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Research article

Stability of nonlinear stochastic systems with delayed impulses under self-triggered impulsive control

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Abstract: This paper investigates the stability problem of nonlinear stochastic systems with delayed impulses based on a self-triggered impulsive control (STIC) strategy. By employing the Lyapunov method, an explicit self-triggering mechanism (STM) with state-dependent waiting time parameters is designed, which ensures system stability while effectively avoiding Zeno behavior. Compared with traditional event-triggered impulsive control (ETIC) methods, this strategy does not require continuous state monitoring and can determine the next triggering instant based on the currently available state information. Furthermore, the developed theoretical results are applied to the STIC problem of nonlinear stochastic systems. Finally, the effectiveness and feasibility of the proposed method are validated through two numerical examples.

Keywords: nonlinear stochastic system; stability; delayed impulses; self-triggered impulsive control
Mathematics Subject Classification: 93C30

1. Introduction

In recent years, impulsive control (IC) has gradually become an important tool for solving complex dynamic system control problems due to its advantages of simple structure, discrete control signals, low consumption of communication resources, and strong robustness. It has been widely applied in fields such as multi-agent systems, neural network synchronization, biological systems, and aerospace [1–5]. As a discontinuous control method, IC achieves effective regulation of system behavior by applying instantaneous impulses to the system at specific moments. It is particularly suitable for dealing with challenges such as system nonlinearity, uncertainty, and limited network bandwidth. Traditional IC is usually based on a time-triggered mechanism, which means that control signals are applied according to a preset and fixed time sequence without relying on real-time changes in system states. Due to its simple analysis and easy implementation, this method has dominated early research [6–9]. However, the fixed time intervals often fail to accurately reflect the dynamic characteristics of the system, which can result in unnecessarily high control frequency and inefficient resource usage, thus limiting both practicality and control performance.

In response to the aforementioned issues, event-triggered impulsive control (ETIC) has emerged. This strategy integrates the merits of event-triggered control (ETC) and IC, applying control only when specific triggering conditions are satisfied. This approach significantly improves the efficiency of resource utilization and effectively alleviates the contradiction between control frequency and performance inherent in traditional methods [10–15]. For example, Kuang et al. [10] investigated the multi-type stability problems of nonlinear stochastic systems within the framework of ETIC; Hu et al. [13] designed two types of event-triggering mechanisms for analyzing the stability of stochastic systems; and Li et al. [15] explored the input-to-state stability of nonlinear systems subject to delayed impulses using the ETIC approach. Despite substantial theoretical progress, implementing this strategy still faces challenges. The triggering conditions typically rely on continuous or periodic monitoring of system states, which places high demands on computational resources and communication capabilities, especially for resource-constrained systems.

To overcome this limitation, the self-triggered impulsive control (STIC) strategy has been proposed. The core idea is to predict the next control triggering time based on the currently available system state at each control execution instant, thereby eliminating the need for continuous sampling and monitoring of the system state. Especially as of recently, STIC has become a research hotspot and has achieved a series of meaningful results [16–21]. For instance, Li et al. [16] designed a self-triggering mechanism (STM) grounded in the comparison principle to analyze the asymptotic stability of nonlinear systems; Tan et al. [18] proposed a periodic STIC strategy for neural network synchronization and image encryption; and Wang [21] investigated the quasi-synchronization problem of parameter-adaptive drive-response systems under the framework of STIC. However, most existing studies focus on deterministic systems or specific structured models and often rely on comparison-based analysis methods, which limits their practicality and scalability.

It is worth noting that most existing studies neglect the influence of time delays [16, 22–25]. However, in practical applications, such as biological signal transmission [26], communication networks [27], and aerospace systems [28], the evolution of the system state is often significantly affected by delays. In IC, such delays are particularly critical, as they may not only interfere with the system's real-time response but also degrade control performance and even threaten system stability. For example, Li et al. [16] studied the stability of nonlinear systems under STIC; Mapui et al. [23] analyzed the Lyapunov-type prescribed-time stability of impulsive systems under two triggering mechanisms; Zhang [25] investigated the cooperative output regulation of linear multi-agent systems via distributed fixed-time ETC. However, none of these studies considered the presence of time delays in the impulses. Although some studies [11, 14, 15] have addressed delayed impulses in the ETIC framework, their triggering mechanisms require continuous state sampling, leading to high computational costs. While [29] investigated the local synchronization of time-delay systems under the STIC framework, the study employed the comparison principle to design the triggering mechanism, which has certain limitations. In [30], a periodic self-triggered intermittent impulsive control strategy with an implicit expression form was designed for the stabilization of complex-valued stochastic complex networks, but this control mechanism is relatively complex and not easily implementable. In addition to delay effects, stochastic disturbances are also a common challenge in practical systems. Their primary sources include sensor measurement noise, external environmental perturbations, and uncertainties in the communication process, all of which significantly increase the complexity of modeling and control design. Despite extensive research on stochastic systems [14, 24, 31–35], certain limitations remain in engineering applications. For example, Li et al. [14] proved the stability of stochastic systems by introducing state-dependent waiting times; however, the associated Lyapunov functionals rely on strongly restrictive conditions, making the theoretical verification process cumbersome. In [24], the stability of nonlinear stochastic systems was analyzed under the STIC framework, but the impact of delays in impulses was neglected, and the mechanism employed fixed waiting times, limiting its flexibility. Mapui et al. [32] focused on consensus in stochastic delayed multi-agent systems with input saturation under STIC, yet the impulse jump design depended solely on current state information without considering potential delay effects. Furthermore, [34,35] investigated the practical exponential mean-square stability from input to state for stochastic nonlinear systems under event-triggered feedback control, taking external disturbances into account. Nevertheless, such methods require continuous state monitoring, which results in high energy consumption, and they are difficult to implement in scenarios where actuators are constrained and cannot maintain output for extended periods. In summary, designing an STIC strategy that guarantees stability for nonlinear stochastic systems in the presence of delayed impulses remains a challenging problem.

Based on the above insights, this paper aims to investigate the p-th moment asymptotic stability (p-AS) and p-th moment exponential stability (p-ES) of nonlinear stochastic systems with delayed impulses using an STIC strategy. By employing the Lyapunov method, a set of verifiable sufficient conditions for system stability are proposed. This paper makes the following key contributions:

(i) An STM with state-dependent waiting times is designed. Compared with the triggering mechanisms with fixed waiting times in [13, 24], it not only enhances flexibility but also overcomes the analytical difficulties brought by the stochastic nature of state-dependent waiting time parameters through the introduction of new conditions, while ensuring the exclusion of Zeno behavior.

