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Research article

A new transform technique for the analysis of the time-fractional water pollution model and Bloch equation

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Abstract: The objective of the present study is to illustrate the significance and applicability of fractional-order derivatives in resolving mathematical models. In this study, we investigated two mathematical models, including the nonlinear time-fractional water pollution model and the time-fractional Bloch model, which arises in bioengineering. These models were successfully solved by using a unique technique known as the q-homotopy analysis Yang transform method, and approximate solutions were obtained. Using the Caputo operator, which accounts for memory effects, the model was examined in fractional order. The results clearly indicate that fractional-order derivatives give a better match for the behavior of the systems under consideration. This method not only allows for improved comprehension of complex phenomena but also provides an opportunity for further research in diverse areas, such as environmental science and bio engineering. The results obtained by the considered method are in the form of a series that converges swiftly and also we proposed convergence analysis for the considered method. This technique helps us modify the convergence area of the series solution by utilizing an auxiliary parameter called $\hbar$, also referred to as the convergence control parameter. The outcomes were analyzed using 3D plots and graphs, and also we conducted numerical simulations. The obtained results demonstrate that the proposed method is highly accurate and successful in examining nonlinear issues that arise in various scientific and technological domains.

Keywords: fractional water pollution model; fractional Bloch equation; fractional derivatives and integrals; q-homotopy analysis Yang transform method
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1. Introduction

Water pollution is an irresistible environmental problem that has a major impact on human health and ecosystems globally. It is caused by the release of harmful substances such as oil, chemicals, agricultural activities, and domestic waste into natural water bodies, which results in unfavorable changes in the water quality. The sources of water pollution are various, like industrial discharges, construction sites, improper drainage systems in towns, and many others. The World Health Organization (WHO) claims that one of the main causes of sickness and death in modern times is waterborne disease. This usually occurs as a result of water pollution caused by human activity. Rivers that are used for drinking water, irrigation, and swimming are impacted by lake contamination. As a result, monitoring and assessing the pollution levels in rivers and lakes is crucial. Consequently, a variety of mathematical models have been constructed to comprehend the dynamics of water pollution. Many researchers worked on these models in order to control them. Sabir et al. applied the Levenberg-Marquardt backpropagation approach to work on the numerical simulations of the novel fractional water contamination model [1]. Ebrahimzadeh et al. [2] employed optimum control methods and an extensive fractional modeling framework to analyze water pollution management. Mosavi et al. [3] worked on effective control strategies of water pollution through mathematical modeling, and many other models have been designed to investigate the dynamics of water pollution and its control [4–6]. The considered fractional water pollution (FWP) model is given by

$$\begin{aligned}
&{}_{0}^{C}D_{\varsigma}^{\alpha}(X) = k_3 Z(\varsigma) + k_1 X(\varsigma)(a - X(\varsigma)) - k_2 Y(\varsigma),\\
&{}_{0}^{C}D_{\varsigma}^{\alpha}(Y) = k_4 X(\varsigma)(a - X(\varsigma)) + k_6 Z(\varsigma) + k_5 Y(\varsigma)(b - Y(\varsigma)),\\
&{}_{0}^{C}D_{\varsigma}^{\alpha}(Z) = k_7 X(\varsigma) - k_8 Y(\varsigma),
\end{aligned} \tag{1.1}$$

with initial conditions $X(0) = S_1, Y(0) = S_2, Z(0) = S_3$, where $k_1, k_2, ..., k_8$ represent constants.

The evolution of a magnetization vector in a magnetic field is described by a vector differential equation called the Bloch equation. It is utilized in chemistry and physics to compute nuclear magnetization in electron spin resonance (ESR), magnetic resonance imaging (MRI), and nuclear magnetic resonance (NMR). The fractional Bloch equation is frequently used in many different areas, especially in quantum mechanics and magnetic resonance studies. This equation with fractional derivatives is much more accurate in the sense of describing systems with memory effects or non-Markovian dynamics, as in real-world systems. It is the best model for complex phenomena like anomalous diffusion and long-range correlations that traditional models usually ignore. MRI and spectroscopy are the two medical fields where the fractional time Bloch equation makes it possible to better the meaning of relaxation processes and the reliability of measurements of diverse tissues or multicomponent materials through complex interactions. The equation plays the role of the most important model in condensed matter physics, statistical mechanics, and the study of chaotic systems, because it permits the acquisition of a deeper understanding of the dynamical process in systems with memory and time-dependent properties. It was first developed by Swiss-born American physicist Felix Bloch in 1946 [7]. This equation is used in physics and bioengineering, and more precisely in magnetic resonance imaging, or NMR, when the relaxation durations are T1 (spin-lattice) and T2 (spin-spin) to compute the nuclear magnetization $M = (M_x(t), M_y(t), M_z(t))$ as a function depending on the temperature [8]. Petras [9] examined the stability and behavior of the Bloch equations. S. Bhalekar et al. [10] proposed a realistic model that included both the fractional derivative and finite time delays. Bain et al. [11] examined the exact solution to the Bloch equation with its applications. Murase and Tanki [12] proposed numerical solutions to the time-dependent Bloch equation. Many more researchers have also worked on this model [13, 14]. The time-fractional Bloch equation is given by

$$\begin{aligned}
{}^{C}_{0}D^{\alpha}_{\varsigma}(M_x(\varsigma)) &= \omega M_y(\varsigma) - \frac{M_x(\varsigma)}{T_1}, \\
{}^{C}_{0}D^{\alpha}_{\varsigma}(M_y(\varsigma)) &= \omega M_x(\varsigma) - \frac{M_y(\varsigma)}{T_2}, \\
{}^{C}_{0}D^{\alpha}_{\varsigma}(M_z(\varsigma)) &= \frac{M_0 - M_z(\varsigma)}{T_1},
\end{aligned} \tag{1.2}$$

with initial conditions $M_x(0) = 0, M_y(0) = 100, M_z(0) = 0$. Here, $M_x(t), M_y(t)$, and $M_z(t)$ show system magnetization in the x, y, and z components, $\omega = \gamma C_0$ is the frequency resonance, C_0 is the static magnetic field and M_0 is the equilibrium magnetization. T_1 and T_2 denote the spin-lattice and spin-spin relaxation times ($T_1 = 1, T_2 = 200, \omega = 1, M_0 = 100$). It is important that the MR signal intensity which is measurable $S(t)$ is directly proportional to the transverse magnetization magnitude $M_{xy}(t)$ at each time point, that is, $S(t) \propto M_{xy}(t)$. This linkage of the model to the measured signal forms the foundation upon which the experimental signal can be connected to the model predictions of transverse magnetization.

