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Research article

Study on integrated management model of *Bactericera gobica* Loginova combining resistance and stage structure

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Abstract: In this paper, according to the biological characteristics of *B. gobica*, it was divided into psyllid nymphs and adults, considering the chemical control of *B. gobica* combined with biological control, according to the resistance of psyllid nymphs and adults to pesticides as well as the predatory role of the dominant natural enemies on it, and we established a model of integrated control of *B. gobica* combined with the resistance to pesticides and the structure of stage. We investigated the dynamical properties of the model developed, theoretically proved the consistent boundedness of the system, and demonstrated sufficient conditions for the existence and global attractiveness of the periodic extinction solution of *B. gobica*. The effect of resistance development on *B. gobica* population density was discussed numerically, and the integrated *B. gobica* management strategy was compared with a single control strategy (chemical or biological), the effect of the number of pesticide sprays on the development of resistance to *B. gobica* was investigated, as well as the timing of pesticide switching according to the threshold theory, and, finally, the effect of the rest of the key parameters on the extinction threshold of *B. gobica* was illustrated by the contour plots as well as by sensitivity analyses. The results showed that shortening the release of natural enemies and the pulse period of spraying, controlling the intrinsic growth rate of nymphs, selecting pesticides with lower killing effect on natural enemies, controlling the killing rate of pesticides on natural enemies, and increasing the number of natural enemies released can be of great help to the management of *B. gobica*.

Keywords: *Bactericera gobica* Loginova; pesticide resistance; integrated management; stage structured; sensitivity analysis
Mathematics Subject Classification: 34C60, 92D25, 92D40

1. Introduction

Lycium barbarum L., which is for the Solanaceae genus *Lycium* plants, is a valuable and highly nutritious medicinal [1] that has become one of the largest cultivated traditional Chinese medicines in China [2, 3]. Ningxia wolfberry industry is based on a 'six new six special six excellent' industrial layout. At the end of 2022, the region retained an area of 380,000 acres of wolfberry cultivation, and fresh fruit production of 300,000 tonnes, to achieve a total output value of the industry chain of 27 billion yuan, which is an important industry and characteristic of the regional economy of Ningxia [4]. In recent years, the rapid development of the wolfberry industry, due to the nutritional growth of wolfberry and reproductive growth at the same time and flowers and fruits at the same time [5], vulnerable to a variety of pests, pests of wolfberry insects in wolfberry cultivation that harm with each passing day, will cause huge economic losses.

The goji berry psyllid, *Bactericera gobica* Loginova (*B. gobica*), is one of the most important pests on goji berry plants (*Lycium barbarum* L.) [6, 7] and mainly harms the young branches, leaves, and fruits of wolfberry with its nymph and adult beetles [8]. The psyllid adults exhibit remarkable jumping abilities and demonstrate strong behavioral tendencies, displaying significant preferences for oviposition and feeding on fresh foliage [9]. Nymphs are often fixed in the abaxial leaf damage, and with the growth of the age of the worm, the resistance to pesticides is increasing [10]. At present, the control of *B. gobica* is mainly chemical control [11], and excessive use of chemical pesticides easily pollutes the environment and will lead to the extinction of natural enemies, caused by *B. gobica* resistance, thus promoting the resurgence of *B. gobica* populations; also, the pesticide residue on the *B. gobica* fruit is large, seriously affecting the quality of *B. gobica* [12–14]. Therefore, the use of biological control methods that make full use of predatory natural enemies of *B. gobica* is one of the effective measures to control *B. gobica*, which is of great significance to improve the quality of *Lycium barbarum L.* [15]. The main predatory natural enemies of *B. gobica* are *Deraeocoris punctulatus* Fallen, ladybugs, etc. When feeding from the side of the body of nymphs, it will be pierced by the oral needle to suck the bodily fluids, which are fed on the only body wall left [16]. Numerous studies have shown that [5, 17, 18], the ability of predatory natural enemies of the *B. gobica* to feed on it belongs to the Holling II-type functional response, and the expression of the Holling II-type functional response function is $\Phi(x) = \frac{cxy}{1+ax}$, which is generally applicable to invertebrates [19–21]. At present, many scholars have conducted in-depth research on the use of natural enemies to control pest populations, for example, the use of *Propylea japonica* to prey on *Diaphorina citri* to achieve Huanglongbing control [22], the use of trichogramma to control the corn borer [23], etc., and they have achieved good results. However, the single use of biological control has the problems of high cost, and natural enemies need to be cultivated and reproduced artificially. At present, the control of *B. gobica* mainly follows the plant protection policy of 'prevention as the mainstay, integrated control' [24], i.e., the integrated pest management (IPM) combining chemical control and biological control.

IPM is a system of scientific management of pests, starting from the overall agro-ecosystem, based on the interconnections between pests and the environment, giving full play to the role of natural control factors, and coordinating the application of the necessary measures in accordance with local conditions, so that pests are controlled below the economically permissible level, to obtain the best economic, ecological, and social benefits [25]. Pest management can be studied with the help of kinetic models, and pest management strategies can be studied through quantitative and qualitative analysis of the models [26]. In the effective control of pests, it is not advisable to utilize too many single control strategies, and pest populations should be controlled in a manner that is as complementary as possible. In 2004, Liu, Zhang, and Chen [27] proposed and investigated a Lotka-Volterra predator-prey model with fixed-moment pulses. In the same year, Tang and Chen [28] proposed an IPM strategy with fixed-moment isoperiodic simultaneous spraying of insecticides and release of natural enemies. In 2010, Tang et al. developed several models to study the effects of many factors on IPM strategies, such as the proportion of natural enemies of pests, pesticide dosage and timing, and the instantaneous kill rate of pesticides on both pests and natural enemies [29]. As we all know, the long-term repeated use of the same pesticide will make the pests produce drug resistance and the natural enemy control ability weakening, which will result in the re-infestation of pests and diseases, and may appear such as pesticide residues exceeding the standard, pollution and harm to the ecological environment, and the destruction of biodiversity and other problems. Resistance is a difficult problem plaguing pest control at present. For the resistance of pests to pesticides, in 2013, Liang [30] et al. established a class of pest growth models with the development of resistance. In the same year, Liang [31] proposed a predator-prey model with the development of pest resistance. In 2018, Yang [32] developed a Holling II-type predator-feeder model with feedback control based on the effects of pesticide dosage and frequency of implementation on the development of pest resistance, and investigated the effects of pesticide dosage on the dynamics behaviors of the model. In 2020, Liu [33] developed a hybrid kinetic model considering pesticide residual effects and delayed effects as well as the pest's resistance, which was used to simulate the process of integrated pest management.

Many scholars have researched integrated pest management, but the study of integrated management of *B. gobica* by dynamic modeling is still in a blank state. Based on the above biological background, this paper classifies *B. gobica* into nymphs and adults according to the biological characteristics of *B. gobica*, combines chemical control with biological control by using a dynamic model, and establishes a model for integrated control of *B. gobica* with pesticide resistance and stage structure according to the resistance of nymphs and adults to pesticides and the predation effect of dominant natural enemies on them. The dynamical properties of the developed model are investigated, consistent boundedness of the system is demonstrated, a periodic solution for the extinction of *B. gobica* is given, and conditions for its existence and global attractiveness are proved. Finally, the effect of resistance development on the population density of *B. gobica* was discussed through numerical simulation, and the integrated management strategy of *B. gobica* was compared with the single control strategy of chemical or biological only, and the effect of the number of pesticide sprays on the development of resistance to *B. gobica* was investigated, as well as the time of pesticide switching according to the threshold theory, and the remaining key parameters were illustrated through the contour plots as well as the sensitivity analyses for the extinction threshold.

