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Research article

Finite-time fuzzy control strategy for nonlinear MASs with actuator faults and deception attacks

Hanen Louati[1], Rohma Arooj[2], Azmat Ullah Khan Niazi[2,*], Mhassen. E. E. Dalam[3], Mohammed M. A. Almazah[3] and Aseel Smerat[4,5]

[1] Department of Mathematics, Faculty of Science, Northern Border University, Arar, Saudi Arabia; Hanen.Louati@nbu.edu.sa

[2] Department of Mathematics and Statistics, University of Lahore, Sargodha 40100, Pakistan; aroojrohman@gmail.com, azmatniazi35@gmail.com

[3] Department of Mathematics, College of Sciences and Arts (Muhyil), King Khalid University, Muhyil 61421, Saudi Arabia; mdalam01@kku.edu.sa, mmalmazah@kku.edu.sa

[4] Faculty of Educational Sciences, Al-Ahliyya Amman University, Amman 19328, Jordan; smerat.2020@gmail.com

[5] Department of Biosciences, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai, 602105, India; smerat.2020@gmail.com

* **Correspondence:** Email: azmatullah.khan@math.uol.edu.pk.

Abstract: In this paper, we addressed the issue of maintaining a desirable level of performance in the presence of actuator faults and deception attacks in nonlinear multi-agent systems (MASs). These problems are critical for the stability and coordination of MASs that are increasingly used in robotics, autonomous vehicles, and industrial automation. Our aim was to design control strategies that, in the presence of these challenges, guarantee practical finite-time stability and robust tracking performance. To accomplish this, a distributed adaptive fuzzy control scheme based on backstepping was developed. Fuzzy logic systems were utilized to capture the complex system's unknown nonlinearities, while adaptive laws were designed to estimate and mitigate actuator gain and bias faults. A Nussbaum-type function was introduced to address unknown control directions resulting from deception attacks. Stability was verified by the Lyapunov theory. The suggested approach ensured that it was a finite-time stable method, and every signal in the closed loop was found to be semi-globally uniformly eventually bounded. Our control strategy, compared to published approaches, improved convergence time by approximately 38% and tracking accuracy of approximately 35% under the same conditions of simultaneous actuator faults and deception attacks.

Keywords: finite-time control; multi-agent systems; adaptive fuzzy control; gain and bias faults; deception attacks
Mathematics Subject Classification: 26A33, 34K37

1. Introduction

In control theory, multi-agent systems (MASs) have emerged as a major area of interest because they provide a means of modeling the distributed, coordinated behavior of multiple autonomous agents. Such systems are used extensively in the domains of autonomous vehicles, smart grids, industrial automation, and aerospace systems. Nonetheless, there are challenges with most of the real-life applications of MASs because of diverse real-life problems such as nonlinear dynamics, actuator failure, external disturbances, cyber-attacks, and communications limitations.

Actuator faults are quite common and some of the worst issues that directly impact system reliability. In response to this, some fault-tolerant control (FTC) strategies have come up. As an instance, Sun et al. [1] introduced an adaptive consensus under which they addressed the actuator faults and output limits in nonlinear MASs. The authors produced a predefined-time controller to be used in the domain of aerospace, which can overcome the actuator fault and rate gyro failure in heavy-lift launch vehicles [2]. Zhao and Yang [3] presented the adaptive FTC mechanism, robust to Denial-of-Service (DoS) attacks that have become an emerging menace on the internet, especially the cyber-physical systems. It has also been found that fault detection with the help of signal analysis [4], can be effectively employed in early diagnosis in the industrial sphere.

The scope of intelligent control techniques has increased, and strategies that deal with fault situations have been implemented as AI-based. A survey presented by Jiang et al. [5] entailed data-driven methods of fault handling in industrial systems, including generative AI and foundation models. Furthermore, to enhance the efficiency in communication of MASs, a new scheme of the double event-triggered control under actuator faults was established by Luo and Ye [6]. High-end Gaussian model approaches have been suggested for risk-conscious motion planning in the intelligent transport systems [7] and automatic tracking systems [8], have enhanced industrial task-based performance. Another field of application of fuzzy finite-time regulators is presented by Ding et al. [9], who developed the fuzzy finite-time controller of heavy trucks to maintain the lane and prevent the vehicle from rolling.

Fuzzy control is useful when it comes to the development of uncertain nonlinear systems. Wu and Tong [10] proposed a fuzzy adaptive controller that can control periodic actuator faults of MASs. Related concepts were generalized to actuator-hysteretic systems through pseudoinverse control [11], and Lurie-type systems had their sampled-data-based stability framework introduced [12]. Due to combined failures of sensors and actuators, Ye et al. [13] used a matrix-pencil-based event-triggered controller to deal with them. Application to prediction and diagnostics, Wavelet-guided neural networks [14], and adaptive sliding mode control [15] have also been applied. Type-3 fuzzy logic control strategies have been applied to offshore systems (e.g., wind turbines) [16], and actor-critic reinforcement learning improved the performance of electro-hydrostatic actuators [17]. In their work, Ye et al. [18] introduced decentralized approaches in prescribed-time control in large-scale systems, and Wang et al. [19] addressed sampled-data output feedback design of aperiodic systems. The idea behind collaborative robots suggested robust constraint-following control schemes, which were experimentally proven in [20]. In systems having a hybrid actuator fault, Zhang et al. [21] proposed finite-time cooperative fault-tolerant control. Moreover, multi-objective fuzzy optimization has been applied in complex aviation scheduling [22].

Real-time diagnostics in mechanical systems [23] are also developed based on a digital twin.

The classical fault-tolerant control procedures continue to be significant (the classic study was by Li and Wang [24]), and the prescribed performance control using fuzzy-based controls has been applied to the robotic manipulators [25]. Deng and Yang [26] investigated adaptive fault-tolerant control of MASs in the nonlinear dynamics case and with multiple failures of the actuators. There has been the emergence of unsupervised methods of deep learning, like fault diagnosis based on transfer learning and transformers [27]. Further, unknown control directions and actuator failure, event-triggered control, and advanced signal processing have enhanced the identification of rotating machinery faults [28, 29]. Faults in PMSM systems have been improved by lightweight fusion models [30], and adaptive fuzzy cooperative control has been implemented on uncertain nonlinear MASs with actuator faults [31].

The security issue has necessitated the study of resilient control algorithms in deception and DoS attacks. Bai et al. [32] created a finite-time fuzzy secure control protocol of MASs against deception attacks. Lin and Qian [33] extended the adaptive control theory to the nonlinearly parameterized systems. Researchers looking at prescribed-time control, [34] and [35], focus on achieving state convergence within a predefined, fixed time regardless of initial conditions. In contrast, the finite-time control approach used in this paper ensures that system states converge in finite time, but the settling time depends on initial conditions and system parameters. Although our method does not enforce a fixed upper bound on convergence time, it achieves practical finite-time stability and is well-suited for nonlinear multi-agent systems affected by actuator faults and deception attacks. Ye et al. [36] introduced higher-order prescribed-time and exponential stabilization algorithms to a system that works on interconnected systems, even in the case of unknown control gains. Additionally, united fault diagnosis methods were used in battery energy storage systems [37], and Zhou and Tong [38] introduced a fuzzy adaptive resilient formation control strategy toward MASs in the event of DoS attacks, where the best fuzzy approximation and attack-resilient logic were together. Most of the available methods either consider perfected communication channels or ignore the combined impact of actuator malfunctions, fuzzy uncertainty, and DoS attacks. Moreover, in these extreme mixed conditions, most designs do not offer finite-time performance guarantees.

In research, the adaptive fault-tolerant controllers have been addressed in nuclear energy systems with a view to enhancing robustness and accuracy of tracking performance in cases of actuator and sensor faults. In order to manage control rod drive mechanism (CRDM) faults, Hui and Yuan [39] developed the adaptive fault-tolerant controller of a modular high-temperature gas-cooled reactor (MHTGR) that includes prescribed performance functions that form part of a neural network. It was based on this development that Hui et al. [40] came up with an adaptive active fault-tolerant control strategy through dynamic surface control that can achieve effective load-following behavior even with the occurrence of both matched and unmatched faults. Further on, a finite-time super-twisting sliding mode control with an extended state observer was used in [41] to control a load follower of a pressurized water reactor (PWR) that achieves faster convergence and fault-resistant control. These studies motivate the creation of frameworks of resilient control schemes in nonlinear multi-agent systems with actuator faults and deception attacks.

The proposed framework is an adaptive fuzzy finite-time control strategy that simultaneously addresses actuator faults and deception in Figure 1.

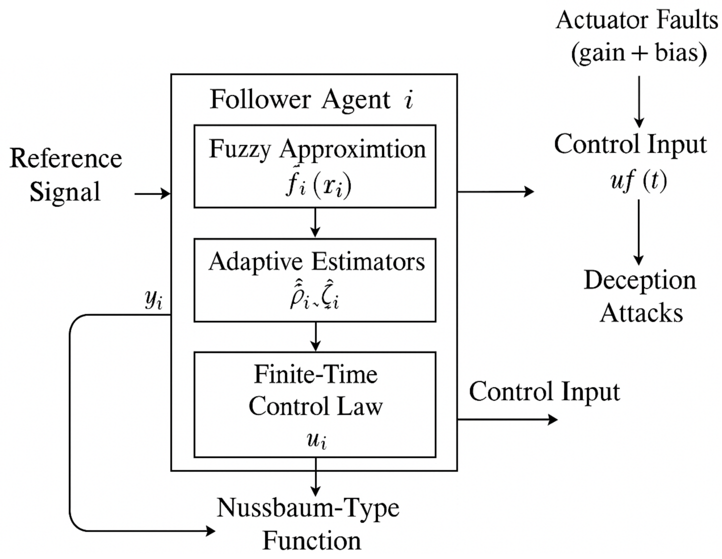

Figure 1. Framework of paper.

The primary contributions of this activity are that the main results formulated are exclusive to current studies and original, not only on the theoretical level but also regarding their application:

- Unlike most researchers who handle either actuator faults (e.g., [10, 24]) or deception attacks (e.g., [3, 38]) in isolation, we propose a distributed adaptive fuzzy finite-time control framework capable of simultaneously handling both actuator faults (gain and bias) and deception attacks in nonlinear MASs.
- To tackle the challenges of unknown control directions introduced by deception attacks, we employ a Nussbaum-type function, a technique not fully exploited in similar works (e.g., [28, 32]). This design significantly enhances robustness without relying on prior knowledge of attack parameters.
- Our adaptive laws separately estimate actuator gain and bias faults, improving upon prior works [10, 26] that use aggregate or partial compensation. This detailed modeling results in more accurate control compensation and enhanced resilience.
- In contrast to many studies that guarantee asymptotic convergence [1, 6, 24]), we prove finite-time convergence of tracking errors using Lyapunov-based techniques with hybrid uncertainties, ensuring semi-global uniform ultimate boundedness (SGUUB).
- We validate the proposed method through simulation of a nonlinear MAS with time-varying actuator faults and deception attacks, demonstrating faster convergence and better tracking accuracy than conventional approaches.

