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A differential game of transboundary pollution in the upper Yangtze river: Ecological compensation and learning-by-doing in the Chongqing-Sichuan region

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Abstract: This paper investigates a differential game modeling transboundary pollution management in the Upper Yangtze River Basin under noncooperative and cooperative scenarios, incorporating emissions trading, learning-by-doing effects, and abatement investment costs. The maximum principle of optimal control theory was employed to derive equilibrium solutions for both models. Numerical simulations identified optimal emission levels and abatement investments for the Chongqing Municipality and Sichuan Province under each scenario, with the pollution stock trajectories computed using the fourth-order Runge-Kutta method. Numerical validation demonstrated that ecological compensation mechanisms enhance mitigation effectiveness. Furthermore, the study revealed that learning-by-doing efficiency gains and reductions in regional abatement investment costs synergistically enhance both pollution mitigation and economic returns within this framework.

Keywords: differential game; transboundary pollution; Runge-Kutta method; learning by doing
Mathematics Subject Classification: 49L20, 91B76

1. Introduction

Rapid industrialization has established China as a global industrial power, with the industrial sector dominating its national economy. This has resulted in massive industrial capacity and world-leading industrial output. However, industrialization has simultaneously exerted significant environmental pressure. China's pollution problems have become increasingly severe, reflecting a "pollution-first, treatment-later" development paradigm that inflicts serious ecological damage and hampers socioeconomic progress. To address this challenge, game theory and ecological-compensation mechanisms have been increasingly applied and refined both domestically and internationally.

In specific regional contexts, such as the Yangtze River Basin, which is crucial for China's water resources and ecological security, numerous pollution control challenges persist. For instance, the water quality of the Tongbo River, a tributary in the Yangtze River Basin, deteriorated to the inferior Class V before 2020 due to inconsistent water environmental functions and insufficient pollution control measures in Sichuan and Chongqing. Yet after Dazhou in Sichuan and Liangping in Chongqing signed the "Joint Prevention and Control Agreement for the Tongbo River" in September 2020, and adopted mechanisms like joint meetings and co-governance, its water quality has remained above Class III since July 2021. This case reflects the role of cross-regional cooperation in pollution control but also implies the lack of exploration in bidirectional pollutant impact mechanisms*.

Another example is the Guanhe River at the junction of the Yunnan and Sichuan Provinces. Once troubled by cross-boundary pollution and ecological damage, it saw improvement after procuratorates from both regions created a cross-regional public interest litigation cooperation model and issued relevant implementation opinions. This again shows the effect of cross-regional collaboration, but similar to the Tongbo River case, the underlying mutual pollutant diffusion dynamics are rarely studied†.

While significant progress has been made in modeling transboundary pollution, including studies on bidirectional diffusion between adjacent regions (e.g., de Frutos & Martín-Herrán, 2019), less attention has been paid to integrating this spatial complexity with the dynamic, time-evolving process of technological learning. Specifically, the interplay between reciprocal pollutant transfer and an endogenous learning-by-doing mechanism—whereby a region's abatement efficiency improves cumulatively through its investment experience—remains underexplored within the framework of ecological compensation.

Therefore, the key innovations of this study are threefold. First, we establish a novel differential game model that synthesizes bidirectional pollution diffusion, endogenous learning-by-doing in abatement investment, and ecological compensation mechanisms into a unified analytical framework. This integration allows us to capture both the spatial interdependence and dynamic technological evolution that characterize real-world transboundary pollution problems. Second, unlike static or unidirectional models, our framework enables the derivation of dynamic optimal strategies for emissions and abatement investment that explicitly account for how a region's evolving efficiency (through learning) affects and is affected by its neighbor's pollution. Finally, we contextualize our theoretical analysis with a numerical simulation grounded in the specific challenges and dynamics of the Chongqing-Sichuan section in the upper Yangtze River Basin, thereby bridging theoretical modeling with region-specific policy insights and offering tangible guidance for policymakers in similar basin ecosystems.

The subsequent structure of this paper is as follows: Section 2 provides a literature review. Section 3 clarifies the basic assumptions of the game model. Section 4 analyzes the model under both cooperative and non-cooperative scenarios, and further derives the optimal emission levels and emission reduction

*https://www.mee.gov.cn/home/ztbd/2023/mlhh2/yxal3/cq/202507/t20250717_1123691.shtml

†https://www.sc.gov.cn/10462/10778/10876/2024/3/26/09e556f05ed14e9b8c04cffb8f5e0ec9.shtml

investments. Section 5 conducts numerical simulations based on transboundary pollution data from the Yangtze River Basin. Section 6 discusses the research results and puts forward relevant policy recommendations.

2. Literature review

As pointed out in the introduction, game theory and ecological compensation mechanisms are key tools for solving cross-regional pollution problems, and the evolution of their related research provides an important reference for this paper. To clearly define the theoretical starting point and innovative direction of this study, it is necessary to systematically sort out the existing relevant research from the dimensions of game theory, ecological compensation mechanisms, and their cross-application.

Game theory, as a key tool, provides mathematical frameworks for analyzing strategic interactions among rational decision-makers. Initially applied to biological population dynamics, the field has expanded to address environmental challenges including watershed management and transboundary pollution. Moslener et al. [1] pioneered work on optimal abatement for cumulative and interacting pollutants, while Li et al. [2] extended Beladi's model to incorporate learning-by-doing effects in abatement costs. Bertinelli et al. [3] examined strategic behaviors in transboundary CO_2 pollution using differential games, with Benchekroun et al. [4] subsequently analyzing foresight impacts in similar settings. Huang et al. [5] and Yeung et al. [6] developed cooperative differential games for industrial pollution, while Jiang et al. [7] constructed stochastic models to analyze transboundary pollution control between compensating regions. Moslener et al. [8] further optimized multi-pollutant abatement considering complementary/substitutable pollutants, and List et al. [9] demonstrated higher combined payoffs under decentralized control using asymmetric dynamic models. In summary, the application of game theory in pollution control has expanded from single pollutants to multiple pollutants and from static analysis to dynamic models. However, existing differential game studies have primarily focused on one-way pollution scenarios, with limited consideration of the two-way feedback characteristics of "mutual diffusion of pollutants" between adjacent regions. Additionally, the dynamic learning process of emission reduction technologies has not been incorporated into the strategy optimization framework, providing a starting point for the model design in this study.

Concurrently, ecological compensation mechanisms have evolved into effective regulatory tools for coordinating cross-regional interests [7]. This has stimulated research on watershed pollution management under such frameworks. Jiang et al. [10] developed differential games for optimal transboundary pollution control under ecological compensation criteria, while Yi et al. [11] investigated Stackelberg differential games for upstream-downstream pollution compensation in river basins. Extending this line of inquiry, Wei et al. [12] modeled government-firm interactions within watershed ecological compensation systems, and Gao et al. [13] examined multilevel governmental decision-making processes in these contexts. The concept of "learning-by-doing" is also being expanded beyond purely technological cost reduction. Recent work by Anjum et al. [14] demonstrates that organizational learning, facilitated by employee empowerment, plays a key role in enhancing environmental process design and performance, offering a valuable micro-level perspective that complements the macro-level technological learning typically modeled in differential games.

Additionally, researchers have integrated learning-by-doing theory into pollution management frameworks. Chang et al. [15] proposed transboundary pollution games incorporating emissions trading and abatement costs under learning effects. Chen et al. [16] analyzed dynamic Stackelberg games for water pollution control with learning-by-doing depreciation, while Wei et al. [17] developed similar models for regulator-oligopoly interactions in green innovation contexts.

