Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(9): 20010–20024.
DOI: 10.3934/math.2025894
Received: 03 July 2025
Revised: 17 August 2025
Accepted: 26 August 2025
Published: 01 September 2025

Research article

Finite-time stochastic synchronization of network systems with intermittent delayed couplings

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Abstract: This article investigates finite-time synchronization (FTS) of network systems (NSs) with intermittent delayed couplings and stochastic perturbations. A channel matrix is introduced to describe the partial couplings of NSs, and state-dependent parameters are also considered to characterize the weight matrices. In order to deal with the intermittent couplings and reduce the control cost, an intermittent control scheme is designed. Based on the 2-norm Lyapunov function and new analytical methods, FTS is achieved, and the settling time is also estimated. Besides, two special cases are presented which indicate that our models are more general than some existing models. Finally, the synchronization criteria are verified by some simulations.

Keywords: finite-time stochastic synchronization; network systems; intermittent couplings; delayed couplings; partial couplings
Mathematics Subject Classification: 93C10, 93D40

1. Introduction

Network systems (NSs) have become useful tools to describe the real networks which include internet, transportation network, communication network, and so on. In NSs, the connections are depend on the couplings of nodes. However, the communication of nodes is inevitably influenced by the external environment. In fact, only partial information is transmitted successfully in many real networks [1], indicating that partial couplings are widespread. As a result, much attention should be attached to partially coupled NSs. However, there are few results on NSs with partial couplings in existing references, see e.g., [2–5].

Due to the fact that real NSs are usually exposed in complex environment, external perturbations always arise stochastically in signal transmission. For instance, reference [6] shows that the probabilistic release of neurotransmitters can result in the randomness of synaptic transmission in neural networks. NSs in production and real life will also be disturbed by many other stochastic factors [7–11]. Hence, it is very natural to consider the stochastic perturbations when NSs are investigated. In addition, some parameters of NSs may be related to the states of NSs. For example, the memristor-based neural network is a typical representative network and event-triggered control also relies on the states of networks, see references [12–16] for more details. It cannot be ignored that state-dependent parameters will bring the jump of parameters, which introduces difficulties to the investigation of NSs.

Note that the nodes of NSs usually exhibit collective behaviors. Synchronization of NSs as a typical collective behavior has been one of the most attractive topics. This is a result of the fact that NSs are usually used to describe many real-life networks. Synchronization implies all the nodes approach same state. Subsequently, people have to care about the convergence time of synchronization. If the synchronization is realized as time tends to infinity, it is clearly impractical owing to the finite lifetime of equipment. Then, finite-time synchronization (FTS) has attracted signficant interest from researchers. FTS implies the synchronization can be obtained within a finite time, known as the settling time. Moreover, it has been demonstrated that FTS exhibits fast convergence and strong robustness [17–19].

Over the past decade, FTS has been investigated extensively, and many results of FTS have been established. However, the FTS of delayed NSs is not studied completely due to the fact that the classic finite-time stability theorem is not suitable for delayed systems, which was pointed out by [20]. Some analytical methods are proposed to solve the FTS of delayed NSs, but those methods still have some limitations [21, 22]. Some references [21, 22] propose and develop effective analytical methods. However, the methods of references [21, 22] are only based on the 1-norm. Recently, some references [23, 24] have developed new analytical strategies to solve the FTS of delayed NSs by considering the activation function with time delays. As is well known, the limited bandwidth of the transmission channel always causes time delays in the couplings between the nodes. Therefore, the FTS of NSs with delayed couplings is also worth considering. Moreover, the couplings of NSs may be discontinuous due to external perturbations, as described in some references, one can refer [25–27] and so on. Motivated by real-world networks, this paper investigates NSs with discontinuous couplings.

As is well known, the synchronization of NSs is not realized spontaneously, and an external controller is necessary. In theoretical investigation, continuous control schemes play an important role, but they are not easy to implement in practical applications. Discontinuous controllers are effective when they are used in real problems. They include impulsive control, intermittent control, sampled-data control, and so on. Intermittent control is a typical type of discontinuous control. In intermittent control, the different control schemes are implemented in different intervals, as shown in references [28–33]. Intermittent control can help reduce control costs, making it a preferred choice among researchers.

Inspired by the above analysis, in order to accurately describe real-world networks, we focus on NSs with state-dependent parameters, intermittent couplings, partial couplings, delayed couplings, and stochastic perturbations. The main contributions include: 1) General models of NSs are structured to describe complex real networks, and two special cases are presented; 2) In order to deal with the intermittent couplings and reduce the control cost, an intermittent control scheme is designed; 3) FTS of NSs is achieved using a 2-norm Lyapunov functional and a new analytical method. In addition, the settling time is also estimated.

The chapter arrangement is given as follows: In Section 2, necessary preliminaries are given. Section 3 establishes finite-time stochastic synchronization based on the models of this paper. In Section 4, some numerical examples are provided to illustrate the obtained results. Finally, the conclusion is summarized in Section 5.

Notations: In this paper, $\Re$ is the real numbers set, $\Re^m$ denotes the corresponding dimensional Euclidean space, and $\Re^{m\times n}$ is the set of all $m\times n$ real matrices. $\mathbb{N}=\{0,1,2,\cdots\}$, I_m represents the m-dimension identity matrix, mathematical expectation is expressed by $\mathcal{E}[\cdot]$.

2. Preliminaries

The NSs with intermittent delayed couplings can be described as follows:

$$\begin{aligned}\mathrm{d}\sigma_i(t)=&[A\sigma_i(t)+D(\sigma_i(t))f(\sigma_i(t))+\theta(t)\sum_{j=1,j\neq i}^{M}g_{ij}\Gamma B_{ij}(\sigma_j(t-\varsigma(t))-\sigma_i(t-\varsigma(t)))]\mathrm{d}t\\&+h_i(\sigma_1(t),\sigma_2(t),\cdots,\sigma_M(t))\mathrm{d}\omega(t)\end{aligned}\tag{2.1}$$