As we know, nonlinear chaos, complex natural phenomena, and real-world problems can be effectively studied by the concept of classical calculus but include some limitations due to which the idea of fractional calculus (FC) to be investigated about three centuries ago by the ideas of two well-known mathematicians, Leibniz and L'Hospital. FC is a rapidly evolving theory that can be defined as derivatives and integrals of non-integer order. The primary benefit of fractional order is that it gives us the solutions in between intervals, which makes it easier for us to understand the results. The primary advantage of FC is that it accurately illustrates a variety of non-linear processes. The mathematical models or problems that possess the properties like solitons, chaos, asymptotic properties, singularity, history, and random motions can be excellently investigated through FC theory. Indeed, this idea gives us a lot of flexibility when it comes to solving different kinds of equations (integral, differential, and integro-differential), mathematical physics issues involving certain special functions and their assumptions, and expansions in one or more variables. Such expansions might be the basis of new ideas and answers, which may turn out to be more deeply rooted in the patterns thus far unrecognized. Therefore, the utilization of FC theory not only deepens our grasp of complex systems but also extends the arsenal of mathematical methods at hand for solving complicated problems in different scientific disciplines. Many researchers contributed to the development of this theory as they proposed their own fractional derivatives and integrals, like the Riemann-Liouville (RL), Caputo-Fabrizio, Atangana-Baleanu, Hilfer, Caputo derivative, and other derivatives. In this work, we use the Caputo operator, which is essential since it is limited, behaves non-locally, and performs with smoother behavior than other fractional operators. It is particularly well-suited for initial value problems, and modeling systems with initial conditions is made simpler by these Caputo operator behaviors [15, 16].

The idea of FC can be applied in various areas of science and technology, like a formulation of nonlocal electrodynamics using integral and integro-differential operators that yield the fractional Luchko form, which was proposed by Tarasov [17]. Waheed et al. [18] proposed a Laplacian discrete operator, which has its significance in data modeling and image processing tasks. Hassan et al. [19] investigated regression analysis of the magnetohydrodynamic flow of Maxwell fluid inside a cylinder using a machine learning technique. The differential equations of fractional order are known as fractional differential equations. The advantages of FDEs include a hereditary property, uncertainty property, and memory effects. Consequently, FDEs are increasingly being used to represent complex and natural phenomena in real-world problems. These equations are solved using various methods, such as Monzon [20], who worked on solving higher-order FDEs using the variational iteration method with a description of the fractional derivative and the iterated derivatives. Ganie et al. [21] investigated fractional partial equations by applying two semi-analytical methods with the Elzaki transform. Oqielat et al. [22] applied the Laplace residual power series method to study the nonlinear time-fractional reaction-diffusion model. Qayyum et al. [23] determined the solution of the non-linear (2 + 1)-dimensional time-fractional Wu-Zhang (WZ) system using homotopy perturbation method with Caputo derivative. Adel et al. [24] studied nonlinear Hilfer fractional stochastic differential equations of the Sobolev type with non-instantaneous impulse in Hilbert space. The Rabotnov fractional-exponential kernel fractional derivative formula was developed by Aboubakr et al. [25] and used to investigate the blood ethanol concentration system. Adel et al. [26] applied the spectal collacation method to study the nine-dimensional fractional chaotic Lorenz system. Tawhari [27] implemented analytical techniques for fractional Schrodinger and Korteweg-de Vries equations. Ibrahim et al. [28] employed Chebyshev approximations to investigate the fractional covid-19 mathematical model. Adel et al. [29] achieved approximate solutions for the ethanol concentration system and predator-prey equations via the VIM, and many numerical and analytical methods were studied, such as [30–33].

This study employs a novel technique termed the q-homotopy analysis Yang transform method (q-HAYTM), which is a combination of the homotopy analysis technique (HAM) [34] and the Yang transform [35]. The Yang transform is an analytical method for solving fractional differential equations (FDEs). The transform in effect reduces a fractional differential equation to an algebraic equation that is easier to manipulate and solve. Especially valuable for physics, engineering, and applied mathematics problems that involve fractional calculus in costly simulations of complex systems, including viscoelastic materials or anomalous diffusion processes. The Yang transform is applied to either convert the fractional derivatives to algebraic terms or, from an analytical perspective,this changes the mathematical treatment of the problem. To obtain the solution in the original domain, the inverse transform is applied after solving the transformed algebraic equation, which provides a systematic approach to solve FDEs. When examining the distinctive features of FDEs that are common in the domains of science and technology, this method is incredibly methodical, efficient, and precise. Additionally, q-HAYTM is known for its simplicity and effectiveness in handling highly nonlinear and singular problems, which are often challenging for other numerical methods. This method do not need the use of any transformation, unrealistic restriction, linearization, complex polynomials, or assumptions. Some of the researchers applied these novel transforms with some homotopy techniques to solve diverse problems, like Naeem et al. [36], who used the Yang transform decomposition method to solve fractional partial differential equations. In order to solve fractional diffusion equations in plasma physics and fluids, Sunthrayuth et al. [37] examined Adomian decomposition techniques utilizing the Jafari-Yang transform. A hybrid Yang transform Adomian decomposition approach was introduced by Bekela and Deresse [38] to solve time-fractional nonlinear partial differential equations.

The current study investigates two considered models using a novel transform technique via the fractional derivative. The obtained results are in the form of a series with rapid convergence. The proposed method is very effective in solving these equations, as it needs less computational work and does not need any linearization or complex polynomials. The solutions obtained are very accurate compared to other methods with more precision. This research aims to convey the importance and usability of fractional derivatives in modeling complicated mathematical models. Two non-linear time-fractional systems have been considered, the water pollution model and the Bloch model, that are the source of bioengineering. The solutions of these models are performed by the q-homotopy analysis Yang transform method (q-HAYTM), a new semi-analytical method that produces the series solutions with a fast convergence rate. A key features of this paper are the identification and incorporation of convergence analysis into the solution process, thereby improving both the precision and trustworthiness of the work. The use of the Caputo fractional operator is the basis for these models to accurately reflect the change in dynamics over time.

The article's outline is organized as follows: The fundamental preliminaries (definitions and lemmas) used in the model's examination are included in Section 2. The method under consideration is illustrated in Section 3, the model's solutions are described with a graphical demonstration in Section 4, and the numerical discussion of the problems under consideration, together with numerical simulations for various fractional orders, are determined in Section 5. The study's findings are explained in Section 6.

2. Preliminaries

Definition 2.1. A function $f(\varsigma) \in C_\mu$ $(\mu \geq -1)$, then the Riemann-Liouville (RL) fractional integral is denoted by $I_\varsigma^\alpha f(\varsigma)$ and it is demonstrated as

$$I_\varsigma^\alpha f(\varsigma) = \frac{1}{\Gamma(\alpha)} \int_0^\varsigma (u-v)^{\alpha-1} f(v) dv. \tag{2.1}$$
$$I_\varsigma^0 f(\varsigma) = f(\varsigma).$$

Definition 2.2. [15, 16] The Caputo fractional operator of the function $f \in C_{-1}^n$ is defined as follows:

$$D_\varsigma^\alpha f(\varsigma) = \begin{cases} \frac{d^n f(\varsigma)}{dt^n}, & \alpha = n \in \mathbb{N}, \\ \frac{1}{\Gamma(n-\alpha)} \int_0^\varepsilon (u-v)^{n-\alpha-1} f^{(n)}(v) dv, & n-1 < \alpha \leqslant n, \ n \in \mathbb{N}. \end{cases} \tag{2.2}$$

