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Research article

Leader-follower consensus of a fractional-order multi-agent system based on event-triggered control and extended state observer

Xingyun Shi and Xuqiong Luo*

School of Mathematics and Statistics, Changsha University of Science and Technology, Changsha, 410114, China

* **Correspondence:** Email: luoxuqiong@csust.edu.cn.

Abstract: A leader-follower consensus of a fractional-order multi-agent system with external disturbances is addressed throughout this paper. To suppress disturbance, we established a predictive extended state observer and designed an event-triggered control. Utilizing the Lyapunov method along with stability theory for fractional-order systems, we constructed a sufficient condition to attain a leader-follower consensus. Furthermore, we demonstrated that Zeno behavior does not appear during the triggering process. Last, we performed some numerical simulations to confirm the effectiveness of the control strategy.

Keywords: fractional-order multi-agent system; leader-follower consensus; Lyapunov method; event-triggered control; extended state observer
Mathematics Subject Classification: 34D23, 34H05, 93A16, 93D05

1. Introduction

Many specialists have described the dynamics of multi-agent systems using integer-order models over the past few decades and have achieved significant results, see for example [1–3] and references therein. However, the fractional-order model is better suited for representing complex systems compared to the integer-order model due to its nonlocality and memorability. For instance, many natural phenomena (stress relaxation in viscoelastic materials, diffusion at the electrode-electrolyte interface, etc.) exhibit behaviors between purely elastic and purely viscous. In this regard, integer-order models are inefficient in accurately capturing the dynamics of non-integer orders, since the integer-order derivatives rely only on the local informations in a neighborhood around the current moment. However, fractional-order differential operators are essentially nonlocal, which enables a more natural description of physical systems with memory, heredity, long range interactions, and full-domain coupling. With additional studies, the fractional-order system has been extensively used in various areas including fluid dynamics, control systems, transportation, and bioengineering. Moreover, consensus research is a fundamental topic in multi-agent systems and has garnered significant interest and investigation [4–8]. To achieve consensus, it is essential to establish a suitable control protocol. Currently, the commonly used control protocols are the distributed control protocols, including adaptive control [9], bipartite containment control [10–13], sliding model control [13, 14], observer control [13, 15, 16], intermittent control [17], and event-triggered control [16, 18–21].

Different from other control methods, event-triggered control can eliminate the need for continuous information exchange and transmit data only after a predetermined event occurs. It not only ensures optimal system performance but also reduces communication costs. Zhang et al. [16] explored the secure practical fault-tolerant output regulation of multi-agent systems with denial-of-servic attacks, where the adaptive edge-event-triggered observer is designed to estimate the leader's matrix and state to overcome the impact of DoS attacks. Chang et al. [21] designed two periodic event-triggered control protocols to achieve the output feedback leaderless consensus and leader-follower consensus for nonlinear multi-agent systems with uniform input delays. Zhao et al. [22] discussed the prescribed-time synchronization issue in networks of piecewise smooth systems, in which a novel event-triggered controller is designed to realize prescribed-time synchronization objectives. You et al. [23] explored the fuzzy tracking problem through an event-triggered adaptive control in a fractional-order nonlinear system. For the above, most of the constructed event-triggered mechanisms ensure the accuracy of triggering by setting the appropriate trigger conditions for different agents. Therefore, inspired by this, we delve into more applicable event-triggered mechanisms.

Notably, the results [16, 18, 20–22] were primarily obtained in ideal environments. In fact, multi-agent systems may suffer from small fluctuations or disturbances that could have negative effects on consensus. For example, Ning et al. [13] concerned the bipartite consensus tracking for multi-agent systems with unknown disturbances. To maintain consensus, external controls are typically used to attenuate or compensate for disturbances. Now, observer control is the main approach for disturbance suppression [24, 25]. Observer controls mostly contain disturbance observer control [26–28] and extended state observer control [29–33]. Compared with disturbance observer, extended state observer can provide greater robustness to the system. Wu et al. [34] focused on trajectory tracking control using an extended state observer in a system affected by model uncertainties and external disturbances. Shen et al. [35] investigated a nonlinear uncertain multi-agent system affected by mismatched uncertainties and disturbances, where a finite-time extended state observer is proposed for each agent to estimate the unavailable state and external disturbances. Thai et al. [36] introduced a fuzzy adaptive finite time extended state observer to enhance system performance. We note that the existing disturbances [10, 13, 26–36] are almost described by constant, nonlinear functions or integer-order generation systems. However, multi-agent systems are often subject to unknown or nonlocal disturbances that may affect all agents. Thus, these descriptions of disturbance are not comprehensive, which motivates us to study more general disturbance generation systems and extended state observers to address the impacts of complex communications.

Current multi-agent systems face disturbance crisis in electric power and unmanned clusters, etc. While existing methods suffer from the inability of traditional integer-order control to model history-dependent disturbances, classical observers are prone to trigger oscillation dispersion due to phase hysteresis. Based on the above discussion, we examined a fractional-order multi-agent system affected by nonlinear external disturbances. On the one hand, the disturbances are more realistically described by a fractional-order disturbance generation system. It can describe most of the existing disturbances such as constant disturbance, fractional harmonic disturbance, and unknown noise disturbance. On the other hand, we take into account the self-resistance factor of agents in the disturbance generation system and the circumstance that all agents are influenced by the disturbances. Thus, the dynamic description of this disturbance is more in line with reality and has greater research significance. Consequently, to achieve the consensus of the system, we constructed a predictive extended observer and used the real-time disturbance estimations obtained from it to compensate for disturbances. Additionally, we design an event-triggered control to reduce output frequency and resource usage. Therefore, we utilize extended state observer along with event-triggered mechanism to realize leader-follower consensus within the system. Moreover, there are two difficulties to be solved: One is the validation of the model observability after extended state, and the other is to prove that Zeno behavior does not occur during the triggering process.

In comparison to previous related research, the key contributions of this study are emphasized in the following areas:

(1) Unlike [10,35–39], from the practical engineering perspective, we explicitly model the leader as being affected by external disturbances. Furthermore, considering that agents inherently possess certain disturbance-rejection capabilities, we incorporate the effect of the agent disturbance-rejection factor into the mathematical representation of external disturbance dynamics. Therefore, this formulation results in a more comprehensive and practically aligned representation of the external disturbance dynamics.

(2) We extend the consensus problem of multi-agent systems to the fractional-order case. To suppress the impact of external disturbances, the corresponding extended state observer design is simultaneously generalized to the fractional-order model. Consequently, whether the fractional-order extended state observer satisfies observability is a problem that needs to be addressed. This provides a novel solution pathway for addressing disturbances with non-local effects. The results proposed in references [35, 36, 38, 39] primarily address integer-order multi-agent systems and integer-order extended state observers, which can be regarded as a particular case of our work.

(3) In this study, observer estimation is used to compensate for the effects of external communication disturbances suffered by a multi-agent system, thus improving the robustness of the system. It also develops control strategies for non-local disturbance compensations to maintain system stability and performance. Moreover, it provides advanced control protocols to cope with complex communication environments.

This paper's structure is summarized as below. In Part 2 and Part 3, we review relevant mathematical concepts and present the dynamics of fractional-order multi-agent system, respectively. Consensus protocol that incorporates an extended state observer and event-triggered mechanisms is constructed in Part 4. Based on this control, we obtain a sufficient condition to realize consensus in the leader-follower system, while ensuring that Zeno behavior is avoided in the triggering process. In the end, we conduct some numerical examples to check the effectiveness of our method.

2. Mathematical knowledge

Notations: Symbols used in this paper are given in Table 1.

Table 1. Notations.

Symbols	Meaning
$\mathbb{R}$	Set of Real Numbers
$\mathbb{R}^n$	n-Dimensional Euclidean Space
$\mathbb{R}^{n\times n}$	n-Order Square Matrix
$\mathbb{S}^{n\times n}$	Symmetric Space
$\text{diag}\{\cdots\}$	Diagonal Matrix
I_n	n-Dimensional Identity Matrix
$S^T\left(S^{-1}\right)$	Transpose (Inverse) of S
$\lambda_{\max}(S)\,(\lambda_{\min}(S))$	Maximal (Minimal) Eigenvalue of S
$S>0\,(S<0)$	Positive (Negative) Definite Matrix
$1_N(0_N)$	N-Dimensional all 1's (0's) Column Vector
$\mathbf{0}$	Null Matrix of Suitable Dimensions
$\Vert\cdot\Vert$	2-Norm vector
$\otimes$	Kronecker product

The interactions between agents are depicted by a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, in which $\mathcal{V}=\{0,1,2,\ldots,N\}$ represents the node set and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ denotes the edge set. $e_{ij}=(i,j)\in\mathcal{E}(0\le i,j\le N,i,j\in\mathbb{Z}^+)$ stands for the directed flow of messages to node i from node j, and j is called a neighbor of i. $\mathcal{A}=\left[a_{ij}\right]\in\mathbb{R}^{(N+1)\times(N+1)}(0\le i,j\le N)$ indicates the adjacency matrix. When $e_{ij}\in\mathcal{E},a_{ij}=1$, otherwise $a_{ij}=0$. In particular, a_{ii} is always assumed to be 0. $\mathcal{L}=\left[l_{ij}\right]\in\mathbb{R}^{(N+1)\times(N+1)}$ stands for the Laplace matrix, with $l_{ij}=-a_{ij}$, when $i\ne j$, otherwise $l_{ij}=\sum_{i,j=0}^{N}a_{ij}$.

Lemma 2.1. *[40] Zero is a simple eigenvalue of $\mathcal{L}$, and the real parts of all non-zero eigenvalues are positive if and only if $\mathcal{G}$ contains a directed spanning tree.*

Definition 2.2. *[41] For $f\in L^1[0,+\infty)$, the α-order integral is given as*

$$ {}_0D_t^{-\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-u)^{\alpha-1}f(u)du, $$

with $\Gamma(\alpha)=\int_0^{+\infty}t^{\alpha-1}e^{-t}dt,\alpha>0$.

