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On the borderline of fields and hyperfields, part II – Enumeration and classification of the hyperfields of order 7

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Abstract: The quotient hyperfield is a landmark on the borderline of fields and hyperfields. In this paper, which is the second part of our previously published paper, all the hyperfields of order 7 are constructed, enumerated, and presented. While doing so, an important family of 7-element canonical hypergroups was revealed. The study of these hyperfields proved the existence of both quotient and non-quotient ones among them. Their construction became feasible because it is based on a new definition of the hyperfield with less axioms, which is introduced in this paper following our proof that the axiom of reversibility can derive from the remaining axioms of the hyperfield. Hence, the processing power needed for a computer to test whether a structure is a hyperfield or not, is much less. This paper also presents properties and provides examples of skew hyperfields, strongly canonical hyperfields/hyperrings, and superiorly canonical hyperfields/hyperrings that wrap up and complete the previously published first part's conclusions and results.

Keywords: hyperfield; field; hyperring; hypergroup; canonical hypergroup
Mathematics Subject Classification: 12E20, 12K99, 16Y20, 20N20

1. Introduction

This paper is the succession of our previous paper [1], and, as such refers to the detailed introduction of [1] when introducing the topics. As described, hypercompositional Algebra began its existence in mathematics with the hypergroup, which was introduced by F. Marty in 1934 [2], and advanced with the hyperfield, which was introduced in 1956 by M. Krasner [3].

The basic concept in hypercompositional Algebra is the hypercomposition. A hypercomposition or a hyperoperation over a non-empty set is a mapping from the cartesian product $E \times E$ into the power set $P(E)$ of E. A *hypergroup* is a non-empty set E enriched with a hypercomposition "·" which satisfies the following two axioms:

(i) The axiom of *associativity*:

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c \text{, for all } a, b, c \in E .$$

(ii) The axiom of *reproductivity*:

$$a \cdot E = E \cdot a = E \text{, for all } a \in E .$$

Papers [4,5] present, in detail, that the group is defined with exactly the same axioms as above. Namely, a group is a non-empty set E that is enriched with a composition (i.e., a mapping from the cartesian product $E \times E$ into the set E) that satisfies axioms (i) and (ii).

If "·" is an internal composition on a set E and A, B are subsets of E, and $A \cdot B$ signifies the set $\{ a \cdot b \mid (a,b) \in A \times B \}$ while if "·" is a hypercomposition, then $A \cdot B$ is the union $\bigcup_{(a,b) \in A \times B} a \cdot b$. $A \cdot b$ and $a \cdot B$ have the same meaning as $A \cdot \{b\}$ and $\{a\} \cdot B$, respectively. In general, the singleton $\{a\}$ is identified with its member a.

Theorem 1. *If either* $A = \varnothing$ *or* $B = \varnothing$, *then* $AB = \varnothing$.

Theorem 1, together with the two axioms of the hypergroup, yields the following theorem:

Theorem 2. [5,6] *The result of the hypercomposition of any two elements in a hypergroup is always non-void.*

Definition 1. [3,7] A *hyperfield* is an algebraic structure $(H, +, \cdot)$ where H is a non-empty set, "·" is a composition on H, and "+" is a hypercomposition on H, which satisfies the axioms:

I. *Multiplicative axiom*
$H = H^* \cup \{0\}$ where $(H^*, \cdot)$ is a multiplicative group and 0 is a bilaterally absorbing element of H, i.e., $0 \cdot a = a \cdot 0 = 0$, for all $a \in H$.

II. *Additive axioms*

i. Associativity:
$a + (b + c) = (a + b) + c$, for all $a, b, c \in H$.

ii. Commutativity:
$a + b = b + a$, for all $a, b \in H$.

iii. For every $a \in H$, there exists one and only one $a' \in H$ such that $0 \in a + a'$. a' is written $-a$ and called the opposite of a; moreover, instead of $a + (-b)$, we write $a - b$.

iv. Reversibility:
If $a \in b + c$, then $c \in a - b$.

III. *Distributive axiom*

$$a\cdot(b+c)=a\cdot b+a\cdot c,\quad (b+c)\cdot a=b\cdot a+c\cdot a\text{, for all } a,b,c\in H.$$

If the multiplicative axiom ***I*** is replaced by the axiom:

I′. $H=H^{*}\cup\{0\}$ where $(H^{*},\cdot)$ is a multiplicative semigroup and 0 is a bilaterally absorbing element of *H,*

then a more general structure is obtained which is called *hyperring* [7].

2. The minimization of the axiomatic framework that defines the hyperfields

Numerous papers have been published regarding the above two hypercompositional structures, as noted in [1]. Some of the most recent works, most of which were published after [1], are those cited in [8–22]. All these papers adopt the definition of a hyperfield as it was originally introduced by M. Krasner, namely the one given in Definition 1. However, the attempts to enumerate the hyperfields quickly led to bigger cardinalities that were difficult to handle by the computational capabilities of our computers. Hence, we decided to reconsider the independence of the axioms in the hope of restricting their number. This was achieved, as demonstrated below.

Theorem 3. [4,23] *A non-empty set H enriched with the additive axioms II is a hypergroup.*

Proof. Associativity holds. Next, observe that $a+b\neq\varnothing$ for every a, $b\in H$. Indeed, if $a+b=\varnothing$, then $a+(b-b)=(a+b)-b=\varnothing-b=\varnothing$, which would imply $a\in\varnothing$, a contradiction. Thus, for every a, $b\in H$ there exists $x\in H$ such that $x\in b-a$ or; equivalently, by reversibility, $b\in a+x$. Therefore, $H\subseteq a+H$. Moreover, it is clear that $a+H\subseteq H$. Consequently, $a+H=H$. Hence, reproductivity is also valid. Thus, *H* is a hypergroup. □

The special hypergroup of the hyperfield was named the *canonical hypergroup* by J. Mittas who studied it in depth and presented his research results through a multitude of papers (e.g., [23–25]).

Theorem 4. [26] *In a hyperfield H, the equality* $a+0=a$ *holds for all a in H.*

Given that the proof of this theorem relies solely on the additive axioms II of the hyperfield, the conclusion remains valid for the canonical hypergroups as well.

Theorem 5. *In a canonical hypergroup H, the axiom of reversibility is equivalent to the validity of the equality*

$$-(a+b)=-a-b$$

for all a,b in H.

Proof. Suppose that the equality $-(a+b)=-a-b$ is valid for all *a,b* in *H*. If $a\in b+c$, then equivalently, we have that

$$0\in a-(b+c)\ \text{ or }\ 0\in a+(-b-c)\ \text{ or }\ 0\in(a-b)-c\,.$$

Therefore, $c\in a-b$, so the reversibility holds. Vice versa now. Suppose that the reversibility holds, and $w\in-(a+b)$. Then, $0\in w+(a+b)$. Applying the associativity and the reversibility, we have:

$$0\in w+(a+b)\Rightarrow 0\in(w+a)+b\Rightarrow -b\in w+a\Rightarrow w\in -a-b .$$

Thus, $-(a+b)\subseteq -a-b$. Next, suppose that $w\in -a-b$. Then, we have the sequence of implications:

$$w\in -a-b\Rightarrow -a\in w+b\Rightarrow 0\in(w+b)+a\Rightarrow 0\in w+(b+a)\Rightarrow w\in-(a+b).$$

Thus, $-a-b\subseteq-(a+b)$. Therefore, $-(a+b)=-a-b$. □

In the hyperfields, due to distributivity, the equality $-(a+b)=-a-b$ is always valid. Therefore, Theorem 5 simplifies the axiomatic structure of hyperfields, as the reversibility property in the additive axioms can be derived from the remaining axioms. Hence, we have the following Definition:

Definition 2. A *hyperfield* is an algebraic structure $(H,+,\cdot)$, where H is a non-empty set, "$\cdot$" is a composition on H, and "+" is a hypercomposition on H that satisfies the axioms:

I. *Multiplicative axiom*
$H=H^*\cup\{0\}$, where $(H^*,\cdot)$ is a multiplicative group and 0 is a bilaterally absorbing element of H, i.e., $0\cdot a=a\cdot 0=0$, for all $a\in H$.

II. *Additive axioms*

i. Associativity:
$a+(b+c)=(a+b)+c$, for all $a,b,c\in H$.

ii. Commutativity:
$a+b=b+a$, for all $a,b\in H$.

iii. For every $a\in H$, there exists one and only one $a'\in H$ such that $0\in a+a'$. a' is written $-a$ and called the opposite of a; moreover, instead of $a+(-b)$, we write $a-b$.

III. *Distributive axiom*

$$a\cdot(b+c)=a\cdot b+a\cdot c,\quad (b+c)\cdot a=b\cdot a+c\cdot a,\quad \text{for all } a,b,c\in H.$$

The elimination of one redundant axiom (reversibility) makes the verification that a structure is a hyperfield more concise with proofs that do not include extra or unnecessary steps, which facilitates the development of computer packages capable of performing it with reduced computational resources. Thus, based on the above definition, it was feasible to enumerate the hyperfields of order 7 and present the 277 non-isomorphic hyperfields in this paper's last Section. The notation used is HF_7^k, $1\le k\le 277$ where HF_7^1 is the field Z_7.

Special notation: In the following pages, in addition to the typical algebraic notations, we use Krasner's notation for the complement and the difference [27]. Thus, $A\cdot\cdot B$ denotes the set of elements that are in A, but not in B. If K is a field or a hyperfield then, K^* denotes the set $K\cdot\cdot\{0\}$.

3. The skew hyperfield

The term division rings, also called skew fields, is used to define nontrivial rings in which every nonzero element has a multiplicative inverse. Historically, division rings were sometimes referred to as fields, while fields were called "commutative fields". Krasner defined the hyperfield considering that its multiplicative group is not necessarily commutative [3,7]. However, in the relevant bibliography, the term hyperfield is used, almost from the very first time, to indicate that its multiplicative group is commutative. Consequently, it is necessary to introduce the term "skew hyperfield" in order to refer to hypercompositional structures that are analogous to skew fields. The constructions of hyperfields given in [26,28] enrich a multiplicative group, which need not be commutative, with a hypercomposition. Thus, we have the following three classes of skew hyperfields:

Theorem 6. *Let G be a multiplicative group and 0 a bilaterally absorbing element, that is* $x\cdot 0=0\cdot x=0$, *for all* $x\in G\cup\{0\}$. *Then,* $G\cup\{0\}$ *equipped with the hypercompositions*:

$$(1)\quad \left|\begin{array}{ll} x+y=\{x,y\}, & \textit{for all } x,y\in G, \textit{ with } x\neq y \\ x+0=0+x=x, & \textit{for all } x\in G\cup\{0\} \\ x+x=G\cup\{0\}, & \textit{for all } x\in G \end{array}\right.$$

$$(2)\quad \left|\begin{array}{ll} x+y=\{x,y\}, & \textit{for all } x,y\in G, \textit{ with } x\neq y \\ x+0=0+x=x, & \textit{for all } x\in G\cup\{0\} \\ x+x=\left[G\cup\{0\}\right]\cdot\cdot\{x\}, & \textit{for all } x\in G \end{array}\right.$$

and

$$(3)\quad \left|\begin{array}{ll} x+y=G\cdot\cdot\{0,x,y\}, & \textit{for all } x,y\in G, \textit{ with } x\neq y \\ x+0=0+x=x, & \textit{for all } x\in G\cup\{0\} \\ x+x=\{0,x\}, & \textit{for all } x\in G \end{array}\right.$$

creates three non-isomorphic skew hyperfields.

For the sake of convenience, these hyperfields are denoted by $SHF_1(G)$, $SHF_2(G)$ and $SHF_3(G)$, respectively. The constructions of $SHF_1(G)$, $SHF_2(G)$ and $SHF_3(G)$ reveal that in the case of hyperfields, the equivalent of Wedderburn's little theorem [29] does not apply, i.e., although every finite skew field is commutative, there exist non-commutative finite hyperfields. Indeed, when the constructions of the above theorem are applied to finite non-commutative multiplicative groups, they produce non-commutative finite hyperfields. For instance, the dihedral group D_3 is the non-Abelian group having the smallest group order. Therefore, $SHF_1(D_3)$, $SHF_2(D_3)$ and $SHF_3(D_3)$ are three skew hyperfields having the smallest order.

Furthermore, both Construction I in [28] and Proposition 1 in [30] do not require the multiplicative group to be abelian. As a result, Theorem 7 holds.

Theorem 7. *Let G be a non-unitary multiplicative group and let* $(H^*,\cdot)$ *be its direct product with the multiplicative group* $\{-1\ ,1\}$. *Consider the set* $H = H^* \cup \{0\}$, *where 0 is a bilaterally absorbing element in H, i.e.* $0w=w0=0$, *for all* $w\in H$. *Then, H, equipped with the hypercompositions:*

$$(1)\quad \left| \begin{array}{ll} (x,i)\ \hat{+}\ (y,j) = \{(x,i),(y,j)\}, & \text{if } (y,j) \neq (x,-i) \\ (x,i)\ \hat{+}\ (x,-i) = H, & \text{for all } (x,i) \in H^* \\ (x,i)\ \hat{+}\ 0 = 0\ \hat{+}\ (x,i) = (x,i) \text{ and } 0\hat{+}0=0, & \text{for all } (x,i) \in H^* \end{array} \right.$$

$$(2)\quad \left| \begin{array}{ll} (x,i)\ \hat{+}\ (w,j) = \{(x,i),(w,j),(x,-i),(w,-j)\}, & \text{if } (w,j) \neq (x,i),\ (x,-i) \\ (x,i)\ \hat{+}\ (x,i) = H \cdot\cdot \{(x,i),(x,-i),0\}, & \text{for all } (x,i) \in H^* \\ (x,i)\ \hat{+}\ (x,-i) = H \cdot\cdot \{(x,i),(x,-i)\}, & \text{for all } (x,i) \in H^* \\ (x,i)\ \hat{+}\ 0 = 0\ \hat{+}\ (x,i) = (x,i) \quad \text{and} \quad 0\hat{+}0=0, & \text{for all } (x,i) \in H^* \end{array} \right.$$

creates two non-isomorphic skew hyperfields.

These hyperfields are denoted by $SHF_4(G)$ and $SHF_5(G)$, respectively. Furthermore, Construction II from [28] can be applied to a skew field or a skew hyperfield; therefore, the following theorems hold:

Theorem 8. *Let* $(K,+,\cdot)$ *be a skew field. If we define the hypercomposition* $\dot{+}$ *on F as follows:*

$$\begin{array}{ll} x \dot{+} y = \{ x, y, x+y \}, & \text{if } y \neq -x,\ x,\ y \neq 0; \\ x \dot{+} (-x) = K, & \text{for all } x\in K^*; \\ x \dot{+} 0 = 0 \dot{+} x = x, & \text{for all } x\in K. \end{array}$$

Then, $(K,\dot{+},\cdot)$ *is a skew hyperfield.*

Theorem 9. *Let* $(H,+,\cdot)$ *be a skew hyperfield. If we define a new hypercomposition* «$\dot{+}$» *on H as follows:*

$$\begin{array}{ll} x \dot{+} y = \{x, y\} \cup x+y, & \text{for all } x,y \in H^*, \text{ with } y \neq -x; \\ x \dot{+} (-x) = H, & \text{for all } x\in H^*; \\ x \dot{+} 0 = 0 \dot{+} x = x, & \text{for all } x\in H. \end{array}$$

Then, $(H,\dot{+},\cdot)$ *is a skew hyperfield.*

As with the hyperfields, the skew hyperfields of Theorems 8 and 9 are constructed over skew fields and hyperfields using an extensive enlargement of their composition or hypercomposition ([5], Example 8) and they will be called *augmented skew hyperfields* in accordance with the terminology established in [1]. In the following, the term *hyperfield* is used to indicate that its multiplicative group is commutative.

4. The quotient hyperfield/hyperring and the non-quotient hyperfields/hyperrings

M. Krasner, generalizing his earlier construction of the residual hyperfield [3], developed the quotient hyperfield and quotient hyperring, built upon a field and a ring, respectively [7]. While this construction is presented in detail in [1], key elements are recalled here to ensure the self-contained nature of this work.

Let *F* be a field and *G* a subgroup of *F*'s multiplicative group *F**. Then, the multiplicative cosets modulo *G* form a partition of *F*. Krasner observed that the product of two such cosets, viewed as subsets of *F*, is again a coset modulo *G*, while their sum is a union of such cosets. Subsequently, he proved that the set *F/G* consisting of the equivalence classes under this partition becomes a hyperfield if the multiplication and the addition are defined as follows:

$$xG \cdot yG = xyG;$$

$$xG \dagger yG = \{(xp + yq)G \mid p, q \in G\}$$

for all $xG, yG \in F/G$.

Moreover, Krasner showed that if *R* is a ring and *G* is a normal subgroup of its multiplicative semigroup, then the above construction gives a hyperring [7]. Ch. Massouros in [30] generalized this construction using multiplicative subgroups that are not necessarily normal. More precisely, he showed that in rings, there exist multiplicative subgroups *G* that satisfy the property $xG \cdot yG = xyG$, even when they are not normal.

A crucial step toward establishing the independence of the theory of hyperfields and hyperrings from the corresponding theory of fields and rings was the discovery of hyperfields and hyperrings that do not belong to the class of quotient hyperfields or hyperrings. The existence of such structures was proved by Ch. Massouros in [26,30,31]. The following are the main theorems that demonstrate the existence of non-quotient hyperfields and hyperrings.

Theorem 10. [30] *Let Θ be a multiplicative group that has more than two elements and let* $(K^*, \cdot)$ *be its direct product with the multiplicative group* $\{-1,1\}$. *Consider the set* $K = K^* \cup \{0\}$, *where 0 is a bilaterally absorbing element in K, i.e.,* $0w=w0=0$, *for all* $w \in K$. *The following hypercomposition is introduced in K:*

$$(x,i) \dotplus (y,j) = \{(x,i), (y,j), (x,-i), (y,-j)\}, \text{ if } (y,j) \neq (x,i), (x,-i);$$

$$(x,i) \dotplus (x,i) = K \dot{-} \{(x,i), (x,-i), 0\};$$

$$(x,i) \dotplus (x,-i) = K \dot{-} \{(x,i), (x,-i)\};$$

$$(x,i) \dotplus 0 = 0 \dotplus (x,i) = (x,i) \text{ and } 0 \dotplus 0 = 0.$$

Then, the triplet $K(\Theta) = (K, \dotplus, \cdot)$ *is a hyperfield that does not belong to the class of quotient hyperfields when Θ is a periodic group.*

Corollary 1. *The hyperfield* HF_7^{81} *(Table 1) is not a quotient hyperfield.*

Table 1. The canonical hypergroup of the non-quotient hyperfield HF_7^{81}.

HF_7^{81}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}

Theorem 11. [26,31] *Let Θ be a multiplicative group that has more than two elements, and let 0 be a multiplicatively bilaterally absorbing element. If a hypercomposition $\dotplus$ is defined on $H=\Theta\cup\{0\}$ as follows:*

$$x \dotplus y = \{x, y\}, \qquad \text{for all } x,y \in \Theta, \text{ with } y \neq x$$

$$x \dotplus x = H \cdot\cdot \{x\}, \qquad \text{for all } x \in \Theta$$

$$x \dotplus 0 = 0 \dotplus x = x, \qquad \text{for all } x \in H$$

then the triplet $H(\Theta) = (\Theta\cup\{0\}, \dotplus, \cdot)$ is a hyperfield that is not isomorphic to a quotient hyperfield when Θ is a periodic group.

Corollary 2. *The hyperfield HF_7^{258} (Table 2) is not a quotient hyperfield.*

Table 2. The canonical hypergroup of the non-quotient hyperfield HF_7^{258}.

HF_7^{258}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a}	{1,b}	{1,c}	{1,d}	{1,e}
a	a	{1,a}	{0,1,b,c,d,e}	{a,b}	{a,c}	{a,d}	{a,e}
b	b	{1,b}	{a,b}	{0,1,a,c,d,e}	{b,c}	{b,d}	{b,e}
c	c	{1,c}	{a,c}	{b,c}	{0,1,a,b,d,e}	{c,d}	{c,e}
d	d	{1,d}	{a,d}	{b,d}	{c,d}	{0,1,a,b,c,e}	{d,e}
e	e	{1,e}	{a,e}	{b,e}	{c,e}	{d,e}	{0,1,a,b,c,d}

Theorem 12. [30] *The direct sum S of the hyperrings S_i, $i\in I$, is not isomorphic to a sub-hyperring of a quotient hyperring if at least one of the S_i is a non-quotient hyperfield.*

Additional classes of non-quotient hyperfields/hyperrings were subsequently made by Nakassis [32].

Theorem 13. [32] *Let $(T,\cdot)$ be a multiplicative group of order m, with m > 3. Also let $H=T\cup\{0\}$, where 0 is a multiplicatively absorbing element. If H is equipped with the hypercomposition:*

$$x \dotplus y = H \setminus \{0,x,y\}, \qquad \text{for all } x,y \in T, \text{ with } y \neq x;$$

$$x \dotplus x = \{0,x\}, \qquad \text{for all } x \in T;$$

$$x \dotplus 0 = 0 \dotplus x = x, \qquad \text{for all } x \in H$$

then, $H(T) = (T\cup\{0\}, \dotplus, \cdot)$ is a hyperfield which is a non-quotient hyperfield if the cardinality of T is properly chosen.

The next two propositions are crucial for proving that hyperfields of the type outlined in Theorem 13 are not quotient hyperfields.

Proposition 1. *In a quotient hyperfield F/Q, the cardinality of the sum of any two elements cannot exceed the cardinality of Q.*

Proof. Suppose that xQ, yQ are two arbitrary elements of F/Q. Then,

$$xQ+yQ=\{(x+yq)Q \mid q\in Q\}$$

and the function $f:Q\to xQ+yQ$ with $f(q)=(x+yq)Q$ is a surjection.□

Proposition 2. *If in a quotient hyperfield F/Q the differences $xQ-xQ$, $xQ\in F/Q$ have only 0 in common, then the cardinality of the sum of any two non-opposite or two non-equal elements is equal to the cardinality of Q.*

Proof. Let xQ, yQ be two non-opposite and non-equal elements in F/Q. Then,

$$xQ+yQ=\{(x+yq)Q \mid q\in Q\}.$$

Next, if $(x+yq)Q=(x+yp)Q$, then

$$x+yq=(x+yp)r \Leftrightarrow x-xr=yq-ypr \Rightarrow (xQ-xQ)\cap(yQ-yQ)\neq\varnothing .$$

From the validity of the equality $(xQ-xQ)\cap(yQ-yQ)=\{0\}$, it follows that $x-xr=0$. Therefore, $r=1$ and consequently $yq-yp=0$ or equivalently $q=p$. Hence,

$$card(xQ+yQ)=cardQ . \ □$$

We subsequently present some non-quotient hyperfields from the list of 277 seven-element hyperfields detailed in Section 10, utilizing these propositions.

Proposition 3.

i. *The hyperfield* HF_7^3 *(Table 3) is not a quotient hyperfield.*

Table 3. The canonical hypergroup of the non-quotient hyperfield HF_7^3.