2. Model formulation

In this paper, it is assumed that the psyllid nymphs population grows in a logistic pattern in the absence of natural enemies. According to the habits, body size, and other characteristics of *B. gobica* in each growth stage, *B. gobica* is divided into two stages, nymphs $x_1(t)$ and adults $x_2(t)$. Since both pesticide spraying and release of natural enemies are instantaneous, assuming that *B. gobica* are controlled by a constant periodic release of natural enemies and pesticide spraying at the same moment, the following model of predator-prey dynamics with fixed-moment impulse control was established

$$\left\{\begin{array}{l}\left.\begin{array}{l}\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}=rx_1(t)\left(1-\frac{x_1(t)}{K}\right)-\frac{c_1x_1(t)y(t)}{1+a_1x_1(t)}-mx_1(t),\\ \frac{\mathrm{d}x_2(t)}{\mathrm{d}t}=mx_1(t)-\frac{c_2x_2(t)y(t)}{1+a_2x_2(t)}-d_2x_2(t),\\ \frac{\mathrm{d}y(t)}{\mathrm{d}t}=\frac{bc_1x_1(t)y(t)}{1+a_1x_1(t)}+\frac{bc_2x_2(t)y(t)}{1+a_2x_2(t)}-d_3y(t),\end{array}\right\} \quad t\neq nT,\\ \left.\begin{array}{l}x_1(nT^+)=(1-p_1)x_1(nT),\\ x_2(nT^+)=(1-p_2)x_2(nT),\\ y(nT^+)=(1-p_3)y(nT)+\tau,\end{array}\right\} \quad t=nT,n\in N,\end{array}\right. \tag{2.1}$$

where $x_1(t)$ and $x_2(t)$ represent the densities of psyllid nymphs and psyllid adults at time t, respectively; $y(t)$ is the population density of natural enemies at time t; r is the intrinsic growth rate of psyllid nymphs population; K is the carrying capacity of psyllid nymphs population; d_2 denotes the death rate of psyllid adults; m is the transformation rate of psyllid nymphs into adults; d_3 denotes the death rate of the natural enemies; $\frac{c_ix_iy}{1+a_ix_i}\,(i=1,2)$ is the Holling II functional response, which responds to the ability of the natural enemies to prey on psyllid nymphs and psyllid adults; b is the rate of conversing prey into the natural enemies; $p_i\,(i=1,2,3)$ represent instantaneous killing efficiency rate of pesticides to psyllid nymphs, psyllid adults, and natural enemies; $\tau>0$ is the release number of natural enemies at time $t=nT$; and T is the pulse period for pesticide spraying and natural enemy release.

The above model did not consider the resistance of *B. gobica* to pesticides. As shown by [34], there are some differences in the resistance of psyllid nymphs and adults of *B. gobica* to pesticides in different production areas of the Ningxia region. Therefore, psyllid nymphs were divided into sensitive to pesticides nymphs $x_{1_s}(t)$ and resistant to pesticides nymphs $x_{1_r}(t)$; psyllid adults were divided into sensitive to pesticides adults $x_{2_s}(t)$ and resistant to pesticides adults $x_{2_r}(t)$. Assuming that the proportion of sensitive to pesticides nymphs $x_{1_s}(t)$ at time t is $q_1(t)$, i.e., $x_{1_s}(t)=q_1(t)x_1(t)$, the proportion of resistant to pesticide nymphs is $1-q_1(t)$,i.e., $x_{1_r}(t)=(1-q_1(t))x_1(t)$. Assuming that the proportion of sensitive to pesticides adults $x_{2_s}(t)$ at time t is $q_2(t)$, the same is true for

$$x_{2_s}(t)=q_2(t)x_2(t),x_{2_r}(t)=(1-q_2(t))x_2(t).$$

When pesticides are sprayed, it is assumed that p_1 denotes the death rate of the sensitive nymphs pests, ξ_1 denotes the death rate of the resistant nymphs, p_2 denotes the death rate of the sensitive adults, and ξ_2 denotes the death rate of the resistant adults, where the mortality rate of sensitive nymphs and adults is higher, i.e., $p_1>\xi_1,p_2>\xi_2$. Thus, sensitive nymphs $x_{1_s}(t)$ and adults $x_{1_r}(t)$ as well as resistant nymphs $x_{2_s}(t)$ and adults $x_{2_r}(t)$ satisfy the following equation :

$$\left\{\begin{array}{l}\frac{\mathrm{d}x_{1_s}(t)}{\mathrm{d}t}=rq_1(t)x_1(t)\left(1-\frac{x_1(t)}{K}\right)-\frac{q_1(t)c_1x_1(t)y(t)}{1+a_1x_1(t)}-q_1(t)mx_1(t)-p_1x_{1_s}(t),\\ \frac{\mathrm{d}x_{1_r}(t)}{\mathrm{d}t}=r(1-q_1(t))x_1(t)\left(1-\frac{x_1(t)}{K}\right)-\frac{(1-q_1(t))c_1x_1(t)y(t)}{1+a_1x_1(t)}-(1-q_1(t))mx_1(t)-\xi_1x_{1_r}(t),\\ \frac{\mathrm{d}x_{2_s}(t)}{\mathrm{d}t}=q_2(t)mx_1(t)-\frac{q_2(t)c_2x_2(t)y(t)}{1+a_2x_2(t)}-q_2(t)d_2x_2(t)-p_2x_{2_s}(t),\\ \frac{\mathrm{d}x_{2_r}(t)}{\mathrm{d}t}=(1-q_2(t))mx_1(t)-\frac{(1-q_2(t))c_2x_2(t)y(t)}{1+a_2x_2(t)}-(1-q_2(t))d_2x_2(t)-\xi_2x_{2_r}(t).\end{array}\right.$$

Resistant pests are assumed to be negligibly affected by pesticides, i.e., $\xi_1,\xi_2=0$; thus, the growth model for psyllid nymphs and adults populations becomes

$$\left\{\begin{array}{l}\frac{\mathrm{d}x_1(t)}{\mathrm{d}t}=\frac{\mathrm{d}x_{1_s}(t)}{\mathrm{d}t}+\frac{\mathrm{d}x_{1_r}(t)}{\mathrm{d}t}=rx_1(t)\left(1-\frac{x_1(t)}{K}\right)-\frac{c_1x_1(t)y(t)}{1+a_1x_1(t)}-mx_1(t)-p_1q_1(t)x_1(t),\\ \frac{\mathrm{d}x_2(t)}{\mathrm{d}t}=\frac{\mathrm{d}x_{2_s}(t)}{\mathrm{d}t}+\frac{\mathrm{d}x_{2_r}(t)}{\mathrm{d}t}=mx_1(t)-\frac{c_2x_2(t)y(t)}{1+a_2x_2(t)}-d_2x_2(t)-p_2q_2(t)x_2(t).\end{array}\right.$$

Calculation based on $x_{1s}(t) = q_1(t)\,x_1(t), x_{2s}(t) = q_2(t)\,x_2(t)$ gives

$$\begin{cases} \frac{dq_1(t)}{dt} = p_1 q_1(t)\left(q_1(t) - 1\right), \\ \frac{dq_2(t)}{dt} = p_2 q_2(t)\left(q_2(t) - 1\right). \end{cases}$$

The initial values are $q_1(0) = q_{10}, q_2(0) = q_{20}$.

Based on the above discussion, the following model for integrated management of *B. gobica* in combination with pesticide resistance was established

$$\begin{cases} \left.\begin{aligned} \frac{dx_1(t)}{dt} &= rx_1(t)\left(1 - \frac{x_1(t)}{K}\right) - \frac{c_1x_1(t)y(t)}{1+a_1x_1(t)} - mx_1(t), \\ \frac{dx_2(t)}{dt} &= mx_1(t) - \frac{c_2x_2(t)y(t)}{1+a_2x_2(t)} - d_2x_2(t), \\ \frac{dy(t)}{dt} &= \frac{bc_1x_1(t)y(t)}{1+a_1x_1(t)} + \frac{bc_2x_2(t)y(t)}{1+a_2x_2(t)} - d_3y(t), \end{aligned}\right\} & t \neq nT, \\ \left.\begin{aligned} x_1(nT^+) &= (1 - p_1q_1(nT))\,x_1(nT), \\ x_2(nT^+) &= (1 - p_2q_2(nT))\,x_2(nT), \\ y(nT^+) &= (1 - p_3)\,y(nT) + \tau, \end{aligned}\right\} & t = nT, n \in N, \\ \frac{dq_1(t)}{dt} = p_1q_1(t)\left(q_1(t) - 1\right), & \\ \frac{dq_2(t)}{dt} = p_2q_2(t)\left(q_2(t) - 1\right). & \end{cases} \tag{2.2}$$