Based on the above research background, this paper intends to realize three aspects of innovation: First, it integrates the bidirectional pollutant diffusion between adjacent regions into the differential game framework, breaking through the limitation that most existing game models focus on unidirectional pollution. Second, it incorporates the learning-by-doing effect of emission reduction investment into the model, and explores how the dynamic improvement of regional emission reduction efficiency affects the formulation of game strategies. Third, it links the dynamic adjustment of ecological compensation standards with the above two elements, constructing a coupled analysis framework that connects spatial pollution characteristics, dynamic technical learning, and inter-regional interest coordination, so as to provide a more comprehensive theoretical reference for transboundary pollution mitigation.

3. The basic model

This section systematically defines the differential game parameters in Table 1, establishing the mathematical foundation for subsequent analysis.

Table 1. Summary of parameters and variables.

Symbol	Description	Type
a, b	Revenue function coefficient	Parameter
p_i	Price of emission permits in region i	Parameter
e_{i0}	Initial emission quota in region i	Parameter
η_1	Pollution diffusion coefficient from region 1 to 2	Parameter
η_2	Pollution diffusion coefficient from region 2 to 1	Parameter
φ	Pollution decay rate	Parameter
b_i	Marginal contribution to experience A_i in region i	Parameter
β_i	Marginal cost reduction due to experience in region i	Parameter
u_i	Learning rate of knowledge accumulation in region i	Parameter
r	Discount rate and memory decay rate	Parameter
ε_j^i	Damage coefficient from region j to i	Parameter
$e_i(t)$	Emission level in region i at time t	Variable (Control)
$k_i(t)$	Abatement investment in region i at time t	Variable (Control)
$s_i(t)$	Pollution stock in region i at time t	Variable (State)
$A_i(t)$	Accumulated experience in region i at time t	Variable (State)

We consider a two-region pollution abatement model, where region i ($i = 1, 2$) generates emissions $e_i(t)$ as production by-products. The output $Q_i(t)$ partially reflects total regional economic development and correlates positively with emissions, expressed as $Q_i(t) = Q_i(e_i(t))$. Following Li et al. [18] and Yeung et al. [6], region i's revenue function takes the concave quadratic form:

$$R_i(e_i(t)) = ae_i(t) - \frac{b}{2}e_i^2(t), \quad (i = 1, 2), \tag{3.1}$$

where a, b are positive constants.

In emission permits trading, p_1 and p_2 denote permit prices, and e_{i0} represents the initial emission quota. Following Lu et al. [19], the emission permits revenue at time t is:

$$E_i(e_i(t)) = p_i(e_i(t) - e_{i0}), \quad (i = 1, 2), \tag{3.2}$$

where $e_i(t) > e_{i0}$ indicates region i purchases permits from the market, while $e_i(t) < e_{i0}$ enables profit from selling excess permits.

In this model, the initial emission quota e_{i0} is assumed to be allocated by the government to region i free of charge based on the grandfathering principle, a common method in the initial phases of many emissions trading systems (ETS) [20, 21]. Under this principle, the allocation is typically tied to historical emissions data. Region i must surrender permits equal to its actual emissions $e_i(t)$. Therefore, $e_i(t) > e_{i0}$ indicates that region i's emissions exceed its free allocation, necessitating the purchase of additional permits from the market. Conversely, $e_i(t) < e_{i0}$ means that the region has a surplus of permits, which it can sell for profit.

Let $s_1(t)$ and $s_2(t)$ denote the pollution stocks in regions 1 and 2 at time t, respectively. Incorporating efficiency-improving learning-by-doing in abatement investments, the pollution stock dynamics follow Yi et al. [11]:

$$\dot{s}_1(t) = e_1(t) - k_1(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) - b_1 A_1(t), \tag{3.3}$$
$$\dot{s}_2(t) = e_2(t) - k_2(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) - b_2 A_2(t), \tag{3.4}$$

where η_1, η_2, and φ are positive constants. Here, η_1 represents the diffusion coefficient from region 1 to region 2, η_2 from region 2 to region 1, and φ the natural decay rate. The term $k_i(t)$ denotes abatement investment, with one unit reducing one unit of pollution. The expression $-b_i A_i$ captures how abatement efficiency increases with accumulated experience $A_i(t)$, where b_i is the marginal contribution to $A_i(t)$.

The abatement investment cost increases concavely while decreasing with accumulated experience. Following Lambertini [22] and Wei et al. [17], region i's investment cost is:

$$C_i(k_i(t), A_i(t)) = k_i^2(t) - \beta_i A_i(t), \quad (i = 1, 2), \tag{3.5}$$

where β_i denotes the marginal cost reduction from knowledge accumulation.

The evolution of accumulated experience $A_i(t)$ from abatement investment follows Chang et al. [15]:

$$\dot{A}_i(t) = u_i k_i(t) - r A_i(t), \quad (i = 1, 2), \tag{3.6}$$

with $u_i > 0$ as the learning rate and $r > 0$ as the experience decay rate.

The pollution damage cost function for region i is additively separable according to Yi et al. [23]:

$$D_1(s_1(t), s_2(t)) = \varepsilon_1^1 s_1(t) + \varepsilon_2^1 s_2(t), \tag{3.7}$$
$$D_2(s_1(t), s_2(t)) = \varepsilon_2^2 s_2(t) + \varepsilon_1^2 s_1(t), \tag{3.8}$$

where ε_j^i represents the damage coefficient from region j to region i.

4. Differential game model

4.1. Noncooperation model

Under noncooperation, each region independently optimizes its decisions, where strategic choices substantially impact the counterpart. Consequently, each region must anticipate the other's reactions during decision-making—a process involving outputs, abatement investments, pollutant emissions, and emission trading. Each region maximizes its net benefits by selecting optimal strategies contingent on the competitor's actions. In the non-cooperative model, each region solves its optimal control problem independently, treating the other region's state trajectories as exogenous. Following the open-loop solution concept, each region solves its optimal control problem independently, taking the time-path of the other region's controls as given. The resulting equilibrium is a pair of strategies such that no region can improve its payoff by unilaterally changing its entire pre-committed strategy path.

Each region maximizes its discounted revenue flow by selecting optimal emission levels and abatement investments over $t \in [0, \infty)$. The objective functional is:

$$W_i = \max_{e_i, k_i} \int_0^\infty [R_i(Q_i(t)) + E_i(e_i(t)) - C_i(k_i(t), A_i(t)) - D_i(s_i(t))]e^{-rt}dt, \quad (i = 1, 2), \tag{4.1}$$

where r denotes the discount rate, which is set here to simplify the model and align it with the empirical decay rate.