Definition 2.3. *[41] For $f\in C^n[0,+\infty)$, the α-order Caputo derivative is defined by*

$$ {}_0D_t^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t\frac{f^{(n)}(u)}{(t-u)^{\alpha-n+1}}du, $$

with $0\le n-1<\alpha<n,n\in\mathbb{Z}^+$.

Lemma 2.4. *[41] If $y(t)$ is a differentiable function in $[a,b]$, then*

$$ {}_aD_t^{-\alpha}\,{}_aD_t^{\alpha}y(t)=y(t)-y(a), $$

where $\alpha\in(0,1)$.

Definition 2.5. *[41] One-parameter Mittag-Leffler function is given by*

$$E_\alpha(z) = \sum_{l=0}^{\infty} \frac{z^l}{\Gamma(\alpha l + 1)},$$

where $\alpha > 0, l \in \mathbb{N}$ *and* $z \in \mathbb{C}$.

Lemma 2.6. *[42] For* $\forall t \geq 0$, *suppose* $s(t) \in \mathbb{R}^n$ *is continuously differentiable, then the following holds*

$${}_0D_t^\alpha \left(s^T(t) P s(t) \right) \leq 2 s^T(t) P {}_0D_t^\alpha s(t),$$

where $\alpha \in (0, 1], P > 0 \in \mathbb{R}^{n \times n}$.

Lemma 2.7. *[43] For* $\forall \zeta \in \mathbb{R}^n, \xi \in \mathbb{R}^n$ *and* $\Omega > 0 \in \mathbb{R}^{n \times n}$, *the following holds*

$$2\zeta^T \xi \leq \zeta^T \Omega \zeta + \xi^T \Omega^{-1} \xi.$$

Lemma 2.8. *[44] For* $H > 0 \in \mathbb{R}^{n \times n}$ *and* $D \in \mathbb{S}^{n \times n}$, *the following holds*

$$\lambda_{\min}\left(H^{-1} D\right) x^T H x \leq x^T D x \leq \lambda_{\max}\left(H^{-1} D\right) x^T H x,$$

for all $x \in \mathbb{R}^n$.

Definition 2.9. *[45] For* $t \geq t_0, \alpha \in (0, 1)$ *and* $s(t) \in \mathbb{R}^n$, *if there exist constants* $\vartheta > 0, M > 0, \sigma > 0$, *s.t.,*

$$\|s(t)\| \leq M \left[E_\alpha \left(-\sigma (t - t_0)^\alpha \right) \right]^\vartheta,$$

then $s(t)$ *is known as Mittag-Leffler convergent to zero.*

Lemma 2.10. *[46] For a differentiable function* $s(t) \in \mathbb{R}^n, 0 < \alpha < 1$ *and* $\sigma > 0$, *if the continuous function* $V : [t_0, +\infty) \times \mathbb{R}^n \to \mathbb{R}$ *satisfies*

$${}_0D_t^\alpha V(t, s(t)) \leq -\sigma V(t, s(t)),$$

then

$$V(t, s(t)) \leq V(t_0, s(t_0)) E_\alpha \left[-\sigma (t - t_0)^\alpha \right].$$

Lemma 2.11. *Assume that a nonlinear system is expressed as follows*

$$\begin{cases} {}_0D_t^\alpha s(t) = A s(t) + \gamma(t), \\ m(t) = \mathrm{Cs}(t), \end{cases} \tag{2.1}$$

with $0 \leq n - 1 < \alpha < n, n \in \mathbb{Z}^+, s(t) \in \mathbb{R}^n, m(t) \in \mathbb{R}^q, \gamma(t) \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}, C \in \mathbb{R}^{q \times n}$. *If the nonlinear system* (2.1) *is observable, then*

$$\mathrm{rank} \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n.$$

Proof. For $m(t)$, calculating its fractional derivative yields

$$\begin{aligned}
{}_0D_t^{\alpha}m(t) &= CAs(t) + C\gamma(t),\\
{}_0D_t^{2\alpha}m(t) &= CA^2s(t) + CA\gamma(t) + C_0D_t^{\alpha}\gamma(t),\\
&\vdots\\
{}_0D_t^{(n-1)\alpha}m(t) &= CA^{n-1}s(t) + CA^{n-2}\gamma(t) + \cdots + C_0D_t^{(n-2)a}\gamma(t),
\end{aligned}$$

with ${}_0D_t^{2\alpha}m(t) = {}_0D_t^{\alpha}\left({}_0D_t^{\alpha}m(t)\right), \cdots, {}_0D_t^{(n-1)\alpha}m(t) = {}_0D_t^{\alpha}\left({}_0D_t^{(n-2)\alpha}m(t)\right)$, and ${}_0D_t^{(n-1)\alpha}s(t), {}_0D_t^{(n-1)\alpha}\gamma(t)$ are in the similar forms.

Let

$$M(t) = \begin{bmatrix} m(t) \\ {}_0D_t^{\alpha}m(t) \\ \vdots \\ {}_0D_t^{(n-1)\alpha}m(t) \end{bmatrix}, \Upsilon(t) = \begin{bmatrix} \gamma(t) \\ {}_0D_t^{\alpha}\gamma(t) \\ \vdots \\ {}_0D_t^{(n-1)\alpha}\gamma(t) \end{bmatrix}.$$

Then

$$M(t) = \Xi s(t) + \Lambda\Upsilon(t),$$

with

$$\Xi = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}, \Lambda = \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ C & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ CA & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \cdots & \vdots & \vdots & \vdots \\ CA^{n-2} & \cdots & CA & C & \mathbf{0} \end{bmatrix}.$$

If $\text{rank}(\Xi) < n$, then $\exists \xi \neq \mathbf{0} \in \mathbb{R}^n$ s.t. $\Xi\xi = \mathbf{0}$, i.e., $\xi \in \text{Ker}(\Xi)$. Thus,

$$\Xi\left(s\left(t_0\right) + \xi\right) = M\left(t_0\right) - \Lambda\Upsilon\left(t_0\right). \tag{2.2}$$

Since

$$\Xi s\left(t_0\right) = M\left(t_0\right) - \Lambda\Upsilon\left(t_0\right), \tag{2.3}$$

this is contrary to the fact that the initial state $m\left(t_0\right)$ uniquely determines the measurement output $s\left(t_0\right)$. Thus, $\text{rank}(\Xi) = n$. □

Lemma 2.12. *The nonlinear system* (2.1) *is observable equivalent to*

$$\text{rank}\begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix} = n, \textit{ for } \forall\lambda \in \mathbb{C}.$$

Proof. **Necessity.** To prove this, we need only show that

$$\text{rank}\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n \Rightarrow \text{rank}\begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix} = n.$$

Suppose that there exists $\lambda \in \mathbb{C}$ s.t. $\operatorname{rank}\begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix} < n$. Then $\begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix}$ is a columnar linear correlation.

Thus, there must be $\varsigma \neq \mathbf{0} \in \mathbb{R}^n$ s.t. $\begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix}\varsigma = \mathbf{0}$, that is $A\varsigma = \lambda\varsigma, C\varsigma = \mathbf{0}$.

Consequently, $CA\varsigma = C\lambda\varsigma = \mathbf{0}, \cdots, CA^{n-1}\varsigma = C\lambda^{n-1}\varsigma = \mathbf{0}$. Then $\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}\varsigma = \mathbf{0}$, since $\varsigma \neq \mathbf{0}$, we have $\operatorname{rank}\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} < n$. This contradicts to $\operatorname{rank}\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$, thus $\operatorname{rank}\begin{bmatrix} \lambda I_n - A \\ C \end{bmatrix} = n$.

Sufficiency. Suppose system (2.1) is unobservable. Then, system (2.1) can be decomposed in terms of observable and unobservable. Thus, there exists non-singular linear transformation $\Phi \in \mathbb{R}^{n\times n}$ acting on matrix pair (A, C) that yields

$$\bar{A} = \Phi^{-1}A\Phi = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \mathbf{0} & \bar{A}_{22} \end{bmatrix}, \bar{C} = C\Phi = \begin{bmatrix} \mathbf{0} & \bar{C}_1 \end{bmatrix}.$$

Here, $\bar{A}_{22} \in \mathbb{R}^{(n-h)\times(n-h)}, \bar{C}_1 \in \mathbb{R}^{q\times(n-h)}$ and $\bar{A}_{11} \in \mathbb{R}^{h\times h}, \bar{A}_{12} \in \mathbb{R}^{h\times(n-h)}, \mathbf{0} \in \mathbb{R}^{q\times h}$ denote the observability and unobservability parts of the system decomposition, respectively. Let $\lambda_* \in \mathbb{C}$ be an eigenvalue of $\bar{A}_{11}$; hence, it is also an eigenvalue of A. Moreover, let $\varsigma_* \in \mathbb{R}^h$ be the eigenvector associated with the eigenvalue λ_*.

Setting $\varsigma = \Phi\begin{bmatrix} \varsigma_* \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^n$ such that

$$C\varsigma = \begin{bmatrix} \mathbf{0} & \bar{C}_1 \end{bmatrix}\Phi^{-1}\Phi\begin{bmatrix} \varsigma_* \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \bar{C}_1 \end{bmatrix}\begin{bmatrix} \varsigma_* \\ \mathbf{0} \end{bmatrix} = \mathbf{0},$$
$$A\varsigma = \Phi\begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \mathbf{0} & \bar{A}_{22} \end{bmatrix}\Phi^{-1}\Phi\begin{bmatrix} \varsigma_* \\ \mathbf{0} \end{bmatrix} = \Phi\begin{bmatrix} \bar{A}_{11}\varsigma_* \\ \mathbf{0} \end{bmatrix} = \Phi\begin{bmatrix} \lambda_*\varsigma_* \\ \mathbf{0} \end{bmatrix} = \lambda_*\varsigma.$$

Then, for $\varsigma \neq \mathbf{0} \in \mathbb{R}^n$, we get $\begin{bmatrix} \lambda_* I_n - A \\ C \end{bmatrix}\varsigma = \mathbf{0}$.