(ii) The coupled effects of delayed impulses and stochastic disturbances are systematically considered, which expands the scope of theoretical applicability. At the control instants, the state jumps of the system integrate information from both the current state and the historical state, and an explicit relationship between the triggering parameters and the impulse intensity is established.

(iii) An explicitly formulated STIC strategy is proposed. Compared with the complex implicit mechanisms in [18, 31, 36], the proposed strategy has a clearer structure, making it more suitable for practical implementation.

The rest of this paper is structured as follows: Section 2 introduces the system model, along with basic definitions and notation. Section 3 presents the main theoretical results in detail. Section 4 demonstrates the application of the proposed results to nonlinear stochastic systems. Section 5 provides two numerical examples for validation. In Section 6, the paper is concluded, and potential future research directions are discussed.

2. Preliminaries

Notation: N, N_+, $\mathbb{R}$, and $\mathbb{R}_+$ denote the sets of non-negative integers, positive integers, real numbers, and non-negative real numbers, respectively. $\mathbb{R}^a$ and $\mathbb{R}^{a\times b}$ represent the a-dimensional real space and the space of $a \times b$ real matrices, respectively. Let $|\cdot|$ denote the Euclidean norm. $\mathbb{E}$ and $\mathcal{P}$ denote the mathematical expectation and probability measure, respectively. $\mathcal{PC}([-\iota, 0]; \mathbb{R}^a) =$

$\Big\{\hbar : [-\iota, 0] \to \mathbb{R}^a$ with norm $\|\cdot\|_\iota$, where $\|\hbar\|_\iota = \sup_{\varsigma\in[-\iota,0]} |\hbar(\varsigma)|\Big\}$. $\mathcal{PC}^d_{\mathcal{F}_0}([-\iota, 0]; \mathbb{R}^a)$ denotes a class of bounded $\mathcal{F}_0$-measurable, $\mathcal{PC}([-\iota, 0]; \mathbb{R}^a)$-valued random variables. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathcal{P})$ is a complete probability space equipped with a filtration $\{\mathcal{F}_t\}_{t\geq t_0}$ that satisfies the usual conditions. For any matrix $\mathcal{S}$, $\mathcal{S}^{-1}$, $\mathcal{S}^T$, and $trace(\mathcal{S})$ denote the inverse, transpose, and trace of $\mathcal{S}$, respectively. Ξ is the identity matrix of appropriate dimension. $G > 0$ (or $G < 0$) indicates that the matrix G is symmetric and positive definite (or negative definite).

The following nonlinear stochastic system influenced by delayed impulses is considered:

$$\begin{cases} \mathrm{d}z(t) = \Phi(t, z(t))\mathrm{d}t + \Psi(t, z(t))\mathrm{d}\omega(t),\ t \neq t_r, \\ z(t_r) = \Pi_r(z(t_r^-), z(t_r - \iota)),\ r \in N_+, \\ z(t_0 + \varsigma) = \hbar(\varsigma),\ \varsigma \in [-\iota, 0], \end{cases} \tag{1}$$

where $z(t) \in \mathbb{R}^a$ is the system state, the initial function $\hbar = \{\hbar(\varsigma), -\iota \leq \varsigma \leq 0\} \in \mathcal{PC}^d_{\mathcal{F}_0}([-\iota, 0]; \mathbb{R}^a)$, $\iota > 0$ is the constant time delay. $\omega(t)$ be a b-dimensional Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathcal{P})$. Let $\{t_r, r \in N_+\}$ denote the sequence of impulse times. Suppose that the functions $\Phi : \mathbb{R}_+ \times \mathbb{R}^a \to \mathbb{R}^a, \Psi : \mathbb{R}_+ \times \mathbb{R}^a \to \mathbb{R}^{a\times b}, \Pi_r : \mathbb{R}^a \times \mathbb{R}^a \to \mathbb{R}^a$ are Borel measurable and satisfy both the Lipschitz condition and the linear growth condition. For any initial state $\hbar \in \mathcal{PC}^d_{\mathcal{F}_0}([-\iota, 0]; \mathbb{R}^a)$, a unique global solution $z(t)$ exists for system (1). Furthermore, assume $\Phi(0, 0) = 0, \Psi(0, 0) = 0$ and $\Pi_r(0, 0) = 0$ for $t \in \mathbb{R}_+, r \in N_+$, which implies that system (1) has a trivial solution $z(t) \equiv 0$.

Definition 1. *[13] Let $\chi^{1,2}$ represent the set of all non-negative functions $\mathcal{V}(t, z) : [t_0, +\infty) \times \mathbb{R}^a$ that are continuously once differentiable at t and twice differentiable at z. For function $\mathcal{V}(t, z) \in \chi^{1,2}$, we define the following operator $\mathcal{L}$ associated with system (1):*

$$\mathcal{L}\mathcal{V}(t, z) = \frac{\partial\mathcal{V}(t, z)}{\partial t} + \frac{\partial\mathcal{V}(t, z)}{\partial z}\Phi(t, z) + \frac{1}{2}trace[\Psi^T(t, z)\frac{\partial^2\mathcal{V}(t, z)}{\partial z^2}\Psi(t, z)].$$

Definition 2. *[12, 14] The trivial solution of system (1) is regarded as:*
$(A1)$ *p-th moment stable: $\forall\epsilon > 0$, $\exists\,\delta > 0$ such that $\|\hbar\|_\iota^p < \delta$, implies*

$$\mathbb{E}\,|z(t)|^p < \epsilon;$$

$(A2)$ *p-th moment asymptotically stable (p-AS): it is p-th moment stable and for any $\hbar \in \mathcal{PC}^d_{\mathcal{F}_0}([-\iota, 0]; \mathbb{R}^a)$,*

$$\mathbb{E}\,|z(t)|^p \to\ 0 \text{ as } t \to +\infty;$$

$(A3)$ *p-th moment exponentially stable (p-ES): exist positive constants α and β such that for any $\hbar \in \mathcal{PC}^d_{\mathcal{F}_0}([-\iota, 0]; \mathbb{R}^a)$,*

$$\mathbb{E}\,|z(t)|^p \leq \alpha e^{-\beta(t-t_0)} E\,\|\hbar\|_\iota^p.$$

In the particular case where $p = 2$, it is referred to as exponential stability in the mean-square sense.