Definition 2.3. [37, 38] The Yang transform (YT) of a function $f(t)$ is given by

$$Y[f(\varsigma)] = \int_0^\infty e^{\frac{-\varsigma}{s}} f(\varsigma) d\varsigma, \ s > 0. \tag{2.3}$$

Definition 2.4. [37, 38] The YT of some basic functions are defined as:

$$Y[1] = s,$$
$$Y[\varsigma] = s^2,$$
$$Y[\varsigma^n] = n! s^{n+1} = \Gamma(n+1) s^{n+1}, n \in \mathrm{N},$$
$$Y[\varsigma^\alpha] = \Gamma(\alpha+1) s^{\alpha+1}, \alpha > 0.$$

3. Projected algorithm for the considered equation

The current section provides the idea of the q-HAYTM. A non-linear fractional differential equation which is non-homogeneous is given by

$$D_{\varsigma}^{\alpha} v(\xi, \varsigma) + \Re\, v(\xi, \varsigma) + \mathcal{N}\, v(\xi, \varsigma) = f(\xi, \varsigma), \; 0 < \alpha < 1, \tag{3.1}$$

where the source term is defined as $f(\xi, \varsigma)$, the Caputo operator of the function $v(\xi, \varsigma)$ is $D_{\varsigma}^{\alpha} v(\xi, \varsigma)$, $\mathcal{N}$ denotes the nonlinear differential operator, and $\Re$ is the bounded linear differential operator in ξ and ς. Performing the YT on Eq (3.1) yields:

$$Y[{}_0^C D_{\varsigma}^{\alpha} v(\xi, \varsigma)] + Y[\Re\, v(\xi, \varsigma)] + Y[\mathcal{N}\, v(\xi, \varsigma)] = Y[f(\xi, \varsigma)]. \tag{3.2}$$

By simplifying Eq (3.2) and using Definition 2.4, we obtain

$$Y[v(\xi, \varsigma)] - sv(\xi, 0) + s^{\alpha}\{Y[\Re\, v(\xi, \varsigma)] + Y[\mathcal{N}\, v(\xi, \varsigma)] - Y[f(\xi, \varsigma)]\} = 0. \tag{3.3}$$

The non-linear operator $\mathcal{N}$ is described via the HAM as

$$\begin{aligned} \mathcal{N}[\varphi(\xi, \varsigma : q)] = {} & Y[\varphi(\xi, \varsigma : q)] - s\varphi^{k}(\xi, \varsigma : q)(0^{+}) \\ & -s^{\alpha} Y[\Re\varphi(\xi, \varsigma : q)] + Y[\mathcal{N}\varphi(\xi, \varsigma : q)] - Y[f\varphi(\xi, \varsigma : q)]. \end{aligned} \tag{3.4}$$

$H(\xi, \varsigma)$ is the auxillary function and the deformation equation is expressed as

$$(1 - nq)Y[\varphi(\xi, \varsigma; q) - v_0(\xi, \varsigma)] = \hbar q \mathcal{N}[\varphi(\xi, \varsigma; q)], \tag{3.5}$$

where $\varphi(\xi, \varsigma; q)$ represents a real function and $v_0(\xi, \varsigma)$ is an initial solution. The embedding parameter is represented by q, which lies within the interval $[0, \frac{1}{r}]$. Y represents the Yang transform, and $\hbar \neq 0$ is the auxiliary parameter. The following terms hold for $q = 0$ and $q = \frac{1}{r}$, respectively:

$$\varphi\Big(\xi, \varsigma; \frac{1}{r}\Big) = v(\xi, \varsigma), \qquad \varphi(\xi, \varsigma; 0) = v_0(\xi, \varsigma). \tag{3.6}$$

By varying q between 0 and $\frac{1}{r}$, the solution $\varphi(\xi, \varsigma; q)$ converges from $v_0(\xi, \varsigma)$ to $v(\xi, \varsigma)$. The series form of the function $\varphi(\xi, \varsigma; q)$ is then extended as follows, using Taylor's theorem for all q.

$$\varphi(\xi, \varsigma; q) = v_0(\xi, \varsigma) + \sum_{m=1}^{\infty} v_m(x, \varsigma) q^m, \tag{3.7}$$

where

$$\varphi_m(\xi, \varsigma) = \frac{1}{m!} \frac{\partial^m \varphi(\xi, \varsigma; q)}{\partial q^m}|_{q=0}. \tag{3.8}$$

For the proper values of $v_0(\xi, \varsigma)$, the series Eq (3.7) converges at $q = \frac{1}{r}$, yielding results for the original discussed Eq (3.1). Let m be the auxiliary linear operator and $\hbar$ indicate a specific choice of auxiliary parameter.

$$v(\xi, \varsigma) = v_0(\xi, \varsigma) + \sum_{m=1}^{\infty} v_m(\xi, \varsigma)\Big(\frac{1}{r}\Big)^m. \tag{3.9}$$

We can now define the m^{th}-order deformation equation for $q = 0$ by m-times differentiating Eq (3.5) with respect to q and dividing it by $m!$.

$$Y[v_m(\xi,\varsigma) - K_m v_{m-1}(\xi,\varsigma)] = \hbar \Re_m(\vec{v}_{m-1}), \tag{3.10}$$

where

$$K_m = \begin{cases} 0 & m \leqslant 1, \\ r & otherwise, \end{cases} \tag{3.11}$$

and

$$\Re_m(\vec{v}_{m-1}) = \frac{1}{m-1}\frac{\partial^{m-1}\mathcal{N}[\varphi(\xi,\varsigma;q)]}{\partial q^m}|_{q=0}. \tag{3.12}$$

The considered vectors are

$$\vec{v}_m = \{v_0(\xi,\varsigma), v_1(\xi,\varsigma)...v_m(\xi,\varsigma)\}. \tag{3.13}$$

The recursive equation is represented as follows by the inverse Yang transform of the deformation equation: Eq (3.10).

$$v_m(\xi,\varsigma) = K_m v_{m-1}(\xi,\varsigma) + \hbar Y^{-1}[\Re_m(\vec{v}_{m-1})]. \tag{3.14}$$

Ultimately, we used Eq (3.14) to achieve a component-wise approximation solution for the q-HAYTM series, which we describe as

$$v(\xi,\varsigma) = \sum_{m=0}^{\infty} v_m(\xi,\varsigma). \tag{3.15}$$

4. Convergence theorem

The solution $v = \sum_{m=0}^{\infty} v_m(\xi,\varsigma)$ of Eq (3.1) via the q-HAYTM converges such that $\|v_{n+1}(\xi,\varsigma)\| \leqslant \lambda \|v_n(\xi,\varsigma)\|$, where $0 \leqslant \lambda < 1$, $m \geqslant 0$ and $\|\cdot\|$ denotes the supremum norm.