where $\sigma_i(t)=(\sigma_{i1}(t),\sigma_{i2}(t),\cdots,\sigma_{im}(t))^T\in\Re^m$ is the state vector for $i\in\mathcal{M}=\{1,2,\cdots,M\}$, and $f(\sigma_i(t))=(f_1(\sigma_{i1}(t)),f_2(\sigma_{i2}(t)),\cdots,f_m(\sigma_{im}(t)))^T\in\Re^m$ is a continuous vector-valued function. If $t\in\hat{\partial}_k$, $\theta(t)=1$; otherwise, $\theta(t)=0$ when $t\in\check{\partial}_k$, where $\hat{\partial}_k=[t_{2k},t_{2k+1})$, $\check{\partial}_k=[t_{2k+1},t_{2k+2})$, and $\{t_k\}_{k\in\mathbb{N}}$ is a strictly increasing time sequence, satisfying $t_0=0$, $\lim_{k\to\infty}t_k=+\infty$. $A=\mathrm{diag}(a_1,a_2,\cdots,a_m)\in\Re^{m\times m}$. $\varsigma(t)$ is a time delay with $\dot{\varsigma}(t)\le\mu<1$ and $0<\varsigma(t)\le\bar{\varsigma}$. $D(\sigma_i(t))=(d_{kj}(\sigma_{ik}(t)))_{n\times n}$, where $d_{kj}(\sigma_{ik}(t))$ represents the state-dependent parameter, which satisfies the following condition:

$$d_{kj}(\sigma_{ik}(t))=\begin{cases}\acute{d}_{kj},|\sigma_{ik}(t)|\le\Xi_k,\\ \grave{d}_{kj},|\sigma_{ik}(t)|>\Xi_k,\end{cases}$$

where $\Xi_k>0$ is the switching jump, $\acute{d}_{kj},\grave{d}_{kj}(k,j\in\{1,2,\dots,m\})$ are constant numbers, and $\acute{d}_{kj}\neq\grave{d}_{kj}$. The elements of $G=(g_{ij})_{M\times M}$ are given as follows: If it has a connection from node j to node $i(j\neq i)$, then $g_{ij}>0$, otherwise $g_{ij}=0$; $g_{ii}=-\sum_{j=1,j\neq i}^{M}g_{ij}$. $\Gamma=\mathrm{diag}\{\gamma_1,\gamma_2,\gamma_m\}$ is an inner coupling matrix. $B_{ij}=\mathrm{diag}(b_{ij}^1,b_{ij}^2,\cdots,b_{ij}^m)$ is a channel matrix with $b_{ij}^s=0$ or 1 for $s=1,2,\cdots,m$. $\omega_i(t)=(\omega_{i1}(t),\omega_{i2}(t),\cdots,\omega_{im}(t))^T$ is an m-dimensional vector Weiner process, and $\omega_i(t)$ is independent of ω_j for $i\neq j$. $h_i(\sigma_1(t),\sigma_2(t),\cdots,\sigma_M(t))\in\Re^{m\times m}$ is the unknown coupling, satisfying $h_i(\tau(t),\tau(t),\cdots,\tau(t))=0$ for $\tau(t)\in\Re^m$. For convenience, $h_i(\sigma_1(t),\sigma_2(t),\cdots,\sigma_M(t))\triangleq h_i(\sigma(t))$ and $h_i(\tau(t),\tau(t),\cdots,\tau(t))\triangleq h_i(\tau(t))$. The initial values of networks (2.1) are $\sigma_i(\iota)=\sigma_{i0}$ for $\iota\in[-\bar{\varsigma},0]$.

Remark 1. *The NSs (2.1) are very general. If $\theta(t)\equiv1$, $\varsigma(t)=0$, and $B_{ij}=I_m$, then the couplings $\theta(t)\sum_{j=1,j\neq i}^{M}g_{ij}\Gamma B_{ij}(\sigma_j(t-\varsigma(t))-\sigma_i(t-\varsigma(t)))$ become $\sum_{j=1}^{M}g_{ij}\Gamma\sigma_j(t)$, which are considered in references [9, 22, 23, 28]. If $\theta(t)\equiv1$, $\varsigma(t)=0$, then the couplings degrade as $\sum_{j=1}^{M}g_{ij}B_{ij}\Gamma\sigma_j(t)$, which are investigated in references [2, 3]. It is noteworthy that none of the above-mentioned references has considered intermittent coupling forms ($\theta(t)=1$ or $\theta(t)=0$ for different cases); instead, they all focus on continuous coupling forms. Besides, in the above references, the state-dependent parameters, for example $D(\sigma_i(t))$, and stochastic perturbations are not involved simultaneously. As is well known, the more factors are considered, the more difficulties of synchronization will be introduced.*

Let $C_{ij} = g_{ij}B_{ij} \triangleq \text{diag}\{c_{ij}^1, c_{ij}^2, \cdots, c_{ij}^m\}$, for $i, j \in \mathcal{M}$ and $C_{ii} = -\sum_{j=1,j\neq i}^{M} C_{ij}$. Then, the networks (2.1) are rewritten as:

$$\mathrm{d}\sigma_i(t) = [A\sigma_i(t) + D(\sigma_i(t))f(\sigma_i(t)) + \theta(t)\sum_{j=1}^{M}\Gamma C_{ij}\sigma_j(t-\varsigma(t))]\mathrm{d}t + h_i(\sigma(t))\mathrm{d}\omega(t).$$

Denote $\underline{d}_{kj} = \min\{\acute{d}_{kj}, \grave{d}_{kj}\}$, $\overline{d}_{kj} = \max\{\acute{d}_{kj}, \grave{d}_{kj}\}, d_{kj} = \frac{1}{2}(\underline{d}_{kj} + \overline{d}_{kj})$, and $\hat{d}_{kj} = \frac{1}{2}(\overline{d}_{kj} - \underline{d}_{kj})$ for $k, j \in \{1, 2, \ldots, m\}$. According to the analysis of references [13, 15], one has

$$\mathrm{d}\sigma_i(t) = [A\sigma_i(t) + (D + D_i(t))f(\sigma_i(t)) + \theta(t)\sum_{j=1}^{M}\Gamma C_{ij}\sigma_j(t-\varsigma(t))]\mathrm{d}t + h_i(\sigma(t))\mathrm{d}\omega(t), \tag{2.2}$$

where $D = (d_{kj})_{m\times m}$, $D_i(t) = (\hat{d}_{kj}\zeta_{kj}^i(t))_{m\times m}$, and $\zeta_{kj}^i(t) \in [-1, 1]$.

