AIMS Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(10): 24514–24547.
DOI: 10.3934/math.20251087
Received: 08 April 2025
Revised: 17 September 2025
Accepted: 22 September 2025
Published: 27 October 2025

Research article

A novel conflict analysis model based on multi-scale information with game theoretic approach

Akrash Tasawar[1] **and M. G. Abbas Malik**[2,*]

[1] Department of Mathematics, Quaid-i-Azam University, Islamabad-45320, Pakistan

[2] Interdisciplinary Sustainable Systems (IS2) research group, College of Computer and Information Science, Prince Sultan University, Riyadh, Saudi Arabia

* **Correspondence:** Email: amaalik@psu.edu.sa.

Abstract: We aimed to design a novel conflict analysis model that incorporates a hybrid information system and gameplay between the involved parties or players. In a hybrid information system, the decision maker can express his opinion from multiple data sets based on his expertise, experience, level of hesitancy, and risk factors involved. From this multiscale data, we carefully compared all players' opinions and designed three measures determining the degree of alliance, neutrality, and conflict. Based on game theory principles, we measured each player's gain for three actions/strategies: Go in alliance, remain neutral, or go against the other player. Based on the highest gains, the players were classified as allies, neutral, or in conflict (Nash equilibrium). With the power of careful multiscale opinions and bilateral study of all three possible scenarios, the proposed model has many advantages in accurately modeling conflict scenarios. A detailed comparative analysis confirms the practical use of the proposed framework.

Keywords: conflict analysis; game theory; hybrid information system; three-way decision
Mathematics Subject Classification: 91A35

1. Introduction

Conflicts are inherent aspects of a human society that frequently arise in economics, politics, and social governance [1]. An effective conflict analysis is crucial for fostering cooperation and developing consensus-building strategies. Traditional models, such as Pawlak's [2] rough set approach, offer valuable insights into simple disputes, but often rely on rigid assumptions. For example, the Pawlak framework [2] encodes each agent's stance as favorable, neutral, or against, typically using the scale $\{-1, 0, +1\}$. Although this study provides a basic overview, it oversimplifies the complexity and uncertainty of real-world conflicts.

Recent advancements have addressed these limitations by incorporating probabilistic and multi-valued techniques. Sun et al. [3] extended the conflict information system to two universes and introduced matrix-based decision models. The three-way decision theory has advanced conflict analysis by categorizing agents into acceptance, rejection, and boundary regions using probabilistic thresholds. For instance, rough set-based conflict models and game-theoretic approaches address uncertainty and feature selection in decision-making [3–5]. Additionally, game theory combined with rough sets has been applied to conflict resolution and recommender systems [6, 7]. Foundational texts and ranking methods provide a theoretical basis for such analyses [8, 9]. Three-way decision frameworks have been enhanced through decision-theoretic rough sets and fuzzy set theory to model complex conflicts [10–12], while seminal works by Pawlak laid the groundwork for conflict analysis using rough sets [13, 14]. Approaches exploring conflict dynamics and hesitant fuzzy sets extend the applicability of these methods [15, 16]. Moreover, Pythagorean fuzzy sets and probabilistic rough sets contribute to refined decision-making [17, 18]. Moreover, researchers investigate hybrid tables and clique consistency for strategy selection in conflicts [19–21], and unification models incorporate formal concept analysis [22]. Extensions into group conflict analysis with Pythagorean fuzzy sets and incomplete information tables further broaden the field [23, 24]. Finally, general models tackle incomplete information scenarios to enhance three-way decision theory [25]. Yao's [26] framework formalized this partitioning, aligning it with alliance, conflict, and neutrality among players. Bayesian decision rules and risk-based strategies have since refined these models, setting optimal thresholds and improving decision accuracy [26].

Classification of existing models:

We classify the existing models into two categories: One is based on the dataset used and the other is based on the number of measures and the methodology applied.

Information system data sets:

The representation of agents' opinions has also undergone significant evolution. Each format offers unique strengths and trade-offs in terms of expressiveness, interpretability, and computational complexity. Discrete scales [2] (e.g., $\{-1, 0, +1\}$) provide simplicity but lack nuance, while continuous scales [7] such as $[-1, 1]$ offer graded assessments, yet assume precise numerical input. More advanced representations include intuitionistic fuzzy sets (IFS) and Pythagorean fuzzy sets (PFS) [23], where the latter relaxes constraints on membership and non-membership to express ambiguity more effectively. Fuzzy sets with interval values (IVFS) [9] assign intervals instead of point values, effectively modeling higher-order uncertainty. Hesitant fuzzy sets [16] enable multiple membership degrees to reflect indecisiveness, though they complicate aggregation and comparison.

Linguistic [27] and probabilistic linguistic term sets (PLTS) [28] have gained popularity for enhancing interpretability, enabling expressions such as "somewhat agree" and "strongly disagree", yet they require careful numerical mapping. The triangular and trapezoidal fuzzy numbers [12,29] capture attitudes within defined ranges, striking a balance between flexibility and simplicity. More recently, q-rung orthopair fuzzy sets (q-ROFS) have generalized earlier models, offering greater tolerance for uncertainty but requiring more complex calibration. There are also other extensions like generalized bipolar neutrosophic sets [30].

To bridge representational gaps, hybrid information systems [19] have emerged that enable multiple opinion formats to co-exist within a single framework. These systems enable agents to express preferences using the most context-appropriate structure, reflecting the growing demand for robust modeling in heterogeneous and uncertain decision environments.

Number of measures and methodologies:

Methodologically, the field has shifted from static, distance-based classifications to a dynamic, decision-driven framework. Pawlak's early conflict distance model [2] introduced single-measure classification into ally, neutral, and conflict groups. This was significantly enhanced by Yao's incorporation of three-way decision theory and probabilistic rough sets [26, 31], which introduced trisection via positive and negative likelihoods. Zhang et al. [32] extended this by incorporating boundary probabilities. Models based on Bayesian risk theory [3], and consistency measures [20] improved rationality and interpretability in strategic choices. Formal concept analysis [33], fuzzy ranking techniques [34], and dual hesitant fuzzy frameworks [35] further enriched the theoretical base for managing vague or conflicting data. Game theory has become an essential tool, utilizing models that employ payoff optimization and evolutionary dynamics to explore strategic behavior and cooperation. For instance, Bashir et al. [6] explored the integration of conflict analysis with game theory. In contrast, studies [7, 19, 34] demonstrated the value of using multiple distinct measures for alliance, neutrality, and conflict, improving precision in decision-making. Bashir et al. [6] applied game-theoretic rough sets to resolve disputes, while Benko et al. [36] used evolutionary game theory to model cooperation in open data environments. These developments mark a shift from static similarity-based frameworks to dynamic, multi-measure, and strategically grounded models, reflecting the complex nature of real-world conflict scenarios.

Motivation and proposed research methodology:

Based on the above discussion, we aim to develop a game-theoretic conflict analysis model that takes advantage of a hybrid information system. Previous studies have primarily relied on single rating scales or types to represent agents' attitudes. For example, Bashir et al. [6, 27] proposed game-theoretic models for conflict analysis using rough sets and linguistic terms, which utilized a single scale. However, real-world conflicts often involve varying levels of uncertainty and hesitation, depending on the complexity of the issue. To address this, Yang et al. [19] introduced hybrid situation tables with multiple rating types, enabling a more comprehensive representation of agents' attitudes. This research builds on their work by incorporating game theory into hybrid information systems, enabling for a detailed examination of player interactions and strategy optimization.

Our proposed model introduces a game-theoretic approach with a hybrid information system, enabling agents to express their attitudes at multiple scales. By defining auxiliary functions for each rating type and establishing measures for alliance, neutrality, and conflict, we formulate a game where the payoff functions evaluate the gains for each player's actions. Players can choose from three strategies: allying, staying neutral, or engaging in conflict with payoffs determined bilaterally. Based on the Nash equilibrium, players are categorized as allied, neutral, or in conflict, providing a more balanced and comprehensive analysis of the situation.

Our major contributions include:

- A hybrid information system integrating seven distinct rating types, enhancing adaptability to diverse, uncertain data.

- A two-player strategic game model with auxiliary functions for each rating type, leading to unified alliance, neutrality, and conflict measures.
- Two case studies are included: One demonstrating the practical application of the model in a government governance scenario, and another in the context of international climate policy analysis. The data set for the International Climate Policy case study is intelligently curated from various sources for the validation of our proposed model.

Our proposed model is extensive and provides a more detailed examination of conflicts. We applied it to the challenges in government governance and climate policy to demonstrate its practical use and validate it with actual data. The rest of the paper is organized as follows: In Section 2 we present the preliminary concepts of game theory and the Pawlak model. In Section 3 introduce the game theory model for hybrid information systems. In Section 4, we examine case studies and offers a detailed comparative analysis. Finally, in Section 5, we presents concluding remarks and outline several future research directions.

2. Preliminaries

In this section, we introduce some basic game theory concepts and the Pawlak model.

2.1. Game theory model

Game theory models typically include a set of players, a set of strategies for each player, and a set of payoffs associated with each strategy. The standard form of game theory is mathematically represented as a triplet (P, S, F) [37], where:

- $P = \{p_1, p_2, ..., p_n\}$ is a finite set of n players;
- $S = \{s_1, s_2, ..., s_m\}$ is a strategy profile for a players;
- $F = \{u_1, u_2, ..., u_n\}$ where $u_i : S \rightarrow \Re$ is a real-valued utility or payoff function for n players.

In a simple game played between two players, where the players are labeled as p_1 and p_2, each player has a set of strategies $\{s_1, s_2, s_3\}$ available for each game.

Table 1 presents the strategy profiles for players p_1 and p_2, with each block showing a pair of payoffs corresponding to the strategies chosen by the players. For instance, the top left block displays the payoffs $u_1(s_1, s_1)$ and $u_2(s_1, s_1)$ when both players select strategy s_1. The best strategy profile is determined by predicting the choices that yield the most favorable outcome. Nash equilibrium is a key concept used to analyze the payoff table and identify stable strategy combinations where no player benefits from changing their strategy unilaterally. If a pair of strategies (s_i, s_j) are the best responses to each other, this scenario is termed a pure strategy Nash equilibrium. Mathematically, it is expressed as follows:

$$\forall s_i^{'} \in S_1, u_1(s_i, s_j) \geq u_1(s_i^{'}, s_j), \text{ where } s_i \in S_1, \text{ and } s_i^{'} \neq s_i, \tag{2.1}$$

$$\forall s_j^{'} \in S_2, u_2(s_i, s_j) \geq u_2(s_i, s_j^{'}), \text{ where } s_i \in S_1, \text{ and } s_j^{'} \neq s_j. \tag{2.2}$$

Since no participant may gain an advantage by changing his or her chosen strategy, Eqs (2.1) and (2.2) could be viewed as a strategy profile. A strategy profile is said to be in a stable condition when a player is unable to outmaneuver another player's plan and comes up with a better one.

Table 1. Payoff table for two players.

		p_2		
		s_1	s_2	s_3
	s_1	$u_1(s_1,s_1),u_2(s_1,s_1)$	$u_1(s_1,s_2),u_2(s_1,s_2)$	$u_1(s_1,s_3),u_2(s_1,s_3)$
p_1	s_2	$u_1(s_2,s_1),u_2(s_2,s_1)$	$u_1(s_2,s_2),u_2(s_2,s_2)$	$u_1(s_2,s_3),u_2(s_2,s_3)$
	s_3	$u_1(s_3,s_1),u_2(s_3,s_1)$	$u_1(s_3,s_2),u_2(s_3,s_2)$	$u_1(s_3,s_3),u_2(s_3,s_3)$

2.2. Pawlak's conflict analysis model

Conflicts are a common phenomenon in the social life. Pawlak [2] studied conflict analysis and developed a method to analyze conflicts based on rough set theory. In this system, there are two major parts: A set of agents (like people or countries) and a set of issues they are arguing about. Each issue can have different outcomes, like support, opposition, or neutrality. The system shows how each agent feels about each issue. The connection between objects and issues of concern is represented as an information system. A conflict information system is defined as a system used to analyze conflicts using rough set theory. This system is represented by a two-tuple $S = (U, A)$, where:

- $U = \{u_1, u_2, \ldots, u_n\}$ is a non-empty finite set of agents (participants in the conflict).
- $A = \{a_1, a_2, \ldots, a_m\}$ is a non-empty finite set of attributes or issues involved in the conflict.

Each attribute $a \in A$ defines a voting function $a : U \to V_a$, where V_a is the range of the function a. The elements in U represent the agents involved in the conflict, and the elements in A represent the dispute issues. Specifically, $V_a = \{-1, 0, +1\}$, where:

- -1 represents opposition,
- 0 represents neutrality, and
- $+1$ represents support towards the issue a that is being disputed.