This paper is organized in the following way: In Section 2, we define the problem and provide preliminaries, which are system modeling, network topology, and fuzzy approximation. Section 3 builds up the proposed adaptive fuzzy finite-time secure design of actuator faults and deception attacks control. Section 4 identifies which system is stable, that is, the closed-loop system, based on the Lyapunov theory. Section 5 contains our simulation results to prove the usefulness and benefits of the proposed method. Lastly, Section 6 is the conclusion of the paper and the description of the potential future research directions. Table 1 shows the nomenclature used throughout this paper.

Table 1. Nomenclature.

Symbol	Description	Unit
x_i	State of the i-th follower agent	Depends on system (e.g., m, m/s)
x_0	State of the leader agent	Same as x_i
$u_i(t)$	Control input for agent i	N or N·m
$u_i^f(t)$	Faulty control input under actuator fault	Same as u_i
$\rho_i(x_i, m_i)$	Actuator gain fault function	Dimensionless
$\zeta_i(t)$	Actuator bias fault	Same as u_i
$y_r(t)$	Reference signal	Depends on system (e.g., m or rad)
S	Upper limit of integration in Nussbaum function	Dimensionless (s, if integrating over time)
$e_i(t)$	Tracking error	Same as x_i
$\Psi(s)$	Nussbaum-type function	Dimensionless
κ_i	Internal variable in Nussbaum function	Dimensionless
$\theta_i(x_i)$	Unknown smooth nonlinear function	Depends on x_i
$\hat{\theta}_i$	Estimated nonlinear function	Same as θ_i
$\mu_l(x_i)$	Gaussian membership function	Dimensionless
$W_{i,l}$	Fuzzy rule weight	Dimensionless
$\gamma_{i,k}$	Adaptive gain parameter	Dimensionless or variable-specific
$s_i(t)$	Sliding surface for agent i	Same as x_i
V_i	Lyapunov candidate function	J (Joules) or dimensionless
α_i, λ_i, c_i, δ_i, ν_i	Design constants	Dimensionless
$\bar{x}_i$	Auxiliary tracking error	Same as x_i
$\Xi_D(t)$	DoS attack active time intervals	s (seconds)
$\Xi_N(t)$	Normal communication intervals	s (seconds)
t, T	Time, DoS attack duration	s (seconds)

2. Problem formulation and preliminaries

2.1. System descriptions

Suppose M followers exist, labeled as agents 1 to M, and a leader. The dynamic models of M followers are considered as

$$\begin{cases} \dot{x}_{i,k} = x_{i,k+1} + \zeta_{i,k}(t)\theta_{i,k}(x_{i,k}), \quad k = 1, \ldots, m_i - 1, \\ \dot{x}_{i,m_i} = u_i^f + \zeta_{i,m_i}(t)\theta_{i,m_i}(x_{i,m_i}), \\ y_i = x_{i,1}, \end{cases} \tag{2.1}$$

where $x_{i,k}$ represents the state variable of follower i at level k, $x_{i,k+1}$ is the next stage in the system dynamics, $\zeta_{i,k}(t)$ is a time-varying uncertainty introducing external disturbances,$\theta_{i,k}(x_{i,k})$ is a nonlinear smooth function, u_i^f is the faulty control input, affected by actuator fault, and $y_i = x_{i,1}$ is the output equation defining the system's measured output.

Deception attacks corrupt the system, states, and control inputs. The state under attack is given by

$$x_{i,k}^{attack} = \Gamma_i(t)x_{i,k} + x_{i,k}, \tag{2.2}$$

where $\Gamma_i(t)$ denotes the deception signal, which captures the effect of external cyber-attacks that tamper with sensor data or communication signals. It corrupts the transmitted information without affecting the physical actuator directly.

The attack-modified state is rewritten as

$$x_{i,k}^{attack} = g_i x_{i,k},$$

where $g_i = \Gamma_i(t) + 1$.

The control input under attack is described as

$$u_i^{attack} = d(t)u_i + Q(x_{i,m_i}), \tag{2.3}$$

where $d(t)$ denotes an unknown and time-varying function that introduces distortion into the control input, whereas $Q(x_{i,m_i})$ represents an unknown yet bounded function.

Actuators may fail due to bias and gain faults, which are the two most common types of actuator faults that commonly occur in practice, expressed as [31]

$$u_i^f(t) = \rho_i(x_{i,m_i})u_i(t) + \zeta_i(t), \tag{2.4}$$

where u_i is the intended control input designed for system stability, $\rho_i(x_{i,m_i})$ represents the actuator gain fault, satisfying $0 < \rho_i(x_{i,m_i}) \leq 1$, and $\zeta_i(t)$ represents the actuator bias fault, which introduces an external disturbance. These faults directly affect the actuator output and are independent of network transmission. Our actuator fault model is nonlinear and state-dependent, using $\rho_i(x_i, t)$ and $\zeta_i(t)$, and our approach is agent-wise and decentralized, making it more suitable for distributed multi-agent systems.

Considering deception attacks and actuator faults, the compromised MASs (2.1) dynamics are rewritten as

$$\begin{cases} \dot{x}_{i,k} = x_{i,k+1} + g_i g_i^{-1} x_{i,k} + g_i \zeta_{i,k}(t)\theta_{i,k}(x_{i,k}), \quad k = 1, \ldots, m_i - 1, \\ \dot{x}_{i,m_i} = g_i d(t)(\rho_i u_i + \zeta_i) + g_i g_i^{-1} x_{i,m_i} + g_i \zeta_{i,m_i}(t)\theta_{i,m_i}(x_{i,m_i}) + g_i Q(x_{i,m_i}), \\ y_i = x_{i,1}. \end{cases} \tag{2.5}$$

Assumption 2.1. *The faulty actuator dynamics can be written as*

$$\tilde{g}_i(t) = g_i(t)\left(1 + \Delta_{g_i}(t)\right) + \Delta_{b_i}(t), \tag{2.6}$$

where $\Delta_{g_i}(t)$ denotes a time-varying gain fault, while $\Delta_{b_i}(t)$ corresponds to a time-varying bias fault. As a result, the control input $\tilde{\Gamma}_i(t)$ is influenced by a non-zero gain distortion $\tilde{g}_i(t)$, which satisfies $0 < |\tilde{g}_i(t)| \leq \tilde{g}_i^*$, with $\tilde{g}_i^*$ representing an upper bound reflecting the extent of the actuator fault. It is also ensured that the rate of gain change is bounded by $\left|\frac{\dot{\tilde{g}}_i(t)}{\tilde{g}_i(t)}\right| \leq \tilde{g}_i^0$, making sure that the gain does not rise too quickly. The adjustment to the output gain $\tilde{d}(t)$ is assumed to be in a known interval, $\underline{\tilde{d}} \leq \tilde{d}(t) \leq \overline{\tilde{d}}$, due to the fault. To keep things concise, we call the bounded gain distortion $\tilde{d}_{g_i}$ for the rest of the paper.

Lemma 2.1. *For a continuous function $Y(u, v) \in \mathbb{R}$, there exist positive functions $\mu(u) > 0$ and $\nu(v) > 0$ such that*

$$|Y(u, v)| \leq \mu(u)\nu(v). \tag{2.7}$$

In the presence of an actuator fault, modeled by a time-varying gain deviation $\Delta_{g_j}(t)$ and a bias disturbance $\Delta_{b_j}(t)$, the affected function is expressed as

$$\hat{Y}(u, v, t) = Y(u, v)(1 + \Delta_{g_j}(t)) + \Delta_{b_j}(t). \tag{2.8}$$

Accordingly, under such actuator faults, the following inequality holds:

$$|\hat{Y}(u,v,t)| \leq \mu(u)v(v)(1+\Delta_{g_j}(t)) + |\Delta_{b_j}(t)|. \tag{2.9}$$

Definition 2.1. *[33] Consider a continuous function $\Psi(s) : \mathbb{R} \to \mathbb{R}$ that satisfies the following conditions:*

$$\lim_{S\to\infty} \sup \left(\frac{1}{S} \int_b^S \Psi(\eta)\, d\eta \right) = +\infty, \tag{2.10}$$

$$\lim_{S\to\infty} \inf \left(\frac{1}{S} \int_b^S \Psi(\eta)\, d\eta \right) = -\infty. \tag{2.11}$$

It is important to note that although the Nussbaum function is commonly used to handle unknown control directions, its effectiveness depends on the assumption that the control gain does not cross or reach zero. In this work, we do not assume that the actuator gain becomes zero. Instead, we assume that the gain affected by faults or deception attacks remains strictly positive but unknown, i.e., it lies within a known positive interval. Therefore, while attacks or faults may alter the magnitude or the sign of the control input, they do not make the gain vanish. This ensures that the use of the Nussbaum function remains valid in our control design.

Definition 2.2. *The smooth saturation function is defined as*

$$sat(s_i) = \frac{s_i}{1+|s_i|}. \tag{2.12}$$

It is differentiable everywhere, continuous, and bounded, meaning that it can be implemented in real-world practical control functions. It makes sure that the control input is smooth and not of the chattering nature that characterizes the discontinuous $sign(s)$ function.

Lemma 2.2. *Consider the nonlinear system affected by actuator faults:*

$$\dot{z}(\tau) = h(z(\tau)) + k(z(\tau))\left[\eta(z(\tau))v(\tau) + \xi(\tau)\right], \tag{2.13}$$

where $h(z)$ is a smooth nonlinear function representing the nominal system dynamics, $k(z)$ is the control input gain matrix, $\eta(x) \in (0, 1]$ denotes the unknown control effectiveness due to gain faults, and $\xi(t)$ represents a bounded bias fault with $\|\xi(t)\| \leq \xi^*$. Consider there exists a continuously differentiable, positive definite Lyapunov function $V(z) : \mathbb{R}^m \to \mathbb{R}_{\geq 0}$, such that its derivative along the system satisfies

$$\dot{V}(z) + \alpha_1 V(z) + \alpha_2 V^r(z) + \Delta \leq 0, \tag{2.14}$$

where $\alpha_1 > 0$, $\alpha_2 > 0$, $\Delta > 0$, $0 < r < 1$, and $0 < \mu < 1$. Hence, the system is considered to exhibit practical finite-time stability, where its trajectories ultimately enter and remain within a bounded region specified as

$$\lim_{t\to T_s} V(x(t)) \leq \min\left\{ \frac{\Delta}{(1-\mu)\alpha_1}, \left(\frac{\Delta}{(1-\mu)\alpha_2} \right)^{\frac{1}{r}} \right\}. \tag{2.15}$$

Furthermore, the finite settling time T_s is bounded by

$$T_s \leq \max\left\{ T_0 + \frac{1}{\mu\alpha_1(1-r)} \ln\left(\frac{\mu\alpha_1 V^{1-r}(T_0) + \alpha_2}{\alpha_2} \right),\ T_0 + \frac{1}{\alpha_1(1-r)} \ln\left(\frac{\alpha_1 V^{1-r}(T_0) + \mu\alpha_2}{\mu\alpha_2} \right) \right\}, \tag{2.16}$$

where T_0 is the initial time and $V(T_0)$ is the initial value of the Lyapunov function.