Substituting Eqs (3.1)–(3.8) yields region 1's optimization problem:

$$\begin{aligned} W_1 = \max_{e_1, k_1} \int_0^\infty \Big[& ae_1(t) - \frac{b}{2}e_1^2(t) + p_1(e_1(t) - e_{10}) - k_1^2(t) \\ & + \beta_1 A_1(t) - \varepsilon_1^1 s_1(t) - \varepsilon_2^1 s_2(t) \Big] e^{-rt}dt, \end{aligned} \tag{4.2}$$

$$s.t. \begin{cases} \dot{s}_1(t) = e_1(t) - k_1(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) - b_1 A_1(t), \\ \dot{A}_1(t) = u_1 k_1(t) - rA_1(t). \end{cases} \tag{4.3}$$

Similarly, region 2's optimization problem is:

$$\begin{aligned} W_2 = \max_{e_2, k_2} \int_0^\infty \Big[& ae_2(t) - \frac{b}{2}e_2^2(t) + p_2(e_2(t) - e_{20}) \\ & - k_2^2(t) + \beta_2 A_2(t) - \varepsilon_2^2 s_2(t) - \varepsilon_1^2 s_1(t) \Big] e^{-rt}dt, \end{aligned} \tag{4.4}$$

$$s.t. \begin{cases} \dot{s}_2(t) = e_2(t) - k_2(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) - b_2 A_2(t), \\ \dot{A}_2(t) = u_2 k_2(t) - rA_2(t). \end{cases} \tag{4.5}$$

The present value Hamiltonian for the optimal control problem in the two regions is:

$$\begin{aligned} H_1 = & ae_1(t) - \frac{b}{2}e_1^2(t) + p_1(e_1(t) - e_{10}) - k_1^2(t) \\ & + \beta_1 A_1(t) - \varepsilon_1^1 s_1(t) - \varepsilon_2^1 s_2(t) + \lambda_1(t)(e_1(t) \\ & - k_1(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) - b_1 A_1(t)) \\ & + \lambda_2(t)(u_1 k_1(t) - rA_1(t)), \end{aligned} \tag{4.6}$$

$$
\begin{aligned}
H_2 =& ae_2(t) - \frac{b}{2}e_2^2(t) - p_2(e_2(t) - e_{20}) - k_2^2(t) \\
&+ \beta_2 A_2(t) - \varepsilon_2^2 s_1(t) - \varepsilon_1^2 s_1(t) + \lambda_3(t)(e_2(t) \\
&- k_2(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) - b_2 A_2(t)) \\
&+ \lambda_4(t)(u_2 k_2(t) - rA_2(t)),
\end{aligned} \tag{4.7}
$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are dynamic co-state variables which measure the shadow prices of the associated state equations $\dot{s}_1(t)$ and $\dot{A}_1(t)$, respectively. Similarly, $\lambda_3(t)$ and $\lambda_4(t)$ are co-state variables for $\dot{s}_2(t)$ and $\dot{A}_2(t)$.

Proposition 1. *The optimal emission levels and abatement investment for the two regions under the non-cooperative model are as follows:*

$$
\begin{aligned}
e_1^n &= \frac{(a + p_1)M_1 - \varepsilon_1^1}{bM_1}, \\
k_1^n &= \frac{(M_1\beta_1 + b_1\varepsilon_1^1)u_1 + 2r\varepsilon_1^1}{4rM_1}, \\
e_2^n &= \frac{(a + p_2)M_2 - \varepsilon_2^2}{bM_2}, \\
k_2^n &= \frac{(M_2\beta_2 + b_2\varepsilon_2^2)u_2 + 2r\varepsilon_2^2}{4rM_2},
\end{aligned}
$$

where $M_1 = r + \eta_1 + \varphi, M_2 = r + \eta_2 + \varphi$.

Proof. For an interior solution of the Hamiltonian (4.6), the equilibrium conditions satisfy:

$$
\frac{\partial H_1}{\partial e_1(t)} = a - be_1(t) + p_1 + \lambda_1(t) = 0, \tag{4.8}
$$

$$
\frac{\partial H_1}{\partial k_1(t)} = -2k_1(t) + \lambda_2(t)u_1 - \lambda_1(t) = 0, \tag{4.9}
$$

and

$$
\dot{\lambda}_1(t) = r\lambda_1(t) - \frac{\partial H_1}{\partial s_1(t)} = (r + \eta_1 + \varphi)\lambda_1(t) + \varepsilon_1^1 = M_1\lambda_1(t) + \varepsilon_1^1, \tag{4.10}
$$

$$
\dot{\lambda}_2(t) = r\lambda_2(t) - \frac{\partial H_1}{\partial A_1(t)} = 2r\lambda_2(t) + b_1\lambda_1(t) - \beta_1, \tag{4.11}
$$

with transversality conditions $\lambda_j^c(T) = 0$ as $T \to \infty$ ($j = 1, 2$). Solving (4.8) and (4.9) yields:

$$
e_1(t) = \frac{a + p_1 + \lambda_1(t)}{b}, \tag{4.12}
$$

$$
k_1(t) = \frac{\lambda_2(t)u_1 - \lambda_1(t)}{2}. \tag{4.13}
$$

Since control variables are functions of state variables, state variables are solved first. From (4.10) and (4.11):

$$
\lambda_1 = -\frac{\varepsilon_1^1}{M_1}, \tag{4.14}
$$

$$\lambda_2 = \frac{M_1\beta_1 + b_1\varepsilon_1^1}{2rM_1}. \tag{4.15}$$

The superscript n denotes noncooperative equilibrium outcomes. Substituting (4.14) and (4.15) into (4.12) and (4.13) gives:

$$e_1^n = \frac{(a + p_1)M_1 - \varepsilon_1^1}{bM_1}, \tag{4.16}$$

$$k_1^n = \frac{(M_1\beta_1 + b_1\varepsilon_1^1)u_1 + 2r\varepsilon_1^1}{4rM_1}. \tag{4.17}$$

The proof for Region 2 is similar, so the proof is omitted here. □

Analytical solutions for pollution stock equations are infeasible. Following Lu et al. [19], we compute numerical solutions using the fourth-order Runge-Kutta method:

$$\begin{aligned}
&u_{n+1} = u_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4),\\
&k_1 = f(t_n, u_n), k_2 = f(t_n + \frac{1}{2}h, u_n + \frac{1}{2}hk_1),\\
&k_3 = f(t_n + \frac{1}{2}h, u_n + \frac{1}{2}hk_2), k_4 = f(t_n + h, u_n + hk_2),
\end{aligned}$$

with $h = 0.2$. Next, we extend this framework to cooperative games to achieve joint optimal solutions at continuous time t.

4.2. Cooperation game

Assuming the two regions establish cooperative agreements for transboundary pollution control, sustainability requires both parties to adhere to agreed decisions. This section derives optimal emission paths and abatement levels under cooperation. The joint objective functional and constraints are:

$$\begin{aligned}
W = \max_{e_i(t),k_i(t)} \int_0^\infty \Big[&\sum_{i=1}^{2}(ae_i(e) - \frac{b}{2}e_i^2(t)) + p_1(e_1(t) - e_{10})\\
&+ p_2(e_2(t) - e_{20}) - k_1^2(t) + \beta_1 A_1(t) - k_2^2(t) + \beta_2 A_2(t)\\
&- \varepsilon_1^1 s_1(t) - \varepsilon_2^1 s_2(t) - \varepsilon_2^2 s_1(t) - \varepsilon_1^2 s_1(t)\Big] e^{-rt}dt,
\end{aligned} \tag{4.18}$$

$$s.t. \begin{cases}
\dot{s}_1(t) = e_1(t) - k_1(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) - b_1 A_1(t),\\
\dot{A}_1(t) = u_1 k_1(t) - rA_1(t),\\
\dot{s}_2(t) = e_2(t) - k_2(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) - b_2 A_2(t),\\
\dot{A}_2(t) = u_2 k_2(t) - rA_2(t).
\end{cases} \tag{4.19}$$

Applying Pontryagin's maximum principle yields the current-value Hamiltonian:

$$H = \sum_{i=1}^{2}\Big(ae_i(e) - \frac{b}{2}e_i^2(t)\Big) + p_1(e_1(t) - e_{10})$$

$$
\begin{aligned}
&+ p_2(e_2(t) - e_{20}) - k_1^2(t) + \beta_1 A_1(t) - k_2^2(t) \\
&+ \beta_2 A_2(t) - \varepsilon_1^1 s_1(t) - \varepsilon_2^1 s_2(t)\varepsilon_2^2 s_2(t) - \varepsilon_1^2 s_1(t) \\
&+ \lambda_1^c(t)(e_1(t) - k_1(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) \\
&- b_1 A_1(t)) + \lambda_2^c(t)(u_1 k_1(t) - rA_1(t)) \\
&+ \lambda_3^c(t)(e_2(t) - k_2(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) \\
&- b_2 A_2(t)) + \lambda_4^c(t)(u_2 k_2(t) - rA_2(t)),
\end{aligned}
\tag{4.20}
$$

where $\lambda_j^c(t)$ $(j = 1, 2, 3, 4)$ measures shadow prices for $\dot{s}_i(t)$ and $\dot{A}_i(t)$.