Definition 2.1. *Let $S = (U, A)$ be an information system. Then the auxiliary function for any issue $a \in A$ can be defined as :*

$$\psi_a(x,y) = \begin{cases} 1, & \text{if } a(x)a(y) = 1 \text{ or } x = y, \\ 0, & \text{if } a(x)a(y) = 0 \text{ and } x \neq y, \\ -1, & \text{if } a(x)a(y) = -1. \end{cases}$$

The auxiliary function $\psi_a(x,y)$ indicates the relationship between players x and y regarding issue $a \in A$:

- $\psi_a(x,y) = 1$*: x and y share the same opinion on issue a.*
- $\psi_a(x,y) = 0$*: At least one of them is neutral on issue a.*
- $\psi_a(x,y) = -1$*:x and y view issue a differently.*

Moreover, Pawlak introduced the concept of a distance function between two objects for conflict analysis.

Definition 2.2. *Consider the information system $IS = (U, A)$. For every two objects $x, y \in U$, the distance function $\sigma_a(x, y)$ is defined as follows:*

$$\sigma_a(x, y) = \frac{\Sigma_{a\in A}\psi_a^*(x, y)}{|A|},$$

where: Since $\psi_a(x, y)$ expresses the qualitative relationship between agents (agreement, neutrality, or disagreement), we transform it into a numerical form that behaves like a distance measure. For this purpose, we define an auxiliary function $\psi_a^*(x, y)$, which maps the values of $\psi_a(x, y)$ into the interval $[0, 1]$, *with 0 representing perfect agreement and 1 representing complete conflict.*

$$\psi_a^*(x, y) = \frac{1 - \psi_a(x, y)}{2} = \begin{cases} 0, & \text{if } a(x)a(y) = 1 \text{ or } x = y, \\ 0.5, & \text{if } a(x)a(y) = 0 \text{ and } x \neq y, \\ 1, & \text{if } a(x)a(y) = -1. \end{cases}$$

Using definition 2.2, the conflict space $CS = (U, \sigma_a)$ is obtained, where $\sigma_A(x, y)$ represent the distance function. Pawlak established the conflict, neutral, and allied relations for conflict analysis utilizing the distance function $\sigma_A(x, y)$ in the following ways:

Definition 2.3. *Consider a conflict space $CS = (U, \sigma_a)$, where $\sigma_A(x, y)$ denotes the distance function. In this context:*
(1) An element $(x, y) \in U$ is classified as a conflict if $\sigma_A(x, y) > 0.5$;
(2) An element $(x, y) \in U$ is classified as neutral if $\sigma_A(x, y) = 0.5$;
(3) An element $(x, y) \in U$ is classified as allied if $\sigma_A(x, y) < 0.5$.

3. Game theoretic model for a hybrid information system

Yang et al. [19] proposed the idea of a hybrid situation table for conflict analysis. It is known that a single data type forms the basis of existing three-way conflict analysis models. However, because the objective world is complex and unpredictable, different situations might be evaluated differently. In decision-making, the complexity of a situation dictates how it should be represented.

However, when it comes to more complex and uncertain decisions, such as determining whether to work in a big city in the future, a more nuanced approach is necessary. These decisions cannot be easily reduced to a simple binary choice; instead, they require more sophisticated data representations, such as interval values or triangular fuzzy numbers, to adequately capture the inherent uncertainty and complexity. To better analyze these intricate decision-making scenarios, hybrid situation tables are employed. These tables combine different types of data to provide a more accurate reflection of real-life situations, particularly when decisions are not straightforward. By incorporating game theory models within these hybrid tables, we can achieve a deeper understanding and more precise analysis of complex decision-making processes.

Definition 3.1. *A hybrid situation table is defined by a 4-tuple $HS = (U, A, \tilde{V}, f)$, where $U = \{x_1, x_2, \ldots, x_n\}$ represents a finite and non-empty set of players, and $A = \{a_1, a_2, \ldots, a_m\}$ represents a finite set of issues. The set $\tilde{V} = \bigcup\{\tilde{V}_a \mid a \in A\}$ contains the ratings of issues, where each $\tilde{V}_a$ corresponds to the ratings of a specific issue a. The rating types are singular, and there are at least two* *distinct issues $a_i, a_j \in A$ (where $a_i \neq a_j$) with different types of ratings. The function f maps each pair from $A \times U$ to $\tilde{V}$.*

A hybrid situation table has multiple rating types. For the proposed model, we use the following seven types of ranking.

$$T = \{I : \text{Three values}, II : \text{Many values}, III : \text{Interval values}, IV : \text{Triangular fuzzy numbers},$$
$$V : \text{Pythagorean fuzzy number}, VI : \text{Trapezoidal fuzzy numbers}, VII : \text{Hesitant fuzzy numbers}\}.$$

Three values are 1, 0, and -1, expressing opinions in favor, neutral, and against the issue, respectively. A significant body of literature on conflict analysis models relies on three values. By many values, we mean choosing multiple values from the interval $[-1, 1]$ following the approach in [7], $(0, 1]$ is the scale for the degree of agreement, $[-1, 0)$ is for disagreement, and 0 is for neutral. Interval values mean interval-valued fuzzy numbers, and the rest of the ranking types are well-known standard terms.

Example 3.2. *Let $\{x_1, x_2, ..., x_6\}$ be set of players and $\{a_1, a_2, \cdots, a_7\}$ be the set of issues. Then, the hybrid information system is given in Table 2.*

Table 2. Hybrid information system.

U/A	a_1	a_2	a_3	a_4	a_5	a_6	a_7
Type	I	II	III	IV	V	VI	VII
x_1	1	{0.2,0.8,-0.3}	[0.78, 0.85]	(0.67, 0.78, 0.89)	$\langle 0.9, 0.3\rangle$	(0.65, 0.79, 0.82, 0.89)	{0.94, 0.8, 0.7}
x_2	0	{-0.6,0.7,0}	[0.90, 0.97]	(0.10, 0.50, 0.61)	$\langle 0.5, 0.5\rangle$	(0.27, 0.34, 0.36, 0.47)	{0.65, 0.31, 0.22}
x_3	−1	{0.4,0,0}	[0.80, 0.95]	(0.15, 0.21, 0.57)	$\langle 0.1, 0.9\rangle$	(0.65, 0.68, 0.72, 0.84)	{0.98, 0.91, 0.81}
x_4	1	{0.1,0.2,0.3}	[0.64, 0.84]	(0.50, 0.60, 0.73)	$\langle 0.1, 0.9\rangle$	(0.44, 0.46, 0.58, 0.65)	{0.66, 0.48, 0.37}
x_5	0	{0.5,-0.8,0.2}	[0.56, 0.65]	(0.22, 0.50, 0.73)	$\langle 0.4, 0.6\rangle$	(0.85, 0.89, 0.95, 0.99)	{0.33, 0.55, 0.63}
x_6	−1	{-0.1,-0.6,0.9}	[0.70, 0.75]	(0.50, 0.50, 0.73)	$\langle 0.9, 0.1\rangle$	(0.85, 0.89, 0.95, 0.99)	{0.72, 0.88, 0.57}

3.1. Opinion comparison mechanism

Assessing and comparing the opinions or plans of different objects involved is very important. However many models do not compare the opinions of players issue-wise, so any independent calculation can lead to a wrong conclusion. The systematic evaluation of the objects' ratings and preferences helps identify potential points of conflict, agreement, and disagreement, making the overall analysis of internal conflicts more effective.

One issue in most of the existing models is the strictness of neutral opinion. For example, 0 is for neutral opinion, but what about 0.01 or $-.01$. They are close to zero, and intuitively they should read as neutral. To resolve this issue, we make adjustments using $\epsilon > 0$ while interpreting the opinions of different rankings. However, the value of ϵ is chosen based on the specific needs of the problem. By setting room for neutrality, the model becomes more adaptable to imprecise data and is better suited to complex real-life scenarios where strict classification may not be appropriate.

From now on, we briefly write the information system as $IS(U, A)$.

Three values

The auxiliary function for three-value to compare the opinions is defined in Definition 2.1, we re-label it as follows.

$$\psi_a^I(x_k, x_l) = \begin{cases} 1, & \text{if } f(a, x_k)f(a, x_l) = 1 \text{ or } x_k = x_y, \\ 0, & \text{if } f(a, x_k)f(a, x_l) = 0 \text{ and } x_k \neq x_l, \\ -1, & \text{if } f(a, x_k)f(a, x_l) = -1. \end{cases}$$

Here $x_k, x_l \in U$ and $a \in A$.

Many values

Let $x \in U$ and $s = \{s_1, s_2, ..., s_m\}$ be its opinion on a issue $a \in A$, where $s_i \in [-1, 1]$ for $i = 1, 2, .., m$. Then, consider

$$\begin{aligned} s^+ =& \{i \in \{1, 2, 3...., m\}; s_i > \epsilon\}, \\ s^- =& \{i \in \{1, 2, 3, ..., m\}; s_i < -\epsilon\}, \\ s^0 =& \{i \in \{1, 2, 3, ..., m\}; s_i \in [-\epsilon, \epsilon]\}. \end{aligned}$$

Evidently, s^+, s^-, and s^0 are the set of indexes with positive/favorable, negative/against and neutral opinions, respectively. Now we compare the opinions of two objects when the opinions are in the form of many values.

Let $x_k, x_l \in U$ and $s = \{s_1, s_2, ..., s_m\}$, $t = \{t_1, t_2, ...t_m\}$ be the opinions for $a \in A$, then opinion comparison function is defined as

$$\psi_a^{II}(x_k, x_l) = \begin{cases} \max\{1 - |s_i - t_j| : & \text{either } \{i \in s^+ \text{ and } j \in t^+\} \text{ or } \{i \in s^- \text{ and } j \in t^-\}\}, \\ \min\{-\frac{|s_i - t_j|}{2} : & \text{either } \{i \in s^+ \text{ and } j \in t^-\} \text{ or } \{i \in s^- \text{ and } j \in t^+\}\}, \\ \{0 : & \text{either } t \in s^0 \text{ or } j \in t^0\}. \end{cases}$$

This piecewise function computes the agreement value by comparing stance pairings across the opinion vectors of the two agents:

- **Agreement case:** When both agents either support (s^+ and t^+) or oppose (s^- and t^-) the issue, the measure takes the maximum of $1 - |s_i - t_j|$ over all such pairings. This value approaches 1 when s_i and t_j are numerically similar, indicating strong agreement.
- **Conflict case:** When one agent supports and the other opposes the issue (that is, s^+ with t^- or s^- with t^+), the measure takes a minimum of $-\frac{|s_i - t_j|}{2}$ across all conflicting pairs, resulting in a strongly negative score to capture severe disagreement.
- **Neutrality case:** If either agent has a neutral position on the issue (i.e., $s_i \in [-\epsilon, \epsilon]$ or $t_j \in [-\epsilon, \epsilon]$), the measure is defined to be zero, reflecting indifference or lack of evaluative stance.

Note that ψ_a^{II} is a multi-valued function due to the nature of multi-dimensional opinion inputs. The output of $\psi_a^{II}(x_k, x_l)$ lies in the interval $[-1, 1]$, where:

$$-1 \leq \psi_a^{II}(x_k, x_l) \leq 1,$$

with $\psi_a^{II}(x_k, x_l) = 1$ indicating complete agreement (i.e., identical positive or negative stances), and $\psi_a^{II}(x_k, x_l) = -1$ indicating maximum conflict.

Interval values

Tran and Duckstein [38] introduced the interval value fuzzy numbers in the conflict analysis model. Let $x \in U$ and $s = [a, b]$ be its opinion on a issue $a \in A$. Then we interpret the opinion as negative/against the issue if $\epsilon + b < 0.5$, that is, the whole interval is on the left side of $0.5 - \epsilon$. If $\epsilon + 0.5 < a$, then it indicates a positive/favorable opinion for the issue, whereas the rest of the cases are interpreted as neutral opinion, i.e., $[a, b] \cap [0.5 - \epsilon, 0.5 + \epsilon] \neq \emptyset$, see Figure 1.

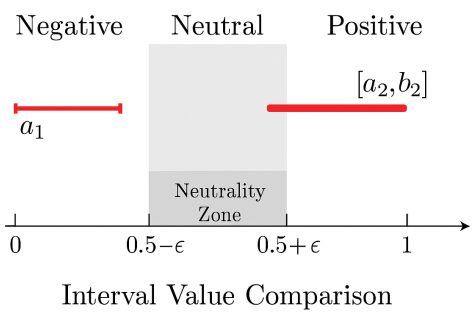

Figure 1. IVFN.

Now we compare the opinions of two objects when the opinions are in the form of an Interval value, Fuzzy numbers. Let $x_k, x_l \in U$ and $s = [a_1, b_1]$, $t = [a_2, b_2]$ be your opinion about issue $a \in A$. The opinion comparison function is

$$\psi_a^{III}(x_k, x_l) = \begin{cases} -|b_2 - a_1|, & \text{if } \epsilon + b_1 < 0.5 \text{ and } \epsilon + 0.5 < a_2, \\ -|a_2 - b_1|, & \text{if } \epsilon + b_2 < 0.5 \text{ and } \epsilon + 0.5 < a_1, \\ 1 - d([a_1, b_1], [a_2, b_2]), & \text{if either } \{\epsilon + b_1 < 0.5 \text{ and } \epsilon + b_2 < 0.5\}, \\ & \text{ or } \{\epsilon + 0.5 < a_1 \text{ and } \epsilon + 0.5 < a_2\}, \\ 0 & \text{otherwise,} \end{cases}$$

where d is the standard distance on the subsets of the real line. The function assigns:

- **Negative values** when the two agents have opposing views (one interval is fully positive, the other fully negative),
- A **similarity score** $\in [0, 1]$ when both intervals lie on the same side of the neutrality zone,
- A score of **zero** when at least one interval is neutral or overlaps the neutrality region.

