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Research article

The influence of stochastic process on some new solutions for the long-short-wave interaction system

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Abstract: The nonlinear long-short wave interaction system serves as a fundamental nonlinear model that characterizes the resonant interactions occurring between high-frequency (short) waves and low-frequency (long) waves. This system is especially useful for comprehending energy transfer, wave modulation, and the development of localized structures such as solitons or breathers. This paper proposes innovative stochastic solutions for the nonlinear long-short wave interaction model within the context of the Brownian motion process. This stochastic process takes into consideration the intrinsic randomness and variations found in real-world systems, including nonlinear optical fibers, Bose-Einstein condensates, fluid dynamics, and plasma environments. We derive stochastic traveling wave solutions using an extended tanh function approach and examine the resulting stochastic dynamics concerning amplitude variation, phase shifts, and noise-induced modulation. We consider the impact of noise intensity on the stability and coherence of wave structures. Our findings suggest that Brownian forcing can fulfill two roles; either aiding in the preservation of localized structures or leading to their collapse and dispersion, depending on the system parameters and initial conditions. To depict the behavior of the designated stochastic solutions, various wave profiles were generated utilizing the MATLAB software. Finally, the proposed method holds the promise of being adapted for various other practical models.

Keywords: Brownian motion process; long-short-wave; soliton solutions; noise intensity
Mathematics Subject Classification: 35C05, 60G07, 35R10, 35R60

1. Introduction

The environment, we inhabit, is fundamentally nonlinear, and nonlinear evolution equations, which are extensively employed to characterize the intricate physical phenomena. These phenomena are frequently explained using nonlinear partial differential equations (NPDEs), which are intimately related to basic ideas in the applied sciences [1–3]. These equations are widely utilized to describe a range of phenomena in applied fields, such as optical fiber communications, economics, manufacturing plants, plasma physics, bio-physics, etc [4–6]. Ultrasound imaging technologies are currently used in medicine to visualize and examine internal human tissues [7]. It is widely recognized that natural science is undergoing significant changes at this time. A traveling wave solution is a sort of solution to the NPDEs that retains its shape while moving at a constant speed. This represents a specific type of solution where the dependent variable (such as temperature, pressure, or displacement) is affected by a blend of spatial and temporal coordinates to demonstrate the wave's movement [8, 9]. The traveling wave solutions serve as a model for a diverse array of phenomena, such as sound waves, Langmuir waves, electrostatic waves, and waves occurring in different physical systems [10, 11]. The study of traveling waves provides a thorough knowledge of the balance between nonlinearity and dispersion that governs the formation of coherent structures, such as solitons, explosive, dark, periodic waves, shock, etc.

The Brownian motion (BM) process acts as a fundamental link between statistics and NPDEs, particularly in the representation of systems affected by randomness and nonlinearity [12]. This relationship signifies a vital aspect in the mathematical modeling of complex, real-world systems. The BM process is a prominent illustration of a stochastic process that combines elements of a Markov process and a martingale [13]. In numerous natural and engineered settings, including optical fibers, plasma physics, crystalline structures, quantum fields, telecommunication engineering, and biological systems, physical processes are regulated not solely by deterministic nonlinear laws, but are also influenced by inherent random fluctuations and environmental noise [14]. A unified framework that blends the probabilistic structure of stochastic processes with the descriptive capability of NPDEs is necessary to capture this dual feature.

The nonlinear long-short wave interaction system (NLSWIS) is a basic nonlinear model that describes resonant interactions between high-frequency (short) and low-frequency (long) waves. Benney originally presented this concept in 1977 to examine the interplay between short and long waves [15]. The NLSWIS was first used to investigate capillary–gravity wave interactions and Langmuir–ion acoustic wave coupling. Since then, it has become a potent framework for examining nonlinear dispersive phenomena in a variety of physical domains including fluid dynamics, oceanography, plasmas, optical media, and Bose-Einstein condensates. From a mathematical perspective, the NLSWIS is composed of a nonlinear Schrödinger equation that describes the short wave envelope, alongside a conservation law or transport equation that regulates the long wave component. The interaction between these two components, characterized by nonlinearity and dispersion, facilitates a diverse array of coherent structures, such as solitons, breathers, and modulated wave trains. These solutions embody the core principles of energy transfer and amplitude modulation within physical media, where an inherent scale disparity exists. Using Hirota's bilinearization approach, Sakkaravarthi et al. [16] examined the nonlinear resonance interactions of many short waves with a long wave in two dimensions. The examination of fundamental physical interactions that form the foundation for further comprehensive exploration into various nonlinear interactions, which are essential to the overall solution framework encompassing analytical, dark, and approximate solutions, is especially intriguing [17]. Triki et al. [18] used the NLSWIS system and obtained soliton solutions along with other solutions such as plane waves and singular periodic solutions by employing the simplest equation approach. Subsequently, the deterministic nonlinear long–short wave interaction model was examined by investigating the transverse linear instability of the one-dimensional solitary wave solution [19]. With the derivative according to the modified Riemann–Liouville definition, the bifurcation and the impact of random interactions on the precise solution to the stochastic fractional long-short wave interactions system with multiplicative Brownian motion were examined [20].