Proposition 2. *The optimal emission levels and abatement investment for the two regions under the cooperative game model are as follows:*

$$
\begin{aligned}
e_1^c &= \frac{a + p_1}{b} + \frac{N_2\eta_1 + M_2 N_1}{(\eta_1\eta_2 - M_1 M_2)b}, \\
k_1^c &= (\frac{\beta_1}{4r} - \frac{(M_2 N_1 - N_2\eta 1)b_1}{(\eta_1\eta_2 - M_1 M_2)4r})u_1 - \frac{N_2\eta_1 + M_2 N_1}{2(\eta_1\eta_2 - M_1 M_2)}, \\
e_2^c &= \frac{a + p_2}{b} + \frac{N_1\eta_2 + M_1 N_2}{(\eta_1\eta_2 - M_1 M_2)b}, \\
k_2^c &= (\frac{\beta_2}{4r} - \frac{(M_1 N_2 - N_1\eta_2)b2}{(\eta_1\eta_2 - M_1 M_2)4r})u_2 - \frac{N_1\eta_2 + M_1 N_2}{2(\eta_1\eta_2 - M_1 M_2)},
\end{aligned}
$$

where $N_1 = \varepsilon_1^1 + \varepsilon_1^2$, $N_2 = \varepsilon_2^1 + \varepsilon_2^2$.

Proof. The first-order conditions for maximizing (4.20) are:

$$\frac{\partial H}{\partial e_1(t)} = a - be_1(t) + p_1 + \lambda_1^c(t) = 0, \tag{4.21}$$

$$\frac{\partial H}{\partial k_1(t)} = -2k_1(t) + \lambda_2^c(t)u_1 - \lambda_1^c(t) = 0, \tag{4.22}$$

$$\frac{\partial H}{\partial e_2(t)} = a - be_2(t) + p_2 + \lambda_3^c(t) = 0, \tag{4.23}$$

$$\frac{\partial H}{\partial k_2(t)} = -2k_2(t) + \lambda_4^c(t)u_2 - \lambda_3^c(t) = 0. \tag{4.24}$$

The current value costate equations are as follows:

$$\dot{\lambda}_1(t) = r\lambda_1(t) - \frac{\partial H}{\partial s_1(t)} = M_1\lambda_1^c(t) + N_1 - \eta_1\lambda_3^c(t), \tag{4.25}$$

$$\dot{\lambda}_2(t) = r\lambda_2(t) - \frac{\partial H}{\partial A_1(t)} = 2r\lambda_2^c(t) + b_1\lambda_1^c(t) - \beta_1, \tag{4.26}$$

$$\dot{\lambda}_3(t) = r\lambda_3(t) - \frac{\partial H}{\partial s_2(t)} = M_2\lambda_3^c(t) + N_2 - \eta_2\lambda_1^c(t), \tag{4.27}$$

$$\dot{\lambda}_4(t) = r\lambda_4(t) - \frac{\partial H}{\partial A_2(t)} = 2r\lambda_4^c(t) + b_2\lambda_3^c(t) - \beta_2, \tag{4.28}$$

$$\dot{s}_1(t) = e_1(t) - k_1(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) - b_1 A_1(t), \tag{4.29}$$

$$\dot{A}_1(t) = u_1k_1(t) - rA_1(t), \tag{4.30}$$
$$\dot{s}_2(t) = e_2(t) - k_2(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) - b_2A_2(t), \tag{4.31}$$
$$\dot{A}_2(t) = u_2k_2(t) - rA_2(t). \tag{4.32}$$

Solving (4.21)–(4.24) yields:

$$e_1^c(t) = \frac{a + p_1 + \lambda_1^c(t)}{b}, \tag{4.33}$$
$$k_1^c(t) = \frac{\lambda_2^c(t)u_1 - \lambda_1^c(t)}{2}, \tag{4.34}$$
$$e_2^c(t) = \frac{a + p_2 + \lambda_3^c(t)}{b}, \tag{4.35}$$
$$k_2^c(t) = \frac{\lambda_4^c(t)u_2 - \lambda_3^c(t)}{2}. \tag{4.36}$$

With transversality conditions $\lambda_j^c(T) = 0$ as $T \to \infty$ ($j = 1, 2, 3, 4$), solving (4.25)–(4.28) gives:

$$\lambda_1^c = \frac{N_2\eta_1 + M_2N_1}{\eta_1\eta_2 - M_1M_2}, \tag{4.37}$$
$$\lambda_2^c = \frac{\beta_1}{2r} - \frac{(M_1N_2 + N_1\eta_2)b_1\eta_1}{(\eta_1\eta_2 - M_1M_2)2rM_1}, \tag{4.38}$$
$$\lambda_3^c = \frac{M_1N_2 + N_1\eta_2}{\eta_1\eta_2 - M_1M_2}, \tag{4.39}$$
$$\lambda_4^c = \frac{\beta_2}{2r} - \frac{(M_1N_2 + N_1\eta_2)b_2}{\eta_1\eta_2 - M_1M_2}. \tag{4.40}$$

The superscript c denotes cooperative equilibrium outcomes. Substituting (4.37)–(4.40) into (4.33)–(4.36) yields Proposition 2. □

4.3. Model extension (Investment compensation models)

The fundamental principle of the ecological compensation mechanism is predicated on the balancing of the rights, responsibilities, and gains/losses of the various stakeholders involved in ecological conservation through reasonable institutional design and resource allocation. This, in turn, serves to incentivize ecological conservation behaviors and promote the sustainable provision of ecological services. Its operational logic typically revolves around the principle of "who benefits, who compensates; who protects, who receives compensation": When the ecological protection efforts of a particular region or group generate ecological benefits for other regions or groups (such as improved water quality, enhanced air quality, or maintained biodiversity), or when they incur opportunity costs due to assuming ecological protection responsibilities, compensation is provided to the protecting party by the benefiting party or the responsible party through various forms of support, including financial assistance, technical aid, and policy preferences. This approach has been demonstrated to facilitate the harmonization of ecological protection and economic development, thereby ensuring the coordination of these two pivotal aspects across various regions and demographic groups. The investment compensation model is an extension of the non-cooperative open-loop framework.

Region 1 (downstream) now has an additional control variable $k_{12}(t)$ for investing in Region 2's abatement. Both regions still pre-commit to their strategy paths at time zero, solving their respective optimization problems simultaneously to reach an open-loop Nash equilibrium.

In the investment compensation model, the downstream region (Region 1) invests in the upstream region (Region 2) to mitigate local pollution and reduce transboundary pollution transfer. The abatement investment costs for Region 1 and Region 2 are:

$$C_{11}(k_{11}(t), A_{11}(t)) = k_{11}^2(t) - \beta_{11}A_{11}(t), \tag{4.41}$$

$$C_{12}(k_{12}(t), A_{12}(t), k_{11}(t), A_{11}(t)) = k_{12}^2(t) - \beta_{12}A_{12}(t) + k_{11}k_{12} - \beta_{11}\beta_{12}A_{11}(t), \tag{4.42}$$

where $k_{11}(t)$ is the pollution abatement investment of Region 1 in the local region, and $k_{12}(t)$ is the pollution abatement investment of Region 1 in Region 2.