HF_7^3	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{b,c,d,e}	{a,c,d,e}	{0,1,c}	{a,b,c,e}	{a,b,c,d}
a	a	{b,c,d,e}	{1,a,b,c,e}	{1,c,d,e}	{1,b,d,e}	{0,a,d}	{1,b,c,d}
b	b	{a,c,d,e}	{1,c,d,e}	{1,a,b,c,d}	{1,a,d,e}	{1,a,c,e}	{0,b,e}
c	c	{0,1,c}	{1,b,d,e}	{1,a,d,e}	{a,b,c,d,e}	{1,a,b,e}	{1,a,b,d}
d	d	{a,b,c,e}	{0,a,d}	{1,a,c,e}	{1,a,b,e}	{1,b,c,d,e}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d}	{0,b,e}	{1,a,b,d}	{1,a,b,c}	{1,a,c,d,e}

ii. *The hyperfield* HF_7^4 *(Table 4) is not a quotient hyperfield.*

Table 4. The canonical hypergroup of the non-quotient hyperfield HF_7^4.

HF_7^4	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{b,c,d,e}	{a,c,d,e}	{0,1,c}	{a,b,c,e}	{a,b,c,d}
a	a	{b,c,d,e}	{1,a,b,c,d,e}	{1,c,d,e}	{1,b,d,e}	{0,a,d}	{1,b,c,d}
b	b	{a,c,d,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,a,d,e}	{1,a,c,e}	{0,b,e}
c	c	{0,1,c}	{1,b,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,b,d}
d	d	{a,b,c,e}	{0,a,d}	{1,a,c,e}	{1,a,b,e}	{1,a,b,c,d,e}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d}	{0,b,e}	{1,a,b,d}	{1,a,b,c}	{1,a,b,c,d,e}

Proof. (***i***) The opposite of 1 is c, the opposite of a is d, the opposite of b is e and the differences $1\text{-}c$, $a\text{-}d$ and $b\text{-}e$ have only 0 in common. Therefore, if HF_7^3 were isomorphic to a quotient hyperfield F/Q, then, according to Proposition 2, the cardinality of Q would be equal to the cardinality of the sum of any two non-opposite and non-equal elements, which is 4. Moreover, according to Proposition 1, the cardinality of the sum of any two elements cannot exceed the cardinality of Q. However, the cardinality of the sum $x+x$ is equal to 5 for any $x \in HF_7^3$. Hence, HF_7^3 is not a quotient hyperfield. Similar is the proof of (***ii***).□

Proposition 4.

i. *The hyperfield* HF_7^5 *(Table 5) is not a quotient hyperfield.*

Table 5. The canonical hypergroup of the non-quotient hyperfield HF_7^5.

HF_7^5	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,e}	{1,b,d,e}	{a,b,c,d,e}	{0,a,d}	{1,a,b,c,e}	{a,c,d,e}
a	a	{1,b,d,e}	{1,b,c,d}	{1,a,c,e}	{1,b,c,d,e}	{0,b,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,e}	{a,c,d,e}	{1,a,b,d}	{1,a,c,d,e}	{0,1,c}
c	c	{0,a,d}	{1,b,c,d,e}	{1,a,b,d}	{1,b,d,e}	{a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,b,e}	{1,a,c,d,e}	{a,b,c,e}	{1,a,c,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c,d}	{0,1,c}	{1,a,b,d,e}	{1,b,c,d}	{1,a,b,d}

ii. *The hyperfield* HF_7^6 *(Table 6) is not a quotient hyperfield.*

Table 6. The canonical hypergroup of the non-quotient hyperfield HF_7^6.

HF_7^6	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d,e}	{1,b,c,d,e}	{b,c,d,e}	{0,a,d}	{1,a,b,c}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d,e}	{1,c,d,e}	{0,b,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c,d,e}	{1,a,c,d,e}	{1,a,b,d,e}	{1,a,d,e}	{0,1,c}
c	c	{0,a,d}	{1,c,d,e}	{1,a,b,d,e}	{1,a,b,d,e}	{1,a,b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,b,e}	{1,a,d,e}	{1,a,b,c,e}	{1,a,b,c,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{a,b,c,d}	{0,1,c}	{1,a,b,e}	{1,a,b,c,d}	{1,a,b,c,d}

iii. *The hyperfield* HF_7^{11} *(Table 7) is not a quotient hyperfield.*

Table 7. The canonical hypergroup of the non-quotient hyperfield HF_7^{11}.

HF_7^{11}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d}	{a,b,c,d,e}	{1,a,d,e}	{0,b,e}	{b,c,d,e}	{1,a,b,c,d}
a	a	{a,b,c,d,e}	{b,c,d,e}	{1,b,c,d,e}	{1,a,b,e}	{0,1,c}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,d,e}	{1,c,d,e}	{1,a,c,d,e}	{1,a,b,c}	{0,a,d}
c	c	{0,b,e}	{1,a,b,e}	{1,a,c,d,e}	{1,a,d,e}	{1,a,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,c}	{1,a,b,c}	{1,a,b,d,e}	{1,a,b,e}	{1,a,b,c,e}
e	e	{1,a,b,c,d}	{1,c,d,e}	{0,a,d}	{a,b,c,d}	{1,a,b,c,e}	{1,a,b,c}

Proof. (***i***) The opposite of 1 is *c*, the opposite of *a* is *d*, the opposite of *b* is *e* and the differences 1-*c*, *a*-*d* and *b*-*e* have only 0 in common. Therefore, if HF_7^5 were isomorphic to a quotient hyperfield F/Q, then, according to Proposition 2, the cardinality of Q would be equal to the cardinality of the sum of any two non-opposite and non-equal elements. Therefore, the cardinalities of the sum of any two non-opposite and non-equal elements would have been equal. However, card(1+*a*) = 4, while card(*b*+*c*)=5. Consequently, HF_7^5 is not a quotient hyperfield. Similar is the proof of (***ii***) and (***iii***). □

Remark 1. In the light of Propositions 1 and 2, it is preferable to keep the first proof that $NQHF_5^2$ in [1] is not a quotient hyperfield when its multiplicative group is the Vierergruppe and it is classified according to Theorem 14 of [1], when its multiplicative group is cyclic.

5. Strongly canonical and superiorly canonical hyperfields/hyperrings

As in the case of fields, a valuation theory has also been developed for hyperfields. The concept of valuation for hyperfields was introduced by M. Krasner [3]. References [33–36] provide more recent studies on valuation in hyperfields, while [37] by A. Linzi offers an extremely detailed and in-depth presentation of hyperfield valuation theory. Among other things in his paper, A. Linzi clarifies several points of J. Mittas' earlier research on valued hyperfields [38–43]. J. Mittas proved [38–42] that a necessary and sufficient condition for a canonical hypergroup, and consequently for a hyperfield, to be valuated or hypervaluated is the validity of certain additional properties of a purely algebraic type, i.e., properties that can be expressed without the intervention of the valuation or the hypervaluation, respectively. This led him to the definition of the following two special canonical hypergroups:

(a) The *strongly canonical hypergroup*, which is a canonical hypergroup that also satisfies the axioms:

S1: If $x \in x + y$, then $x + y = x$, for all $x, y \in H$;

S2: If $(x+y)\cap(z+w) \neq \varnothing$, then either $x+y \subseteq z+w$ or $z+w \subseteq x+y$.

(b) The *superiorly canonical hypergroup*, which is a strongly canonical hypergroup that also satisfies the axioms:

S3: If $z, w \in x-y$ and $x \neq y$, then $z-z=w-w$;

S4: If $x \in z-z$ and $y \notin z-z$ then $x-x \subseteq y-y$.

Thus, depending on the type of their additive hypergroup, strongly canonical and superiorly canonical hyperfields are defined, respectively. Mittas proved the following theorem (e.g., [44], Theorem 4.2]:

Theorem 14. *A hyperfield can be valuated if and only if its additive hypergroup is superiorly canonical.*

A detailed and thorough proof of this theorem is given by A. Linzi in [37] (Theorem 4.22).

Proposition 5. *In a strongly canonical hyperfield, if* $x \neq y$, *then*:

$$(x-x)\cap(y-x)=\varnothing \text{ and } (y-y)\cap(y-x)=\varnothing.$$

Proof. Suppose that $x \neq y$ and $(x-x)\cap(y-x)\neq\varnothing$. Let $w\in(x-x)\cap(y-x)$. Then, $w \in y-x$ implies that $y \in x+w$. Moreover, $w \in x-x$ implies that $x \in x+w$. Therefore, $x = x+w$. Thus $x=y$, which contradicts our assumption. □

Proposition 6. *In a strongly canonical hyperfield K, the following statements hold.*

i. $x+(x-x)=x$, *for all* $x \in K$;

ii. $x \notin x-x$ *and equivalently* $x \notin x+x$, *for all* $x \in K \cdot\cdot \{0\}$.

Proof. (i) Suppose that $w \in x-x$. Then, $x \in x+w$ and therefore, $x = x+w$. Thus,

$$x+(x-x)=\bigcup_{w\in x-x} x+w = x.$$

(ii) Suppose that x is a non-zero element such that $x \in x-x$. Then, $0 \in x+(x-x)$. However, according to (i), $x = x+(x-x)$. Consequently, $x=0$, which is absurd. □

Lemma 1. [43] *In a canonical hypergroup H, the following property:*

$$(x+y)\cap(z+w)\neq\varnothing \Rightarrow (x-z)\cap(w-y)\neq\varnothing$$

is satisfied for all $x, y, z, w \in H$.

Theorem 15. *Let K be a strongly canonical hyperfield. Then, for any fixed point* $a \in K$, *the sets* $a+x$, $x \in K$ *form a partition of K.*

Proof. Let $x \neq y$ and suppose that $(a+x)\cap(a+y)\neq\varnothing$. Then, because of Lemma 1, $(x-y)\cap(a-a)\neq\varnothing$. If $w\in(x-y)\cap(a-a)$, then:

$$w \in x-y \Rightarrow x \in w+y \text{ and } w \in a-a \Rightarrow a \in w+a \Rightarrow a = w+a.$$

Hence,

$$a+x \subseteq a+(w+y)=(a+w)+y=a+y\,.$$

Moreover,

$$w \in x-y \Rightarrow y \in x-w \;\text{ and }\; w \in a-a \Rightarrow a \in a-w \Rightarrow a=a-w\,.$$

Thus,

$$a+y \subseteq a+(x-w)=(a-w)+x=a+x\,.$$

Therefore,

$$a+x=a+y\,.$$

Example 1. Let *H* be a set that is totally ordered and symmetric around a center, denoted by $0 \in H$. Then, *H*, equipped with the hypercomposition:

$$x+y=\begin{cases} y, & \text{if } |x|<|y| \\ x+x=x \\ x-x=\left[-|x|,\ |x|\right] \end{cases}$$

is a canonical hypergroup. Now, if $H \setminus \{0\}$ is an abelian multiplicative group and 0 is bilaterally absorbing with regard to the multiplication, then *H* becomes a hyperfield. Moreover, if the hypercomposition on *H* is defined as follows:

$$x+y=\begin{cases} y, & \text{if } |x|<|y| \\ x+x=-x \\ x-x=\left(-|x|,|x|\right) \end{cases}$$

then *H* is a strongly canonical hyperfield.

Example 2. Let $(E,\cdot)$ be a totally ordered multiplicative semigroup, having a minimum element 0, which is bilaterally absorbing with regard to the multiplication. The following hypercomposition is defined on *E*:

$$x \hat{+} y=\begin{cases} \max\{x,y\}, & \text{if } x \neq y \\ \{z \in E \mid z \leq x\}, & \text{if } x=y \end{cases}$$

Then, $(E,\hat{+},\cdot)$ is a hyperring. If $E \setminus \{0\}$ is a multiplicative group, then $(E,\hat{+},\cdot)$ is a skew hyperfield, while if it is an abelian group, then $(E,\hat{+},\cdot)$ is a hyperfield. This hyperfield was introduced by J. Mittas (see [25] page 86 and [45] page 370); nowadays, it is called tropical hyperfield (see e.g., [46–50]). Moreover, if the hypercomposition on *E* is defined as follows:

$$x \breve{+} y=\begin{cases} \max\{x,y\}, & \text{if } x \neq y \\ \{z \in E \mid z < x\}, & \text{if } x=y \end{cases}$$

then $(E,\breve{+},\cdot)$ is a strongly canonical hyperring. If $E \cdot\cdot \{0\}$ is a multiplicative group, then $(E,\breve{+},\cdot)$ is a strongly canonical skew hyperfield, while if it is an abelian group, $(E,\breve{+},\cdot)$ is a strongly canonical hyperfield.

Corrigendum on [1]

Theorem 12 in [1] claims that the above hyperring $(E,\breve{+},\cdot)$ is a non-quotient hyperring. However, this is not accurate due to a slight confusion in the proof. Specifically, while it is initially shown that the assumption that *E* is isomorphic to a quotient hyperfield *R/Q* implies that «$4Q$ *is a distinct class from both* Q *and* $2Q$» just six lines below it is mistakenly assumed that *4Q=Q*, which led to an incorrect reductio ad absurdum, concluding that $7 \in Q$ and $7 \notin Q$. This was observed by David Hobby and Jaiung Jun, who subsequently examined whether the family of these hyperfields contains quotient hyperfields and constructed an interesting example that answers this question in the affirmative [51].

6. Construction of the hyperfields of order 7

This Section presents the algorithmic method, which is used for the construction of the hyperfields that have 7 elements. There exist 277 such hyperfields, detailed in Section 10 and they are denoted as HF_7^k, $1 \le k \le 277$. HF_7^1 is the field Z_7. The multiplicative group HF_7^* of these hyperfields consists of six elements. As it is known, there are two groups with six elements: The cyclic group Z_6 and the dihedral group D_3. Since Z_6 is cyclic, it is abelian, while D_3 is not abelian. Consequently, the multiplicative group of hyperfields of order 7 is the cyclic group Z_6. Of course, D_3 can be used to form skew hyperfields, as it is indicated in Section 3.

Since the multiplicative group HF_7^* of hyperfields of order 7 is the cyclic group Z_6, we have that $HF_7^* = \{1, a, a^2, a^3, a^4, a^5\}$, where *a* is a generator of HF_7^*. Note that the generators of the multiplicative group of Z_7 are *a*=3 and *a*=5.

Theorem 16. *For the hyperfields of order 7, the following apply:*

i. $0 \notin a+1$,

ii. $0 \notin a^2+1$,

iii. $0 \notin a^4+1$,

iv. $0 \notin a^5+1$

where a is the generator of the multiplicative group.

Proof. i. Suppose that $0 \in a+1$, then $a=-1$ and therefore $a^2=1$. Hence, *a* is not a generator of HF_7^*, which is a contradiction.

ii. If $0 \in a^2+1$, then $a^2=-1$. Hence, $a^4=1$ and therefore *a* is not a generator of HF_7^*, which contradicts the assumption for *a*.

iii. If $0 \in a^4+1$, then $a^4=-1$. Also, $a^8=a^6a^2=a^2$. Thus, $a^2=1$ and therefore *a* is not a generator of HF_7^*, which is absurd.

iv. If $0 \in a^5 + 1$, then $a^5 = -1$. From $a^{10} = a^6 a^4 = a^4$, it derives that $a^4 = 1$ and, therefore, a is not a generator of HF_7^*, absurd. □

Corollary 1. *For the hyperfields of order 7, it holds that either* $0 \in 1+1$ *or* $0 \in a^3 + 1$*, where a is the generator of the multiplicative group.*

Corollary 2. *For hyperfields of order 7, the following properties hold:*

i. $1 + a \subseteq \{1, a, a^2, a^3, a^4, a^5\}$;

ii. $1 + a^2 \subseteq \{1, a, a^2, a^3, a^4, a^5\}$.

where a is the generator of the multiplicative group.

Proposition 7. *The addition in* HF_7^k *is completely defined by the sums:*

$$1+1, \quad 1+a, \quad 1+a^2, \quad 1+a^3$$

where a is the generator of the multiplicative group.

Proof. Distributivity implies the following:

i. $1+a^4 = a^4(a^2+1)$	vii. $a+a^5 = a(a^4+1)$	xiii. $a^3+a^4 = a^3(a+1)$
ii. $1+a^5 = a^5(a+1)$	viii. $a^2+a^2 = a^2(1+1)$	xiv. $a^3+a^5 = a^3(a^2+1)$
iii. $a+a = a(1+1)$	ix. $a^2+a^3 = a^2(a+1)$	xv. $a^4+a^4 = a^4(1+1)$
iv. $a+a^2 = a(a+1)$	x. $a^2+a^4 = a^2(a^2+1)$	xvi. $a^4+a^5 = a^4(a+1)$
v. $a+a^3 = a(a^2+1)$	xi. $a^2+a^5 = a^2(a^3+1)$	xvii. $a^5+a^5 = a^5(1+1)$
vi. $a+a^4 = a(a^3+1)$	xii. $a^3+a^3 = a^3(1+1)$	

Using the notation b, c, d, and e for the elements a^2, a^3, a^4 and a^5, respectively, the following Cayley Table (Table 8) summarizes the above results:

Table 8. The addition in HF_7^k.

	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	$1+1 \subseteq \{0\} \cup \{1,a,b,c,d,e\}$	$1+a \subseteq \{1,a,b,c,d,e\}$	$1+b \subseteq \{1,a,b,c,d,e\}$	$1+c \subseteq \{0\} \cup \{1,a,b,c,d,e\}$	$d(b+1)$	$e(a+1)$
a	a	$1+a \subseteq \{1,a,b,c,d,e\}$	$a(1+1)$	$a(a+1)$	$a(b+1)$	$a(c+1)$	$a(d+1)$
b	b	$1+b \subseteq \{1,a,b,c,d,e\}$	$a(a+1)$	$b(1+1)$	$b(a+1)$	$b(b+1)$	$b(c+1)$
c	c	$1+c \subseteq \{0\} \cup \{1,a,b,c,d,e\}$	$a(b+1)$	$b(a+1)$	$c(1+1)$	$c(a+1)$	$c(b+1)$
d	d	$d(b+1)$	$a(c+1)$	$b(b+1)$	$c(a+1)$	$d(1+1)$	$d(a+1)$
e	e	$e(a+1)$	$a(d+1)$	$b(c+1)$	$c(b+1)$	$d(a+1)$	$e(1+1)$

7. Classification of the hyperfields of order 7

Most of the 277 seven-element hyperfields that are listed in Section 10 are classified here. Krasner's question of whether there exist non-quotient hyperfields [7] or not, led to the construction of the class of the *monogene hyperfields* [28,52]. The study of the isomorphism of these hyperfields to the quotient ones brought forth the question of whether a field F can be expressed as the difference $G-G$, where G is a subgroup of its multiplicative group [30,31,52–55]. The conditions under which this occurs are detailed in the same referenced works. Thus, if a hyperfield of order 7 is isomorphic to a quotient hyperfield F/G, then G is a multiplicative subgroup of F having index 6 and, in this case, the following theorem ([1], Theorem 15) holds:

Theorem 17. [1] *If F is a finite field and G is a subgroup of its multiplicative group of index 6 and order* m*, then G–G=F, if and only if:*

$-1 \notin G$ *and* $m \geq 11$,

$-1 \in G$, $charF = 11$ *and* $m \geq 20$,

$-1 \in G$, $charF = 13$ *and* $m \geq 28$,

$-1 \in G$, $charF \neq 11,13$ *and* $m \geq 30$.

In the above theorem, the notation *charF* denotes the characteristic of the field F. Moreover, it is known that if G is a subgroup of finite index in the multiplicative group of an infinite field F, then the equality $G–G=F$ holds [1,56,57]. Consequently, the quotient hyperfields F/G for which $G–G\neq F/G$ are fully and explicitly determined by Theorem 17. Therefore, the following theorem holds:

Theorem 18. *The quotient hyperfields of order 7 that satisfy the condition G−G≠F/G are as follows:*

i. Z_7, Z_{19}/G, Z_{31}/G, Z_{43}/G, $GF\left[5^2\right]/G$ *and* $GF\left[7^2\right]/G$, *when the multiplicative subgroup G does not contain* -1.

ii. Z_{13}/G, Z_{37}/G, Z_{61}/G, Z_{73}/G, Z_{97}/G *and* Z_{109}/G, *when the multiplicative subgroup G contains* -1.

Remark 2. As shown below, in Proposition 9.v, Z_{157}/G is isomorphic to Z_{97}/G, and for this reason, it is not included in case (ii) of the above theorem.

The following subsections focus on the study of the isomorphisms of the hyperfields presented in Theorem 18 in relation to the seven-element hyperfields listed in Section 10. The objective is to identify which hyperfields from this list are classified as quotient hyperfields and which are not.

7.1. The quotient hyperfields F/G of order 7 that are constructed from the prime fields and for which G − G ≠ F/G and −1 ∉ G hold

Field Z_7, which may be regarded as a trivial hyperfield, could be considered the initial member of this category. The remaining members, according to Theorem 18, are Z_{19}/G, Z_{31}/G, and Z_{43}/G.

Proposition 8.

i. *The multiplicative subgroup of index 6 of the field* Z_{19} *is*

$$G = \{1,\ 7,\ 11\}$$

and the hyperfield Z_{19}/G *is isomorphic to* HF_7^9 *(Table 9).*

Table 9. The canonical hypergroup of the hyperfield Z_{19}/G.

HF_7^9	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,c}	{a,b,e}	{1,d,e}	{0,b,e}	{b,c,d}	{1,a,d}
a	a	{a,b,e}	{b,d}	{1,b,c}	{1,a,e}	{0,1,c}	{c,d,e}
b	b	{1,d,e}	{1,b,c}	{c,e}	{a,c,d}	{1,a,b}	{0,a,d}
c	c	{0,b,e}	{1,a,e}	{a,c,d}	{1,d}	{b,d,e}	{a,b,c}
d	d	{b,c,d}	{0,1,c}	{1,a,b}	{b,d,e}	{a,e}	{1,c,e}
e	e	{1,a,d}	{c,d,e}	{0,a,d}	{a,b,c}	{1,c,e}	{1,b}

ii. *The multiplicative subgroup of index 6 of the field* Z31 *is*

$$G = \{1,\ 2,\ 4,\ 8,\ 16\}$$

and the hyperfield Z31/G *is isomorphic to* HF_7^{13} *(Table 10).*

Table 10. The canonical hypergroup of the hyperfield Z31/G.

HF_7^{13}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b}	{1,b,d,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,c,d,e}
a	a	{1,b,d,e}	{a,b,c}	{1,a,c,e}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,e}	{b,c,d}	{1,a,b,d}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{1,a,b,d}	{c,d,e}	{a,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{a,b,c,e}	{1,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,b,c,d}	{1,a,e}

iii. *The multiplicative subgroup of index 6 of the field* Z43 *is*

$$G = \{1,\ 4,\ 11,\ 16,\ 21,\ 35,\ 41\}$$

and the hyperfield Z43/G *is isomorphic to* HF_7^{61} *(Table 11).*

Table 11. The canonical hypergroup of the hyperfield Z43/G.

HF_7^{61}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c}	{1,a,b,d,e}	{1,a,b,d,e}	{0,a,b,d,e}	{1,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{b,c,d}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,b,c,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,e}	{c,d,e}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,e}	{1,a,b,c,d}	{1,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d}	{a,b,c,d,e}	{1,a,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,c,d,e}	{0,1,a,c,d}	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,b}

7.2. The quotient hyperfields F/G of order 7 that are constructed from the prime fields and for which $G-G \neq F/G$ and $-1 \in G$ hold

The members of this category, according to Theorem 18, are Z_{13}/G, Z_{37}/G, Z_{61}/G, Z_{73}/G, Z_{97}/G, and Z_{109}/G.