The incorporation of the BM process into NPDEs allows for the modeling of both spatial and temporal uncertainty, including fluctuating boundary pressures, noise-driven dispersion, and random wave collapse. This results in enhanced dynamical behavior that is not represented by standard deterministic models, such as probability distributed solitons, mean field deviations, and stochastic bifurcations. The incorporation of stochastic elements, particularly multiplicative or additive noise influenced by the BM process, converts conventional NPDEs into systems that are more physically realistic and amenable to statistical analysis. On other hand, nonlinear stochastic partial differential equations (NSPDEs) are equations that involve both random noise and nonlinear terms. These formulations enable researchers to thoroughly examine the statistical characteristics of nonlinear waves, calculate expectation values and probability distributions of solution profiles, and simulate collections of behaviors instead of individual trajectories. In the context of NSPDEs, stochastic components that incorporate BM can be understood in two primary manners, namely the Itô and Stratonovich interpretations [12]. By establishing the NLSWIS within a stochastic framework, researchers are provided with both probabilistic tools and nonlinear dynamical techniques, which enhance the comprehension of the relationship between nonlinearity and randomness. This methodology paves the way for advanced modeling, statistical characterization, and control of nonlinear wave systems across various fields. We examine the NLSWIS in the Itô sense that are compelled by multiplicative noise as follows:

$$\begin{aligned} &i\chi_t + \chi_{xx} - q\chi + \sigma\chi\,\Xi_t = 0\,, \\ &q_t + q_x + (|\,\chi\,|^2)_x = 0, \end{aligned} \tag{1.1}$$

where $\chi(x,t)$ denotes the complex function that characterizes the slowly varying envelope of the short transverse wave, and $q(x,t)$ represents the real function that characterizes the slowly varying envelope of the long transverse wave. The parameter σ represents the noise strength, whereas the noise Ξ_t represents the time derivative of the Brownian motion process $\Xi(t)$ [12]. Most authors explore the NLSWIS model in the deterministic case, i.e., at $\sigma = 0$, and use various analytical approaches [18, 19, 21, 22]. In contrast, we consider the stochastic version of this model to investigate the new direction about stochastic behaviour of the presented solitary waves. Specifically, we captures the stochastic modulation of nonlinear waves.

Currently, there is a deficiency of study on the extended tanh function method's (ETFM) applicability [23] to stochastic nonlinear partial differential equations (SNPDEs), particularly those that include noise-driven characteristics, such as the BM process. The primary goal of this work is to analyze the solutions of the NLSWIS influenced by multiplicative noise in the Itô sense. This approach produces rational, hyperbolic, trigonometric, and hybrid solutions through the Itô framework. We offer some innovative stochastic solutions based on the physical properties in the Itô sense. We also illustrate the influence of stochastic process on the prosperities of presented stochastic solutions. To our knowledge, the ETFM has not before been used to handle the stochastic NLSWIS.

The following arrangement is the structure of this work. A brief explanation of the Brownian motion mechanism and its prosperities is shown in Section 2. Section 3 presents the mathematical analysis for the NLSWIS. Additionally, we introduce a crucial traveling wave solution for this system within the framework of the Itô sense. Section 4 represents the graphical illustration for some stochastic solutions. Section 5 shows the influence of stochastic process on the behavior of stochastic solutions presented. Section 6 presents the conclusions and future works.

2. Brownian motion process

A stochastic process represents the mathematical representation of the potential manifestation of a random phenomenon at any specific moment in time after its initial event [12]. This text outlines the temporal evolution of a random phenomenon. A specific example of a stochastic process in continuous time is the Brownian motion process. This process represents a continuous variation of the fundamental random walk [13]. It is interconnected with various other stochastic processes and holds significant importance in stochastic calculus and martingales. The Brownian motion process also has numerous applications. For example, it is crucial in engineering and quantitative finance. Additionally, it serves as a mathematical model for various random events in physical sciences and several fields within social sciences. Thus, the Brownian motion process is a widely-used random process.