Furthermore, the pollution stock dynamics for both regions evolve as follows:

$$\dot{s}_1(t) = e_1(t) - k_{11}(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) - b_{11}A_{11}(t) - b_{12}A_{12}(t), \tag{4.43}$$

$$\dot{s}_2(t) = e_2(t) - k_{12}(t) - k_{22}(t) - \eta_2 s_2(t) - \varphi s_1(t) + \eta_1 s_1(t) - b_{22}A_{22}(t). \tag{4.44}$$

Region 1 maximizes its instantaneous net profit under optimal emission and abatement investment paths. For brevity, we omit explicit time dependence t; thus, Region 1's objective function and constraints are:

$$\begin{aligned} W_1 = \max_{e_1,k_{11},k_{12}} \int_0^\infty \Big[& ae_1 - \frac{b}{2}e_1^2 + p_1(e_1 - e_{10}) - k_{11}^2 + \beta_{11}A_{11} - k_{12}^2 + \beta_{12}A_{12} - k_{11}k_{12} \\ & + \beta_{11}\beta_{12}A_{11} - \varepsilon_1^1 s_1 - \varepsilon_2^1 s_2 \Big] e^{-rt}dt, \end{aligned} \tag{4.45}$$

$$s.t. \begin{cases} \dot{s}_1(t) = e_1(t) - k_{11}(t) - \eta_1 s_1(t) - \varphi s_1(t) + \eta_2 s_2(t) - b_{11}A_{11}(t) - b_{12}A_{12}(t), \\ \dot{s}_2(t) = e_2(t) - k_{12}(t) - k_{22}(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) - b_{22}A_{22}(t), \\ \dot{A}_{11}(t) = u_{11}k_{11}(t) - rA_{11}(t), \\ \dot{A}_{12}(t) = u_{12}k_{12}(t) - rA_{12}(t). \end{cases} \tag{4.46}$$

Similarly, Region 2's objective function and constraints are:

$$W_2 = \max_{e_2,k_{11},k_{22}} \int_0^\infty \Big[ae_2 - \frac{b}{2}e_2^2 + p_2(e_2 - k_{12} - e_{20}) - k_{22}^2 + \beta_{22}A_{22} - \varepsilon_2^2 s_2 - \varepsilon_1^2 s_1 \Big] e^{-rt}dt, \tag{4.47}$$

$$s.t. \begin{cases} \dot{s}_2(t) = e_2(t) - k_{12}(t) - k_{22}(t) - \eta_2 s_2(t) - \varphi s_2(t) + \eta_1 s_1(t) - b_{22}A_{22}(t), \\ \dot{A}_{22}(t) = u_{22}k_{22}(t) - rA_{22}(t). \end{cases} \tag{4.48}$$

The current-value Hamiltonian for (4.45) is:

$$\begin{aligned} H_1 =& ae_1 - \frac{b}{2}e_1^2 + p_1(e_1 - e_{10}) - k_{11}^2 + \beta_{11}A_{11} - k_{12}^2 + \beta_{12}A_{12} - k_{11}k_{12} + \beta_{11}\beta_{12}A_{11} \\ & - \varepsilon_1^1 s_1 - \varepsilon_2^1 s_2 + \lambda_1(e_1 - k_{11} - \eta_1 s_1 - \varphi s_1 + \eta_2 s_2 - b_{11}A_{11} - b_{12}A_{12}) \\ & + \lambda_2(e_2 - k_{12} - k_{22} - \eta_2 s_2 - \varphi s_2 + \eta_1 s_1 - b_{22}A_{22}) + \lambda_3(u_{11}k_{11} - rA_{11}) + \lambda_4(u_{12}k_{12} - rA_{12}). \end{aligned} \tag{4.49}$$

The current-value Hamiltonian for (4.47) is:

$$
\begin{aligned}
H_2 = {} & ae_2 - \frac{b}{2}e_2^2 + p_2(e_2 - k_{12} - e_{20}) - k_{22}^2 + \beta_{22}A_{22} - \varepsilon_2^2 s_2 - \varepsilon_1^2 s_1 \\
& + \lambda_5(e_2 - k_{12} - k_{22} - (\eta_2 + \varphi)s_2 + \eta_1 s_1 - b_{22}A_{22}) + \lambda_6(u_{22}k_{22} - rA_{22}),
\end{aligned} \tag{4.50}
$$

where λ_j $(j = 1, 2, \ldots, 6)$ denotes adjoint variables associated with the state equations.

Proposition 3. *The optimal emission levels and abatement investment for the two regions under the investment compensation model are as follows:*

$$
e_1^* = \frac{a + p_1}{b} + \frac{\varepsilon_1^1(M_2 + \eta_2) + \varepsilon_2^1(M_1 + \eta_1)}{b(\eta_1\eta_2 - M_1M_2)}, \tag{4.51}
$$

$$
e_2^* = \frac{a + p_2}{b} - \frac{\varepsilon_2^2}{bM_2}, \tag{4.52}
$$

$$
\begin{aligned}
k_{11}^* = {} & \frac{1}{6r(\eta_1\eta_2 - M_1M_2)}[\varepsilon_1^1(8rM_2 - 4r\eta_2 - 2u_{11}b_{11}M_2 + 2u_{12}b_{12}M_2) \\
& + \varepsilon_2^1(8r\eta_1 - 4rM_1 - 2u_{11}b_{11}\eta_1 + u_{12}b_{12}\eta_1)] - \frac{u_{11}\beta_{11}(1 + \beta_{12}) - u_{12}\beta_{12}}{3r},
\end{aligned} \tag{4.53}
$$

$$
\begin{aligned}
k_{12}^* = {} & \frac{1}{6r(\eta_1\eta_2 - M_1M_2)}[\varepsilon_1^1(2rM_2 - 4r\eta_2 + u_{11}b_{11}M_2 + 2u_{12}b_{12}M_2) \\
& + \varepsilon_2^1(2r\eta_1 - 4rM_1 + u_{11}b_{11}\eta_1 + u_{12}b_{12}\eta_1)] - \frac{u_{11}\beta_{11}(1 + \beta_{12})}{6r} - \frac{u_{12}\beta_{12}}{3r},
\end{aligned} \tag{4.54}
$$

$$
k_{22}^* = \frac{\varepsilon_2^2(2r - u_{22}b_{22})}{4rM_2}. \tag{4.55}
$$

Proof. Maximizing (4.49) and (4.50) yields the first-order conditions:

$$
\frac{\partial H_1}{\partial e_1(t)} = a - be_1 + p_1 + \lambda_1 = 0, \tag{4.56}
$$

$$
\frac{\partial H_1}{\partial k_{11}(t)} = -2k_{11} - k_{12} - \lambda_1 + u_{11}\lambda_3 = 0, \tag{4.57}
$$

$$
\frac{\partial H_1}{\partial k_{12}(t)} = -2k_{12} - k_{11} - \lambda_2 + u_{12}\lambda_4 = 0, \tag{4.58}
$$

$$
\frac{\partial H_2}{\partial e_2(t)} = a - be_2 + p_2 + \lambda_5 = 0, \tag{4.59}
$$

$$
\frac{\partial H_2}{\partial k_{22}(t)} = -2k_{22} - \lambda_5 + u_{22}\lambda_6 = 0. \tag{4.60}
$$