Proposition 9.

i. *The multiplicative subgroup of index 6 of field* Z_{13} *is*

$$G = \{1,12\}$$

and the hyperfield Z_{13}/G *is isomorphic to* HF_7^{143} *(Table 12).*

Table 12. The canonical hypergroup of the hyperfield Z_{13}/G.

HF_7^{143}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a}	{1,d}	{c,d}	{b,e}	{a,b}	{c,e}
a	a	{1,d}	{0,b}	{a,e}	{d,e}	{1,c}	{b,c}
b	b	{c,d}	{a,e}	{0,c}	{1,b}	{1,e}	{a,d}
c	c	{b,e}	{d,e}	{1,b}	{0,d}	{a,c}	{1,a}
d	d	{a,b}	{1,c}	{1,e}	{a,c}	{0,e}	{b,d}
e	e	{c,e}	{b,c}	{a,d}	{1,a}	{b,d}	{0,1}

ii. *The multiplicative subgroup of index 6 of the field* Z_{37} *is*

$$G = \{1, 10, 11, 26, 27, 36\}$$

and the hyperfield Z_{37}/G *is isomorphic to* HF_7^{160} *(Table 13).*

Table 13. The canonical hypergroup of the hyperfield Z_{37}/G.

HF_7^{160}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,d}	{1,b,c,d,e}	{a,b,c,e}	{a,b,d,e}	{1,a,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,e}	{1,a,c,d,e}	{1,b,c,d}	{1,b,c,e}	{1,a,b,d}
b	b	{a,b,c,e}	{1,a,c,d,e}	{0,1,b,c}	{1,a,b,d,e}	{a,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,b,c,d}	{1,a,b,d,e}	{0,a,c,d}	{1,a,b,c,e}	{1,b,d,e}
d	d	{1,a,c,e}	{1,b,c,e}	{a,c,d,e}	{1,a,b,c,e}	{0,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d}	{1,a,c,d}	{1,b,d,e}	{1,a,b,c,d}	{0,1,c,e}

iii. *The multiplicative subgroup of index 6 of the field* Z_{61} *is*

$$G = \{1,\ 3,\ 9,\ 20,\ 27,\ 34,\ 41,\ 52,\ 58,\ 60\}$$

and the hyperfield Z_{61}/G *is isomorphic to* HF_7^{234} *(Table 14).*

Table 14. The canonical hypergroup of the hyperfield Z_{61}/G.

HF_7^{234}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,1,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,d}

iv. *The multiplicative subgroup of index 6 of field* Z_{73} *is*

$$G = \{1,\ 3,\ 8,\ 9,\ 24,\ 27,\ 46,\ 49,\ 64,\ 65,\ 70,\ 72\}$$

and the hyperfield Z_{73}/G *is isomorphic to* HF_7^{245} *(Table 15).*

Table 15. The canonical hypergroup of the hyperfield Z_{73}/G.

HF_7^{245}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,b,c,d,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{0,1,a,c,d,e}	{1,a,b,c,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,e}

v. *The multiplicative subgroups of index 6 of fields* Z_{97} and Z_{157} *are, respectively:*

$$G = \{1,\ 8,\ 12,\ 18,\ 22,\ 27,\ 33,\ 47,\ 50,\ 64,\ 70,\ 75,\ 79,\ 85,\ 89,\ 96\}$$

$$G = \begin{Bmatrix} 1,\ 4,\ 14,\ 16,\ 27,\ 39,\ 46,\ 49,\ 56,\ 58,\ 64,\ 67,\ 75,\ 82, \\ 90,\ 93,\ 99,\ 101,\ 108,\ 111,\ 118,\ 130,\ 141,\ 143,\ 153,\ 156 \end{Bmatrix}$$

and the hyperfields Z_{97}/G, Z_{157}/G *are isomorphic to* HF_7^{267} *(Table 16).*

Table 16. The canonical hypergroup of the hyperfields Z_{97}/G and Z_{157}/G.

HF_7^{267}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d}

vi. *The multiplicative subgroup of index 6 of field* Z_{109} *is*

$$G = \{1,\ 4,\ 16,\ 27,\ 34,\ 38,\ 43,\ 45,\ 46,\ 63,\ 64,\ 66,\ 71,\ 75,\ 82,\ 93,\ 105,\ 108\}$$

and the hyperfield Z_{109}/G *is isomorphic to* HF_7^{246} *(Table 17).*

Table 17. The canonical hypergroup of the hyperfield Z_{109}/G.

HF_7^{246}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,e}

7.3. The quotient hyperfields F/G of order 7 that are constructed from the finite fields $GF[p^n]$, *n>1 and for which* $G-G\neq F/G$ *holds*

According to Theorem 18, this category has two members, which are the hyperfields $GF\left[5^2\right]/G$ and $GF\left[7^2\right]/G$.

Field $GF[5^2]$ consists of all the linear polynomials with coefficients in the field of residues modulo 5. In $GF[5^2]$, the polynomial x^2+3x+4 is irreducible. Thus, in the multiplication of the polynomials, we set $x^2 = -3x-4 = 2x+1$ and then they are combined according to the ordinary rules, working modulo 5. The multiplicative subgroup of index 6 in $GF[5^2]$ is $G = \{1, 2, 3, 4\}$ and its cosets are:

$$G, xG, (x+1)G, (2x+1)G, (3x+1)G, (4x+1)G$$

The results of the hypercomposition in the above hyperfield lead to the following proposition:

Proposition 10. *The hyperfield*

$$GF\left[5^2\right]\Big/G = \{0, G, xG, (x+1)G, (2x+1)G, (3x+1)G, (4x+1)G\}$$

is isomorphic to HF_7^{142} *(Table 18).*

Table 18. The canonical hypergroup of the hyperfield $GF[5^2]/G$.

HF_7^{142}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1}	{b,c,d,e}	{a,c,d,e}	{a,b,d,e}	{a,b,c,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a}	{1,c,d,e}	{1,b,d,e}	{1,b,c,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,c,d,e}	{0,b}	{1,a,d,e}	{1,a,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,b,d,e}	{1,a,d,e}	{0,c}	{1,a,b,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,b,c,e}	{1,a,c,e}	{1,a,b,e}	{0,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d}	{1,a,c,d}	{1,a,b,d}	{1,a,b,c}	{0,e}

In field $GF[7^2]$ of all linear polynomials with coefficients from Z_7, the addition and the multiplication of the polynomials are defined in the usual way; by replacing x^2 with 6, since x^2+1 is the irreducible polynomial of degree 2. The multiplicative subgroup of index 6 in field $GF[7^2]$ is

$$G = \{1, 6, x, 6x, 2x+2, 2x+5, 5x+2, 5x+5\},$$

and its cosets are:

$$G, (x+2)G, (x+2)^2G = (4x+3)G, (x+2)^3G = (4x+2)G, (x+2)^4G = (3x)G \text{ and } (x+2)^5G = (6x+4)G$$

The results of the hypercomposition in hyperfield $GF[7^2]$/G lead to the following proposition:

Proposition 11. *The hyperfield*

$$GF\left[7^2\right]/G = \left\{ 0, G, (x+2)G, (4x+3)G, (4x+2)G, (3x)G, (6x+4)G \right\}$$

is isomorphic to HF_7^{225} *(Table 19).*

Table 19. The canonical hypergroup of the hyperfield $GF[7^2]/G$.

HF_7^{225}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}
b	b	{1,a,b,c,d}	{1,a,c,e}	{0,1,c,d,e}	{1,a,b,d}	{1,b,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,d}	{0,1,a,d,e}	{a,b,c,e}	{1,a,c,d,e}
d	d	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,c,e}	{0,1,a,b,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,d}	{0,1,a,b,c}

7.4. Quotient hyperfields F/G of order 7 for which G – G = F/G.

This subsection, presents some seven-element hyperfields that derive as quotients of finite fields, in which the sum of two opposite elements yields the whole hyperfield.

Proposition 12. *The multiplicative subgroup of index 6 of fields* Z_{67}, Z_{79}, *and* Z_{139} *are, respectively:*

$$G = \{1,\ 9,\ 14,\ 15,\ 22,\ 24,\ 25,\ 40,\ 59,\ 62,\ 64\}$$

$$G = \{1,\ 8,\ 10,\ 21,\ 24,\ 25,\ 38,\ 46,\ 52,\ 62,\ 64,\ 65,\ 67\}$$

$$G = \begin{Bmatrix} 1,\ 6,\ 34,\ 36,\ 44,\ 45,\ 52,\ 55,\ 57,\ 63,\ 64,\ 65, \\ 77,\ 79,\ 80,\ 91,\ 100,\ 106,\ 112,\ 116,\ 125,\ 129,\ 131 \end{Bmatrix}$$

and hyperfields Z_{67}/G, Z_{79}/G *and* Z_{139}/G *are isomorphic to* HF_7^{137} *(Table 20).*

Table 20. The canonical hypergroup of the hyperfields Z_{67}/G, Z_{79}/G and Z_{139}/G.

HF_7^{137}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}

Proposition 13.

i. *The multiplicative subgroups of index 6 of fields* Z_{103}, Z_{127}, Z_{151} and Z_{163} *are respectively:*

$$G = \{1,\ 8,\ 9,\ 13,\ 14,\ 23,\ 30,\ 34,\ 61,\ 64,\ 66,\ 72,\ 76,\ 79,\ 81,\ 93,\ 100\}$$

$$G = \begin{Bmatrix} 1,\ 2,\ 4,\ 8,\ 16,\ 19,\ 25,\ 32,\ 38,\ 47,\ 50, \\ 61,\ 64,\ 73,\ 76,\ 87,\ 94,\ 100,\ 107,\ 117,\ 122 \end{Bmatrix}$$

$$G = \begin{Bmatrix} 1,\ 8,\ 9,\ 19,\ 20,\ 29,\ 44,\ 50,\ 59,\ 64,\ 68,\ 72,\ 78,\ 81, \\ 84,\ 86,\ 91,\ 94,\ 98,\ 110,\ 123,\ 124,\ 125,\ 127,\ 148 \end{Bmatrix}$$

$$G = \begin{Bmatrix} 1,\ 6,\ 21,\ 22,\ 25,\ 36,\ 38,\ 40,\ 53,\ 58,\ 61,\ 64,\ 65,\ 77,\ 85, \\ 104,\ 115,\ 126,\ 132,\ 133,\ 135,\ 136,\ 140,\ 146,\ 150,\ 155,\ 158 \end{Bmatrix}$$

and the hyperfields Z_{103}/G, Z_{127}/G, Z_{151}/G and Z_{163}/G *are isomorphic to* HF_7^{141} *(Table 21).*

Table 21. The canonical hypergroup of the hyperfields Z_{103}/G, Z_{127}/G, Z_{151}/G and Z_{163}/G.

HF_7^{141}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}

ii. *The multiplicative subgroup of index 6 of field* Z_{181} *is*

$$G = \begin{Bmatrix} 1, 5, 25, 27, 29, 36, 42, 46, 48, 49, 56, 59, 64, 67, 82, 99, 114, \\ 117, 122, 125, 132, 133, 135, 139, 145, 152, 154, 156, 176, 180 \end{Bmatrix}$$

and the hyperfield Z_{181}/G *is isomorphic to* HF_7^{277} *(Table 22).*

Table 22. The canonical hypergroup of the hyperfield Z_{181}/G.

HF_7^{277}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}

Field $GF[11^2]$ consists of all the linear polynomials with coefficients from the field of residues modulo 11. In $GF[11^2]$, the polynomial x^2+1 is irreducible. Thus, the polynomials are combined according to the ordinary rules, working modulo 11, by setting $x^2=-1=10$. The multiplicative subgroup of index 6 in $GF[11^2]$ is

$G = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, x, 2x, 3x, 4x, 5x, 6x, 7x, 8x, 9x, 10x\}$

and its cosets are:

$$G,\quad (3x+1)G,\quad (3x+1)^2G = (6x+3)G,\quad (3x+1)^3G = (4x+7)G,$$
$$(3x+1)^4G = (3x+6)G,\quad (3x+1)^5G = (10x+8)G.$$

The outcomes of the hypercomposition within the hyperfield $GF\left[11^2\right]/G$ give rise to the next proposition:

Proposition 14. *The hyperfield* $GF\left[11^2\right]/G$ *is isomorphic to* HF_7^{277}.

In the field $GF[13^2]$, the irreducible polynomial is x^2+2. Thus, in the multiplication of the polynomials, we set $x^2 = -2 = 11$. The multiplicative subgroup of index 6 in $GF[13^2]$ is

$$G = \begin{Bmatrix} 1,\ 5,\ 8,\ 12, \\ 5x+1,\ 8x+1,\ 2x+2,\ 11x+2,\ 3x+3,\ 10x+3,\ 5x+4,\ 8x+4,\ x+5,\ 12x+5, \\ x+6,\ 12x+6,\ x+7,\ 12x+7,\ x+8,\ 12x+8,\ 5x+9,\ 8x+9, \\ 3x+10,\ 10x+10,\ 2x+11,\ 11x+11,\ 5x+12,\ 8x+12 \end{Bmatrix},$$

which has 28 elements, and its cosets are:

$$G,\quad xG,\quad x^2G = 11G,\quad x^3G = 11xG,\quad x^4G = 4G,\quad x^5G = 11xG.$$

Therefore, the proposition holds:

Proposition 15. *The hyperfield* $GF\left[13^2\right]/G$ *is isomorphic to* HF_7^{277}.

A hyperfield of type HF_7^{277} or HF_7^{141} is called a *total hyperfield*.

Example 3. The multiplicative subgroup of index 6 of the field Z_{193} is:

$$G = \begin{Bmatrix} 1,3,8,9,14,23,24,27,42,43,50,64,67,69,72,81,112,121,124, \\ 126,129,143,150,151,166,169,170,179,184,185,190,192 \end{Bmatrix}$$

Its cosets are:

$$G,\ 5G,\ 5^2G,\ 5^3G,\ 5^4G \text{ and } 5^5G,$$

and the hyperfield Z_{193}/G is the total with self-opposite elements, which is isomorphic to HF_7^{277}.

Example 4. The multiplicative subgroup of index 6 of field Z_{199} is:

$$G = \begin{Bmatrix} 1,5,8,18,25,28,40,52,61,62,63,64,90,92,98,103,106,111,114, \\ 116,117,121,123,125,132,139,140,144,157,172,182,187,188 \end{Bmatrix}$$

Its cosets are:

$$G,\ 3G,\ 3^2G,\ 3^3G,\ 3^4G \text{ and } 3^5G,$$

and the hyperfield Z199/*G* is the total with no self-opposite elements, which is isomorphic to HF_7^{141}.

The combination of Propositions 13, 14 and 15 along with Examples 3 and 4, gives rise to the conjecture:

Conjecture: *If the order of the multiplicative subgroup of a finite field exceeds a certain number, then the generated quotient hyperfield is total*.

The next theorems refer to the augmented hyperfields of the above quotient hyperfields. Bear in mind that if $(H,+,\cdot)$ is a field or a hyperfield, then its augmented hyperfield is the hyperfield in which the sum of any two non-zero and non-opposite elements is extended (augmented) to include the two addends (extensive enlargement of the hypercomposition [5, example 8]). Proposition 1 in [28] (see also Proposition 2 in [1]) shows that in the augmented hyperfields the sum of two opposite elements is equal to the entire set *H*. Thus, the augmented hyperfield of a hyperfield is endowed with the following hypercomposition «$\dot{+}$»:

$x \dot{+} y = \{x, y\} \cup x+y,$ for all $x,y \in H^*$, with $y \neq -x$,

$x \dot{+} (-x) = H,$ for all $x \in H^*$,

$x \dot{+} 0 = 0 \dot{+} x = x,$ for all $x \in H$.

The augmented hyperfield of a hyperfield *H* is denoted by [*H*] and, as proved in [28] (Proposition 4) and in [1] (Theorems 4 and 5), if *H* is a field or a quotient hyperfield, then [*H*] is also a quotient hyperfield.

Theorem 19. *The augmented hyperfield of the field* Z7 *is isomorphic to* HF_7^2 *(Table 23)*.

Table 23. The canonical hypergroup of the augmented hyperfield of the field Z7.

$[\mathbb{Z}_7] \cong HF_7^2$	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b}	{1,a,d}	{1,a,b}	{0,1,a,b,c,d,e}	{1,d,e}	{1,c,e}
a	a	{1,a,d}	{a,c}	{a,b,e}	{a,b,c}	{0,1,a,b,c,d,e}	{1,a,e}
b	b	{1,a,b}	{a,b,e}	{b,d}	{1,b,c}	{b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,b,c}	{1,b,c}	{c,e}	{a,c,d}	{c,d,e}
d	d	{1,d,e}	{0,1,a,b,c,d,e}	{b,c,d}	{a,c,d}	{1,d}	{b,d,e}
e	e	{1,c,e}	{1,a,e}	{0,1,a,b,c,d,e}	{c,d,e}	{b,d,e}	{a,e}

Theorem 20. *If G is the multiplicative subgroup of index 6 in the following fields, then:*

i. *The augmented hyperfield of the hyperfield* Z_{13}/G *is isomorphic to* HF_7^{271}, *i.e.,*

$$\left[Z_{13}/G\right] \cong \left[HF_7^{143}\right] \cong HF_7^{271};$$

ii. *The augmented hyperfield of the hyperfield* Z_{19}/G *is isomorphic to* HF_7^{96}, *i.e.,*

$$\left[Z_{19}/G\right] \cong \left[HF_7^{9}\right] \cong HF_7^{96};$$

iii. *The augmented hyperfield of the hyperfield* Z_{31}/G *is isomorphic to* HF_7^{95}, *i.e.,*

$$\left[Z_{31}/G\right] \cong \left[HF_7^{13}\right] \cong HF_7^{95};$$

iv. *The augmented hyperfield of the hyperfield* Z_{37}/G *is isomorphic to* HF_7^{276}, *i.e.,*

$$\left[Z_{37}/G\right] \cong \left[HF_7^{160}\right] \cong HF_7^{276};$$

v. *The augmented hyperfield of the hyperfield* Z_{43}/G *is isomorphic to* HF_7^{110}, *i.e.,*

$$\left[Z_{43}/G\right] \cong \left[HF_7^{61}\right] \cong HF_7^{110};$$

vi. *The augmented hyperfield of the hyperfield* Z_{61}/G *is isomorphic to* HF_7^{277}, *i.e.,*

$$\left[Z_{61}/G\right] \cong \left[HF_7^{234}\right] \cong HF_7^{277};$$

vii. *For the hyperfields* Z_{67}/G, Z_{79}/G *and* Z_{139}/G, *it holds that:*

$$\left[Z_{67}/G\right] \cong \left[Z_{79}/G\right] \cong \left[Z_{139}/G\right] \cong \left[HF_7^{137}\right] \equiv HF_7^{137};$$

viii. *The augmented hyperfield of the hyperfield* Z_{73}/G *is isomorphic to* HF_7^{276}, *i.e.,*

$$\left[Z_{73}/G\right] \cong \left[HF_7^{245}\right] \cong HF_7^{276};$$

ix. *The augmented hyperfield of:*

(a) *the isomorphic hyperfields* Z_{97}/G *and* Z_{157}/G,

(b) *the hyperfield* Z_{109}/G *and*

(c) *the hyperfield* Z_{181}/G

is isomorphic to HF_7^{277}, *that is*

$$\left[Z_{97}/G\right] \cong \left[Z_{157}/G\right] \cong \left[HF_7^{267}\right] \cong HF_7^{277},$$

$$\left[Z_{109}/G\right] \cong \left[HF_7^{246}\right] \cong HF_7^{277},$$

and

$$\left[Z_{181}/G\right]\cong\left[HF_7^{277}\right]\equiv HF_7^{277};$$

x. *For the hyperfields* $Z_{103}/G,\ Z_{127}/G,\ Z_{151}/G,\ Z_{163}/G$ *it holds that:*

$$\left[Z_{103}/G\right]\cong\left[\ Z_{127}/G\right]\cong\left[\ Z_{151}/G\right]\cong\left[\ Z_{163}/G\right]\cong\left[HF_7^{141}\right]\equiv HF_7^{141};$$

xi. *The augmented hyperfield of the hyperfield* $GF\left[5^2\right]/G$ *is isomorphic to* HF_7^{277}*, i.e.,*

$$\left[GF\left[5^2\right]/G\right]\cong\left[HF_7^{142}\right]\equiv HF_7^{277};$$

xii. *The augmented hyperfield of the hyperfield* $GF\left[7^2\right]/G$ *is isomorphic to* HF_7^{275}*, i.e.,*

$$\left[GF\left[7^2\right]/G\right]\cong\left[HF_7^{225}\right]\equiv HF_7^{275}.$$

The proofs for cases (i) through (xi) of the aforementioned theorem are straightforward; however, the proof for the last case is included in Section 10 along with the necessary analysis.

7.5. Classification of hyperfields of order 7

The analysis and study of the various cases, conducted through Propositions 8, 9, 10 and 11, lead to the following theorem, which provides a classification of all the hyperfields of subsections A1–A3 and B1–B5 of Section 10:

Theorem 21.

i. *Subsection A1 consists of the field* Z_7 *and its augmented hyperfield.*

ii. *All hyperfields in subsection A2i are non-quotient hyperfields.*

iii. *All hyperfields in subsection A2ii are non-quotient hyperfields with the exception of* HF_7^{9}*, which is a quotient hyperfield.*

iv. *All hyperfields in subsection A3i are non-quotient hyperfields with the exception of* HF_7^{13}*, which is a quotient hyperfield.*

v. *All hyperfields in subsection A3ii are non-quotient hyperfields with the exception of* HF_7^{61}*, which is a quotient hyperfield.*

vi. *Subsection B1 consists of quotient hyperfields.*

vii. *Subsection B2 consists of non-quotient hyperfields.*

viii. *All hyperfields in subsection B3i are non-quotient hyperfields with the exception of* HF_7^{160}*, which is a quotient hyperfield.*

ix. *All hyperfields in subsections B3ii and B4i are non-quotient hyperfields.*

x. *All hyperfields in subsection B4ii are non-quotient hyperfields with the exception of* HF_7^{225} *and* HF_7^{234}*, which are quotient hyperfields.*

xi. *All hyperfields in subsection B5i are non-quotient hyperfields with the exception of* HF_7^{245} *and* HF_7^{246}*, which are quotient hyperfields.*

xii. *All hyperfields in subsection B5ii are non-quotient hyperfields with the exception of* HF_7^{267}*, which is a quotient hyperfield.*

8. Discussion

In mathematics, a set of axioms is any collection of formally stated assertions that are consistent, can be used to prove other formally stated assertions and none of which is redundant. In this sense, the four additive axioms of the hyperfield are reduced in this paper by one, as it is proved that the axiom of reversibility can derive from the remaining axioms.

This reduction, combined with the development of a suitable algorithm based on the newly established reduced set of axioms, enabled the construction and enumeration of the hyperfields of order 7. Through the construction of these hyperfields, an important family of canonical hypergroups was simultaneously obtained, specifically, those capable of serving as the additive parts of hyperfields with 7 elements. Moreover, since canonical hypergroups are join hypergroups [82] and, when their elements are idempotent, they form join spaces [83,84], a corresponding family of finite join hypergroups was also constructed. The construction of all these hypercompositional structures not only provides insights into their internal algebraic properties but also establishes a solid foundation for further theoretical developments and applications in areas to which they are directly related, such as projective geometry, matroid theory, and tropical geometry.