Additionally, it is important to mention that Brownian motion, commonly known as the Brownian motion process, represents a type of Markov stochastic process. The Brownian motion process is comprised of a set of random variables $\Xi(t)$ that correspond to the continuous variable t within the range $[0, \infty]$. It is a stochastic process $\{\Xi(t)\}_{t\geq 0}$ that satisfies:

(i) $\Xi(t), t \geq 0$ is a continuous function of t, and $\Xi(t) \sim N(0, t)$.
(ii) For $s < t < u < c$, $\Xi(t) - \Xi(s)$, $\Xi(c) - \Xi(u)$ are assumed to be independent.
(iii) $\Xi(t) - \Xi(s)$ follows a normal distribution with zero mean and variance $t - s$, i.e., $\Xi(t) - \Xi(s) \sim \sqrt{t-s}\, N(0, 1)$, $N(0, 1)$ represents a standard normal distribution.
(iv) The distribution of $\Xi(t + s) - \Xi(s)$ is not dependent on s. Therefore, it has the same distribution as $\Xi(t)$.

The time derivative of the Brownian motion process $\Xi(t)$ is defined as $\Xi_t = \frac{d\,\Xi(t)}{dt}$.

The above properties indicate that the likelihood of the process advancing a specific distance within a defined time frame is governed by the characteristics of the normal distribution. Therefore, Brownian motion can be interpreted as a limiting case of random walks, in which the steps decrease in size and increase in frequency. This relationship further highlights the significance of the normal distribution, as the central limit theorem posits that the distribution of the sum of independent, identically distributed random variables (such as those found in a random walk) converges to a normal distribution.So, we briefly describe the normal distribution as follows. First, it is important to recognize that the normal distribution displays both symmetry and continuity. Its probability density function is generally depicted as

$$f(z) = \frac{1}{\delta\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\delta^2}}, \qquad -\infty < z < \infty,$$

where μ indicates the mean and δ indicates the standard deviation. The standard normal distribution illustrates a particular example with a mean of zero and a variance of one.

The Itô lemma calculates the time derivative of stochastic processes. The Itô differential is defined as follows:

$$df = f(\Xi_{t+dt}, t + dt) - f(\Xi_t, t),$$

where $f(\Xi_t, t)$ refers to the time derivative of stochastic processes, and $f(b, t)$ refers to a differentiable function. Thus, this demonstrates the change of f during a brief time interval dt. The Itô lemma pertaining to Brownian motion can be articulated as

$$df(\Xi_t, t) = \partial_b f(\Xi_t, t)\, d\Xi_t + \frac{1}{2}\partial_b^2 f(\Xi_t, t)\, dt \, + \, \partial_t f(\Xi_t, t)\, dt.$$

3. Mathematical investigation

Using the wave transformation

$$\chi(x,t) = \Upsilon(\zeta)e^{i(cx+\beta t+\sigma\Xi(t))}, q(x,t) = q(\zeta), \tag{3.1}$$

$$\zeta = v\, x + \rho\, t,$$

we produce the real part

$$-c^2\Upsilon(\zeta) + v^2\Upsilon''(\zeta) - \beta\Upsilon(\zeta) - q(\zeta)\Upsilon(\zeta) = 0 \tag{3.2}$$

with the constraint condition $\rho = -2cv$ from the imaginary part. Also, ρ, c, p, and v denote constants. On the other hand, the second equation of model (1.1) yields

$$2v\Upsilon(\zeta)\Upsilon'(\zeta) + (\rho + v)q'(\zeta) = 0.$$

Solving the last equation gives

$$q(\zeta) = -\left(\frac{v}{\rho + v}\right)\Upsilon^2(\zeta) = -\left(\frac{1}{1-2c}\right)\Upsilon^2(\zeta). \tag{3.3}$$

Equation (3.2) becomes

$$v^2\Upsilon''(\zeta) + \frac{1}{(1-2c)}\Upsilon^3(\zeta) - \left(\beta + c^2\right)\Upsilon(\zeta) = 0. \tag{3.4}$$

We apply the ETFM for solving Eq (3.4). The balance between the higher nonlinear term Υ^3 and the highest-order derivative Υ'' yields $M = 1$. According to the ETFM [23], the solution of Eq (3.4) takes the form

$$\begin{aligned}\Upsilon(\zeta) &= \sum_{j=0}^{j=M} a_j\psi^j(\zeta) + \sum_{j=1}^{j=M} b_j\psi^{-j}(\zeta)\\ &= a_0 + a_1\psi(\zeta) + \frac{b_1}{\psi(\zeta)},\end{aligned} \tag{3.5}$$

where the function $\psi(\zeta)$ satisfies the Riccati equation

$$\psi' = \varrho + \psi^2(\zeta), \tag{3.6}$$

and ϱ is a constant.

$$\Upsilon'(\zeta) = a_1\varrho + a_1\psi^2 - \frac{b_1\varrho}{\psi^2} - b_1. \tag{3.7}$$

Substituting Eq (3.5) and its derivative into Eq (3.4) and then assembling all terms according to their respective powers of ψ^3, ψ^2, ψ, ψ^0, ψ^{-1}, ψ^{-2}, and ψ^{-3} gives a set of algebraic equations. Solving these equations with Maple yields the following solutions:

Family I.

$$a_0 = 0, \quad a_1 = \pm\sqrt{2v^2(2c-1)},\ b_1 = \pm\frac{\sqrt{(2c-1)}(\beta+c^2)}{2\sqrt{2}v}, \varrho = \frac{-(\beta+c^2)}{4v^2}. \tag{3.8}$$

For $\beta + c^2 > 0$, the solutions of (3.4) are given as

$$\Upsilon_{1,2}(\zeta) = \pm\sqrt{-2(\beta+c^2)(1-2c)}\, csch\left(\sqrt{\frac{\beta+c^2}{v^2}}\,\zeta\right),\ c > \frac{1}{2}. \tag{3.9}$$

Consequently, the solutions to (1.1) are given as

$$\chi_{1,2}(x,t) = \pm\sqrt{-2(\beta+c^2)(1-2c)}\, csch\left(\sqrt{\frac{\beta+c^2}{v^2}}\,(v\,x+\rho\,t)\right)e^{i(cx+\beta t+\sigma\Xi(t))}, \tag{3.10}$$

$c > \frac{1}{2}$.

For $\beta + c^2 < 0$, the solutions of (3.4) are given as

$$\Upsilon_{3,4}(\zeta) = \pm\sqrt{2(\beta+c^2)(1-2c)}\, csc\left(\sqrt{\frac{-(\beta+c^2)}{v^2}}\,\xi\right),\ c > \frac{1}{2}. \tag{3.11}$$

Consequently, the solutions to (1.1) are given as

$$\chi_{3,4}(x,t) = \pm\sqrt{2(\beta+c^2)(1-2c)}\, csc\left(\sqrt{\frac{-(\beta+c^2)}{v^2}}\,(v\,x+\rho\,t)\right)e^{i(cx+\beta t+\sigma\Xi(t))}, \tag{3.12}$$

$c > \frac{1}{2}$.

For $\beta + c^2 = 0$, the solutions of (3.4) are given as

$$\Upsilon_{5,6}(\zeta) = \pm\sqrt{2v^2(2c-1)}\frac{-1}{\zeta},\ c > \frac{1}{2}. \tag{3.13}$$

Consequently, the solutions to (1.1) are given as

$$\chi_{5,6}(x,t) = \pm\sqrt{2v^2(2c-1)}\frac{-1}{v\,x+\rho\,t}e^{i(cx+\beta t+\sigma\Xi(t))},\ c > \frac{1}{2}. \tag{3.14}$$

Family II.

$$a_0 = 0, \quad a_1 = \pm\sqrt{2v^2(2c-1)},\ b_1 = 0, \varrho = \frac{\beta+c^2}{2v^2}. \tag{3.15}$$

Putting Eq (3.15) into Eq (3.5), we get the following solutions:

For $\beta + c^2 < 0$, the solutions of (3.4) are given as

$$\begin{aligned}\Upsilon_{7,8}(\zeta) &= \pm\sqrt{(\beta+c^2)(1-2c)}\,tanh(\sqrt{\frac{-(\beta+c^2)}{2v^2}}\,\zeta),\ \ c>\frac{1}{2},\\ \Upsilon_{9,10}(\zeta) &= \pm\sqrt{(\beta+c^2)(1-2c)}\,coth(\sqrt{\frac{-(\beta+c^2)}{2v^2}}\,\zeta),\ \ c>\frac{1}{2}.\end{aligned}\tag{3.16}$$

Thus, the solution of (1.1) are given as

$$\begin{aligned}\chi_{7,8}(x,t) &= \pm\sqrt{(\beta+c^2)(1-2c)}\,tanh(\sqrt{\frac{-(\beta+c^2)}{2v^2}}\,(v\,x+\rho\,t))e^{i(cx+\beta t+\sigma\Xi(t))},\\ \chi_{9,10}(x,t) &= \pm\sqrt{(\beta+c^2)(1-2c)}\,coth(\sqrt{\frac{-(\beta+c^2)}{2v^2}}\,(v\,x+\rho\,t))e^{i(cx+\beta t+\sigma\Xi(t))},\end{aligned}\tag{3.17}$$

$c > \frac{1}{2}$.