The adjoint equations are:

$$
\dot{\lambda}_1 = r\lambda_1 - \frac{\partial H_1}{\partial s_1} = M_1\lambda_1 + \varepsilon_1^1 - \eta_1\lambda_2, \tag{4.61}
$$

$$
\dot{\lambda}_2 = r\lambda_2 - \frac{\partial H_1}{\partial s_2} = M_2\lambda_2 + \varepsilon_2^1 - \eta_2\lambda_1, \tag{4.62}
$$

$$
\dot{\lambda}_3 = r\lambda_3 - \frac{\partial H_1}{\partial A_{11}} = 2r\lambda_3 + b_{11}\lambda_1 - \beta_{11} - \beta_{11}\beta_{12}, \tag{4.63}
$$

$$\dot{\lambda}_4 = r\lambda_4 - \frac{\partial H_1}{\partial A_{12}} = 2r\lambda_4 + b_{12}\lambda_1 - \beta_{12}, \tag{4.64}$$

$$\dot{\lambda}_5 = r\lambda_5 - \frac{\partial H_2}{\partial s_2} = A_2\lambda_5 + \varepsilon_2^2, \tag{4.65}$$

$$\dot{\lambda}_6 = r\lambda_6 - \frac{\partial H_2}{\partial A_{22}} = 2r\lambda_6 + b_{22}\lambda_5. \tag{4.66}$$

With transversality conditions $\lambda_j^c(T) = 0$ as $T \to \infty$ ($j = 1, 2, 3, 4, 5, 6$), solving (4.61)–(4.66) yields the adjoint variables:

$$\lambda_1 = \frac{M_2\varepsilon_1^1 + \varepsilon_2^1\eta_1}{\eta_1\eta_2 - M_1M_2}, \tag{4.67}$$

$$\lambda_2 = \frac{M_1\varepsilon_2^1 + \varepsilon_1^1\eta_2}{\eta_1\eta_2 - M_1M_2}, \tag{4.68}$$

$$\lambda_3 = \frac{\beta_{11}}{2r}(1 + \beta_{12}) - \frac{b_{11}(M_2\varepsilon_1^1 + \varepsilon_2^1\eta_1)}{2r(\eta_1\eta_2 - M_1M_2)}, \tag{4.69}$$

$$\lambda_4 = \frac{\beta_{12}}{2r} - \frac{b_{12}(M_2\varepsilon_1^1 + \varepsilon_2^1\eta_1)}{2r(\eta_1\eta_2 - M_1M_2)}, \tag{4.70}$$

$$\lambda_5 = -\frac{\varepsilon_2^2}{M_2}, \tag{4.71}$$

$$\lambda_6 = -\frac{b_{22}\varepsilon_2^2}{2rM_2}. \tag{4.72}$$

Substituting $\lambda_i(t)$ into (4.56)–(4.60) yields Proposition 3. □

5. Numerical example

Section 4 derived optimal emission and investment levels for both regions under noncooperative and cooperative scenarios. To derive additional insights, we conduct numerical experiments using Chongqing Municipality (Region 1) and Sichuan Province (Region 2) in the Upper Yangtze River as case studies. This section analyzes emission levels, abatement investments, and pollution stock trajectories. We further examine how the learning rate of knowledge accumulation affects emissions, investments, pollution stocks, and net profits.

Carbon emission trading prices in Chongqing and Sichuan range from 24–28 Yuan/ton and 30–38 Yuan/ton, respectively. We select permit prices $p_1 = 24$ Yuan/ton and $p_2 = 34$ Yuan/ton. Given 2020 carbon emissions of 170 million tons (Chongqing) and 220 million tons (Sichuan), the initial pollution stock is set to $e_0 = 150$. Other parameter values are listed in Table 2.

Using these parameters, we compute emission levels and abatement investments for Chongqing and Sichuan under both scenarios (Table 3). Table 3 reveals that noncooperative emission levels exceed cooperative levels in both regions, indicating that cooperation reduces pollution. Sichuan's abatement investments are significantly higher than Chongqing's under both scenarios. Cooperative investments surpass noncooperative levels for both regions, implying enhanced abatement efforts under cooperation. Given that abatement investments directly reflect emission reduction efforts, Sichuan achieves superior abatement performance compared to Chongqing in both scenarios.

Table 2. Parameter values used in numerical simulation.

Symbol	φ	η_1	η_2	b_1	b_2	β_1	β_2	u_1	u_2	r	ε_1^1	ε_1^2	ε_2^1	ε_2^2	a	b
Value	0.1	0.05	0.065	0.01	0.013	1	1.3	0.5	0.55	0.08	0.2	0.15	0.26	0.18	2	4

Table 3. The emission levels and the abatement investment of the noncooperative and cooperative transboundary pollution problem.

Chongqing		Sichuan	
e_1^n	6.2826	e_2^n	8.8163
e_1^c	5.9927	e_2^c	8.4164
k_1^n	2.0109	k_2^n	2.6181
k_1^c	2.5959	k_2^c	3.4345

5.1. Pollution stock trajectories

Using fourth-order Runge-Kutta methods, we compute pollution stock trajectories for both scenarios (Figure 1). As illustrated in Figure 1, the dynamic evolution of pollution stock levels in Chongqing and Sichuan is depicted under two game scenarios: non-cooperation and cooperation. The figure's curves demonstrate that, irrespective of the region, the pollution stock curve under the cooperative scenario is consistently lower than that under the non-cooperative scenario. Furthermore, the discrepancy between the two curves increases over time, thereby underscoring the long-term benefits of cooperation in the context of transboundary pollution control. For Chongqing, at $t = 10$, the pollution stock under the cooperative path decreases more rapidly than under the non-cooperative path; a similar trend is exhibited by Sichuan. This finding suggests that regional coordination has the potential to not only mitigate the pollution peaks in each region but also expedite the rate of pollution reduction. Furthermore, the two cooperative curves tend to stabilize in the later stages, indicating that the system is gradually approaching a steady state. This outcome reflects the sustainability of pollution control under the cooperative mechanism. Conversely, the non-cooperative path curve demonstrates a more gradual decline and exhibits no indications of convergence, indicating that in the absence of coordination, pollution concerns may persist over an extended duration.

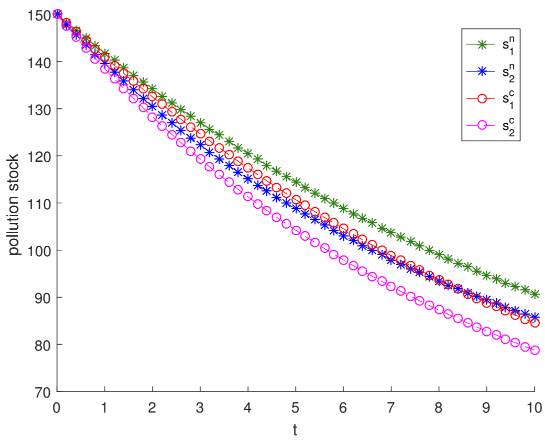


Figure 1. Pollution stock under noncooperative (n) and cooperative (c) scenarios.

5.2. *Net benefit analysis*

Figures 2 and 3 compare regional and total net benefits under both scenarios.