Assumption 1. *Assume that there exists a function $\mathcal{V}(t, z(t)) \in \chi^{1,2}$ and a function $\theta(t) : [t_0, +\infty) \to \mathbb{R}$, as well as some positive constants u_1, u_2 and $\xi_{1,r} \in [0, 1)$, $\xi_{2,r} \in [0, 1)$ for each $r \in N_+$, which are not simultaneously zero, such that the following inequality conditions are satisfied:*
$(B1)$ $u_1\,|z(t)|^p \leq \mathcal{V}(t, z) \leq u_2\,|z(t)|^p,\ \forall z(t) \in \mathbb{R}^a$;
$(B2)$ $\mathbb{E}\mathcal{L}\mathcal{V}(t, z(t)) \leq \theta(t)\mathbb{E}\mathcal{V}(t, z(t))$, *where $\theta(t)$ is bounded on $[t_0, +\infty)$, and we define* $\theta = \sup_{s\in[t_0,+\infty)} \theta(s) > 0$;
$(B3)$ $\mathbb{E}\mathcal{V}(t, \Pi_r(z(t_r^-), z(t_r - \iota)) \leq \xi_{1,r}\mathbb{E}\mathcal{V}(t_r^-, z(t_r^-)) + \xi_{2,r}\mathbb{E}\mathcal{V}(t_r - \iota, z(t_r - \iota)),\ r \in N_+$.

Remark 1. *Condition (B2) characterizes the evolution trend of the Lyapunov function during the continuous dynamics of the system, ensuring that its expected value does not grow unboundedly. Condition (B3) describes the jump characteristics of the Lyapunov function under the effect of impulses, indicating that the impulses may be influenced by the current or previous system states, or possibly by only one of them. Here,* $\xi_{1,r}$ *and* $\xi_{2,r}$ *are the scaling factors of the impulse on the current state and the delayed state, respectively, reflecting the magnitude of the impulse strength and the delay effect. This condition is used to restrict the expected value of the Lyapunov function under the effect of impulses from exceeding the weighted sum of the Lyapunov values corresponding to the current and delayed states (typically requiring* $\xi_{1,r}+\xi_{2,r} < 1$*), thereby suppressing non-physical energy growth and ensuring system stability.*

3. Main results

This section aims to develop a class of STM for the stability analysis of system (1). To address the potential occurrence of Zeno behavior, a waiting time parameter is introduced into the mechanism. The detailed design is as follows:

$$t_{r+1} = \inf\left\{ t > t_r + \pi(z(t_r)) : t - t_r - \pi(z(t_r)) - \frac{\ell_r + \ln \frac{\mathcal{V}(t_r, z(t_r))}{\mathcal{V}(\tilde{t}_r, z(\tilde{t}_r))}}{\theta} \geq 0 \right\}, \tag{2}$$

where $\pi(z(t_r))$ denotes the waiting time parameter associated with the system state $z(t_r)$, and $\pi(z(t_r)) : \mathbb{R}^a \to (0, \mathfrak{A}]$ $(\mathfrak{A} > 0)$. $\ell_r > 0$ is the triggering parameter and $\ell \doteq \max_{r\in N}\{\ell_r\}$. Here we set $\tilde{t}_r = t_r + \pi(z(t_r))$. According to the STM (2), the next triggering instant t_{r+1} must occur strictly after $t_r + \pi(z(t_r))$, which inherently prevents the emergence of Zeno behavior.

Theorem 1. *Under Assumption 1, if the parameters of STM (2) satisfy:*

$$\mathcal{P}(\iota < \pi(z(t_r)) \leq \hat{\iota}) = 1, \; for\ each\ r \in N, \tag{3}$$

$$\theta\hat{\iota} < \ell_r, \quad e^{(r+1)\ell}\prod_{i=1}^{r}(\xi_{1,i} + \xi_{2,i}) \quad \to 0 \quad as \quad r \to +\infty, \tag{4}$$

where $\hat{\iota} > 0$ denotes the upper bound of the waiting time, then system (1) is p-AS under STM (2).

Proof. By integrating both sides of condition (B2) over the interval $t \in [t_r, t_{r+1})$, we derive

$$\int_{t_r}^{t} \frac{\mathbb{E}\mathcal{L}\mathcal{V}(t, z(s))\mathrm{d}s}{\mathbb{E}\mathcal{V}(t, z(s))} \leq \int_{t_r}^{t} \theta(s)\mathrm{d}s \leq \theta(t - t_r).$$

Applying Itô's formula yields

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-t_r)}\mathbb{E}\mathcal{V}(t_r, z(t_r)), \tag{5}$$

this implies that

$$\mathbb{E}\mathcal{V}(\tilde{t}_r, z(\tilde{t}_r)) \leq e^{\theta(\tilde{t}_r - t_r)}\mathbb{E}\mathcal{V}(t_r, z(t_r)) \leq e^{\theta\pi(z(t_r))}\mathbb{E}\mathcal{V}(t_r, z(t_r)),$$

then, from conditions (3) and (4), we have

$$\frac{\ell_r + \ln \frac{\mathcal{V}(t_r, z(t_r))}{\mathcal{V}(\tilde{t}_r, z(\tilde{t}_r))}}{\theta} > 0.$$

Clearly, inequality (5) holds for all $t \in [t_r, \tilde{t}_r)$. Based on STM (2), on the interval $t \in [\tilde{t}_r, t_{r+1})$, one has $t - t_r - \pi(z(t_r)) - \frac{\ell_r + \ln \frac{\mathcal{V}(t_r, z(t_r))}{\mathcal{V}(\tilde{t}_r, z(\tilde{t}_r))}}{\theta} < 0$, from which it follows that

$$e^{\theta(t-\tilde{t}_r)}\mathbb{E}\mathcal{V}(\tilde{t}_r, z(\tilde{t}_r)) < e^{\ell_r}\mathbb{E}\mathcal{V}(t_r, z(t_r)). \tag{6}$$

Then, for $t \in [t_0, \tilde{t}_0)$, by (3)–(5), we get that

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-t_0)}\mathbb{E}\mathcal{V}(t_0, z(t_0)) \leq e^{\theta\hat{\iota}}\mathbb{E}\mathcal{V}(t_0, z(t_0)) \leq e^{\ell_0}\mathbb{E}\mathcal{V}(t_0, z(t_0)), \tag{7}$$

and if $t \in [\tilde{t}_0, t_1)$, then according to (5) and (6), one can derive that

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-\tilde{t}_0)}\mathbb{E}\mathcal{V}(\tilde{t}_0, z(\tilde{t}_0) \leq e^{\ell_0}\mathbb{E}\mathcal{V}(t_0, z(t_0))). \tag{8}$$