This contradicts to $\operatorname{rank}\begin{bmatrix} \lambda_* I_n - A \\ C \end{bmatrix} = n$. Thus, system (2.1) is observable. □

Lemma 2.13. *[47] For a matrix* $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix}$*, it satisfies* $M_{11} = M_{11}^T, M_{22} = M_{22}^T$*, then the following terms are equivalent*

(1) $M < 0$,
(2) $M_{22} < 0, M_{11} - M_{12}M_{22}^{-1}M_{12}^T < 0$,
(3) $M_{11} < 0, M_{22} - M_{12}^T M_{11}^{-1} M_{12} < 0$.

3. Problem formulation

We consider a fractional-order multi-agent system that comprises N followers and a single leader. The i-th follower's dynamic can be characterized by

$$\begin{cases} {}_0D_t^\alpha s_i(t) = As_i(t) + Bc_i(t) + Jw_i(t), \\ m_i(t) = Cs_i(t), 1 \le i \le N, i \in \mathbb{Z}^+. \end{cases} \tag{3.1}$$

Leader's dynamic can be considered by

$$\begin{cases} {}_0D_t^\alpha s_0(t) = As_0(t) + Jw_0(t), \\ m_0(t) = Cs_0(t), \end{cases} \tag{3.2}$$

with $0 < \alpha < 1$, $\mathrm{s}_i(t) \in \mathbb{R}^n, \mathrm{c}_i(t) \in \mathbb{R}^p, m_i(t) \in \mathbb{R}^q, w_i(t) \in \mathbb{R}^h$ standing for the i-th follower's state, control input, measured output, and external disturbance. $s_0(t) \in \mathbb{R}^n, m_0(t) \in \mathbb{R}^q, w_0(t) \in \mathbb{R}^h$ represent the leader's state, measured output, and external disturbance, respectively. $n, p, q, h \in \mathbb{Z}^+$. A, B, C, and J are suitable matrices.

The external disturbance $w_i(t) \in \mathbb{R}^h$ is generated by

$$\begin{cases} {}_0D_t^\alpha d_i(t) = \Pi d_i(t), \\ w_i(t) = \delta_i d_i(t), 0 \le i \le N, i \in \mathbb{Z}^+, \end{cases} \tag{3.3}$$

with $0 < \alpha < 1, d_i(t) \in \mathbb{R}^h$ is the factor of disturbance, $\Pi \in \mathbb{R}^{h\times h}$ is a constant matrix, and δ_i is the self-resistance factor.

The following assumptions must be met for systems (3.1)–(3.3).

Assumption 3.1. *$w_i(t) \in \mathbb{R}^h (0 \le i \le N, i \in \mathbb{Z}^+)$ and $s_0(t) \in \mathbb{R}^n$ are bounded, i.e., $\exists M_i > 0, \overline{M_i} > 0, \phi > 0, \phi_1 > 0$, such that $\|w_i(t)\| \le M_i, \left\|{}_0D_t^\alpha w_i(t)\right\| \le \overline{M_i}, \|s_0(t)\| < \phi, \left\|{}_0D_t^\alpha s_0(t)\right\| < \phi_1$.*

Assumption 3.2. *Matrix pairs are stable and observable for (A, B) and (Π, J), respectively. C is a column-full rank matrix. The eigenvalues of Π are distinct and on the imaginary axis.*

Assumption 3.3. *The external disturbance $w_i(t) \in \mathbb{R}^h (0 \le i \le N, i \in \mathbb{Z}^+)$ is matched, i.e., $\exists K \in \mathbb{R}^{p\times h}$ s.t. $J = BK$.*

Assumption 3.4. *A directed spanning tree is included in the communication topology $\mathcal{G}$ with the leader as the root.*

By Assumption 3.4, the leader is without neighbors, and $\mathcal{L}$ is split into a block matrix as follows

$$\mathcal{L} = \begin{bmatrix} 0 & 0_N^T \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}, \tag{3.4}$$

with $\mathcal{L}_1 \in \mathbb{R}^{N\times N}, \mathcal{L}_2 \in \mathbb{R}^{N\times 1}$. Furthermore, Lemma 2.1 shows that $\mathcal{L}_1$ is a non-singular M-matrix.

Lemma 3.5. *[48] For the non-singular M-matrix $\mathcal{L}_1$, there exists $\Omega = \mathrm{diag}\{\omega_1, \cdots, \omega_N\} > 0$ such that*

$$\overline{\mathcal{L}} = \Omega\mathcal{L}_1 + \mathcal{L}_1^T\Omega \ge \lambda_0 I_N > 0,$$

in which λ_0 is the minimal non-zero eigenvalue of $\overline{\mathcal{L}}$, and Ω can be generated from $[\omega_1, \cdots, \omega_N]^T = \left(\mathcal{L}_1^T\right)^{-1} 1_N$.

Definition 3.6. *For* $\forall s_i(t_0)(1 \le i \le N, i \in \mathbb{Z}^+)$*, if the tracking error* $\hat{q}_i(t) = s_i(t) - s_0(t)$ *satisfies*

$$\lim_{t\to\infty} \|\hat{q}_i(t)\| = \lim_{t\to\infty} \|s_i(t) - s_0(t)\| = 0,$$

then the systems (3.1)–(3.3) *realize leader-follower consensus.*

The design of control protocol is the key of this paper for achieving system consensus. It is difficult to achieve system consensus by relying solely on a general control protocol of weighted sums of state differences between agents, as the system is subject to external disturbances. To cope with this challenge, we design an extended state observer to compensate for the effects of disturbances, and construct an event-triggered mechanism to reduce output frequency and conserve computational storage resources.

4. Major results

Define $e_i(t) = \left[s_i^T(t), w_i^T(t)\right]^T$, then the systems (3.1)–(3.3) can be rewritten as below

$$\begin{cases} {}_0D_t^{\alpha} e_i(t) = \bar{A}e_i(t) + \bar{B}c_i(t), \\ m_i(t) = \bar{C}e_i(t), 0 \le i \le N, i \in \mathbb{Z}^+, \end{cases} \tag{4.1}$$

with

$$\bar{A} = \begin{bmatrix} A & J \\ \mathbf{0} & \Pi \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ \mathbf{0} \end{bmatrix}, \bar{C} = \begin{bmatrix} C & \mathbf{0} \end{bmatrix}, c_0(t) = 0.$$

Theorem 4.1. *The system* (4.1) *is observable if Assumptions 3.1–3.3 hold.*

Proof. By Lemma 2.12, assume that system (4.1) is unobservable, so there has to be an eigenvalue $\lambda_* \in \mathbb{C}$ of $\bar{A}$, s.t.

$$\text{rank}\begin{bmatrix} \lambda_* I_n - A & -J \\ \mathbf{0} & \lambda_* I_h - \Pi \\ C & \mathbf{0} \end{bmatrix} < n + h,$$

i.e., $\exists x = \left[x_1^T, x_2^T\right]^T \neq \mathbf{0} \in \mathbb{R}^{n+h}$ with $x_1 \in \mathbb{R}^n, x_2 \in \mathbb{R}^h$, s.t.

$$\begin{bmatrix} \lambda_* I_n - A & -J \\ \mathbf{0} & \lambda_* I_h - \Pi \\ C & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{0}.$$

Thus, $Cx_1 = \mathbf{0}$, and from Assumption 3.2, we have $\text{rank}(C) = n$, so it follows that $x_1 = \mathbf{0}$. In addition, $x \neq \mathbf{0}$, thus $x_2 \neq \mathbf{0}$ and $\begin{bmatrix} -J \\ \lambda_* I_h - \Pi \end{bmatrix} x_2 = \mathbf{0}$.

This is inconsistent with the observability of (Π, J), which means that system (4.1) is observable. □

In the sequel, we suppose that the extended state observer is constructed by

$$\begin{cases} {}_0D_t^{\alpha} \bar{e}_i(t) = \bar{A}\bar{e}_i(t) + \bar{B}c_i(t) + G\left(\bar{m}_i(t) - \bar{C}e_i(t)\right), \\ \bar{m}_i(t) = \bar{C}\bar{e}_i(t), 0 \le i \le N, i \in \mathbb{Z}^+, \end{cases} \tag{4.2}$$

with $\bar{e}_i(t) = \left[\bar{s}_i^T(t), \bar{w}_i^T(t)\right]^T, \bar{s}_i(t)$ and $\bar{w}_i(t)$ being the estimations of $e_i(t), s_i(t)$ and $w_i(t)$, respectively. $G = \left[G_s^T, G_w^T\right]^T \in \mathbb{R}^{(n+h)\times q}$ is a gain matrix with $G_s \in \mathbb{R}^{n\times q}$ and $G_w \in \mathbb{R}^{h\times q}$. The operation mechanism of the extended state observer (4.2) is shown in Figure 1.

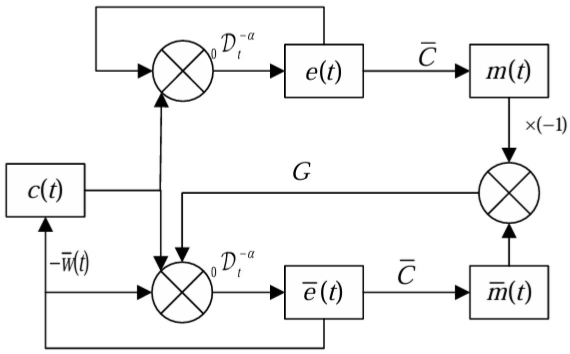


Figure 1. Observer (4.2) operation mechanism.

Remark 4.2. *Extended state observer* (4.2) *is essentially a mathematical model parallel to the real system. The core mechanism is to generate an error signal* $(\bar{m}_i(t) - m_i(t))$ *by comparing the observer output* $\bar{m}_i(t)$ *with the actual system output* $m_i(t)$*, and then dynamically correct the state estimate to ensure that* $\bar{s}_i(t)$ *asymptotically converges to* $s_i(t)$*. The synchronously generated estimation of disturbance* $\bar{w}_i(t)$ *is injected into the control channel to form a feed-forward compensation, which realizes the suppression of disturbance actively.*

Define $p_i(t) = \bar{e}_i(t) - e_i(t) = \left[p_{s_i}^T(t), p_{w_i}^T(t)\right]^T, q_i(t) = \bar{s}_i(t) - \bar{s}_0(t)$ and $h_i(t) = \bar{s}_i\left(t_l^i\right) - \bar{s}_i(t), t \in \left[t_l^i, t_{l+1}^i\right)$ as the observer error, estimated tracking error, and measurement error, respectively, where t_l^i stands for the i-th follower's l-th triggering instant.