For $\beta + c^2 > 0$, the solutions of (3.4) are given as

$$\begin{aligned}\Upsilon_{11,12}(\zeta) &= \pm\sqrt{(\beta+c^2)(2c-1)}\,tan(\sqrt{\frac{\beta+c^2}{2v^2}}\,\zeta),\ \ c>\frac{1}{2},\\ \Upsilon_{13,14}(\zeta) &= \pm\sqrt{(\beta+c^2)(2c-1)}\,cot(\sqrt{\frac{\beta+c^2}{2v^2}}\,\zeta),\ \ c>\frac{1}{2}.\end{aligned}\tag{3.18}$$

Therefore, the solution of (1.1) is given as

$$\begin{aligned}\chi_{11,12}(x,t) &= \pm\sqrt{(\beta+c^2)(2c-1)}\,tan(\sqrt{\frac{\beta+c^2}{2v^2}}\,(v\,x+\rho\,t))e^{i(cx+\beta t+\sigma\Xi(t))},\\ \chi_{13,14}(x,t) &= \pm\sqrt{(\beta+c^2)(2c-1)}\,cot(\sqrt{\frac{\beta+c^2}{2v^2}}\,(v\,x+\rho\,t))e^{i(cx+\beta t+\sigma\Xi(t))},\end{aligned}\tag{3.19}$$

$c > \frac{1}{2}$.

Family III.

$$a_0 = 0,\quad a_1 = \pm\sqrt{2v^2(2c-1)},\ \ b_1 = \pm\frac{(\beta+c^2)\sqrt{2c-1}}{4\sqrt{2}v}, \varrho = \frac{\beta+c^2}{8v^2}.\tag{3.20}$$

For $\beta + c^2 > 0$, the solutions of (3.4) are given as

$$\Upsilon_{15,16}(\zeta) = \pm\sqrt{\frac{(\beta+c^2)(2c-1)}{4}}\left(tan\left(\sqrt{\frac{\beta+c^2}{8v^2}}\,\zeta\right) - cot\left(\sqrt{\frac{\beta+c^2}{8v^2}}\,\zeta\right)\right),\ \ c>\frac{1}{2}.\tag{3.21}$$

Thus, the solutions of (1.1) are given as

$$\chi_{15,16}(x,t) = \pm\sqrt{\frac{(\beta+c^2)(2c-1)}{4}}\,e^{i(cx+\beta t+\sigma\Xi(t))}\times$$

$$\left(tan\left(\sqrt{\frac{\beta+c^2}{8v^2}}\,\zeta\right)-cot\left(\sqrt{\frac{\beta+c^2}{8v^2}}\,\zeta\right)\right),\ c>\frac{1}{2}. \tag{3.22}$$

For $\beta + c^2 < 0$, the solutions of (3.4) are given as

$$\Upsilon_{17,18}(\zeta)=\pm\sqrt{\frac{(\beta+c^2)(1-2c)}{4}}\left(tanh\left(\sqrt{\frac{-(\beta+c^2)}{8v^2}}\,\zeta\right)+coth\left(\sqrt{\frac{-(\beta+c^2)}{8v^2}}\,\zeta\right)\right),\ c>\frac{1}{2}. \tag{3.23}$$

Thus, the solutions of (1.1) are

$$\begin{aligned}\chi_{17,18}(x,t) &= \pm\sqrt{\frac{(\beta+c^2)(1-2c)}{4}}\,e^{i(cx+\beta t+\sigma\Xi(t))}\times\\ &\left(tanh\left(\sqrt{\frac{-(\beta+c^2)}{8v^2}}\,\zeta\right)+coth\left(\sqrt{\frac{-(\beta+c^2)}{8v^2}}\,\zeta\right)\right),\ c>\frac{1}{2}.\end{aligned} \tag{3.24}$$

Using relation (3.3), one can easily get the coupled solution $q(x,t)$ for each family.

4. Waves behaviour

Comprehending the wave dynamics of the NLSWIS is essential for elucidating the intricate mechanisms that regulate nonlinear wave coupling in physical media. This model encapsulates the resonant interaction between a high-frequency (short) wave and a low-frequency (long) wave, a phenomenon that can be observed across various domains including fluid dynamics, nonlinear optics, plasma physics, and stratified geophysical flows. Examining the behavior of these waves offers essential understanding regarding energy transfer, modulation instability, and the formation of coherent structures including solitons, periodic, and modulated wave packets. Nonlinear systems, particularly those with cubic or higher-order terms, can result in finite-time blow-up, where the solution becomes unbounded (i.e., singular) within a finite time frame. This is often indicative of strong interactions or self-focusing effects.

The motivation for this part derives from the need to carefully investigate the evolution of the amplitude, speed, and stability of wave components during nonlinear interactions. By investigating these behaviors, we may get a better understanding of how nonlinear wave systems self-organize, propagate, and respond to disturbances, strengthening mathematical theory and broadening its real-world applications. The solution $\chi_1(x,t)$ of Eq (3.10) shows the profiles of 2D explosive pulse wave solutions without noise ($\sigma = 0$), as depicted in Figure 1. The solution $\chi_5(x,t)$ of Eq (3.14) shows the 2D rational traveling wave solution, as illustrated in Figure 2. The solution $\chi_7(x,t)$ of Eq (3.17) shows the 2D periodic wave solution $\chi_7(x,t)$ in (3.17) with different values of t while maintaining constant amplitude, without reversal of direction, and accompanied by a phase shift, as depicted in Figure 3. The solution $\chi_{11}(x,t)$ of Eq (3.19) shows the profiles of the 2D periodic pulse wave solution, as depicted in Figure 4. The solution $\chi_{17}(x,t)$ of Eq (3.24) shows the profiles of the 2D periodic wave solution, as illustrated in Figure 5.