Lemma 1. [24] *The continuous and positive definite function $v(t)(t \in [0, +\infty))$ satisfies*

$$\dot{v}(t) \leq \begin{cases} -av(t) - c, & t \in \hat{\partial}_k, \\ bv(t), & t \in \check{\partial}_k. \end{cases}$$

Let the condition $\chi(k) = (v(0)e^{\sum_{s=0}^{k-1}[-a(t_{2s+1}-t_{2s})+b(t_{2s+2}-t_{2s})]} + \frac{c}{a})e^{-\rho(t_{2k+1}-t_{2k})} - \frac{c}{a} \leq 0$ hold. Then $v(t) \equiv 0$ for $t \geq \mathcal{T}$. $\mathcal{T}$ is given as

$$\mathcal{T} = t_{2k_*} + \frac{1}{a}\ln\left(\frac{a}{c}v(0)e^{\sum_{s=0}^{k_*-1}[-a(t_{2s+1}-t_{2s})+b(t_{2s+2}-t_{2s})]} + 1\right),$$

where $a > 0, b > 0, c > 0$, $v(0) \geq 0$ are constants, and $k_* = \min\{k|\chi(k) \leq 0, k \in \mathbb{N}\}$.

In this paper, we are devoted to establishing some conditions for ensuring all the nodes of NSs (2.1) synchronize with the node (2.3).

$$\mathrm{d}\tau(t) = [A\tau(t) + D(\tau(t))f(\tau(t))]\mathrm{d}t, \tag{2.3}$$

and $\tau(0) \in \Re^m$ is the initial value of network (2.3). By the same analysis with (2.2), we also obtain

$$\mathrm{d}\tau(t) = [A\tau(t) + (D + D_\tau(t))f(\tau(t))]\mathrm{d}t, \tag{2.4}$$

where $D_\tau(t) = (\hat{d}_{kj}\zeta_{kj}^\tau(t))_{m\times m}$ and $\zeta_{kj}^\tau(t) \in [-1, 1]$.

Let $\delta_i(t) = \sigma_i(t) - \tau(t)$, $\bar{f}(\delta_i(t)) = f(\sigma_i(t)) - f(\tau(t))$. Then from (2.2) and (2.4), we have

$$\begin{aligned}\mathrm{d}\delta_i(t) =& [A\delta_i(t) + D\bar{f}(\delta_i(t)) + \theta(t)\sum_{j=1}^{M}\Gamma C_{ij}\delta_j(t-\varsigma(t)) + D_i(t)f(\sigma_i(t)) - D_\tau(t)f(\tau(t)) + \mathfrak{U}_i(t)]\mathrm{d}t \\ &+ \tilde{h}_i(\delta(t))\mathrm{d}\omega(t),\end{aligned} \tag{2.5}$$

where $\tilde{h}_i(\delta(t)) = h_i(\sigma(t)) - h_i(\tau(t))$, and $\mathfrak{U}_i(t)$ is a controller which need to be designed.

The following Assumptions 1–3 are needed to realize the goal of synchronization.

Assumption 1. *There exists a $\ell \geq 0$ satisfying $\|f(x) - f(y)\| \leq \ell\|x - y\|$ for any $x, y \in \Re^m$.*

Assumption 2. *There are constants $L_i \geq 0$ satisfying $|f_i(\tau)| \leq L_i$ for $\tau \in \Re$, $i = 1, 2, \cdots, m$.*

Assumption 3. *There exist some $\hbar_{ij} \geq 0$ such that*

$$\text{trace}\{(\tilde{h}_i(\delta(t)))^T \tilde{h}_i(\delta(t))\} \leq \sum_{j=1}^{M} \hbar_{ij} \|\delta_j(t)\|^2, \text{ for } i \in \mathcal{M}.$$

In this paper, the following definition of finite-time stochastic synchronization is considered.

Definition 1. *Network (2.1) is said to realize finite-time stochastic synchronization with network (2.3) if it has a settling time $\mathcal{T}$ such that $\lim_{t\to\mathcal{T}} \mathcal{E}[\|\delta_i(t)\|] = 0$ and $\mathcal{E}[\|\delta_i(t)\|] \equiv 0$ for $t > \mathcal{T}$, where $\mathcal{T} > 0$ is a constant which is said to be settling time.*

3. Main results

This section establishes the results of FTS. The following controller (3.1) plays a pivotal role in the synchronization of NSs (2.1) and (2.3):

$$\mathfrak{U}_i(t) = \begin{cases} -\xi_i \delta_i(t) - \eta_i \text{sign}\delta_i(t), & t \in \hat{\partial}, \\ -\eta_i \text{sign}\delta_i(t), & t \in \check{\partial}, \end{cases} \tag{3.1}$$

where $\xi_i > 0, \eta_i > 0$ are constants.

Next, based on the controller (3.1), we establish the main results in Theorem 1.

Theorem 1. *Let Assumptions 1–3 hold. If the constants $\varepsilon > 0$, ξ_i, and η_i satisfy the following conditions*

$$\Xi \otimes I_m \geq I_M \otimes A + (\frac{1}{2}\|D\|(1 + \ell^2) + \frac{\rho}{\varepsilon} + \rho)(I_M \otimes I_m) + \mathcal{H} \otimes I_m, \tag{3.2}$$

$$\eta_i > 2\|\hat{D}\|_\infty L_{\max}, \tag{3.3}$$

$$\chi(k) = \Big(V(0)e^{\sum_{s=0}^{k-1}\Delta_s} + \frac{\eta}{\rho}\Big)e^{-\rho(t_{2k+1}-t_{2k})} - \frac{\eta}{\rho} \leq 0, \tag{3.4}$$