Proof. The sequence of partial sums is given by S_n where $S_n = \sum\limits_{m=0}^{\infty} v_m(\xi,\varsigma)$. We have to show $\{S_n\}_{n=0}^{\infty}$ is Cauchy. Let us consider

$$\|S_{n+1} - S_n\| = \|v_{n+1}\| \leqslant \lambda \|v_n\| \leqslant \lambda^2 \|v_{n-1}\| \leqslant ... \leqslant \lambda^{n+1} \|v_0\|.$$

For all $n, m \in \mathbb{N}$ and $n \geqslant m$, we get

$$\begin{aligned} \|S_n - S_m\| &= \|S_n - S_{n-1} + S_{n-1} - S_{n-2} + S_{n-2}..... - S_{m+1} + S_{m+1} - S_m\| \\ &\leqslant \|S_n - S_{n-1}\| + \|S_{n-1} - S_{n-2}\| + ... + \|S_{m+1} - S_m\| \\ &\leqslant \lambda^n \|v_0\| + \lambda^{n-1} \|v_0\| + ... + \lambda^{m+1} \|v_0\| \\ &\leqslant \|v_0\| (\lambda^n + \lambda^{n-1} + ... + \lambda^{m+1}), \end{aligned}$$

which gives

$$\|S_n - S_m\| \leqslant \frac{\lambda^{m+1}}{1-\lambda} \|v_0\| \to 0, \ as \ m, n \to \infty.$$

Therefore S_n is a Cauchy sequence and converges uniformly to $v(\xi,\varsigma)$.

5. Solution of the model via the proposed method

Example 1. Consider the time-fractional FWP model [1].

$$\begin{aligned} {}_0^C D_\varsigma^\alpha(X) &= k_3 Z(\varsigma) + k_1 X(\varsigma)(a - X(\varsigma)) - k_2 Y(\varsigma), \\ {}_0^C D_\varsigma^\alpha(Y) &= k_4 X(\varsigma)(a - X(\varsigma)) + k_6 Z(\varsigma) + k_5 Y(\varsigma)(b - Y(\varsigma)), \\ {}_0^C D_\varsigma^\alpha(Z) &= k_7 X(\varsigma) - k_8 Y(\varsigma). \end{aligned} \tag{5.1}$$

The initial conditions are given by

$$\begin{aligned} X_0 &= 2, \\ Y_0 &= 4, \\ Z_0 &= 3. \end{aligned} \tag{5.2}$$

Performing a Yang transform on Eq (5.1), we get

$$\begin{aligned} &Y[X(\varsigma)] - sX[0] + s^\alpha\{k_3 Z(\varsigma) + k_1 X(\varsigma)a - k_1 X(\varsigma)X(\varsigma) - k_2 Y(\varsigma)\} = 0, \\ &Y[Y(\varsigma)] - sY[0] + s^\alpha\{k_4 X(\varsigma)a - k_4 X(\varsigma)X(\varsigma) + k_6 Z(\varsigma) + k_5 Y(\varsigma)b - k_5 Y(\varsigma)Y(\varsigma)\} = 0, \\ &Y[Z(\varsigma)] - sZ[0] + s^\alpha\{k_7 X(\varsigma) - k_8 Y(\varsigma)\} = 0. \end{aligned} \tag{5.3}$$

The non-linear operator is given by

$$\begin{aligned} &N^1[\varphi_1(\xi,\varsigma;q), \varphi_2(\xi,\varsigma;q), \varphi_3(\xi,\varsigma;q)] = Y[\varphi_1(\xi,\varsigma;q)] - s(X_0) \\ &-s^\alpha Y\{k_3\varphi_3(\xi,\varsigma;q) + k_1 a\varphi_1(\xi,\varsigma;q) - k_1\varphi_1(\xi,\varsigma;q)\varphi_1(\xi,\varsigma;q) - k_2\varphi_2(\xi,\varsigma;q)\}, \\ &N^2[\varphi_1(\xi,\varsigma;q), \varphi_2(\xi,\varsigma;q), \varphi_3(\xi,\varsigma;q)] = Y[\varphi_1(\xi,\varsigma;q)] - s(Y_0) \\ &-s^\alpha Y\{k_4 a\varphi_1(\xi,\varsigma;q) - k_4\varphi_1(\xi,\varsigma;q)\varphi_1(\xi,\varsigma;q) - k_6\varphi_3(\xi,\varsigma;q) + k_5 b\varphi_2(\xi,\varsigma;q) - k_5\varphi_2(\xi,\varsigma;q)\varphi_2(\xi,\varsigma;q)\}, \\ &N^3[\varphi_1(\xi,\varsigma;q), \varphi_2(\xi,\varsigma;q), \varphi_3(\xi,\varsigma;q)] = Y[\varphi_1(\xi,\varsigma;q)] - s(Z_0) \\ &-s^\alpha Y\{k_7\varphi_1(\xi,\varsigma;q) - k_8\varphi_2(\xi,\varsigma;q)\}. \end{aligned} \tag{5.4}$$

The deformation equation at $H(\xi,\varsigma) = 1$ is given by

$$\begin{aligned} Y[X(\xi,\varsigma) - k_m X_{m-1}(\xi,\varsigma)] &= h\{R_{1,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}]\}, \\ Y[Y_m(\xi,\varsigma) - k_m Y_{m-1}(\xi,\varsigma)] &= h\{R_{2,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}]\}, \\ Y[Z_m(\xi,\varsigma) - k_m Z_{m-1}(\xi,\varsigma)] &= h\{R_{3,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}]\}, \end{aligned} \tag{5.5}$$

where

$$R_{1,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}] = Y[X_{m-1}(\xi, \varsigma)] - \left(1 - \frac{k_m}{n}\right) s(X_0)$$
$$-s^{\alpha}[k_3 Z_{m-1}(\varsigma) + k_1 X_{m-1}(\varsigma)a - k_1 \sum_{i=0}^{m-1} X_i(\varsigma) X_{m-1-i}(t) - k_2 Y_{m-1}(\varsigma)],$$
$$R_{2,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}] = Y[Y_{m-1}(\xi, \varsigma)] - \left(1 - \frac{k_m}{n}\right) s(Y_0) \tag{5.6}$$
$$-s^{\alpha}[k_4 X_{m-1}(t)a - k_4 \sum_{i=0}^{m-1} X_i(\varsigma) X_{m-1-i}(\varsigma) + k_6 Z_{m-1}(\varsigma) + k_5 Y_{m-1}(\varsigma)b - k_5 \sum_{i=0}^{m-1} Y_i(\varsigma) Y_{m-1-i}(\varsigma)],$$
$$R_{3,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}] = Y[Z_{m-1}(\xi, \varsigma)] - \left(1 - \frac{k_m}{n}\right) s(Z_0) - s^{\alpha}[k_7 X_{m-1} - k_8 Y_{m-1}(\varsigma)].$$

Applying an inverse Yang transform we have

$$\begin{aligned} Y[X_m(\xi, \varsigma) - k_m X_{m-1}(\xi, \varsigma)] &= hY^{-1} R_{1,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}], \\ Y[Y_m(\xi, \varsigma) - k_m Y_{m-1}(\xi, \varsigma)] &= hY^{-1} R_{2,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}], \\ Y[Z_m(\xi, \varsigma) - k_m Z_{m-1}(\xi, \varsigma)] &= hY^{-1} R_{3,m}[\overrightarrow{X}_{m-1}, \overrightarrow{Y}_{m-1}, \overrightarrow{Z}_{m-1}]. \end{aligned} \tag{5.7}$$