Actually, a strong and realistic framework for researching nonlinear wave interactions under uncertainty is provided by the stochastic nonlinear long-short-wave interaction system, which offers important insights for both theoretical advancement and real-world applications in physics and engineering.

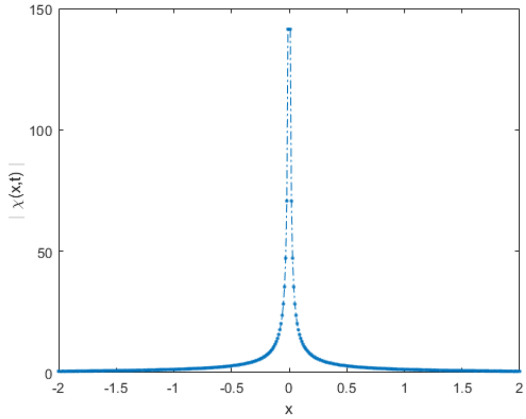

Figure 1. 2D explosive pulse wave solution $\chi_1(x,t)$ in Eq (3.10) with $\beta = -0.6, c = 1$, and $v = 0.8$.

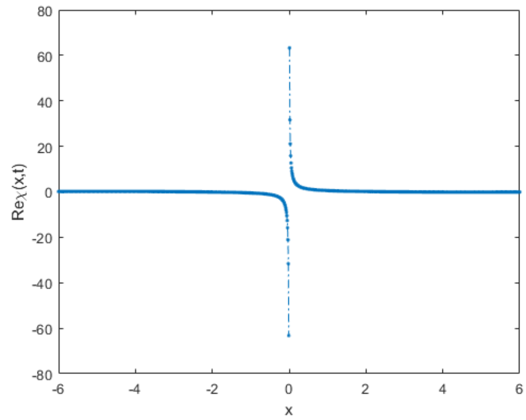

Figure 2. 2D rational traveling wave solution $\chi_5(x,t)$ in Eq (3.14) with $\beta = -0.36, c = 0.6$, and $v = 1$.

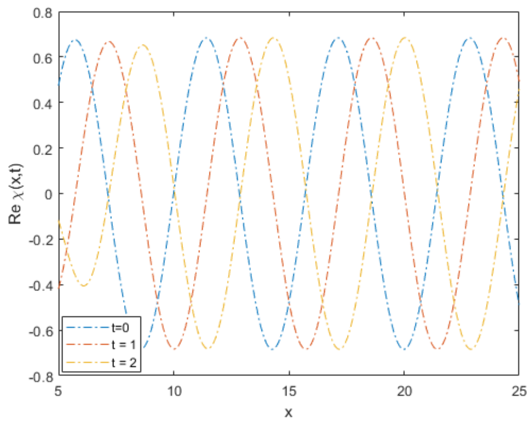

Figure 3. 2D periodic wave solution $\chi_7(x,t)$ in Eq (3.17) with different values of t with $\beta = -1.7, c = 1.2$, and $v = 1$.

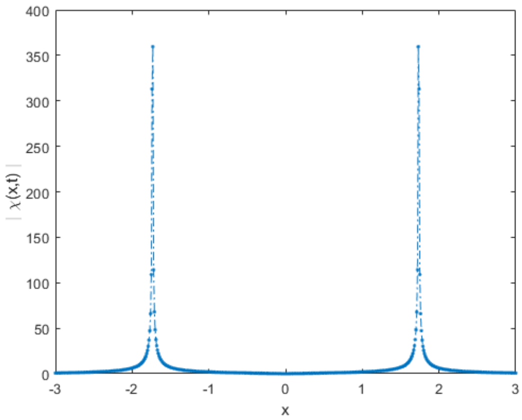

Figure 4. 2D periodic pulse wave solution $\chi_{11}(x,t)$ in Eq (3.19) with $\beta = 0.2, c = 1.2$, and $v = 0.5$.

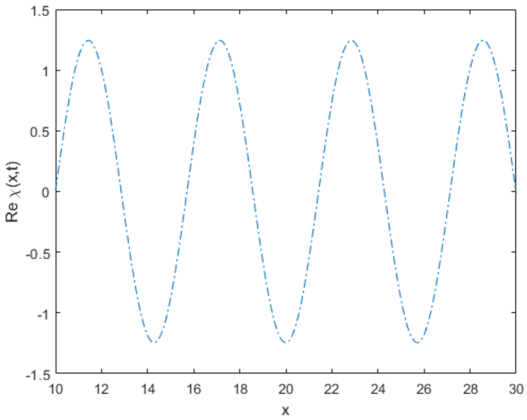

Figure 5. 2D periodic wave solution $\chi_{17}(x,t)$ in Eq (3.24) with $\beta = -2.5, c = 1.1$, and $v = 1$.