Furthermore, the study of the isomorphism of the hyperfields of order 7 to the quotient hyperfields supports the conjecture that the multiplicative subgroups of a field have additive properties that other, arbitrary subsets with the same cardinality do not have. In simpler terms, we can reasonably assert that the elements of a finite field's multiplicative subgroups have a specific distribution within the field, such that, when these subgroups exceed a certain minimum order, they can be added to, or subtracted from themselves or from their cosets and generate the field. In light of this conjecture and given that if G is a subgroup of finite index in the multiplicative group of an infinite field F, then $G–G=F$, it is reasonable to raise the question of whether most finite hyperfields are non-quotient ones.

In this paper, also appear properties and examples of skew hyperfields, strongly canonical hyperfields/hyperrings, and superiorly canonical hyperfields/hyperrings that round off and complete the conclusions and results of [1].

9. Conclusions

The development of an equivalent and reduced axiom system for hyperfields is important in both theoretical and practical aspects of the theory of hypercompositional structures. A more concise set of axioms not only simplifies the process of formal proofs but also significantly improves the efficiency of algorithmic approaches, particularly in the enumeration and classification of algebraic structures such as hyperfields. This simplification is of considerable value in computational algebra, where exhaustive searches and structural verifications must be implemented efficiently.

Regarding the enumeration of the hyperfields of order 7, the findings presented in this study provide a complete enumeration of such structures for the first time. It establishes that there exist 277 hyperfields of order 7, including the field Z_7, all of which are presented in Section 10.

Among these 277 hyperfields, 64 are monogene. Using the propositions and theorems established in this paper, the 213 non-monogene hyperfields were classified. It is shown that, apart from the field Z_7, there exist 11 quotient hyperfields and 201 non-quotient hyperfields. Furthermore, it is shown that among the monogene hyperfields, 9 are quotient hyperfields. The analytical classification of these hyperfields is presented in Tables 26 and 27 of Section 10.

The classification of the remaining monogene hyperfields necessitates the development of new theorems, and this remains an open area for further research.

The challenge of constructing, enumerating, and classifying hyperfields with an order exceeding 7 continues to be unresolved.

10. The list of 7-element hyperfields and their classifications

The enumeration of hypercompositional structures has been a research area since the 1980s. The initial publication addressing this subject was authored by M. De Salvo and D. Freni [58]. Subsequently, R. Migliorato focused on the enumeration of 3-element hypergroups [59], with his findings being validated by later studies [60,61], which progressively refined the computational techniques employed in the enumeration process. Since then, this topic has been the subject of numerous papers (e.g., [62–81]). The hyperfields of order less than 6 are enumerated in [1,69–72], from which [1] presents their detailed classification with the necessary extensions and corrections of the previous results and summarizes all of them in its Table 26. Considering the field as a special case of the hyperfield, there are 2 hyperfields of order 2, 5 hyperfields of order 3, and 27 hyperfields of order 5. Interestingly, all the hyperfields of order 2 and 3 are quotient hyperfields [1].

In this Section, we present the hyperfields of order 7, which are produced as described in Section 6 and with the use of the Mathematica [85] packages that are developed in [61,73–76]. A collateral direct consequence is that the corresponding family of canonical 7-element hypergroups is also revealed. Moreover, since every canonical hypergroup is a join one [82], a familly of 277 seven-element join hypergroups is constructed as well.

The symbols **0**, **1**, **a**, **b**, **c**, **d** and **e** are used to denote these hyperfields' elements, where **a** is the generator of their multiplicative subgroup and **b**, **c**, **d** and **e** represent the elements a^2, a^3, a^4, and a^5, respectively.

The table of order 7 hyperfields' multiplicative subgroup is the following (Table 24):

Table 24. The multiplicative subgroup of the hyperfields of order 7.

$\left(HF_7^*,\cdot\right)$	1	a	b=α2	c=α3	d=α4	e=α5
1	1	a	b	c	d	e
a	a	b	c	d	e	1
b=α2	b	c	d	e	1	a
c=α3	c	d	e	1	a	b
d=α4	d	e	1	a	b	c
e=α5	e	1	a	b	c	d

Note that the isomorphic for each hyperfield in this Section can be obtained by interchanging **a** with **e,** and **b** with **d**. For example, the augmented hyperfield of HF_7^{225}, referenced in Theorem 20.xii, has the canonical additive hypergroup of Table 25:

Table 25. The canonical hypergroup of the augmented hyperfield $\left[HF_7^{225}\right]$ of HF_7^{225}.

$\left[HF_7^{225}\right]$	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}
b	b	{1,a,b,c,d}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d,e}
d	d	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,d,e}	{0,1,a,b,c,d,e}

This hyperfield is not among the ones in the list below. However, the list contains its isomorphic hyperfield HF_7^{275}, which is obtained from $\left[HF_7^{225}\right]$ by interchanging **a** with **e** and **b** with **d** in the above Cayley table.

The categories in which the hyperfields of order 7 are classified in the following list are:

A. The hyperfields of order 7, which do not have self-opposite elements. For these, $0 \in 1+c$.

B. The hyperfields of order 7 with self-opposite elements. For these, $0 \in 1+1$.

The above categories were divided into subcategories according to the number of elements included in the difference *x*-*x*. Observe that when card(*x*-*x*)=7, then the hypercomposition is closed, i.e., $x,y \in x+y$ (see Proposition 1 in [28], Proposition 2 in [1]). The following Tables 26 and 27 summarize the number of hyperfields of order 7 in each category and subcategory.

Table 26. The class of hyperfields of order 7 with no self-opposite elements.

A. The class of hyperfields with no self-opposite elements	Number of elements: 141		
Cardinality of x-x	Number of hyperfields	Number of quotient hyperfields	Number of non-quotient hyperfields
1	1 (the field $\mathbb{Z}_7$)	-	-
2	0	0	0
3	9	1	8
4	0	0	0
5	77	2	75
6	0	0	0
7 *(the monogene hyperfields)*	54	at least 5	unknown number

Table 27. The class of hyperfields of order 7 with self-opposite elements.

B. The class of hyperfields with self-opposite elements	Number of elements: 136		
Cardinality of x-x	Number of hyperfields	Number of quotient hyperfields	Number of non-quotient hyperfields
1	0	0	0
2	2	2	0
3	9	0	9
4	40	1	39
5	46	2	44
6	29	3	26
7 *(the monogene hyperfields)*	10	at least 4	unknown number

A. The hyperfields which do not have self-opposite elements

A1. The field $\mathbb{Z}_7$ and its augmented hyperfield.

$HF_7^1 = \mathbb{Z}_7$	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	b	d	a	0	e	c
a	a	d	c	e	b	0	1
b	b	a	e	d	1	c	0
c	c	0	b	1	e	a	d
d	d	e	0	c	a	1	b
e	e	c	1	0	d	b	a

$HF_7^2 = [\mathbb{Z}_7]$	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b}	{1,a,d}	{1,a,b}	{0,1,a,b,c,d,e}	{1,d,e}	{1,c,e}
a	a	{1,a,d}	{a,c}	{a,b,e}	{a,b,c}	{0,1,a,b,c,d,e}	{1,a,e}
b	b	{1,a,b}	{a,b,e}	{b,d}	{1,b,c}	{b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,b,c}	{1,b,c}	{c,e}	{a,c,d}	{c,d,e}
d	d	{1,d,e}	{0,1,a,b,c,d,e}	{b,c,d}	{a,c,d}	{1,d}	{b,d,e}
e	e	{1,c,e}	{1,a,e}	{0,1,a,b,c,d,e}	{c,d,e}	{b,d,e}	{a,e}

A2i. Hyperfields for which $card(x-x)=3$ and $x,-x \in x-x$, for every non-zero element x.

HF_7^3	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{b,c,d,e}	{a,c,d,e}	{0,1,c}	{a,b,c,e}	{a,b,c,d}
a	a	{b,c,d,e}	{1,a,b,c,e}	{1,c,d,e}	{1,b,d,e}	{0,a,d}	{1,b,c,d}
b	b	{a,c,d,e}	{1,c,d,e}	{1,a,b,c,d}	{1,a,d,e}	{1,a,c,e}	{0,b,e}
c	c	{0,1,c}	{1,b,d,e}	{1,a,d,e}	{a,b,c,d,e}	{1,a,b,e}	{1,a,b,d}
d	d	{a,b,c,e}	{0,a,d}	{1,a,c,e}	{1,a,b,e}	{1,b,c,d,e}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d}	{0,b,e}	{1,a,b,d}	{1,a,b,c}	{1,a,c,d,e}

HF_7^4	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{b,c,d,e}	{a,c,d,e}	{0,1,c}	{a,b,c,e}	{a,b,c,d}
a	a	{b,c,d,e}	{1,a,b,c,d,e}	{1,c,d,e}	{1,b,d,e}	{0,a,d}	{1,b,c,d}
b	b	{a,c,d,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,a,d,e}	{1,a,c,e}	{0,b,e}
c	c	{0,1,c}	{1,b,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,b,d}
d	d	{a,b,c,e}	{0,a,d}	{1,a,c,e}	{1,a,b,e}	{1,a,b,c,d,e}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d}	{0,b,e}	{1,a,b,d}	{1,a,b,c}	{1,a,b,c,d,e}

A2ii. Hyperfields for which $card(x-x)=3$ and $x,-x \notin x-x$, for every non-zero element x.

HF_7^5	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,e}	{1,b,d,e}	{a,b,c,d,e}	{0,a,d}	{1,a,b,c,e}	{a,c,d,e}
a	a	{1,b,d,e}	{1,b,c,d}	{1,a,c,e}	{1,b,c,d,e}	{0,b,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,e}	{a,c,d,e}	{1,a,b,d}	{1,a,c,d,e}	{0,1,c}
c	c	{0,a,d}	{1,b,c,d,e}	{1,a,b,d}	{1,b,d,e}	{a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,b,e}	{1,a,c,d,e}	{a,b,c,e}	{1,a,c,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c,d}	{0,1,c}	{1,a,b,d,e}	{1,b,c,d}	{1,a,b,d}

HF_7^6	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d,e}	{1,b,c,d,e}	{b,c,d,e}	{0,a,d}	{1,a,b,c}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d,e}	{1,c,d,e}	{0,b,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c,d,e}	{1,a,c,d,e}	{1,a,b,d,e}	{1,a,d,e}	{0,1,c}
c	c	{0,a,d}	{1,c,d,e}	{1,a,b,d,e}	{1,a,b,d,e}	{1,a,b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,b,e}	{1,a,d,e}	{1,a,b,c,e}	{1,a,b,c,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{a,b,c,d}	{0,1,c}	{1,a,b,e}	{1,a,b,c,d}	{1,a,b,c,d}

HF_7^7	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d,e}	{1,b,c,d,e}	{a,b,c,d,e}	{0,a,d}	{1,a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d,e}	{1,b,c,d,e}	{0,b,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,d,e}	{1,a,c,d,e}	{1,a,b,d,e}	{1,a,c,d,e}	{0,1,c}
c	c	{0,a,d}	{1,b,c,d,e}	{1,a,b,d,e}	{1,a,b,d,e}	{1,a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,b,e}	{1,a,c,d,e}	{1,a,b,c,e}	{1,a,b,c,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d}	{0,1,c}	{1,a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,d}

HF_7^8	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,e}	{1,b,d,e}	{b,c,d,e}	{0,a,d}	{1,a,b,c}	{a,c,d,e}
a	a	{1,b,d,e}	{1,b,c,d}	{1,a,c,e}	{1,c,d,e}	{0,b,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c,e}	{a,c,d,e}	{1,a,b,d}	{1,a,d,e}	{0,1,c}
c	c	{0,a,d}	{1,c,d,e}	{1,a,b,d}	{1,b,d,e}	{a,b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,b,e}	{1,a,d,e}	{a,b,c,e}	{1,a,c,e}	{1,b,c,d}
e	e	{a,c,d,e}	{a,b,c,d}	{0,1,c}	{1,a,b,e}	{1,b,c,d}	{1,a,b,d}

HF_7^9	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,c}	{a,b,e}	{1,d,e}	{0,b,e}	{b,c,d}	{1,a,d}
a	a	{a,b,e}	{b,d}	{1,b,c}	{1,a,e}	{0,1,c}	{c,d,e}
b	b	{1,d,e}	{1,b,c}	{c,e}	{a,c,d}	{1,a,b}	{0,a,d}
c	c	{0,b,e}	{1,a,e}	{a,c,d}	{1,d}	{b,d,e}	{a,b,c}
d	d	{b,c,d}	{0,1,c}	{1,a,b}	{b,d,e}	{a,e}	{1,c,e}
e	e	{1,a,d}	{c,d,e}	{0,a,d}	{a,b,c}	{1,c,e}	{1,b}

HF_7^{10}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,c,d}	{a,b,c,e}	{1,a,d,e}	{0,b,e}	{b,c,d,e}	{1,a,b,d}
a	a	{a,b,c,e}	{b,d,e}	{1,b,c,d}	{1,a,b,e}	{0,1,c}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,d}	{1,c,e}	{a,c,d,e}	{1,a,b,c}	{0,a,d}
c	c	{0,b,e}	{1,a,b,e}	{a,c,d,e}	{1,a,d}	{1,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,c}	{1,a,b,c}	{1,b,d,e}	{a,b,e}	{1,a,c,e}
e	e	{1,a,b,d}	{1,c,d,e}	{0,a,d}	{a,b,c,d}	{1,a,c,e}	{1,b,c}

HF_7^{11}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d}	{a,b,c,d,e}	{1,a,d,e}	{0,b,e}	{b,c,d,e}	{1,a,b,c,d}
a	a	{a,b,c,d,e}	{b,c,d,e}	{1,b,c,d,e}	{1,a,b,e}	{0,1,c}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,d,e}	{1,c,d,e}	{1,a,c,d,e}	{1,a,b,c}	{0,a,d}
c	c	{0,b,e}	{1,a,b,e}	{1,a,c,d,e}	{1,a,d,e}	{1,a,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,c}	{1,a,b,c}	{1,a,b,d,e}	{1,a,b,e}	{1,a,b,c,e}
e	e	{1,a,b,c,d}	{1,c,d,e}	{0,a,d}	{a,b,c,d}	{1,a,b,c,e}	{1,a,b,c}

A3i. Hyperfields for which $card(x-x)=5$ and $x,-x \in x-x$, for every non-zero element x.

HF_7^{12}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b}	{1,b,d,e}	{b,d,e}	{0,1,a,c,d}	{1,b,c}	{a,c,d,e}
a	a	{1,b,d,e}	{a,b,c}	{1,a,c,e}	{1,c,e}	{0,a,b,d,e}	{a,c,d}
b	b	{b,d,e}	{1,a,c,e}	{b,c,d}	{1,a,b,d}	{1,a,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,e}	{1,a,b,d}	{c,d,e}	{a,b,c,e}	{a,b,e}
d	d	{1,b,c}	{0,a,b,d,e}	{1,a,d}	{a,b,c,e}	{1,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{a,c,d}	{0,1,b,c,e}	{a,b,e}	{1,b,c,d}	{1,a,e}

HF_7^{13}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b}	{1,b,d,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,c,d,e}
a	a	{1,b,d,e}	{a,b,c}	{1,a,c,e}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,e}	{b,c,d}	{1,a,b,d}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{1,a,b,d}	{c,d,e}	{a,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{a,b,c,e}	{1,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,b,c,d}	{1,a,e}

HF_7^{14}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c}	{1,b,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,d,e}
a	a	{1,b,e}	{a,b,d}	{1,a,c}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c}	{b,c,e}	{a,b,d}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{a,b,d}	{1,c,d}	{b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{b,c,e}	{a,d,e}	{1,c,d}
e	e	{a,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,c,d}	{1,b,e}

HF_7^{15}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,d}	{1,b,c,e}	{b,d,e}	{0,1,a,c,d}	{1,b,c}	{a,b,d,e}
a	a	{1,b,c,e}	{a,b,e}	{1,a,c,d}	{1,c,e}	{0,a,b,d,e}	{a,c,d}
b	b	{b,d,e}	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{1,a,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}	{a,b,e}
d	d	{1,b,c}	{0,a,b,d,e}	{1,a,d}	{1,b,c,e}	{b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{a,c,d}	{0,1,b,c,e}	{a,b,e}	{1,a,c,d}	{1,c,e}

HF_7^{16}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}
a	a	{1,b,c,e}	{a,b,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,c,e}

HF_7^{17}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,e}	{1,b,e}	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,d,e}
a	a	{1,b,e}	{1,a,b}	{1,a,c}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c}	{a,b,c}	{a,b,d}	{1,a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d,e}	{a,b,d}	{b,c,d}	{b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,a,b,d,e}	{1,a,d,e}	{b,c,e}	{c,d,e}	{1,c,d}
e	e	{a,d,e}	{a,b,c,d}	{0,1,b,c,e}	{1,a,b,e}	{1,c,d}	{1,d,e}

HF_7^{18}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,e}	{1,b,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,d,e}
a	a	{1,b,e}	{1,a,b}	{1,a,c}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c}	{a,b,c}	{a,b,d}	{1,a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d,e}	{a,b,d}	{b,c,d}	{b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{b,c,e}	{c,d,e}	{1,c,d}
e	e	{a,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,c,d}	{1,d,e}

HF_7^{19}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,c}	{1,b,d,e}	{a,b,d}	{0,1,a,c,d}	{1,b,e}	{a,c,d,e}
a	a	{1,b,d,e}	{a,c,d}	{1,a,c,e}	{b,c,e}	{0,a,b,d,e}	{1,a,c}
b	b	{a,b,d}	{1,a,c,e}	{b,d,e}	{1,a,b,d}	{1,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{b,c,e}	{1,a,b,d}	{1,c,e}	{a,b,c,e}	{a,d,e}
d	d	{1,b,e}	{0,a,b,d,e}	{1,c,d}	{a,b,c,e}	{1,a,d}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,c}	{0,1,b,c,e}	{a,d,e}	{1,b,c,d}	{a,b,e}

HF_7^{20}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,d}	{1,b,c,d,e}	{b,d}	{0,1,a,c,d}	{1,b}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{a,c,e}	{1,a,c,d,e}	{c,e}	{0,a,b,d,e}	{a,c}
b	b	{b,d}	{1,a,c,d,e}	{1,b,d}	{1,a,b,d,e}	{1,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{c,e}	{1,a,b,d,e}	{a,c,e}	{1,a,b,c,e}	{a,e}
d	d	{1,b}	{0,a,b,d,e}	{1,d}	{1,a,b,c,e}	{1,b,d}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{a,c}	{0,1,b,c,e}	{a,e}	{1,a,b,c,d}	{a,c,e}

HF_7^{21}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c}	{1,b,d,e}	{b,d,e}	{0,1,a,c,d}	{1,b,c}	{a,c,d,e}
a	a	{1,b,d,e}	{a,b,c,d}	{1,a,c,e}	{1,c,e}	{0,a,b,d,e}	{a,c,d}
b	b	{b,d,e}	{1,a,c,e}	{b,c,d,e}	{1,a,b,d}	{1,a,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,e}	{1,a,b,d}	{1,c,d,e}	{a,b,c,e}	{a,b,e}
d	d	{1,b,c}	{0,a,b,d,e}	{1,a,d}	{a,b,c,e}	{1,a,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{a,c,d}	{0,1,b,c,e}	{a,b,e}	{1,b,c,d}	{1,a,b,e}

HF_7^{22}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c}	{1,b,d,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,c,d,e}
a	a	{1,b,d,e}	{a,b,c,d}	{1,a,c,e}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,e}	{b,c,d,e}	{1,a,b,d}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{1,a,b,d}	{1,c,d,e}	{a,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{a,b,c,e}	{1,a,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,b,c,d}	{1,a,b,e}

HF_7^{23}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d}	{1,b,c,d,e}	{b,d,e}	{0,1,a,c,d}	{1,b,c}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{a,b,c,e}	{1,a,c,d,e}	{1,c,e}	{0,a,b,d,e}	{a,c,d}
b	b	{b,d,e}	{1,a,c,d,e}	{1,b,c,d}	{1,a,b,d,e}	{1,a,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,e}	{1,a,b,d,e}	{a,c,d,e}	{1,a,b,c,e}	{a,b,e}
d	d	{1,b,c}	{0,a,b,d,e}	{1,a,d}	{1,a,b,c,e}	{1,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{a,c,d}	{0,1,b,c,e}	{a,b,e}	{1,a,b,c,d}	{1,a,c,e}

HF_7^{24}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d}	{1,b,c,d,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{a,b,c,e}	{1,a,c,d,e}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,d}	{1,a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{1,a,b,d,e}	{a,c,d,e}	{1,a,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,a,b,c,e}	{1,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{1,a,c,e}

HF_7^{25}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{1,b,d,e}	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,c,d,e}
a	a	{1,b,d,e}	{1,a,b,c}	{1,a,c,e}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c,e}	{a,b,c,d}	{1,a,b,d}	{1,a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d,e}	{1,a,b,d}	{b,c,d,e}	{a,b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,a,b,d,e}	{1,a,d,e}	{a,b,c,e}	{1,c,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{a,b,c,d}	{0,1,b,c,e}	{1,a,b,e}	{1,b,c,d}	{1,a,d,e}

HF_7^{26}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{1,b,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,c,d,e}
a	a	{1,b,d,e}	{1,a,b,c}	{1,a,c,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,e}	{a,b,c,d}	{1,a,b,d}	{1,a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d,e}	{1,a,b,d}	{b,c,d,e}	{a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{a,b,c,e}	{1,c,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,b,c,d}	{1,a,d,e}

HF_7^{27}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,d}	{1,b,c,e}	{b,d,e}	{0,1,a,c,d}	{1,b,c}	{a,b,d,e}
a	a	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,c,e}	{0,a,b,d,e}	{a,c,d}
b	b	{b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,e}
d	d	{1,b,c}	{0,a,b,d,e}	{1,a,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{a,c,d}	{0,1,b,c,e}	{a,b,e}	{1,a,c,d}	{1,b,c,e}

HF_7^{28}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}
a	a	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}

HF_7^{29}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,e}	{1,b,e}	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,d,e}
a	a	{1,b,e}	{1,a,b,d}	{1,a,c}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c}	{a,b,c,e}	{a,b,d}	{1,a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d,e}	{a,b,d}	{1,b,c,d}	{b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,a,b,d,e}	{1,a,d,e}	{b,c,e}	{a,c,d,e}	{1,c,d}
e	e	{a,d,e}	{a,b,c,d}	{0,1,b,c,e}	{1,a,b,e}	{1,c,d}	{1,b,d,e}

HF_7^{30}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,e}	{1,b,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,d,e}
a	a	{1,b,e}	{1,a,b,d}	{1,a,c}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c}	{a,b,c,e}	{a,b,d}	{1,a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d,e}	{a,b,d}	{1,b,c,d}	{b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{b,c,e}	{a,c,d,e}	{1,c,d}
e	e	{a,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,c,d}	{1,b,d,e}

HF_7^{31}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,c,d}	{1,b,c,d,e}	{a,b,d}	{0,1,a,c,d}	{1,b,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{a,c,d,e}	{1,a,c,d,e}	{b,c,e}	{0,a,b,d,e}	{1,a,c}
b	b	{a,b,d}	{1,a,c,d,e}	{1,b,d,e}	{1,a,b,d,e}	{1,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{b,c,e}	{1,a,b,d,e}	{1,a,c,e}	{1,a,b,c,e}	{a,d,e}
d	d	{1,b,e}	{0,a,b,d,e}	{1,c,d}	{1,a,b,c,e}	{1,a,b,d}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,c}	{0,1,b,c,e}	{a,d,e}	{1,a,b,c,d}	{a,b,c,e}