From the observer (4.2), the setup of an event-triggered control protocol is shown as below

$$\begin{aligned} c_i(t) &= kHz_i(t) - K\bar{w}_i(t) + K\bar{w}_0(t), 1 \le i \le N, i \in \mathbb{Z}^+, \\ z_i(t) &= \sum_{j=1}^{N} a_{ij}\left(\bar{s}_i\left(t_l^i\right) - \bar{s}_j\left(t_l^i\right)\right) + a_{i0}\left(\bar{s}_i\left(t_l^i\right) - \bar{s}_0(t)\right), \end{aligned} \tag{4.3}$$

where $t \in \left[t_l^i, t_{l+1}^i\right), k \ge \frac{4\lambda_{\max}(\Omega)}{\lambda_0}, H \in \mathbb{R}^{p\times n}$ is a column-full rank gain matrix.

The order of triggering instants is set up as follows

$$t_{l+1}^i = \inf_{l\in\mathbb{Z}^+}\left\{t > t_l^i \mid g_i(t) > 0\right\}, 1 \le i \le N, i \in \mathbb{Z}^+, \tag{4.4}$$

in which the triggering function is built as

$$g_i(t) = \left(\frac{1}{\rho_1} - 1 + \rho_2\right)\|h_i(t)\|^2 - \frac{1-\rho_1}{\|\mathcal{L}_1\|^2}\|z_i(t)\|^2, \tag{4.5}$$

with ρ_1, ρ_2 being constants.

Remark 4.3. *With respect to the i-th agent control protocol* (4.3)*, the first term is set to be a weighted sum of the differences between the states of this agent and all other agents. This weighted sum drives the convergence of the agents' states through local interactions, which is consistent with the general design of control protocols [26, 27]. Subsequently, in the second setting we compensate the external disturbance effects with the estimation of disturbances obtained from extend state observer* (4.2)*. Furthermore, the leaders are also affected by external disturbances; thus the third setting is based on the fact that the disturbances suffered by leader indirectly affect followers.*

Remark 4.4. *For the triggering instant t_l^i, there are only two scenarios that can trigger the i-th follower: One is that the i-th follower gets the current estimation from observer* (4.2) *and passes it to adjacent agents. The other is that the i-th follower receives the new messages from the whole system [27, 31], and the corresponding trigger conditions are designed for different agents. Therefore, each agent transmits only the estimated information of the extended observer to the adjacent agent at its own trigger time, which can reduce unnecessary communication and save resources. In contrast to [28, 29], the triggering mechanism is easier to design and is similar to the mechanism in [26], which is more widely applicable.*

For $p_i(t)$ and $q_i(t)$, we get

$$\begin{aligned} {}_0D_t^{\alpha} q_i(t) =& Aq_i(t) + G_s C p_{s_i}(t) + kBH\Bigg[\sum_{j=1}^{N} a_{ij}\Big(q_i(t) - q_j(t)\Big) + a_{i0}q_i(t) \\ &+ \sum_{j=1}^{N} a_{ij}\Big(h_i(t) - h_j(t)\Big) + a_{i0}h_i(t)\Bigg], \end{aligned} \tag{4.6}$$

$${}_0D_t^{\alpha} p_i(t) = (\bar{A} + G\bar{C})p_i(t). \tag{4.7}$$

Define

$$q(t) = \begin{bmatrix} q_1(t) \\ \vdots \\ q_N(t) \end{bmatrix}, p(t) = \begin{bmatrix} p_0(t) \\ p_1(t) \\ \vdots \\ p_N(t) \end{bmatrix}, p_s(t) = \begin{bmatrix} p_{s_1}(t) \\ \vdots \\ p_{s_N}(t) \end{bmatrix}, h(t) = \begin{bmatrix} h_1(t) \\ \vdots \\ h_N(t) \end{bmatrix}.$$

Then, from (4.6) and (4.7), we have

$${}_0D_t^{\alpha} q(t) = (I_N \otimes A)\, q(t) + (k\mathcal{L}_1 \otimes BH)\, q(t) + (k\mathcal{L}_1 \otimes BH)\, h(t) + (I_N \otimes G_s C)\, p_s(t), \tag{4.8}$$

$${}_0D_t^{\alpha} p(t) = \Big[I_{N+1} \otimes (\bar{A} + G\bar{C})\Big]\, p(t). \tag{4.9}$$

Based on the above discussion, we can provide a sufficient condition for achieving a leader-follower consensus within systems (3.1)–(3.3), as well as Zeno behavior being avoided during the triggering process.

Theorem 4.5. *If Assumptions 3.1–3.4 hold and for $\sigma > 0$, there exist $E > 0 \in \mathbb{S}^{n\times n}$ and $\Theta > 0 \in \mathbb{S}^{(n+h)\times(n+h)}$ such that*

$$\begin{bmatrix} AE^{-1} + E^{-1}A^T - 2BB^T + I_n & E^{-1} \\ E^{-1} & -\frac{1}{\sigma}I_n \end{bmatrix} < 0 \tag{4.10}$$

and

$$\Theta\bar{A}+\bar{A}^T\Theta-2\bar{C}^T\bar{C}+\sigma I_{n+h}<0 \tag{4.11}$$

hold, then the systems (3.1)–(3.3) *realize leader-follower consensus. Here,* $H=-B^TE$ *and* $G=-\Theta^{-1}\bar{C}^T$.

Proof. Constructing function

$$V(t,q(t),p(t))=q^T(t)(\Omega\otimes E)q(t)+p^T(t)\left(I_{N+1}\otimes\bar{\Theta}\right)p(t), \tag{4.12}$$

with $\Omega>0, E>0$ being defined in Lemma 3.5 and (4.10), $\bar{\Theta}=\frac{\lambda_{\max}(\Omega)\|G_sC\|^2}{(1-\mu_1)\sigma}\Theta, 0<\mu_1<1$.

For (4.12), according to Lemma 2.6, we obtain

$$\begin{aligned}{}_0D_t^{\alpha}V(t,q(t),p(t))\le&2q^T(t)(\Omega\otimes E){}_0D_t^{\alpha}q(t)+2p^T(t)\left(I_N\otimes\bar{\Theta}\right){}_0D_t^{\alpha}p(t)\\ =&\,q^T(t)\left[\Omega\otimes\left(EA+A^TE\right)\right]q(t)+2q^T(t)(k\Omega\mathcal{L}_1\otimes EBH)q(t)\\ &+2q^T(t)(k\Omega\mathcal{L}_1\otimes EBH)h(t)+2q^T(t)(\Omega\otimes EG_sC)p_s(t)\\ &+2p^T(t)\left[I_{N+1}\otimes(\bar{\Theta}\bar{A}+\bar{\Theta}G\bar{C})\right]p(t).\end{aligned} \tag{4.13}$$

Combining Lemma 2.8 with Lemma 3.5, we get

$$\begin{aligned}2q^T(t)(k\Omega\mathcal{L}_1\otimes EBH)q(t)&=-kq^T(t)\left[(\Omega\mathcal{L}_1+\mathcal{L}_1^T\Omega)\otimes EBB^TE\right]q(t)\\ &\le-kq^T(t)\left(\lambda_0I_N\otimes EBB^TE\right)q(t)\\ &\le-\frac{k\lambda_0}{\lambda_{\max}(\Omega)}q^T(t)\left(\Omega\otimes EBB^TE\right)q(t).\end{aligned} \tag{4.14}$$

Using Lemma 2.7, it yields that

$$\begin{aligned}2q^T(t)(k\Omega\mathcal{L}_1\otimes EBH)h(t)&=-2q^T(t)\left(k\Omega\mathcal{L}_1\otimes EBB^TE\right)h(t)\\ &\le\frac{k\lambda_0}{2\lambda_{\max}(\Omega)}q^T(t)\left(\Omega\otimes EBB^TE\right)q(t)\\ &\quad+\frac{2k\lambda_{\max}(\Omega)}{\lambda_0}h^T(t)\left(\mathcal{L}_1^T\Omega\mathcal{L}_1\otimes EBB^TE\right)h(t),\end{aligned} \tag{4.15}$$

and

$$2q^T(t)(\Omega\otimes EG_sC)p_s(t)\le q^T(t)\left(\Omega\otimes E^2\right)q(t)+p_s^T(t)\left\{\Omega\otimes\left[(G_sC)^T(G_sC)\right]\right\}p_s(t). \tag{4.16}$$

Based on (4.11) and Lemma 2.8, one can derive

$$\begin{aligned}2p^T(t)\left[I_{N+1}\otimes(\bar{\Theta}\bar{A}+\bar{\Theta}G\bar{C})\right]p(t)=&\frac{\lambda_{\max}(\Omega)\|G_sC\|^2}{(1-\mu_1)\sigma}p^T(t)\left[I_{N+1}\otimes(\Theta\bar{A}+\bar{A}^T\Theta-2\bar{C}^T\bar{C})\right]p(t)\\ \le&-\frac{\lambda_{\max}(\Omega)\|G_sC\|^2}{(1-\mu_1)}p^T(t)(I_{N+1}\otimes I_{n+h})q(t)\\ \le&-\frac{\mu_1\sigma}{\lambda_{\max}(\Theta)}p^T(t)\left(I_{N+1}\otimes\bar{\Theta}\right)p(t)-\lambda_{\max}(\Omega)\|G_sC\|^2\|p(t)\|^2.\end{aligned} \tag{4.17}$$