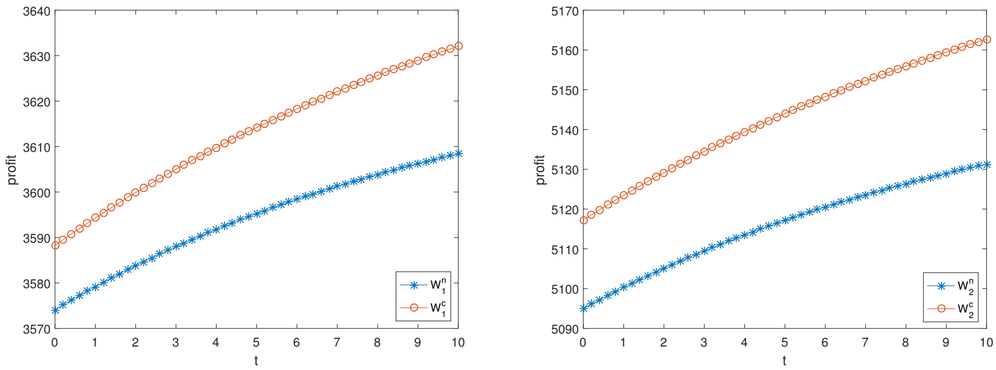


Figure 2. Net profits for Chongqing and Sichuan under n and c scenarios.

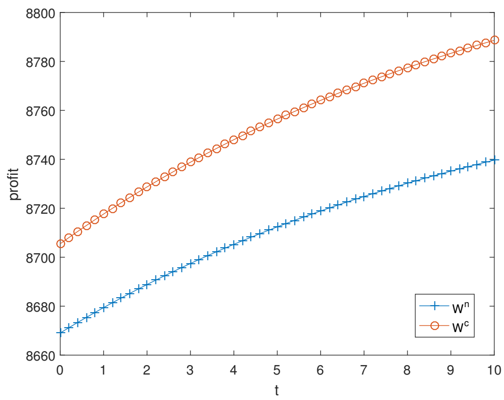


Figure 3. Total net profit under n and c scenarios.

Figures 2 and 3 illustrate the dynamic trends of interest distribution and overall benefits under different game scenarios. As illustrated in Figure 2, irrespective of the specific region, whether Chongqing or Sichuan, the cooperative line consistently maintains a position above the non-cooperation line. Both lines demonstrate an accelerating upward convex trajectory. This finding suggests that cooperation yields immediate benefits and also reduces future costs by decreasing synergistic emissions, thereby resulting in a cumulative increase in net benefits. Figure 3 combines the benefits of the two provinces, showing that the total cooperative benefit curve steadily rises, while the non-cooperative curve remains slightly lower with a gentler slope. The findings indicate that cooperative games internalize cross-border externalities, concurrently enhancing regional self-interest and overall welfare. This provides "win-win" evidence for basin ecological compensation mechanisms and clear value signals for policymakers. The findings also suggest that the earlier coordinated action is taken, the greater the benefits.

5.3. *Impact of the learning rate*

We analyze how the learning rate of knowledge accumulation (u_i, $i = 1, 2$) influences emissions, investments, pollution stocks, and net profits over time t. Emission levels remain unaffected by u_i under both scenarios.

As illustrated in Figure 4, an increase in the rates of knowledge accumulation, designated u_1 and u_2, results in a nearly linear rise in emissions reduction investments across both regions. Notably, the slope in the cooperative scenario exhibits a significantly greater increase compared to the non-cooperative scenario. This suggests that the combination of learning by doing and cooperative incentives leads to an increased willingness of both upstream Sichuan and downstream Chongqing to persist in their efforts to enhance pollution control measures.

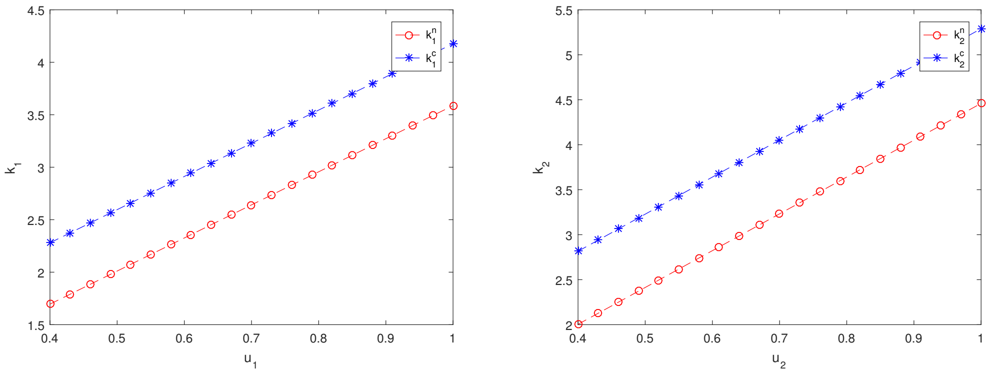
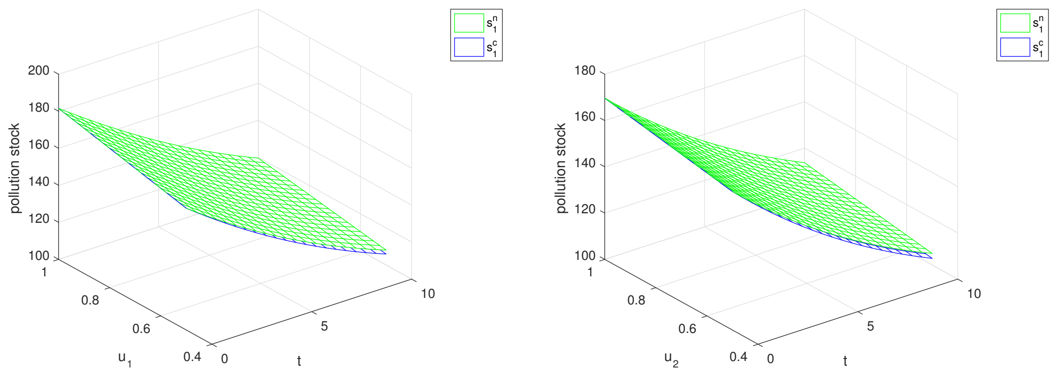


Figure 4. Effect of u_i on abatement investments in Chongqing and Sichuan.

Figures 5 and 6 provide a more detailed representation of the aforementioned investment increment as it pertains to the pollution stock. It is evident that as the parameter u increases from a value of 0.4 to 1, there is a concomitant shift in both pollution curves, resulting in a downward convergence at an accelerated rate. The cooperative path remains below the non-cooperative path. However, as the learning rate increases, the gap between the two paths first narrows and then stabilizes. This suggests that "learning faster" can partially offset the disadvantages of lacking coordination. However, it cannot fully replace institutional cooperation.



(a) Chongqing **(b)** Sichuan

Figure 5. Pollution stock dynamics under the noncooperative scenario.

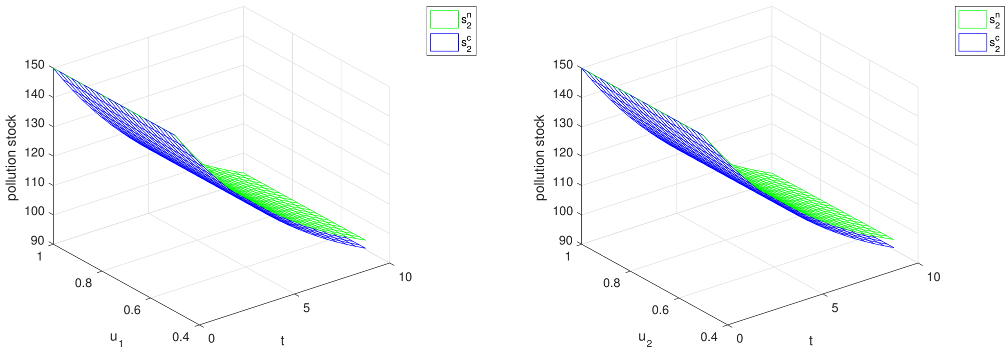


Figure 6. Effect of u_i on Sichuan's pollution stock under cooperation.