Lemma 2.3. *[33] Let $\alpha, \beta \in \mathbb{R}$, $0 < \gamma \leq 1$, and $\delta \in [0, 1]$. Assume that the control input applied to the actuator is affected by an additive fault term $\phi(t)$. Under these assumptions, the following inequalities are satisfied:*

$$|\alpha^\gamma - \beta^\gamma| \leq 2^{1-\gamma}|\alpha - \beta|^\gamma, \tag{2.17}$$

$$|\alpha + \beta|^\delta \leq |\alpha|^\delta + |\beta|^\delta. \tag{2.18}$$

Suppose the actual input to the system is given by

$$\hat{v}(t) = v(t) + \phi(t). \tag{2.19}$$

Then, for any continuously differentiable Lyapunov function $W(z)$, the time derivative along the system trajectories satisfies the following inequality:

$$\dot{W}(z) \leq \nabla W(z)^\top g(z, v(t)) + \|\nabla W(z)\| \cdot |\phi(t)|. \tag{2.20}$$

Lemma 2.4. *[33] Let $a, b, \theta > 0$ be given positive constants. For any real variables $\chi, \psi \in \mathbb{R}$ with actuator fault, the following inequality holds:*

$$|\chi|^a|\psi|^b \leq (1 + \rho_i(x_i, m_i))\left(\frac{a\theta}{a+b}|\chi|^{a+b} + \frac{b(1-\theta)}{a+b}|\psi|^{a+b}\right) + |\zeta_i(t)|. \tag{2.21}$$

2.2. Network topology

Consider a directed graph $G = (H, D)$ representing the communication topology in the MAS, where $H = \{1, 2, \ldots, N, N+1, \ldots, M\}$ is the set of all agents, with the first N nodes representing followers, and the remaining $M - N$ nodes representing leaders, and $D \subseteq H \times H$ is the edge set. A directed edge $(u, k) \in D$ means that agent k can receive messages from agent u. Followers are connected in a closed ring such that each follower i communicates with its neighbor $i + 1 \mod N$, ensuring redundancy. Each leader is directly connected to a subset of followers, forming a star structure where leaders act as control reference sources, shown in Figure 2.

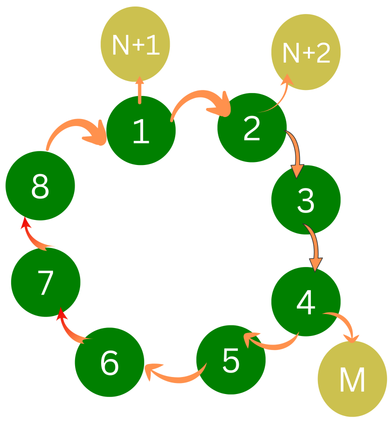

Figure 2. Network topology.

The adjacency matrix $\bar{A} = [a_{uk}] \in \mathbb{R}^{M\times M}$ is defined as:

Let $b_{uv} > 0$ if the pair (v, u) belongs to the set $\mathcal{D}$, and $b_{uv} = 0$ otherwise. The Laplacian matrix can be expressed as

$$\mathcal{L} = \mathcal{D} - \mathcal{B}, \quad \mathcal{D} = \operatorname{diag}(d_1, \ldots, d_M), \quad d_i = \sum_{\substack{j=1 \\ j\neq i}}^{M} b_{ij}. \tag{2.22}$$

Next, we define the Laplacian matrix $\mathcal{L}$ in block form, which can be written as

$$\mathcal{L} = \begin{bmatrix} \tilde{L}_{\text{cycle}} + \tilde{L}_f & -\tilde{L}_f \\ \mathbf{0}_{(M-N)\times N} & \mathbf{0}_{(M-N)\times(M-N)} \end{bmatrix}, \tag{2.23}$$

where $\tilde{L}_{\text{cycle}}$ represents the cyclic communication topology among follower nodes, $\tilde{L}_f$ characterizes the influence from leader nodes to followers. The bottom blocks are zero matrices, reflecting that leader nodes do not receive information from others.

2.3. Fuzzy approximation model (FAM)

Since the non-linear functions in system (2.5) are not explicitly known, fuzzy approximation models are utilized to approximate them. The rule base is constructed using fuzzy IF-THEN rules, which take the following form:

$$\mathcal{R}_j : \text{If } \iota_1 \text{ is } \mathcal{I}_1^j, \iota_2 \text{ is } \mathcal{I}_2^j, \ldots, \iota_m \text{ is } \mathcal{I}_m^j, \text{ then } y \text{ is } \mathcal{P}^j. \tag{2.24}$$

In this formulation, $\iota = (\iota_1, \ldots, \iota_m)^\top$ and y denote the input and output of the FAM, respectively. The fuzzy sets $\mathcal{I}_i^j$ and $\mathcal{P}^j$ are defined by their corresponding membership functions $\eta_{\mathcal{I}_i^j}(\iota_i)$ and $\eta_{\mathcal{P}^j}(y)$, where $i = 1, \ldots, m$.

The system receives its actual control input $u^f(t)$ through the combination of an actuator fault signal with the original input.

$$u^f(t) = (1 - \delta(t))u(t), \tag{2.25}$$

where $u(t)$ is the designed control input and $\delta(t) \in [0, 1]$ represents the unknown time-varying fault signal. When $\delta(t) = 0$, the actuator is fully functional; when $\delta(t) = 1$, a complete loss of actuator effectiveness occurs. The fuzzy approximation model (FAM) with an actuator fault is written as

$$y(\boldsymbol{\iota}, u) = \delta_a(u) \frac{\sum_{j=1}^{N} y_{hj}\, \omega_j(\boldsymbol{\iota})}{\sum_{j=1}^{N} \omega_j(\boldsymbol{\iota})}, \tag{2.26}$$

where fuzzy basis functions are expressed as

$$\omega_j(\boldsymbol{\iota}) = \frac{\bigotimes_{i=1}^{n} \mu_{\mathcal{I}_j^i}(\iota_i)}{\sum_{j=1}^{N} \bigotimes_{i=1}^{n} \mu_{\mathcal{I}_\ell^i}(\iota_i)}, \tag{2.27}$$

where $\boldsymbol{\iota} = (\iota_1, \iota_2, \ldots, \iota_n)^T$ is the input vector to the fuzzy logic system (FLS), y_{hj} is the representative output for rule j, $\mu_{\mathcal{I}_j^i}(\iota_i)$ is the membership function for the i-th input corresponding to rule j, and $\delta_a(u)$ is the actuator fault factor. When $\delta_a(u) = 1$, the actuator is fault-free; deviations from 1 indicate the presence and level of fault (e.g., $\delta_a(u) < 1$ may indicate actuator degradation).

Lemma 2.5. *[33] For any continuous function $\phi(\upsilon)$ defined over a compact domain $\mathcal{D}$, and for any given positive scalar δ, there exists a fuzzy approximation model representation $\hat{\phi}(\upsilon|\Theta) = \Theta^\top \varphi(\upsilon)$, such that*

$$\sup_{\upsilon\in\mathcal{D}} \left|\phi(\upsilon) - \Theta^{*\top}\varphi(\upsilon)\right| \leq \delta. \tag{2.28}$$

Accordingly, when considering actuator faults, the MAS dynamics are represented by Eq (2.5):

$$\dot{x}_i = f_i(x_i) + \varphi_i(t)u_i(t), \quad i = 1, 2, \ldots, N, \tag{2.29}$$

where $f_i(x_i)$ denotes the intrinsic dynamics of an agent i. The actual control input applied to agent i becomes $\varphi_i(t)u_i(t)$, where $u_i(t)$ is the nominal control input.

Our control objective is to design a finite-time secure control strategy for MASs (2.5) against actuator faults and spoofing attacks. The control scheme proposed operates so that all system signals remain semi-globally uniformly bounded, and the output y_i of each agent converges to the reference trajectory y_r within a finite time. Furthermore, the proposed approach effectively avoids the singularity issues typically associated with certain classes of finite-time controllers.

3. Major results

In this section, a novel decentralized adaptive fuzzy security control scheme is proposed for the i-th subsystem, which employs an adaptive backstepping technique to guarantee the stability of the system (2.5). This scheme addresses communication channels that are susceptible to deception attacks and actuator faults. For the i-th subsystem, the control design process can be divided into m steps, where each step is based on coordinate transformation and filters as follows:

$$\begin{cases} e_{i,1} = \sum\limits_{j=1,\, j\neq i}^{M} a_{ij}(y_i - y_j) + b_i(y_i - y_r) + \phi_i(t)b_i(y_i - y_r) + \delta_i(t), \\ e_{i,k} = x_{i,k} - r_{i,k}, \\ w_{i,k} = r_{i,k} - \phi_i(t)\alpha_{i,k-1} - \delta_i(t), \end{cases} \tag{3.1}$$

where $e_{i,k}$ denotes the tracking error, while $\alpha_{i,k-1}$ and $r_{i,k}$ represent the intermediate control law and the associated filtered control input for the i-th subsystem, respectively. The term $\phi_i(t) \in [0, 1]$ characterizes a multiplicative fault that captures the effect of gain reduction, and $\delta_i(t)$ denotes an additive fault that accounts for actuator bias.

The error regulation function is adapted to account for the presence of actuator faults, affecting the error decay function:

$$\bar{h}_{i,k}(e_{i,k}) = \begin{cases} \mathrm{sgn}(e_{i,k}), & \text{if } |e_{i,k}| \geq \sigma_{i,k}(t), \\ \dfrac{e_{i,k}^3}{(e_{i,k}^2 - \sigma_{i,k}^2)^2 + |e_{i,k}^3|}, & \text{otherwise}, \end{cases} \tag{3.2}$$

$$h_{i,k}(e_{i,k}) = \begin{cases} 1, & \text{if } |e_{i,k}| \geq \sigma_{i,k}(t), \\ 0, & \text{otherwise}, \end{cases} \tag{3.3}$$

where $\sigma_{i,k}(t) = \pi_{i,k}e^{-\rho_{i,k}t} + \bar{\pi}_{i,k} + \eta_{i,k}(t)$, $\pi_{i,k} > 0$ is the initial threshold magnitude,$\rho_{i,k} > 0$ is the exponential decay rate, and $\eta_{i,k}(t) = \gamma_{i,k}\left(|1 - \phi_i(t)| + |\delta_i(t)|\right)$, $\gamma_{i,k} > 0$ is a scaling factor that determines the sensitivity to actuator faults, $\phi_i(t) \in [0,1]$ represents the gain fault if $\phi_i(t) = 1$, there is no gain fault, a value less than 1 indicates a loss of effectiveness, and $\delta_i(t)$ denotes the bias fault. According to the system model shown in (2.5) and the coordinate transformation defined in (3.1), one has

$$\dot{e}_{i,1} = \sum_{\substack{j=1 \\ j\neq i}}^{M} a_{ij}(\dot{y}_i - \dot{y}_j) + b_i\dot{\varphi}_i(t)(y_i - y_r) + b_i(1+\varphi_i(t))(\dot{y}_i - \dot{y}_r) + \dot{\delta}_i(t),$$

$$\dot{e}_{i,1} = v_i\left(e_{i,2} + w_{i,2} + \alpha_{i,1} + g_i x_{i,1} + \pi_{i,1}(x_i, x_j)\right) - b_i(1+\varphi_i(t))\dot{y}_r + b_i\dot{\varphi}_i(t)(y_i - y_r) + \dot{\delta}_i(t), \tag{3.4}$$

where

$$v_i = \sum_{j\neq i} a_{ij} + b_i(1+\varphi_i(t)), \quad \alpha_{i,1} = \zeta_{i,1}(t)\theta_{i,1}(x_{i,1}), \quad \pi_{i,1}(x_i, x_j): \text{ terms involving } x_{j,2}, x_{j,1}, \ldots,$$
$$e_{i,2} = x_{i,2} - r_{i,2}, \quad w_{i,2} = r_{i,2} - \varphi_i(t)\alpha_{i,1} - \delta_i(t).$$

Design

$$\Xi_{i,1}(\chi_{i,1}) = \varphi_i^T \xi_i(\chi_{i,1}) + \psi_i(\chi_{i,1}). \tag{3.5}$$

The function $\Xi_{i,1}(\chi_{i,1})$ can be obtained by introducing a fault parameter which can be written as

$$\Xi_{i,1}(\chi_{i,1}) = \varphi_i^T \xi_i(\chi_{i,1}) + \psi_i(\chi_{i,1}) + \Delta\mathbf{F}_i(\chi_{i,1}). \tag{3.6}$$

By using a fuzzy approximation model $\Xi_{i,1}(\chi_{i,1})$, the continuous nonlinear function is approximated using a nonlinear estimator developed in Lemma 2.5.