Computing the above equations and taking parameter values as $k_1 = 0.4, k_2 = 0.2, k_3 = 0.3, k_4 = 0.2, k_5 = 0.3, k_6 = 0.3, k_7 = -0.3, k_8 = 0.4$, we have

$$\begin{aligned} &\mathrm{X}(0) = 2, \\ &\mathrm{Y}(0) = 4, \\ &\mathrm{Z}(0) = 3, \\ &\mathrm{X}(1) = -\ 3.2 * \frac{\hbar\varsigma^{\alpha}}{\Gamma(1+\alpha)}, \\ &\mathrm{Y}(1) = -\ 5.3 * \frac{\hbar\varsigma^{\alpha}}{\Gamma(1+\alpha)}, \\ &\mathrm{Z}(1) = 0.4 * \frac{\hbar\varsigma^{\alpha}}{\Gamma(1+\alpha)}, \\ &. \\ &. \\ &. \end{aligned} \tag{5.8}$$

Example 2. Consider the time-fractional Bloch equation [10].

$$\begin{aligned} {}_0^C D_{\varsigma}^{\alpha}(M_x(\varsigma)) &= \omega M_y(\varsigma) - \frac{M_x(\varsigma)}{T_1}, \\ {}_0^C D_{\varsigma}^{\alpha}(M_y(\varsigma)) &= \omega M_x(\varsigma) - \frac{M_y(\varsigma)}{T_2}, \\ {}_0^C D_{\varsigma}^{\alpha}(M_z(\varsigma)) &= \frac{M_0 - M_z(\varsigma)}{T_1}, \end{aligned} \tag{5.9}$$

with initial conditons

$$\begin{aligned}
&\mathrm{M_x}(0) = 0,\\
&\mathrm{M_y}(0) = 100,\\
&\mathrm{M_z}(0) = 0.
\end{aligned} \tag{5.10}$$

Applying Yang transform on Eq (5.9), we get

$$\begin{aligned}
&Y[M_x(\varsigma)] - sM_x(0) + s^{\alpha}\{\omega M_y(\varsigma) - \frac{M_x(\varsigma)}{T_1}\} = 0,\\
&Y[M_y(\varsigma)] - sM_y(0) + s^{\alpha}\{\omega M_x(\varsigma) - \frac{M_y(\varsigma)}{T_2}\} = 0,\\
&Y[M_z(\varsigma)] - sM_y(0) + s^{\alpha}\{\frac{M_0 - M_z(\varsigma)}{T_1}\} = 0.
\end{aligned} \tag{5.11}$$

The non-linear operator is defined as

$$\begin{aligned}
&N^1[\varphi_1(\xi,\varsigma;q), \varphi_2(\xi,\varsigma;q), \varphi_3(\xi,\varsigma;q)] = Y[\varphi_1(\xi,\varsigma;q)] - s(M_x(0))\\
&-s^{\alpha}Y\{\omega\varphi_2(\xi,\varsigma;q) - \frac{\varphi_1(\xi,\varsigma;q)}{T_1}\},\\
&N^2[\varphi_1(\xi,\varsigma;q), \varphi_2(\xi,\varsigma;q), \varphi_3(\xi,\varsigma;q)] = Y[\varphi_2(\xi,\varsigma;q)] - s(M_y(0))\\
&-s^{\alpha}Y\{\omega\varphi_1(\xi,\varsigma;q) - \frac{\varphi_2(\xi,\varsigma;q)}{T_2}\},\\
&N^3[\varphi_1(\xi,\varsigma;q), \varphi_2(\xi,\varsigma;q), \varphi_3(\xi,\varsigma;q)] = Y[\varphi_3(\xi,\varsigma;q)] - s(M_z(0))\\
&-s^{\alpha}Y\{\varphi_2(\xi,\varsigma;q) - \frac{\varphi_3(\xi,\varsigma;q)}{T_1}\}.
\end{aligned} \tag{5.12}$$

The deformation equation is at $H(\xi,\varsigma) = 1$ is

$$\begin{aligned}
&Y[M_x(\xi,\varsigma) - k_m M_{x_{m-1}}(\xi,\varsigma)] = \hbar\{R_{1,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}]\},\\
&Y[M_y(\xi,\varsigma) - k_m M_{y_{m-1}}(\xi,\varsigma] = \hbar\{R_{2,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}]\},\\
&Y[M_z(\xi,\varsigma) - k_m M_{z_{m-1}}(\xi,\varsigma)] = \hbar\{R_{3,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}]\},
\end{aligned} \tag{5.13}$$

where

$$\begin{aligned}
&R_{1,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}] - \left(1 - \frac{k_m}{n}\right)s(M_x(0)) - s^{\alpha}\{\omega M_{y_{m-1}}(\varsigma) - \frac{M_{x_{m-1}}(\varsigma)}{T_1}\},\\
&R_{2,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}] - \left(1 - \frac{k_m}{n}\right)s(M_y(0)) - s^{\alpha}\{\omega M_{x_{m-1}}(\varsigma) - \frac{M_{y_{m-1}}(\varsigma)}{T_2}\},\\
&R_{3,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}] - \left(1 - \frac{k_m}{n}\right)s(M_z(0)) - s^{\alpha}\{\frac{M_0 - M_{z_{m-1}}(\varsigma)}{T_1}\}.
\end{aligned} \tag{5.14}$$

Applying an inverse Yang transform, we get

$$
\begin{aligned}
&M_x(\xi,\varsigma) - k_m M_{x_{m-1}}(\xi,\varsigma) = Y^{-1}\hbar\{R_{1,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}]\},\\
&M_y(\xi,\varsigma) - k_m M_{y_{m-1}}(\xi,\varsigma) = Y^{-1}\hbar\{R_{2,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}]\},\\
&M_z(\xi,\varsigma) - k_m M_{z_{m-1}}(\xi,\varsigma) = Y^{-1}\hbar\{R_{3,m}[\overrightarrow{M}_{x_{m-1}}, \overrightarrow{M}_{y_{m-1}}, \overrightarrow{M}_{z_{m-1}}]\}.
\end{aligned} \tag{5.15}
$$

Computing the above equations and taking the parameter values as $\omega = 1, T_1 = 1, T_2 = 20$, we have

$$
\begin{aligned}
&M_x(0) = 0,\\
&M_y(0) = 100,\\
&M_z(0) = 0,\\
&M_x(1) = -100 \times \hbar \times \frac{\varsigma^{\alpha}}{\Gamma(1+\alpha)},\\
&M_y(1) = 5 \times \hbar \times \frac{\varsigma^{\alpha}}{\Gamma(1+\alpha)},\\
&M_z(1) = -100 \times \hbar \times \frac{\varsigma^{\alpha}}{\Gamma(1+\alpha)}.
\end{aligned} \tag{5.16}
$$