then under the controller (3.1), the NSs (2.1) are synchronized with network (2.3) within the settling time $\mathcal{T} = t_{2k_*} + \frac{1}{\rho}\ln\big(\frac{\rho}{\eta}V(0)e^{\sum_{s=0}^{k_*-1}\Delta_s} + 1\big)$, where $k_* = \min\{k|\chi(k) \leq 0\}$, $\eta = \min_{i\in\mathcal{M}}\{\eta_i - 2\|D\|_\infty L_{\max}\}$. The related parameters are given as follows: $\rho = \frac{\varepsilon e^{\varepsilon\bar{\varsigma}}\|\mathfrak{C}\|}{1-\mu}$ with $\mathfrak{C} = (I_M \otimes \Gamma)C$ and $C = (C_{ij})_{M\times M}$, $\Delta_s = -\rho(t_{2s+1} - t_{2s}) + \lambda(t_{2s+2} - t_{2s+1})$, $\lambda = \|I_M \otimes A + (\frac{1}{2}\|D\|(1 + \ell^2) + \frac{\rho}{\varepsilon})(I_M \otimes I_m) + \mathcal{H} \otimes I_m\| > 0$, $\sum_{i=1}^{M} \hbar_{ij} \triangleq \hbar_j$, $\mathcal{H} = diag(\hbar_1, \hbar_2, \cdots, \hbar_M)$, $\Xi = diag(\xi_1, \xi_2, \cdots, \xi_M)$, $\|\hat{D}\|_\infty = \max_{j=1,2,\cdots,m} \sum_{k=1}^{m} |\hat{d}_{jk}|$, $L_{\max} = \max\{L_1, L_2, \cdots, L_m\}$ and $V(0) = \|\delta(0)\| + \frac{\rho}{\varepsilon}\int_{-\varsigma(0)}^{0} e^{\varepsilon\aleph}\|\delta(\aleph)\|\mathrm{d}\aleph$.

Proof. We consider the Lyapunov functional $V(t) = V_1(t) + V_2(t)$ and

$$V_1(t) = \|\delta(t)\|, V_2(t) = \frac{\rho}{\varepsilon}\int_{t-\varsigma(t)}^{t} e^{\varepsilon(\aleph - t)}\|\delta(\aleph)\|\mathrm{d}\aleph,$$

where $\delta(t) = (\delta_1^T(t), \delta_2^T(t), \cdots, \delta_M^T(t))^T$. For convenience, $(\delta_1^T(t-\varsigma(t)), \delta_2^T(t-\varsigma(t)), \cdots, \delta_M^T(t-\varsigma(t)))^T$ is also denoted as $\delta(t-\varsigma(t))$.

When $\|\delta(t)\| \neq 0$ and $t \in \hat{\partial}$, differentiating $V_1(t)$ along the trajectories of error system (2.5) gives

$$\mathrm{d}V_1(t) = \mathcal{L}V_1(t) + \frac{1}{V_1(t)} \sum_{i=1}^{M} \delta_i^T(t)\tilde{h}_i(\delta(t))\mathrm{d}\omega(t),$$

where

$$\begin{aligned}
\mathcal{L}V_1(t) =& \frac{1}{V_1(t)} \sum_{i=1}^{M} \Big[\delta_i^T(t)(A\delta_i(t) + D\bar{f}(\delta_i(t)) + \sum_{j=1}^{M} \Gamma C_{ij}\delta_j(t-\varsigma(t)) - \xi_i\delta_i(t) - \eta_i \mathrm{sign}(\delta_i(t)) \\
&+ D_i(t)f(\sigma_i(t)) - D_\tau(t)f(\tau(t))) + \frac{1}{2}\mathrm{trace}\{(\tilde{h}_i(\delta(t)))^T\tilde{h}_i(\delta(t))\} \\
&- \frac{1}{2\|\delta(t)\|^2}\delta_i^T(t)\tilde{h}_i(\delta(t))(\tilde{h}_i(\delta(t)))^T\delta_i(t)\Big] \\
\leq& \frac{1}{V_1(t)} \sum_{i=1}^{M} \Big[\delta_i^T(t)(A\delta_i(t) + D\bar{f}(\delta_i(t)) + \sum_{j=1}^{M} \Gamma C_{ij}\delta_j(t-\varsigma(t)) - \xi_i\delta_i(t) - \eta_i \mathrm{sign}(\delta_i(t)) \\
&+ D_i(t)f(\sigma_i(t)) - D_\tau(t)f(\tau(t))) + \frac{1}{2}\mathrm{trace}\{(\tilde{h}_i(\delta(t)))^T\tilde{h}_i(\delta(t))\}. \qquad (3.5)
\end{aligned}$$

Meanwhile, the following inequality (3.6) is derived:

$$\sum_{i=1}^{M} \delta_i^T(t)A\delta_i(t) = \delta^T(t)(I_M \otimes A)\delta(t). \qquad (3.6)$$

Utilizing Assumption 1, it generates

$$\begin{aligned}
\sum_{i=1}^{M} \delta_i^T(t)D\bar{f}(\delta_i(t)) &\leq \frac{1}{2}\|D\| \sum_{i=1}^{M} \|\delta_i(t)\|^2 + \frac{1}{2}\|D\| \sum_{i=1}^{M} \ell^2\|\delta_i(t)\|^2 \\
&= \frac{1}{2}\|D\|(1+\ell^2)\delta^T(t)(I_M \otimes I_m)\delta(t). \qquad (3.7)
\end{aligned}$$

From Assumption 2, the mismatched parts are handled as follows:

$$\begin{aligned}
&\sum_{i=1}^{M} \delta_i^T(t)(D_i(t)f(\sigma_i(t)) - D_\tau(t)f(\tau(t))) \\
&\leq \sum_{i=1}^{M}\sum_{j=1}^{m}\sum_{k=1}^{m} |\delta_{ij}(t)||\hat{d}_{jk}||f_k(\sigma_{ik}(t))| + \sum_{i=1}^{M}\sum_{j=1}^{m}\sum_{k=1}^{m} |\delta_{ij}(t)||\hat{d}_{jk}||f_k(\tau_k(t))| \\
&\leq 2\sum_{i=1}^{M}\sum_{j=1}^{m}\sum_{k=1}^{m} |\delta_{ij}(t)||\hat{d}_{jk}|L_k \\
&\leq 2\|\hat{D}\|_\infty L_{\max} \sum_{i=1}^{M}\sum_{j=1}^{m} |\delta_{ij}(t)|. \qquad (3.8)
\end{aligned}$$

The following inequality is related to the controller (3.1):

$$-\delta_i^T(t)\eta_i \text{sign}(\delta_i(t)) \leq -\eta_i \sum_{j=1}^{m} |\delta_{ij}(t)|. \tag{3.9}$$

By utilizing Assumption 3, it follows that

$$\frac{1}{2}\sum_{i=1}^{M} \text{trace}\{(\tilde{h}_i(\delta(t)))^T \tilde{h}_i(\delta(t))\} \leq \frac{1}{2}\delta^T(t)(\mathcal{H} \otimes I_m)\delta(t). \tag{3.10}$$

