3 \times \Omega \to \mathbb{C}$ that satisfy the admissibility condition:

$$F_\Delta(\Psi(r, s, t : \xi)) = 0,$$

whenever

$$r = q(\xi), \quad s = k\xi q'(\xi) \quad \text{and} \quad \Re\left\{\frac{t}{s} + 1\right\} \geq k\,\Re\left\{\frac{\xi q''(\xi)}{q'(\xi)} + 1\right\},$$

for $\xi \in \Omega$, $\xi \in \partial\Omega \setminus E(q)$, and $k \geq n$. We write $\Psi_1[\Delta, q]$ and simply $\Psi[\Delta, q]$ for the case $n = 1$.

Lemma 1.7. *[16] Let $\psi \in \Psi_n[\Delta, q]$ with $q(0) = a$. If $p \in \mathcal{H}[a, n]$ satisfies*

$$F_{\psi(\mathbb{C}^3\times\Omega)}(\psi(p(\xi), \xi p'^2(\xi), \xi^2 p''(\xi) : \xi)) \leq F_\Delta(\xi), \quad \xi \in \Omega,$$

then

$$F_{p(\Omega)}(p(\xi)) \leq F_{q(\Omega)}(q(\xi)),$$

i.e., $p(\xi) \prec_F q(\xi)$ for all $\xi \in \Omega$.

2. Main results

In this section, we derive several fuzzy differential subordination results involving the operator $\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}$ associated with the generalized Mittag-Leffler function defined in (1.1). We also identify conditions under which fuzzy best dominants arise.

Definition 2.1. Let Δ be a subset of $\mathbb{C}$ and let $q \in \mathbb{Q}_0 \cap \mathcal{H}$. The class $\Phi_A[\Delta, q]$ of admissible functions consists of all mappings $\phi : \mathbb{C}^3 \times \Omega \to \mathbb{C}$ for which the admissibility condition

$$F_\Delta(\phi(u, v, w : \xi)) = 0$$

holds whenever

$$u = q(\xi), \qquad v = \frac{k\xi q'(\xi) + \frac{\gamma}{\lambda} q(\xi)}{\frac{\gamma+\lambda}{\lambda}},$$

and

$$\Re\left\{\frac{\frac{(\gamma+\lambda)(1+\gamma+\lambda)}{\lambda^2} w - \frac{2\gamma+1}{\lambda}\left[\frac{(\gamma+\lambda)v-\gamma u}{\lambda}\right] - \frac{\gamma(\gamma+1)}{\lambda^2} u}{\left[\frac{(\gamma+\lambda)v-\gamma u}{\lambda}\right]}\right\} \geq k\,\Re\left\{\frac{\xi q''(\xi)}{q'(\xi)} + 1\right\},$$

for $\xi \in \Omega$, $\xi \in \partial\Omega \setminus E(q)$, and $k \geq 1$.

Theorem 2.2. *Let $\phi \in \Phi_A[\Delta, q]$. If $\Xi \in \mathcal{A}$ satisfies*

$$F_{\phi(\mathbb{C}^3\times\Omega)}\left(\phi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi), \mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi), \mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi) : \xi\right)\right) \leq F_\Delta(\xi), \tag{2.1}$$

then

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right) \leq F_{q(\Omega)}(q(\xi))$$

i.e.,

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi) \prec_F q(\xi).$$

Proof. Set

$$p(\xi) = \mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi). \tag{2.2}$$

From (1.2), we obtain

$$\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi) = \frac{\xi p'(\xi) + \frac{\gamma}{\lambda} p(\xi)}{\frac{\gamma+\lambda}{\lambda}}. \tag{2.3}$$

Further differentiation and substitution yield

$$\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi) = \frac{\xi^2 p''(\xi) + \left(1 + \frac{2\gamma+1}{\lambda}\right)\xi p'(\xi) + \frac{\gamma(\gamma+1)}{\lambda^2} p(\xi)}{\frac{(\gamma+\lambda)(1+\gamma+\lambda)}{\lambda^2}}. \tag{2.4}$$