Since the frequency of pesticide spraying, the time of spraying, as well as the concentration and the dosage of pesticide spraying D_n will have a certain effect on the development of resistance, it is known from [30], assuming that the dosage of pesticide for each spraying is a constant, i.e., $D_n = 1\ (n \in N)$, and defining the frequency function of pesticide spraying as $q_n = \frac{n}{T}$, the system (2.2) can be written as

$$\begin{cases} \left.\begin{aligned} \frac{dx_1(t)}{dt} &= rx_1(t)\left(1 - \frac{x_1(t)}{K}\right) - \frac{c_1x_1(t)y(t)}{1+a_1x_1(t)} - mx_1(t), \\ \frac{dx_2(t)}{dt} &= mx_1(t) - \frac{c_2x_2(t)y(t)}{1+a_2x_2(t)} - d_2x_2(t), \\ \frac{dy(t)}{dt} &= \frac{bc_1x_1(t)y(t)}{1+a_1x_1(t)} + \frac{bc_2x_2(t)y(t)}{1+a_2x_2(t)} - d_3y(t), \end{aligned}\right\} & t \neq nT, \\ \left.\begin{aligned} x_1(nT^+) &= (1 - p_1q_1(nT))\,x_1(nT), \\ x_2(nT^+) &= (1 - p_2q_2(nT))\,x_2(nT), \\ y(nT^+) &= (1 - p_3)\,y(nT) + \tau, \end{aligned}\right\} & t = nT, n \in N, \\ \frac{dq_1(t)}{dt} = p_1q_1(t)\left(q_1^{q_n}(t) - 1\right), & \\ \frac{dq_2(t)}{dt} = p_2q_2(t)\left(q_2^{q_n}(t) - 1\right). & \end{cases} \tag{2.3}$$

By calculation, when $t \in [nT, (n+1)\,T]$, the analytic solution of $q_1(t), q_2(t)$ is

$$\begin{cases} q_1(t) = \left(1 + e^{q_np_1(t-nT)}\left((q_1(nT))^{-q_n} - 1\right)\right)^{-\frac{1}{q_n}}, \\ q_2(t) = \left(1 + e^{q_np_2(t-nT)}\left((q_2(nT))^{-q_n} - 1\right)\right)^{-\frac{1}{q_n}}. \end{cases}$$

Thus we have

$$\begin{cases} q_1((n+1)\,T) = \left(1 + e^{np_1}\left(q_1(nT)^{-\frac{n}{T}} - 1\right)\right)^{-\frac{T}{n}}, \\ q_2((n+1)\,T) = \left(1 + e^{np_2}\left(q_2(nT)^{-\frac{n}{T}} - 1\right)\right)^{-\frac{T}{n}}. \end{cases} \tag{2.4}$$

In particular, when $q_n = 1$,

$$\begin{cases} q_1(t) = \frac{q_{10}}{q_{10}+e^{p_1t}(1-q_{10})}, \\ q_2(t) = \frac{q_{20}}{q_{20}+e^{p_2t}(1-q_{20})}. \end{cases}$$

3. Extinction threshold condition of *B. gobica*

Let $R_+ = [0, \infty), R_+^3 = \left\{z \in R_+^3 \mid z > 0\right\}$, $\Omega = \text{int } R_+^3$, Z_+ be the set of all nonnegative integers. Denote as $f = (f_1, f_2, f_3)$ the map defined by the righthand side of the first three equations of system (2.2). Let V_0 be the set of V that satisfies the following conditions, $V = \left\{V : R_+ \times R_+^3 \to R_+\right\}$, then $V \in V_0$ if

(1) V is continuous in $(nT, (n+1)T] \times R_+^3$, and for each $x \in R_+^3, n \in Z_+, \lim_{(t,y)\to(nT^+,z)} V(t,y) = V(nT^+, z)$ exists;

(2) V is locally Lipschitzian in z.

Lemma 3.1. *Let $z(t) = (x_1(t), x_2(t), y(t))$ be a solution of system (2.2) with $z(0^+) \geq 0$ for all $t \geq 0$. Further $z(t) > 0$, $t > 0$ if $z(0^+) > 0$.*

This indicates that any solution for system (2.2) with nonnegative initial values is nonnegative. The following lemma shows that the solutions of system (2.2) are uniformly and ultimately bounded.

Theorem 3.2. *For each positive solution $z(t) = (x_1(t), x_2(t), y(t))$ of system (2.2), there exists a constant $M > 0$ such that $x_i(t) \leq M, i = 1, 2, y(t) \leq M$ with t large enough.*

Proof. Let $z(t)$ be any solution of the system (2.2). Define a function $V(t)$ such that

$$V(t) = \sum_{i=1}^{2} bx_i(t) + y(t).$$

Then, $V \in V_0$, and there are

$$\begin{aligned} D^+V(t) &= brx_1\left(1 - \tfrac{x_1}{K}\right) - \tfrac{bc_1x_1y}{1+a_1x_1} - bmx_1 + bmx_1 - \tfrac{bc_2x_2y}{1+a_2x_2} - bd_2x_2 + \tfrac{bc_1x_1y}{1+a_1x_1} + \tfrac{bc_2x_2y}{1+a_2x_2} - d_3y \\ &= brx_1\left(1 - \tfrac{x_1}{K}\right) - bd_2x_2 - d_3y. \end{aligned}$$

For any $\lambda > 0$, we have

$$\begin{aligned} D^+V(t) + \lambda V(t) &= brx_1\left(1 - \tfrac{x_1}{K}\right) - bd_2x_2 - d_3y + \lambda bx_1 + \lambda bx_2 + \lambda y \\ &= bx_1\left(r + \lambda - \tfrac{r}{K}x_1\right) + bx_2(\lambda - d_2) + y(\lambda - d_3). \end{aligned}$$

When taking $\lambda = \min\{d_2, d_3\}$, the right end of the above equation is bounded for all $x_1(t), x_2(t), y(t) \in R_+^3$, so there exists the constant $M_0 > 0$ as the right end boundary of the equation, which gives us

$$D^+V(t) + \lambda V(t) \leq M_0, \quad t \neq nT.$$

When $t = nT$, it can be obtained that

$$V(nT^+) \leq V(nT) + \tau.$$

From Lemma 2.4 in the literature [35], the comparison theorem for the impulse differential equations, it follows that

$$V(t) \leq \left(V(0^+) - \frac{M_0}{\lambda}\right)e^{-\lambda t} - \frac{\tau e^{-\lambda t}}{1 - e^{-\lambda T}} + \frac{\tau e^{-\lambda(t-nT)}}{1 - e^{-\lambda T}} + \frac{M_0}{\lambda}, \quad t \to \infty.$$

Therefore, $V(t)$ is uniformly bounded, and there exists a constant $M > 0$ such that each solution $z(t) = (x_1(t), x_2(t), y(t))$ of the system (2.2) has $x_1(t) \leq M, x_2(t) \leq M, y(t) \leq M$ when t large enough. The proof is complete. □

Next, we prove the global attraction of the periodic extinction solution of *B. gobica*.

$$\begin{cases} \frac{\mathrm{d}y(t)}{\mathrm{d}t} = -d_3 y(t), & t \neq nT, \\ y(nT^+) = (1-p_3)\, y(nT) + \tau, & t = nT. \end{cases} \tag{3.1}$$

Theorem 3.3. *System (3.1) has a periodic solution $y^*(t)$ with T, and for any solution $y(t)$ of system (3.1), we have $|y(t) - y^*(t)| \to 0$ as $t \to \infty$, where*

$$y^*(t) = y_0 e^{-d_3(t-nT)} = \frac{\tau e^{-d_3(t-nT)}}{1-(1-p_3)\, e^{-d_3T}}, \quad nT < t \leq (n+1)T,$$

$$y_0 = \frac{\tau}{1-(1-p_3)\, e^{-d_3T}}.$$

Proof. In the impulse interval $(nT, (n+1)T]$, according to the first equation of system (3.1) using the method of separated variables, we can get

$$y(t) = y(nT^+)\, e^{-d_3(t-nT)}, \quad nT < t \leq (n+1)T,$$

where $y(nT^+)$ denotes the number of natural enemies y released at time $t = nT$. Pesticide spraying and one natural enemy release at $t = (n+1)T$,

$$y((n+1)T^+) = (1-p_3)\, y((n+1)T) + \tau = (1-p_3)\, y(nT^+)\, e^{-d_3T} + \tau.$$

Denote $y(nT^+) = y_n$, then we obtain the following difference equation:

$$y_{n+1} = (1-p_3)\, y_n e^{-d_3T} + \tau.$$

Using the remaining equations of (3.1), we deduce the stroboscopic map

$$y_{n+1} = (1-p_3)\, y_n e^{-d_3T} + \tau \overset{\triangle}{=} F(y_n). \tag{3.2}$$

Then, we obtain that there exists a unique fixed point of y_0 for the above stroboscopic map

$$y_0 = \frac{\tau}{1-(1-p_3)\, e^{-d_3T}}.$$

We first discuss the existence of a positive steady state of (3.2).