Here, $\Delta\mathbf{F}_i(\chi_{i,1})$ denotes the combined effect of actuator gain and bias faults.

When using a FLS $\Xi_{i,1}(\chi_{i,1})$, the continuous non-linear function can be handled by a non-linear approximator designed in Lemma 2.5.

$$\Xi_{i,1}(\chi_{i,1}) = \varphi_i^T \xi_i(\chi_{i,1}) + \psi_i(\chi_{i,1}) + \mathbf{F}_{i,\text{gain}}(\chi_{i,1}) \cdot \Delta u_i + \mathbf{F}_{i,\text{bias}}(\chi_{i,1}). \tag{3.7}$$

The Lyapunov function can be designed as

$$V_{i,1} = \frac{1}{3}\left(|e_{i,1}| - \sigma_{i,1}\right)^3 h_{i,1} + \frac{1}{2}\tilde{W}_{i,1}^T \tilde{W}_{i,1} + \frac{1}{2}\zeta_i^T(t)\zeta_i(t), \tag{3.8}$$

where $\tilde{W}_{i,1} = W_{i,1}^* - \hat{W}_{i,1}$, and $\zeta_i(t)$ represents the actuator bias fault.

From Eqs (3.4)–(3.6), the time derivative of $V_{i,1}$ can be written as

$$\begin{aligned} \dot{V}_{i,1} = \left(|e_{i,1}| - \sigma_{i,1}\right)^2 h_{i,1}\hat{h}_{i,1} \cdot \Big[v_i\Big(& e_{i,2} + w_{i,2} + \alpha_{i,1}^{\text{fault}} + \vartheta_{i,1}^{*T}\xi_{i,1}(\chi_{i,1}) + \psi_{i,1} \\ &+ \dot{g}_i g_i^{-1} e_{i,1} + \pi_{i,1}\rho_{i,1}e^{-\rho_{i,1}t}\Big) + \pi_{j,1}\rho_{j,1}e^{-\rho_{j,1}t}\Big] + \tilde{W}_{i,1}^T \dot{\hat{W}}_{i,1} + \zeta_i^T(t)\dot{\zeta}_i(t), \end{aligned} \tag{3.9}$$

where the fault-influenced term $\alpha_{i,1}^{\text{fault}} = \rho_i(x_i, m_i)u_i(t) + \zeta_i(t)$ for both gain and bias faults.

Using Lemma 2.1, there exists a constant $\tau_{i,k} \geq \tau_{i,k}(g_i^{-1})$ and a known function $\gamma_{i,k}(x_{i,k})$ such that

$$\|\theta_{i,k}(x_{i,k})\| \leq \tau_{i,k}(g_i^{-1})\gamma_{i,k}(x_{i,k}) \leq \tau_{i,k}\gamma_{i,k}(x_{i,k}). \tag{3.10}$$

Since $0 < \xi_{i,1}^T(\cdot)\xi_{i,1}(\cdot) \leq 1$, applying Young's inequality for any $s > 0$ gives

$$\begin{aligned}\vartheta_{i,1}^{*T}\xi_{i,1}(\chi_{i,1}) &\leq \frac{1}{2s}\|\vartheta_{i,1}^*\|^2\|\xi_{i,1}(\chi_{i,1})\|^2 + \frac{s}{2} = \frac{W_{i,1}^*}{2s}\|\xi_{i,1}(\chi_{i,1})\|^2 + \frac{s}{2} \\ &\leq \frac{W_{i,1}^*}{4s}\|\xi_{i,1}(\chi_{i,1})\|^2 + s.\end{aligned} \tag{3.11}$$

Substituting (3.10) and (3.11) into (3.9), the upper bound of $\dot{V}_{i,1}$ becomes

$$\begin{aligned}\dot{V}_{i,1} \leq &(|e_{i,1}| - \sigma_{i,1})^2 h_{i,1}\hat{h}_{i,1} \cdot \Big[v_i\Big(\frac{1}{2}W_{i,1,1}(e_{i,1}^2 + 1) + w_{i,2} + \rho_i(x_i, m_i)u_i(t) + \zeta_i(t) \\ &+ W_{i,1,2}\gamma_{i,1}(x_{i,1}) + \pi_{i,1}\rho_{i,1} + \frac{W_{i,1}^*}{4s}\|\xi_{i,1}(\chi_{i,1})\|^2 + s\Big)\Big] \\ &+ \tilde{W}_{i,1}^T\dot{\tilde{W}}_{i,1} + \zeta_i^T(t)\dot{\zeta}_i(t),\end{aligned} \tag{3.12}$$

where $W_{i,1,1} = \dot{g}_i g_i^{-1}$ and $W_{i,1,2} = g_i\zeta_{i,1}(t)\tau_{i,1}$.

The intermediate control law $\alpha_{i,1}$ for actuator faults and unknown dynamics written as

$$\begin{aligned}\alpha_{i,1} = \frac{1}{v_i}\Big[&- c_{i,1}(|e_{i,1}| - \sigma_{i,1})^{3\omega-2}h_{i,1}\hat{h}_{i,1} - \pi_{i,1}\rho_{i,1} - c_{i,1}(|e_{i,1}| - \sigma_{i,1})h_{i,1}\hat{h}_{i,1} \\ &- (|e_{i,1}| - \sigma_{i,1})^2 h_{i,1}\hat{h}_{i,1} - \hat{h}_{i,1}\hat{W}_{i,1,1}\frac{1}{2}(e_{i,1}^2 + 1) - \hat{h}_{i,1}\hat{W}_{i,1,2}\gamma_{i,1}(x_{i,1}) \\ &- \sqrt{(1 + \sigma_{i,2})^2 + \iota_i} - \rho_i(x_i, m_i)u_i(t) - \zeta_i(t) - \frac{\hat{W}_{i,1}^*}{4s}\|\xi_{i,1}(\chi_{i,1})\|^2 - s\Big].\end{aligned} \tag{3.13}$$

The adaptive laws are defined as

$$\begin{aligned}\dot{\hat{W}}_{i,1,1} &= (|e_{i,1}| - \sigma_{i,1})^2 h_{i,1}\hat{h}_{i,1} \cdot \frac{1}{2}(e_{i,1}^2 + 1) - \eta_{i,1,1}\hat{W}_{i,1,1}, \\ \dot{\hat{W}}_{i,1,2} &= (|e_{i,1}| - \sigma_{i,1})^2 h_{i,1}\hat{h}_{i,1} \cdot \gamma_{i,1}(x_{i,1}) - \eta_{i,1,2}\hat{W}_{i,1,2}, \\ \dot{\hat{W}}_{i,1}^* &= \frac{1}{4s}\|\xi_{i,1}(\chi_{i,1})\|^2 - \rho_{i,1}\hat{W}_{i,1}^*.\end{aligned} \tag{3.14}$$

By applying Young's inequality, substituting (3.13) and (3.14) into (3.9) becomes

$$\begin{aligned}\dot{V}_{i,1} \leq &- c_{i,1}(|e_{i,1}| - \sigma_{i,1})^{3\omega}h_{i,1}^2\hat{h}_{i,1}^2 - c_{i,1}(|e_{i,1}| - \sigma_{i,1})^3 h_{i,1}^2\hat{h}_{i,1}^2 - \frac{\eta_{i,1,1}}{2}\|\tilde{W}_{i,1,1}\|^2 \\ &- \frac{\eta_{i,1,2}}{2}\|\tilde{W}_{i,1,2}\|^2 - \frac{\rho_{i,1}}{2}(\tilde{W}_{i,1}^*)^2 + \frac{1}{2\eta_{i,1,1}}\left((|e_{i,1}| - \sigma_{i,1})^2 h_{i,1}\hat{h}_{i,1} \cdot \frac{1}{2}(e_{i,1}^2 + 1)\right)^2 \\ &+ \frac{1}{2\eta_{i,1,2}}\left((|e_{i,1}| - \sigma_{i,1})^2 h_{i,1}\hat{h}_{i,1} \cdot \gamma_{i,1}(x_{i,1})\right)^2 + \frac{1}{2\rho_{i,1}}\left(\frac{1}{4s}\|\xi_{i,1}(\chi_{i,1})\|^2\right)^2 + \zeta_i^T(t)\dot{\zeta}_i(t).\end{aligned} \tag{3.15}$$

Step k. $(2 \leq m \leq m_i - 1)$ The time derivative of $e_{i,k}$ is computed along with the corresponding dynamics (3.1) are rewritten as

$$e_{i,k} = \dot{x}_{i,k} - \dot{r}_{i,k} = e_{i,k+1} + (\alpha_{i,k} + \Delta\alpha_i) + w_{i,k+1} + (g_i + \hat{\Delta}g_i)^{-1}\dot{x}_{i,k} + (g_i + \hat{\Delta}g_i)\zeta_{i,k}\theta_{i,k}(\dot{x}_{i,k}) + (b_i + \hat{\Delta}b_i) - \dot{r}_{i,k}. \tag{3.16}$$

The adaptation laws for fault are written as

$$\dot{\hat{\Delta}}g_i = -\gamma_g e_{i,k}\dot{x}_{i,k}, \quad \dot{\hat{\Delta}}b_i = -\gamma_b e_{i,k}. \tag{3.17}$$

The first-order filter is written as

$$\dot{B}_{i,k}\dot{r}_{i,k} + r_{i,k} = \alpha_{i,k-1} + \hat{\Delta\alpha}_i, \quad r_{i,k}(0) = \alpha_{i,k-1}(0). \tag{3.18}$$

According to (3.18), we have

$$\dot{w}_{i,k} = -\frac{w_{i,k}}{B_{i,k}} - \dot{\alpha}_{i,k-1} + \hat{\Delta\alpha}_i. \tag{3.19}$$

The control input with fault compensation is

$$u_i = (g_i + \hat{\Delta g}_i)\dot{x}_{i,k} + (b_i + \hat{\Delta b}_i). \tag{3.20}$$

The Lyapunov function can be written as

$$V_{i,k} = \frac{1}{3}(\|e_{i,k}\| - \sigma_{i,k})^3 h_{i,k} + \frac{1}{2}\tilde{W}_{i,1}^2 + \frac{1}{2}w_{i,k}^2 + \frac{1}{2}\tilde{\rho}_i^2 + \frac{1}{2}\tilde{\zeta}_i^2, \tag{3.21}$$

where $\tilde{\rho}_i = \hat{\rho}_i - \rho_i$ is the estimation error of the gain fault and $\tilde{\zeta}_i = \hat{\zeta}_i - \zeta_i$ is the estimation error of the bias fault.