Next, the following inequality is analyzed:

$$\begin{aligned}\sum_{i=1}^{M} \delta_i^T(t) \sum_{j=1}^{M} \Gamma C_{ij}\delta_j(t-\varsigma(t)) &= \delta^T(t)\mathfrak{C}\delta(t-\varsigma(t)) \\ &\leq \|\delta(t)\|\|\mathfrak{C}\|\|\delta(t-\varsigma(t))\|.\end{aligned} \tag{3.11}$$

Taking inequalities (3.6)–(3.11) into (3.5), $\mathcal{L}V_1(t)$ can be simplified to

$$\begin{aligned}\mathcal{L}V_1(t) \leq &\frac{1}{V_1(t)}\Big[\delta^T(t)\Big(I_M \otimes A + \frac{1}{2}\|D\|(1+\ell^2)(I_M \otimes I_m) + \mathcal{H} \otimes I_m - \Xi\Big)\delta(t) \\ &+ \|\delta(t)\|\|\mathfrak{C}\|\|\delta(t-\varsigma(t))\| - \sum_{i=1}^{M}\sum_{j=1}^{m}(\eta_i - 2\|D\|_\infty L_{\max})|\delta_{ij}(t)|\Big].\end{aligned} \tag{3.12}$$

Moreover, the infinitesimal operator of $V_2(t)$ is given as

$$\mathcal{L}V_2(t) \leq -\rho V_2(t) + \frac{\rho}{\varepsilon}\|\delta(t)\| - \|\mathfrak{C}\|\|\delta(t-\varsigma(t))\|. \tag{3.13}$$

Then, from inequalities (3.12) and (3.13), it follows that

$$\begin{aligned}\mathcal{L}V(t) \leq &\frac{1}{V_1(t)}\delta^T(t)\Big(I_M \otimes A + \frac{1}{2}\|D\|(1+\ell^2)(I_M \otimes I_m) + \mathcal{H} \otimes I_m - \Xi\Big)\delta(t) + \|\mathfrak{C}\|\|\delta(t-\varsigma(t))\| \\ &- \frac{1}{V_1(t)}\sum_{i=1}^{M}\sum_{j=1}^{m}(\eta_i - 2\|D\|_\infty L_{\max})|\delta_{ij}(t)| - \rho V_2(t) + \frac{\rho}{\varepsilon}\|\delta(t)\| - \|\mathfrak{C}\|\|\delta(t-\varsigma(t))\| \\ \leq &\frac{1}{V_1(t)}\delta^T(t)\Big(I_M \otimes A + (\frac{1}{2}\|D\|(1+\ell^2) + \frac{\rho}{\varepsilon})(I_M \otimes I_m) + \mathcal{H} \otimes I_m - \Xi\Big)\delta(t) \\ &- \rho V_2(t) - \frac{\eta\sum_{i=1}^{M}\sum_{j=1}^{m}|\delta_{ij}(t)|}{\|\delta(t)\|}.\end{aligned}$$

By considering $\|\delta(t)\| \leq \sum_{i=1}^{M}\sum_{j=1}^{m}|\delta_{ij}(t)|$ and conditions (3.2) and (3.3), furthermore, one has

$$\begin{aligned}\mathcal{L}V(t) \leq &-\rho V(t) - \frac{\eta\sum_{i=1}^{M}\sum_{j=1}^{m}|\delta_{ij}(t)|}{\|\delta(t)\|} \\ \leq &-\rho V(t) - \eta.\end{aligned} \tag{3.14}$$

Similarly, for $t \in \check{\partial}, k \in \mathbb{N}$, one can obtain

$$\begin{aligned}\mathcal{L}V(t) \leq & \frac{\delta^T(t)\Big(I_M \otimes A + (\frac{1}{2}\|D\|(1+\ell^2) + \frac{\rho}{\varepsilon})(I_M \otimes I_m) + \mathcal{H} \otimes I_m\Big)\delta(t)}{V_1(t)} - \rho V_2(t) \\ \leq & \lambda V(t) - (\lambda+\rho)V_2(t) \\ \leq & \lambda V(t). \end{aligned} \tag{3.15}$$

In view of the inequalities (3.14) and (3.15), it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}[V(t)] \leq \begin{cases} -\rho\mathcal{E}[V(t)] - \eta, & t \in \hat{\partial}_k, \\ \lambda\mathcal{E}[V(t)], & t \in \check{\partial}_k. \end{cases}$$

Based on condition (3.4) and Lemma 1, $\lim_{t\to\mathcal{T}}\mathcal{E}[V(t)] = 0$ and $\mathcal{E}[V(t)] \equiv 0$ for $t \geq \mathcal{T}$ are obtained. Obviously, $\lim_{t\to\mathcal{T}}\mathcal{E}[\|\delta_i(t)\|] = 0$ and $\mathcal{E}[\|\delta_i(t)\|] \equiv 0$ for $t \geq \mathcal{T}$ are also acquired, where

$$\mathcal{T} = t_{2k_*} + \frac{1}{\rho}\ln\Big(\frac{\rho}{\eta}V(0)e^{\sum_{s=0}^{k_*-1}[-\rho(t_{2s+1}-t_{2s})+\lambda(t_{2s+2}-t_{2s+1})]} + 1\Big).$$

Then, according to Definition 1, the finite-time stochastic synchronization of networks (2.1) and (2.3) is realized. □

Remark 2. *From the proof of Theorem 1, one can see that the controllers of [23, 24, 29] are invalid to realize synchronization since the mismatched parameters can not be handled if $\mathfrak{U}_i(t) = 0$ when $t \in \check{\partial}$. Therefore, $-\eta_i sign\delta_i(t)$ plays an important role in realizing synchronization of this paper.*

Remark 3. *Inspired by [23, 24], this paper employs 2-norm analytical techniques to establish the FTS criteria for NSs with time delays, which differs from the approaches in references [21, 22]. In those works, the authors derived FTS of delayed NSs using 1-norm analytical techniques. Moreover, NSs can be formed using some multi-agent systems such as [11] and so on. When modeling multi-agent systems with intermittent variables and channel matrices, the results of Theorem 1 can serve as a reference for solving the consensus problem of multi-agent systems subject to communication constraints and intermittent connectivity.*