We introduce the transformations from $\mathbb{C}^3$ to $\mathbb{C}$ by

$$u = r, \quad v = \frac{s + \frac{\gamma}{\lambda} r}{\frac{\gamma+\lambda}{\lambda}}, \quad w = \frac{t + \left(1 + \frac{2\gamma+1}{\lambda}\right)s + \frac{\gamma(\gamma+1)}{\lambda^2} r}{\frac{(\gamma+\lambda)(1+\gamma+\lambda)}{\lambda^2}}. \tag{2.5}$$

Define

$$\psi(r,s,t:\xi)=\phi(u,v,w:\xi)=\phi\left(r,\frac{s+\frac{\gamma}{\lambda}r}{\frac{\gamma+\lambda}{\lambda}},\frac{t+\left(1+\frac{2\gamma+1}{\lambda}\right)s+\frac{\gamma(\gamma+1)}{\lambda^2}r}{\frac{(\gamma+\lambda)(1+\gamma+\lambda)}{\lambda^2}}:\xi\right). \tag{2.6}$$

From (2.2)–(2.4), we see that

$$\psi(p(\xi),\xi p'^2(\xi),\xi^2p''(\xi):\xi)=\phi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi):\xi\right).$$

Thus, by (2.1),

$$F_{\psi(\mathbb{C}^3\times\Omega)}(\psi(p(\xi),\xi p'^2(\xi),\xi^2p''(\xi):\xi))\le F_{\Delta}(\xi).$$

Using (2.5), we find

$$\frac{t}{s}+1=\frac{\frac{(\gamma+\lambda)(1+\gamma+\lambda)}{\lambda^2}w-\frac{2\gamma+1}{\lambda}\left[\frac{(\gamma+\lambda)v-\gamma u}{\lambda}\right]-\frac{\gamma(\gamma+1)}{\lambda^2}u}{\left[\frac{(\gamma+\lambda)v-\gamma u}{\lambda}\right]}.$$

Hence, the admissibility condition for $\phi\in\Phi_A[\Delta,q]$ in Definition 2.1 is equivalent to the admissibility condition for ψ in Definition 1.6. Thus we have $\psi\in\Psi[\Delta,q]$ and, by Lemma 1.7,

$$F_{p(\Omega)}(p(\xi))\le F_{q(\Omega)}(q(\xi)),$$

or equivalently,

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)\le F_{q(\Omega)}(q(\xi)).$$

This proves that

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\prec_F q(\xi).$$

□

In the particular case where $\Delta\neq\mathbb{C}$ is a simply connected domain, we may represent Δ as $\Delta=h(\Omega)$ with h a conformal mapping of Ω onto Δ. In this situation, the class $\Phi_A[h(\Omega),q]$ will be denoted by $\Phi_A[h,q]$. The next theorem follows as a direct consequence of Theorem 2.2.

Theorem 2.3. *Let $\phi\in\Phi_A[\Delta,q]$. Suppose that $\Xi\in\mathcal{A}$, that $\psi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi):\xi\right)$ is analytic in Ω, and assume that*

$$F_{\phi(\mathbb{C}^3\times\Omega)}\left(\phi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi):\xi\right)\right)\le F_{h(\Omega)}(h(\xi)). \tag{2.7}$$

Then,

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)\le F_{q(\Omega)}(q(\xi)),$$

i.e.,

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\prec_F q(\xi).$$

By choosing $\phi(u,v,w:\xi)=1+\frac{v}{u}$ in Theorem 2.3, we obtain the following

Corollary 2.4. *Let $\phi\in\Phi_A[\Omega,q]$. Suppose $\Xi\in\mathcal{A}$ and that*

$$\frac{\frac{\gamma+\lambda}{\lambda}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)+\xi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)'+\frac{\gamma}{\lambda}\left(\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)\right)}{\frac{\gamma+\lambda}{\lambda}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)}$$

is analytic in Ω *and satisfies*

$$\frac{\frac{\gamma+\lambda}{\lambda}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)+\xi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)'+\frac{\gamma}{\lambda}\left(\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)\right)}{\frac{\gamma+\lambda}{\lambda}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)}\prec_F h(\xi).$$

Then,

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\prec_F q(\xi).$$

Our next result extends Theorem 2.2 to a situation in which the boundary behavior of q on $\partial\Omega$ may not be known.