At the first triggering instant t_1, it can be deduced from condition (B3) and inequalities (7) and (8) that

$$\begin{aligned} \mathbb{E}\mathcal{V}(t_1, z(t_1)) &\leq \xi_{1,1}\mathbb{E}\mathcal{V}(t_1^-, z(t_1^-)) + \xi_{2,1}\mathbb{E}\mathcal{V}(t_1 - \iota, z(t_1 - \iota)) \\ &\leq e^{\ell_0}(\xi_{1,1} + \xi_{2,1})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \end{aligned} \tag{9}$$

For $t \in (t_1, \tilde{t}_1)$, by combining (3)–(5) and (9), we obtain

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-t_1)}\mathbb{E}\mathcal{V}(t_1, z(t_1)) \leq e^{\theta\hat{\iota}}\mathbb{E}\mathcal{V}(t_1, z(t_1)) \leq e^{\ell_1+\ell_0}(\xi_{1,1} + \xi_{2,1})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \tag{10}$$

If $t \in [\tilde{t}_1, t_2)$, in accordance with (5), (6), and (9), it follows that

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-\tilde{t}_1)}\mathbb{E}\mathcal{V}(\tilde{t}_1, z(\tilde{t}_1)) \leq e^{\ell_1}\mathbb{E}\mathcal{V}(t_1, z(t_1)) \leq e^{\ell_1+\ell_0}(\xi_{1,1} + \xi_{2,1})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \tag{11}$$

At the second triggering time t_2, the combination of (B3), (10), and (11) yields

$$\begin{aligned} \mathbb{E}\mathcal{V}(t_2, z(t_2)) &\leq \xi_{1,2}\mathbb{E}\mathcal{V}(t_2^-, z(t_2^-)) + \xi_{2,2}\mathbb{E}\mathcal{V}(t_2 - \iota, z(t_2 - \iota)) \\ &\leq e^{\ell_1+\ell_0}\prod_{i=1}^{2}(\xi_{1,i} + \xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \end{aligned} \tag{12}$$

For $t \in (t_2, \tilde{t}_2)$, based on (3)–(5) and (12), one can have

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-t_2)}\mathbb{E}\mathcal{V}(t_2, z(t_2)) \leq e^{\theta\hat{\iota}}\mathbb{E}\mathcal{V}(t_2, z(t_2)) \leq e^{\sum_{j=0}^{2}\ell_j}\prod_{i=1}^{2}(\xi_{1,i} + \xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \tag{13}$$

If $t \in [\tilde{t}_2, t_3)$, by applying (5), (6), and (12), we obtain the following

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-\tilde{t}_2)}\mathbb{E}\mathcal{V}(\tilde{t}_2, z(\tilde{t}_2)) \leq e^{\ell_2}\mathbb{E}\mathcal{V}(t_2, z(t_2)) \leq e^{\sum_{j=0}^{2}\ell_j}\prod_{i=1}^{2}(\xi_{1,i} + \xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \tag{14}$$

By recursively repeating this process for any $r \in N_+$, we obtain, when $t = t_r$,

$$\begin{aligned} \mathbb{E}\mathcal{V}(t_r, z(t_r)) &\leq \xi_{1,r}\mathbb{E}\mathcal{V}(t_r^-, z(t_r^-)) + \xi_{2,r}\mathbb{E}\mathcal{V}(t_r - \iota, z(t_r - \iota)) \\ &\leq e^{\sum_{j=0}^{r-1}\ell_j}\prod_{i=1}^{r}(\xi_{1,i} + \xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0)), \end{aligned} \tag{15}$$

and for $t \in (t_r, \tilde{t}_r)$, in accordance with the argument used in (13), it follows that

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-t_r)}\mathbb{E}\mathcal{V}(t_r, z(t_r)) \leq e^{\theta\hat{\iota}}\mathbb{E}\mathcal{V}(t_r, z(t_r)) \leq e^{\sum_{j=0}^{r}\ell_j}\prod_{i=1}^{r}(\xi_{1,i}+\xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \quad (16)$$

Similarly to the derivation in (14), for $t \in [\tilde{t}_r, t_{r+1})$, we have

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{\theta(t-\tilde{t}_r)}\mathbb{E}\mathcal{V}(\tilde{t}_r, z(\tilde{t}_r)) \leq e^{\ell_r}\mathbb{E}\mathcal{V}(t_r, z(t_r)) \leq e^{\sum_{j=0}^{r}\ell_j}\prod_{i=1}^{r}(\xi_{1,i}+\xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0)). \quad (17)$$

Hence, from $\ell = \max_{r\in N}\{\ell_r\}$, for $t \in [t_r, t_{r+1})$, $r \in N_+$, the inequality

$$\mathbb{E}\mathcal{V}(t, z(t)) \leq e^{(r+1)\ell}\prod_{i=1}^{r}(\xi_{1,i}+\xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0))$$

always holds. Furthermore, by applying condition (B1), we obtain

$$\mathbb{E}\,|z(t)|^p \leq u_1^{-1}e^{(r+1)\ell}\prod_{i=1}^{r}(\xi_{1,i}+\xi_{2,i})\mathbb{E}\mathcal{V}(t_0, z(t_0)),\ \forall t \geq t_0,$$

in conjunction with (4), it then follows that

$$\mathbb{E}\,|z(t)|^p \to 0 \quad as \quad t \to +\infty.$$

Accordingly, system (1) can be concluded to be p-AS under STM (2). The proof is completed.

Remark 2. *To ensure the effectiveness of the triggering mechanism, it is crucial to first exclude the possibility of Zeno behavior. In the STM (2), this is achieved by introducing a state-dependent waiting time* $\pi(z(t_r))$ *and by imposing a new condition (3), which prevents the case* $\pi(z(t_r)) = 0$ *from occurring. Consequently, Zeno behavior is effectively avoided. In fact, since* $t_{r+1} - t_r - \pi(z(t_r)) - \frac{\ell_r + \ln\frac{\mathcal{V}(t_r, z(t_r))}{\mathcal{V}(\tilde{t}_r, z(\tilde{t}_r))}}{\theta} = 0$*, it follows from inequality (6) that* $e^{\theta(t_{r+1}-t_r-\pi(z(t_r)))}\mathbb{E}\mathcal{V}(t_r + \pi(z(t_r)), z(t_r + \pi(z(t_r)))) = e^{\ell_r}\mathbb{E}\mathcal{V}(t_r, z(t_r))$*. On the other hand, inequality (5) yields* $\mathbb{E}\mathcal{V}(t_r + \pi(z(t_r)), z(t_r + \pi(z(t_r)))) \leq e^{\theta(t_r+\pi(z(t_r))-t_r)}\mathbb{E}\mathcal{V}(t_r, z(t_r))$*. Combining the above, we obtain* $e^{-\theta(t_{r+1}-t_r-\pi(z(t_r)))+\ell_r}\mathbb{E}\mathcal{V}(t_r, z(t_r)) \leq e^{\theta(t_r+\pi(z(t_r))-t_r)}\mathbb{E}\mathcal{V}(t_r, z(t_r))$*, which further implies* $t_{r+1} - t_r \geq \frac{\ell_r}{\theta}$*. This establishes a strictly positive lower bound between any two consecutive triggering instants, thereby ensuring the exclusion of Zeno behavior.*