5. The influence of stochastic process

The noise component profoundly influences the development of wave fields, resulting in variations in amplitude, unpredictable phase shifts, and, in certain instances, initiating or inhibiting collapse events that would not take place within a deterministic framework. Conventional deterministic models are inadequate in representing the intricate behavior that emerges when random external disturbances affect both the short-wave envelope and the long-wave background field. By incorporating Brownian motion into the analysis of the NLSWIS, a stochastic framework is established to allow for the examination of wave evolution from both dynamic and statistical perspectives.

This section attempts to close the gap between theoretical modeling and real-world uncertainty by concentrating on the statistical implications of Brownian perturbations. This will give a probabilistic viewpoint on nonlinear wave dynamics and support more reliable predictions for systems whose physical behavior is inherently random. Brownian motion $\Xi(t)$, is an important stochastic process. This approach converts the NLSWIS model with multiplicative noise into the usual nonlinear ordinary differential form. Figure 6 shows how the stochastic solution $\chi_7(x,t)$ varies depending on the spatial variable x and noise strength σ with different values of t. This figure shows that the wave is periodic with phase shift and without reverse in direction. Figures 7 and 8 present, respectively, the three-dimensional graphs and the trajectories of the stochastic solution $\chi_7(x,t)$ for different values of the noise intensity σ. The graphs depict the stochastic solutions of the long-short-wave interaction system, offering information on the impact of stochastic perturbations on wave dynamics. These representations are crucial for researching soliton interaction because they highlight areas of high and low density, offering insight into processes such as soliton contact, soliton merging, energy exchange, and modulation. Furthermore, the inclusion of a stochastic element in nonlinear wave equations introduces unpredictability, which can significantly alter the behavior of solitary waves such as optical solitons or hydrodynamic solitons. Shape distortion, location jitter, and eventually death result from stochastic perturbations caused by thermal noise, impurities, or environmental variations that upset the delicate balance between nonlinearity and dispersion that underpins solitons. A more realistic framework for explaining the interaction between long and short waves is provided by including stochastic effects, which captures issues like turbulence, ambient noise, and irregular forcing that are insufficiently addressed by deterministic models alone. However, under some situations, modest noise might result in stochastic resonance, which could stabilize solitons or perhaps allow for their production.

In summary, the ETFM for extracting solitary wave solutions is effective, straightforward, succinct, and robust. Hence, it may be used on a variety of NPDEs in mathematical physics and other natural science fields. Lastly, among other things, the stochastic character of solutions can explain a lot of complex events in these fields.