This design ensures that similarity is emphasized when opinions are aligned, conflict is penalized when positions are opposed, and neutrality is treated as indifference.

Triangular fuzzy numbers

Let $x \in U$ and $s = (a_1, b_1, c_1)$ be its opinion on a issue $a \in A$. Lang et al. [29] introduced the concept of the relative area of a triangle to interpret the opinion. Let s be a TFN. The straight line $x = 0.5$ divides the triangle into two regions: s_R (right region) and s_L (left region) as shown in Figure 2.

If $s_R + \epsilon > s_L$, it indicates a positive attitude of s. Conversely, if $s_R + \epsilon < s_L$, it indicates a negative attitude of s. In the case where $|S_R - S_L| < \epsilon$, it reflects a neutral attitude of s.

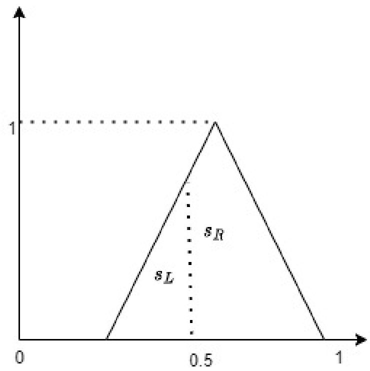

Figure 2. TFN.

Now we compare the opinions of two objects when the opinions are in the form of Triangular Fuzzy numbers. For $x_k, x_l \in U$ and $s = (s_R, s_L)$, $t = (t_R, t_L)$ are the opinions regarding the issue $a \in A$, then opinion comparison function can be defined as

$$\psi_a^{IV}(x_k, x_l) = \begin{cases} -|s_R + t_L|, & \text{if } s_R > s_L + \epsilon \text{ and } \epsilon + t_R < t_L, \\ 1 - |s_R - t_R|, & \text{if } s_R > s_L + \epsilon \text{ and } t_L + \epsilon < t_R, \\ 1 - |s_L - t_L|, & \text{if } s_L > \epsilon + s_R \text{ and } \epsilon + t_R < t_L, \\ -|s_L + t_R|, & \text{if } s_L > \epsilon + s_R \text{ and } t_L + \epsilon < t_R, \\ 0, & \text{if } \textit{either } |s_R - s_L| < \epsilon \text{ or } |t_R - t_L| < \epsilon. \end{cases}$$

The function $\psi_a^{IV}(x_k, x_l)$ assesses the agreement or conflict between two agents whose opinions are represented as triangular fuzzy numbers (s_R, s_L) and (t_R, t_L). When both agents have clearly favorable ($s_R > s_L + \epsilon$ and $t_R > t_L + \epsilon$) or clearly unfavorable ($s_L > s_R + \epsilon$ and $t_L > t_R + \epsilon$) stances, the function returns a similarity score based on the proximity of their right or left endpoints, respectively. If one agent is clearly favorable while the other is clearly unfavorable, the function returns a negative value reflecting a strong conflict. If either opinion has a narrow support ($|s_R - s_L| < \epsilon$ or $|t_R - t_L| < \epsilon$), the stance is treated as neutral, and the function returns 0. This piecewise definition ensures directionally consistent interpretation of polarized or aligned fuzzy opinions.

Pythagorean fuzzy numbers

Furthermore, we work with Pythagorean Fuzzy Numbers. Lang et al. [23] proposed the concept of a Pythagorean fuzzy information system. Let $x \in U$ and $s = \langle \mu_1, \nu_1 \rangle$ represent its opinion on an issue $a \in A$, where for every $x \in U$, $\mu_1(x)$ and $\nu_1(x)$ denote the membership and non-membership degrees, respectively. When $\mu_1 + \epsilon > \nu_1$, it indicates a positive attitude of $x \in U$ toward issue $a \in A$. Conversely, if $\mu_1 + \epsilon < \nu_1$, it reflects a negative attitude. If $|\mu_1 - \nu_1| < \epsilon$, it represents a neutral attitude.

We then compare the opinions of two objects, $x_k, x_l \in U$, expressed as Pythagorean fuzzy numbers $s = \langle \mu_1, \nu_1 \rangle$ and $t = \langle \mu_2, \nu_2 \rangle$ regarding the issue $a \in A$. The opinion comparison function is then defined as:

$$\psi_a^V(x_k, x_l) = \begin{cases} 1 - |\mu_1 - \mu_2|, & \text{if } \epsilon + \nu_1 < \mu_1 \text{ and } \epsilon + \nu_2 < \mu_2, \\ 1 - |\nu_1 - \nu_2|, & \text{if } \epsilon + \mu_1 < \nu_1 \text{ and } \epsilon + \mu_2 < \nu_2, \\ -\frac{|\mu_1+\nu_2|}{2}, & \text{if } \nu_1 + \epsilon < \mu_1 \text{ and } \epsilon + \mu_2 < \nu_2, \\ -\frac{|\mu_2+\nu_1|}{2}, & \text{if } \epsilon + \mu_1 < \nu_1 \text{ and } \epsilon + \nu_2 < \mu_2, \\ 0. & \text{if } |\mu_1 - \nu_1| < \epsilon \text{ and } |\mu_2 - \nu_2| < \epsilon. \end{cases}$$

The function $\psi_a^V(x_k, x_l)$ compares the fuzzy opinions of two agents x_k and x_l on issue $a \in A$, where each opinion is represented as a pair (μ, ν) denoting favorability and opposition, respectively. When both agents express strong positive opinions ($\mu_i > \nu_i + \epsilon$), their similarity is given by $1 - |\mu_1 - \mu_2|$. Similarly, if both express strong negative opinions ($\nu_i > \mu_i + \epsilon$), the function returns $1 - |\nu_1 - \nu_2|$.

In cases of direct opposition, where one agent supports while the other opposes, the function returns a negative value: Either $-\frac{|\mu_1+\nu_2|}{2}$ or $-\frac{|\mu_2+\nu_1|}{2}$, depending on the configuration. When neither opinion is distinct (i.e., $|\mu_i - \nu_i| < \epsilon$), the function outputs 0, indicating neutrality.

This piecewise formulation captures alignment, conflict, or neutrality between agents' fuzzy stances, while the parameter ϵ ensures tolerance for minor deviations.

Trapezoidal fuzzy numbers

Let $x \in U$ and $s = (a_1, b_1, c_1, d_1)$ be a trapezoidal fuzzy number as its opinion. Yang et al. [12] used the concept of the expectation of trapezoidal fuzzy numbers $E(s)$ to represent an agent's particular stance on an issue, defined as

$$E(s) = \frac{\int_a^d x f_s(x) dx}{\int_a^d f_s(x) dx},$$

where $f_s(x)$ is its membership function. By simple integration, we get

$$\int_a^d x f_s(x) dx = \frac{(-a^2 - b^2 + c^2 + d^2 - ab + cd)}{6}, \tag{3.1}$$

$$\int_a^d f_s(x) dx = \frac{-a - b + c + d}{2}. \tag{3.2}$$

Then

$$E(s) = \frac{(-a^2 - b^2 + c^2 + d^2 - ab + cd)}{3(-a - b + c + d)},$$

which expresses the specific attitude of an object toward an issue. $E(s)$ lies between a_1 and d_1. If s is a symmetric trapezoidal fuzzy number, then $E(s) = \frac{b_1+c_1}{2}$.

Now we compare the opinions of two objects when the opinions are in the form of a trapezoidal fuzzy number. Let $x_k, x_l \in U$ and $s = (a_1, b_1, c_1, d_1)$, and $t = (a_2, b_2, c_2, d_2)$ be the opinions for $a \in A$, then opinion comparison function can be defined as

$$\psi_a^{VI}(x_k, x_l) = \begin{cases} 1 - |E_s - E_t|, & \text{if } \epsilon + E_s < 0.5 \text{ and } \epsilon + E_t < 0.5, \\ 1 - |E_s - E_t|, & \text{if } 0.5 + \epsilon < E_s \text{ and } 0.5 + \epsilon < E_t, \\ -\frac{|E_s + E_t|}{2}, & \text{if } \epsilon + E_s < 0.5 \text{ and } 0.5 + \epsilon < E_t, \\ -\frac{|E_s + E_t|}{2}, & \text{if } 0.5 + \epsilon < E_s \text{ and } \epsilon + E_t < 0.5, \\ 0 & \text{if } E_s \text{ or } E_t \in [0.5 - \epsilon, 0.5 + \epsilon]. \end{cases}$$

The comparison is based on the positions of E_s and E_t relative to the neutrality threshold 0.5, adjusted by a tolerance ϵ. If both expected values lie significantly below or above the threshold, the function returns a similarity score $1 - |E_s - E_t|$. If one lies below and the other above, it indicates opposition and returns $-\frac{|E_s+E_t|}{2}$. When either expected value is within the neutral zone $[0.5 - \epsilon, 0.5 + \epsilon]$, the function assigns a score of 0, indicating neutrality or uncertainty.

This formulation effectively distinguishes agreement, opposition, and neutrality in trapezoidal fuzzy opinions while accounting for small deviations via ϵ.

Hesitant fuzzy numbers

Yang et al. [39] proposed a conflict analysis model in the framework of a hesitant fuzzy set. Let $s = (s_1, s_2, \ldots, s_n)$ and $t = (t_1, t_2, \ldots, t_m)$ represent the opinions of two objects x_k and $x_l \in U$. The hesitant Euclidean distance between these opinions is given by.

$$d_e(s, t) = (\frac{1}{n} \sum_{i=1}^{n} |s_i - t_i|^2)^{\frac{1}{2}}.$$

The auxiliary function for hesitant fuzzy numbers can be defined as

$$\psi_a^{VII}(x_k, x_l) = \begin{cases} 0, & \text{if } d_e(s, t) \in [0.5 - \epsilon, 0.5 + \epsilon], \\ 1 - 2d_e(s, t). & \text{if } otherwise. \end{cases}$$

This function evaluates the similarity or opposition between hesitant fuzzy opinions. If the distance between them lies within a neutral tolerance zone centered at 0.5 (controlled by parameter ϵ), the function assigns a value of 0, indicating a neutral or indistinct relationship. Outside this zone, it returns $1 - 2d_e(s, t)$, providing a similarity score that decreases as the hesitant distance increases. This ensures the function distinguishes aligned and divergent opinions while accommodating uncertainty through ϵ.

Discussion:

We compare the opinions issue by issue to correctly find the ideal position for players. We like to summarize the properties and explanations for auxiliary functions for all types.

- For $x_k, x_l \in U$, $a \in A$ and for all $ty \in T$ $-1 \leq \psi_a^{ty}(x_k, x_l) \leq 1$.
- When $\psi_a^{ty}(x_k, x_l) > 0$, the players have similar opinion. The similarity increases as the value approaches 1 and equal to 1 means exact same opinion for both players.
- When $\psi_a^{ty}(x_k, x_l) < 0$, the players have opposite stances. The dissimilarity increases as the value decreases to -1. It is equal to -1 when players have the maximum possible opposition.

- When $\psi_a^{ty}(x_k, x_l) = 0$, one of the players is neutral on a given issue.
- Auxiliary function is symmetric i.e., $\psi_a^{ty}(x_k, x_l) = \psi_a^{ty}(x_l, x_k)$.

The primary advantage of a hybrid information system is the flexibility to select any dataset for providing opinions. Different people have different preferences.

Example 3.3. *(Continued from Example 3.2) We present some computations for understanding auxiliary function in Table 3.*

Table 3. Auxiliary function values for information system of Table 2.

$\Psi_a(x_1, x_j)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7
$\Psi_a(x_1, x_2)$	0	{0.89,-0.7,0}	0.95	0.9	0	-0.57	0
$\Psi_a(x_1, x_3)$	-1	{0.8,-0.35,0}	1	0.9	-0.9	0.94	0.81
$\Psi_a(x_1, x_4)$	1	{1,-0.3,0}	1	0.995	-0.9	0	0.37
$\psi_a(x_1, x_5)$	0	{1,-0.8}	0	0	-0.75	0.86	0.23
$\Psi_a(x_1, x_6)$	-1	{0.9,-0.7,0}	0.97	0.995	1	0.86	0.69

3.2. Conflict, alliance, and neutrality measures

In this section, we introduce three essential measures: The alliance measure M_A, the neutrality measure M_N, and the conflict measure M_C. These measures are based on an auxiliary function that evaluates the level of agreement or disagreement between two players or objects, represented by x_k and x_l, across all issues.