As widely recognized, secure communication is an important application of chaos synchronization. By Theorem 1, one can construct a simple secure communication strategy. System (2.1) can be applied to transmit a message to system (2.3) in the following form:

$$\text{Transmitter}: \begin{cases} \mathrm{d}\sigma_i(t) = & [A\sigma_i(t) + D(\sigma_i(t))f(\sigma_i(t)) + \theta(t)\sum_{j=1,j\neq i}^{M} g_{ij}\Gamma B_{ij}(\sigma_j(t-\varsigma(t)) \\ & -\sigma_i(t-\varsigma(t))) + \mathfrak{U}_i(t)]\mathrm{d}t + h_i(\sigma_1(t), \sigma_2(t), \cdots, \sigma_M(t))\mathrm{d}\omega(t), \\ \varrho(t) = & \frac{1}{M}\sum_{i=1}^{M} Q\sigma_i(t) + r(t), \end{cases} \tag{3.16}$$

$$\text{Receiver}: \begin{cases} \mathrm{d}\tau(t) = & [A\tau(t) + D(\tau(t))f(\tau(t))]\mathrm{d}t, \\ \hat{\varrho}(t) = & \varrho(t) - Q\tau(t), \end{cases} \tag{3.17}$$

where $Q \in \Re^{n\times m}$ is a known matrix, $\varrho(t) \in \Re^n$ serves as the chaotic carrier that masks the true message $r(t)$, and $\hat{\varrho}(t) \in \Re^n$ is the recovered signal from $r(t)$. Let the conditions of Theorem 1 be satisfied.

Then systems (3.16) and (3.17) can reach the FTS. As a result, the recovered signal $\hat{\varrho}(t)$ is achieved by $\hat{\varrho}(t) = \frac{1}{M}\sum_{i=1}^{M} Q\sigma_i(t) + r(t) - Q\tau(t) = \frac{1}{M}\sum_{i=1}^{M} Q(\sigma_i(t) - \tau(t)) + r(t) \to r(t)$ when $t \to \mathcal{T}$.

If stochastic perturbations are not considered, then the following model of NSs is investigated:

$$\begin{cases} \dot{\sigma}_i(t) = A\sigma_i(t) + D(\sigma_i(t))f(\sigma_i(t)) + \theta(t)\sum_{j=1}^{M} g_{ij}\Gamma B_{ij}(\sigma_j(t-\varsigma(t)) - \sigma_i(t-\varsigma(t))), & (3.18) \\ \dot{\tau}(t) = A\tau(t) + D(\tau(t))f(\tau(t)). & (3.19) \end{cases}$$

Corollary 1. *Let Assumptions 1 and 2 hold. If the constants $\varepsilon > 0$, ξ_i, and η_i satisfy the conditions*

$$\Xi \geq I_M \otimes A + (\frac{1}{2}\|D\|(1+\ell^2) + \frac{\rho}{\varepsilon} + \rho)(I_M \otimes I_m),$$
$$\eta_i > 2\|\hat{D}\|_\infty L_{\max},$$
$$\chi(k) = \Big(V(0)e^{\sum_{s=0}^{k-1}[-\rho(t_{2s+1}-t_{2s})+\bar{\lambda}(t_{2s+2}-t_{2s+1})]} + \frac{\eta}{\rho}\Big)e^{-\rho(t_{2k+1}-t_{2k})} - \frac{\eta}{\rho} \leq 0,$$

then under the controller (3.1), the NSs (3.18) are synchronized with network (3.19) within the settling time $\mathcal{T} = t_{2k_*} + \frac{1}{\rho}\ln(\frac{\rho}{\eta}V(0)e^{\sum_{s=0}^{k_*-1}[-\rho(t_{2s+1}-t_{2s})+\bar{\lambda}(t_{2s+2}-t_{2s+1})]} + 1)$. The related parameters $\bar{\lambda} = \|I_M \otimes A + (\frac{1}{2}\|D\|(1+\ell^2) + \frac{\rho}{\varepsilon})(I_M \otimes I_m)\| > 0$, and the other parameters are given as those in Theorem 1.

If the couplings are based on complete information, then the following model of NSs is investigated.

$$\mathrm{d}\sigma_i(t) = [A\sigma_i(t) + D(\sigma_i(t))f(\sigma_i(t)) + \theta(t)\sum_{j=1}^{M} g_{ij}\Gamma(\sigma_j(t-\varsigma(t)) - \sigma_i(t-\varsigma(t)))]\mathrm{d}t$$
$$+ h_i(\sigma(t))\mathrm{d}\omega(t). \tag{3.20}$$

Corollary 2. *Let Assumptions 1–3 hold. If the constants $\varepsilon > 0$, ξ_i, and η_i satisfy*

$$\Xi \geq I_M \otimes A + (\frac{1}{2}\|D\|(1+\ell^2) + \frac{\tilde{\rho}}{\varepsilon} + \tilde{\rho})(I_M \otimes I_m) + \mathcal{H} \otimes I_m,$$
$$\eta_i > 2\|\hat{D}\|_\infty L_{\max},$$
$$\chi(k) = \Big(V(0)e^{\sum_{s=0}^{k-1}[-\tilde{\rho}(t_{2s+1}-t_{2s})+\tilde{\lambda}(t_{2s+2}-t_{2s+1})]} + \frac{\eta}{\tilde{\rho}}\Big)e^{-\tilde{\rho}(t_{2k+1}-t_{2k})} - \frac{\eta}{\tilde{\rho}} \leq 0,$$

then under controller (3.1), the NSs (3.20) are synchronized with network (2.3) within the settling time $\mathcal{T} = t_{2k_*} + \frac{1}{\tilde{\rho}}\ln(\frac{\tilde{\rho}}{\eta}V(0)e^{\sum_{s=0}^{k_*-1}[-\tilde{\rho}(t_{2s+1}-t_{2s})+\tilde{\lambda}(t_{2s+2}-t_{2s+1})]} + 1)$. The parameter $\tilde{\rho} = \frac{\varepsilon e^{\varepsilon\bar{\varsigma}}\|\mathcal{G}^T\mathcal{G}\|^{\frac{1}{2}}}{1-\mu}$ with $\mathcal{G} = (I_M \otimes \Gamma)G$, $\tilde{\lambda} = \|I_M \otimes A + (\frac{1}{2}\|D\|(1+\ell^2) + \frac{\tilde{\rho}}{\varepsilon})(I_M \otimes I_m) + \mathcal{H} \otimes I_m\| > 0$ and the other parameters are given as those in Theorem 1.