6. Results and discussion

The current study demonstrates the application of a novel transform technique known as the q-homotopy analysis Yang transform method. This technique is employed to address the fractional water pollution model and the Bloch equation of arbitrary order, utilizing the Caputo fractional derivative. Here we demonstrate the behavior of the obtained outcomes with the values of the parameters. Figures 1 and 4 elucidate the nature of the approximate solutions obtained for both the considered models through the q-haytm, which is shown by 3D plots. In Figures 2 and 5, the behavior of the obtained solutions for diverse alpha values is depicted. To examine how the derived solution behaves with respect to the homotopy parameter $\hbar$, the $\hbar$-curves are depicted with varying α in Figures 3 and 6. These curves help manage and modify the q-HAYTM solution's convergence zone. Tables 1–3 denote the conducted numerical stimulations for the fractional water pollution model for diverse fractional orders. The numerical stimulations for the fractional Bloch equation have been presented in Tables 4–6, for diverse fractional orders. Results in this section demonstrate that q-haytm is not only a method, but also a concept of successful and precise mathematical tools of complex fractional models. The $\hbar$-curves reveal how quickly the series solutions come to their end, with the auxiliary parameter $\hbar$ playing the role of the conductor, perfectly controlling the convergence domain and also ensuring that the calculations remain stable. Only in fractional-order derivatives, the use of the Caputo operator is extremely suitable to perfectly explain the memory effects and nonlocal behaviors of the matter that are indispensable for very precise models of nature. The time-fractional water pollution model and the time-fractional Bloch model are two examples where the differences of the systems from the classical one, due to the fractional derivatives, can be observed, and at the same time, these differences give more profound insights into the systems' processing never-before-seen environmental and bioengineering processes. The shift to a fractional framework allows the representation of the system development over time to be more accurate than with classical integer-order methods. In addition, the method illuminated here is turning out to be one of the most valuable tools in analytical problems of high-precision, nonlinear fractional dynamics, giving a more thorough understanding of the systems with a wide range of possible applications in various science and engineering fields.

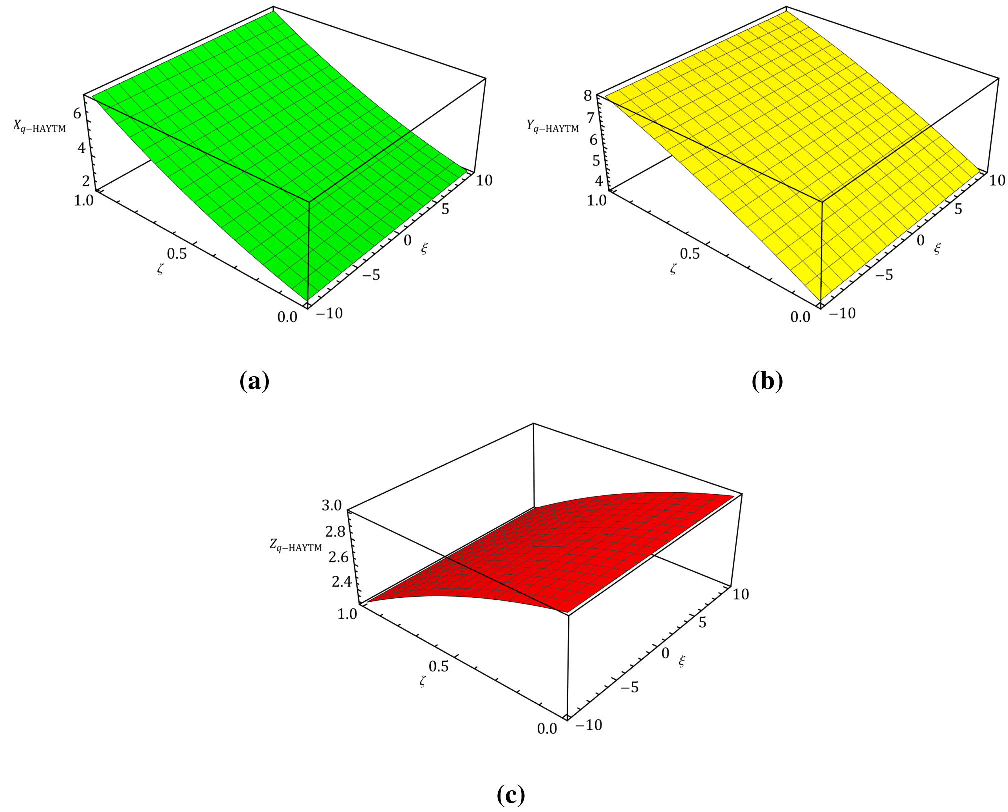
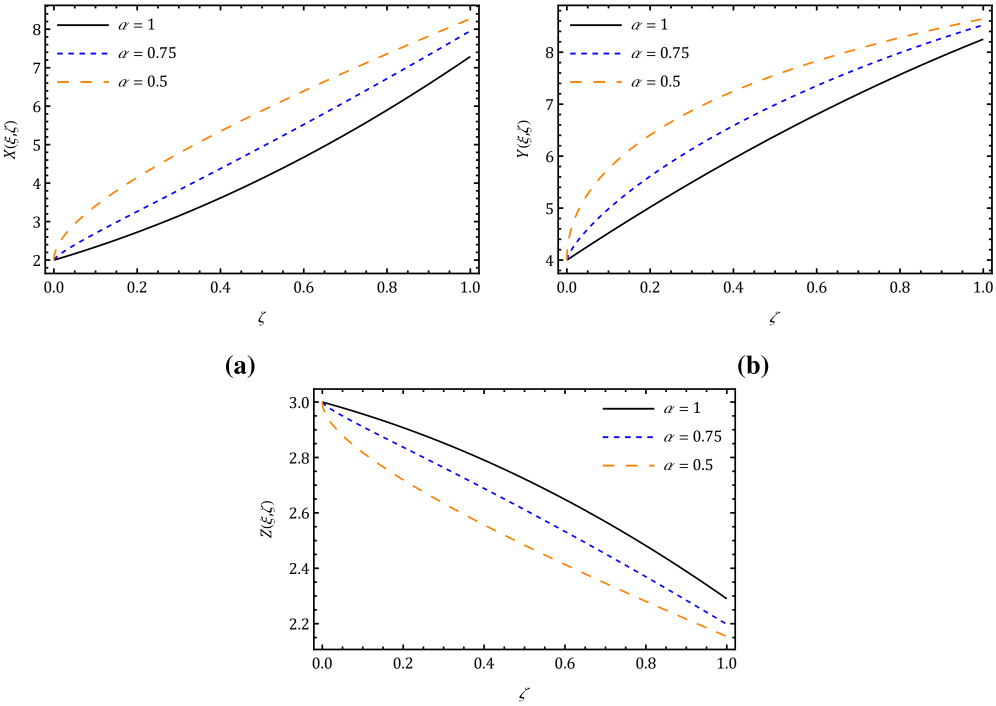