Remark 3. *Based on Theorem 1, condition (3) ensures that the waiting time* $\pi(z(t_r))$ *at each triggering instant almost surely falls within the interval* $(\iota, \hat{\iota}]$*. This implies the existence of a strictly positive minimum inter-event time between any two consecutive triggering instants, hence eliminating the possibility of Zeno phenomena. Moreover, since* $\pi(z(t_r))$ *is a state-dependent waiting time, the length of the triggering interval* $[t_r, t_r + \pi(z(t_r)))$ *dynamically varies with the evolution of the system state. As a result,* $\pi(z(t_r))$ *can be suitably designed based on the system state to flexibly regulate the "sleep time" of the observer: The interval can be prolonged when the system approaches stability or shortened when the system deviates from equilibrium. This state-dependent characteristic grants the proposed STM greater flexibility, which helps to reduce communication and computational costs while preserving system performance.*

Remark 4. *Condition (4) characterizes the connection between the triggering parameters and the impulse intensity. Its primary purpose is to leverage this coupling effect to ensure that the Lyapunov function decreases over the system's evolution and asymptotically converges to zero, thereby guaranteeing the asymptotic stability of the system. In addition, if the strict inequality* $\theta\hat{\iota} < \ell_r$ *in condition (4) is relaxed to* $\theta\hat{\iota} \leq \ell_r$*, the stability conclusion still holds. However, in the critical case* $\theta\hat{\iota} = \ell_r$, $\frac{\ell_r + \ln\frac{\mathcal{V}(t_r,z(t_r))}{\mathcal{V}(\tilde{t}_r,z(\tilde{t}_r))}}{\theta}$ *may become zero, which causes the interval* $[\tilde{t}_r, t_{r+1})$ *to degenerate into a single point. Nevertheless, this does not affect the inductive estimation process or the conclusion of p-AS. To avoid such a situation and to simplify the derivation, this paper adopts the strict inequality form.*

Corollary 1. *On the basis of Theorem 1, if condition (4) is substituted with* $\theta\hat{\iota} < \ell_r$, $e^{2r\check{\ell}\hat{\iota}+(r+1)\ell}\prod_{i=1}^{r}(\xi_{1,i}+\xi_{2,i}) \leq \Im$, $r \in N_+$*. And the STM (2) is replaced by*

$$t_{r+1} = \inf\left\{t > t_r + \pi(z(t_r)) : t - t_r - \pi(z(t_r)) - \frac{\ell_r + \ln\frac{\mathcal{V}(t_r,z(t_r))}{\mathcal{V}(\tilde{t}_r,z(\tilde{t}_r))}}{\theta + \check{\ell}} \geq 0\right\}, \tag{18}$$

where $\Im$ *is a positive constant,* $\ell_r > 0$ *and* $\check{\ell} > 0$ *are both triggering parameters, then system (1) is p-ES under STM (18).*

Proof. Based on STM (18) and following a similar line of reasoning as in Theorem 1, the detailed proof is not repeated here.

Remark 5. *In Corollary 1, the parameter* $\check{\ell} > 0$ *is used to set the exponential convergence rate of the system and, through the STM (18), affects the event triggering frequency. Under the condition* $e^{2r\check{\ell}\hat{\iota}+(r+1)\ell}\prod_{i=1}^{r}(\xi_{1,i}+\xi_{2,i}) \leq \Im$*, the parameter* $\check{\ell}$ *together with* ℓ, $\hat{\iota}$, $\xi_{1,i}$, $\xi_{2,i}$ *determines the overall convergence behavior of the Lyapunov function* $\mathcal{V}(t, z(t))$*. Apart from this condition, there are no additional constraints, and* $\check{\ell}$ *can be flexibly chosen to balance the desired stability rate and triggering frequency.*

Remark 6. *Compared with the ETIC strategies proposed in [11–15], which require continuous or periodic monitoring of the system state, the STIC strategy proposed in this paper does not require such continuous monitoring. This is because the adopted STM can predict the next triggering instant based on the available information at the previous triggering moment, thereby effectively reducing communication costs. Although STIC methods have also been studied in [16, 19–21], they mainly focus on deterministic systems and primarily use comparison-principle-based approaches, making their applicability relatively limited. In contrast, this paper introduces stochastic disturbances under the framework of nonlinear impulsive systems and designs a state-dependent waiting time parameter to effectively eliminate Zeno behavior. Although [24] addresses a similar problem, its method is based on a constant waiting time, which lacks flexibility and does not consider delay effects during the impulsive process. In summary, this paper designs a Lyapunov-based STM, which offers a rigorous assurance of the stability of system (1).*

4. Applications

Next, we apply the previously proposed theoretical findings to a class of nonlinear stochastic systems affected by delayed impulses in order to confirm the effectiveness of the theoretical results.

Consider the following system:

$$\begin{cases} \mathrm{d}z(t) = [Pz(t) + Q\Phi(z(t))]\mathrm{d}t + \Psi(t, z(t))\mathrm{d}\omega(t), \ t \neq t_r, \\ z(t_r) = (\Xi + \mathcal{H})z(t_r - \iota), \ r \in N_+, \\ z(t_0 + \varsigma) = z(\varsigma), \ \varsigma \in [-\iota, 0], \end{cases} \tag{19}$$

where $\Phi(z(t)) : \mathbb{R}^a \to \mathbb{R}^a$ is a Lipschitz function with $\Phi(0) = 0$ and a Lipschitz matrix $\aleph$. P and Q are two $a \times a$ real matrices. $\Psi(\cdot)$ is locally Lipschitz continuous, and there exists a compatible dimension matrix $\mathcal{A}$ such that $trace[\Psi^T(z)\Psi(z)] \leq z(t)^T \mathcal{A}^T \mathcal{A} z(t)$. The sequence $\{t_r, r \in N_+\}$ represents the impulse times. Ξ is defined as an identity matrix with requisite dimensions. $\mathcal{H}$ is an $a \times a$ control gain matrix, and ι is the time delay.