On the other hand, the derivative for time of $v_{i,k}$ can be expressed as

$$\begin{aligned}\dot{V}_{i,k} \le & (\|e_{i,k}\| - \sigma_{i,k})^2 h_{i,k}\hat{h}_{i,k} W_{i,k,1}\left(\tilde{\rho}_i(x_{i,m_i})u_{i,k} + \tilde{\zeta}_i(t)\right) + \tilde{W}_{i,k}^{\top}\dot{\hat{W}}_{i,k} \\ & + w_{i,k}\dot{w}_{i,k} + (\|e_{i,k}\| - \sigma_{i,k})^2 f_{i,k}\sqrt{e_{i,k+1}^2 + \iota_i} + \tilde{\rho}_i\dot{\hat{\rho}}_i + \tilde{\zeta}_i\dot{\hat{\zeta}}_i.\end{aligned} \tag{3.22}$$

The intermediate control function $\alpha_{i,k}$ and adaptive laws $\dot{\hat{W}}_{i,k}$ are designed as follows:

$$\begin{aligned}\alpha_{i,k} = & - c_{i,k}\left(\|e_{i,k}\| - \sigma_{i,k}\right)^{3\omega-2} h_{i,k}\hat{h}_{i,k} - \pi_{i,k}\rho_{i,k} - c_{i,k}\left(\|e_{i,k}\| - \sigma_{i,k}\right) h_{i,k}\hat{h}_{i,k} \\ & - \left(\|e_{i,k}\| - \sigma_{i,k}\right)^2 h_{i,k}\hat{h}_{i,k} - \frac{1}{2}h_{i,k}\hat{h}_{i,k}\hat{W}_{i,k,1}(x_{i,k}^2 + 1) - \hat{h}_{i,k}\hat{W}_{i,k,2}\gamma_{i,k}(x_{i,k}) \\ & - \sqrt{(1 + \sigma_{i,k})^2 + \iota_i} + \nu_{i,k} + \hat{h}_{i,k}\left(\frac{1 - \rho_i(x_i, m_i)}{\rho_i(x_i, m_i)}\bar{\alpha}_{i,k} + \frac{1}{\rho_i(x_i, m_i)}\zeta_i(t)\right),\end{aligned} \tag{3.23}$$

$$\dot{\hat{W}}_{i,k,1} = \left(\|e_{i,k}\| - \sigma_{i,k}\right)^2 h_{i,k}\hat{h}_{i,k} \cdot \frac{1}{2}(x_{i,k}^2 + 1) - \eta_{i,k,1}\hat{W}_{i,k,1}, \tag{3.24}$$

$$\dot{\hat{W}}_{i,k,2} = \left(\|e_{i,k}\| - \sigma_{i,k}\right)^2 h_{i,k}\hat{h}_{i,k} \cdot \gamma_{i,k}(x_{i,k}) - \eta_{i,k,2}\hat{W}_{i,k,2}. \tag{3.25}$$

Therefore, it yields

$$\begin{aligned}\dot{V}_{i,k} \le & - c_{i,k}\left(\|e_{i,k}\| - \sigma_{i,k}\right)^{3\omega} h_{i,k}^2\hat{h}_{i,k}^2 - c_{i,k}\left(\|e_{i,k}\| - \sigma_{i,k}\right)^3 h_{i,k}^2\hat{h}_{i,k}^2 \\ & + \left(\|e_{i,k}\| - \sigma_{i,k}\right)^2 h_{i,k}\sqrt{e_{i,k+1}^2 + \iota_i} - \left(\|e_{i,k}\| - \sigma_{i,k}\right)^2 h_{i,k}\sqrt{(1 + \sigma_{i,k+1})^2 + \iota_i} \\ & + \frac{1}{4}w_{i,k+1}^2 + \frac{1}{4} - \left(\frac{1}{B_{i,k}} - N_{i,k}^2\right)w_{i,k}^2 - \frac{\eta_{i,k}}{2}\tilde{W}_{i,k}^T\tilde{W}_{i,k} + \frac{\eta_{i,k}}{2}\left\|W_{i,k}^*\right\|^2 \\ & + \left(\|e_{i,k}\| - \sigma_{i,k}\right) h_{i,k}\left|\frac{1}{\rho_i(x_i, m_i)}\zeta_i(t)\right|.\end{aligned} \tag{3.26}$$

Step m_i. Assume that the controller-to-sensor communication channels are compromised by deception attacks and that the actuators experience faults caused by gain-and-bias uncertainties. Based on these conditions, the secure control strategy is developed as follows:

$$\dot{e}_{i,m_i} = dg_i\left(\rho_i(x_{i,m_i})u_i(t) + \zeta_i(t)\right) + g_i Q(x_{i,m_i}) + \dot{g}_i g_i^{-1} x_{i,m_i} + g_i \zeta_{i,m_i}(t)\theta_{i,m_i}(x_{i,m_i}). \tag{3.27}$$

Consider the following Lyapunov function:

$$V_{i,m_i} = \frac{1}{3}\left(\|e_{i,m_i}\| - \sigma_{i,m_i}\right)^3 h_{i,m_i} + \frac{1}{2}\tilde{W}_{i,m_i}^2 + \frac{1}{2}w_{i,m_i}^2 + \frac{1}{2}\tilde{\Upsilon}_{i,m_i}^2 + \frac{1}{2\gamma_{g_i}}\tilde{\Delta}_{g_i}^2 + \frac{1}{2\gamma_{b_i}}\tilde{\Delta}_{b_i}^2. \tag{3.28}$$

The derivative for time of Eq (3.28) can be expressed as

$$\begin{aligned}\dot{V}_{i,m_i} \le{}& \left(\|e_{i,m_i}\| - \sigma_{i,m_i}\right)^2 h_{i,m_i}\hat{h}_{i,m_i} \times \Big(dg_i(1+\Delta_{g_i})u_i + dg_i\Delta_{b_i} + \widehat{\Pi}_i^\top \phi_i + \varsigma_i + \dot{g}_i g_i^{-1} x_{i,m_i} \\ &+ g_i\zeta_{i,m_i}(t)\theta_{i,m_i}(x_{i,m_i}) - \dot{r}_{i,m_i-1} + \pi_{i,m_i}\rho_{i,m_i}e^{-\rho_{i,m_i}t}\Big) \\ &+ \tilde{W}_{i,m_i}^\top \dot{\tilde{W}}_{i,m_i} + \tilde{\Upsilon}_{i,m_i}^\top \dot{\tilde{\Upsilon}}_{i,m_i} + w_{i,m_i}\dot{w}_{i,m_i} + \frac{1}{\gamma_{g_i}}\tilde{\Delta}_{g_i}\dot{\tilde{\Delta}}_{g_i} + \frac{1}{\gamma_{b_i}}\tilde{\Delta}_{b_i}\dot{\tilde{\Delta}}_{b_i}.\end{aligned} \tag{3.29}$$

The intermediate control signal u_i can be defined as follows for actuator faults:

$$u_i = \frac{1}{\hat{\rho}_i(x_i, m_i)}\begin{bmatrix} N_i(\kappa_i)\Big(- c_{i,m_i}(\|e_{i,m_i}\| - \sigma_{i,m_i})^{3\omega-2} h_{i,m_i}\hat{h}_{i,m_i} - \pi_{i,m_i}\rho_{i,m_i} \\ \quad - c_{i,m_i}(\|e_{i,m_i}\| - \sigma_{i,m_i}) h_{i,m_i}\hat{h}_{i,m_i} \\ \quad - (\|e_{i,m_i}\| - \sigma_{i,m_i})^2 h_{i,m_i}\hat{h}_{i,m_i} - \frac{1}{2}\hat{h}_{i,m_i}\hat{W}_{i,m_i,1}(x_{i,m_i}^2 + 1) \\ \quad - \hat{h}_{i,m_i}\hat{W}_{i,m_i,2}\gamma_{i,m_i}(x_{i,m_i}) - \hat{\Pi}_i^{*T}\phi_i + \dot{v}_{i,m_i-1}\Big) - \dot{\hat{\zeta}}_i(t) \end{bmatrix}. \tag{3.30}$$

The adaptive law can be written as

$$\dot{\hat{W}}_{i,m_i,1} = \left(\left\|z_{i,m_i}\right\| - \sigma_{i,m_i}\right)^{2h_{i,m_i}} \cdot \hat{h}_{i,m_i} \cdot \frac{1}{2}\left(x_{i,m_i}^2 + 1\right) \cdot \rho_i(x_{i,m_i}) - \eta_{i,m_i,1}\hat{W}_{i,m_i,1}, \tag{3.31}$$

$$\dot{\hat{W}}_{i,m_i,2} = \left(\left\|z_{i,m_i}\right\| - \sigma_{i,m_i}\right)^{2h_{i,m_i}} \cdot \hat{h}_{i,m_i} \cdot \gamma_i(x_{i,m_i}) \cdot \rho_i(x_{i,m_i}) - \eta_{i,m_i,2}\hat{W}_{i,m_i,2}, \tag{3.32}$$

$$\dot{\hat{W}}_{i,m_i} = \left(\left\|z_{i,m_i}\right\| - \sigma_{i,m_i}\right)^2 h_{i,m_i} \cdot \hat{h}_{i,m_i} \cdot \gamma_i(x_{i,m_i}) \cdot \rho_i(x_{i,m_i}) - \xi_i\hat{W}_i, \tag{3.33}$$

$$\dot{\hat{\zeta}}_i = \lambda_{\zeta_i} \cdot \phi_i(x_{i,m_i}) \cdot s_i(t), \tag{3.34}$$

where $\lambda_{\zeta_i} > 0$ is the adaptation gain, $\phi_i(x_{i,m_i})$ is a known basis function, and $s_i(t)$ is a stabilizing error signal.