Let x_k and x_l represent two objects and $w = (w_1, w_2, ...w_m)$ be the weight vector of issues, which satisfies $w_i \in [0, 1]$ also $\Sigma w_i = 1$. First, we divide issues into three parts:

$$A^+ = \{a_i \mid \psi_{a_i}(x_k, x_l) > 0\},$$

$$A^- = \{a_i \mid \psi_{a_i}(x_k, x_l) < 0\},$$

$$A^0 = \{a_i \mid \psi_{a_i}(x_k, x_l) = 0\}.$$

Fundamentally, A^+, A^-, and A^0 collect the issues for which players x_k and x_l have similar, opposite, and neutral opinions, respectively. Then

$$M_A(x_k, x_l) = \sum_{a_i \in A^+} w_i \psi_{a_i}^{ty_i}(x_k, x_l), \tag{3.3}$$

$$M_N(x_k, x_l) = \sum_{a_i \in A^0} w_i, \tag{3.4}$$

$$M_c(x_k, x_l) = -\sum_{a_i \in A^-} w_i \psi_{a_i}^{ty_i}(x_k, x_l), \tag{3.5}$$

where $ty_i \in T$ is type of ranking used for issue $a_i \in A$.

The degrees of agreement (positive values), neutrality (zero values), and disagreement (negative values) are collected individually in this instance. We can, therefore, calculate the player's winnings for each of the three potential strategies. It should be noted that these measures (M_A, M_N, M_C) range from 0 to 1.

3.3. Game formulation

This subsection describes how the game between two players is conducted and details the creation of payoff functions and decision-making rules. Let U represent the set of objects and A represent the set of issues, with $IS(U, A)$ being the hybrid information system as defined in Definition 3.1.

Set of players: U is considered as the set of players. The designed game will involve these players with two competing at a time.

Set of Strategies: Let $S = \{s_{all}, s_{neu}, s_{con}\}$ be the set of strategies available to every player in U, where:

s_{all} signifies the strategy of allying with another player;

s_{neu} denotes the strategy of remaining neutral towards another player;

s_{con} Indicates the tactic of putting another player in conflict.

The Payoff Functions: For two players x_k and x_l in U, the payoff functions are represented as u_{x_k} and u_{x_l} for the respective players.

During the gameplay, the payoffs for each player are calculated based on the specific strategy, they choose from set S. Below are the two sets of payoff functions.

$$\begin{aligned} u_{x_k} = \{&u_{x_k}(s_{all}, s_{all}), u_{x_k}(s_{all}, s_{neu}), u_{x_k}(s_{all}, s_{con}),\\ &u_{x_k}(s_{neu}, s_{all}), u_{x_k}(s_{neu}, s_{neu}), u_{x_k}(s_{neu}, s_{con}),\\ &u_{x_k}(s_{con}, s_{all}), u_{x_k}(s_{con}, s_{neu}), u_{x_k}(s_{con}, s_{con})\}. \end{aligned}$$

$$\begin{aligned} u_{x_l} = \{&u_{x_l}(s_{all}, s_{all}), u_{x_l}(s_{all}, s_{neu}), u_{x_l}(s_{all}, s_{con}),\\ &u_{x_l}(s_{neu}, s_{all}), u_{x_l}(s_{neu}, s_{neu}), u_{x_l}(s_{neu}, s_{con}),\\ &u_{x_l}(s_{con}, s_{all}), u_{x_l}(s_{con}, s_{neu}), u_{x_l}(s_{con}, s_{con})\}. \end{aligned}$$

We determine each player's payoffs (or gains) for all possible strategies during the gameplay. This approach enables a player to select the optimal strategy that yields the highest benefit, using the concept of Nash equilibrium.

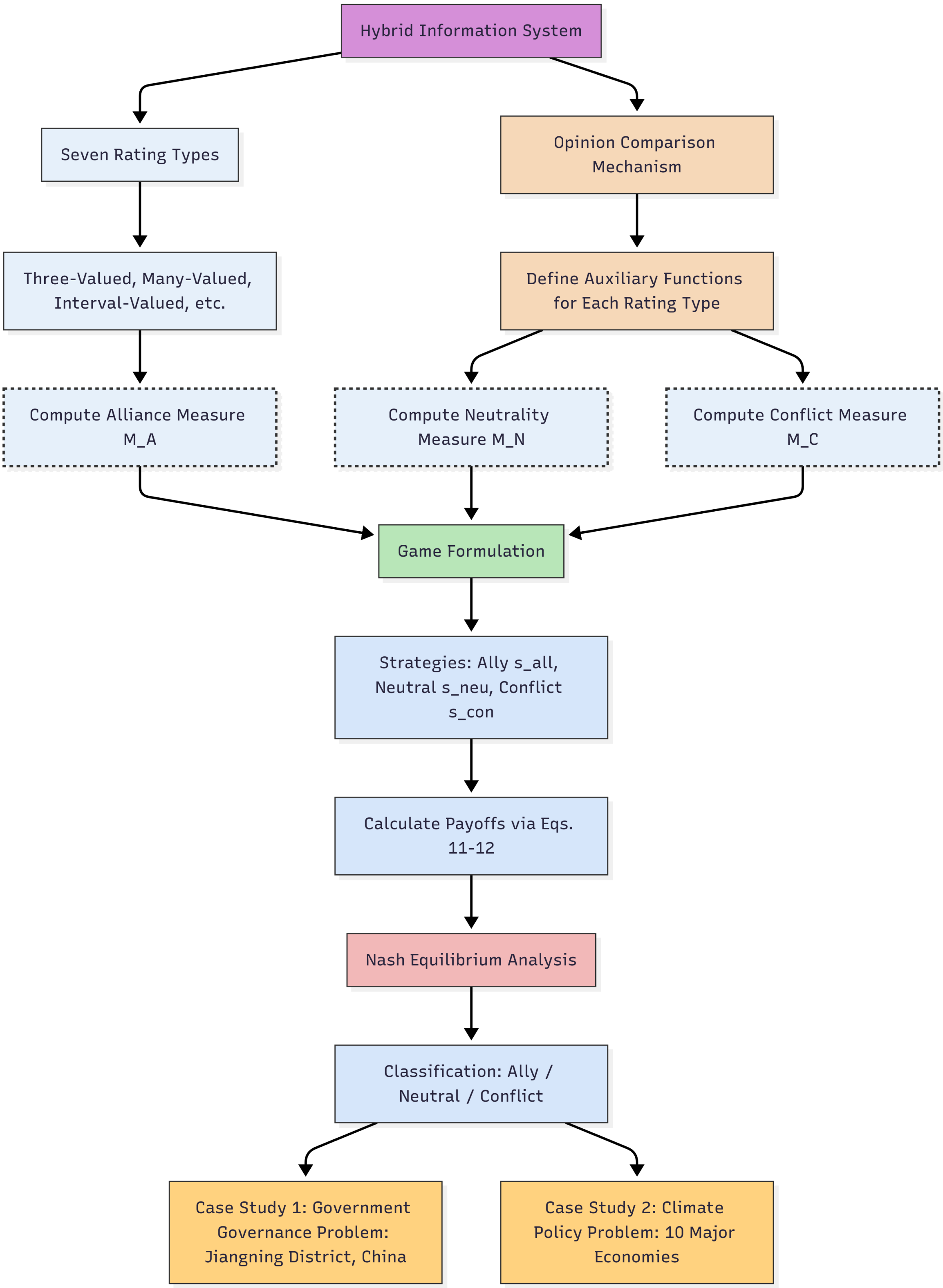

Figure 3. Workflow of the proposed conflict analysis model.

To make the proposed model more flexible, we use three pairs of threshold values (α_A, β_A),

(α_N,β_N), and (α_C,β_C) that decision makers choose to determine the weight given to forming alliances, maintaining neutrality, and engaging in conflict, respectively. For players x_k and x_l in U, the initial weighted payoff, denoted by Δ_{x_k,x_l}, for each strategy is defined as follows:

$$\Delta_{x_k,x_l}(s_{all}) = \begin{cases} 1, & M_A(x_k, x_l) \geq \alpha_A, \\ 0.5, & \beta_A < M_A(x_k, x_l) < \alpha_A, \\ 0, & M_A(x_k, x_l) \leq \beta_A. \end{cases} \tag{3.6}$$

$$\Delta_{x_k,x_l}(s_{neu}) = \begin{cases} 1, & M_N(x_k, x_l) \geq \alpha_N, \\ 0.5, & \beta_N < M_N(x_k, x_l) < \alpha_N, \\ 0, & M_N(x_k, x_l) \leq \beta_N. \end{cases} \tag{3.7}$$

$$\Delta_{x_k,x_l}(s_{con}) = \begin{cases} 1, & M_C(x_k, x_l) \geq \alpha_C, \\ 0.5, & \beta_C < M_C(x_k, x_l) < \alpha_C, \\ 0, & M_C(x_k, x_l) \leq \beta_C. \end{cases} \tag{3.8}$$

In light of the definitions provided, the threshold pairs of values (α_A,β_A), (α_N,β_N), and (α_C,β_C) determine the preferences for alliance, neutrality, and conflict, respectively. For example, when the value of α_A decreases, the value of Δ for the alliance strategy increases, leading to greater gains for players who choose this strategy. Consequently, if players aim to resolve a conflict, they can lower α_A while increasing α_N and α_C. Similarly, a smaller value of α_C will yield higher payoffs for those adopting a conflict strategy.

The utility function $\Delta_{x_k,x_l}(s_{all})$ measures the alliance preference of player x_k with player x_l. It reaches 1 when $M_A(x_k, x_l)$ is at or above α_A, showing a strong desire for alliance. It drops to 0 when $M_A(x_k, x_l)$ is at or below β_A, indicating a preference for conflict. A value of 0.5 suggests uncertainty, where $M_A(x_k, x_l)$ is between α_A and β_A. The remaining two functions, $\Delta_{x_k,x_l}(s_{con})$ and $\Delta_{x_k,x_l}(s_{neu})$, can also be explained in a similar manner. Here, communication between several players inside the game is necessary. The other players' favored strategies would have an impact on the players' own beliefs and utility. We account for this by using the average utility of the participating players to determine their payoff functions. This approach encourages participants to coordinate decisions on whether objects fall into allied, neutral, or conflict categories. Both players' opinions are considered equally valuable. The preliminary payoff serves as the basis for defining the payoffs for players x_k and x_l, with strategies s_1 and s_2 in $S = \{s_{all}, s_{neu}, s_{con}\}$. The player's payoff is defined as follows:

$$u_{x_k}(s_1, s_2) = \frac{\Delta_{x_k,x_l}(s_1) + \Delta_{x_k,x_l}(s_2)}{2}, \tag{3.9}$$

$$u_{x_l}(s_1, s_2) = \frac{\Delta_{x_k,x_l}(s_1) + \Delta_{x_k,x_l}(s_2)}{2}. \tag{3.10}$$

To ensure fairness and interpretability, the model adopts a symmetric Δ average payoff, treating the views of both players equally and delivering a common payoff game where any jointly optimal profile (e.g., mutual neutrality) is a Nash equilibrium. As an alternative, assigning each player a distinct payoff based solely on their own Δ (i.e., $u_{x_k}(s_k, s_l) = \Delta_{x_k,x_l}(s_k)$, $u_{x_l}(s_k, s_l) = \Delta_{x_k,x_l}(s_1)$) restores individual utility functions and introduces richer, potentially conflicting equilibria, aligning more closely with classical game-theoretic principles. A game mechanism is designed between two players, x_k and x_l, with payoffs presented in Table 4. Each block in the table represents a strategy profile along with its corresponding payoffs. For example, the top-left block shows the payoffs $u_{x_k}(s_{all}, s_{all})$ and $u_{x_l}(s_{all}, s_{all})$ when both players choose the strategy (s_{all}, s_{all}). Every cell in Table 4 displays a pair of payoffs $(u_{x_k}(s_p, s_q), u_{x_l}(s_p, s_q))$, corresponding to the strategies s_p for player x_k and s_q for player x_l. Table 4 lists the nine pairs of payoffs that we used to design this game. A formal rule known as the game solution is used to predict the optimal strategy profile in which participants adopt the behaviors they find most comfortable. Three-way decisions are then made using the game solutions.