HF_7^{32}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d}	{1,b,c,d,e}	{b,d,e}	{0,1,a,c,d}	{1,b,c}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d,e}	{1,c,e}	{0,a,b,d,e}	{a,c,d}
b	b	{b,d,e}	{1,a,c,d,e}	{1,b,c,d,e}	{1,a,b,d,e}	{1,a,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,e}	{1,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,e}	{a,b,e}
d	d	{1,b,c}	{0,a,b,d,e}	{1,a,d}	{1,a,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{a,c,d}	{0,1,b,c,e}	{a,b,e}	{1,a,b,c,d}	{1,a,b,c,e}

HF_7^{33}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d}	{1,b,c,d,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,a,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,e}

HF_7^{34}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{1,b,d,e}	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,c,d,e}
a	a	{1,b,d,e}	{1,a,b,c,d}	{1,a,c,e}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}	{1,a,b,d}	{1,a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d,e}	{1,a,b,d}	{1,b,c,d,e}	{a,b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,a,b,d,e}	{1,a,d,e}	{a,b,c,e}	{1,a,c,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{a,b,c,d}	{0,1,b,c,e}	{1,a,b,e}	{1,b,c,d}	{1,a,b,d,e}

HF_7^{35}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{1,b,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,c,d,e}
a	a	{1,b,d,e}	{1,a,b,c,d}	{1,a,c,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}	{1,a,b,d}	{1,a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d,e}	{1,a,b,d}	{1,b,c,d,e}	{a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{a,b,c,e}	{1,a,c,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,b,c,d}	{1,a,b,d,e}

HF_7^{36}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,c,d}	{b,c,e}	{0,1,a,c,d}	{1,a,c}	{b,c,e}
a	a	{1,c,d}	{1,a,b,c,e}	{a,d,e}	{1,c,d}	{0,a,b,d,e}	{a,b,d}
b	b	{b,c,e}	{a,d,e}	{1,a,b,c,d}	{1,b,e}	{a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d}	{1,b,e}	{a,b,c,d,e}	{1,a,c}	{1,b,e}
d	d	{1,a,c}	{0,a,b,d,e}	{a,d,e}	{1,a,c}	{1,b,c,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d}	{0,1,b,c,e}	{1,b,e}	{a,b,d}	{1,a,c,d,e}

HF_7^{37}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,c,d}	{a,b,c,e}	{0,1,a,c,d}	{1,a,c,e}	{b,c,e}
a	a	{1,c,d}	{1,a,b,c,e}	{a,d,e}	{1,b,c,d}	{0,a,b,d,e}	{1,a,b,d}
b	b	{a,b,c,e}	{a,d,e}	{1,a,b,c,d}	{1,b,e}	{a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d}	{1,b,e}	{a,b,c,d,e}	{1,a,c}	{1,b,d,e}
d	d	{1,a,c,e}	{0,a,b,d,e}	{a,c,d,e}	{1,a,c}	{1,b,c,d,e}	{a,b,d}
e	e	{b,c,e}	{1,a,b,d}	{0,1,b,c,e}	{1,b,d,e}	{a,b,d}	{1,a,c,d,e}

HF_7^{38}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,b,c,d,e}	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{1,a,b,c,e}	{1,a,c,d,e}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{1,a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,a,b,d,e}	{1,a,d,e}	{1,a,b,c,e}	{1,b,c,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{a,b,c,d}	{0,1,b,c,e}	{1,a,b,e}	{1,a,b,c,d}	{1,a,c,d,e}

HF_7^{39}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,b,c,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{1,a,b,c,e}	{1,a,c,d,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{1,a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,e}	{1,b,c,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d}	{1,a,c,d,e}

HF_7^{40}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,c,d}	{b,c,e}	{0,1,a,c,d}	{1,a,c}	{b,c,e}
a	a	{1,c,d}	{1,a,b,c,d,e}	{a,d,e}	{1,c,d}	{0,a,b,d,e}	{a,b,d}
b	b	{b,c,e}	{a,d,e}	{1,a,b,c,d,e}	{1,b,e}	{a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d}	{1,b,e}	{1,a,b,c,d,e}	{1,a,c}	{1,b,e}
d	d	{1,a,c}	{0,a,b,d,e}	{a,d,e}	{1,a,c}	{1,a,b,c,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d}	{0,1,b,c,e}	{1,b,e}	{a,b,d}	{1,a,b,c,d,e}

HF_7^{41}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,c,d}	{a,b,c,e}	{0,1,a,c,d}	{1,a,c,e}	{b,c,e}
a	a	{1,c,d}	{1,a,b,c,d,e}	{a,d,e}	{1,b,c,d}	{0,a,b,d,e}	{1,a,b,d}
b	b	{a,b,c,e}	{a,d,e}	{1,a,b,c,d,e}	{1,b,e}	{a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d}	{1,b,e}	{1,a,b,c,d,e}	{1,a,c}	{1,b,d,e}
d	d	{1,a,c,e}	{0,a,b,d,e}	{a,c,d,e}	{1,a,c}	{1,a,b,c,d,e}	{a,b,d}
e	e	{b,c,e}	{1,a,b,d}	{0,1,b,c,e}	{1,b,d,e}	{a,b,d}	{1,a,b,c,d,e}

HF_7^{42}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,b,c,d,e}	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}
b	b	{b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,e}
d	d	{1,a,b,c}	{0,a,b,d,e}	{1,a,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{a,b,c,d}	{0,1,b,c,e}	{1,a,b,e}	{1,a,b,c,d}	{1,a,b,c,d,e}

HF_7^{43}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,b,c,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d,e}	{0,1,b,c,e}
c	c	{0,1,a,c,d}	{1,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}

HF_7^{44}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b}	{a,b,d,e}	{1,d}	{0,1,b,c,e}	{b,d}	{1,a,c,d}
a	a	{a,b,d,e}	{a,c}	{1,b,c,e}	{a,e}	{0,1,a,c,d}	{c,e}
b	b	{1,d}	{1,b,c,e}	{b,d}	{1,a,c,d}	{1,b}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{a,e}	{1,a,c,d}	{c,e}	{a,b,d,e}	{a,c}
d	d	{b,d}	{0,1,a,c,d}	{1,b}	{a,b,d,e}	{1,d}	{1,b,c,e}
e	e	{1,a,c,d}	{c,e}	{0,a,b,d,e}	{a,c}	{1,b,c,e}	{a,e}

HF_7^{45}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b}	{a,b,d,e}	{1,a,d,e}	{0,1,b,c,e}	{b,c,d,e}	{1,a,c,d}
a	a	{a,b,d,e}	{a,b,c}	{1,b,c,e}	{1,a,b,e}	{0,1,a,c,d}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,e}	{b,c,d}	{1,a,c,d}	{1,a,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,e}	{1,a,c,d}	{c,d,e}	{a,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{1,d,e}	{1,b,c,e}
e	e	{1,a,c,d}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}	{1,b,c,e}	{1,a,e}

HF_7^{46}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c}	{a,b,e}	{1,a,d,e}	{0,1,b,c,e}	{b,c,d,e}	{1,a,d}
a	a	{a,b,e}	{a,b,d}	{1,b,c}	{1,a,b,e}	{0,1,a,c,d}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c}	{b,c,e}	{a,c,d}	{1,a,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,e}	{a,c,d}	{1,c,d}	{b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{b,d,e}	{a,d,e}	{1,c,e}
e	e	{1,a,d}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}	{1,c,e}	{1,b,e}

HF_7^{47}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,d}	{a,b,c,e}	{1,a,d,e}	{0,1,b,c,e}	{b,c,d,e}	{1,a,b,d}
a	a	{a,b,c,e}	{a,b,e}	{1,b,c,d}	{1,a,b,e}	{0,1,a,c,d}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,d}	{1,b,c}	{a,c,d,e}	{1,a,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,e}	{a,c,d,e}	{a,c,d}	{1,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{1,b,d,e}	{b,d,e}	{1,a,c,e}
e	e	{1,a,b,d}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}	{1,a,c,e}	{1,c,e}

HF_7^{48}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,c}	{a,b,d,e}	{1,a,d}	{0,1,b,c,e}	{b,d,e}	{1,a,c,d}
a	a	{a,b,d,e}	{a,c,d}	{1,b,c,e}	{a,b,e}	{0,1,a,c,d}	{1,c,e}
b	b	{1,a,d}	{1,b,c,e}	{b,d,e}	{1,a,c,d}	{1,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{a,b,e}	{1,a,c,d}	{1,c,e}	{a,b,d,e}	{a,c,d}
d	d	{b,d,e}	{0,1,a,c,d}	{1,b,c}	{a,b,d,e}	{1,a,d}	{1,b,c,e}
e	e	{1,a,c,d}	{1,c,e}	{0,a,b,d,e}	{a,c,d}	{1,b,c,e}	{a,b,e}

HF_7^{49}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}	{0,1,b,c,e}	{b,c,d,e}	{1,a,c,d}
a	a	{a,b,d,e}	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{0,1,a,c,d}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}	{1,a,c,d}	{1,a,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}	{1,b,c,e}
e	e	{1,a,c,d}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}

HF_7^{50}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d}	{a,b,c,d,e}	{1,a,d,e}	{0,1,b,c,e}	{b,c,d,e}	{1,a,b,c,d}
a	a	{a,b,c,d,e}	{a,b,c,e}	{1,b,c,d,e}	{1,a,b,e}	{0,1,a,c,d}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,d,e}	{1,b,c,d}	{1,a,c,d,e}	{1,a,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,e}	{1,a,c,d,e}	{a,c,d,e}	{1,a,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{1,a,b,d,e}	{1,b,d,e}	{1,a,b,c,e}
e	e	{1,a,b,c,d}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}	{1,a,b,c,e}	{1,a,c,e}

HF_7^{51}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}	{0,1,b,c,e}	{a,b,c,d}	{1,a,c,d}
a	a	{a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,d,e}	{0,1,a,c,d}	{b,c,d,e}
b	b	{1,c,d,e}	{1,b,c,e}	{a,b,c,d}	{1,a,c,d}	{1,a,b,e}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}
d	d	{a,b,c,d}	{0,1,a,c,d}	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}	{1,b,c,e}
e	e	{1,a,c,d}	{b,c,d,e}	{0,a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,d,e}

HF_7^{52}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{a,b,d,e}	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}
a	a	{a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,b,d,e}	{0,1,a,c,d}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d}	{1,a,c,d}	{1,a,b,c,e}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,c,d,e}	{1,b,c,e}
e	e	{1,a,c,d}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,d,e}

HF_7^{53}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,d}	{a,b,c,e}	{1,a,d,e}	{0,1,b,c,e}	{b,c,d,e}	{1,a,b,d}
a	a	{a,b,c,e}	{a,b,d,e}	{1,b,c,d}	{1,a,b,e}	{0,1,a,c,d}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,d}	{1,b,c,e}	{a,c,d,e}	{1,a,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,e}	{a,c,d,e}	{1,a,c,d}	{1,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{1,b,d,e}	{a,b,d,e}	{1,a,c,e}
e	e	{1,a,b,d}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}	{1,a,c,e}	{1,b,c,e}

HF_7^{54}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d}	{a,b,c,d,e}	{1,a,d,e}	{0,1,b,c,e}	{b,c,d,e}	{1,a,b,c,d}
a	a	{a,b,c,d,e}	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,e}	{0,1,a,c,d}	{1,c,d,e}
b	b	{1,a,d,e}	{1,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,e}	{1,a,c,d,e}	{1,a,c,d,e}	{1,a,b,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{0,1,a,c,d}	{1,a,b,c}	{1,a,b,d,e}	{1,a,b,d,e}	{1,a,b,c,e}
e	e	{1,a,b,c,d}	{1,c,d,e}	{0,a,b,d,e}	{a,b,c,d}	{1,a,b,c,e}	{1,a,b,c,e}

HF_7^{55}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{a,b,d,e}	{1,c,d,e}	{0,1,b,c,e}	{a,b,c,d}	{1,a,c,d}
a	a	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,d,e}	{0,1,a,c,d}	{b,c,d,e}
b	b	{1,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,e}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c}
d	d	{a,b,c,d}	{0,1,a,c,d}	{1,a,b,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}
e	e	{1,a,c,d}	{b,c,d,e}	{0,a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,b,d,e}

HF_7^{56}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}
a	a	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{0,1,a,c,d}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{0,a,b,d,e}
c	c	{0,1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}
e	e	{1,a,c,d}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}

A3ii. Hyperfields for which $card(x-x)=5$ and $x,-x \notin x-x$, for every non-zero element x.

HF_7^{57}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{b,d}	{1,a,c,d}	{1,b}	{0,a,b,d,e}	{1,d}	{1,b,c,e}
a	a	{1,a,c,d}	{c,e}	{a,b,d,e}	{a,c}	{0,1,b,c,e}	{a,e}
b	b	{1,b}	{a,b,d,e}	{1,d}	{1,b,c,e}	{b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{a,c}	{1,b,c,e}	{a,e}	{1,a,c,d}	{c,e}
d	d	{1,d}	{0,1,b,c,e}	{b,d}	{1,a,c,d}	{1,b}	{a,b,d,e}
e	e	{1,b,c,e}	{a,e}	{0,1,a,c,d}	{c,e}	{a,b,d,e}	{a,c}

HF_7^{58}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{b,d}	{1,a,b,c,d,e}	{1,b,d}	{0,a,b,d,e}	{1,b,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{c,e}	{1,a,b,c,d,e}	{a,c,e}	{0,1,b,c,e}	{a,c,e}
b	b	{1,b,d}	{1,a,b,c,d,e}	{1,d}	{1,a,b,c,d,e}	{1,b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{a,c,e}	{1,a,b,c,d,e}	{a,e}	{1,a,b,c,d,e}	{a,c,e}
d	d	{1,b,d}	{0,1,b,c,e}	{1,b,d}	{1,a,b,c,d,e}	{1,b}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,c,e}	{0,1,a,c,d}	{a,c,e}	{1,a,b,c,d,e}	{a,c}

HF_7^{59}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c}	{1,a,d}	{1,a,b,e}	{0,a,b,d,e}	{1,c,d,e}	{1,c,e}
a	a	{1,a,d}	{b,c,d}	{a,b,e}	{1,a,b,c}	{0,1,b,c,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,e}	{c,d,e}	{1,b,c}	{a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c}	{1,b,c}	{1,d,e}	{a,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,b,c,e}	{a,b,c,d}	{a,c,d}	{1,a,e}	{b,d,e}
e	e	{1,c,e}	{1,a,d,e}	{0,1,a,c,d}	{b,c,d,e}	{b,d,e}	{1,a,b}

HF_7^{60}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c}	{1,a,b,d,e}	{1,b,d,e}	{0,a,b,d,e}	{1,b,c,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{b,c,d}	{1,a,b,c,e}	{1,a,c,e}	{0,1,b,c,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,e}	{c,d,e}	{1,a,b,c,d}	{1,a,b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,e}	{1,a,b,c,d}	{1,d,e}	{a,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,b,c,e}	{1,a,b,d}	{a,b,c,d,e}	{1,a,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,c,d,e}	{0,1,a,c,d}	{a,b,c,e}	{1,b,c,d,e}	{1,a,b}

HF_7^{61}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c}	{1,a,b,d,e}	{1,a,b,d,e}	{0,a,b,d,e}	{1,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{b,c,d}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,b,c,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,e}	{c,d,e}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,e}	{1,a,b,c,d}	{1,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d}	{a,b,c,d,e}	{1,a,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,c,d,e}	{0,1,a,c,d}	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,b}

HF_7^{62}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d}	{1,a,c,d}	{1,b,e}	{0,a,b,d,e}	{1,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{b,c,e}	{a,b,d,e}	{1,a,c}	{0,1,b,c,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d,e}	{1,c,d}	{1,b,c,e}	{a,b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c}	{1,b,c,e}	{a,d,e}	{1,a,c,d}	{b,c,e}
d	d	{1,c,d}	{0,1,b,c,e}	{a,b,d}	{1,a,c,d}	{1,b,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,d,e}	{0,1,a,c,d}	{b,c,e}	{a,b,d,e}	{1,a,c}

HF_7^{63}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d}	{1,a,c,d}	{1,a,b,e}	{0,a,b,d,e}	{1,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{b,c,e}	{a,b,d,e}	{1,a,b,c}	{0,1,b,c,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,d,e}	{1,c,d}	{1,b,c,e}	{a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{a,d,e}	{1,a,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,b,c,e}	{a,b,c,d}	{1,a,c,d}	{1,b,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,d,e}	{0,1,a,c,d}	{b,c,d,e}	{a,b,d,e}	{1,a,c}

HF_7^{64}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{0,a,b,d,e}	{1,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{0,1,b,c,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,d,e}	{1,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,d,e}	{1,a,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,b,c,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,c,d,e}	{0,1,a,c,d}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c}

HF_7^{65}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,a,b,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,b,c,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{a,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,b,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,c,d}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c}

HF_7^{66}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,e}	{1,a,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,c,e}
a	a	{1,a,d}	{1,b,c}	{a,b,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,e}	{a,c,d}	{1,b,c}	{a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d}	{1,b,c}	{b,d,e}	{a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{a,c,d}	{1,c,e}	{b,d,e}
e	e	{1,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{b,d,e}	{1,a,d}

HF_7^{67}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,e}	{1,a,d}	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,c,e}
a	a	{1,a,d}	{1,b,c}	{a,b,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,e}	{a,c,d}	{1,b,c}	{a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d}	{1,b,c}	{b,d,e}	{a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}	{a,c,d}	{1,c,e}	{b,d,e}
e	e	{1,c,e}	{1,a,b,d,e}	{0,1,a,c,d}	{1,b,c,d,e}	{b,d,e}	{1,a,d}

HF_7^{68}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,e}	{1,a,b,d,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,b,c}	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,e}	{a,c,d}	{1,a,b,c,d}	{1,a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{b,d,e}	{a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,c,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{1,b,c,d,e}	{1,a,d}

HF_7^{69}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,b,c}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,c,d}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{b,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,c,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,d}

HF_7^{70}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,c,d}	{1,a,b,c,e}	{1,b,d,e}	{0,a,b,d,e}	{1,b,c,d}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{b,d,e}	{1,a,b,c,d}	{1,a,c,e}	{0,1,b,c,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,d}	{1,c,e}	{a,b,c,d,e}	{1,a,b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,e}	{a,b,c,d,e}	{1,a,d}	{1,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,b,c,e}	{1,a,b,d}	{1,b,c,d,e}	{a,b,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{a,c,d,e}	{0,1,a,c,d}	{a,b,c,e}	{1,a,c,d,e}	{1,b,c}

HF_7^{71}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,c,d}	{1,a,b,c,e}	{1,a,b,d,e}	{0,a,b,d,e}	{1,b,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{b,d,e}	{1,a,b,c,d}	{1,a,b,c,e}	{0,1,b,c,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d}	{1,c,e}	{a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,e}	{a,b,c,d,e}	{1,a,d}	{1,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d}	{1,b,c,d,e}	{a,b,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,a,c,d,e}	{0,1,a,c,d}	{a,b,c,d,e}	{1,a,c,d,e}	{1,b,c}

HF_7^{72}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{0,a,b,d,e}	{1,b,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{0,1,b,c,e}	{1,a,c,e}
b	b	{1,a,b,d}	{1,a,b,c,d,e}	{1,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,e}	{1,a,b,c,d,e}	{a,c,d,e}
d	d	{1,b,d,e}	{0,1,b,c,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,c,e}	{0,1,a,c,d}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c}

HF_7^{73}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d}	{1,a,c,d}	{1,b,e}	{0,a,b,d,e}	{1,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{b,c,d,e}	{a,b,d,e}	{1,a,c}	{0,1,b,c,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d,e}	{1,c,d,e}	{1,b,c,e}	{a,b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c}	{1,b,c,e}	{1,a,d,e}	{1,a,c,d}	{b,c,e}
d	d	{1,c,d}	{0,1,b,c,e}	{a,b,d}	{1,a,c,d}	{1,a,b,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,d,e}	{0,1,a,c,d}	{b,c,e}	{a,b,d,e}	{1,a,b,c}

HF_7^{74}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d}	{1,a,c,d}	{1,a,b,e}	{0,a,b,d,e}	{1,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{0,1,b,c,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}	{1,b,c,e}	{a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,b,c,e}	{a,b,c,d}	{1,a,c,d}	{1,a,b,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,d,e}	{0,1,a,c,d}	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}

HF_7^{75}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d}	{1,a,b,c,d,e}	{1,b,d,e}	{0,a,b,d,e}	{1,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,e}	{0,1,b,c,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,d,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,b,c,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,c,d,e}	{0,1,a,c,d}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c}

HF_7^{76}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,a,b,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,b,c,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,c,d}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}

HF_7^{77}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,e}	{1,a,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,c,e}
a	a	{1,a,d}	{1,b,c,d}	{a,b,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,e}	{a,c,d,e}	{1,b,c}	{a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d}	{1,b,c}	{1,b,d,e}	{a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{a,c,d}	{1,a,c,e}	{b,d,e}
e	e	{1,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{b,d,e}	{1,a,b,d}

HF_7^{78}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,e}	{1,a,d}	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,c,e}
a	a	{1,a,d}	{1,b,c,d}	{a,b,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,e}	{a,c,d,e}	{1,b,c}	{a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d}	{1,b,c}	{1,b,d,e}	{a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}	{a,c,d}	{1,a,c,e}	{b,d,e}
e	e	{1,c,e}	{1,a,b,d,e}	{0,1,a,c,d}	{1,b,c,d,e}	{b,d,e}	{1,a,b,d}

HF_7^{79}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,e}	{1,a,b,d,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,b,c,d}	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,e}	{a,c,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,b,d,e}	{a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,c,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{1,b,c,d,e}	{1,a,b,d}

HF_7^{80}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,b,c,d}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,d}

HF_7^{81}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}

HF_7^{82}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d,e}	{1,a,c,d}	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,b,d,e}	{0,1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,c,d}

HF_7^{83}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d}

HF_7^{84}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}

HF_7^{85}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d}

HF_7^{86}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,b,d,e}	{0,1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}

HF_7^{87}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d}

HF_7^{88}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}
c	c	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}

A4. Hyperfields for which $card(x-x)=7$, for every non-zero element x.