Substituting (4.14)–(4.17) in (4.13) and by Lemma 2.13, then

$$
\begin{aligned}
{}_0D_t^{\alpha}V(t,q(t),p(t)) \le\, & q^T(t)\left[\Omega\otimes\left(EA+A^TE-\frac{k\lambda_0}{2\lambda_{\max}(\Omega)}EBB^TE+E^2\right)\right]q(t)\\
&+\frac{2k\lambda_{\max}(\Omega)}{\lambda_0}h^T(t)\left(\mathcal{L}_1^T\Omega\mathcal{L}_1\otimes EBB^TE\right)h(t)-\frac{\mu_1\sigma}{\lambda_{\max}(\Theta)}p^T(t)\left(I_{N+1}\otimes\bar{\Theta}\right)p(t)\\
&+p_s^T(t)\left\{\Omega\otimes\left[(G_sC)^T(G_sC)\right]\right\}p_s(t)-\lambda_{\max}(\Omega)\|G_sC\|^2\|p(t)\|^2\\
\le & -\frac{\mu_2\sigma}{\lambda_{\max}(E)}q^T(t)(\Omega\otimes E)q(t)-\frac{\mu_1\sigma}{\lambda_{\max}(\Theta)}p^T(t)\left(I_{N+1}\otimes\bar{\Theta}\right)p(t)\\
&-(1-\mu_2)\sigma q^T(t)(\Omega\otimes I_n)q(t)+\frac{2k\lambda_{\max}(\Omega)\lambda_{\max}\left(\mathcal{L}_1^T\Omega\mathcal{L}_1\otimes EBB^TE\right)}{\lambda_0}\|h(t)\|^2,
\end{aligned}
\tag{4.18}
$$

with $0<\mu_2<1$.

Setting $\bar{q}(t)=h(t)+q(t)=\begin{bmatrix}\bar{q}_1(t)\\ \vdots\\ \bar{q}_N(t)\end{bmatrix}$, and from Lemmas 2.7 and 2.8, it can be deduced that

$$
\begin{aligned}
-q^T(t)(\Omega\otimes I_n)q(t) &\le -\lambda_{\min}(\Omega)q^T(t)q(t)\\
&=-\lambda_{\min}(\Omega)\left(\bar{q}^T(t)\bar{q}(t)-\bar{q}^T(t)h(t)-h^T(t)\bar{q}(t)+h^T(t)h(t)\right)\\
&\le -\lambda_{\min}(\Omega)\left[(1-\rho_1)\|\bar{q}(t)\|^2+\left(1-\frac{1}{\rho_1}\right)\|h(t)\|^2\right],
\end{aligned}
\tag{4.19}
$$

with $0<\rho_1<1$.

From (4.19), then

$$
\begin{aligned}
{}_0D_t^{\alpha}V(t,q(t),p(t)) \le & -\frac{\mu_2\sigma}{\lambda_{\max}(E)}q^T(t)(\Omega\otimes E)q(t)-\frac{\mu_1\sigma}{\lambda_{\max}(\Theta)}p^T(t)\left(I_{N+1}\otimes\bar{\Theta}\right)p(t)\\
&-(1-\mu_2)\sigma\lambda_{\min}(\Omega)\sum_{i=1}^{N}\left[(1-\rho_1)\|\bar{q}_i(t)\|^2+\left(1-\frac{1}{\rho_1}-\rho_2\right)\|h_i(t)\|^2\right],
\end{aligned}
\tag{4.20}
$$

with $\rho_1=\frac{2k\lambda_{\max}(\Omega)\lambda_{\max}\left(\mathcal{L}_1^T\Omega\mathcal{L}_1\otimes EBB^TE\right)}{(1-\mu_2)\sigma\lambda_{\min}(\Omega)\lambda_0}$.

With the help of (4.4) and (4.5), we obtain

$$
\left(\frac{1}{\rho_1}-1+\rho_2\right)\|h_i(t)\|^2\le\frac{1-\rho_1}{\|\mathcal{L}_1\|^2}\|z_i(t)\|^2\,. \tag{4.21}
$$

Setting $z(t)=\begin{bmatrix}z_1(t)\\ \vdots\\ z_N(t)\end{bmatrix}$, it yields

$$
\|z(t)\|\le\|\mathcal{L}_1\otimes I_n\|\,\|\bar{q}(t)\|,
$$

hence

$$
\sum_{i=1}^{N}\|z_i(t)\|^2\le\|\mathcal{L}_1\|^2\sum_{i=1}^{N}\|\bar{q}_i(t)\|^2. \tag{4.22}
$$

Let $\varphi = \min\left\{\frac{\mu_2\sigma}{\lambda_{\max}(E)}, \frac{\mu_1\sigma}{\lambda_{\max}(\Theta)}\right\}$, substituting (4.21) and (4.22) into (4.20), it can be asserted that

$$ {}_0D_t^{\alpha}V(t,q(t),p(t)) \le -\varphi V(t,q(t),p(t)). $$

Applying Lemma 2.10 yields

$$ V(t,q(t),p(t)) \le V(0,q(0),p(0))E_{\alpha}\left(-\varphi t^{\alpha}\right), \tag{4.23} $$

and utilizing Lemma 2.8, it has

$$ \lambda_{\min}(\Omega)\lambda_{\min}(E)\|q(t)\|^2 + \frac{\lambda_{\max}(\Omega)\,\|G_sC\|^2\,\lambda_{\min}(\Theta)}{(1-\mu_1)\sigma}\|p(t)\|^2 \le V(t,q(t),p(t)). \tag{4.24} $$

Let $\vartheta = \min\left\{\lambda_{\min}(\Omega)\lambda_{\min}(E), \frac{\lambda_{\max}(\Omega)\|G_sC\|^2\lambda_{\min}(\Theta)}{(1-\mu_1)\sigma}\right\}$ and combining (4.23) with (4.24), it follows

$$ \vartheta\left(\|q(t)\|^2 + \|p(t)\|^2\right) \le V(t,q(t),p(t)) \le V(0,q(0),p(0))E_{\alpha}\left(-\varphi t^{\alpha}\right), $$

then

$$ \|q(t)\| + \|p(t)\| \le \left[\frac{2V(0,q(0),p(0))}{\vartheta}\right]^{\frac{1}{2}} \left[E_{\alpha}\left(-\varphi t^{\alpha}\right)\right]^{\frac{1}{2}}. $$

From $\hat{q}_i(t) = s_i(t) - s_0(t), 1 \le i \le N, i \in \mathbb{Z}^+$, we get

$$ \|\hat{q}_i(t)\| = \|q_i(t) - p_{s_i}(t) + p_{s_0}(t)\| \le 2\left(\|q(t)\| + \|p(t)\|\right) \le \left[\frac{V(0,q(0),p(0))}{2\vartheta}\right]^{\frac{1}{2}} \left[E_{\alpha}\left(-\varphi t^{\alpha}\right)\right]^{\frac{1}{2}}. $$

Thus, $p(t)$, $q(t)$, and $\hat{q}_i(t)$ are Mittag-Leffler convergent to zero, and systems (3.1)–(3.3) can achieve leader-follower consensus in external disturbance rejection. □

Theorem 4.6. *During the triggering process, Zeno behavior can be avoided, i.e., the duration between triggers is consistently greater than zero.*

Proof. To exclude Zeno behavior, it is necessary to show that the time interval between two events is greater than zero, that is $t_{l+1}^i - t_l^i \ne 0, 1 \le i \le N, i \in \mathbb{Z}^+$. In accordance with the established triggering conditions, we need to discuss two cases within the closed-loop systems (3.1)–(3.3).

Case 1. During the time interval $\left[t_l^i, t_{l+1}^i\right]$ ($1 \le i \le N, i \in \mathbb{Z}^+$), all adjacent agents of the i-th follower are not triggered.

For $h_i(t) = \bar{s}_i(t_l^i) - \bar{s}_i(t), t \in \left[t_l^i, t_{l+1}^i\right]$ and Lemma 2.3, we have

$$ \begin{aligned} \|h_i(t)\| = \|h_i(t) - h_i(t_l^i)\| &= \left\|{}_{t_l^i}D_t^{-\alpha}\,{}_{t_l^i}D_t^{\alpha}h_i(t)\right\| \\ &= \left\|\frac{1}{\Gamma(\alpha)}\int_{t_l^i}^{t}(t-\tau)^{\alpha-1}\,{}_{\tau_l^i}D_{\tau}^{\alpha}h_i(\tau)d\tau\right\| \\ &\le \frac{1}{\Gamma(\alpha)}\int_{t_l^i}^{t}(t-\tau)^{\alpha-1}\left\|{}_{\tau_l^i}D_{\tau}^{\alpha}h_i(\tau)\right\|d\tau \\ &\le \frac{1}{\Gamma(\alpha)}\int_{t_l^i}^{t}(t-\tau)^{\alpha-1}\left(\left\|{}_{\tau_l^i}D_{\tau}^{\alpha}q_i(\tau)\right\| + \left\|{}_{\tau_l^i}D_{\tau}^{\alpha}\bar{s}_0(\tau)\right\|\right)d\tau. \end{aligned} $$

From Theorem 4.2, ${}_{\tau_l^i}D_\tau^\alpha q_i(\tau)$ is bounded, which means that $\exists\eta_1 > 0$ s.t. $\left\|{}_{\tau_l^i}D_\tau^\alpha q_i(\tau)\right\| \leq \eta_1$ holds. Thus, by Assumption 3.1 $\left\|{}_{\tau_l^i}D_\tau^\alpha s_0(\tau)\right\| \leq \phi_1$, we can obtain $\left\|{}_{\tau_l^i}D_\tau^\alpha \bar{s}_0(\tau)\right\| \leq \phi_1$. Let $\eta = \eta_1 + \phi_1$, we get

$$\begin{aligned}\|h_i(t)\| &\leq \frac{\eta}{\Gamma(\alpha)}\int_0^{t-t_l^i} u^{\alpha-1}du\\ &= \frac{\eta}{\Gamma(\alpha+1)}\left(t - t_l^i\right)^\alpha.\end{aligned}$$

Thus,

$$\left\|h_i\left(t_{l+1}^i\right)\right\| \leq \frac{\eta}{\Gamma(\alpha+1)}\left(t_{l+1}^i - t_l^i\right)^\alpha.$$

Consequently,

$$\left[\frac{1-\rho_1}{\left(\frac{1}{\rho_1} - 1 + \rho_2\right)\|\mathcal{L}_1\|^2}\right]^{\frac{1}{2}} \|z_i(t)\| < \|h_i(t)\|.$$

Henceforth,

$$\left[\frac{1-\rho_1}{\left(\frac{1}{\rho_1} - 1 + \rho_2\right)\|\mathcal{L}_1\|^2}\right]^{\frac{1}{2}} \left\|z_i(t_{l+1}^i)\right\| < \frac{\eta\left(t_{l+1}^i - t_l^i\right)^\alpha}{\Gamma(\alpha+1)}.$$

However, the inequality does not hold when $t_{l+1}^i = t_l^i$. That is $t_{l+1}^i \neq t_l^i$, the minimum time interval of the event-triggered is positive.