Lemma 1. *[12] For any given real matrices $\mathcal{B}_1, \mathcal{B}_2, C$ with $C > 0$, and a positive scalar γ, the inequality given below holds:*

$$\mathcal{B}_1^T \mathcal{B}_2 + \mathcal{B}_2^T \mathcal{B}_1 \leq \gamma \mathcal{B}_1^T C \mathcal{B}_1 + \gamma^{-1} \mathcal{B}_2^T C^{-1} \mathcal{B}_2.$$

Theorem 2. *Given matrices $\mathcal{A}$ and $\aleph$, if there exists an $a \times a$ matrix $\mathcal{D} > 0$, $a \times a$ diagonal matrix $\mathcal{J} > 0$, $a \times a$ real matrix $\mathcal{M}$, and positive constants ζ, ξ, θ, ι, ℓ, $\breve{\ell}$ with $\breve{\ell} > 0$ such that $\mathcal{D} \leq \zeta \Xi$ and the following linear matrix inequalities conditions are satisfied:*

$$\begin{pmatrix} P^T \mathcal{D} + \mathcal{D}P + \aleph \mathcal{J} \aleph + \zeta \mathcal{A}^T \mathcal{A} - \theta \mathcal{D} & \mathcal{D}Q \\ * & -\mathcal{J} \end{pmatrix} < 0, \tag{20}$$

$$\begin{pmatrix} -\xi \mathcal{D} & \mathcal{D} + \mathcal{M} \\ * & -\mathcal{D} \end{pmatrix} < 0, \tag{21}$$

then system (19) is mean-square exponentially stable under the control gain $\mathcal{H} = \mathcal{D}^{-1}\mathcal{M}^T$ and STM:

$$t_{r+1} = \inf \left\{ t > t_r + \pi(z(t_r)) : t - t_r - \pi(z(t_r)) - \frac{\ell + \ln \frac{z(t_r)\mathcal{D}z(t_r)}{z(t_r + \pi(z(t_r)))\mathcal{D}z(t_r + \pi(z(t_r)))}}{\theta + \breve{\ell}} \geq 0 \right\}. \tag{22}$$

Proof. Choose an Lyapunov function $\mathcal{V}(t, z(t)) = z(t)^T \mathcal{D} z(t)$. According to (19), (20), and Lemma 1, it can be readily obtained that

$$\begin{aligned} \mathcal{L}\mathcal{V}(t, z(t)) &= 2z^T(t)\mathcal{D}(Pz(t) + Q\Phi(z(t))) + \mathrm{trace}[\Psi^T(t, z(t))\mathcal{D}\Psi(t, z(t))] \\ &\leq z^T(t)(\mathcal{D}P + P^T\mathcal{D})z(t) + 2z^T(t)\mathcal{D}Q\Phi(z(t)) + \zeta z^T(t)\mathcal{A}^T\mathcal{A}z(t) \\ &\leq z^T(t)(\mathcal{D}P + P^T\mathcal{D} + \mathcal{D}Q\mathcal{J}^{-1}Q^T\mathcal{D} + \aleph\mathcal{J}\aleph + \zeta\mathcal{A}^T\mathcal{A})z(t) \\ &\leq \theta z^T(t)\mathcal{D}z(t) \\ &= \theta\mathcal{V}(t, z(t)). \end{aligned} \tag{23}$$

Taking the expectation on both sides of formula (23) yields

$$\mathbb{E}\mathcal{L}\mathcal{V}(t, z(t)) \leq \theta\mathbb{E}\mathcal{V}(t, z(t)).$$

According to (19) and (21), we have at the impulse time $t = t_r$,

$$\begin{aligned} \mathbb{E}\mathcal{V}(t_r, z(t_r)) &= \mathbb{E}\left[z^T(t_r)\mathcal{D}z(t_r)\right] \leq \mathbb{E}\left[z^T(t_r - \iota)(\Xi + \mathcal{H})^T\mathcal{D}(\Xi + \mathcal{H})z(t_r - \iota)\right] \\ &\leq \xi\mathbb{E}[z^T(t_r - \iota)\mathcal{D}z(t_r - \iota)] = \xi\mathbb{E}\mathcal{V}(z(t_r - \iota)). \end{aligned}$$

Therefore, system (19) achieves mean-square exponential stability under the impulsive control gain $\mathcal{H}$ and the STM given in (22).

5. Numerical examples

This section verifies the feasibility and effectiveness of the proposed theoretical findings through two numerical simulations.

Example 1. *Investigate a stochastic nonlinear system with delayed impulsive effects:*

$$\begin{cases} \dot{z}(t) = 0.2z(t)\mathrm{d}t + 0.5z(t)\mathrm{d}\omega(t), \ t \neq t_r, \\ z(t) = 0.69z(t-0.2), \ t = t_r, \\ z(t_0) = 1, \end{cases} \tag{24}$$

where $\iota = 0.2$, $\{t_r, r \in N_+\}$ *be the set of self-triggered impulse instants to be determined. In the absence of control input, system (24) is clearly unstable, as shown in Figure 1.*

To achieve mean-square exponential stability of system (24), we choose the Lyapunov function $\mathcal{V}(t, z(t)) = |z(t)|^2$, *and design the following STM:*

$$t_{r+1} = \inf\left\{ t > t_r + \pi(z(t_r)) : t - t_r - \pi(z(t_r)) - \frac{\ell + \ln \frac{|z(t_r)|^2}{|z(t_r+\pi(z(t_r)))|^2}}{\theta + \breve{\ell}} \geq 0 \right\}, \tag{25}$$

where the waiting time is given by $\pi(z(t_r)) = 0.4e^{-100|z(t_r)|} + 0.2$ *and set* $\hat{\iota} = 0.6$. *The parameters are selected as* $\ell = 0.39$, $\theta = 0.65$, $\breve{\ell} = 0.01$, $\xi = 0.48$. *Based on Corollary 1, Figure 2 demonstrates that system (24) is mean-square exponentially stable under STM (25).*

Moreover, to verify the control performance of the STIC mechanism proposed in this paper, the ETIC method presented in [13] is applied to the system (24) considered herein, and a comparative simulation is conducted. The results are shown in Figure 3. It should be noted that although the ETIC method in [13] was originally designed for systems without impulsive delays and adopts an event-triggered mechanism with a fixed waiting time, its control objective is consistent with that of this paper, namely, to achieve mean-square exponential stability of the system. Therefore, under the same system model and parameter settings, a comparison between Figures 2 and 3 shows that the STIC strategy exhibits better performance in terms of convergence speed and state fluctuations, further verifying its effectiveness in dealing with complex impulsive structures and state-dependent triggering mechanisms.