Table 4. Payoff table for the two-player game.

		x_l		
		s_{all}	s_{neu}	s_{con}
x_k	s_{all}	$u_{x_k}(s_{\text{all}}, s_{\text{all}}), u_{x_l}(s_{\text{all}}, s_{\text{all}})$	$u_{x_k}(s_{\text{all}}, s_{\text{neu}}), u_{x_l}(s_{\text{all}}, s_{\text{neu}})$	$u_{x_k}(s_{\text{all}}, s_{\text{con}}), u_{x_l}(s_{\text{all}}, s_{\text{con}})$
	s_{neu}	$u_{x_k}(s_{\text{neu}}, s_{\text{all}}), u_{x_l}(s_{\text{neu}}, s_{\text{all}})$	$u_{x_k}(s_{\text{neu}}, s_{\text{neu}}), u_{x_l}(s_{\text{neu}}, s_{\text{neu}})$	$u_{x_k}(s_{\text{neu}}, s_{\text{con}}), u_{x_l}(s_{\text{neu}}, s_{\text{con}})$
	s_{con}	$u_{x_k}(s_{\text{con}}, s_{\text{all}}), u_{x_l}(s_{\text{con}}, s_{\text{all}})$	$u_{x_k}(s_{\text{con}}, s_{\text{neu}}), u_{x_l}(s_{\text{con}}, s_{\text{neu}})$	$u_{x_k}(s_{\text{con}}, s_{\text{con}}), u_{x_l}(s_{\text{con}}, s_{\text{con}})$

Nash equilibrium is a strategic portfolio and a stable state of a game in which each player is aware of every potential prediction regarding the move of every other player. It is also the best answer to the opposing player's potential option. In game theory, the Nash equilibrium is commonly utilized to determine potential game outcomes through game solutions. For the players x_k and x_l in the suggested two-player game a payoff pair $(u'_{x_k}(s_{all}, s_{all}), u'_{x_l}(s_{all}, s_{all}))$ is an equilibrium point if for any strategy s_e:

$$u'_{x_k}(s_{all}, s_{all}) \geq u_{x_k}(s_e, s_v) \ \forall \ s_e, s_v \in S, \tag{3.11}$$

$$u'_{x_l}(s_{all}, s_{all}) \geq u_{x_l}(s_e, s_o) \ \forall \ s_e, s_o \in S. \tag{3.12}$$

It is stated that this payoff pair $(u'_{x_k}(s_{all}, s_{all}), u'_{x_l}(s_{all}, s_{all}))$ is the ideal choice or a balanced point in deciding the players' actions and any deviation in it would not be advantageous to any player. The other two payoff pairings $(u'_{x_k}(s_{neu}, s_{neu}), u'_{x_l}(s_{neu}, s_{neu}))$ and $(u'_{x_k}(s_{con}, s_{con}), u'_{x_l}(s_{con}, s_{con}))$ are similarly described. The game theory model analyzes payoff tables using Nash equilibrium to determine optimal solutions in games.

3.4. Proposed conflict analysis model

We systematically compare the opinions of players for all issues and devise ways to measure the degree of similarity, dissimilarity, and neutrality. The three measures M_A, M_N, and M_C are the basis for computing payoffs for all three strategies given in game formulation subsection, see the flow chart 3.

Now, we provide the algorithm to practically apply the proposed theory. We play games between all the players taking two at a time and as a result classify them as allies, in conflict, neutral (see Algorithm 1, where we design its code using Python language).

Algorithm 1 Conflict analysis model

Input: Information system $IS(P, A)_{n\times m}$

for $i \in \{1, 2, \ldots, n\}$ **do**
    Assign three empty sets $ALL(i) = \emptyset$, $NEU(i) = \emptyset$, and $CON(i) = \emptyset$;
    if $i < n$ **then**
        for $j \in \{i + 1, i + 2, \ldots, n\}$ **do**
            for $v \in \{1, 2, \ldots, m\}$ **do**
                Compute auxiliary function $\psi_{a_v}^{ty_v}(x_i, x_j)$ according to ranking type ty_v;
            end for
            Compute alliance measure $M_A(x_i, x_j)$ according to Eq (3.3);
            Compute neutrality measure $M_N(x_i, x_j)$ according to Eq (3.4);
            Compute conflict measure $M_C(x_i, x_j)$ according to Eq (3.5);
            Compute utility function $\Delta_{x_i,x_j}(s_{all})$ based on (α_A, β_A) and Eq (3.6);
            Compute utility function $\Delta_{x_i,x_j}(s_{new})$ based on (α_N, β_N) and Eq (3.7);
            Compute utility function $\Delta_{x_i,x_j}(s_{con})$ based on (α_C, β_C) and Eq (3.8);
            for $s_1 \in \{s_{all}, s_{new}, s_{con}\}$ **do**
                for $s_2 \in \{s_{all}, s_{new}, s_{con}\}$ **do**
                    Compute payoff $u_{x_i}(s_1, s_2)$ according to Eq (3.9);
                    Compute payoff $u_{x_j}(s_1, s_2)$ according to Eq (3.10);
                end for
            end for
        end for
        Put p_j in $ALL(i)$ or $NEU(i)$ or $CON(i)$ based on Nash equilibrium;
    end if
    if $i > 1$ **then**
        for $j \in \{1, 2, \ldots, i - 1\}$ **do**
            if $p_i \in ALL(j)$ **then**
                Put p_j in $ALL(i)$;
            end if
            if $p_i \in NEU(j)$ **then**
                Put p_j in $NEU(i)$;
            end if
            if $p_i \in CON(j)$ **then**
                Put p_j in $CON(i)$;
            end if
        end for
    end if
end for

4. Case studies and comparative analysis

In this section, we present two case studies: Jiangning District development plan [19,40] and climate policy conflict analysis. For the Government Governance Analysis, we compare the results of our algorithm with that of Yang et al. [26]. Additionally, we apply our algorithm on the climate policy of 10 big economies of the world and report our results.

4.1. Government governance analysis - Jiangning District development plan

The case of Jiangning District development plan [19, 40] is used here to illustrate a government governance problem in a more complex scenario, highlighting the practical applicability of our proposed model. Local governments in China play a key role in implementing national policies, promoting democratic values, and driving socioeconomic growth, acting as a bridge between the people and the central government. Due to rapid modernization, they face increasing pressure to enhance their legal frameworks, decision-making processes, coordination, crisis management, and resource allocation. Effective governance is critical in managing conflicts that arise during economic development. We analyze Jiangning District, a major hub for science, education, and advanced manufacturing in southeast Nanjing, known for its integration of highway, rail, maritime, and air transportation, to assess the effectiveness of the proposed approach. Consider $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ as a set of objects (street offices) involved in the local government, where
x_1: Dongshan
x_2: Moling
x_3: Tangshan
x_4: Dunhua
x_5: Lukou
x_6: Jiangning
x_7: Guli
x_8: Hushu
x_9: Hengxi
x_{10}: Binjiang

The set $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ represents the conflict issues that Jiangning District's administration must resolve to achieve sustainable economic development. where,
a_1: Employment
a_2: Environment
a_3: Transportation
a_4: Housing
a_5: Education
a_6: Entertainment
a_7: Internet services

We will evaluate governance challenges using the information system $IS(U, A)$ shown in Table 5.

Table 5. A hybrid situation table based on a government governance problem.

U/A	a_1	a_2	a_3	a_4	a_5	a_6	a_7
Type	I	II	III	IV	V	VI	VII
x_1	-1	-0.2	$[0.00, 0.50]$	$(0.11, 0.22, 0.33)$	$\langle 1.00, 0.00\rangle$	$(0.12, 0.21, 0.33, 0.40)$	$\{0.29, 0.16, 0.16\}$
x_2	$+1$	0	$[0.15, 0.30]$	$(0.33, 0.50, 0.85)$	$\langle 0.95, 0.15\rangle$	$(0.03, 0.18, 0.29, 0.40)$	$\{0.98, 0.91, 0.81\}$
x_3	$+1$	$+0.3$	$[0.26, 0.45]$	$(0.42, 0.72, 0.83)$	$\langle 0.10, 0.90\rangle$	$(0.57, 0.67, 0.69, 0.75)$	$\{0.88, 0.73, 0.73\}$
x_4	0	$+0.7$	$[0.45, 0.55]$	$(0.50, 0.50, 0.50)$	$\langle 0.55, 0.45\rangle$	$(0.42, 0.48, 0.52, 0.58)$	$\{0.54, 0.43, 0.43\}$
x_5	$+1$	-0.9	$[0.78, 0.85]$	$(0.50, 0.56, 0.61)$	$\langle 0.90, 0.20\rangle$	$(0.79, 0.85, 0.87, 0.96)$	$\{0.95, 0.85, 0.85\}$
x_6	0	$+0.6$	$[0.90, 0.97]$	$(0.22, 0.41, 0.93)$	$\langle 0.00, 1.00\rangle$	$(0.87, 0.89, 0.93, 0.94)$	$\{0.66, 0.48, 0.37\}$
x_7	-1	$+0.4$	$[0.3, 0.55]$	$(0.70, 0.76, 0.81)$	$\langle 0.81, 0.35\rangle$	$(0.66, 0.75, 0.77, 0.86)$	$\{0.33, 0.55, 0.63\}$
x_8	$+1$	$+0.5$	$[0.24, 0.34]$	$(0.62, 0.71, 0.93)$	$\langle 1.00, 0.00\rangle$	$(0.27, 0.30, 0.33, 0.40)$	$\{0.72, 0.88, 0.57\}$
x_9	0	$+0.8$	$[0.56, 0.65]$	$(0.52, 0.56, 0.61)$	$\langle 0.05, 0.75\rangle$	$(0.56, 0.65, 0.67, 0.76)$	$\{0.92, 0.95, 0.95\}$
x_{10}	-1	-0.3	$[0.45, 0.60]$	$(0.42, 0.51, 0.83)$	$\langle 0.70, 0.10\rangle$	$(0.77, 0.83, 0.86, 0.92)$	$\{0.16, 0.28, 0.37\}$

Game play for x_1: We show the numerical data for gameplay between player x_1 and the other players x_j, where $j = 2, 3, ..., 10$. This serves to demonstrate the practical use of the proposed model.

Calculating auxiliary functions: The auxiliary function $\psi_a(x_1, x_j)$ is then computed for each attribute a in A. Table 6 displays the values obtained from these auxiliary functions for player x_1 and the other players.

Table 6. Auxiliary functions of the Hybrid situation table.

$\Psi_a(x_1, x_j)$	a_1	a_2	a_3	a_4	a_5	a_6	a_7
$\Psi_a(x_1, x_2)$	-1	0	0	0	0.95	0.96	-0.395
$\Psi_a(x_1, x_3)$	-1	-0.25	0	0.915	-0.95	-0.46	0
$\Psi_a(x_1, x_4)$	0	-0.44	0	0	0	0	0.47
$\psi_a(x_1, x_5)$	-1	0.3	0	0	0.9	-0.56	-0.36
$\Psi_a(x_1, x_6)$	0	-0.4	0	0	-1	-0.58	0.38
$\Psi_a(x_1, x_7)$	1	-0.3	0	0	0.81	-0.51	0.29
$\Psi_a(x_1, x_8)$	-1	-0.35	0	-0.53	1	0.93	0
$\Psi_a(x_1, x_9)$	0	-0.5	0	0	-0.87	-0.46	-0.48
$\Psi_a(x_1, x_{10})$	1	0.9	0	0.94	0.7	-0.55	0.68

Calculating Perception Measures: Next, using Eq (3.3) through (3.5), we compute the opinion measures M_A, M_N, and M_C. Specifically, the weight vector w is defined as:

$$w = (w_1, w_2, \dots, w_7),$$

where each weight corresponds to an issue $a_i \in A$ and is given by:

$$w = [0.2,\ 0.12,\ 0.12,\ 0.12,\ 0.2,\ 0.12,\ 0.12]\,.$$

This satisfies the normalization condition:

$$\sum_{i=1}^{7} w_i = 1.$$

Naturally, some issues are more important than others, like employment and education in the conflict analysis of Jiangning District's governance scenario.

The computation is done for the three opinion measures: M_A, M_N, and M_C. For example, we compute $M_C(x_1, x_2)$ as a conflict measure for objects x_1 and x_2, given that their total disagreement or conflict measure is 0.247. Similarly, $\sum_{a \in A^+} w_i \psi_a(x_1, x_2) = 0.272$ and $\sum_{a \in A^0}(w_i) = 0.4285$ are the values obtained for $M_A(x_1, x_2)$, and $M_N(x_1, x_2)$, respectively. Thus, the values of $M_C(x_1, x_2)$, $M_N(x_1, x_2)$, and $M_A(x_1, x_2)$ can be observed in Table 7.

Table 7. Opinion measures of the hybrid situation.

(x_1, x_j)	M_A	M_N	M_C
(x_1, x_2)	0.305	0.36	0.247
(x_1, x_3)	0.109	0.24	0.475
(x_1, x_4)	0.056	0.76	0.053
(x_1, x_5)	0.216	0.24	0.31
(x_1, x_6)	0.046	0.44	0.318
(x_1, x_7)	0.397	0.24	0.097
(x_1, x_8)	0.312	0.24	0.305
(x_1, x_9)	0	0.44	0.348
(x_1, x_{10})	0.64	0.12	0.065

Calculating the evaluation function: The evaluation function $\Delta_{x_i,x_j}(s)$ is calculated for each s in S using Eqs (3.6)–(3.8). The values of $\Delta_{x_i,x_j}(s)$ depend on the threshold values (α_A,β_A) and (α_C,β_C). After examining the opinion measures, a set of thresholds can be established for each player. If the desire for peace is strong, the value of α_A increases, leading to higher payoffs for alliance strategies. Depending on players' opinion measures, they can adopt different strategies: Alliance, neutrality, or conflict. In this case, the threshold values are taken as $(\alpha_A,\beta_A) = (\alpha_N,\beta_N) = (\alpha_C,\beta_C) = (0.7, 0.3)$. From Table 8, $\Delta_{x_1,x_2}(s_{neu})$ is 0.5 as $\beta_N < M_N(x_1, x_2) < \alpha = 0.36$. This indicates that the player x_1 is neutral for player x_2. Similarly, the utility function for the player x_2 within the neutral range is 0.5 as $\beta_N < M_N(x_1, x_2) < \alpha = 0.36$, suggesting that x_2 is neutral with respect to x_1.