Since $F'(y_n) = (1-p_3)\, e^{-d_3T}$, it follows that $0 < F'(y_n) < 1$ holds for all y_n. Therefore, from the theory of differential equations, it can be obtained that the system has a fixed point and is globally stable.

The solutions of (3.1) on the interval $(nT, (n+1)T]$ initially from the fixed points y_0 follow

$$y^*(t) = y_0 e^{-d_3(t-nT)} = \frac{\tau e^{-d_3(t-nT)}}{1-(1-p_3)\, e^{-d_3T}}.$$

Moreover, $y^*(t)$ is periodic in time, thus, $y^*(t+T) = y^*(t)$.

From the above discussion, system (3.1) has a periodic solution $y^*(t)$ with T

$$y^*(t) = y_0 e^{-d_3(t-nT)} = \frac{\tau e^{-d_3(t-nT)}}{1-(1-p_3)\, e^{-d_3T}}, \quad nT < t \leq (n+1)T,$$

where $y_0 = \frac{\tau}{1-(1-p_3)e^{-d_3T}}$. This completes the proof. □

So, system (2.2) has a periodic solution of the extinction of *B. gobica* $E_0(0,0,y^*(t))$.

Define the following threshold conditions for population dynamics of *B. gobica*

$$R_1(n,T) = R_0(n,T)\,M(T),$$

where

$$\begin{aligned} R_0(n,T) &= (1-p_1q_1(nT))\,e^{(r-m)T}, \\ M(T) &= \exp\left[\int_{nT}^{(n+1)T}\left(-\tfrac{c_1y^*(t)}{1+a_1M}\right)\mathrm{d}t\right] \\ &= \exp\left[\tfrac{c_1\tau\left(e^{-d_3T}-1\right)}{d_3(1+a_1M)\left[1-(1-p_3)e^{-d_3T}\right]}\right]. \end{aligned}$$

Theorem 3.4. *If* $R_1(n,T) < 1$ *holds, then the periodic solution* $E_0(0,0,y^*(t))$ *of system (2.2) for the extinction of B. gobica is globally attractive.*

Proof. Since $R_1(n,T) < 1$, we can choose a $\varepsilon > 0$ small enough such that

$$\sigma = R_1(n,T)\,e^{\frac{c_1\varepsilon T}{1+a_1M}} < 1,$$

holds. From the third equation of the system (2.2), we know that

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \ge -d_3y(t).$$

Consider the following nonautonomous impulsive system:

$$\begin{cases} \frac{\mathrm{d}y_1(t)}{\mathrm{d}t} = -d_3y_1(t), & t \ne nT, \\ y_1(nT^+) = (1-p_3)\,y_1(nT) + \tau, & t = nT. \end{cases} \tag{3.3}$$

By comparing the theorem of impulsive differential equations and Theorem 3.3, we get $y(t) \ge y_1(t) > y^*(t) - \varepsilon$ for t large enough. For simplification, we assume $y(t) > y^*(t) - \varepsilon$ for all $t > 0$. Then, from the first equation of system (2.2), we have

$$\begin{cases} \frac{\mathrm{d}x_1(t)}{\mathrm{d}t} \le x_1(t)\left[r - \frac{c_1(y^*(t)-\varepsilon)}{1+a_1M} - m\right], & t \ne nT, \\ x_1(nT^+) = (1-p_1q_1(nT))\,x_1(nT), & t = nT. \end{cases} \tag{3.4}$$

For $t \in (nT,(n+1)T]$, since $q(t)$ decreases with time t, we can obtain

$$\begin{aligned} & x_1((n+1)T) \le x_1(nT^+)\exp\left[\int_{nT}^{(n+1)T}\left(r - \tfrac{c_1(y^*(t)-\varepsilon)}{1+a_1M} - m\right)\mathrm{d}t\right] \\ &= (1-p_1q_1(nT))\,x_1(nT)\exp\left[\int_{nT}^{(n+1)T}\left(r - \tfrac{c_1(y^*(t)-\varepsilon)}{1+a_1M} - m\right)\mathrm{d}t\right] \\ &= (1-p_1q_1(nT))\,x_1(nT)\,e^{(r-m)T}\exp\left[\int_{nT}^{(n+1)T}\left(-\tfrac{c_1y^*(t)}{1+a_1M}\right)\mathrm{d}t\right]\exp\left(\tfrac{c_1\varepsilon T}{1+a_1M}\right) \\ &= x_1(nT)\,(1-p_1q_1(nT))\,e^{(r-m)T}\exp\left[\tfrac{c_1\tau\left(e^{-d_3T}-1\right)}{d_3(1+a_1M)\left[1-(1-p_3)e^{-d_3T}\right]}\right]\exp\left(\tfrac{c_1\varepsilon T}{1+a_1M}\right) \\ &= x_1(nT)\,R_0(n,T)\,M(T)\exp\left(\tfrac{c_1\varepsilon T}{1+a_1M}\right) \\ &= x_1(nT)\,R_1(n,T)\exp\left(\tfrac{c_1\varepsilon T}{1+a_1M}\right) \\ &= x_1(nT)\,\sigma, \end{aligned} \tag{3.5}$$

so we have $x\left((n+1)T\right) \le x_1(0)\sigma^{n+1} \to 0$ as $n \to \infty$. For $\forall t \in (nT, (n+1)T]$, there are

$$0 < x_1(t) \le x_1\left(nT^+\right)\exp\left[\int_{nT}^{t}\left[r - \frac{c_1\left(y^*(t)-\varepsilon\right)}{1+a_1M_1} - m\right]\mathrm{d}t\right] \le x_1(nT)\,e^{rT}.$$

So, $x_1(t) \to 0$ when $t \to \infty$. Thus, for t large enough, i.e., there exists $T_1 > 0$, and when $t > T_1$, we know that $x_1(t) < \varepsilon$. From the second equation of the system (2.2), we get

$$\begin{cases} \frac{\mathrm{d}x_2(t)}{\mathrm{d}t} \le m\varepsilon - \frac{c_2x_2(t)(y^*(t)-\varepsilon)}{1+a_2M} - d_2x_2(t), & t \ne nT, \\ x_2\left(nT^+\right) = \left(1 - p_2q_2(nT)\right)x_2(nT), & t = nT. \end{cases} \tag{3.6}$$

Integrating the above equation over $(nT, (n+1)T]$ yields

$$\begin{aligned} x_2\left((n+1)T\right) &\le x_2\left(nT^+\right)\frac{m\varepsilon}{\frac{c_2\left(y^*(t)-\varepsilon\right)}{1+a_2M}+d_2} \\ &= x_2(nT)\left(1 - p_1q_2(nT)\right)\frac{m\varepsilon}{\frac{c_2\left(y^*(t)-\varepsilon\right)}{1+a_2M}+d_2}. \end{aligned} \tag{3.7}$$

Thus, by the arbitrariness of ε, it follows that $x_2(t) \to 0$ when $t \to \infty$, i.e., there exists $T_2 > 0$, and when $t > T_2$, we have $x_1(t) < \varepsilon$.