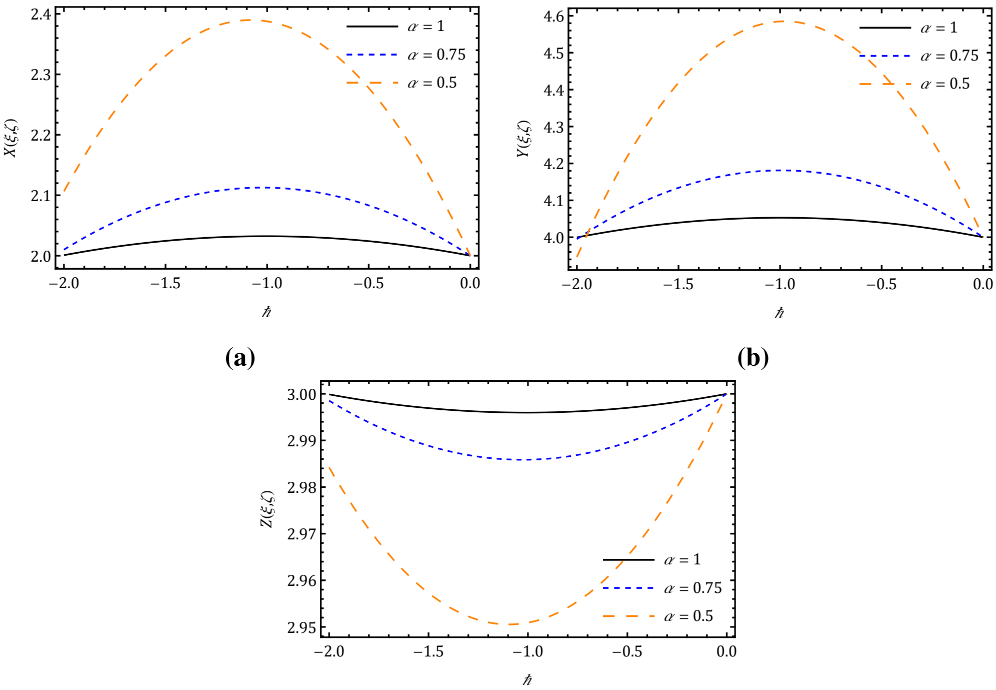
Figure 1. Variations of $X(\xi,\varsigma), Y(\xi,\varsigma), Z(\xi,\varsigma)$ via the q-haytm with respect to ξ,ς.



(a) (b)

(c)

Figure 2. Nature of α-trajectories with respect to time under diverse α.



(a) (b)

(c)

Figure 3. Graphical demonstration of $\hbar$ curves, with time for various α values.

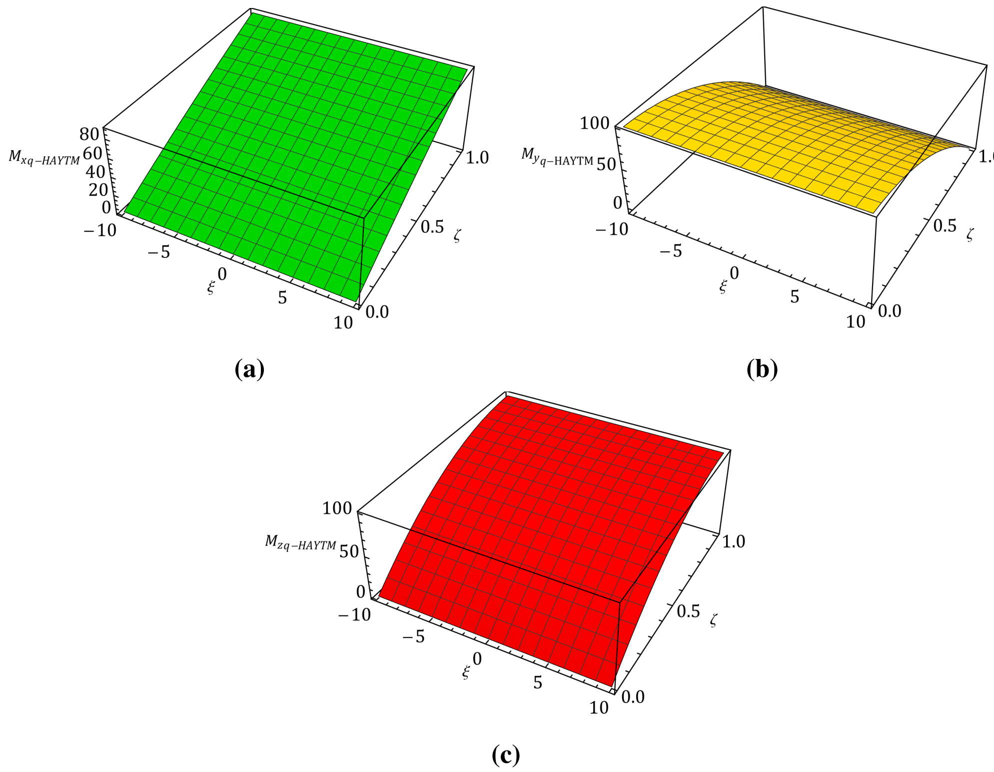


Figure 4. Variations of $M_x(\xi, \varsigma)$, $M_y(\xi, \varsigma)$, $M_z(\xi, \varsigma)$ via the q-haytm with respect to ξ, ς.