Table 8. Payoffs for players x_1 and x_2.

		x_2		
		s_{all}	s_{neu}	s_{con}
x_1	s_{all}	(0.5, 0.5)	(0.5,0.5)	(0.25,0.25)
	s_{neu}	(0.5, 0.5)	(0.5, 0.5)	(0.25, 0.25)
	s_{con}	(0.25, 0.25)	(0.25, 0.25)	(0.00, 0.00)

Computation of payoffs for all strategies: For each strategy in S, the payoffs for both players, u_{x_k} and u_{x_l}, are determined using Eqs (3.9) and (3.10). The payoff function for the pair $(u_{x_1}(s_{con}, s_{con}), u_{x_2}(s_{con}, s_{con}))$ can be computed as follows:

$$u_{x_1}(s_{con}, s_{con}) = \frac{\Delta_{x_1,x_2}(s_{con}) + \Delta_{p_2,p_1}(s_{con})}{2} = \frac{0.5 + 0.5}{2} = 0.5,$$

$$u_{x_2}(s_{con}, s_{con}) = \frac{\Delta_{x_2,x_1}(s_{con}) + \Delta_{x_1,x_2}(s_{con})}{2} = \frac{0.5 + 0.5}{2} = 0.5.$$

Table 9 shows that other payoff pairs can be computed similarly. Playing a game between two players also enables you to generalize this strategy; see Tables 10–16. Furthermore, based on Nash equilibrium, players x_k and x_l can be classified as allies, neutrals, or engaged in conflict. Using Eqs (3.9) and (3.10), we find the equilibrium point to classify x_k and x_l as allies, neutrals, or at odds based on the Nash equilibrium. A closer look at Table 8 shows that x_1 and x_2 achieve the greatest gain when allied. Consequently, they are classified as allied under the Nash equilibrium principle. Similarly, as in the games between players x_1 and x_j, we also analyze the interactions of player x_1 with every other participant. For players x_3 and x_5, the hybrid system (HS) exhibits the highest level of conflict, i.e., $M_C(x_1, x_3) = 0.475$, whereas the measures for alliance $M_A(x_1, x_3) = 0.109$ and neutrality $M_N(x_1, x_3) = 0.24$ are comparatively low. The payoffs for players x_1 and x_3 in the corresponding game are shown in Table 9. Likewise, for players x_4, x_6, and x_9, the hybrid system indicates neutrality, with the measures for conflict and alliance being relatively balanced but lower in magnitude. Similarly, for players x_2, x_7, x_8, and x_{10}, the hybrid system indicates a strong alliance, while the measures for conflict and neutrality remain comparatively low. Overall, the analysis suggests that player x_1 maintains strong alliances with x_2, x_7, x_8, and x_{10}, neutral relationships with x_4, x_6, and x_9, and conflicting interactions with x_3 and x_5, as shown in Table 17. For the first object x_1, the Conflict, Neutral, and Allied sets can be described through the lens of Nash equilibrium analysis as follows:

- **Conflict set**: $\mathrm{CON}(x_1) = \{x_3, x_5\}$;
- **Neutral set**: $\mathrm{NEU}(x_1) = \{x_4, x_6, x_9\}$;
- **Allied set**: $\mathrm{ALL}(x_1) = \{x_2, x_7, x_8, x_{10}\}$.

Whereas, Table 17 and Figure 4 show complete solution.

Table 9. Payoffs for players x_1 and x_3.

		x_3		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.00, 0.00)	(0.00, 0.00)	(0.25, 0.25)
x_1	s_{neu}	(0.00, 0.00)	(0.00, 0.00)	(0.25, 0.25)
	s_{con}	(0.25, 0.25)	(0.25, 0.25)	(0.5, 0.5)

Table 10. Payoffs for players x_1 and x_4.

		x_4		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.00, 0.00)	(0.5, 0.5)	(0.00, 0.00)
x_1	s_{neu}	(0.5, 0.5)	(1, 1)	(0.5, 0.5)
	s_{con}	(0.00, 0.00)	(0.5, 0.5)	(0.00, 0.00)

Table 11. Payoffs for players x_1 and x_5.

		x_5		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.00, 0.00)	(0.00, 0.00)	(0.25, 0.25)
x_1	s_{neu}	(0.00, 0.00)	(0.00, 0.00)	(0.25, 0.25)
	s_{con}	(0.25, 0.25)	(0.25, 0.25)	(0.5, 0.5)

Table 12. Payoffs for players x_1 and x_6.

		x_6		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.00, 0.00)	(0.25, 0.25)	(0.25, 0.25)
x_1	s_{neu}	(0.25, 0.25)	(0.5, 0.5)	(0.5, 0.5)
	s_{con}	(0.25, 0.25)	(0.5, 0.5)	(0.5, 0.5)

Table 13. Payoffs for players x_1 and x_7.

		x_7		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.5, 0.5)	(0.25, 0.25)	(0.25, 0.25)
x_1	s_{neu}	(0.25, 0.25)	(0.00, 0.00)	(0.00, 0.00)
	s_{con}	(0.25, 0.25)	(0.00, 0.00)	(0.00, 0.00)

Table 14. Payoffs for players x_1 and x_8.

		x_8		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.5, 0.5)	(0.25, 0.25)	(0.5, 0.5)
x_1	s_{neu}	(0.25, 0.25)	(0.00, 0.00)	(0.25, 0.25)
	s_{con}	(0.5, 0.5)	(0.25, 0.25)	(0.5, 0.5)

Table 15. Payoffs for players x_1 and x_9.

		x_9		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.00, 0.00)	(0.25, 0.25)	(0.25, 0.25)
x_1	s_{neu}	(0.25, 0.25)	(0.5, 0.5)	(0.5, 0.5)
	s_{con}	(0.25, 0.25)	(0.5, 0.5)	(0.5, 0.5)

Table 16. Payoffs for players x_1 and x_{10}.

		x_{10}		
		s_{all}	s_{neu}	s_{con}
	s_{all}	(0.5, 0.5)	(0.25, 0.25)	(0.25, 0.25)
x_1	s_{neu}	(0.25, 0.25)	(0.00, 0.00)	(0.00, 0.00)
	s_{con}	(0.25, 0.25)	(0.00, 0.00)	(0.00, 0.00)

Table 17. Solution of the government governance problem with the proposed model.

x	ALL(x_i)	NEU(x_i)	CON(x_i)
x_1	$\{x_2, x_7, x_8, x_{10}\}$	$\{x_4, x_6, x_9\}$	$\{x_3, x_5\}$
x_2	$\{x_5, x_8\}$	$\{x_2, x_3, x_4, x_6, x_7, x_9, x_{10}\}$	$\emptyset$
x_3	$\{x_5, x_6, x_8, x_9\}$	$\{x_2, x_4\}$	$\{x_1, x_7, x_{10}\}$
x_4	$\emptyset$	$\{x_1, x_2, ..., x_{10}\}$	$\emptyset$
x_5	$\{x_2, x_3, x_{10}\}$	$\{x_4, x_6, x_7, x_8, x_9\}$	$\{x_1\}$
x_6	$\{x_3, x_7, x_9\}$	$\{x_1, x_2, x_5, x_4, x_7, x_8, x_{10}\}$	$\emptyset$
x_7	$\{x_1, x_8, x_{10}\}$	$\{x_2, x_4, x_5, x_6, x_9\}$	$\{x_3\}$
x_8	$\{x_1, x_2, x_3\ x_5, x_7\}$	$\{x_4, x_6, x_9\}$	$\{x_{10}\}$
x_9	$\{x_3, x_6\}$	$\{x_1, x_2, ..., x_{10}\}$	$\emptyset$
x_{10}	$\{x_1, x_5, x_7\}$	$\{x_2, x_4, x_6, x_7, x_9\}$	$\{x_3, x_8\}$

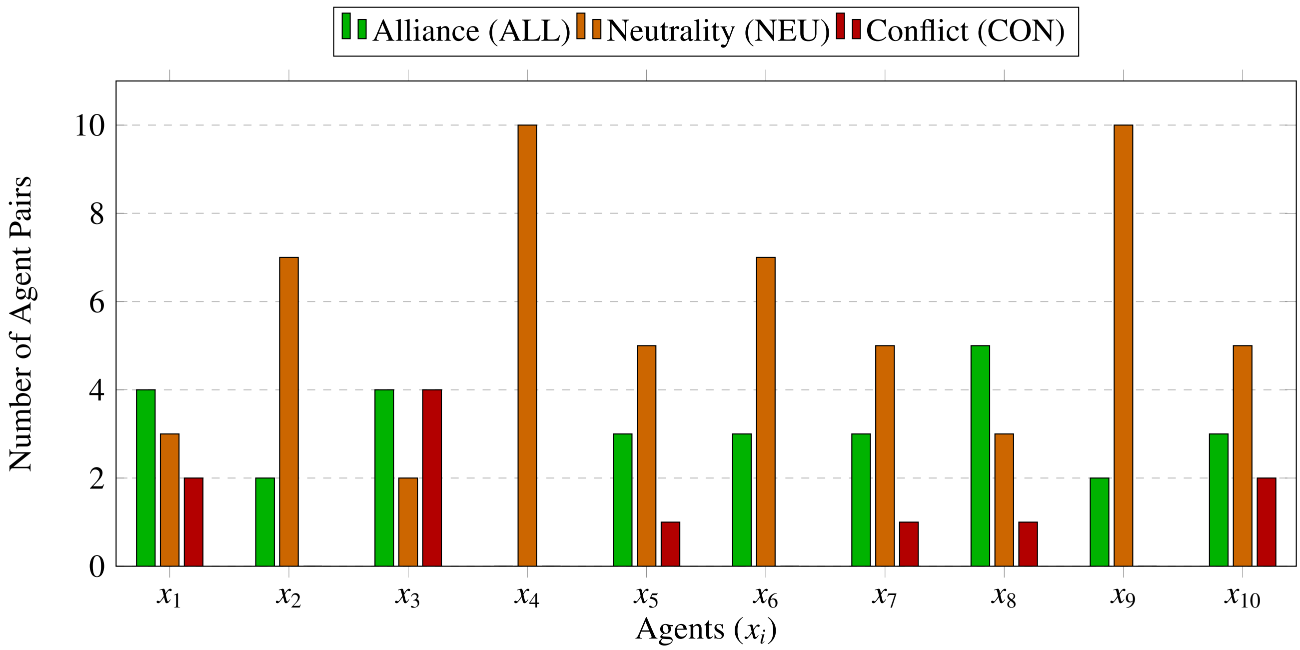

Figure 4. Conflict, alliance, and neutrality relationships among players $(x_1, \ldots, x_{10})$ computed from Table 17.

4.2. *Climate policy analysis*

Understanding the complex dynamics of international climate policy requires a nuanced approach that moves beyond simple binary agreements. To effectively model the spectrum of alliance, neutrality, and conflict between major global and economic powers, we employ a multi-attribute decision-making framework. The data is curated to capture not only the formal stances of nations but also the intensity, uncertainty, and trade-offs inherent in their climate policies. The selection of G7 nations, plus Russia, China, and India, is deliberate. This group represents a critical mass of the global economy, historical and contemporary emissions, and geopolitical influence. Their collective actions, conflicts, and alliances will largely determine the world's ability to mitigate climate change. Analyzing their interrelationships provides invaluable insights into the feasibility of international cooperation. This climate policy case study is used to illustrate the practical applicability and effectiveness of our proposed model.

Consider $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ as set of objects (countries), where:
x_1: Canada
x_2: China
x_3: France
x_4: Germany
x_5: India
x_6: Italy
x_7: Japan
x_8: Russia
x_9: United Kingdom
x_{10}: United States of America

The set $A = \{a_1, a_2, a_3, a_4, a_5\}$ represents the conflict issues that must be resolved to achieve a sustainable climate policy for the world, where:
a_1: **Support for Net-Zero by 2050**: a three-value classification of national net-zero commitments [41].
a_2: **Renewable Energy Investment Portfolio**: The vector of investment scores [solar, wind, other] mainly constructed using quantitative data on installed capacity, projected growth and policy momentum [42].
a_3: **Emission Reduction Target by 2030**: The interval data [min_reduction, max_reduction] was extracted directly from the quantified components of each country's National Determined Contribution [43].
a_4: **Climate Risk Perception**: The triangular fuzzy number [pessimistic, likely, optimistic] for perceived climate risk was informed by two sources: (1) The qualitative risk assessments for each world region [44] and (2) the Notre Dame Global Adaptation Initiative (ND-GAIN) Index [45].
a_5: **Economic Vs. Environmental Priority**: The pair [economic_priority, environmental_priority] is a proxy variable constructed from two quantitative metrics: (1) the level of fossil fuel subsidies (a proxy for economic priority on incumbent industries) and (2) the level of green stimulus spending (a proxy for environmental priority) [46, 47].