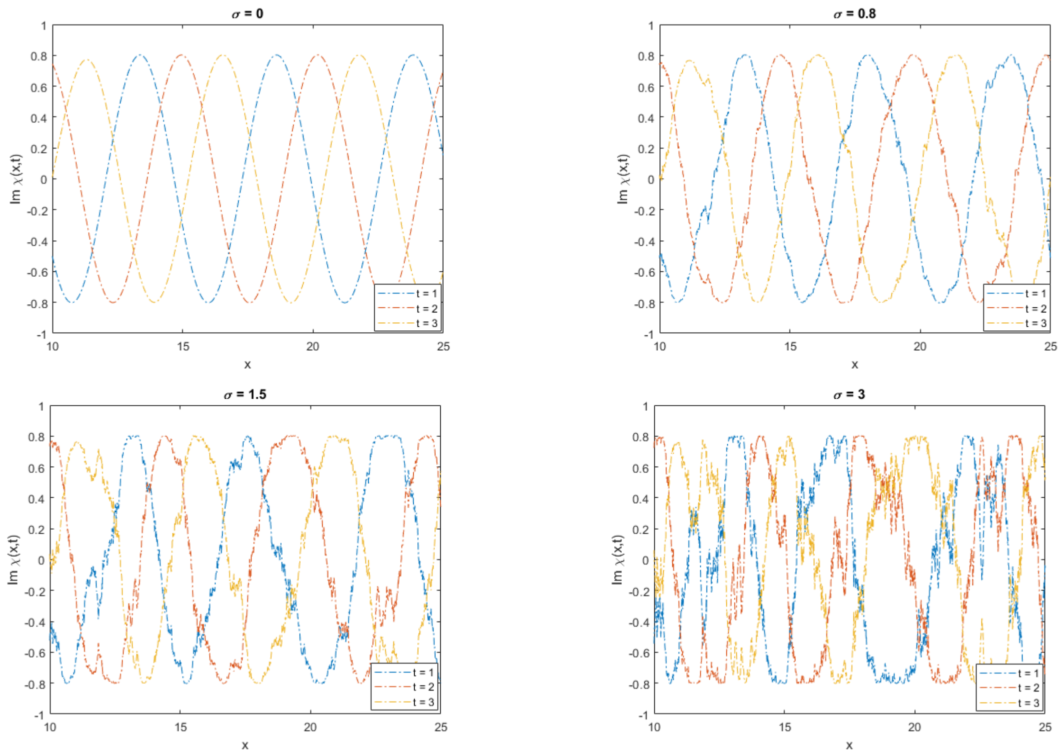

Figure 6. Two-dimensional plots of solution $\chi_7(x,t)$ in Eq (3.17) with $\beta = -1.9, c = 1.2$, and $v = 1$.

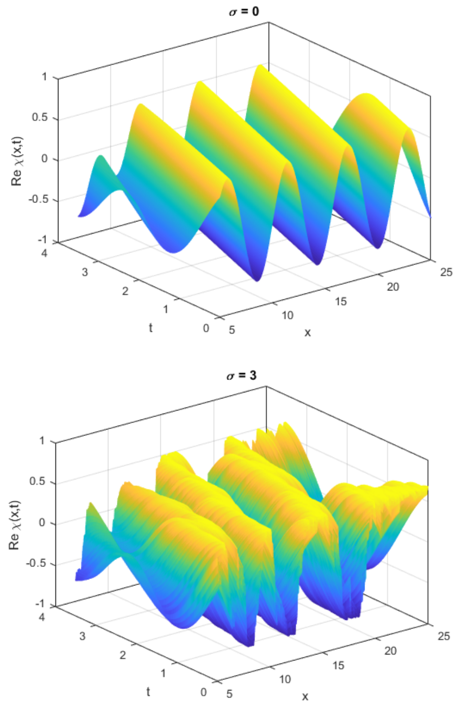

Figure 7. Three-dimensional plots of solution $\chi_7(x,t)$ in Eq (3.17) with $\beta = -1.6, c = 1.1$, and $v = 1$.

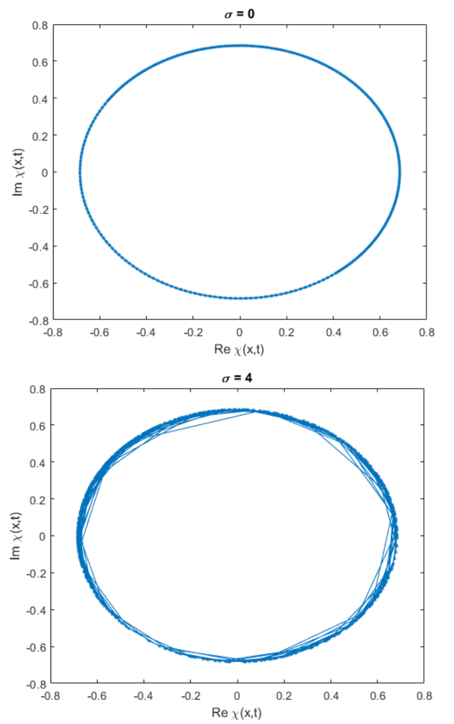

Figure 8. Phase trajectories plots of solution $\chi_7(x,t)$ in Eq (3.17) with different values of σ for $\beta = -1.6, c = 1.1$, and $v = 1$.

6. Conclusions

Using ETFM, we investigated the NLSWIS caused by multiplicative noise in the Itô sense. We employ this strategy to create novel traveling wave solutions. This approach has an advantage over others in that it can handle a broader variety of scientific difficulties while eliminating time-consuming and expensive computations. Among other essential features, we have incorporated rational waves, periodic waves, periodic pulse waves, and hybrid wave structures into the stochastic NLSWIS that is defined by multiplicative noise intensity. We demonstrate how multiplicative noise influences the behavior of the given solutions. The 2D and 3D plots illustrated the behavior of the solutions when the appropriate values for the free parameters were applied. The discovered solutions have consequences for the growing field of optical fiber communications, plasma physics, coastal engineering, etc. Our findings open up new possibilities for investigating high-dimensional stochastic integrable systems, potentially useful in applied research due to the unexpected nature of the real world. In future investigations, we will use various analytical strategies to generate more stochastic traveling waves for NLSWIS and related models. It is also possible to investigate the inclusion of fractional-order derivatives in order to account for memory effects and nonlocal interactions, which are frequently found in complex media and viscoelastic materials.

Author contributions

Mahmoud A. E. Abdelrahman: Conceptualization, data curation, formal analysis, writing-original draft; Yousef F. Alharbi: Conceptualization, data curation, formal analysis, writing-original draft. All authors have read and agreed to the submitted version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no competing interests.

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