Next, we will prove $y(t) \to y^*(t)$ as $t \to \infty$. From the above conclusion, we know that there exists $T_1, T_2 > 0$, and when $t > T_1, T_2$, there are $0 < x_1(t) < \varepsilon, 0 < x_2(t) < \varepsilon$. Without loss of generality, we assume $0 < x_1(t) < \varepsilon, 0 < x_2(t) < \varepsilon$ for all $t \ge 0$. From the third equation of the system (2.2), we obtain

$$\begin{cases} \frac{\mathrm{d}y(t)}{\mathrm{d}t} \le \left(\frac{bc_1\varepsilon}{1+a_1\varepsilon} + \frac{bc_2\varepsilon}{1+a_1\varepsilon} - d_3\right)y(t), & t \ne nT, \\ y\left(nT^+\right) = (1-p_3)\,y(nT) + \tau, & t = nT. \end{cases}$$

Consider the following comparison systems:

$$\begin{cases} \frac{\mathrm{d}y_2(t)}{\mathrm{d}t} = \left(\frac{bc_1\varepsilon}{1+a_1\varepsilon} + \frac{bc_2\varepsilon}{1+a_1\varepsilon} - d_3\right)y_2(t), & t \ne nT, \\ y_2\left(nT^+\right) = (1-p_3)\,y_2(nT) + \tau, & t = nT. \end{cases} \tag{3.8}$$

Then we have $y_1(t) \le y(t) \le y_2(t)$, and when $t \to \infty$, $y_1(t) \to y^*(t), y_2(t) \to y_2^*(t)$, where $y_1(t)$ is a solution of the system (3.3) and the system (3.8), respectively, and similar to the system (3.1), it is easy to see that there exists a global asymptotically stabilized periodic solution $y_2^*(t)$ of the system (3.8),

$$y_2^*(t) = y_{20}\exp\left[\left(\frac{bc_1\varepsilon}{1+a_1M} + \frac{bc_2\varepsilon}{1+a_2M} - d_3\right)(t-nT)\right], t \in (nT, (n+1)T],$$

where

$$y_{20} = \frac{\tau}{1-(1-p_3)\exp\left[\left(\frac{bc_1\varepsilon}{1+a_1M} + \frac{bc_2\varepsilon}{1+a_2M} - d_3\right)T\right]}.$$

Thus, for $\forall\varepsilon_0$, there exists a $T_0 > 0$ such that when $t > T_0$, we have

$$y^*(t) - \varepsilon_0 \le y(t) \le y_2^*(t) + \varepsilon_0.$$

Let $\varepsilon \to 0$, and we have

$$y^*(t) - \varepsilon_0 \le y(t) \le y^*(t) + \varepsilon_0,$$

for t large enough, which implies $y(t) \to y^*(t)$ as $t \to \infty$.

Therefore, if $R_1(n, T) < 1$ holds, then the periodic solution $E_0\left(0, 0, y^*(t)\right)$ of system (2.2) for the extinction of *B. gobica* is globally attractive. This completes the proof. $\square$

Remark 1. *From the literature [31, 33, 36], the first part* $R_0(n,T)$ *of* $R_1(n,T)$ *shows the effects of the development of B. gobica resistance on the threshold; the second part* $M(T)$ *reflects the effects of natural enemies such as ladybugs on the control of B. gobica, including the maximum capture rate of natural enemies to B. gobica* c_i*, parameters of natural enemy selectivity for B. gobica* a_i*, natural enemy release* τ*, etc. Without biological control, the threshold conditions for the extinction of B. gobica becomes* $R_0(n,T)$*, and it is clear that* $R_1(n,T) < R_0(n,T)$*. Since* $q_1(t)$ *is decreasing with time t,* $R_1(n,T)$ *and* $R_0(n,T)$ *are increasing functions with respect to n, and* $R_1(n,T)$ *tends to* $e^{(r-m)T}M(T)$ *(see Fig.3), which means that, at last, chemical control does not work. The threshold condition for the extinction of B. gobica without chemical control is* $e^{(r-m)T}M(T)$*, which is also greater than* $R_1(n,T)$*. Thus, the integrated control strategy is more likely to keep the threshold* $R_1(n,T)$ *below 1 and to be more effective in the short term than any single biological or chemical control. Nevertheless, B. gobica eventually breaks out due to its resistance to pesticides. Therefore, more effective measures are needed to control the population density of B. gobica.*

4. Numerical simulation

The above mainly discuss the dynamical behavior of the system (2.2), and through theoretical analysis, a sufficient condition for the global attraction of the periodic solution $E_0(0,0,y^*(t))$ for the extinction of *B. gobica* is obtained. Next, we will investigate the dynamic complexity of the system (2.2) and the effect of key parameters on the extinction threshold of *B. gobica* through numerical simulations.

To illustrate the effect of pesticide spraying period T on the development of resistance of *B. gobica*, the change curves of $q_1(t)$ and $q_2(t)$ with time t were simulated, where $q_1(t)$ is the proportion of pesticide-sensitive psyllid nymphs at the moment of t, and $q_2(t)$ is the proportion of pesticide-sensitive psyllid adults *B. gobica* at the moment of t, it is thus clear how different pesticide spraying cycles T and different pesticide instantaneous kill rates p_i affect the evolution of resistance in *B. gobica*. The results showed that the higher the frequency of pesticide spraying, the faster the development of resistance. Since the instantaneous killing rate of pesticides was different for psyllid nymphs and adults, from Figure 1, it could be seen that the higher the instantaneous killing rate of pesticides, the faster the development of resistance.

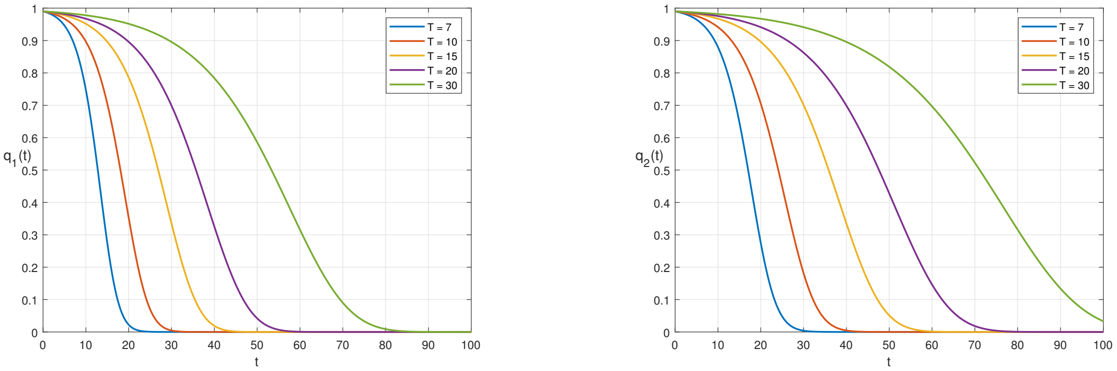

Figure 1. The dynamics of $q_1(t)$, $q_2(t)$ over time t is plotted, where $p_1 = 0.8, p_2 = 0.6$.

To assess the effect of pesticide resistance on the population densities of psyllid nymphs and adults, consider the following system M_0 that does not carry resistance:

$$M_0:\begin{cases}\left.\begin{aligned}\dot{x}_1 &= rx_1\left(1-\frac{x_1}{K}\right)-\frac{c_1x_1y}{1+a_1x_1}-mx_1,\\ \dot{x}_2 &= mx_1-\frac{c_2x_2y}{1+a_2x_2}-d_2x_2,\\ \dot{y} &= \frac{b_1c_1x_1y}{1+a_1x_1}+\frac{b_2c_2x_2y}{1+a_2x_2}-d_3y,\end{aligned}\right\}\quad t\neq nT,\\ \left.\begin{aligned}x_1\left(nT^+\right) &= \left(1-p_1\right)x_1\left(nT\right),\\ x_2\left(nT^+\right) &= \left(1-p_2\right)x_2\left(nT\right),\\ y\left(nT^+\right) &= \left(1-p_3\right)y(nT)+\tau,\end{aligned}\right\}\quad t=nT, n\in N.\end{cases}\tag{4.1}$$

Through numerical simulation, we obtained graphs of trends in psyllid nymph and adult densities with and without resistance. From Figure 2, it can be seen that the number of *B. gobica* in the system (2.2) increased significantly compared to M_0, i.e., the system (4.1), which suggests that ignoring the resistance of *B. gobica* to pesticides will underestimate the population of *B. gobica* and, thus, effect the wolfberry industry. In addition, it can be seen from the above figure that the population density of psyllid nymphs and adults after pesticide spraying first decreased due to the high efficacy of the pesticide, and then increased due to the evolution of pesticide resistance of *B. gobica*.