Case 2. During the time interval $\left[t_l^i, t_{l+1}^i\right](1 \leq i \leq N, i \in \mathbb{Z}^+)$, at least one adjacent agent is triggered.

Assume that the adjacent agent triggered by the i-th follower is the j-th follower ($j \neq i$), that is, the triggering instant t_l^j is after t_l^i. Thus, $t_l^j - t_l^i > 0$ holds. This means that $t_{l+1}^i - t_l^i$ is bigger than $t_l^j - t_l^i$. It is concluded that $t_{l+1}^i - t_l^i$ is strictly positive.

To sum up, the proof has been completed. □

5. Numerical simulation

In order to illustrate the effectiveness of the method, some examples are provided in this part.

Fractional-order mutil-agent systems (3.1)–(3.3) are constructed to fit the dynamic description of many engineering devices, such as DC-DC energy converters. DC-DC converters are the simple power electronic devices that switch one level of input voltage to another level of output voltage [49–51]. The fractional-order DC-DC boost converter is shown in Figure 2, in which both the inductor L and the capacitor C are fractional order energy-storage components, where the switch S_T and the diode S_D are ideal. If S_T is off and S_D is on, $(n + D)Ts < t \leq (n + 1)Ts$, where n is an integer, Ts is the switching period, and D is the duty ratio, which is defined as the ratio of the turn-on time of S_T to the switching period Ts. Thus the fractional-order boost converter can be described by

$$\begin{aligned}\frac{d^\alpha i_L}{dt^\alpha} &= \frac{1}{L}U_{in} - \frac{1}{L}v_C,\\ \frac{d^\beta v_C}{dt^\beta} &= \frac{1}{L}i_L - \frac{1}{RL}v_C,\end{aligned} \tag{5.1}$$

in which, i_L and v_C are states, and α and β are orders of derivations.

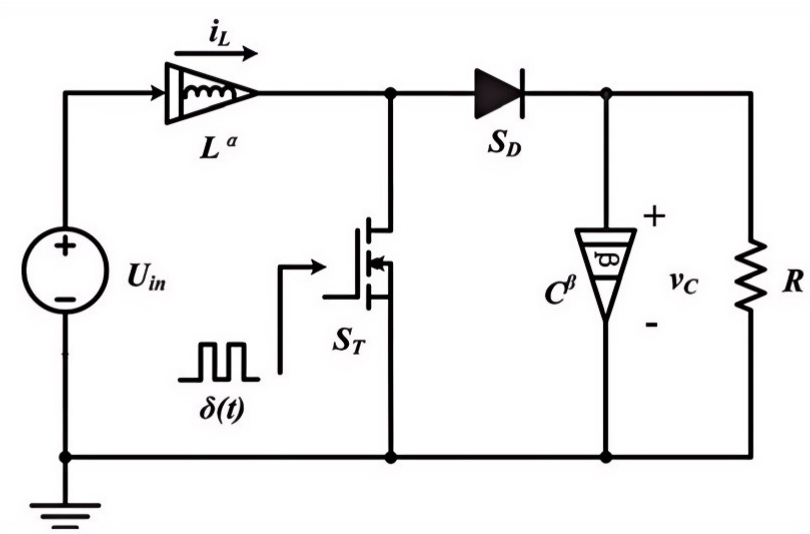


Figure 2. DC-DC boost converter.

Now, consider the fractional-order DC-DC boost converter with $\alpha = \beta$. Thus, formula (5.1) can be transformed to the form of formula (3.1) via the extended variable $x_i(t) = [i_L^T, v_C^T]^T$. In particular, when $\alpha = \beta = 1$, the inductor and capacitor are conventional integer-order components. The construction of the DC-DC converter into a multi-agent system is the current cutting-edge research direction in the field of power electronics, which is especially valuable in distributed energy systems.

Next, numerical simulations are performed, and Figure 3 expresses the fixed communication topology among all agents. In this setup, node 0 acts as the leader, and nodes 1 to 4 are four followers.

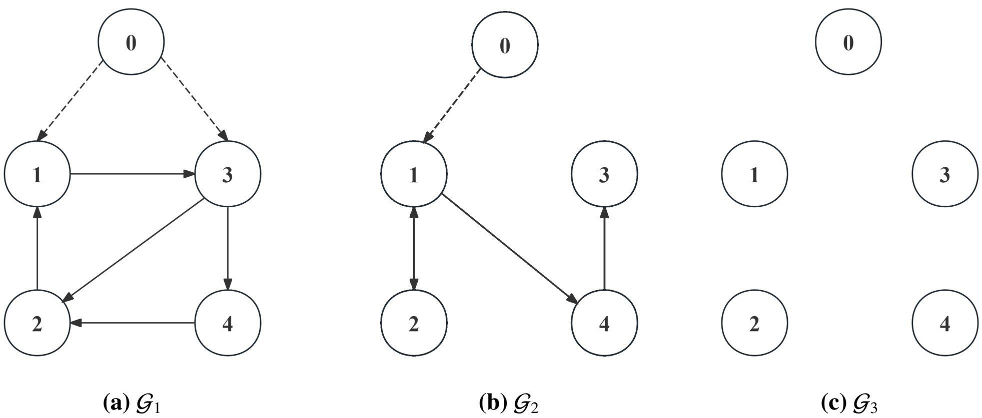


Figure 3. Communication topology.

According to Figure 3 , we can obtain

$$\mathcal{L}_{\mathcal{G}_1}=\begin{bmatrix}0&0&0&0&0\\-1&2&-1&0&0\\0&0&2&-1&-1\\-1&-1&0&2&0\\0&0&0&-1&1\end{bmatrix},\mathcal{L}_{\mathcal{G}_2}=\begin{bmatrix}0&0&0&0&0\\-1&2&-1&0&0\\0&-1&1&0&0\\0&0&0&1&-1\\0&-1&0&0&1\end{bmatrix},\mathcal{L}_{\mathcal{G}_3}=\mathbf{0}_{5\times5},$$

and the matrices of systems (3.1)–(3.3) satisfy

$$A_{\mathcal{G}_1/\mathcal{G}_3}=\begin{bmatrix}-1&0.4&-2.5\\-0.4&0&-1\\2.5&0.1&0\end{bmatrix},\ A_{\mathcal{G}_2}=\begin{bmatrix}0&1&0\\0&-1&1\\-1&0&0\end{bmatrix},\ C=\begin{bmatrix}1.2&0&0\\0&1.2&0\\0&0&1.2\end{bmatrix},$$

$$B=\begin{bmatrix}0.8&0.6\\1.3&-0.5\\-1.1&-0.5\end{bmatrix},\ J=\begin{bmatrix}1.6&0.54\\2.6&-0.45\\-2.2&-0.45\end{bmatrix},\ \Pi=\begin{bmatrix}0&1.6\\-1&0\end{bmatrix},$$

$$\delta_1=\delta_4=0.5,\ \delta_2=0.8,\ \delta_3=\delta_0=0.6,\ k_{\mathcal{G}_1}=5.2,\ k_{\mathcal{G}_2}=14,\ k_{\mathcal{G}_3}=0.$$

Setting $\sigma = 1.1$ and using LMI toolbox, we can get

$$H=\begin{bmatrix}-7.6799&-3.9773&-0.9588\\-3.0425&0.8626&0.7520\end{bmatrix},G=\begin{bmatrix}-26.0117&-14.9504&3.9093\\-14.9504&-11.0956&1.8201\\3.9093&1.8201&-2.3918\\-19.8967&-11.5699&3.2263\\-7.5146&3.6106&1.8415\end{bmatrix}.$$

Initial value choice 1:

$$s_0^T(0)=\begin{bmatrix}-12&0&3\end{bmatrix},\ d_0^T(0)=\begin{bmatrix}2.4&-3.6\end{bmatrix},\ \alpha=0.95,$$

$$\begin{bmatrix}d_1^T(0)\\d_2^T(0)\\d_3^T(0)\\d_4^T(0)\end{bmatrix}=\begin{bmatrix}3.2&2\\1&2.5\\8&8\\12.8&8\end{bmatrix},\ \begin{bmatrix}s_1^T(0)\\s_2^T(0)\\s_3^T(0)\\s_4^T(0)\end{bmatrix}=\begin{bmatrix}-17&7&2\\-4&-8&6\\3&-4&-1\\16&7&-2\end{bmatrix}.$$

Initial value choice 2:

$$s_0^T(0)=\begin{bmatrix}-15&14&-13\end{bmatrix},\ d_0^T(0)=\begin{bmatrix}2.4&-3.6\end{bmatrix},\ \alpha=0.98,$$

$$\begin{bmatrix}d_1^T(0)\\d_2^T(0)\\d_3^T(0)\\d_4^T(0)\end{bmatrix}=\begin{bmatrix}3.2&2\\1&2.5\\8&8\\12.8&8\end{bmatrix},\ \begin{bmatrix}s_1^T(0)\\s_2^T(0)\\s_3^T(0)\\s_4^T(0)\end{bmatrix}=\begin{bmatrix}12&-11&10\\-9&8&-7\\6&-5&4\\-3&2&-1\end{bmatrix}.$$