4. Numerical example

This section presents some simulations to illustrate the synchronization criteria.

The parameters of network (2.3) are given as follows: $\tau(t) = (\tau_1(t), \tau_2(t), \tau_3(t))^T$, $A = -0.5I_3$, $f(\tau(t)) = (\sin(\tau_1(t)), \tanh(\tau_2(t)), \tanh(\tau_3(t)))^T$, and $D(\tau(t))$ with

$$d_{11}(\tau_1(t)) = \begin{cases} 0.8, & |\tau_1(t)| \leq 4, \\ 0.6, & |\tau_1(t)| > 4, \end{cases} d_{12}(\tau_1(t)) = \begin{cases} 1, & |\tau_1(t)| \leq 4, \\ 0.5, & |\tau_1(t)| > 4, \end{cases} d_{13}(\tau_1(t)) = \begin{cases} 0.5, & |\tau_1(t)| \leq 4, \\ -0.1, & |\tau_1(t)| > 4, \end{cases}$$

$$d_{21}(\tau_2(t)) = \begin{cases} -0.9, & |\tau_2(t)| \le 4, \\ -1.3, & |\tau_2(t)| > 4, \end{cases} d_{22}(\tau_2(t)) = \begin{cases} 0.8, & |\tau_2(t)| \le 4, \\ 1, & |\tau_2(t)| > 4, \end{cases} d_{23}(\tau_2(t)) = \begin{cases} 0.1, & |\tau_2(t)| \le 4, \\ 0.5, & |\tau_2(t)| > 4, \end{cases}$$

$$d_{31}(\tau_3(t)) = \begin{cases} 1.5, & |\tau_3(t)| \le 4, \\ 1.6, & |\tau_3(t)| > 4, \end{cases} d_{32}(\tau_3(t)) = \begin{cases} -0.9, & |\tau_3(t)| \le 4, \\ -0.7, & |\tau_3(t)| > 4, \end{cases} d_{33}(\tau_3(t)) = \begin{cases} 0.8, & |\tau_3(t)| \le 4, \\ 1.5, & |\tau_3(t)| > 4. \end{cases}$$

The initial value is taken as $\tau(0) = (-0.8, 0.5, -0.7)^T$. Then the network (2.3) displays the chaotic trajectory which can be seen in Figure 1. Assumption 1 is satisfied with $\ell = 1$. Moreover, one can obtain that Assumption 2 is satisfied with $L_1 = L_2 = L_3 = 1$.

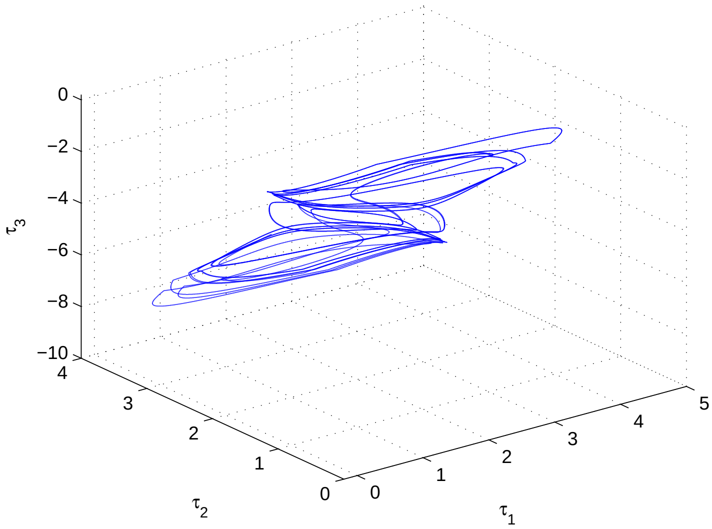

Figure 1. Chaotic trajectory of system (2.3) with $\tau(0) = (-0.8, 0.5, -0.7)^T$.

Besides the same parameters with network (2.3), the other parameters of NSs (2.1) are taken as follows: $\Gamma = I_3$,

$$G = \begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 \\ 2 & 0 & -3 & 1 & 0 \\ 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 1 & -2 \end{pmatrix}.$$

$h_i(\sigma_1, \sigma_2, \cdots, \sigma_5) = \mathrm{diag}(\sigma_{i1} - \sigma_{i+1,1}, \sigma_{i2} - \sigma_{i+1,2}, \sigma_{i3} - \sigma_{i+1,3})$ with $i = 1, 2, \cdots, 5$, and $\sigma_6(t) = \sigma_1(t)$. According to the analysis of reference [9], one can obtain $\mathrm{trace}\{(\tilde{h}_i(\delta(t)))^T \tilde{h}_i(\delta(t))\} \le 2(\|\delta_i(t)\|^2 + \|\delta_{i+1}\|^2)$, that is, Assumption 3 is satisfied. In addition, let the channel matrices be
$B_{11} = \mathrm{diag}\{0, 0, 1\}$, $B_{12} = \mathrm{diag}\{0, 1, 0\}$, $B_{13} = \mathrm{diag}\{1, 0, 1\}$, $B_{14} = \mathrm{diag}\{0, 0, 1\}$, $B_{15} = \mathrm{diag}\{1, 0, 0\}$,
$B_{21} = \mathrm{diag}\{0, 1, 1\}$, $B_{22} = \mathrm{diag}\{1, 0, 0\}$, $B_{23} = \mathrm{diag}\{1, 1, 0\}$, $B_{24} = \mathrm{diag}\{0, 0, 1\}$, $B_{25} = \mathrm{diag}\{1, 0, 1\}$,
$B_{31} = \mathrm{diag}\{1, 0, 1\}$, $B_{32} = \mathrm{diag}\{1, 0, 1\}$, $B_{33} = \mathrm{diag}\{0, 1, 0\}$, $B_{34} = \mathrm{diag}\{1, 0, 0\}$, $B_{35} = \mathrm{diag}\{0, 0, 1\}$,
$B_{41} = \mathrm{diag}\{1, 1, 0\}$, $B_{42} = \mathrm{diag}\{0, 1, 0\}$, $B_{43} = \mathrm{diag}\{1, 1, 0\}$, $B_{44} = \mathrm{diag}\{1, 1, 0\}$, $B_{45} = \mathrm{diag}\{1, 1, 0\}$,
$B_{51} = \mathrm{diag}\{0, 1, 1\}$, $B_{52} = \mathrm{diag}\{0, 1, 0\}$, $B_{53} = \mathrm{diag}\{1, 1, 0\}$, $B_{54} = \mathrm{diag}\{1, 0, 1\}$, $B_{55} = \mathrm{diag}\{0, 1, 0\}$.