$HF_7^2 = [\mathbb{Z}_7]$	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b}	{1,a,d}	{1,a,b}	{0,1,a,b,c,d,e}	{1,d,e}	{1,c,e}
a	a	{1,a,d}	{a,c}	{a,b,e}	{a,b,c}	{0,1,a,b,c,d,e}	{1,a,e}
b	b	{1,a,b}	{a,b,e}	{b,d}	{1,b,c}	{b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,b,c}	{1,b,c}	{c,e}	{a,c,d}	{c,d,e}
d	d	{1,d,e}	{0,1,a,b,c,d,e}	{b,c,d}	{a,c,d}	{1,d}	{b,d,e}
e	e	{1,c,e}	{1,a,e}	{0,1,a,b,c,d,e}	{c,d,e}	{b,d,e}	{a,e}

HF_7^{89}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	1	{1,a}	{1,b}	{0,1,a,b,c,d,e}	{1,d}	{1,e}
a	a	{1,a}	a	{a,b}	{a,c}	{0,1,a,b,c,d,e}	{a,e}
b	b	{1,b}	{a,b}	b	{b,c}	{b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,c}	{b,c}	c	{c,d}	{c,e}
d	d	{1,d}	{0,1,a,b,c,d,e}	{b,d}	{c,d}	d	{d,e}
e	e	{1,e}	{a,e}	{0,1,a,b,c,d,e}	{c,e}	{d,e}	e

HF_7^{90}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,c}	{1,a}	{1,b}	{0,1,a,b,c,d,e}	{1,d}	{1,e}
a	a	{1,a}	{a,d}	{a,b}	{a,c}	{0,1,a,b,c,d,e}	{a,e}
b	b	{1,b}	{a,b}	{b,e}	{b,c}	{b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,c}	{b,c}	{1,c}	{c,d}	{c,e}
d	d	{1,d}	{0,1,a,b,c,d,e}	{b,d}	{c,d}	{a,d}	{d,e}
e	e	{1,e}	{a,e}	{0,1,a,b,c,d,e}	{c,e}	{d,e}	{b,e}

HF_7^{91}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	1	{1,a}	{1,a,b}	{0,1,a,b,c,d,e}	{1,d,e}	{1,e}
a	a	{1,a}	a	{a,b}	{a,b,c}	{0,1,a,b,c,d,e}	{1,a,e}
b	b	{1,a,b}	{a,b}	b	{b,c}	{b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,b,c}	{b,c}	c	{c,d}	{c,d,e}
d	d	{1,d,e}	{0,1,a,b,c,d,e}	{b,c,d}	{c,d}	d	{d,e}
e	e	{1,e}	{1,a,e}	{0,1,a,b,c,d,e}	{c,d,e}	{d,e}	e

HF_7^{92}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b}	{1,a,b,d,e}	{1,b,d}	{0,1,a,b,c,d,e}	{1,b,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{a,c}	{1,a,b,c,e}	{a,c,e}	{0,1,a,b,c,d,e}	{a,c,e}
b	b	{1,b,d}	{1,a,b,c,e}	{b,d}	{1,a,b,c,d}	{1,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,c,e}	{1,a,b,c,d}	{c,e}	{a,b,c,d,e}	{a,c,e}
d	d	{1,b,d}	{0,1,a,b,c,d,e}	{1,b,d}	{a,b,c,d,e}	{1,d}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,c,e}	{0,1,a,b,c,d,e}	{a,c,e}	{1,b,c,d,e}	{a,e}

HF_7^{93}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b}	{1,a,d}	{1,a,b,e}	{0,1,a,b,c,d,e}	{1,c,d,e}	{1,c,e}
a	a	{1,a,d}	{a,b,c}	{a,b,e}	{1,a,b,c}	{0,1,a,b,c,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,e}	{b,c,d}	{1,b,c}	{a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c}	{1,b,c}	{c,d,e}	{a,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d}	{a,c,d}	{1,d,e}	{b,d,e}
e	e	{1,c,e}	{1,a,d,e}	{0,1,a,b,c,d,e}	{b,c,d,e}	{b,d,e}	{1,a,e}

HF_7^{94}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b}	{1,a,b,d,e}	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{a,b,c}	{1,a,b,c,e}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,e}	{b,c,d}	{1,a,b,c,d}	{1,a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d}	{c,d,e}	{a,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d}	{a,b,c,d,e}	{1,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,b,c,d,e}	{1,a,e}

HF_7^{95}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b}	{1,a,b,d,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{a,b,c}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,e}	{b,c,d}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d}	{c,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{a,b,c,d,e}	{1,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,e}

HF_7^{96}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c}	{1,a,b,e}	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,d,e}
a	a	{1,a,b,e}	{a,b,d}	{1,a,b,c}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c}	{b,c,e}	{a,b,c,d}	{1,a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,e}	{a,b,c,d}	{1,c,d}	{b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d}	{b,c,d,e}	{a,d,e}	{1,c,d,e}
e	e	{1,a,d,e}	{a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,c,d,e}	{1,b,e}

HF_7^{97}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c}	{1,a,b,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,d,e}
a	a	{1,a,b,e}	{a,b,d}	{1,a,b,c}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c}	{b,c,e}	{a,b,c,d}	{1,a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d}	{1,c,d}	{b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{b,c,d,e}	{a,d,e}	{1,c,d,e}
e	e	{1,a,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,c,d,e}	{1,b,e}

HF_7^{98}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,d}	{1,a,c}	{1,b,e}	{0,1,a,b,c,d,e}	{1,c,d}	{1,b,e}
a	a	{1,a,c}	{a,b,e}	{a,b,d}	{1,a,c}	{0,1,a,b,c,d,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d}	{1,b,c}	{b,c,e}	{a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c}	{b,c,e}	{a,c,d}	{1,c,d}	{b,c,e}
d	d	{1,c,d}	{0,1,a,b,c,d,e}	{a,b,d}	{1,c,d}	{b,d,e}	{a,d,e}
e	e	{1,b,e}	{a,d,e}	{0,1,a,b,c,d,e}	{b,c,e}	{a,d,e}	{1,c,e}

HF_7^{99}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,d}	{1,a,c}	{1,a,b,e}	{0,1,a,b,c,d,e}	{1,c,d,e}	{1,b,e}
a	a	{1,a,c}	{a,b,e}	{a,b,d}	{1,a,b,c}	{0,1,a,b,c,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,d}	{1,b,c}	{b,c,e}	{a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c}	{b,c,e}	{a,c,d}	{1,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d}	{1,c,d}	{b,d,e}	{a,d,e}
e	e	{1,b,e}	{1,a,d,e}	{0,1,a,b,c,d,e}	{b,c,d,e}	{a,d,e}	{1,c,e}

HF_7^{100}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,d}	{1,a,b,c,e}	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{a,b,e}	{1,a,b,c,d}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,d}	{1,b,c}	{a,b,c,d,e}	{1,a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}	{a,c,d}	{1,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d}	{1,b,c,d,e}	{b,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,a,c,d,e}	{1,c,e}

HF_7^{101}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,d}	{1,a,b,c,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{a,b,e}	{1,a,b,c,d}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d}	{1,b,c}	{a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d,e}	{a,c,d}	{1,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,d,e}	{b,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d,e}	{1,c,e}

HF_7^{102}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,e}	{1,a,b,e}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,d,e}
a	a	{1,a,b,e}	{1,a,b}	{1,a,b,c}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c}	{a,b,c}	{a,b,c,d}	{1,a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,c,d}	{b,c,d}	{b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}	{b,c,d,e}	{c,d,e}	{1,c,d,e}
e	e	{1,a,d,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,c,d,e}	{1,d,e}

HF_7^{103}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,e}	{1,a,b,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,d,e}
a	a	{1,a,b,e}	{1,a,b}	{1,a,b,c}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c}	{a,b,c}	{a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}	{b,c,d}	{b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{b,c,d,e}	{c,d,e}	{1,c,d,e}
e	e	{1,a,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,c,d,e}	{1,d,e}

HF_7^{104}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,c}	{1,a,b,d,e}	{1,a,b,d}	{0,1,a,b,c,d,e}	{1,b,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{a,c,d}	{1,a,b,c,e}	{a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,e}
b	b	{1,a,b,d}	{1,a,b,c,e}	{b,d,e}	{1,a,b,c,d}	{1,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d}	{1,c,e}	{a,b,c,d,e}	{a,c,d,e}
d	d	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{a,b,c,d,e}	{1,a,d}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}	{1,b,c,d,e}	{a,b,e}

HF_7^{105}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,d}	{1,a,c,d}	{1,b}	{0,1,a,b,c,d,e}	{1,d}	{1,b,c,e}
a	a	{1,a,c,d}	{a,c,e}	{a,b,d,e}	{a,c}	{0,1,a,b,c,d,e}	{a,e}
b	b	{1,b}	{a,b,d,e}	{1,b,d}	{1,b,c,e}	{b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,c}	{1,b,c,e}	{a,c,e}	{1,a,c,d}	{c,e}
d	d	{1,d}	{0,1,a,b,c,d,e}	{b,d}	{1,a,c,d}	{1,b,d}	{a,b,d,e}
e	e	{1,b,c,e}	{a,e}	{0,1,a,b,c,d,e}	{c,e}	{a,b,d,e}	{a,c,e}

HF_7^{106}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,d}	{1,a,c,d}	{1,a,b}	{0,1,a,b,c,d,e}	{1,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{a,c,e}	{a,b,d,e}	{a,b,c}	{0,1,a,b,c,d,e}	{1,a,e}
b	b	{1,a,b}	{a,b,d,e}	{1,b,d}	{1,b,c,e}	{b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,b,c}	{1,b,c,e}	{a,c,e}	{1,a,c,d}	{c,d,e}
d	d	{1,d,e}	{0,1,a,b,c,d,e}	{b,c,d}	{1,a,c,d}	{1,b,d}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,e}	{0,1,a,b,c,d,e}	{c,d,e}	{a,b,d,e}	{a,c,e}

HF_7^{107}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,d}	{1,a,b,c,d,e}	{1,b,d}	{0,1,a,b,c,d,e}	{1,b,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{a,c,e}	{1,a,b,c,d,e}	{a,c,e}	{0,1,a,b,c,d,e}	{a,c,e}
b	b	{1,b,d}	{1,a,b,c,d,e}	{1,b,d}	{1,a,b,c,d,e}	{1,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,c,e}	{1,a,b,c,d,e}	{a,c,e}	{1,a,b,c,d,e}	{a,c,e}
d	d	{1,b,d}	{0,1,a,b,c,d,e}	{1,b,d}	{1,a,b,c,d,e}	{1,b,d}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,c,e}	{0,1,a,b,c,d,e}	{a,c,e}	{1,a,b,c,d,e}	{a,c,e}

HF_7^{108}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c}	{1,a,d}	{1,a,b,e}	{0,1,a,b,c,d,e}	{1,c,d,e}	{1,c,e}
a	a	{1,a,d}	{a,b,c,d}	{a,b,e}	{1,a,b,c}	{0,1,a,b,c,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,e}	{b,c,d,e}	{1,b,c}	{a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c}	{1,b,c}	{1,c,d,e}	{a,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d}	{a,c,d}	{1,a,d,e}	{b,d,e}
e	e	{1,c,e}	{1,a,d,e}	{0,1,a,b,c,d,e}	{b,c,d,e}	{b,d,e}	{1,a,b,e}

HF_7^{109}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c}	{1,a,b,d,e}	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{a,b,c,d}	{1,a,b,c,e}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,e}	{b,c,d,e}	{1,a,b,c,d}	{1,a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d}	{1,c,d,e}	{a,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d}	{a,b,c,d,e}	{1,a,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,b,c,d,e}	{1,a,b,e}

HF_7^{110}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c}	{1,a,b,d,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{a,b,c,d}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,e}	{b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d}	{1,c,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{a,b,c,d,e}	{1,a,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,e}

HF_7^{111}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d}	{1,a,c,d}	{1,b,e}	{0,1,a,b,c,d,e}	{1,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{a,b,c,e}	{a,b,d,e}	{1,a,c}	{0,1,a,b,c,d,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d,e}	{1,b,c,d}	{1,b,c,e}	{a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c}	{1,b,c,e}	{a,c,d,e}	{1,a,c,d}	{b,c,e}
d	d	{1,c,d}	{0,1,a,b,c,d,e}	{a,b,d}	{1,a,c,d}	{1,b,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,d,e}	{0,1,a,b,c,d,e}	{b,c,e}	{a,b,d,e}	{1,a,c,e}

HF_7^{112}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d}	{1,a,c,d}	{1,a,b,e}	{0,1,a,b,c,d,e}	{1,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{a,b,c,e}	{a,b,d,e}	{1,a,b,c}	{0,1,a,b,c,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,d,e}	{1,b,c,d}	{1,b,c,e}	{a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c}	{1,b,c,e}	{a,c,d,e}	{1,a,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d}	{1,a,c,d}	{1,b,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,d,e}	{0,1,a,b,c,d,e}	{b,c,d,e}	{a,b,d,e}	{1,a,c,e}

HF_7^{113}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}

HF_7^{114}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,e}

HF_7^{115}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{1,a,d}	{1,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,c,e}
a	a	{1,a,d}	{1,a,b,c}	{a,b,e}	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,e}	{a,b,c,d}	{1,b,c}	{a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c}	{b,c,d,e}	{a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}	{a,c,d}	{1,c,d,e}	{b,d,e}
e	e	{1,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,e}	{b,d,e}	{1,a,d,e}

HF_7^{116}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{1,a,d}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,c,e}
a	a	{1,a,d}	{1,a,b,c}	{a,b,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,e}	{a,b,c,d}	{1,b,c}	{a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c}	{b,c,d,e}	{a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{a,c,d}	{1,c,d,e}	{b,d,e}
e	e	{1,c,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{b,d,e}	{1,a,d,e}

HF_7^{117}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{1,a,b,d,e}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,a,b,c}	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d}	{1,a,b,c,d}	{1,a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,c,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,b,c,d,e}	{1,a,d,e}

HF_7^{118}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,a,b,c}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,c,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,d,e}

HF_7^{119}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,d}	{1,a,c}	{1,b,e}	{0,1,a,b,c,d,e}	{1,c,d}	{1,b,e}
a	a	{1,a,c}	{a,b,d,e}	{a,b,d}	{1,a,c}	{0,1,a,b,c,d,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d}	{1,b,c,e}	{b,c,e}	{a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c}	{b,c,e}	{1,a,c,d}	{1,c,d}	{b,c,e}
d	d	{1,c,d}	{0,1,a,b,c,d,e}	{a,b,d}	{1,c,d}	{a,b,d,e}	{a,d,e}
e	e	{1,b,e}	{a,d,e}	{0,1,a,b,c,d,e}	{b,c,e}	{a,d,e}	{1,b,c,e}

HF_7^{120}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,d}	{1,a,c}	{1,a,b,e}	{0,1,a,b,c,d,e}	{1,c,d,e}	{1,b,e}
a	a	{1,a,c}	{a,b,d,e}	{a,b,d}	{1,a,b,c}	{0,1,a,b,c,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,d}	{1,b,c,e}	{b,c,e}	{a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c}	{b,c,e}	{1,a,c,d}	{1,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d}	{1,c,d}	{a,b,d,e}	{a,d,e}
e	e	{1,b,e}	{1,a,d,e}	{0,1,a,b,c,d,e}	{b,c,d,e}	{a,d,e}	{1,b,c,e}

HF_7^{121}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,d}	{1,a,b,c,e}	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,a,c,d,e}	{1,b,c,e}

HF_7^{122}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,d}	{1,a,b,c,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,e}

HF_7^{123}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,e}	{1,a,b,e}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,d,e}
a	a	{1,a,b,e}	{1,a,b,d}	{1,a,b,c}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c}	{a,b,c,e}	{a,b,c,d}	{1,a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,c,d}	{1,b,c,d}	{b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}	{b,c,d,e}	{a,c,d,e}	{1,c,d,e}
e	e	{1,a,d,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,c,d,e}	{1,b,d,e}

HF_7^{124}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,c,e}	{1,a,b,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,d,e}
a	a	{1,a,b,e}	{1,a,b,d}	{1,a,b,c}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c}	{a,b,c,e}	{a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}	{1,b,c,d}	{b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{b,c,d,e}	{a,c,d,e}	{1,c,d,e}
e	e	{1,a,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,c,d,e}	{1,b,d,e}

HF_7^{125}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{0,1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,e}
b	b	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}
d	d	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}

HF_7^{126}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d}	{1,a,c,d}	{1,b,e}	{0,1,a,b,c,d,e}	{1,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{a,b,c,d,e}	{a,b,d,e}	{1,a,c}	{0,1,a,b,c,d,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d,e}	{1,b,c,d,e}	{1,b,c,e}	{a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c}	{1,b,c,e}	{1,a,c,d,e}	{1,a,c,d}	{b,c,e}
d	d	{1,c,d}	{0,1,a,b,c,d,e}	{a,b,d}	{1,a,c,d}	{1,a,b,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,d,e}	{0,1,a,b,c,d,e}	{b,c,e}	{a,b,d,e}	{1,a,b,c,e}

HF_7^{127}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d}	{1,a,c,d}	{1,a,b,e}	{0,1,a,b,c,d,e}	{1,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{a,b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{0,1,a,b,c,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{a,b,d,e}	{1,b,c,d,e}	{1,b,c,e}	{a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,c,d,e}	{1,a,c,d}	{b,c,d,e}
d	d	{1,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d}	{1,a,c,d}	{1,a,b,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,d,e}	{0,1,a,b,c,d,e}	{b,c,d,e}	{a,b,d,e}	{1,a,b,c,e}

HF_7^{128}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,e}	{0,1,a,b,c,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}
d	d	{1,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,e}

HF_7^{129}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}

HF_7^{130}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{1,a,d}	{1,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,c,e}
a	a	{1,a,d}	{1,a,b,c,d}	{a,b,e}	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,e}	{a,b,c,d,e}	{1,b,c}	{a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c}	{1,b,c,d,e}	{a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}	{a,c,d}	{1,a,c,d,e}	{b,d,e}
e	e	{1,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,e}	{b,d,e}	{1,a,b,d,e}

HF_7^{131}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{1,a,d}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,c,e}
a	a	{1,a,d}	{1,a,b,c,d}	{a,b,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,e}	{a,b,c,d,e}	{1,b,c}	{a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c}	{1,b,c,d,e}	{a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{a,c,d}	{1,a,c,d,e}	{b,d,e}
e	e	{1,c,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{b,d,e}	{1,a,b,d,e}

HF_7^{132}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{1,a,b,d,e}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,b,c,d,e}	{1,a,b,d,e}

HF_7^{133}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,d,e}

HF_7^{134}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d,e}

HF_7^{135}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,a,c,d}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{a,b,d,e}	{1,a,c,d,e}

HF_7^{136}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}

HF_7^{137}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}

HF_7^{138}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}

HF_7^{139}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,e}
a	a	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}

HF_7^{140}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}

HF_7^{141}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}
c	c	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}

B. The hyperfields with self-opposite elements

B1i. Hyperfields for which $card(x-x)=2$ and $x \in x-x$, for every non-zero element *x*.

HF_7^{142}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1}	{b,c,d,e}	{a,c,d,e}	{a,b,d,e}	{a,b,c,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a}	{1,c,d,e}	{1,b,d,e}	{1,b,c,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,c,d,e}	{0,b}	{1,a,d,e}	{1,a,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,b,d,e}	{1,a,d,e}	{0,c}	{1,a,b,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,b,c,e}	{1,a,c,e}	{1,a,b,e}	{0,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d}	{1,a,c,d}	{1,a,b,d}	{1,a,b,c}	{0,e}

B1ii. Hyperfields for which $card(x-x)=2$ and $x \notin x-x$, for every non-zero element *x*.

HF_7^{143}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a}	{1,d}	{c,d}	{b,e}	{a,b}	{c,e}
a	a	{1,d}	{0,b}	{a,e}	{d,e}	{1,c}	{b,c}
b	b	{c,d}	{a,e}	{0,c}	{1,b}	{1,e}	{a,d}
c	c	{b,e}	{d,e}	{1,b}	{0,d}	{a,c}	{1,a}
d	d	{a,b}	{1,c}	{1,e}	{a,c}	{0,e}	{b,d}
e	e	{c,e}	{b,c}	{a,d}	{1,a}	{b,d}	{0,1}

B2i. Hyperfields for which $card(x-x)=3$ and $x \in x-x$, for every non-zero element x.

HF_7^{144}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a}	{1,b,c,d,e}	{a,c,d,e}	{a,b,d,e}	{a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b}	{1,a,c,d,e}	{1,b,d,e}	{1,b,c,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,a,c,d,e}	{0,b,c}	{1,a,b,d,e}	{1,a,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,b,d,e}	{1,a,b,d,e}	{0,c,d}	{1,a,b,c,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,b,c,e}	{1,a,c,e}	{1,a,b,c,e}	{0,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,c,d}	{1,a,c,d}	{1,a,b,d}	{1,a,b,c,d}	{0,1,e}

HF_7^{145}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b}	{b,c,d,e}	{1,a,c,e}	{a,b,d,e}	{a,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c}	{1,c,d,e}	{1,a,b,d}	{1,b,c,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,c,d,e}	{0,b,d}	{1,a,d,e}	{a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d}	{1,a,d,e}	{0,c,e}	{1,a,b,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,b,c,e}	{a,b,c,e}	{1,a,b,e}	{0,1,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,d,e}	{1,a,c,d}	{1,b,c,d}	{1,a,b,c}	{0,a,e}

HF_7^{146}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b}	{b,c,d,e}	{1,a,c,d,e}	{a,b,d,e}	{a,b,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c}	{1,c,d,e}	{1,a,b,d,e}	{1,b,c,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,c,d,e}	{0,b,d}	{1,a,d,e}	{1,a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d,e}	{1,a,d,e}	{0,c,e}	{1,a,b,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,e}	{1,a,b,e}	{0,1,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d}	{1,a,b,c}	{0,a,e}

HF_7^{147}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,c}	{b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,d}	{1,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,c,d,e}	{0,b,e}	{1,a,d,e}	{1,a,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,d,e}	{0,1,c}	{1,a,b,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,e}	{0,a,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c}	{0,b,e}

B2ii. Hyperfields for which $card(x-x)=3$ and $x \notin x-x$, for every non-zero element x.

HF_7^{148}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b}	{1,c,d}	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,b,c}	{a,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,d,e}	{0,c,d}	{1,b,e}	{1,a,b,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,d,e}	{1,b,e}	{0,d,e}	{1,a,c}	{1,a,b,c}
d	d	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c}	{0,1,e}	{a,b,d}
e	e	{b,c,e}	{b,c,d,e}	{1,a,c,d}	{1,a,b,c}	{a,b,d}	{0,1,a}

HF_7^{149}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b}	{1,b,c,d,e}	{1,a,c,e}	{a,b,d,e}	{a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c}	{1,a,c,d,e}	{1,a,b,d}	{1,b,c,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,c,d,e}	{0,c,d}	{1,a,b,d,e}	{a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d}	{1,a,b,d,e}	{0,d,e}	{1,a,b,c,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,b,c,e}	{a,b,c,e}	{1,a,b,c,e}	{0,1,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,d,e}	{1,a,c,d}	{1,b,c,d}	{1,a,b,c,d}	{0,1,a}

HF_7^{150}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b}	{1,b,c,d,e}	{1,a,c,d,e}	{a,b,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c}	{1,a,c,d,e}	{1,a,b,d,e}	{1,b,c,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,c,d,e}	{0,c,d}	{1,a,b,d,e}	{1,a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d,e}	{1,a,b,d,e}	{0,d,e}	{1,a,b,c,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a}

HF_7^{151}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,b,d}	{b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,c,e}	{1,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,c,d,e}	{0,1,d}	{1,a,d,e}	{a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d}	{1,a,d,e}	{0,a,e}	{1,a,b,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,e}	{0,1,b}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c}	{0,a,c}

HF_7^{152}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,b,d}	{b,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,c,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,c,d,e}	{0,1,d}	{1,a,d,e}	{1,a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,d,e}	{0,a,e}	{1,a,b,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,e}	{0,1,b}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c}	{0,a,c}

B3i. Hyperfields for which $card(x-x)=4$ and $x \in x-x$, for every non-zero element *x*.