The Nussbaum function is defined as

$$N_i(\kappa_i) = e^{\kappa_i^2}\cos(\pi\kappa_i), \tag{3.35}$$

$$\begin{aligned}\dot{\kappa}_i ={}& - c_{i,m_i}\left(\|e_{i,m_i}\| - \sigma_{i,m_i}\right)^{3\omega-2} h_{i,m_i}\hat{h}_{i,m_i} - \pi_{i,m_i}\rho_{i,m_i} - c_{i,m_i}\left(\|e_{i,m_i}\| - \sigma_{i,m_i}\right) h_{i,m_i}\hat{h}_{i,m_i} - \left(\|e_{i,m_i}\| - \sigma_{i,m_i}\right)^{2h_{i,m_i}} \bar{h}_{i,m_i} \\ &- \frac{1}{2}\hat{h}_{i,m_i}\hat{W}_{i,m_i,1}\left(x_{i,m_i}^2 + 1\right)\rho_i(x_{i,m_i}) - \hat{h}_{i,m_i}\hat{W}_{i,m_i,2}\gamma_i(x_{i,m_i})\rho_i(x_{i,m_i}) - \dot{\hat{\zeta}}_i - \hat{\Pi}_i^{*T}\phi_i + \dot{v}_{i,m}.\end{aligned} \tag{3.36}$$

Using Young's inequality, we have

$$\tilde{\xi}_i^T M_i \hat{M}_i \leq -\frac{1}{2}\tilde{\xi}_i^T M_i \tilde{\xi}_i + \frac{1}{2}||M_i^*||^2 + \frac{1}{2}||\rho_i(x_{i,mi}) - 1||^2||u_i(t)||^2 + \frac{1}{2}||\zeta_i(t)||^2. \tag{3.37}$$

In summary, we can obtain

$$\begin{aligned}\dot{V}_{i,mi} \leq & - c_{i,mi}\left(||e_{i,mi}|| - \sigma_{i,mi}\right)^{3\omega} h_{i,mi}^2 \hat{h}_{i,mi}^2 - c_{i,mi}\left(||e_{i,mi}|| - \sigma_{i,mi}\right)^3 h_{i,mi}^2 \hat{h}_{i,mi}^2 \\ & - \left(\frac{1}{B_{i,mi}} - N_{i,mi}^2\right) w_{i,mi}^2 - \frac{\mu_i}{2}\tilde{\Pi}_{i,mi}^T \tilde{\Pi}_{i,mi} - \frac{\eta_{i,mi}}{2}\tilde{W}_{i,mi}^T \tilde{W}_{i,mi} \\ & + \left(dg_i N(\kappa_i) - 1\right)\dot{\kappa}_i + \frac{\eta_{i,mi}}{2}||W_{i,mi}^*||^2 + \frac{\mu_{i,mi}}{2}||\Pi_{i,mi}^*||^2 + \frac{1}{4}(\omega_i^{*2} + 1) \\ & + \frac{1}{2}||\rho_i(x_{i,mi}) - 1||^2||u_i(t)||^2 + \frac{1}{2}||\zeta_i(t)||^2.\end{aligned} \tag{3.38}$$

The selection of control parameters in the proposed algorithm is given as:

- $(\sigma_{i,k}, \eta_{i,k})$ should be selected as positive constants to balance learning speed and robustness. They can be tuned through simulation-based trial and error.
- $(c_k,\ \lambda_i,\ \mu)$ must satisfy the conditions derived from the Lyapunov analysis to ensure finite-time convergence. These are chosen to be positive and increase convergence speed.
- Nussbum function $\Psi(\kappa)$ typically adopts a standard form such as $\Psi(\kappa) = e^{\kappa^2}\cos(\pi\kappa)$, and no tuning is required.
- The parameters (e.g., number of membership functions, center, and widths) are chosen relative to the range of state variables. Three to five Gaussian functions per dimension are usually accurate enough.
- Backstepping sliding gains must be tuned in a way to overcome the uncertainties and make the errors converge quickly without chattering.

4. System stability

Theorem 4.1. *Consider the design of a secure controller that guarantees convergence within a finite time (3.13), (3.23), and (3.30), and the adaptive laws are designed as (3.14), (3.24), (3.25), and (3.31) to (3.34), incorporating compensation for actuator faults. In that case, the proposed secure control method can ensure that all signals of the closed-loop system are semiglobally uniformly bounded for the nonlinear MASs (2.5) in this paper, even in the presence of actuator faults.*

Proof. To analyze the stability of the complete multi-agent system, we construct a Lyapunov function defined as

$$V = \sum_{i=1}^{M}\sum_{k=1}^{m_i}\left(\frac{1}{2}e_{i,k}^2 + \frac{1}{2\eta_{i,k}}\tilde{W}_{i,k}^\top \tilde{W}_{i,k} + \frac{1}{2\mu_i}\tilde{\Pi}_i^\top \tilde{\Pi}_i\right) + \frac{b_i^2(\rho_i - 1)^2}{2\hat{h}_{i,m_i}^2}e_{i,m_i}^2 + \frac{b_i^2}{2\hat{\bar{h}}_{i,m_i}^2}\zeta_i^2. \tag{4.1}$$

The time derivative (4.1) can be expressed as

$$\dot{V} \leq \sum_{i=1}^{M}\left\{\sum_{k=1}^{m_i}\left(-c_{i,k}||e_{i,k} - \sigma_{i,k}||^{3\beta} h_{i,k}^2 \hat{h}_{i,k}^2 - c_{i,k}||e_{i,k} - \sigma_{i,k}||^3 h_{i,k}^2 \hat{h}_{i,k}^2 - \frac{\eta_{i,k}}{2}\tilde{W}_{i,k}^\top \tilde{W}_{i,k}\right)\right. \tag{4.2}$$

$$+\sum_{k=1}^{m_i-1}\left(\|e_{i,k}-\sigma_{i,k}\|^2 h_{i,m}\left(\frac{1}{\sqrt{e_{i,k+1}^2+\iota_i}}-\frac{1}{\sqrt{(1+\sigma_{i,k+1})^2+\iota_i}}\right)\right)$$
$$-\sum_{k=2}^{m_i}\left(\left(\frac{1}{B_{i,k}}-N_{i,k}^2-\frac{1}{4}\right)w_{i,k}^2\right)+(dg_iN_i(\chi_i)-1)\dot{\chi}_i-\frac{\mu_i}{2}\tilde{\Pi}_i^{\top}\tilde{\Pi}_i+\sum_{k=1}^{m_i}\frac{\eta_{i,k}}{2}\|W_{i,k}^*\|^2$$
$$+\frac{\mu_i}{2}\|\Pi_i\|^2+\frac{1}{4}\omega_i^{*2}+\frac{b_i^2(\rho_i-1)^2}{2\hat{h}_{i,m_i}^2}e_{i,m_i}^2+\frac{1}{2\hat{h}_{i,m_i}^2}u_i^2+\frac{b_i^2}{2\hat{h}_{i,m_i}^2}\zeta_i^2+\frac{1}{2\hat{h}_{i,m_i}^2}e_{i,m_i}^2\Bigg\}+\frac{N-1}{4}.$$

The following inequality is derived by using Young's inequality:

$$\left(\left|\rho_i(x_i,m_i)(e_{i,k}-\sigma_{i,k})\right|\right)^2 h_{i,k}\left(\sqrt{\rho_i^2(x_i,m_i)e_{i,k+1}^2+\iota_i}-\sqrt{\rho_i^2(x_i,m_i)(1+\sigma_{i,k+1})^2+\iota_i}\right)$$
$$\leq\frac{2}{3}\left(\left|\rho_i(x_i,m_i)(e_{i,k}-\sigma_{i,k})\right|\right)^3 h_{i,k}+\frac{1}{3}\left(\left|\rho_i(x_i,m_i)(e_{i,k+1}-\sigma_{i,k+1})\right|\right)^3 h_{i,k+1}. \tag{4.3}$$

According to Lemma 2.4, suppose that $\lambda=\frac{\omega^{\omega}}{1-\omega}$, we have

$$\left(\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(\tilde{W}_{i,k}^f)^T\tilde{W}_{i,k}^f\right)^{\omega}\leq(1-\omega)\lambda+\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(\tilde{W}_{i,k}^f)^T\tilde{W}_{i,k}^f, \tag{4.4}$$

$$\left(\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(w_{i,k}^f)^2\right)^{\omega}\leq(1-\omega)\lambda+\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(w_{i,k}^f)^2, \tag{4.5}$$

$$\left(\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(\tilde{\Pi}_{i,k}^f)^T\tilde{\Pi}_{i,k}^f\right)^{\omega}\leq(1-\omega)\lambda+\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(\tilde{\Pi}_{i,k}^f)^T\tilde{\Pi}_{i,k}^f. \tag{4.6}$$

Accordingly, we have

$$\dot{V}\leq\sum_{i=1}^{M}\left(\sum_{k=1}^{m_i}\left[-c_{i,k}h_{i,k}^2\hat{h}_{i,k}^2\|e_{i,k}-\sigma_{i,k}\|^{\beta}-\frac{1}{2}(\eta_{i,k}-1)(\tilde{W}_{i,k}^f)^{\top}\tilde{W}_{i,k}^f\right]\right.$$
$$-\sum_{k=2}^{m_i}\frac{1}{2}\left(2B_{i,k}-2N_{i,k}^2-\frac{3}{2}\right)(w_{i,k}^f)^2-\frac{1}{2}(\mu_i-1)(\tilde{\Pi}_{i,k}^f)^{\top}\tilde{\Pi}_{i,k}^f+\sum_{k=1}^{m_i}\frac{\eta_{i,k}}{2}\|W_{i,k}^*\|^2$$
$$\left.+\frac{\mu_i}{2}\|\Pi_i\|^2+\frac{1}{4}(\omega_i^*)^2+(d_{g_i}N_i(\chi_i)-1)\dot{\chi}_i+\frac{1}{2}\|\rho_i(x_i,m_i)-1\|^2\|u_i\|^2+\frac{1}{2}\|\zeta_i(t)\|^2\right) \tag{4.7}$$
$$-\left(\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(\tilde{W}_{i,k}^f)^{\top}\tilde{W}_{i,k}^f\right)^{\omega}-\sum_{i=1}^{M}\sum_{k=1}^{m_i}c_{i,k}h_{i,k}^2\hat{h}_{i,k}^2\|e_{i,k}-\sigma_{i,k}\|^{\beta}$$
$$-\left(\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(w_{i,k}^f)^2\right)^{\omega}-\left(\sum_{i=1}^{M}\sum_{k=1}^{m_i}\frac{1}{2}(\tilde{\Pi}_{i,k}^f)^{\top}\tilde{\Pi}_{i,k}^f\right)^{\omega}+3(1-\omega)\lambda+\frac{N-1}{4},$$
$$\dot{V}\leq-C_1V-C_2V^{\omega}+D,$$

where

$$C_1=\min\left\{\min_{i,k}\{c_{i,k}\},\min_{i,k}\{2\eta_{i,k}-1\},\min_{i}\{\mu_i-1\},\min_{i,k\geq2}\left(\frac{2}{B_{i,k}}-2N_{i,k}^2-\frac{3}{2}\right)\right\},$$

$$C_2 = \min_{i,k} \{c_{i,k}\},$$

$$D = \sum_{i=1}^{M} \sum_{k=1}^{m_i} \left(\frac{\eta_{i,k}}{2} \left\| W_{i,k}^{*} \right\|^2 \right) + \sum_{i=1}^{M} \left(\frac{\mu_i}{2} \|\Pi_i\|^2 + \frac{1}{4} \omega_i^{*2} \right) + \sum_{i=1}^{M} \left(\frac{1}{2} \|\rho_i(x_i, m_i) - 1\|^2 \|u_i\|^2 + \frac{1}{2} \|\zeta_i(t)\|^2 \right)$$
$$+ \max_i \left\{ (d_{g_i} N_i(\chi_i) - 1)\dot{\chi}_i \right\} + \frac{N-1}{4} + 3(1-\omega)\lambda.$$