The initial values of $\sigma_i(t)$ are $\sigma_i(\iota) = \sigma_{i0}$ for $\iota \in [-\bar{\varsigma}, 0]$, and σ_{i0} are randomly chosen from $(-5, 5)$. Take $\varsigma(t) = 1$, $\mu = 0$, $\bar{\varsigma} = 1$, and $\varepsilon = 1$. In order to illustrate Theorem 1, we also need to validate the synchronization conditions. By simple computation, $\rho = 10.3325$, $\lambda = 16.2180$, $\xi_i \geq 26.5505$, and $\eta_i \geq 1.3$ satisfy the conditions (3.2)–(3.4). Take $\xi_i = 27$ and $\eta_i = 2$. The time sequence $\{t_k\}_{k\in\mathbb{N}}$ is taken as $t_{2k+1} - t_{2k} = 0.07$ and $t_{2k+2} - t_{2k+1} = 0.03$. Then, one can obtain $k_* = 31$. It follows that the settling time is computed as $\mathcal{T} = 3.1594$. By Theorem 1, the FTS of NSs (2.1) and (2.3) can be realized within $\mathcal{T}$, which is presented by Figures 2–4. From Figures 2–4, it can be observed that as time progresses, the time response curves of the error variables converge to the vicinity of zero within finite time $\mathcal{T}$. Additionally, the chattering of the curves indicates that stochastic disturbances significantly affect the system.

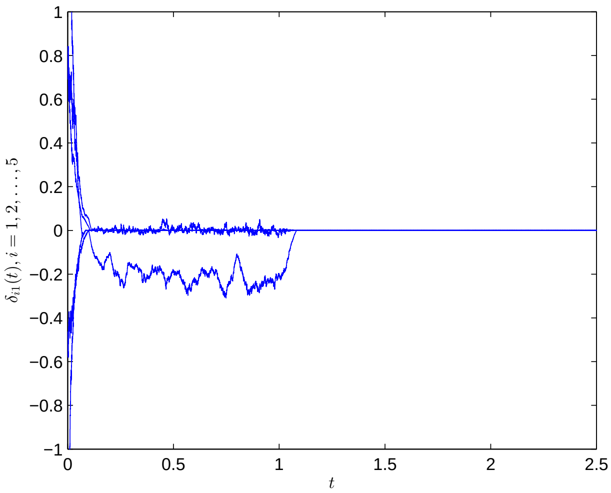

Figure 2. Time response of the error variables $\delta_{i1}(t)$ via controller (3.1).

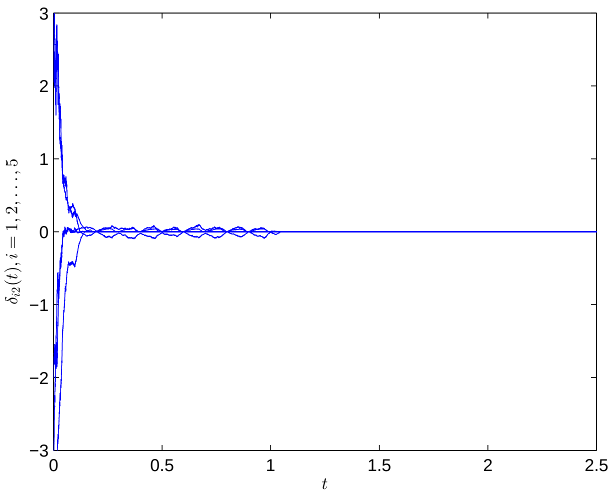

Figure 3. Time response of the error variables $\delta_{i2}(t)$ via controller (3.1).

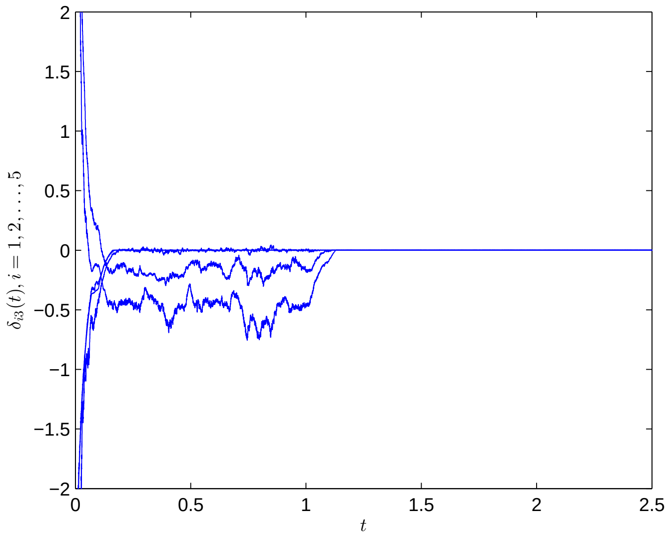

Figure 4. Time response of the error variables $\delta_{i3}(t)$ via controller (3.1).

5. Conclusions

In this paper, we consider the FTS of the general NSs model. This model includes intermittent delayed couplings, partial couplings, stochastic perturbations, and state-dependent parameters. In order to achieve synchronization, an intermittent control scheme is designed. Then, FTS is achieved, and the settling time is also estimated via the Lyapunov-based analytical method. The corresponding control method is applied to two simplified NSs as special cases. Moreover, we conduct numerical simulations to validate the obtained results.

Note that faults are inevitable in NSs, and fault diagnosis using observers has garnered significant research interest, as seen in [34]. Building on our work in this paper, future research will explore how to develop effective fault estimation methods for NSs. In addition, while considering partial couplings with node communication states, this work naturally raises an important question: How might channel access rates affect nodal coupling dynamics? This will be another research topic for our future work.

Use of Generative-AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

Natural Science Foundation of Chongqing, China (Grant No. CSTB2023NSCQ-MSX0771), the Science and Technology Research Program of Chongqing Municipal Education Commission of China (Grant No. KJZD-K202300502).

Conflict of interest

The author declares no conflicts of interest in this paper.

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