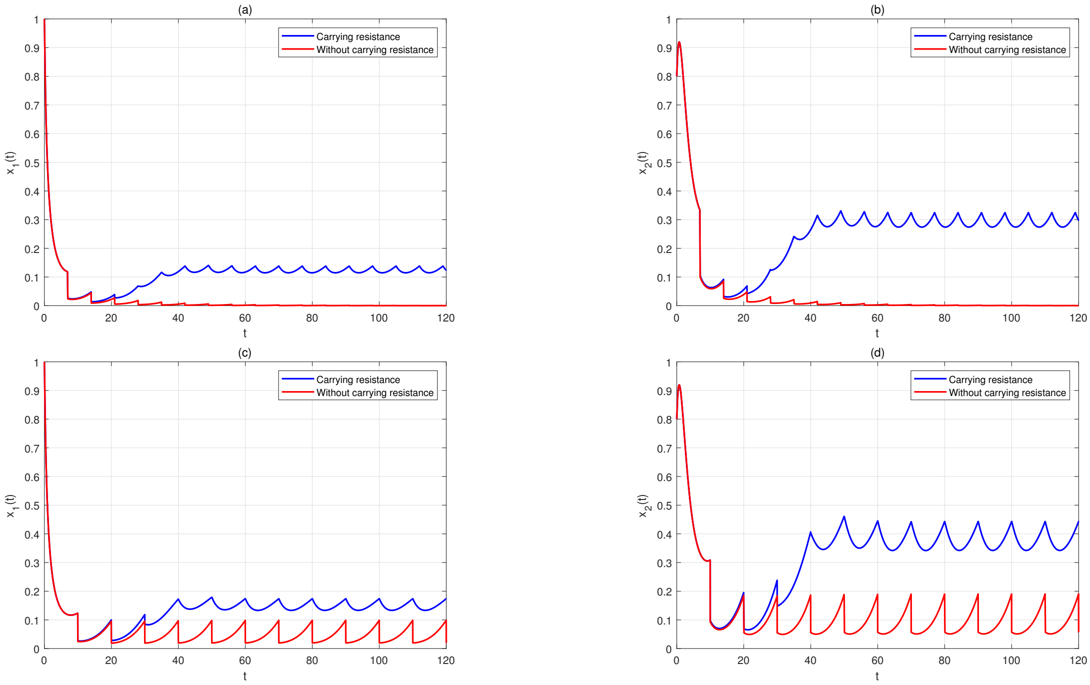

Figure 2. $(a),(b)$ are time series plots of nymphs and adults densities of the system (2.2) versus the system (4.1) at $T = 7$; $(c),(d)$ are time series plots of nymphs and adults densities of the system (2.2) versus the system (4.1) at $T = 10$, where $r = 1.1, K = 1, c_1 = 0.6327, a_1 = 0.3428, m = 0.8, c_2 = 0.5373, a_2 = 0.2283, d_2 = 0.2, d_3 = 0.3, b = 0.5, p_2 = 0.7, \tau = 0.4, p_1 = 0.8, p_1 = 0.7, p_3 = 0.2$.

From the expression of $R_1(n,T)$, it can be seen that $\frac{\partial R_1(n,T)}{\partial\tau} < 0$, $R_1(n,T)$ is a decreasing function with respect to τ, which means that the greater the number of τ released by natural enemies, the smaller the threshold $R_1(n,T)$, and the more beneficial biological control measures are to control pest population density. On this basis, the effect of chemical control on $R_1(n,T)$ was emphasized.

From the Figure 3, it can be seen that since $R_1(n,T)$ and $R_0(n,T)$ are increasing functions of n, as the number of sprays n increases, $R_1(n,T)$ tends to $e^{(r-m)T}M(T)$, which means that, finally, chemical control does not work. As shown in Figure 3, the extinction threshold condition of *B. gobica* in the absence of chemical control is $e^{(r-m)T}M(T) > R_1(n,T)$. Thus, it can be seen that the integrated control strategy is more effective than any single biological or chemical control in the short term.

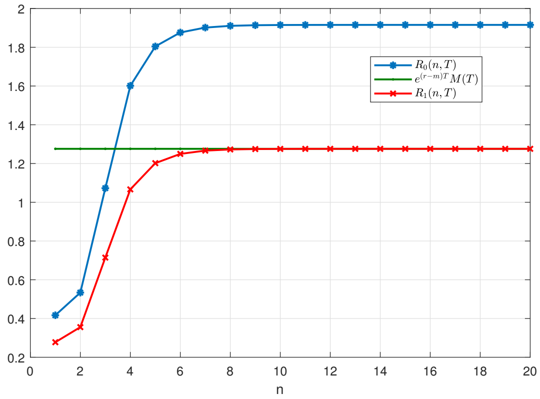

Figure 3. Plot of dynamics of $R_0(n,T), e^{(r-m)T}M(T), R_1(n,T)$ with time t, where $r = 1.1, c_1 = 0.6327, m = 0.8, d_3 = 0.3, a_1 = 0.3428, T = 2, \tau = 0.4, p_3 = 0.2, p_1 = 0.8$.

In fact, from the expression of $R_1(n,T)$, it can be seen that $R_1(n,T)$ is a monotonically increasing function with respect to n, and Figure 4 gives a graphical representation of the trend of changes in $R_1(n,T)$ with increasing n, which shows that for the same pesticide spraying times $n \leq 4$ when $R_1(n,T) < 1$ holds, but with the increase of time t, due to the evolution of the resistance of *B. gobica*, *B. gobica* gradually develops resistance, and after spraying 5 times, pesticide $R_1(n,T)$ will be greater than 1. Figure 4 provides an example of how the density of *B. gobica* populations and the threshold $R_1(n,T)$ change as the number of times the same pesticide is sprayed increases.

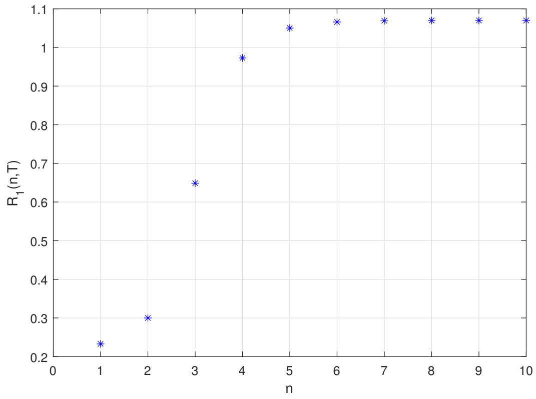

Figure 4. Plot of $R_1(n,T)$ with n, where $r = 1.1, c_1 = 0.6327, m = 0.8, d_3 = 0.3, a_1 = 0.3428, T = 2, \tau = 0.4, p_3 = 0.2, p_1 = 0.8$.

Next, we used the contours of $R_1(n,T)$ to illustrate the effects of the remaining key parameters on the control of *B. gobica*. From Figure 5(a–d), it can be seen that the threshold $R_1(n,T)$ is a monotonically increasing function with respect to T, r, a_1, p_3 and a monotonically decreasing function with respect to τ, c_1, m, which suggests that shortening the T, controlling the intrinsic growth rate of r of psyllid nymphs population, selecting pesticides with a lower killing rate of natural enemies to control the killing rate of pesticides on natural enemies p_3, and increase the number of natural enemies put in τ, etc., are very helpful for the management of *B. gobica*.