HF_7^{153}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b}	{1,c,d}	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,a,b,c}	{a,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,d,e}	{0,b,c,d}	{1,b,e}	{1,a,b,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,d,e}	{1,b,e}	{0,c,d,e}	{1,a,c}	{1,a,b,c}
d	d	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c}	{0,1,d,e}	{a,b,d}
e	e	{b,c,e}	{b,c,d,e}	{1,a,c,d}	{1,a,b,c}	{a,b,d}	{0,1,a,e}

HF_7^{154}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b}	{1,b,c,d,e}	{1,a,c,e}	{a,b,d,e}	{a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c}	{1,a,c,d,e}	{1,a,b,d}	{1,b,c,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,c,d,e}	{0,b,c,d}	{1,a,b,d,e}	{a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d}	{1,a,b,d,e}	{0,c,d,e}	{1,a,b,c,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,b,c,e}	{a,b,c,e}	{1,a,b,c,e}	{0,1,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,d,e}	{1,a,c,d}	{1,b,c,d}	{1,a,b,c,d}	{0,1,a,e}

HF_7^{155}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b}	{1,b,c,d,e}	{1,a,c,d,e}	{a,b,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c}	{1,a,c,d,e}	{1,a,b,d,e}	{1,b,c,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,c,d,e}	{0,b,c,d}	{1,a,b,d,e}	{1,a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d,e}	{1,a,b,d,e}	{0,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a,e}

HF_7^{156}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c}	{1,b,c,e}	{a,d,e}	{1,a,c,d}	{b,c,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,a,b,d}	{1,a,c,d}	{1,b,e}	{a,b,d,e}	{1,c,d}
b	b	{a,d,e}	{1,a,c,d}	{0,b,c,e}	{a,b,d,e}	{1,a,c}	{1,b,c,e}
c	c	{1,a,c,d}	{1,b,e}	{a,b,d,e}	{0,1,c,d}	{1,b,c,e}	{a,b,d}
d	d	{b,c,e}	{a,b,d,e}	{1,a,c}	{1,b,c,e}	{0,a,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,c,d}	{1,b,c,e}	{a,b,d}	{1,a,c,d}	{0,1,b,e}

HF_7^{157}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c}	{1,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,d}	{1,a,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,a,c,d,e}	{0,b,c,e}	{1,a,b,d,e}	{1,a,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,d,e}	{0,1,c,d}	{1,a,b,c,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,e}	{0,a,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d}	{0,1,b,e}

HF_7^{158}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,d}	{1,c,d}	{b,c,e}	{a,b,d,e}	{1,a,c}	{b,c,e}
a	a	{1,c,d}	{0,a,b,e}	{a,d,e}	{1,c,d}	{1,b,c,e}	{a,b,d}
b	b	{b,c,e}	{a,d,e}	{0,1,b,c}	{1,b,e}	{a,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,c,d}	{1,b,e}	{0,a,c,d}	{1,a,c}	{1,b,e}
d	d	{1,a,c}	{1,b,c,e}	{a,d,e}	{1,a,c}	{0,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d}	{1,a,c,d}	{1,b,e}	{a,b,d}	{0,1,c,e}

HF_7^{159}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,d}	{1,c,d}	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{b,c,e}
a	a	{1,c,d}	{0,a,b,e}	{a,d,e}	{1,c,d,e}	{1,b,c,e}	{a,b,c,d}
b	b	{b,c,d,e}	{a,d,e}	{0,1,b,c}	{1,b,e}	{1,a,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,c,d,e}	{1,b,e}	{0,a,c,d}	{1,a,c}	{1,a,b,e}
d	d	{1,a,b,c}	{1,b,c,e}	{1,a,d,e}	{1,a,c}	{0,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,c,d}	{1,a,c,d}	{1,a,b,e}	{a,b,d}	{0,1,c,e}

HF_7^{160}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,d}	{1,b,c,d,e}	{a,b,c,e}	{a,b,d,e}	{1,a,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,e}	{1,a,c,d,e}	{1,b,c,d}	{1,b,c,e}	{1,a,b,d}
b	b	{a,b,c,e}	{1,a,c,d,e}	{0,1,b,c}	{1,a,b,d,e}	{a,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,b,c,d}	{1,a,b,d,e}	{0,a,c,d}	{1,a,b,c,e}	{1,b,d,e}
d	d	{1,a,c,e}	{1,b,c,e}	{a,c,d,e}	{1,a,b,c,e}	{0,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d}	{1,a,c,d}	{1,b,d,e}	{1,a,b,c,d}	{0,1,c,e}

HF_7^{161}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,d}	{1,b,c,d,e}	{a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,e}	{1,a,c,d,e}	{1,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,d,e}	{0,1,b,c}	{1,a,b,d,e}	{1,a,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,b,c,d,e}	{1,a,b,d,e}	{0,a,c,d}	{1,a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{1,b,c,e}	{1,a,c,d,e}	{1,a,b,c,e}	{0,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d}	{1,a,c,d}	{1,a,b,d,e}	{1,a,b,c,d}	{0,1,c,e}

HF_7^{162}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,e}	{1,a,b,c,d,e}	{a,c,d,e}	{a,b,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b}	{1,a,b,c,d,e}	{1,b,d,e}	{1,b,c,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,a,b,c,d,e}	{0,a,b,c}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{0,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,b,c,e}	{1,a,c,e}	{1,a,b,c,d,e}	{0,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,c,d}	{1,a,b,d}	{1,a,b,c,d,e}	{0,1,d,e}

HF_7^{163}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,c}	{b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{a,c,d}
a	a	{b,d,e}	{0,a,c,d}	{1,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
b	b	{1,a,c,d}	{1,c,e}	{0,b,d,e}	{1,a,d}	{1,b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d,e}	{1,a,d}	{0,1,c,e}	{a,b,e}	{1,a,c,d}
d	d	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,e}	{0,1,a,d}	{1,b,c}
e	e	{a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c}	{0,a,b,e}

HF_7^{164}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,c}	{b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c,d}	{1,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,c,d,e}	{0,b,d,e}	{1,a,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,d,e}	{0,1,c,e}	{1,a,b,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,e}	{0,1,a,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c}	{0,a,b,e}

HF_7^{165}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,c}	{b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c,d}	{1,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,c,d,e}	{0,b,d,e}	{1,a,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,d,e}	{0,1,c,e}	{1,a,b,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,e}	{0,1,a,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c}	{0,a,b,e}

HF_7^{166}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,d}	{b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c,e}	{1,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,c,d,e}	{0,1,b,d}	{1,a,d,e}	{a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d}	{1,a,d,e}	{0,a,c,e}	{1,a,b,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,e}	{0,1,b,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c}	{0,a,c,e}

HF_7^{167}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,d}	{b,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,c,d,e}	{0,1,b,d}	{1,a,d,e}	{1,a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,d,e}	{0,a,c,e}	{1,a,b,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,e}	{0,1,b,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c}	{0,a,c,e}

B3ii. Hyperfields for which $card(x-x)=4$ and $x \notin x-x$, for every non-zero element x.

HF_7^{168}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c}	{1,c,d}	{1,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,b,c,d}	{a,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,d,e}	{0,c,d,e}	{1,b,e}	{1,a,b,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,d,e}	{1,b,e}	{0,1,d,e}	{1,a,c}	{1,a,b,c}
d	d	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,c}	{0,1,a,e}	{a,b,d}
e	e	{b,c,e}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}	{a,b,d}	{0,1,a,b}

HF_7^{169}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c}	{1,b,c,e}	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,b,c,d}	{1,a,c,d}	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}
b	b	{1,a,d,e}	{1,a,c,d}	{0,c,d,e}	{a,b,d,e}	{1,a,b,c}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,e}	{a,b,d,e}	{0,1,d,e}	{1,b,c,e}	{a,b,c,d}
d	d	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{0,1,a,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,c,d,e}	{1,b,c,e}	{a,b,c,d}	{1,a,c,d}	{0,1,a,b}

HF_7^{170}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c}	{1,b,d,e}	{1,a,c}	{1,b,c,e}	{a,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,b,c,d}	{1,a,c,e}	{a,b,d}	{1,a,c,d}	{1,b,e}
b	b	{1,a,c}	{1,a,c,e}	{0,c,d,e}	{1,a,b,d}	{b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d}	{1,a,b,d}	{0,1,d,e}	{a,b,c,e}	{1,c,d}
d	d	{a,d,e}	{1,a,c,d}	{b,c,e}	{a,b,c,e}	{0,1,a,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,b,e}	{a,b,d,e}	{1,c,d}	{1,b,c,d}	{0,1,a,b}

HF_7^{171}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c}	{1,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,b,c,d}	{1,a,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
b	b	{1,a,c,d}	{1,a,c,e}	{0,c,d,e}	{1,a,b,d}	{1,b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d,e}	{1,a,b,d}	{0,1,d,e}	{a,b,c,e}	{1,a,c,d}
d	d	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,c,e}	{0,1,a,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,d}	{0,1,a,b}

HF_7^{172}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c}	{1,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c,d}	{1,a,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,c,d,e}	{0,c,d,e}	{1,a,b,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,d,e}	{0,1,d,e}	{1,a,b,c,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,e}	{0,1,a,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d}	{0,1,a,b}

HF_7^{173}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c}	{1,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c,d}	{1,a,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,c,d,e}	{0,c,d,e}	{1,a,b,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,d,e}	{0,1,d,e}	{1,a,b,c,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,a,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a,b}

HF_7^{174}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d}	{1,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{b,c,e}
a	a	{1,c,d}	{0,b,c,e}	{a,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,d,e}	{0,1,c,d}	{1,b,e}	{a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d}	{1,b,e}	{0,a,d,e}	{1,a,c}	{1,b,c,e}
d	d	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c}	{0,1,b,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d}	{0,1,a,c}

HF_7^{175}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d}	{1,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,b,c,e}	{a,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,d,e}	{0,1,c,d}	{1,b,e}	{1,a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d,e}	{1,b,e}	{0,a,d,e}	{1,a,c}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c}	{0,1,b,e}	{a,b,d}
e	e	{b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d}	{0,1,a,c}

HF_7^{176}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d}	{1,b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,c,d}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{0,a,d,e}	{1,a,b,c,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,e}	{0,1,b,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c,d}	{0,1,a,c}

HF_7^{177}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,c,d}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,a,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,b,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,c}

HF_7^{178}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,e}	{1,a,c,d}	{1,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c}	{a,b,d,e}	{1,a,d}	{1,b,c,e}	{b,d,e}
b	b	{1,c,e}	{a,b,d,e}	{0,a,c,d}	{1,b,c,e}	{a,b,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,d}	{1,b,c,e}	{0,b,d,e}	{1,a,c,d}	{1,b,c}
d	d	{a,c,d}	{1,b,c,e}	{a,b,e}	{1,a,c,d}	{0,1,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,d,e}	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{0,1,a,d}

HF_7^{179}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c}	{a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,b,d,e}	{0,a,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{0,b,d,e}	{1,a,c,d}	{1,a,b,c}
d	d	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c,d}	{0,1,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,c,d,e}	{1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{0,1,a,d}

HF_7^{180}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,e}	{1,a,b,c,d,e}	{1,a,c,e}	{a,b,d,e}	{a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c}	{1,a,b,c,d,e}	{1,a,b,d}	{1,b,c,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,b,c,d,e}	{0,a,c,d}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{0,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,b,c,e}	{a,b,c,e}	{1,a,b,c,d,e}	{0,1,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,c,d}	{1,b,c,d}	{1,a,b,c,d,e}	{0,1,a,d}

HF_7^{181}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,b,c,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,d}

HF_7^{182}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,c,d}	{1,b,c,e}	{a,b,e}	{1,a,c,d}	{1,c,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,b,d,e}	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{1,a,d}
b	b	{a,b,e}	{1,a,c,d}	{0,1,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{0,1,a,d}	{1,b,c,e}	{b,d,e}
d	d	{1,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}	{0,a,b,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,d}	{1,b,c,e}	{b,d,e}	{1,a,c,d}	{0,1,b,c}

HF_7^{183}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,d}	{0,1,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,d}	{1,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c}

HF_7^{184}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,c,d}	{1,b,d,e}	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,b,d,e}	{1,a,c,e}	{b,c,d,e}	{1,a,c,d}	{1,a,b,c}
b	b	{a,b,c,d}	{1,a,c,e}	{0,1,c,e}	{1,a,b,d}	{1,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{b,c,d,e}	{1,a,b,d}	{0,1,a,d}	{a,b,c,e}	{1,a,d,e}
d	d	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,c,e}	{0,a,b,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}	{1,b,c,d}	{0,1,b,c}

HF_7^{185}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,c,d}	{1,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,d,e}	{1,a,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}
b	b	{a,b,c,e}	{1,a,c,d,e}	{0,1,c,e}	{1,a,b,d,e}	{a,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,d,e}	{0,1,a,d}	{1,a,b,c,e}	{1,b,d,e}
d	d	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,e}	{0,a,b,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d}	{0,1,b,c}

HF_7^{186}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,c,d}	{1,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,d,e}	{1,a,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,d,e}	{0,1,c,e}	{1,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,d,e}	{0,1,a,d}	{1,a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,e}	{0,a,b,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d}	{0,1,b,c}

HF_7^{187}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,c,e}	{1,a,b,c,e}	{a,d,e}	{1,a,c,d}	{b,c,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,b,d}	{1,a,b,c,d}	{1,b,e}	{a,b,d,e}	{1,c,d}
b	b	{a,d,e}	{1,a,b,c,d}	{0,a,c,e}	{a,b,c,d,e}	{1,a,c}	{1,b,c,e}
c	c	{1,a,c,d}	{1,b,e}	{a,b,c,d,e}	{0,1,b,d}	{1,b,c,d,e}	{a,b,d}
d	d	{b,c,e}	{a,b,d,e}	{1,a,c}	{1,b,c,d,e}	{0,a,c,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,c,d}	{1,b,c,e}	{a,b,d}	{1,a,c,d,e}	{0,1,b,d}

HF_7^{188}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,a,b,c,d,e}	{0,a,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{0,1,b,d}	{1,a,b,c,d,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{0,a,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{0,1,b,d}

HF_7^{189}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,b,c,d}	{b,c,e}	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,d}
a	a	{b,c,e}	{0,c,d,e}	{1,c,d}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{1,c,d}	{0,1,d,e}	{a,d,e}	{a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c}	{a,d,e}	{0,1,a,e}	{1,b,e}	{b,c,d,e}
d	d	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,e}	{0,1,a,b}	{1,a,c}
e	e	{a,b,d}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}	{1,a,c}	{0,a,b,c}

HF_7^{190}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,b,c,d}	{b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d}
a	a	{b,c,e}	{0,c,d,e}	{1,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,c,d}	{0,1,d,e}	{a,d,e}	{1,a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c,e}	{a,d,e}	{0,1,a,e}	{1,b,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,e}	{0,1,a,b}	{1,a,c}
e	e	{a,b,d}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c}	{0,a,b,c}

HF_7^{191}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,b,c,d}	{b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,c,d,e}	{1,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,c,d,e}	{0,1,d,e}	{1,a,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,d,e}	{0,1,a,e}	{1,a,b,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,e}	{0,1,a,b}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c}	{0,a,b,c}

HF_7^{192}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,b,c,d}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,c,d,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,c,d,e}	{0,1,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,d,e}	{0,1,a,e}	{1,a,b,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,e}	{0,1,a,b}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}	{0,a,b,c}

B4i. Hyperfields for which $card(x-x)=5$ and $x \in x-x$, for every non-zero element x.

HF_7^{193}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c}	{1,c,d}	{1,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,a,b,c,d}	{a,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,d,e}	{0,b,c,d,e}	{1,b,e}	{1,a,b,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,d,e}	{1,b,e}	{0,1,c,d,e}	{1,a,c}	{1,a,b,c}
d	d	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,c}	{0,1,a,d,e}	{a,b,d}
e	e	{b,c,e}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}	{a,b,d}	{0,1,a,b,e}

HF_7^{194}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c}	{1,b,c,e}	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,a,b,c,d}	{1,a,c,d}	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}
b	b	{1,a,d,e}	{1,a,c,d}	{0,b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,e}	{a,b,d,e}	{0,1,c,d,e}	{1,b,c,e}	{a,b,c,d}
d	d	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{1,b,c,e}	{0,1,a,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,c,d,e}	{1,b,c,e}	{a,b,c,d}	{1,a,c,d}	{0,1,a,b,e}

HF_7^{195}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c}	{1,b,d,e}	{1,a,c}	{1,b,c,e}	{a,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,a,b,c,d}	{1,a,c,e}	{a,b,d}	{1,a,c,d}	{1,b,e}
b	b	{1,a,c}	{1,a,c,e}	{0,b,c,d,e}	{1,a,b,d}	{b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d}	{1,a,b,d}	{0,1,c,d,e}	{a,b,c,e}	{1,c,d}
d	d	{a,d,e}	{1,a,c,d}	{b,c,e}	{a,b,c,e}	{0,1,a,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,b,e}	{a,b,d,e}	{1,c,d}	{1,b,c,d}	{0,1,a,b,e}

HF_7^{196}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c}	{1,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,a,b,c,d}	{1,a,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
b	b	{1,a,c,d}	{1,a,c,e}	{0,b,c,d,e}	{1,a,b,d}	{1,b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d,e}	{1,a,b,d}	{0,1,c,d,e}	{a,b,c,e}	{1,a,c,d}
d	d	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,c,e}	{0,1,a,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,d}	{0,1,a,b,e}

HF_7^{197}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c}	{1,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,d}	{1,a,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,c,d,e}	{0,b,c,d,e}	{1,a,b,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,d,e}	{0,1,c,d,e}	{1,a,b,c,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,e}	{0,1,a,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d}	{0,1,a,b,e}

HF_7^{198}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c}	{1,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,d}	{1,a,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,c,d,e}	{0,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,d,e}	{0,1,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,e}	{0,1,a,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d}	{0,1,a,b,e}

HF_7^{199}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d}	{1,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{b,c,e}
a	a	{1,c,d}	{0,a,b,c,e}	{a,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,d,e}	{0,1,b,c,d}	{1,b,e}	{a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d}	{1,b,e}	{0,a,c,d,e}	{1,a,c}	{1,b,c,e}
d	d	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c}	{0,1,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d}	{0,1,a,c,e}

HF_7^{200}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d}	{1,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,a,b,c,e}	{a,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,d,e}	{0,1,b,c,d}	{1,b,e}	{1,a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d,e}	{1,b,e}	{0,a,c,d,e}	{1,a,c}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c}	{0,1,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d}	{0,1,a,c,e}

HF_7^{201}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d}	{1,b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,b,c,d}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{0,a,c,d,e}	{1,a,b,c,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,e}	{0,1,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c,d}	{0,1,a,c,e}

HF_7^{202}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,b,c,d}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,a,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,c,e}

HF_7^{203}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,e}	{1,a,c,d}	{1,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c}	{a,b,d,e}	{1,a,d}	{1,b,c,e}	{b,d,e}
b	b	{1,c,e}	{a,b,d,e}	{0,a,b,c,d}	{1,b,c,e}	{a,b,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,d}	{1,b,c,e}	{0,b,c,d,e}	{1,a,c,d}	{1,b,c}
d	d	{a,c,d}	{1,b,c,e}	{a,b,e}	{1,a,c,d}	{0,1,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,d,e}	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{0,1,a,d,e}

HF_7^{204}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c}	{a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,b,d,e}	{0,a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{0,b,c,d,e}	{1,a,c,d}	{1,a,b,c}
d	d	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c,d}	{0,1,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,c,d,e}	{1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{0,1,a,d,e}

HF_7^{205}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,e}	{1,a,b,c,d,e}	{1,a,c,e}	{a,b,d,e}	{a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c}	{1,a,b,c,d,e}	{1,a,b,d}	{1,b,c,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,b,c,d,e}	{0,a,b,c,d}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{0,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,b,c,e}	{a,b,c,e}	{1,a,b,c,d,e}	{0,1,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,c,d}	{1,b,c,d}	{1,a,b,c,d,e}	{0,1,a,d,e}

HF_7^{206}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,b,c,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,d,e}

HF_7^{207}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,d}	{1,c,d}	{b,c,e}	{1,a,b,c,d,e}	{1,a,c}	{b,c,e}
a	a	{1,c,d}	{0,a,b,d,e}	{a,d,e}	{1,c,d}	{1,a,b,c,d,e}	{a,b,d}
b	b	{b,c,e}	{a,d,e}	{0,1,b,c,e}	{1,b,e}	{a,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,c,d}	{1,b,e}	{0,1,a,c,d}	{1,a,c}	{1,b,e}
d	d	{1,a,c}	{1,a,b,c,d,e}	{a,d,e}	{1,a,c}	{0,a,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d}	{1,a,b,c,d,e}	{1,b,e}	{a,b,d}	{0,1,b,c,e}

HF_7^{208}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,d}	{1,c,d}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}	{b,c,e}
a	a	{1,c,d}	{0,a,b,d,e}	{a,d,e}	{1,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}
b	b	{b,c,d,e}	{a,d,e}	{0,1,b,c,e}	{1,b,e}	{1,a,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,c,d,e}	{1,b,e}	{0,1,a,c,d}	{1,a,c}	{1,a,b,e}
d	d	{1,a,b,c}	{1,a,b,c,d,e}	{1,a,d,e}	{1,a,c}	{0,a,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,e}	{a,b,d}	{0,1,b,c,e}

HF_7^{209}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,d}	{1,b,c,e}	{a,b,e}	{1,a,c,d}	{1,c,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{1,a,d}
b	b	{a,b,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,b,c}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{b,d,e}
d	d	{1,c,e}	{a,b,d,e}	{a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,d}	{1,b,c,e}	{b,d,e}	{1,a,c,d}	{0,1,b,c,e}

HF_7^{210}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}
b	b	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}
d	d	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}

HF_7^{211}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,d}	{1,b,d,e}	{a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,a,b,d,e}	{1,a,c,e}	{b,c,d,e}	{1,a,c,d}	{1,a,b,c}
b	b	{a,b,c,d}	{1,a,c,e}	{0,1,b,c,e}	{1,a,b,d}	{1,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{b,c,d,e}	{1,a,b,d}	{0,1,a,c,d}	{a,b,c,e}	{1,a,d,e}
d	d	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,c,e}	{0,a,b,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}	{1,b,c,d}	{0,1,b,c,e}

HF_7^{212}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,d}	{1,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}
b	b	{a,b,c,e}	{1,a,c,d,e}	{0,1,b,c,e}	{1,a,b,d,e}	{a,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{1,b,d,e}
d	d	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d}	{0,1,b,c,e}

HF_7^{213}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,d}	{1,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,d,e}	{1,a,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
b	b	{a,b,c,d,e}	{1,a,c,d,e}	{0,1,b,c,e}	{1,a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,d,e}	{0,1,a,c,d}	{1,a,b,c,e}	{1,a,b,d,e}
d	d	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,e}	{0,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d}	{0,1,b,c,e}

HF_7^{214}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,e}	{1,a,b,c,e}	{a,d,e}	{1,a,c,d}	{b,c,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,a,b,d}	{1,a,b,c,d}	{1,b,e}	{a,b,d,e}	{1,c,d}
b	b	{a,d,e}	{1,a,b,c,d}	{0,a,b,c,e}	{a,b,c,d,e}	{1,a,c}	{1,b,c,e}
c	c	{1,a,c,d}	{1,b,e}	{a,b,c,d,e}	{0,1,b,c,d}	{1,b,c,d,e}	{a,b,d}
d	d	{b,c,e}	{a,b,d,e}	{1,a,c}	{1,b,c,d,e}	{0,a,c,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,c,d}	{1,b,c,e}	{a,b,d}	{1,a,c,d,e}	{0,1,b,d,e}

HF_7^{215}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
b	b	{a,c,d,e}	{1,a,b,c,d,e}	{0,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{0,1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}
d	d	{a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{0,a,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{0,1,b,d,e}

HF_7^{216}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,c,d}	{b,c,e}	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,d}
a	a	{b,c,e}	{0,a,c,d,e}	{1,c,d}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{1,c,d}	{0,1,b,d,e}	{a,d,e}	{a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c}	{a,d,e}	{0,1,a,c,e}	{1,b,e}	{b,c,d,e}
d	d	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,e}	{0,1,a,b,d}	{1,a,c}
e	e	{a,b,d}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}	{1,a,c}	{0,a,b,c,e}

HF_7^{217}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,c,d}	{b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d}
a	a	{b,c,e}	{0,a,c,d,e}	{1,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,c,d}	{0,1,b,d,e}	{a,d,e}	{1,a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c,e}	{a,d,e}	{0,1,a,c,e}	{1,b,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,e}	{0,1,a,b,d}	{1,a,c}
e	e	{a,b,d}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c}	{0,a,b,c,e}

HF_7^{218}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,c,d}	{b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c,d,e}	{1,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,c,d,e}	{0,1,b,d,e}	{1,a,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,d,e}	{0,1,a,c,e}	{1,a,b,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,e}	{0,1,a,b,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c}	{0,a,b,c,e}

HF_7^{219}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,b,c,d}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}
a	a	{b,c,d,e}	{0,a,c,d,e}	{1,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,c,d,e}	{0,1,b,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,d,e}	{0,1,a,c,e}	{1,a,b,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,e}	{0,1,a,b,d}	{1,a,b,c}
e	e	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}	{0,a,b,c,e}

B4ii. Hyperfields for which $card(x-x)=5$ and $x \notin x-x$, for every non-zero element x.