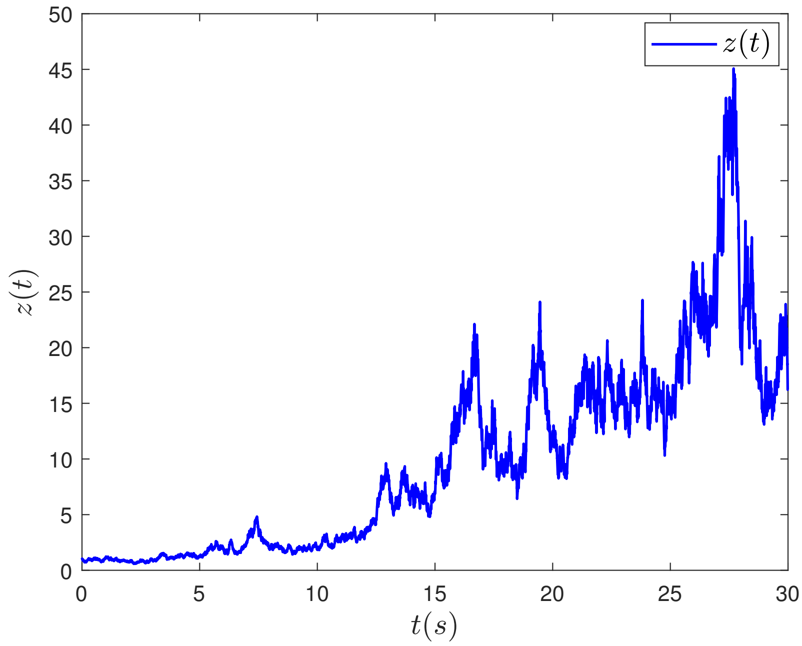

Figure 1. State trajectories of system (24) without control input.

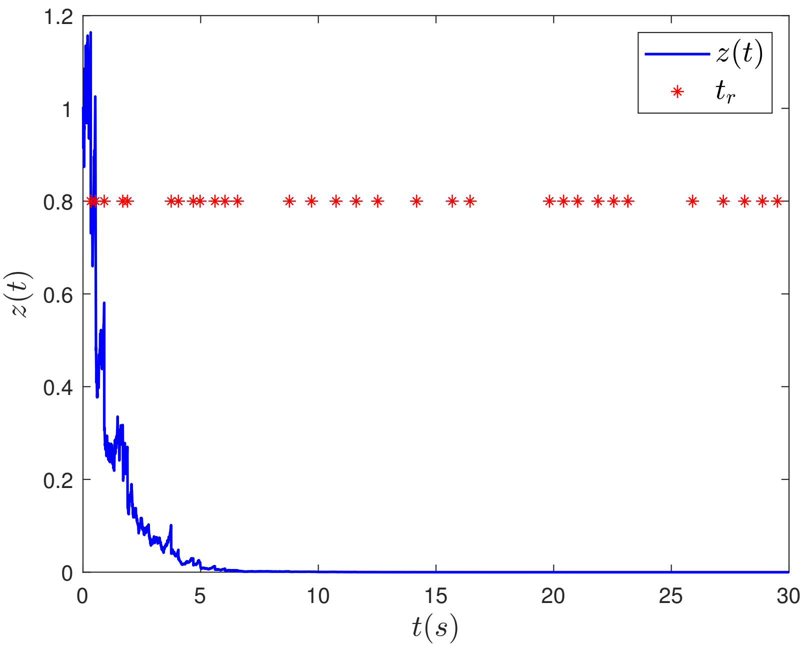

Figure 2. State trajectories of system (24) under STM (25).

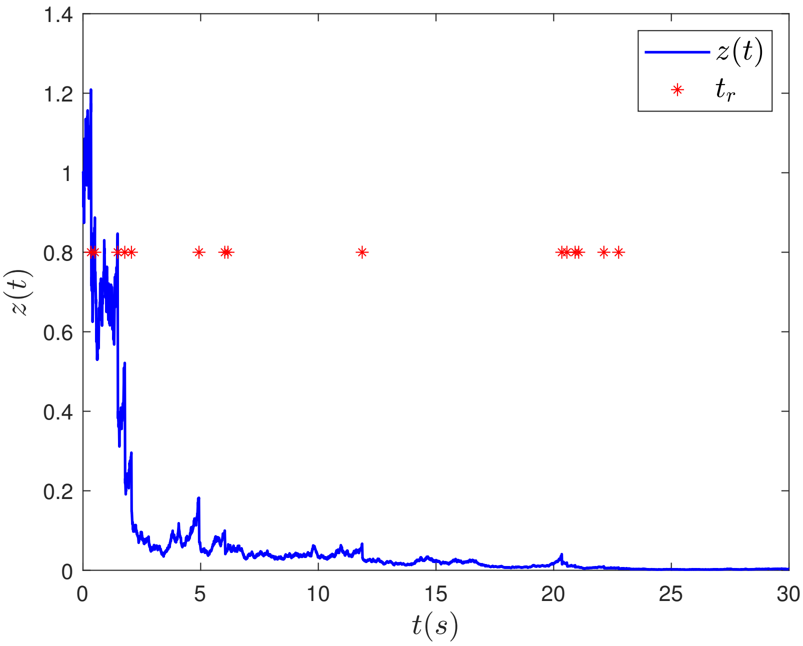

Figure 3. State trajectories of system (24) under the continuous ETM proposed in [13].