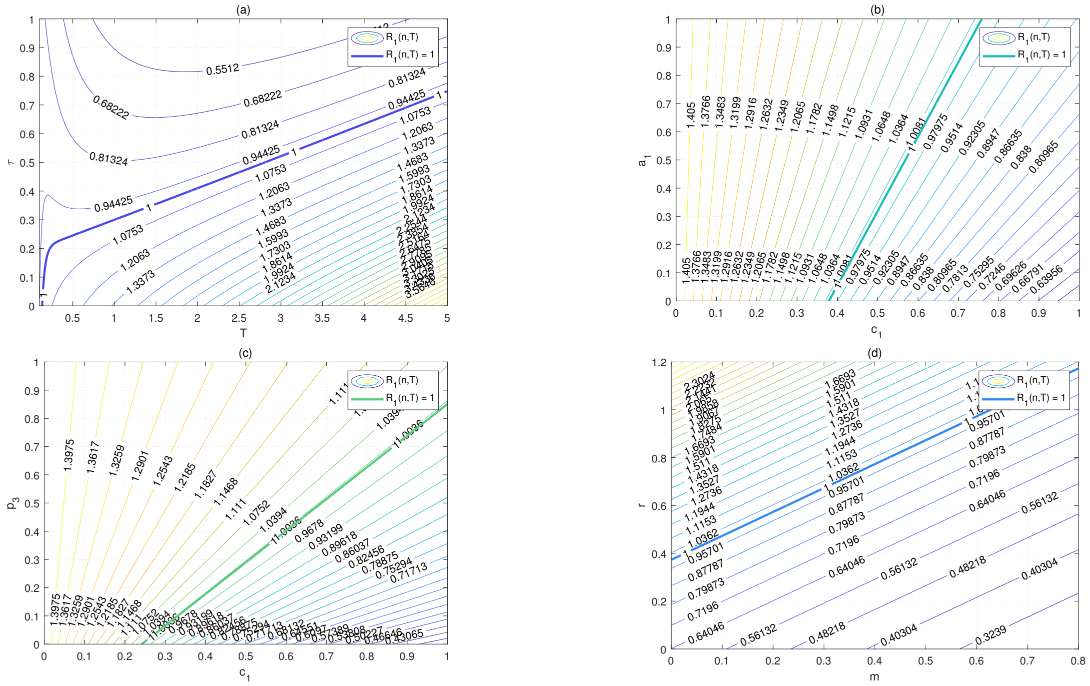

Figure 5. (a) Plot contours of $R_1(n, T)$ verse τ and T, where $c_1 = 0.6327, d_3 = 0.2, m = 0.8, p_1 = 0.8, p_3 = 0.3, a_1 = 0.3428, r = 1.1$; (b) Plot contours of $R_1(n, T)$ verse c_1 and a_1, where $\tau = 0.4, T = 1.2, d_3 = 0.2, m = 0.8, p_1 = 0.8, p_3 = 0.3, r = 1.1$; (c) Plot contours of $R_1(n, T)$ verse c_1 and p_3, where $\tau = 0.4, T = 1.2, d_3 = 0.2, m = 0.8, p_1 = 0.8, a_1 = 0.3428, r = 1.1$; (d) Plot contours of $R_1(n, T)$ verse r and m, where $c_1 = 0.6327, \tau = 0.4, T = 1.2, d_3 = 0.2, a_1 = 0.3428, p_1 = 0.8, p_3 = 0.3$

As shown in previous studies [13,34,36], if the same pesticide is sprayed frequently, the resistance of *B. gobica* to the pesticide grows rapidly and may eventually lead to another outbreak of *B. gobica*. Therefore, people usually switch pesticides at specific times, and how to choose the best switching time is an urgent problem. In the following study, the optimal time for switching pesticides will be selected based on the system (2.2).

Remark 2. *For each novel pesticide switched, we assume that the evolution of pesticide resistance in B. gobica, i.e.,* $q_i(t)$*, follows the same equation and has the same initial conditions* q_{i0}.

From Figure 6, it can be seen that the dynamic threshold value $R_1(n, T)$ is increasing relative to the number of application times n, so the population of *B. gobica* has become extinct, and it is necessary to make $R_1(n, T) < 1, n \in N$. That is to say, to maintain the killing effect of pesticides on *B. gobica*, it is necessary to switch pesticides before the dynamic threshold $R_1(n, T)$ reaches 1. Figure 4 is an example, and if the parameter in Figure 4 is used, it is necessary to switch to a different pesticide after the same pesticide is sprayed up to five times. Thus, without loss of generality, we borrow the idea from literature [30] and assume that after repeated spraying of pesticide $n_1^{(1)}$ times, the dynamic threshold $R_1(n,T)$ will be greater than 1 for the first time. Define

$$n_1^{(1)} = \max\left\{n : R_1(n,T) \leq 1\right\}.$$

To get $n_1^{(1)}$, first let $R_1(n,T) = 1$. From this, we can get

$$q_1(nT) = \frac{1 - \exp\left[\frac{c_1\tau\left(1-e^{-d_3T}\right)+(m-r)T}{d_3(1+a_1M)\left[1-(1-p_3)e^{-d_3T}\right]}\right]}{p_1},$$

where $q_1(nT)$ is already given in (2.3). Therefore,

$$n_1^{(1)} = \left[\left\{n : q_1(nT) = \frac{1 - \exp\left[\frac{c_1\tau\left(1-e^{-d_3T}\right)+(m-r)T}{d_3(1+a_1M)\left[1-(1-p_3)e^{-d_3T}\right]}\right]}{p_1}\right\}\right],$$

and $[n]$ denotes the greatest integer no larger than n.

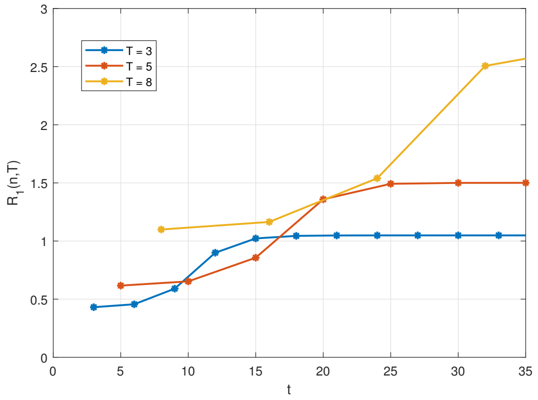

Figure 6. Plot of $R_1(n,T)$ versus n, where $r = 1.1, c_1 = 0.6327, m = 0.8, d_3 = 0.3, a_1 = 0.3428, \tau = 0.4, p_3 = 0.2, p_1 = 0.8$.

If the dynamic threshold $R_1(n,T)$ is used as the indicator for switching pesticides in this strategy, then *B. gobica* can be completely extinct after several times of pesticide switching. The numerical simulation under this strategy is given in Figure 7, from which it can be seen that by switching pesticides, *B. gobica* populations will be extinct.

Partial rank correlation coefficients PRCCs quantify the strength of associations between model parameters and outcomes, thereby elucidating the relative contribution of each parameter to the variability in the results [37–39]. To investigate the extent of influence brought about by parameter variations of the system (2.2) on the control strategy of *B. gobica*, we carried out PRCCs sensitivity analyses on the threshold value $R_1(n,T)$. Thus, several important parameters affecting the threshold $R_1(n,T)$ were identified from the expression of $R_1(n,T)$. For PRCCs values in Figures 8 and 9, a parameter is positively correlated with the threshold if the sign of PRCCs for that parameter is positive, and conversely, a parameter is negatively correlated with the threshold if the sign of PRCCs for that parameter is negative. If the absolute value of its PRCCs is greater than 0.6, it indicates that the parameter is strongly correlated with $R_1(n, T)$ and has a large effect on $R_1(n, T)$. If the absolute value of its PRCCs is between 0.4 and 0.6, it indicates that the parameter is moderately correlated with $R_1(n, T)$. When the absolute value of PRCCs is less than 0.4, it indicates that the parameter has a weak correlation with $R_1(n, T)$ and a small effect on $R_1(n, T)$.

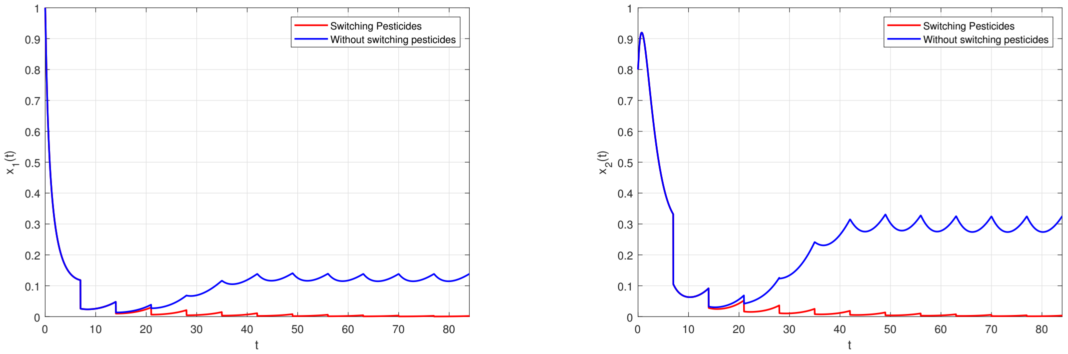

Figure 7. The effects of switching pesticides on nymphs and adults populations, which are $r = 1.1, K = 1, c_1 = 0.6327, c_2 = 0.5373, m = 0.8, b = 0.5, d_2 = 0.2, d_3 = 0.3, a_1 = 0.3428, a_2 = 0.2283, T = 7, \tau = 0.4, p_1 = 0.8, p_2 = 0.7, p_3 = 0.3$.