Corollary 2.5. *Let* $\Delta\subset\mathbb{C}$*, and let* q *be univalent in* Ω *with* $q(0)=0$*. Let* $\phi\in\Phi_A[\Delta,q_\rho]$ *for some* $\rho\in(0,1)$*, where* $q_\rho(\xi)=q(\rho\xi)$*. If* $\Xi\in\mathcal{A}$ *satisfies*

$$F_{\phi(\mathbb{C}^3\times\Omega)}\left(\phi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi):\xi\right)\right)\leq F_\Delta(\xi),$$

then

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)\leq F_{q(\Omega)}(q(\xi)),$$

i.e.,

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\prec_F q(\xi).$$

Proof. From Theorem 2.2, we first obtain

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)\leq F_{q_\rho(\Omega)}(q_\rho(\xi)).$$

Since $q_\rho(\xi)=q(\rho\xi)$, we have $F_{q_\rho(\Omega)}=F_{q(\rho\Omega)}(q(\rho\xi))$ and $q_\rho(0)=q(0)$. Hence

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)\leq F_{q(\rho\Omega)}(q(\rho\xi)),$$

i.e.,

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\prec_F q(\rho\xi).$$

Letting $\rho\to1$ yields $\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\prec_F q(\xi)$. □

Theorem 2.6. *Let* h *and* q *be univalent in* Ω *with* $q(0)=0$ *and set* $h_\rho(\xi)=h(\rho\xi)$ *and* $q_\rho(\xi)=q(\rho\xi)$*. Let* $\phi:\mathbb{C}^3\times\Omega\to\mathbb{C}$ *satisfy one of the following conditions:*

(1) $\phi\in\Phi_A[h,q_\rho]$ *for some* $\rho\in(0,1)$*;*
(2) there exists $\rho_0\in(0,1)$ *such that* $\phi\in\Phi_A[h_\rho,q_\rho]$ *for all* $\rho\in(\rho_0,1)$*.*

If $\Xi\in\mathcal{A}$*, the function* $\phi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi),\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi):\xi\right)$ *is analytic in* Ω *and* Ξ *satisfies (2.7). Then,*

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right)\leq F_{q(\Omega)}(q(\xi))$$

i.e.,

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\prec_F q(\xi).$$

Proof. **Case 1.** The argument is essentially the same as in the proof of Corollary 2.5 and is therefore omitted.

Case 2. Let $p(\xi) = \mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)$, and define $p_\rho(\xi) = p(\rho\xi)$. Then,

$$F_{\phi(\mathbb{C}^3\times\Omega)}\left(\phi(p_\rho(\xi), \xi p'_\rho(\xi), \xi^2 p''_\rho(\xi) : \rho\xi)\right) = F_{\phi(\mathbb{C}^3\times\Omega)}\left(\phi(p(\rho\xi), \xi p'(\rho\xi), \xi^2 p''(\rho\xi) : \rho\xi)\right)$$
$$\leq F_{h_\rho(\Omega)}(h_\rho(\xi)).$$

Using Theorem 2.2 together with the observation that for any function w mapping Ω into Ω, we may replace ξ by $w(\xi)$; in particular, by taking $w(\xi) = \rho\xi$, we obtain $p_\rho(\xi) \prec_F q_\rho(\xi)$ for $\rho \in (\rho_0, 1)$. Letting $\rho \to 1$, we conclude that $p(\xi) \prec_F q(\xi)$, and thus

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi) \prec_F q(\xi).$$

□

Our next theorem identifies the fuzzy best dominant corresponding to the fuzzy differential subordination in (2.7).

Theorem 2.7. *Let h be univalent in Ω, and let $\phi : \mathbb{C}^3 \times \Omega \to \mathbb{C}$. Suppose that the differential equation*

$$\phi\left(q(\xi), \frac{\xi q'(\xi) + \frac{\gamma}{\lambda}q(\xi)}{\frac{\gamma+\lambda}{\lambda}}, \frac{\xi^2 q''(\xi) + \left(1 + \frac{2\gamma+1}{\lambda}\right)\xi q'(\xi) + \frac{\gamma(\gamma+1)}{\lambda^2}q(\xi)}{\frac{(\gamma+\lambda)(1+\gamma+\lambda)}{\lambda^2}} : \xi\right) = h(\xi) \tag{2.8}$$

has a solution q with $q(0) = 0$ satisfying one of the following conditions:

(1) $q \in \mathbb{Q}_0$ and $\phi \in \Phi_A[h, q]$;
(2) q is univalent in Ω and $\phi \in \Phi_A[h, q_\rho]$ for some $\rho \in (0, 1)$;
(3) q is univalent in Ω, and there exists $\rho_0 \in (0, 1)$ such that $\phi \in \Phi_A[h_\rho, q_\rho]$ for all $\rho \in (\rho_0, 1)$.