We will evaluate climate policy using the information system $IS(U, A)$ shown in Table 18.

Table 18. A hybrid situation table based on a climate policy problem.

U/A	a_1	a_2	a_3	a_4	a_5
Type	I	II	III	IV	V
x_1 : $Canada$	+1	{0.6, 0.7, 0.8}	[0.4, 0.45]	(0.6, 0.7, 0.8)	⟨0.6, 0.4⟩
x_2 : $China$	0	{0.9, 0.9, 0.8}	[0.1, 0.15]	(0.6, 0.7, 0.8)	⟨0.7, 0.3⟩
x_3 : $France$	+1	{0.5, 0.6, 0.7}	[0.5, 0.55]	(0.6, 0.7, 0.8)	⟨0.5, 0.5⟩
x_4 : $Germany$	+1	{0.8, 0.9, 0.7}	[0.55, 0.65]	(0.7, 0.8, 0.9)	⟨0.4, 0.6⟩
x_5 : $India$	0	{0.8, 0.7, 0.6}	[0.05, 0.1]	(0.7, 0.8, 0.9)	⟨0.7, 0.3⟩
x_6 : $Italy$	+1	{0.7, 0.6, 0.5}	[0.5, 0.55]	(0.6, 0.7, 0.8)	⟨0.5, 0.5⟩
x_7 : $Japan$	+1	{0.6, 0.5, 0.4}	[0.4, 0.46]	(0.5, 0.6, 0.7)	⟨0.6, 0.4⟩
x_8 : $Russia$	0	{0.3, 0.2, 0.4}	[0.2, 0.25]	(0.3, 0.4, 0.5)	⟨0.8, 0.2⟩
x_9 : $UnitedKingdom$	+1	{0.7, 0.8, 0.6}	[0.6, 0.68]	(0.6, 0.7, 0.8)	⟨0.5, 0.5⟩
x_{10} : $UnitedStatesofAmerica$	+1	{0.8, 0.8, 0.7}	[0.5, 0.52]	(0.5, 0.6, 0.7)	⟨0.5, 0.5⟩

To ensure neutrality and avoid bias in the analysis, different weights are assigned to all five issues. Issue a_1 ($w_1 = 0.25$) represents the fundamental commitment to climate goals. Issue a_2 ($w_2 = 0.20$) represents concrete actions toward transition. The issue a_3 ($w_3 = 0.30$) represents near-term targets and is crucial for immediate impact. Issue a_4 ($w_4 = 0.15$) represents the policy orientation but is less directly actionable. The issue a_5 ($w_5 = 0.10$) is important but perception does not directly translate into action. Specifically, the weight vector w is defined as:

$$w = (w_1, w_2, \ldots, w_5),$$

where each weight corresponds to an issue $a_i \in A$ and is given by:

$$w = [0.25,\ 0.20,\ 0.30,\ 0.15,\ 0.10]\,.$$

This satisfies the normalization condition:

$$\sum_{i=1}^{5} w_i = 1.$$

Table 19 reports the results of the Climate Policy Analysis, classifying each country x_i into alliance (ALL), neutrality (NEU), or conflict (CON) sets. All CON sets are empty, indicating the absence of direct conflicts among the ten economies. Canada (x_1) allies broadly except with Russia (x_8), while China (x_2) balances alliances with neutrality toward Japan (x_7) and the USA (x_{10}). The USA forms strong ties with Canada, European countries, and Japan, but remains neutral toward China, India, and Russia. Russia allies only with China and India, underscoring its relative isolation and divergent climate policies. Overall, the findings highlight a cohesive Western coalition, selective neutrality by emerging economies, and a negotiation landscape shaped more by cooperation than confrontation.

Table 19. Solution of the Climate Policy Analysis problem with the proposed model.

x	ALL(x_i)	NEU(x_i)	CON(x_i)
x_1	$\{x_2, x_3, x_4, x_5, x_6, x_7, x_9, x_{10}\}$	$\{x_8\}$	$\emptyset$
x_2	$\{x_1, x_3, x_4, x_5, x_6, x_8, x_9\}$	$\{x_7, x_{10}\}$	$\emptyset$
x_3	$\{x_1, x_2, x_4, x_5, x_6, x_7, x_9, x_{10}\}$	$\{x_8\}$	$\emptyset$
x_4	$\{x_1, x_2, x_3, x_5, x_6, x_7, x_9, x10\}$	$\{x_8\}$	$\emptyset$
x_5	$\{x_1, x_2, x_3, x_4, x_6, x_8, x_9\}$	$\{x_7, x_{10}\}$	$\emptyset$
x_6	$\{x_1, x_2, x_3, x_4, x_5, x_7, x_9, x_{10}\}$	$\{x_8\}$	$\emptyset$
x_7	$\{x_1, x_3, x_4, x_6, x_9, x_{10}\}$	$\{x_2, x_5, x_8\}$	$\emptyset$
x_8	$\{x_2, x_5\}$	$\{x_1, x_3, x_4, x_6, x_7, x_9, x_{10}\}$	$\emptyset$
x_9	$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_{10}\}$	$\{x_8\}$	$\emptyset$
x_{10}	$\{x_1, x_3, x_4, x_6, x_7, x_9\}$	$\{x_2, x_5, x_8\}$	$\emptyset$

4.3. Comparative analysis of the proposed model vs. Yang et al.

The government governance case study in Jiangning District yields markedly different alliance, neutrality, and conflict outcomes under our game-theoretic model, compared to the three-way decision model by Yang et al. [26] (Table 17 vs. Table 20). Table 17 shows far more alliances and fewer conflicts than Table 20.

Table 20. Decision table for the government governance problem based on Yang et al. [19].

x	ALL(x_i)	NEU(x_i)	CON(x_i)
x_1	$\{x_{10}\}$	$\{x_7\}$	$\{x_2, x_3, x_4, x_5, x_6, x_8, x_9\}$
x_2	$\{x_8\}$	$\{x_3, x_4, x_5\}$	$\{x_1, x_6, x_7, x_8, x_9, x_{10}\}$
x_3	$\{x_4, x_6, x_9\}$	$\{x_2, x_8\}$	$\{x_1, x_5, x_7, x_{10}\}$
x_4	$\{x_3, x_7, x_9\}$	$\{x_2, x_6, x_8, x_{10}\}$	$\{x_1, x_5\}$
x_5	$\emptyset$	$\{x_2\}$	$\{x_1, x_3, x_4, x_6, x_7, x_8, x_9, x_{10}\}$
x_6	$\emptyset$	$\{x_4\}$	$\{x_1, x_2, x_5, x_7, x_8, x_{10}\}$
x_7	$\{x_4, x_{10}\}$	$\{x_3, x_4, x_6, x_{10}\}$	$\{x_1, x_2, x_5, x_8\}$
x_8	$\{x_2\}$	$\{x_3, x_4, x_7\}$	$\{x_1, x_2, x_3, x_4, x_6, x_9, x_{10}\}$
x_9	$\{x_3, x_8\}$	$\{x_4\}$	$\{x_1, x_2, x_5, x_6, x_8, x_{10}\}$
x_{10}	$\{x_1, x_7\}$	$\{x_4\}$	$\{x_1, x_2, x_3, x_4, x_5, x_7, x_8, x_9\}$

For example, player x_1 is allied with $\{x_2, x_7, x_8, x_{10}\}$, neutral with $\{x_4, x_6, x_9\}$, and in conflict only with $\{x_3, x_5\}$ in our model. In contrast, Yang et al.'s approach classifies x_1 as allied with only $\{x_{10}\}$, neutral only with $\{x_7\}$, and in conflict with $\{x_2, x_3, x_4, x_5, x_6, x_8, x_9\}$. Thus, Yang et al.'s [26] method clusters most relationships into conflict, while our model distinguishes many as alliances or neutral bonds. This reflects fundamental methodological differences:

Modeling approach: Our model uses bilateral game-theoretic analysis, computing payoffs for every pair of players across all strategies and classifying relationships via Nash equilibrium. This explores all strategy combinations and uses separate weighted measures of alliance and conflict (not a single distance measure). In contrast, Yang et al. [26] apply a three-way decision framework based on an aggregated conflict distance metric. They compute a single conflict distance for each pair across all issues and then trisect relationships (alliance/neutral/conflict) using fixed thresholds. This single distance assumption is more restrictive: It treats all issues equally and does not distinguish alliance and conflict measures independently.

Alliance vs. Conflict sensitivity: In our model, alliances, neutralities, and conflicts are computed with distinct auxiliary functions and weighted issue importance, as suggested by recent three-way conflict models [26]. This yields more nuanced triaging of relationships. By assigning higher weight to some issues (e.g., employment and education) and lower weight to others, we allow a $(+1, +1)$ agreement on key issues to count more strongly toward alliance than $(0, 0)$ agreement.

By contrast, Yang et al. [26] conflict-distance approach treats all issue differences uniformly, often pushing many pairs into the conflict category. For example, x_1 agrees strongly with x_2 on key issues, leading our model to mark them as allies, while Yang's method places x_1–x_2 in conflict.

Threshold tuning: The payoff thresholds in our model (α, β) give decision makers control over the sensitivity of alliances/conflicts/neutralities. For instance, adjusting α_A upward places more emphasis on peace, potentially converting neutral ties to allies if desired. In Table 17, we use $(\alpha_A, \beta_A) = (0.7, 0.3)$ uniformly, but other settings could shift outcomes. Yang et al.'s approach uses fixed trisection thresholds based on conflict distance distribution, offering less flexibility. Thus, our model can be calibrated for different governance priorities, while Yang's model yields a static partition of relations.

Outcome differences (Table 17 vs. Table 20): The practical effect is a more cooperative governance picture under our model. For x_1, our allied set $\{x_2, x_7, x_8, x_{10}\}$ is four times larger than Yang's $\{x_{10}\}$. Our neutral set (four regions) is also larger than Yang's (one region), and our conflict set shrinks (only x_3, x_5 versus Yang's seven regions).

This trend persists for other players. For example, x_5 allies with $\{x_2, x_3, x_{10}\}$ in our model but has no allies in Yang's model. Similarly, x_8 allies with $\{x_1, x_2, x_3, x_5, x_7\}$ and is neutral with many, whereas Yang's method allies only x_8 with $\{x_2\}$ and classifies most others as conflict. In general, Table 17 shows more allied pairs and fewer conflict pairs than Table 20, indicating greater potential for cooperation in our model.

Computational complexity vs. Precision: Our model requires computing payoffs for each ordered pair (x_i, x_j), with time complexity $O(n^2 m)$ for n players and m issues, and solving small 3×3 games. Although this is more complex than a simple distance calculation, game-theoretic methods provide more realistic and accurate conflict resolutions by considering all players' strategies. The extra computation is justified by the richer insights: The additional alliances identified suggest where joint policies can be formed. Yang et al.'s distance-based method is computationally lighter but yields a more adversarial picture.

Governance implications: The tripartition of our model enables for a multilevel response. Players classified as neutral (NEU) are candidates for alliance-building strategies, given their moderate stance. For instance, x_4, x_6, x_9 are neutral with x_1 in our results, suggesting they could be brought into cooperative initiatives. Yang's classification would treat most of these as conflicts with x_1, potentially overlooking opportunities for collaboration. Thus, the game-theoretic approach provides better support for nuanced policymaking: Identifying where collaboration is already strong (alliances) and where it can be cultivated (neutrals), instead of only highlighting conflict zones.

5. Conclusions

In this study, we have presented a novel conflict analysis framework that integrates multi-scale hybrid information with a game-theoretic decision model. The proposed approach enables agents to express opinions using seven rating scales (e.g., fuzzy numbers, and interval values), capturing varying levels of uncertainty and hesitation. By defining auxiliary functions for each scale and deriving numerical measures of alliance, neutrality, and conflict, we construct a bilateral game between any two agents with three possible strategies: Ally, neutral, and oppose. The resulting payoff functions quantify the gains for each pair of strategies. Solving for Nash equilibria then yields an automated classification of agents into allied, neutral, or adversarial groups.

The key contributions and findings of this work are as follows:

- Unlike traditional single-scale models, our hybrid system adapts to heterogeneous input data, making the analysis more flexible and realistic.
- The game-theoretic component ensures that agent classifications emerge from strategic payoff maximization, offering a dynamic and balanced decision rule.
- In an empirical case study of local government governance (Jiangning District), the model effectively identified plausible alliances and conflicts among stakeholders.
- Compared to previous models, our framework reduced unnecessary adversarial classification and fostered more cooperative outcomes in a complex socio-political setting.
- The proposed model also demonstrates that international climate policy interactions are predominantly shaped by alliances and neutrality, with no direct conflicts, highlighting a cooperative yet strategically balanced global landscape.