Example 2. *The analysis of system (19) is carried out under the following conditions:*

$$P=\begin{pmatrix}-1.3 & 1.1 & 0.6\\ 0.9 & -1.6 & 1.2\\ -0.8 & 1 & -1.2\end{pmatrix},\quad Q=\begin{pmatrix}0.5 & -0.7 & 0.2\\ -0.4 & 0.9 & 0.6\\ 0.7 & -0.3 & -0.5\end{pmatrix},\quad \aleph=\mathcal{A}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix},$$

$\Phi_j(z) = 0.5(|z+1|-|z-1|)$, $j=1,2,3$, $\Psi = \frac{\sqrt{2}}{2}diag\{z_1(t), z_2(t), z_3(t)\}$, $z(\varsigma) = [-0.5, 0.3, -0.1]^T$. *As shown in Figure 4, system (19) is unstable without the implementation of control input.*

To achieve mean-square exponential stability of system (19), an STM is designed. We set the parameters as $\iota = 0.15$, $\xi = 0.95$, $\theta = 2$, $\zeta = 2$, $\ell = 0.5$, $\breve{\ell} = 0.01$, *choose* $\pi(z(t_r)) = 0.1e^{-100|z(t_r)|}+0.15$ *and let* $\hat{\iota} = 0.25$. *By solving the linear matrix inequalities (20) and (21) using MATLAB, the following STM can be designed:*

$$t_{r+1} = \inf\left\{t > t_r + \pi(z(t_r)) : t - t_r - \pi(z(t_r)) - \frac{\ell + \ln\frac{z(t_r)\mathcal{D}z(t_r)}{z(t_r+\pi(z(t_r)))\mathcal{D}z(t_r+\pi(z(t_r)))}}{\theta+\breve{\ell}} \geq 0\right\}, \tag{26}$$

where $\mathcal{D} = \begin{pmatrix}1.3958 & 0.3619 & -0.0095\\ 0.3619 & 1.4150 & 0.5926\\ -0.0095 & 0.5926 & 1.6127\end{pmatrix}$, $\mathcal{M} = \begin{pmatrix}-0.7453 & -0.3619 & 0.0095\\ -0.3619 & -0.7645 & -0.5926\\ 0.0095 & -0.5926 & -0.9621\end{pmatrix}$. *Thus, from* $\mathcal{H} = \mathcal{D}^{-1}\mathcal{M}^T$, *we obtain* $\mathcal{H} = \begin{pmatrix}-0.4934 & -0.1546 & 0.0598\\ -0.1546 & -0.4094 & -0.2179\\ 0.0598 & -0.2179 & -0.5162\end{pmatrix}$. *Based to Theorem 2, the system (19) is mean-square exponentially stable under STM (26), as shown in Figure 5.*

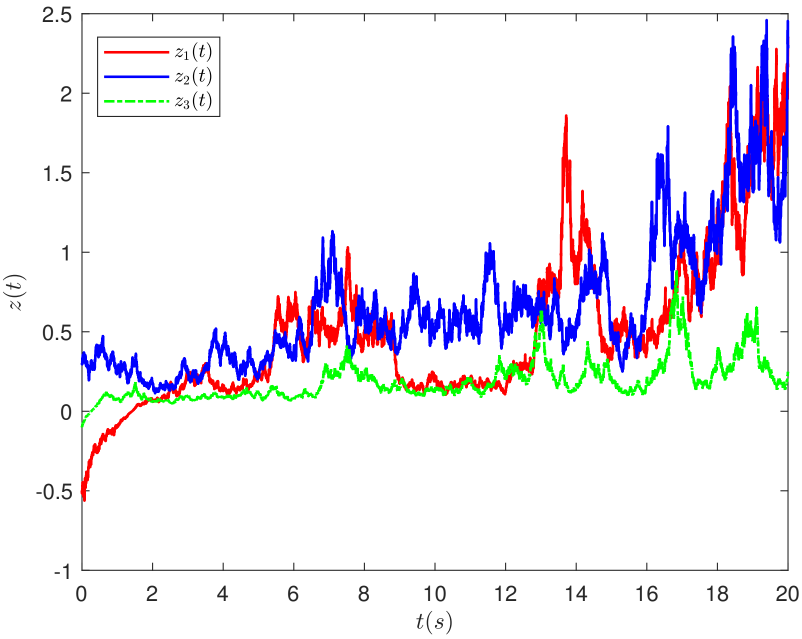

Figure 4. State trajectories of system (19) without control input.

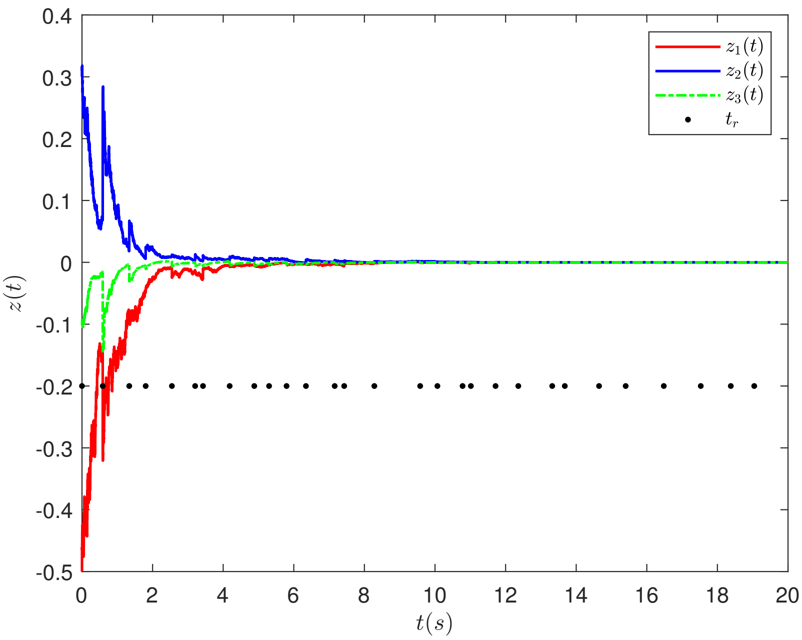

Figure 5. State trajectories of system (19) under STIC (26).

6. Conclusions and outlook

Based on Lyapunov theory, this paper had designed an explicit STIC mechanism, which had not only derived sufficient conditions for ensuring the p-th moment stability of the system but also effectively avoided the occurrence of Zeno behavior. Compared with traditional ETIC strategies, this mechanism had eliminated the need for continuous monitoring of system states, significantly reducing the consumption of communication and computational resources. Moreover, unlike STM constructed based on the comparison principle, this paper had introduced state-dependent waiting time parameters, enhancing the flexibility and adaptability of the triggering mechanism. The theoretical findings established were applied to nonlinear stochastic systems, and a feasible joint design scheme for the STM and impulsive controller was proposed based on the linear matrix inequalities. Finally, the effectiveness and practicality of the proposed method were verified through two numerical examples. Future research can further extend to state-dependent impulsive stochastic systems with state-dependent delays, or explore the input-to-state stability of time-delay systems under the STIC framework.

Author contributions

Bing Shang: Writing-original draft; Jin-E Zhang: Supervision, writing-review and editing. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare there is no conflict of interest.

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