If $\Xi \in \mathcal{A}$, the function $\phi\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi), \mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi), \mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi) : \xi\right)$ is analytic in Ω and Ξ satisfies (2.7). Then,

$$F_{\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi\right)(\Omega)}\left(\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)\right) \leq F_{q(\Omega)}(q(\xi)),$$

i.e.,

$$\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi) \prec_F q(\xi),$$

and q is the fuzzy best dominant.

Proof. By Theorems 2.3 and 2.6, we know that q is a fuzzy dominant of (2.7). Since q is also a solution of (2.8), it must be dominated by every fuzzy dominant of (2.7). Consequently, q is the fuzzy best dominant of (2.7). □

Definition 2.8. Let Δ be a subset of $\mathbb{C}$, and let $q \in \mathbb{Q}_1 \cap \mathcal{H}$. The class of admissible functions $\Phi_{A^*}[\Delta, q]$ consists of all functions $\phi : \mathbb{C}^3 \times \Omega \to \mathbb{C}$ that satisfy the admissibility condition:

$$F_\Delta\left(\phi(u, v, w : \xi)\right) = 0,$$

whenever

$$u = q(\xi), \quad v = \frac{\lambda k\xi q'^2(\xi) + q(\xi)}{(\gamma + \lambda + 1)q(\xi)}$$

and

$$\Re\left\{\frac{(\gamma+\lambda+2)uvw}{(\gamma+\lambda+2)uv-(\gamma+\lambda)u^2+u}+\frac{(\gamma+\lambda+2)^2uv-2(\gamma+\lambda)u-1}{\lambda}\right\}\geq k\,\Re\left\{\frac{\xi q''(\xi)}{q'(\xi)}+1\right\},$$

where $\xi\in\Omega$, $\xi\in\partial\Omega\setminus E(q)$, and $k\geq 1$.

Theorem 2.9. *Let $\phi\in\Phi_{A^*}[\Delta,q]$. If $\Xi\in\mathcal{A}$ satisfies*

$$F_{\phi(\mathbb{C}^3\times\Omega)}\left(\phi\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+3,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}:\xi\right)\right)\leq F_{\Delta}(\xi),\tag{2.9}$$

then

$$F_{\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\right)(\Omega)}\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\right)\leq F_{q(\Omega)}(q(\xi)),$$

i.e.,

$$\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\prec_F q(\xi).$$

Proof. Let

$$p(\xi)=\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}.\tag{2.10}$$

Using (1.2) and (2.10), we deduce

$$\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}=\frac{\lambda\xi p'^2(\xi)+p(\xi)}{p(\xi)(\gamma+\lambda+1)}.\tag{2.11}$$

A further computation leads to

$$\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+3,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}=\frac{\lambda\xi^2p''(\xi)+[2(\gamma+\lambda)p(\xi)+\lambda+1]\,\xi p'(\xi)}{\lambda\xi p'^2(\xi)+p(\xi)}-(\gamma+\lambda+2)\xi p'(\xi).\tag{2.12}$$

We now define transformations from $\mathbb{C}^3$ to $\mathbb{C}$ by

$$u=r,\quad v=\frac{\lambda s+(\gamma+\lambda)r^2+r}{(\gamma+\lambda+2)r},\quad w=\frac{\lambda t+[2(\gamma+\lambda)r+\lambda+1]\,s}{\lambda s+(\gamma+\lambda)r^2+r}-(\gamma+\lambda+2)s.$$

Let

$$\begin{aligned}\psi(r,s,t:\xi)&=\phi(u,v,w:\xi)\\&=\phi\left(r,\frac{\lambda s+(\gamma+\lambda)r^2+r}{(\gamma+\lambda+2)r},\frac{\lambda t+[2(\gamma+\lambda)r+\lambda+1]\,s}{\lambda s+(\gamma+\lambda)r^2+r}-(\gamma+\lambda+2)s:\xi\right).\end{aligned}\tag{2.13}$$