Remark 4.1. *Parameter D in Eq (4.7) reflects the upper bound of uncertainties, fuzzy approximation errors, and fault-related residual terms in the Lyapunov analysis. To minimize D, one can increase the number of fuzzy rules to enhance approximation accuracy, properly tune the adaptive gains to speed up convergence, and impose tighter bounds on disturbances and fault terms. A smaller D leads to a tighter bound on the tracking error and improves control accuracy. However, overly aggressive reduction of D may result in higher control effort or reduced robustness.*

Remark 4.2. *All closed-loop signals are shown to be semi-globally uniformly ultimately bounded (SGUUB) using Lyapunov analysis, and the tracking errors approach a small neighborhood of zero in finite time. Thus, the suggested control approach rather guarantees practical finite-time stability than precise convergence to the origin.*

5. Simulation example

In this section, we illustrate the effectiveness of the proposed sliding mode control method in actuator faults and deception attacks through numerical simulation studies. Consider a MAS consisting of one leader and four followers. The dynamics of each agent $i = 1, \ldots, 5$ are given by

$$\begin{cases} \dot{x}_{i,1} = f_{i,1}(x_{i,1}) + x_{i,2} \\ \dot{x}_{i,2} = f_{i,2}(x_{i,2}) + u_i^f(t) \\ y_i = x_{i,1}, \end{cases} \tag{5.1}$$

where $f_{i,1}(x_{i,1}) = x_{i,1}^2 \sin(3x_{i,1})$, $f_{i,2}(x_{i,2}) = x_{i,1}\cos(x_{i,2})$, the leader's output is $y_r(t) = 0.5\cos(0.5t)$, $\rho_i(x_i, m_i) = 1 + 0.3\sin(x_{i,1} + m_i t)$, and $\zeta_i(t) = 0.15\cos(1.5t)$, and the simulation parameters are $\lambda_i = 2.5$, $k_i = 5$, $\gamma_{\rho i} = 0.2$, $\gamma_{\zeta i} = 0.1$, and $\gamma_{Hi,m} = 0.3$ with states $x_{i,1}, x_{i,2} \in [-5, 5]$. Control parameter values are represented in the following:

$$\text{Simulation parameters} = \begin{bmatrix} \lambda_i & = & 3.0 \\ \mu & = & 2.0 \\ \eta_{i,k} & = & 5.0 \\ \gamma_{i,k}(x_i) & = & 1.0 \text{ (constant)} \\ \text{Number of fuzzy rules} & = & 5 \\ \text{Membership function type} & = & \text{Gaussian} \\ \text{Nussbaum function} & = & \Psi(\kappa) = e^{\kappa^2}\cos(\pi\kappa) \\ \text{Step size} & = & 0.001 \text{ s} \end{bmatrix}.$$

The controller and adaptive laws are designed as shown in Eqs (3.9)–(3.11) and (3.27). Deception attacks affecting the communication channel are given as

$$\Gamma_i(t) = \begin{cases} 0.1\sin(t)\cos(2t), & 10 < t \le 18, \\ 0, & \text{otherwise}. \end{cases} \tag{5.2}$$

The FLS is constructed using IF-THEN rules to approximate nonlinear functions $f_{i,m}(x_i)$, $m = 1, 2$. Define Gaussian membership functions for the fuzzy sets $\mathcal{M}_l^m$ as

$$\mu_l^m(x_{i,m}) = \exp\left(-\frac{(x_{i,m} - c_l)^2}{2\sigma^2}\right), \quad l = 1, \ldots, 11, \tag{5.3}$$

$c_l = \{-5, -4, \ldots, 5\}$ and $\sigma = 1$.

The fuzzy IF-THEN rules for each l are

$$\textit{If} \;\; x_{i,1} \text{ is } \mathcal{I}_j^1 \;\; \textit{and} \;\; x_{i,2} \text{ is } \mathcal{I}_j^2, \;\; \textit{then} \;\; f_{i,m}(x_i) \text{ is } D_j^m, \quad \text{for } j = 1, \ldots, 11, m = 1, 2.$$

Using the fuzzy rules, approximate $f_{i,m}(x_i)$ as

$$\hat{f}_{i,m}(x_i) = H_{i,m}^\top \eta_{i,m}(x_i), \tag{5.4}$$

where $\eta_{i,m}(x_i) = [\mu_1^1(x_{i,1})\mu_1^2(x_{i,2}), \ldots, \mu_{11}^1(x_{i,1})\mu_{11}^2(x_{i,2})]^\top$ and $H_{i,m}$ are adjustable parameter vectors.

Define the tracking error and sliding surface as

$$e_i = x_{i,1} - y_r(t), \quad s_i = \dot{e}_i + \lambda_i e_i = x_{i,2} - \dot{y}_r + \lambda_i(x_{i,1} - y_r), \tag{5.5}$$

where $\lambda_i > 0$ is a design parameter.

The sliding mode control law for actuator faults is designed using Definition 2.2:

$$u_i = \frac{1}{\hat{\rho}_i}\left(-\hat{f}_{i,2}(x_i) - \lambda_i\left(\hat{f}_{i,1}(x_i) + x_{i,2} - \dot{y}_r\right) - \hat{\zeta}_i - k_i \cdot \frac{s_i}{1 + |s_i|}\right), \tag{5.6}$$

where $k_i > 0$ is the sliding gain, and $\hat{\rho}_i$, $\hat{\zeta}_i$ are adaptive estimates of the fault parameters.

The adaptive laws to estimate the actuator fault parameters and fuzzy parameters are

$$\begin{cases} \dot{\hat{\rho}}_i = \gamma_{\rho i} s_i u_i, \\ \dot{\hat{\zeta}}_i = \gamma_{\zeta i} s_i, \\ \dot{H}_{i,m} = \gamma_{Hi,m} s_i \eta_{i,m}(x_i), \end{cases} \tag{5.7}$$

where $\gamma_{\rho i}, \gamma_{\zeta i}, \gamma_{Hi,m} > 0$ are adaptation gains.

Quantitative analysis and comparisons with existing work are given in Table 2:

- **Settling time (ST):** Time required for the tracking error to converge within a small neighborhood of the reference signal.
- **Steady-state error (SSE):** Maximum absolute tracking error after convergence.
- **Control effort (CE):** Norm of control input over time.

Table 2. Quantitative comparison between the proposed method and existing work.

Method	Settling time (s)	Steady-state error (SSE)	Control effort (CE)
Proposed method	6.2	0.078	12.5
Li & Wang (2020)	10.0	0.120	18.7

In Figure 3, Gaussian membership functions are used in the fuzzy logic system to approximate unknown nonlinearities. The performance of the two methods is shown in tracking in Figure 4. The proposed controller has a quicker convergence compared to the benchmark method, along with a smaller steady-state error. The control input signals can be seen in Figure 5, and it can be observed that the proposed controller produces smoother and more efficient inputs.

Figure 6 gives the comparison of the actual control inputs when actuator faults occur, which proves that the proposed method has superior robustness and fault compensation ability. Comparing Figure 6 with Figure 8 in [25], the consensus behavior among faulty follower agents shows noticeable transient fluctuations and a relatively slow synchronization process. While the original control protocol leads to consensus, it exhibits higher control activity and variability across agents during the initial period. However, in our simulation, we see a much more aligned and faster converging behavior, with convergence among all four followers not taking long to occur. This enhancement is due to the incorporation of an efficient though simplified consensus control technique, which directly combines the effect of actuator faults by estimating gain and bias parameters. The method decreases the oscillation of control and minimizes the deviation among the agents, and provides a more robust and stable formation response to adverse fault conditions.

Comparing Figure 5 with Figure 9 in [25] illustrates consensus among healthy followers, but displays an initial divergence and slower coordination rate. In comparison, our fault-free control design achieves rapid and tightly grouped convergence of all follower states. Moreover, the produced control inputs are smooth and bounded, denoting enhanced efficiency and efficiency of the control, as well as appropriateness of the control implementation in the real world. Overall, our strategy leads to faster consensus, lower control effort, and more consistent cooperative behavior across agents compared to the results shown in [25].

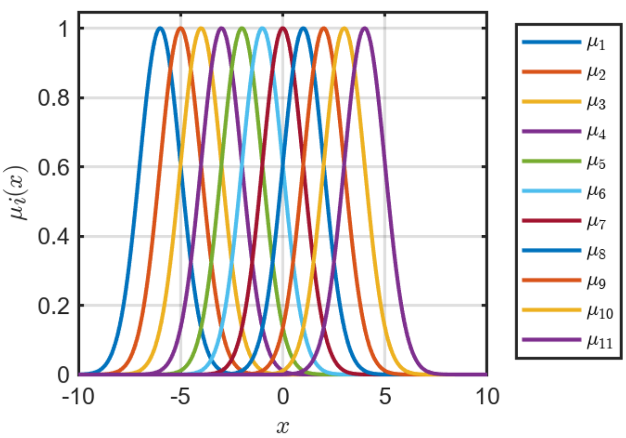

Figure 3. Gaussian membership functions.

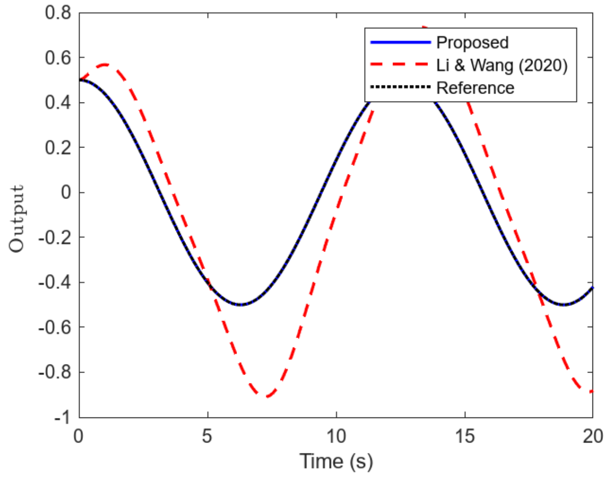

Figure 4. Tracking performance of the proposed method vs. Li & Wang (2020).

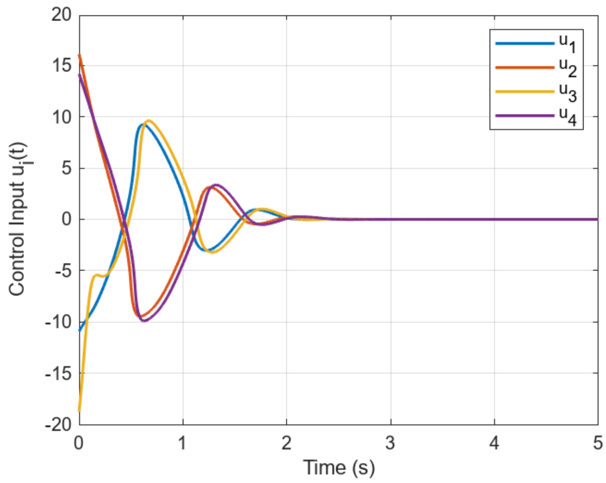

Figure 5. Control input without actuator fault.