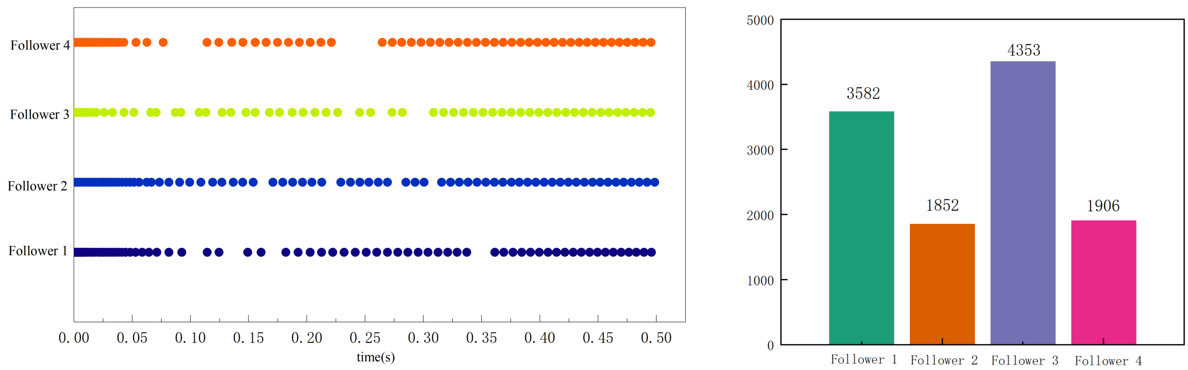
Initial value choice 3:

$$s_0^T(0)=\begin{bmatrix}-3&2&-5\end{bmatrix},\ d_0^T(0)=\begin{bmatrix}3.4&-6.7\end{bmatrix},\ \alpha=0.95,$$

$$\begin{bmatrix}d_1^T(0)\\d_2^T(0)\\d_3^T(0)\\d_4^T(0)\end{bmatrix}=\begin{bmatrix}4&2\\1.25&2.5\\8.4&8.4\\12&8\end{bmatrix},\ \begin{bmatrix}s_1^T(0)\\s_2^T(0)\\s_3^T(0)\\s_4^T(0)\end{bmatrix}=\begin{bmatrix}-4&3&-4\\-4&2&-5\\3&4&-1\\-3&2&-5\end{bmatrix}.$$

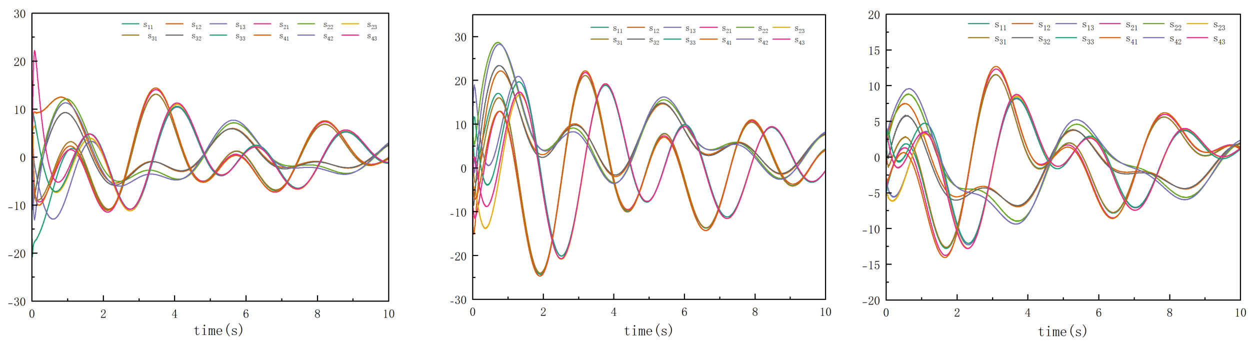
Based on the above conditions, we construct the corresponding mathematical model and simulate it using the SIMULINK platform in MATLAB.

The simulation results are as follows. Figure 4 shows the triggering instant and triggering times of the event-triggered mechanism, respectively. It displays the triggering instant of four followers from 0 to 0.5s as well as the total triggering times of the system with intercepted step size of 0.0001 and simulation time of 10s. The state trajectory of the followers with different initial values is portrayed in Figure 5. It can be seen that despite the different initial values, the state of followers remain convergent. The state trajectories of followers under different communication conditions are illustrated in Figures 6–8. Figure 6 depicts the state trajectory of followers without control, where the system has neither an observer to compensate for disturbances nor event triggers to regulate the interactions among agents. Figure 7 exhibits the state trajectory of followers in $\mathcal{G}_3$, where $k_{\mathcal{G}_3} = 0$ due to the absence of interactions between agents. In this case, only the estimation of disturbance obtained by the observer is used for compensation. Figure 8 depicts the state trajectory of followers under $\mathcal{G}_1$, in which the observer-based event-triggered control (4.3) performs the system. It can be concluded from Figures 6–8 that the composite control protocol we constructed is effective in achieving the consensus of the system.



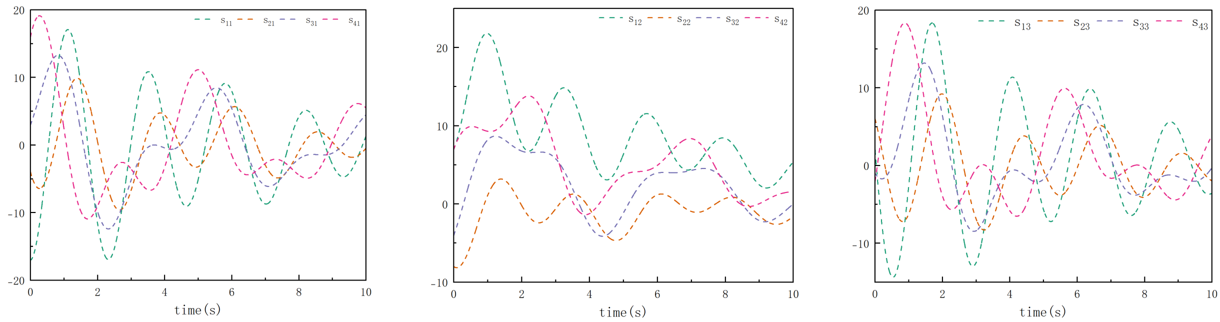
(a) Triggering instants **(b)** Triggering times

Figure 4. Trigger mechanism.



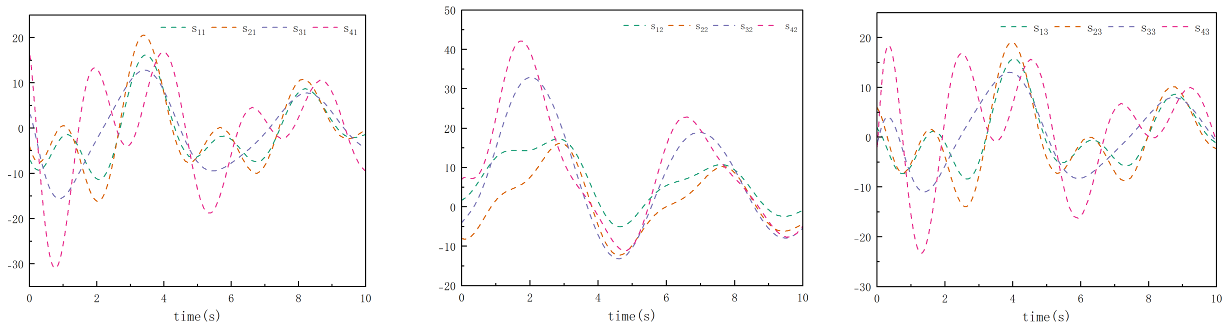
(a) Initial value choice 1 **(b)** Initial value choice 2 **(c)** Initial value choice 3

Figure 5. State trajectory of followers with different initial values.



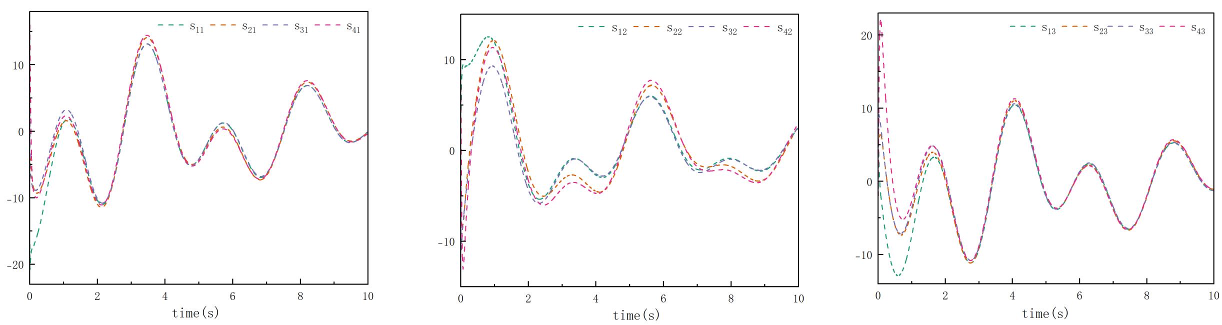
(a) $s_{i1}(t)$ **(b)** $s_{i2}(t)$ **(c)** $s_{i3}(t)$

Figure 6. State trajectory of followers without control.



(a) $s_{i1}(t)$ **(b)** $s_{i2}(t)$ **(c)** $s_{i3}(t)$

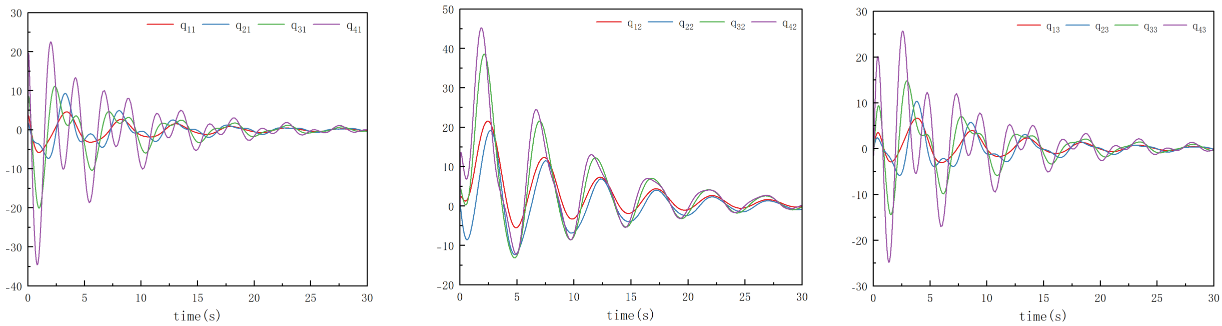
Figure 7. State trajectory of followers under communication topology $\mathcal{G}_3$.