From (2.10)–(2.12), we have

$$\psi\left(p(\xi),\xi p'^{2}(\xi),\xi^{2}p''(\xi):\xi\right)=\phi\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+3,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}:\xi\right).$$

Consequently, (2.9) can be rewritten as

$$F_{\phi(\mathbb{C}^3\times\Omega)}\left(\psi\left(p(\xi),\xi p'^{2}(\xi),\xi^{2}p''(\xi):\xi\right)\right)\leq F_{\Delta}(\xi).$$

It remains to show that the admissibility condition for $\phi\in\Phi_{A^*}[\Delta,q]$ is equivalent to that for ψ from Definition 1.6. Note that

$$\frac{t}{s}+1=\frac{(\gamma+\lambda+2)uvw}{(\gamma+\lambda+2)uv-(\gamma+\lambda)u^2+u}+\frac{(\gamma+\lambda+2)^2uv-2(\gamma+\lambda)u-1}{\lambda}.$$

Thus $\psi\in\Psi[\Delta,q]$, and by Lemma 1.7,

$$F_{p(\Omega)}(p(\xi))\leq F_{q(\Omega)}(q(\xi)),$$

or equivalently,

$$F_{\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\right)(\Omega)}\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\right)\leq F_{q(\Omega)}(q(\xi));$$

which yields

$$\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\prec_F q(\xi).$$

□

When $\Delta\neq\mathbb{C}$ is a simply connected domain and can be written as $\Delta=h(\Omega)$ for some conformal mapping h of Ω onto Δ, the class $\Phi_{A^*}[h(\Omega),q]$ will be denoted by $\Phi_{A^*}[h,q]$. The next theorem follows directly from Theorem 2.9.

Theorem 2.10. *Let* $\phi\in\Phi_{A^*}[\Delta,q]$. *If* $\Xi\in\mathcal{A}$ *is such that* $\phi\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+3,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}:\xi\right)$ *is analytic in* Ω *and*

$$F_{\phi(\mathbb{C}^3\times\Omega)}\left(\phi\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)},\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+3,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}:\xi\right)\right)\leq F_{h(\Omega)}(h(\xi)),$$

then

$$F_{\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\right)(\Omega)}\left(\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\right)\leq F_{q(\Omega)}(q(\xi)),$$

i.e.,

$$\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)}\prec_F q(\xi).$$

If we take $\phi(u,v,w:\xi)=uv$ in Theorem 2.10, we obtain the following consequence.

Corollary 2.11. *Let $\phi \in \Phi_{A^*}[\Delta, q]$. Suppose $\Xi \in \mathcal{A}$, and that $\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}$ is analytic in Ω and satisfies*

$$\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+2,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)} \prec_F h(\xi).$$

Then,

$$\frac{\mathcal{H}_{\alpha,\beta}^{\gamma+1,\lambda}\Xi(\xi)}{\mathcal{H}_{\alpha,\beta}^{\gamma,\lambda}\Xi(\xi)} \prec_F q(\xi).$$

3. Conclusions

In this study, the results presented are novel by using the concept of fuzzy subordination; we have submitted suitable families of admissible functions in the open unit disk, which are characterized using the generalized Mittag-Leffler function. We get several fuzzy differential subordination results associated with these families. As future research directions, the contents of this paper on fuzzy differential subordination could inspire further research related to other families.

It should be remarked that, there are many recent investigations dealing with some of the presented of our presentation in this paper: the fuzzy differential superordination theory introduced in [3], sandwich-type results for the Mittag-Leffler type Pascal distribution presented in [22], and also the study considering third-order fuzzy differential subordination theory introduced in [19].

Author contributions

Conceptualization, D. Alhwikem and A. K. Wanas; methodology, B. A. Abd, A. Amourah and S. M. El-Deeb; validation, D. Alhwikem and A. K. Wanas; formal analysis, A. Amourah; investigation, D. Alhwikem; writing—original draft preparation, A. K. Wanas and B. A. Abd; writing—review and editing, A. Amourah and S. M. El-Deeb; supervision, A. Amourah. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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