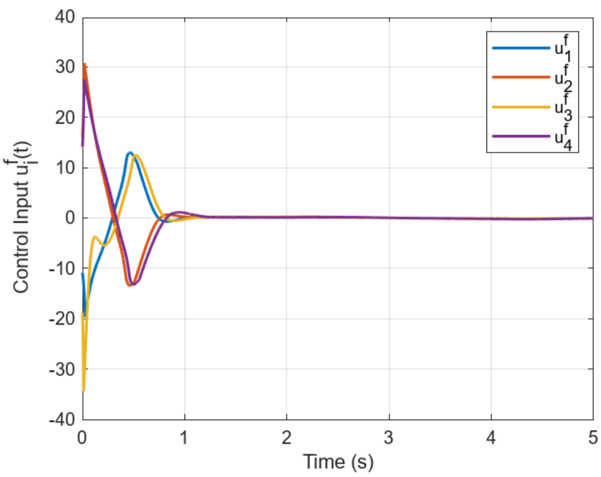

Figure 6. Control input u_i^f with actuator fault.

Figures 7 and 8 demonstrate that gain and bias faults are bounded in nature, developing and fading over time. Between t= 10s and t= 18s, Figure 9 shows the period of deception attacks.

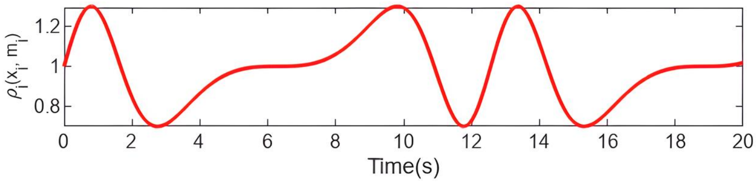

Figure 7. Gain fault over time.

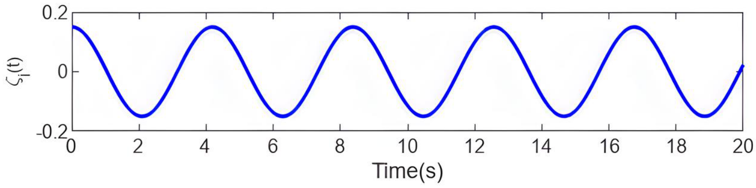

Figure 8. Bias fault over time.

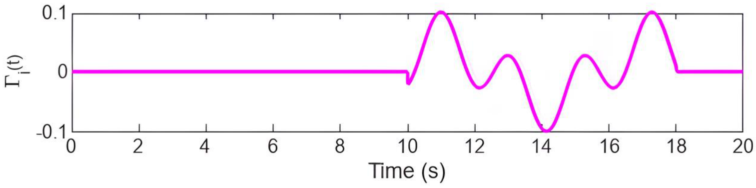

Figure 9. Deception attack over time.

From Figures 10 and 11 that the fuzzy models are very close to the true nonlinear functions everywhere. Figures 12 and 13 demonstrate that the system remains stable, as the sliding surfaces reach zero without attacks or faults present.

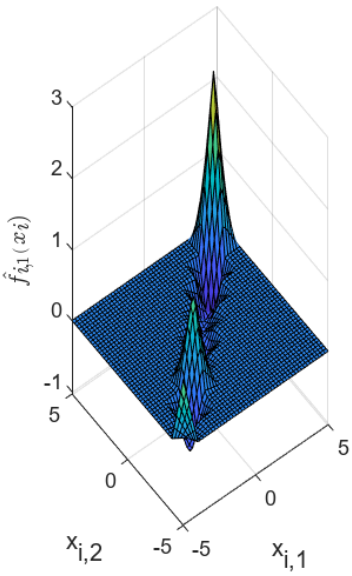

Figure 10. Fuzzy approximation of $\hat{f_i, 1}(x_i)$.

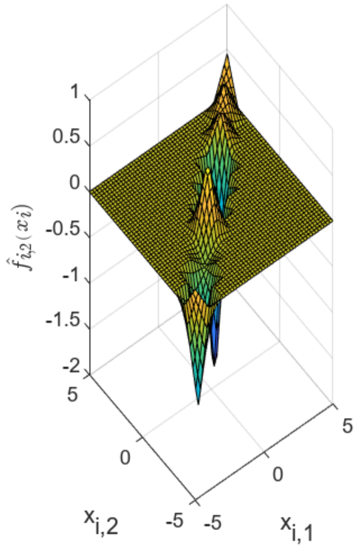

Figure 11. Fuzzy approximation of $f_i,\hat{}2(x_i)$.

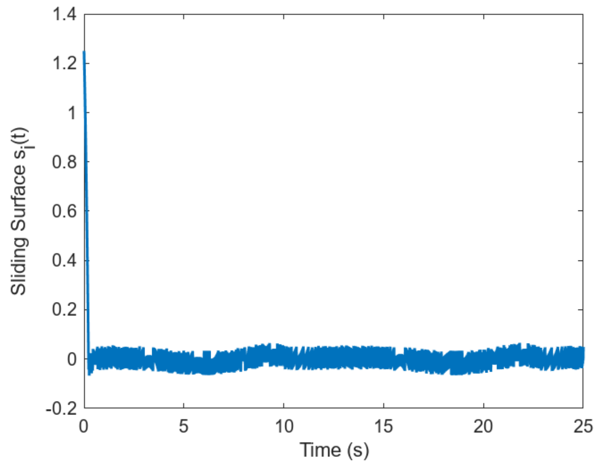

Figure 12. Sliding surface without attack and actuator fault.

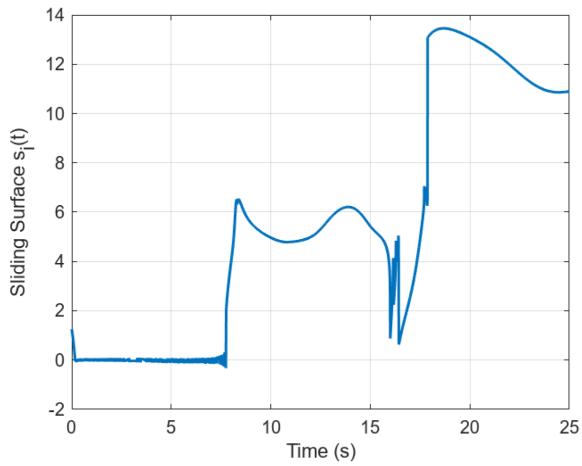

Figure 13. Sliding surface with attack and actuator fault.

Finally, comparing the results in Figures 14–17, that with or without faults and attacks, the designed controller guarantees stability by maintaining bounded errors and achieving convergence within a finite time, thereby confirming the effectiveness of the control approach.

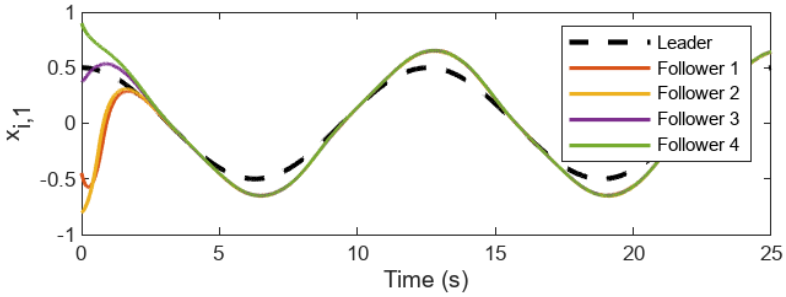

Figure 14. Follower-leader trajectories $x_{i,1}$ without actuator fault and attack.

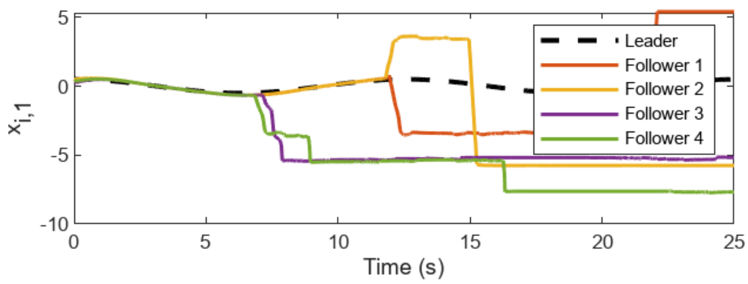

Figure 15. Follower-leader trajectories $x_{i,1}$ with actuator fault and attack.

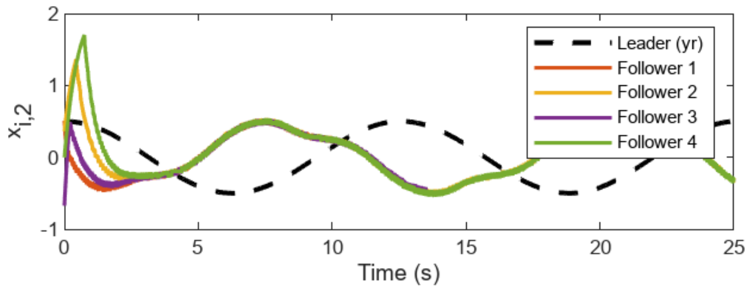

Figure 16. Follower-leader trajectories $x_{i,2}$ without actuator fault and attack.

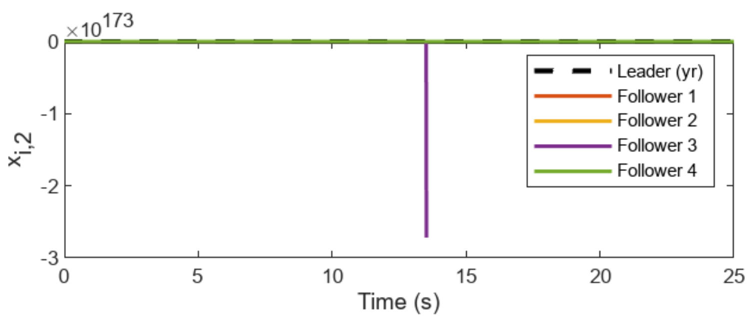

Figure 17. Follower-leader trajectories $x_{i,2}$ with actuator fault and attack.

6. Summary

In this paper, an adaptive fuzzy practical finite-time secure control method was developed for MASs that may experience actuator failures and deception. With this approach, backstepping design, fuzzy logic systems, and adaptive laws are combined to identify and correct both gain and bias errors in the actuator. A Nussbaum function was applied to address challenges that arose when the control was unknown during a deception attack. Using Lyapunov theory, a rigorous proof verified that our control strategy ensures the system signals are semi-globally uniformly bounded and drives the tracking error to a small residual set in a finite period. The proposed method demonstrated good performance, but there are certain limitations. One is that the complexity of the controller becomes higher with fuzzy rule expansion and having several adaptive laws. Additionally, the method is based on the full-state availability, which is not likely to realistically exist in large-scale and partially visible systems. In the future, the controller will be extended to an output-feedback scenario in which only partial measurements are considered. Moreover, efforts to increase computational efficiency through fuzzy rule base reduction and examining event-triggered or data-driven adaptations will be addressed.

Author contributions

H. Louati: Software, Project administration; R. Arooj: Data curation; A. U. K. Niazi: Conceptualization, Supervision; R. Arooj and A. U. K. Niazi: Writing–original draft, Writing–review & editing; M. E. E. Dalam: Software, Resources; M. M. A. Almazah: Validation; A. Smerat: Formal analysis. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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