(a) $s_{i1}(t)$ **(b)** $s_{i2}(t)$ **(c)** $s_{i3}(t)$

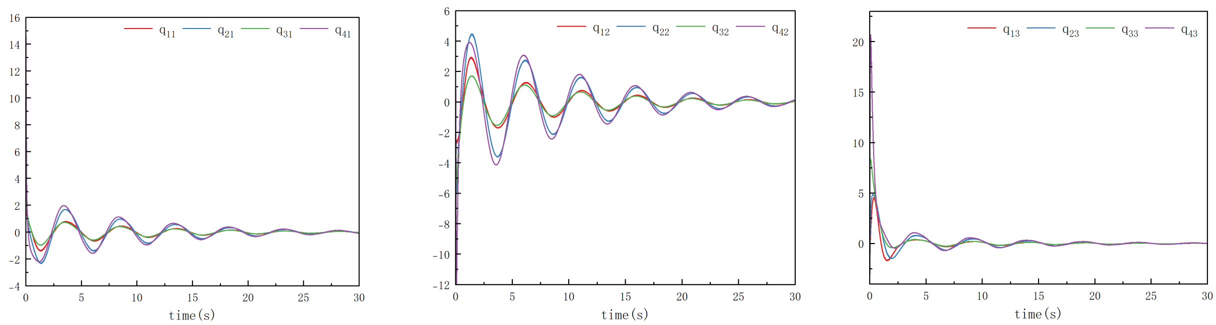
Figure 8. State trajectory of followers under communication topology $\mathcal{G}_1$.

The trajectories of the estimated tracking errors under $\mathcal{G}_1$ and $\mathcal{G}_3$ are presented in Figures 9 and 10, respectively. They show that although the communication topology is different, each follower can track the leader. Figure 11 shows the observer error of the leader and the followers and verifies the effectiveness of the extended state observer (4.2). Figure 12 characterizes the state trajectory of followers at different fractional orders. By comparison, it can be seen that the state trajectories of followers under different orders all converge to 0, and the state trajectory is more fluctuated when the fractional order is higher. Figure 13 demonstrates the comparison of the state of agents between the present systems (3.1)–(3.3) and the system in [26] with the same communication topology $\mathcal{G}_2$ and initial values . As can be seen, the two systems all achieve the leader-follower consensus. However, the value of the state trajectory is tens of times more than that of the literature [26], and this is caused by the presence of external disturbance affecting. Finally, we compare the tracking error paradigm trajectories with the desired result displayed in Figure 14. From this, we can get that the system tracking error is convergent to 0.



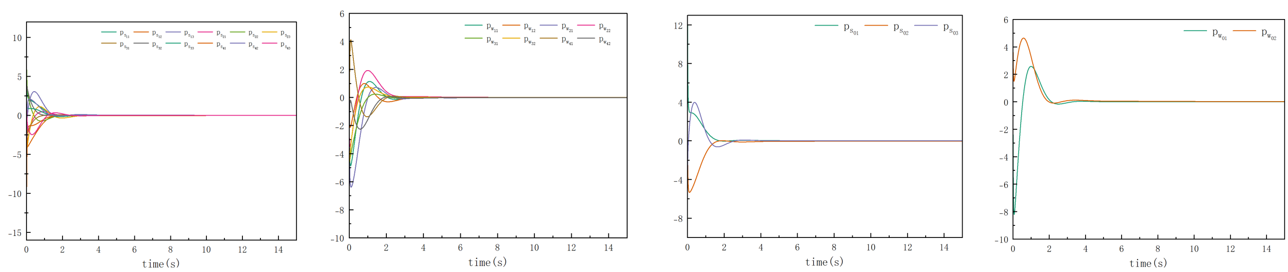
(a) $q_{i1}(t)$ **(b)** $q_{i2}(t)$ **(c)** $q_{i3}(t)$

Figure 9. Estimated tracking error under communication topology $\mathcal{G}_3$.



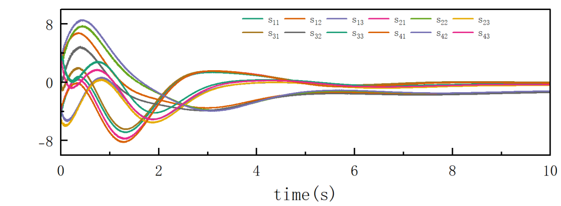
(a) $q_{i1}(t)$ **(b)** $q_{i2}(t)$ **(c)** $q_{i3}(t)$

Figure 10. Estimated tracking error under communication topology $\mathcal{G}_1$.

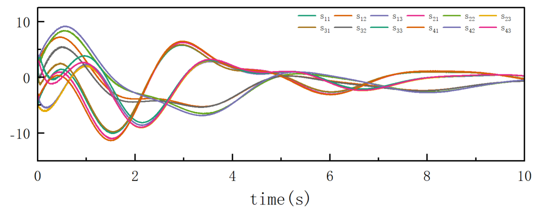


(a) $p_{s_i}(t)$ **(b)** $p_{w_i}(t)$ **(c)** $p_{s_0}(t)$ **(d)** $p_{w_0}(t)$

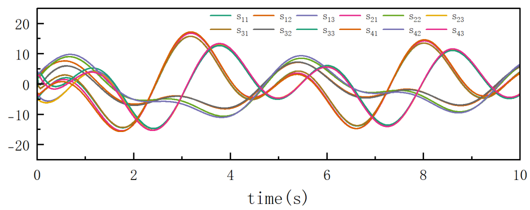
Figure 11. Observer errors.



(a) $\alpha = 0.79$

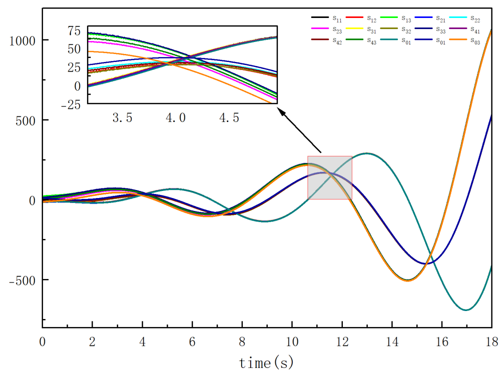


(b) $\alpha = 0.89$

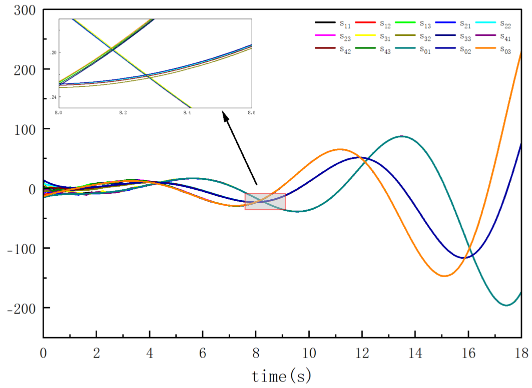


(c) $\alpha = 0.98$

Figure 12. State trajectory of followers at different fractional orders.



(a) the systems (3.1)–(3.3)



(b) the system of the literature [26]

Figure 13. State trajectories of agents under $\mathcal{G}_2$ and initial value choice 3.

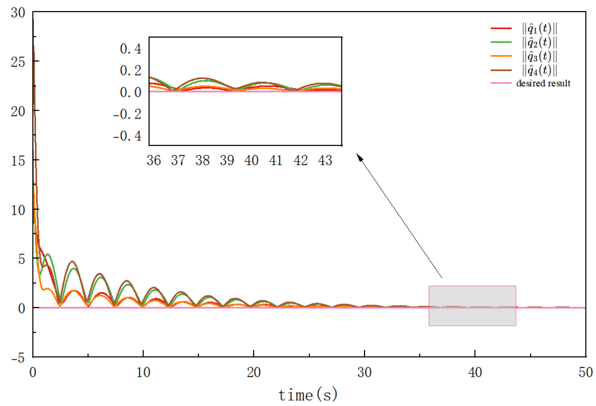


Figure 14. Comparison of tracking errors with desired result.

6. Conclusions

In this paper, we investigate the leader-follower consensus problem in a fractional-order multi-agent system with external disturbances. We consider that all agents in the system are affected by nonlinear external disturbances whose fractional-order generating model is in line with the principles governing the effects of most disturbances. In particular, the self-resistance factor of agents in the external disturbance generation system is considered, resulting in a more comprehensive and meaningful study of disturbance. To realize disturbance rejection and a leader-follower consensus, a composite control that combines an extended state observer with event-triggered control is proposed. The disturbance suppression is performed by applying the estimation of disturbance derived from an extended state observer to the control channel. The corresponding triggering conditions are tailored for different agents in the event-triggered mechanism, so each agent transmits information to its adjacent agents solely at the triggering instant. This reduces the output frequency and storage capacity. Finally, numerical simulations are utilized to verify the validity of the observer and the reasonableness of the sufficient condition for the leader-follower consensus. Next, we will focus on the leader-follower consensus problem in more complex situations, such as communication delay, communication interruption, and the collaboration of heterogeneous order system ($\alpha_i \neq \alpha_j$).

Author contributions

Xuqiong Luo: Wrote the manuscript, conducted simulations, revised the manuscript and edited it, prepared figures and visualizations; Xingyun Shi: Wrote the manuscript, conducted simulations, performed validation, formal analysis, drafted the manuscript, prepared figures and visualizations. All authors have read and agreed to the published version of the manuscript.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no competing interests.

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