A detailed comparative analysis confirmed that the proposed model provides superior handling of multi-scale inputs and delivers a robust conflict assessment across scenarios, see Table 21. The novelty of this work lies in uniting multi-scale/hybrid opinion representations with a strategic game framework. To our knowledge, this is the first conflict analysis model that simultaneously:

(1) Accommodates a wide variety of data types in a single information system.
(2) Applies a bilateral game to determine alliance and conflict relationships.

Table 21. Comparative analysis of conflict analysis models.

Models	Research environment	Issue weighting method	Calculation thresholds	Subjectivity /Objectivity	Scale type
Pawlak [2]	Three-valued situation tables	Arithmetic method	Average	Subjective	Single-scale
Yao [26]	Many-valued situation tables	Arithmetic method	Average	Subjective	Single-scale
Bashir et al. [7]	Fuzzy situation tables	Arithmetic method	Average	Objective	Single-scale
Wang et al. [28]	Interval-valued Pythagorean fuzzy situation tables	Maximizing deviation method	–	Objective	Single-scale
Feng et al. [35]	Dual hesitant fuzzy situation tables	CRITIC method	–	Objective	Single-scale
Luo et al. [21]	Three-valued situation tables	N-bounded symmetric concave reciprocal*	–	Objective	Single-scale
Our model	Hybrid multi-scale situation tables	Maximizing deviation method	Dynamic thresholds	Objective	Multi-scale

This integration enables richer modeling of real-world uncertainty and provides a principled basis for decision-making. Its practical relevance is demonstrated by the governance case study, where the model produced insights aligned with expert expectations and helped foster more harmonious inter-agent relations. Future research may explore several promising directions:

- **Extending to multi-agent and dynamic conflicts:** Generalize the two-player game to multi-party or repeated interactions, accounting for strategic evolution over time.
- **Adaptive and learning-based enhancements:** Incorporate machine learning methods to dynamically adjust auxiliary functions or thresholds based on real-time conflict data.
- **Scalability and efficiency:** Develop fast approximation algorithms or parallel computing

frameworks for large-scale decision problems with many agents and complex rating data.
- **Broader applications:** Apply the framework to new domains, such as international diplomacy, organizational strategy, or environmental disputes, to validate its versatility and refine context-specific scales and payoffs.

In summary, the proposed model bridges granular opinion data with strategic analysis, offering a powerful and practical tool for conflict resolution. By addressing both representational and decision-theoretic gaps, this work paves the way for more advanced and context-aware conflict management strategies in theoretical and applied settings.

Author contributions

Akrash Tasawar: Conceptualization, methodology, formal analysis, writing - Original Draft; M. G. Abbas Malik: Supervision, funding acquisition, resources, software. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare they did not use Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

We are grateful to the editor and anonymous reviewers for reviewing the manuscript and providing constructive comments for improvement. The authors would like to acknowledge the support of Prince Sultan University for paying the Article Processing Charges (APC) of this publication.

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. L. Boulaasair, H. Bouzahir, N. S. Rao, D. Santina, N. Mlaiki, Mathematical analysis of competitive dynamics between two regional areas, *J. Inequal. Appl.*, **2025** (2025), 71. https://doi.org/10.1186/s13660-025-03326-2
2. Z. Pawlak, An inquiry into the anatomy of conflicts, *Inf. Sci.*, **109** (1998), 65–78. https://doi.org/10.1016/S0020-0255(97)10072-X
3. B. Sun, W. Ma, H. Zhao, Rough set-based conflict analysis model and method over two universes, *Inf. Sci.*, **372** (2016), 111–125. https://doi.org/10.1016/j.ins.2016.08.030
4. N. Azam, J. Yao, Game-theoretic rough sets for feature selection, In: *Rough sets and intelligent systems – Professor Zdzisław Pawlak in memoriam*, **43** (2013), 61–78. https://doi.org/10.1007/978-3-642-30341-8_5

5. N. Azam, J. Yao, Game-theoretic rough sets for recommender systems, *Knowl. Based Syst.*, **72** (2014), 96–107. https://doi.org/10.1016/j.knosys.2014.08.030
6. Z. Bashir, S. Mahnaz, M. G. Abbas Malik, Conflict resolution using game theory and rough sets, *Int. J. Intell. Syst.*, **36** (2021), 237–259. https://doi.org/10.1002/int.22298
7. Z. Bashir, A. Wahab, T. Rashid, Three-way decision with conflict analysis approach in the framework of fuzzy set theory, *Soft Comput.*, **26** (2022), 309–326. https://doi.org/10.1007/s00500-021-06509-3
8. K. Leyton-Brown, Y. Shoham, *Essentials of game theory: A concise multidisciplinary introduction*, Cham: Springer, 2008. https://doi.org/10.1007/978-3-031-01545-8
9. J. S. Liu, X. Z. Wang, B. Y. Zhang, Ranking of interval numbers, *Chinese J. Eng. Math.*, **18** (2001), 103–109.
10. G. Lang, D. Miao, M. Cai, Three-way decision approaches to conflict analysis using decision-theoretic rough set theory, *Inf. Sci.*, **406** (2017), 185–207. https://doi.org/10.1016/j.ins.2017.04.030
11. G. Lang, Three-way conflict analysis: Alliance, conflict, and neutrality reducts of three-valued situation tables, *Cogn. Comput.*, **14** (2022), 2040–2053. https://doi.org/10.1007/s12559-021-09905-x
12. X. Li, Y. Yang, H. Yi, Q. Yu, Conflict analysis based on three-way decision for trapezoidal fuzzy information systems, *Int. J. Mach. Learn. Cybern.*, **13** (2022), 929–945. https://doi.org/10.1007/s13042-021-01427-0
13. Z. Pawlak, On conflicts, *Int. J. Man-Mach. Stud.*, **21** (1984), 127–134. https://doi.org/10.1016/S0020-7373(84)80062-0
14. Z. Pawlak, Some remarks on conflict analysis, *Eur. J. Oper. Res.*, **166** (2005), 649–654. https://doi.org/10.1016/j.ejor.2003.09.038
15. S. Ramanna, J. F. Peters, A. Skowron, Approaches to conflict dynamics based on rough sets, *Fundam. Inform.*, **75** (2007), 453–468.
16. V. Torra, Hesitant fuzzy sets, *Int. J. Intell. Syst.*, **25** (2010), 529–539. https://doi.org/10.1002/int.20418
17. R. R. Yager, A. M. Abbasov, Pythagorean membership grades, complex numbers, and decision-making, *Int. J. Intell. Syst.*, **28** (2013), 436–452. https://doi.org/10.1002/int.21584
18. Y. Yao, Three-way decisions with probabilistic rough sets, *Inf. Sci.*, **180** (2010), 341–353. https://doi.org/10.1016/j.ins.2009.09.021
19. H. L. Yang, Y. Wang, Z. L. Guo, Three-way conflict analysis based on hybrid situation tables, *Inf. Sci.*, **628** (2023), 522–541. https://doi.org/10.1016/j.ins.2023.02.006
20. F. Xu, M. Cai, H. Song, J. Dai, The selection of feasible strategies is based on the consistency measurement of cliques, *Inf. Sci.*, **583** (2022), 33–55. https://doi.org/10.1016/j.ins.2021.10.080
21. J. Luo, M. Hu, G. Lang, X. Yang, K. Qin, Three-way conflict analysis based on alliance and conflict functions, *Inf. Sci.*, **594** (2022), 322–359. https://doi.org/10.1016/j.ins.2022.01.067
22. G. Lang, J. Luo, Y. Yao, Three-way conflict analysis: A unification of models based on rough sets and formal concept analysis, *Knowl. Based Syst.*, **194** (2020), 105556. https://doi.org/10.1016/j.knosys.2020.105556

23. G. Lang, D. Miao, H. Fujita, Three-way group conflict analysis based on Pythagorean fuzzy set theory, *IEEE Trans. Fuzzy Syst.*, **28** (2019), 447–461. https://doi.org/10.1109/TFUZZ.2019.2908123
24. L. W. Suo, H. L. Yang, Three-way conflict analysis based on incomplete situation tables: A tentative study, *Int. J. Approx. Reason.*, **145** (2022), 51–74. https://doi.org/10.1016/j.ijar.2022.03.004
25. H. L. Yang, S. Y. Xue, Y. H. She, General three-way decision models on incomplete information tables, *Inf. Sci.*, **605** (2022), 136–158. https://doi.org/10.1016/j.ins.2022.05.002
26. Y. Yao, Three-way conflict analysis: Reformulations and extensions of the Pawlak model, *Knowl. Based Syst.*, **180** (2019), 26–37. https://doi.org/10.1016/j.knosys.2019.05.016
27. Z. Gillani, Z. Bashir, S. Aquil, A game theoretic conflict analysis model with linguistic assessments and two levels of game play, *Inf. Sci.*, **677** (2024), 120840. https://doi.org/10.1016/j.ins.2024.120840
28. T. Wang, B. Huang, H. Li, D. Liu, H. Yu, Three-way decision for probabilistic linguistic conflict analysis via compounded risk preference, *Inf. Sci.*, **631** (2023), 65–90. https://doi.org/10.1016/j.ins.2023.02.067
29. X. Li, X. Wang, G. Lang, H. Yi, Conflict analysis based on the three-way decision for triangular fuzzy information systems, *Int. J. Approx. Reason.*, **132** (2021), 88–106. https://doi.org/10.1016/j.ijar.2020.12.004
30. A. Fahmi, A. Khan, T. Abdeljawad, A novel approach to group decision-making using generalized bipolar neutrosophic sets, *PLoS One*, **20** (2025), e0317746. https://doi.org/10.1371/journal.pone.0317746
31. Y. Yao, The superiority of three-way decisions in probabilistic rough set models, *Inf. Sci.*, **181** (2011), 1080–1096. https://doi.org/10.1016/j.ins.2010.11.019
32. X. Zhang, J. Chen, Three-hierarchical three-way decision models for conflict analysis: A qualitative improvement and a quantitative extension, *Inf. Sci.*, **587** (2022), 485–514. https://doi.org/10.1016/j.ins.2021.12.012
33. H. Zhi, J. Qi, T. Qian, R. Ren, Conflict analysis under one-vote veto based on an approximate three-way concept lattice, *Inf. Sci.*, **516** (2020), 316–330. https://doi.org/10.1016/j.ins.2019.12.065
34. X. Li, X. Wei, Z. Qian, A probability-measure-based approach to ranking fuzzy numbers via three-way decision and its application to conflict analysis, *Fuzzy Sets Syst.*, **465** (2023), 108539. https://doi.org/10.1016/j.fss.2023.108539
35. Y. Fan, J. Qi, L. Wei, A conflict analysis model based on three-way decisions, In: *Rough sets*, **11103** (2018), 522–532. https://doi.org/10.1007/978-3-319-99368-3_41
36. T. P. Benko, B. Pi, Q. Li, M. Feng, M. Perc, H. B. Vošner, Evolutionary games for cooperation in open data management, *Appl. Math. Comput.*, **496** (2025), 129364. https://doi.org/10.1016/j.amc.2025.129364
37. J. Von Neumann, O. Morgenstern, *Theory of games and economic behavior*, Princeton University Press, 1944.

38. L. Tran, L. Duckstein, Comparison of fuzzy numbers using a fuzzy distance measure, *Fuzzy Sets Syst.*, **130** (2002), 331–341. https://doi.org/10.1016/S0165-0114(01)00195-6

39. H. Yi, H. Zhang, X. Li, Y. Yang, Three-way conflict analysis based on hesitant fuzzy information systems, *Int. J. Approx. Reason.*, **139** (2021), 12–27. https://doi.org/10.1016/j.ijar.2021.09.002

40. J. Du, S. Liu, Y. Liu, J. Yi, A novel approach to three-way conflict analysis and resolution with Pythagorean fuzzy information, *Inf. Sci.*, **584** (2022), 65–88. https://doi.org/10.1016/j.ins.2021.10.051

41. UNFCCC, *United nations framework convention on climate change*, United Nations, 2024.

42. European Climate, Infrastructure and Environment Executive Agency, *Renewables 2023: Analysis and forecast to 2028*, 2024.

43. World Resources Institute, *Climate watch: NDC data*, 2025.

44. H. O. Portner, D. C. Roberts, M. M. B. Tignor, E. Poloczanska, K. Mintenbeck, A. Alegria, et al., *Climate change 2022: Impacts, adaptation and vulnerability*, Cambridge University Press, 2023. https://doi.org/10.1017/9781009325844

45. Notre Dame Global Adaptation Initiative, *ND-GAIN Country Index*, University of Notre Dame, 2023.

46. International Energy Agency, *Government Energy Spending Tracker*, IEA, 2023.

47. OECD, *OECD inventory of support measures for fossil fuels 2024: Policy trends up to 2023*, Paris: OECD Publishing, 2024. https://doi.org/10.1787/87dc4a55-en