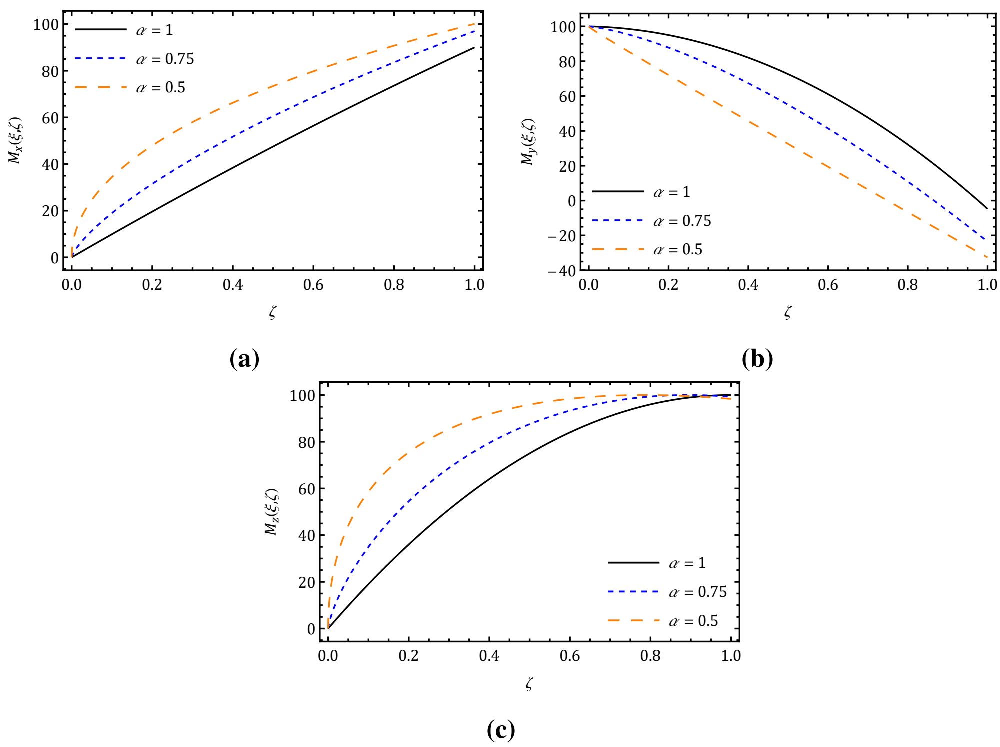
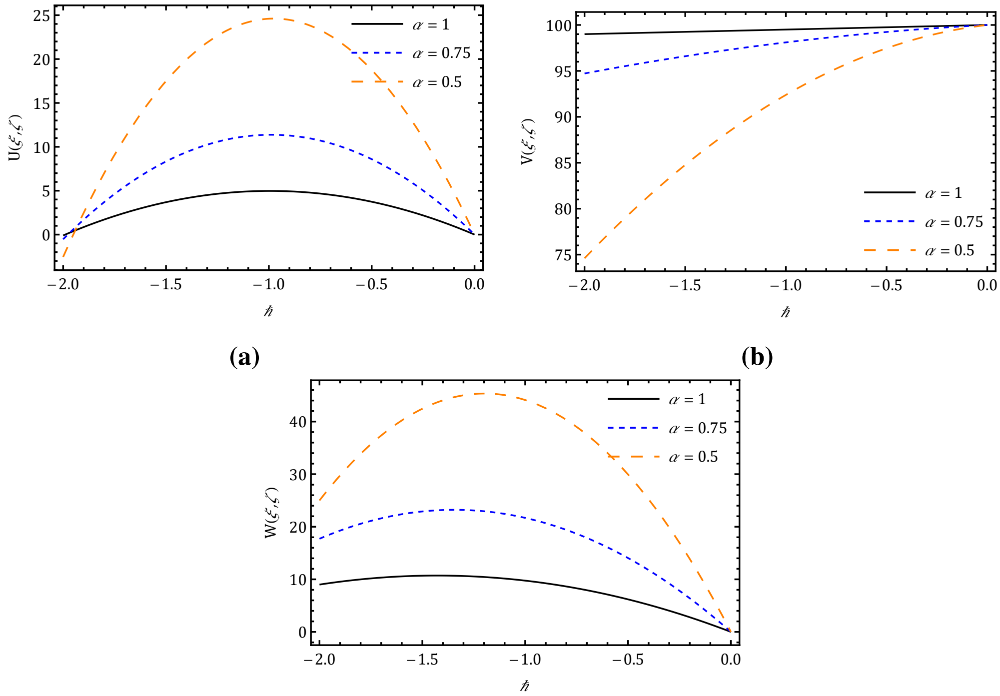


Figure 5. Nature of α-trajectories with respect to time under diverse α.



(a) (b) (c)

Figure 6. Nature of α-trajectories with respect to time under diverse α.

Table 1. Numerical simulations for $X(\varsigma)$ in Example 1 for diverse values of α.

$\varsigma = 0$	$\alpha = 0.3$	$\alpha = 0.7$	$\alpha = 1$
0	2	2	2
0.2	4.743	3.4074	2.7236
0.4	3.9385	4.5562	3.6144
0.6	3.0541	4.7011	4.6724
0.8	2.1354	4.8646	5.1132
1	1.1991	4.0531	5.2976

Table 2. Numerical results for $Y(\varsigma)$ in Example 1 for diverse values of α.

$\varsigma = 0$	$\alpha = 0.3$	$\alpha = 0.7$	$\alpha = 1$
0	4	4	4
0.2	6.1475	5.7770	5.018
0.4	7.7338	6.7187	5.952
0.6	8.1069	6.4573	6.802
0.8	9.3828	6.0588	6.568
1	10.6018	7.5611	7.258

Table 3. Numerical results for $Z(\varsigma)$ in Example 1 for diverse values of α.

$\varsigma = 0$	$\alpha = 0.3$	$\alpha = 0.7$	$\alpha = 1$
0	3	3	3
0.2	2.5789	2.8178	2.9076
0.4	2.4393	2.6641	2.7904
0.6	2.3343	2.5084	2.6484
0.8	1.2465	2.3487	2.4816
1	1.694	2.1884	2.2917

.

Table 4. Obtained results for $M_x(\varsigma)$ in Example 2 for different α values.

$\varsigma = 0$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1$
0	0	0	0
0.2	39.7948	28.7497	15.6
0.4	52.3412	48.9236	38.4321
0.6	60.1732	60.2589	56.4181
0.8	61.5224	69.1605	73.1618
1	63.1218	75.1136	81.2461

Table 5. Obtained results for $M_y(\varsigma)$ in Example 2 for different α values.

$\varsigma = 0$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1$
0	100	100	100
0.2	79.7584	89.7627	95.0100
0.4	70.5162	80.0877	88.2392
0.6	56.8196	66.5651	81.0932
0.8	48.5621	54.0475	64.1091
1	30.5384	40.0357	50.2713

Table 6. Obtained results for $M_z(\varsigma)$ in Example 2 for different α values.

$\varsigma = 0$	$\alpha = 0.6$	$\alpha = 0.8$	$\alpha = 1$
0	0	0	0
0.2	34.7064	26.4771	20.6412
0.4	43.7458	36.6559	31.6889
0.6	50.8853	49.1791	48.1253
0.8	55.9956	56.8962	54.7164
1	62.8579	62.9457	65.6413

7. Conclusions

The considered models under study offer fundamental as well as advantageous implications for introducing new information in the physical sciences. Additionally, fresh research might be done to examine its applicability for wider and more universal applications. In the present investigation, the approximate analytical solutions are achieved for the fractional Bloch equations model in bioengineering and fractional water pollution (FWP) model using an effective novel technique called the q-homotopy analysis Yang transform method via the Caputo fractional derivative. This study reveals fresh insights on the time-fractional order and its impact on the considered models. Also numerical simulations with great exactness of the approximate solutions are demonstrated. The dynamics of both the proposed models are influenced by both the time history and the time instant, which can be effectually analyzed using the concept of FC. The advantage of the considered method is that it is a mixture of the homotopy analysis method and the Yang transform, which provides an efficient framework to investigate various epidemic, biological, economical, and financial models. This method gives accurate and systematic solutions to these equations, overcoming the difficulties posed by traditional methods, particularly in solving fractional models. This method does not need the use of any transformation, unrealistic restriction, linearization, complex polynomials, or assumptions. As a result, the q-haytm is simpler, more practical, and more effective than other current techniques. Future studies could use suitable numerical techniques to investigate other fractional derivatives. Hence, we can conclude that the proposed method is effective in solving the considered finance model, and it can be applied to solve various mathematical models.

Author contributions

Naveed Iqbal: Conceptualization, validation, investigation, data curation, writing-review and editing; H. B. Chethan: Conceptualization, methodology, software, resources, writing the original draft, writing-review and editing; Doddabhadrappla Gowda Prakasha: Conceptualization, methodology, formal analysis, validation, resources, visualization, writing the original draft; Meraj Ali Khan: Methodology, formal analysis, investigation, visualization, writing the original draft, writing-review and editing, supervision. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors have no conflict of interest regarding the publication of this paper.

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