HF_7^{220}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,c,d}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d}	{b,c,e}
a	a	{1,c,d}	{0,b,c,d,e}	{a,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,d,e}	{0,1,c,d,e}	{1,b,e}	{a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,e}	{0,1,a,d,e}	{1,a,c}	{1,b,c,e}
d	d	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,c}	{0,1,a,b,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{a,b,d}	{0,1,a,b,c}

HF_7^{221}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,b,c,d,e}	{a,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,d,e}	{0,1,c,d,e}	{1,b,e}	{1,a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,b,e}	{0,1,a,d,e}	{1,a,c}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,c}	{0,1,a,b,e}	{a,b,d}
e	e	{b,c,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,d}	{0,1,a,b,c}

HF_7^{222}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,b,c,d,e}	{1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{1,a,c,d}	{0,1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{0,1,a,d,e}	{1,b,c,e}	{b,c,d,e}
d	d	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,e}	{0,1,a,b,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}	{1,a,c,d}	{0,1,a,b,c}

HF_7^{223}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,c,d}	{0,1,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{0,1,a,d,e}	{1,b,c,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{0,1,a,b,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{0,1,a,b,c}

HF_7^{224}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,b,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,b,c,d,e}	{1,a,c,e}	{a,b,c,d}	{1,a,c,d}	{1,a,b,e}
b	b	{1,a,b,c}	{1,a,c,e}	{0,1,c,d,e}	{1,a,b,d}	{b,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,c,d}	{1,a,b,d}	{0,1,a,d,e}	{a,b,c,e}	{1,c,d,e}
d	d	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}	{a,b,c,e}	{0,1,a,b,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}	{1,b,c,d}	{0,1,a,b,c}

HF_7^{225}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}
b	b	{1,a,b,c,d}	{1,a,c,e}	{0,1,c,d,e}	{1,a,b,d}	{1,b,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,d}	{0,1,a,d,e}	{a,b,c,e}	{1,a,c,d,e}
d	d	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,c,e}	{0,1,a,b,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,d}	{0,1,a,b,c}

HF_7^{226}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,c,d,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{0,1,a,d,e}	{1,a,b,c,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c}

HF_7^{227}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c}

HF_7^{228}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,c,d}	{1,c,e}	{1,a,b,c,d,e}	{a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c,d}	{a,b,d,e}	{1,a,d}	{1,a,b,c,d,e}	{b,d,e}
b	b	{1,c,e}	{a,b,d,e}	{0,a,c,d,e}	{1,b,c,e}	{a,b,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,d}	{1,b,c,e}	{0,1,b,d,e}	{1,a,c,d}	{1,b,c}
d	d	{a,c,d}	{1,a,b,c,d,e}	{a,b,e}	{1,a,c,d}	{0,1,a,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,d,e}	{1,a,b,c,d,e}	{1,b,c}	{a,b,d,e}	{0,1,a,b,d}

HF_7^{229}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,c,d}	{1,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c,d}	{a,b,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,b,d,e}	{0,a,c,d,e}	{1,b,c,e}	{1,a,b,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,d,e}	{1,b,c,e}	{0,1,b,d,e}	{1,a,c,d}	{1,a,b,c}
d	d	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,c,d}	{0,1,a,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}	{a,b,d,e}	{0,1,a,b,d}

HF_7^{230}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,b,c,e}	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,b,c,d}	{1,a,b,c,d}	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}
b	b	{1,a,d,e}	{1,a,b,c,d}	{0,a,c,d,e}	{a,b,c,d,e}	{1,a,b,c}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,e}	{a,b,c,d,e}	{0,1,b,d,e}	{1,b,c,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{1,b,c,d,e}	{0,1,a,c,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,c,d,e}	{1,b,c,e}	{a,b,c,d}	{1,a,c,d,e}	{0,1,a,b,d}

HF_7^{231}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,b,d,e}	{1,a,c}	{1,b,c,e}	{a,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{0,1,b,c,d}	{1,a,b,c,e}	{a,b,d}	{1,a,c,d}	{1,b,e}
b	b	{1,a,c}	{1,a,b,c,e}	{0,a,c,d,e}	{1,a,b,c,d}	{b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d}	{1,a,b,c,d}	{0,1,b,d,e}	{a,b,c,d,e}	{1,c,d}
d	d	{a,d,e}	{1,a,c,d}	{b,c,e}	{a,b,c,d,e}	{0,1,a,c,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,b,e}	{a,b,d,e}	{1,c,d}	{1,b,c,d,e}	{0,1,a,b,d}

HF_7^{232}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{0,1,b,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
b	b	{1,a,c,d}	{1,a,b,c,e}	{0,a,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{0,1,b,d,e}	{a,b,c,d,e}	{1,a,c,d}
d	d	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,c,d,e}	{0,1,a,c,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{0,1,a,b,d}

HF_7^{233}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,b,c,d,e}	{0,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{0,1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,d}

HF_7^{234}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,1,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,d}

HF_7^{235}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d}

HF_7^{236}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,b,d,e}	{0,1,a,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{0,a,b,d,e}	{1,a,c,d}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{0,1,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{0,1,a,c,d}

HF_7^{237}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}

HF_7^{238}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d}

B5i. Hyperfields for which $card(x-x)=6$ and $x \in x-x$, for every non-zero element x.

HF_7^{239}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,c,d}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d}	{b,c,e}
a	a	{1,c,d}	{0,a,b,c,d,e}	{a,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,d,e}	{0,1,b,c,d,e}	{1,b,e}	{a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,e}	{0,1,a,c,d,e}	{1,a,c}	{1,b,c,e}
d	d	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,c}	{0,1,a,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{a,b,d}	{0,1,a,b,c,e}

HF_7^{240}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{b,c,e}
a	a	{1,c,d}	{0,a,b,c,d,e}	{a,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,d,e}	{0,1,b,c,d,e}	{1,b,e}	{1,a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,b,e}	{0,1,a,c,d,e}	{1,a,c}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,c}	{0,1,a,b,d,e}	{a,b,d}
e	e	{b,c,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,d}	{0,1,a,b,c,e}

HF_7^{241}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,c,e}	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{1,a,c,d}	{0,1,b,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c}	{a,b,d,e}	{0,1,a,c,d,e}	{1,b,c,e}	{b,c,d,e}
d	d	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,e}	{0,1,a,b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}	{1,a,c,d}	{0,1,a,b,c,e}

HF_7^{242}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}
a	a	{1,b,c,e}	{0,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,c,d}	{0,1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{0,1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{0,1,a,b,d,e}	{1,a,c,d}
e	e	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{0,1,a,b,c,e}

HF_7^{243}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,d,e}	{1,a,b,c}	{1,b,c,e}	{1,a,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,a,b,c,d,e}	{1,a,c,e}	{a,b,c,d}	{1,a,c,d}	{1,a,b,e}
b	b	{1,a,b,c}	{1,a,c,e}	{0,1,b,c,d,e}	{1,a,b,d}	{b,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,c,d}	{1,a,b,d}	{0,1,a,c,d,e}	{a,b,c,e}	{1,c,d,e}
d	d	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}	{a,b,c,e}	{0,1,a,b,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}	{1,b,c,d}	{0,1,a,b,c,e}

HF_7^{244}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{a,c,d,e}
a	a	{1,b,d,e}	{0,a,b,c,d,e}	{1,a,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}
b	b	{1,a,b,c,d}	{1,a,c,e}	{0,1,b,c,d,e}	{1,a,b,d}	{1,b,c,d,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,d}	{0,1,a,c,d,e}	{a,b,c,e}	{1,a,c,d,e}
d	d	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,c,e}	{0,1,a,b,d,e}	{1,b,c,d}
e	e	{a,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,d}	{0,1,a,b,c,e}

HF_7^{245}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,c,d,e}	{0,1,b,c,d,e}	{1,a,b,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,d,e}	{0,1,a,c,d,e}	{1,a,b,c,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,e}

HF_7^{246}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
a	a	{1,b,c,d,e}	{0,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,c,d,e}	{0,1,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,d,e}	{0,1,a,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{0,1,a,b,d,e}	{1,a,b,c,d}
e	e	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{0,1,a,b,c,e}

HF_7^{247}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,e}	{1,a,c,d}	{1,c,e}	{1,a,b,c,d,e}	{a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c,d}	{a,b,d,e}	{1,a,d}	{1,a,b,c,d,e}	{b,d,e}
b	b	{1,c,e}	{a,b,d,e}	{0,a,b,c,d,e}	{1,b,c,e}	{a,b,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,d}	{1,b,c,e}	{0,1,b,c,d,e}	{1,a,c,d}	{1,b,c}
d	d	{a,c,d}	{1,a,b,c,d,e}	{a,b,e}	{1,a,c,d}	{0,1,a,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,d,e}	{1,a,b,c,d,e}	{1,b,c}	{a,b,d,e}	{0,1,a,b,d,e}

HF_7^{248}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,e}	{1,a,c,d}	{1,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c,d}	{a,b,d,e}	{1,a,d,e}	{1,a,b,c,d,e}	{b,c,d,e}
b	b	{1,c,d,e}	{a,b,d,e}	{0,a,b,c,d,e}	{1,b,c,e}	{1,a,b,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,d,e}	{1,b,c,e}	{0,1,b,c,d,e}	{1,a,c,d}	{1,a,b,c}
d	d	{a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,e}	{1,a,c,d}	{0,1,a,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c}	{a,b,d,e}	{0,1,a,b,d,e}

HF_7^{249}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,e}	{1,a,b,c,e}	{1,a,d,e}	{1,a,c,d}	{b,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,a,b,c,d}	{1,a,b,c,d}	{1,a,b,e}	{a,b,d,e}	{1,c,d,e}
b	b	{1,a,d,e}	{1,a,b,c,d}	{0,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,e}	{a,b,c,d,e}	{0,1,b,c,d,e}	{1,b,c,d,e}	{a,b,c,d}
d	d	{b,c,d,e}	{a,b,d,e}	{1,a,b,c}	{1,b,c,d,e}	{0,1,a,c,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,c,d,e}	{1,b,c,e}	{a,b,c,d}	{1,a,c,d,e}	{0,1,a,b,d,e}

HF_7^{250}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,e}	{1,a,b,d,e}	{1,a,c}	{1,b,c,e}	{a,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{0,1,a,b,c,d}	{1,a,b,c,e}	{a,b,d}	{1,a,c,d}	{1,b,e}
b	b	{1,a,c}	{1,a,b,c,e}	{0,a,b,c,d,e}	{1,a,b,c,d}	{b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d}	{1,a,b,c,d}	{0,1,b,c,d,e}	{a,b,c,d,e}	{1,c,d}
d	d	{a,d,e}	{1,a,c,d}	{b,c,e}	{a,b,c,d,e}	{0,1,a,c,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,b,e}	{a,b,d,e}	{1,c,d}	{1,b,c,d,e}	{0,1,a,b,d,e}

HF_7^{251}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
a	a	{1,a,b,d,e}	{0,1,a,b,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
b	b	{1,a,c,d}	{1,a,b,c,e}	{0,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{a,b,d,e}
c	c	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d}	{0,1,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d}
d	d	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,c,d,e}	{0,1,a,c,d,e}	{1,b,c,d,e}
e	e	{1,a,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{0,1,a,b,d,e}

HF_7^{252}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,e}	{1,a,b,c,d,e}	{a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{1,b,d,e}
b	b	{1,a,c,e}	{1,a,b,c,d,e}	{0,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d}	{1,a,b,c,d,e}	{0,1,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d}
d	d	{a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,d,e}

HF_7^{253}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
b	b	{1,a,c,d,e}	{1,a,b,c,d,e}	{0,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{0,1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}
d	d	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,d,e}

HF_7^{254}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{0,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{0,1,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{1,a,c,d}	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d,e}

HF_7^{255}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,b,d,e}	{0,1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{0,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{0,1,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}	{a,b,d,e}	{0,1,a,c,d,e}

HF_7^{256}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d,e}

HF_7^{257}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}
c	c	{a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d,e}

B5ii. Hyperfields for which $card(x-x)=6$ and $x \notin x-x$, for every non-zero element x.

HF_7^{258}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a}	{1,b}	{1,c}	{1,d}	{1,e}
a	a	{1,a}	{0,1,b,c,d,e}	{a,b}	{a,c}	{a,d}	{a,e}
b	b	{1,b}	{a,b}	{0,1,a,c,d,e}	{b,c}	{b,d}	{b,e}
c	c	{1,c}	{a,c}	{b,c}	{0,1,a,b,d,e}	{c,d}	{c,e}
d	d	{1,d}	{a,d}	{b,d}	{c,d}	{0,1,a,b,c,e}	{d,e}
e	e	{1,e}	{a,e}	{b,e}	{c,e}	{d,e}	{0,1,a,b,c,d}

HF_7^{259}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a}	{1,b,d}	{1,c}	{1,b,d}	{1,e}
a	a	{1,a}	{0,1,b,c,d,e}	{a,b}	{a,c,e}	{a,d}	{a,c,e}
b	b	{1,b,d}	{a,b}	{0,1,a,c,d,e}	{b,c}	{1,b,d}	{b,e}
c	c	{1,c}	{a,c,e}	{b,c}	{0,1,a,b,d,e}	{c,d}	{a,c,e}
d	d	{1,b,d}	{a,d}	{1,b,d}	{c,d}	{0,1,a,b,c,e}	{d,e}
e	e	{1,e}	{a,c,e}	{b,e}	{a,c,e}	{d,e}	{0,1,a,b,c,d}

HF_7^{260}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,c}	{1,b,e}	{1,a,c,d}	{1,c,d}	{1,b,e}
a	a	{1,a,c}	{0,1,b,c,d,e}	{a,b,d}	{1,a,c}	{a,b,d,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d}	{0,1,a,c,d,e}	{b,c,e}	{a,b,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,c}	{b,c,e}	{0,1,a,b,d,e}	{1,c,d}	{b,c,e}
d	d	{1,c,d}	{a,b,d,e}	{a,b,d}	{1,c,d}	{0,1,a,b,c,e}	{a,d,e}
e	e	{1,b,e}	{a,d,e}	{1,b,c,e}	{b,c,e}	{a,d,e}	{0,1,a,b,c,d}

HF_7^{261}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,c}	{1,b,d,e}	{1,a,c,d}	{1,b,c,d}	{1,b,e}
a	a	{1,a,c}	{0,1,b,c,d,e}	{a,b,d}	{1,a,c,e}	{a,b,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{a,b,d}	{0,1,a,c,d,e}	{b,c,e}	{1,a,b,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,c,e}	{b,c,e}	{0,1,a,b,d,e}	{1,c,d}	{a,b,c,e}
d	d	{1,b,c,d}	{a,b,d,e}	{1,a,b,d}	{1,c,d}	{0,1,a,b,c,e}	{a,d,e}
e	e	{1,b,e}	{a,c,d,e}	{1,b,c,e}	{a,b,c,e}	{a,d,e}	{0,1,a,b,c,d}

HF_7^{262}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{0,1,a,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{0,1,a,b,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{0,1,a,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d}

HF_7^{263}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,b,c,d,e}	{a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,b,d,e}	{0,1,a,c,d,e}	{1,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,e}	{0,1,a,b,d,e}	{1,a,c,d}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d}	{0,1,a,b,c,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d}

HF_7^{264}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{1,a,b,c,d}	{0,1,a,c,d,e}	{a,b,c,d,e}	{a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c}	{a,b,c,d,e}	{0,1,a,b,d,e}	{1,b,c,d,e}	{b,c,d,e}
d	d	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,d,e}	{0,1,a,b,c,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d}

HF_7^{265}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d}	{0,1,a,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c,e}	{a,b,c,d,e}	{0,1,a,b,d,e}	{1,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,d,e}	{0,1,a,b,c,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d}

HF_7^{266}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d}

HF_7^{267}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d}

B6. Hyperfields for which $card(x-x)=7$, for every non-zero element x.

HF_7^{268}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a}	{1,b}	{1,c}	{1,d}	{1,e}
a	a	{1,a}	{0,1,a,b,c,d,e}	{a,b}	{a,c}	{a,d}	{a,e}
b	b	{1,b}	{a,b}	{0,1,a,b,c,d,e}	{b,c}	{b,d}	{b,e}
c	c	{1,c}	{a,c}	{b,c}	{0,1,a,b,c,d,e}	{c,d}	{c,e}
d	d	{1,d}	{a,d}	{b,d}	{c,d}	{0,1,a,b,c,d,e}	{d,e}
e	e	{1,e}	{a,e}	{b,e}	{c,e}	{d,e}	{0,1,a,b,c,d,e}

HF_7^{269}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a}	{1,b,d}	{1,c}	{1,b,d}	{1,e}
a	a	{1,a}	{0,1,a,b,c,d,e}	{a,b}	{a,c,e}	{a,d}	{a,c,e}
b	b	{1,b,d}	{a,b}	{0,1,a,b,c,d,e}	{b,c}	{1,b,d}	{b,e}
c	c	{1,c}	{a,c,e}	{b,c}	{0,1,a,b,c,d,e}	{c,d}	{a,c,e}
d	d	{1,b,d}	{a,d}	{1,b,d}	{c,d}	{0,1,a,b,c,d,e}	{d,e}
e	e	{1,e}	{a,c,e}	{b,e}	{a,c,e}	{d,e}	{0,1,a,b,c,d,e}

HF_7^{270}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,c}	{1,b,e}	{1,a,c,d}	{1,c,d}	{1,b,e}
a	a	{1,a,c}	{0,1,a,b,c,d,e}	{a,b,d}	{1,a,c}	{a,b,d,e}	{a,d,e}
b	b	{1,b,e}	{a,b,d}	{0,1,a,b,c,d,e}	{b,c,e}	{a,b,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,c}	{b,c,e}	{0,1,a,b,c,d,e}	{1,c,d}	{b,c,e}
d	d	{1,c,d}	{a,b,d,e}	{a,b,d}	{1,c,d}	{0,1,a,b,c,d,e}	{a,d,e}
e	e	{1,b,e}	{a,d,e}	{1,b,c,e}	{b,c,e}	{a,d,e}	{0,1,a,b,c,d,e}

HF_7^{271}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,c}	{1,b,d,e}	{1,a,c,d}	{1,b,c,d}	{1,b,e}
a	a	{1,a,c}	{0,1,a,b,c,d,e}	{a,b,d}	{1,a,c,e}	{a,b,d,e}	{a,c,d,e}
b	b	{1,b,d,e}	{a,b,d}	{0,1,a,b,c,d,e}	{b,c,e}	{1,a,b,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,c,e}	{b,c,e}	{0,1,a,b,c,d,e}	{1,c,d}	{a,b,c,e}
d	d	{1,b,c,d}	{a,b,d,e}	{1,a,b,d}	{1,c,d}	{0,1,a,b,c,d,e}	{a,d,e}
e	e	{1,b,e}	{a,c,d,e}	{1,b,c,e}	{a,b,c,e}	{a,d,e}	{0,1,a,b,c,d,e}

HF_7^{272}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}
b	b	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,e}
d	d	{1,a,c,d}	{1,a,b,c,d,e}	{a,b,d,e}	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}

HF_7^{273}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}
a	a	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}
b	b	{1,b,c,d,e}	{a,b,d,e}	{0,1,a,b,c,d,e}	{1,b,c,e}	{1,a,b,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,b,c,e}	{0,1,a,b,c,d,e}	{1,a,c,d}	{1,a,b,c,e}
d	d	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,c,d}	{0,1,a,b,c,d,e}	{a,b,d,e}
e	e	{1,b,c,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{a,b,d,e}	{0,1,a,b,c,d,e}

HF_7^{274}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,e}	{1,a,c,d}	{1,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c}	{a,b,d,e}	{1,a,d,e}
b	b	{1,a,b,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{b,c,d,e}
d	d	{1,c,d,e}	{a,b,d,e}	{a,b,c,d}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,a,d,e}	{1,b,c,e}	{b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}

HF_7^{275}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,d,e}	{1,a,c,d}	{1,b,c,d,e}	{1,a,b,d,e}
a	a	{1,a,b,c,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,e}	{a,b,d,e}	{1,a,c,d,e}
b	b	{1,a,b,d,e}	{1,a,b,c,d}	{0,1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d}	{1,b,c,e}
c	c	{1,a,c,d}	{1,a,b,c,e}	{a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,b,c,d,e}	{a,b,c,d,e}
d	d	{1,b,c,d,e}	{a,b,d,e}	{1,a,b,c,d}	{1,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,c,d,e}
e	e	{1,a,b,d,e}	{1,a,c,d,e}	{1,b,c,e}	{a,b,c,d,e}	{1,a,c,d,e}	{0,1,a,b,c,d,e}

HF_7^{276}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,e}	{1,a,b,c,d,e}	{1,a,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{1,a,b,d,e}
b	b	{1,a,b,c,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}
d	d	{1,a,c,d,e}	{1,a,b,c,d,e}	{a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,d,e}	{1,a,b,c,d,e}	{1,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}

HF_7^{277}	0	1	a	b	c	d	e
0	0	1	a	b	c	d	e
1	1	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
a	a	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
b	b	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
c	c	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}
d	d	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}	{1,a,b,c,d,e}
e	e	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{1,a,b,c,d,e}	{0,1,a,b,c,d,e}

Author contributions

Both authors contributed equally to all phases of this work, like conceptualization, methodology, visualization, software development, writing-review.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this paper.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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