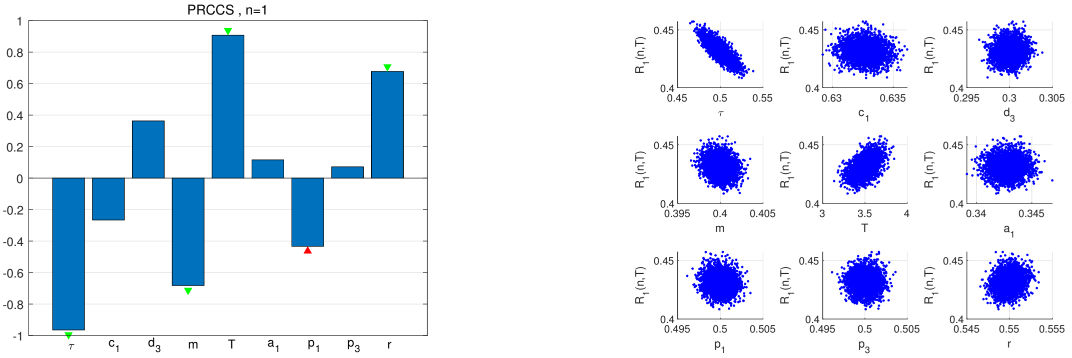

Figure 8. Sensitivity analysis of $R_1(n, T)$ when $n = 1$.

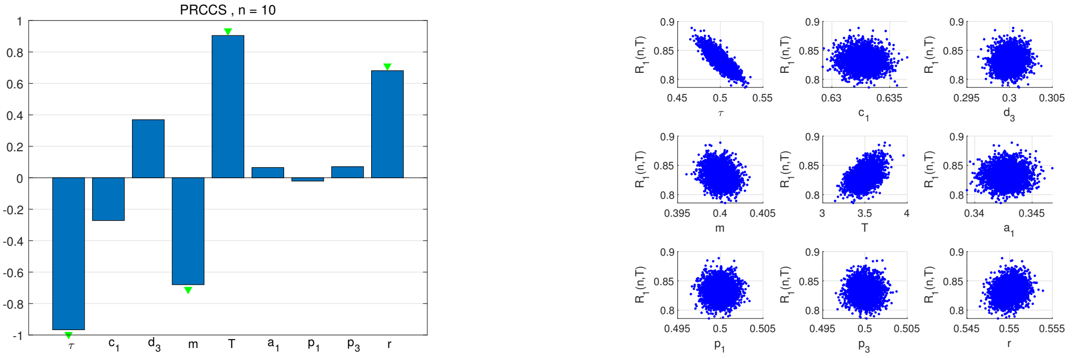

Figure 9. Sensitivity analysis of $R_1(n, T)$ when $n = 10$.

From Figures 8 and 9, we can see that the parameters T, r, d_3, a_1, p_3 are positively correlated with the threshold value $R_1(n,T)$, and that an increase in these parameters will lead to an outbreak of *B. gobica*. From Figures 8 and 9, we can see that the parameters T, r, d_3, a_1, p_3 are positively correlated with the threshold value $R_1(n,T)$, and that an increase in these parameters will lead to an outbreak of *B. gobica*. However, the parameters τ, c_1, m were negatively correlated with the threshold value $R_1(n,T)$, so the increase of these parameters was favorable to the control of *B. gobica*, and the effects of τ, T, r, and m on the $R_1(n,T)$ were great, which were important control parameters. Therefore, for the management of *B. gobica*, increasing the number of natural enemies τ and shortening the release period of natural enemies and pesticides T are effective and feasible control strategies. By comparing the Figures 8 and 9, we can see that the PRCC value of the parameter p_1 is variable. When $n = 1$ (as in Figure 8), the correlation of p_1 to $R_1(n,T)$ is stronger, indicating that pesticides have a greater killing effect on the *B. gobica* in the early stage of its control, and that spraying of pesticides is extremely favorable for *B. gobica* control. When $n = 10$ (Figure 9), due to the development of pesticide resistance of the woodlouse of Lycium barbarum, p_1 to $R_1(n,T)$ correlation reduces to very small. At this time, the *B. gobica* woodlouse on the same pesticide produces a strong resistance. Repeated spraying of the same pesticide is not conducive to the control of *B. gobica*, so we should switch pesticides to take a strategy to prevent the *B. gobica* resistance.

5. Conclusions

B. gobica is a great threat to the production of wolfberry, seriously affecting the yield and quality of Lycium barbarum, and has become an important factor restricting the development of the wolfberry industry [40]. In this paper, based on the characteristics of *B. gobica* in each growth stage, they were divided into nymphs and adults; a dynamic model was established based on their resistance to pesticides and predation by dominant natural enemies, and an integrated management model for *B. gobica* combining pesticide resistance and stage structure was put forward, proving the existence of the periodic solution for the extinction of *B. gobica* and demonstrating the consistent boundedness and global attraction of the system by using the comparison theorem of the impulsive differential equations; and the dynamic threshold conditions for extinction of *B. gobica* were given as $R_1(n,T)$. The results show that the periodic solution for the extinction of *B. gobica* is globally attractive when $R_1(n,T) < 1$. Through numerical simulation, we first discuss the effect of resistance development on the population density of *B. gobica*, and it can be seen that ignoring the resistance of *B. gobica* to pesticides underestimates the population density of *B. gobica* and, thus, has an effect on the *B. gobica* industry. After spraying pesticides, the population density of psyllid nymphs and adults first decreased due to the high efficacy of pesticides, and then increased due to the evolution of resistance to pesticides by *B. gobica*. Next, the integrated management strategy of *B. gobica* was compared with a single control strategy (chemical or biological), and it can be obtained that the integrated management strategy is more likely to keep the threshold value $R_1(n,T)$ below 1 and is more effective than any single biological or chemical control in the short term. In addition, the effect of the number of pesticide spraying on the development of resistance in *B. gobica* was also investigated by numerical simulation, and the results showed that the higher the frequency of pesticide spraying, the faster the development of resistance to pesticides by *B. gobica*. The pesticide switching time was discussed with the threshold value $R_1(n,T)$ as the condition of pesticide switching. Finally, the effects of the remaining key parameters on the extinction threshold of *B. gobica* were illustrated by contour plots and sensitivity analyses, which shortened the release of natural enemies and the pulse period of spraying T, controlling the endowment growth rate of *B. gobica* r. Selecting pesticides with a lower killing effect on natural enemies to control the killing rate of natural enemies by pesticides p_3 and increasing the number of natural enemies released τ was helpful for the management of *B. gobica*. However, the size of the PRCC value of the parameter p_1 is changing, indicating that in the early stage of the control of *B. gobica*, the pesticide kills the *B. gobica* more. Spraying pesticides for the control of *B. gobica* is extremely effective, with the increase in the number of pesticide spraying, due to the *B. gobica* to pesticide resistance to the development of pesticide resistance. At this time, *B. gobica* produces strong resistance to the same pesticide, and multiple sprayings of the same pesticide are not conducive to *B. gobica* control. We should adopt a switching pesticide strategy to prevent *B. gobica* resistance. Combined with the actual production, the eradication of *B. gobica* is extremely difficult to realize, but also from the pesticide for the *B. gobica* residual effect and control of *B. gobica* economic costs and other different control strategies, trying to control *B. gobica* in an acceptable level does affect the Lycium barbarum yield and quality.

Author contributions

Mengge Zhao: Writing original draft, Formal analysis, Validation, Parameter estimation; Jinyan Wang: Supervision, Validation, Writing-review & editing, Formal analysis. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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