Figure 7 corroborates the aforementioned logic from the revenue perspective: the net revenue curves of the two provinces shift upward as u increases, with the Sichuan curve rising more than the Chongqing curve. This indicates that upstream regions benefit more significantly from "learning by doing," and explains why upstream regions are willing to accept financial and technological inputs from downstream regions in the ecological compensation mechanism.

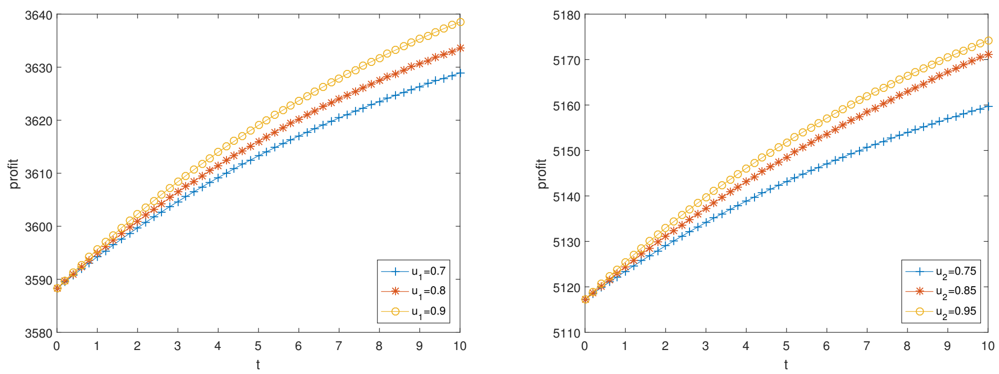


Figure 7. Effect of u_i on net profits in Chongqing and Sichuan under cooperation.

In summary, the learning rate emerges as a pivotal catalyst for concurrently augmenting regional emission reduction levels and benefits by diminishing marginal pollution control expenditures and enhancing investment efficiency. However, its impact is subject to a threshold. Once the learning dividend has been fully exploited, cooperative mechanisms are still required to ensure the long-term benefits are secured and to prevent "free-riding" behavior from reemerging.

5.4. The investment compensation

We conduct numerical experiments to analyze transboundary pollution under ecological compensation using the same parameters as previously defined.

Table 4 shows that implementing ecological compensation reduces Chongqing's emission level below both cooperative and noncooperative scenarios. However, Sichuan's emission level exceeds that of both regions under cooperation. This confirms the mechanism's effectiveness in reducing emissions specifically in the downstream region (Chongqing).

Table 4. The emission levels and the abatement investment of the noncooperative and cooperative transboundary pollution problem.

Chongqing		Sichuan	
e_1^n	6.2826	e_2^n	8.8163
e_1^c	5.9927	e_2^c	8.4164
e_1^*	5.8635	e_2^*	8.8163

As illustrated in Figure 8, Chongqing, in its capacity as the compensating party, exhibits a substantially higher local emissions reduction investment, designated as k_{11}, within the ecological compensation scenario (k_1^*) as compared to the pure cooperation (k_1^c) and non-cooperation (k_1^n) scenarios. Sichuan, as the compensated party, has k_2^* and k_2^n that are lower than under the cooperation scenario, reflecting a structural change characterized by "more investment downstream and less investment upstream." As the learning rates u_1 and u_2 increase, both investment curves shift upward to the right. However, Chongqing's slope is steeper, indicating that the compensation mechanism amplifies the positive feedback of "the faster you learn, the more you invest" in the downstream region. In contrast, the upstream region reduces its reliance on local investment due to the acquisition of funds and technology transfer.

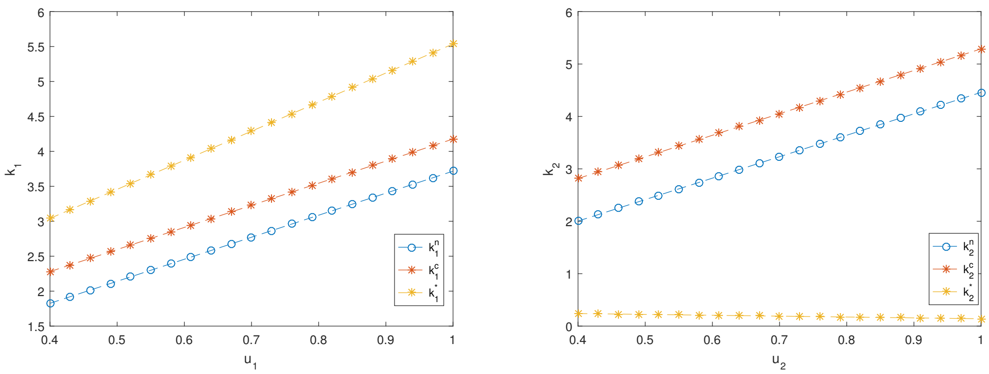


Figure 8. Sensitivity analysis of the abatement investment level to u_i.

Figure 9 focuses on Chongqing's pollution stock s_1. The curve under the compensation scenario exhibits the lowest overall value and undergoes an acceleration downward as u_1 and u_2 increase. As the parameter u_1 increases from 0.4 to 1, the parameter s_1^* experiences a decrease of approximately 10% compared to s_1^c and nearly 30% compared to s_1^n after 10 periods. This indicates that the dual mechanism of "learning + compensation" significantly reduces the cycle required for pollution control to take effect. It is noteworthy that the supplementary enhancement in s_1 resulting from an augmentation in u_2 gradually diminishes, indicating that the spillover effects of upstream learning on downstream regions adhere to a law of diminishing marginal returns.

As illustrated in Figure 10, the mirrored results for Sichuan's pollution stock s_2 demonstrate a slightly higher value under the compensation scenario, a value that is slightly lower under the cooperation scenario, and a significantly lower value under the non-cooperation scenario. As u_1 and u_2 increase, the gap between s_2^* and s_2^c rapidly converges, indicating that once upstream regions improve their learning efficiency, the "investment slack" brought about by compensation will be offset more quickly. When $u_2 > 0.8$, the three curves decline almost parallel to each other, indicating that a high learning rate can compensate for institutional differences, enabling the upstream region to achieve similar emission reduction effects even with reduced investment.

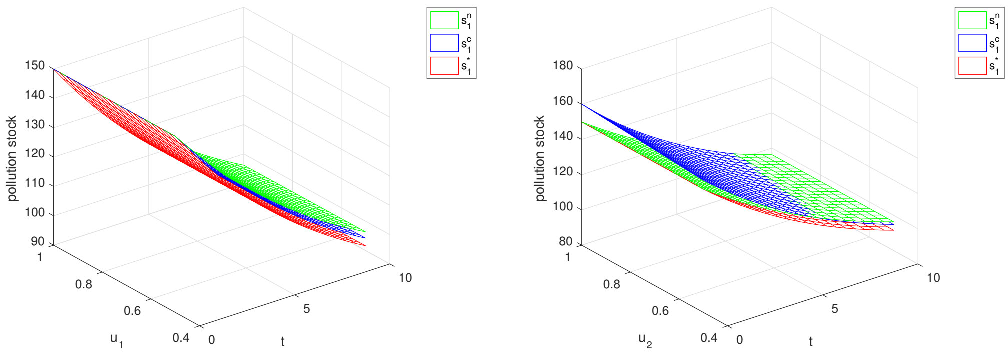


Figure 9. The effects of u_i on Chongqing Municipality's pollution stocks under the ecological compensation mechanism.

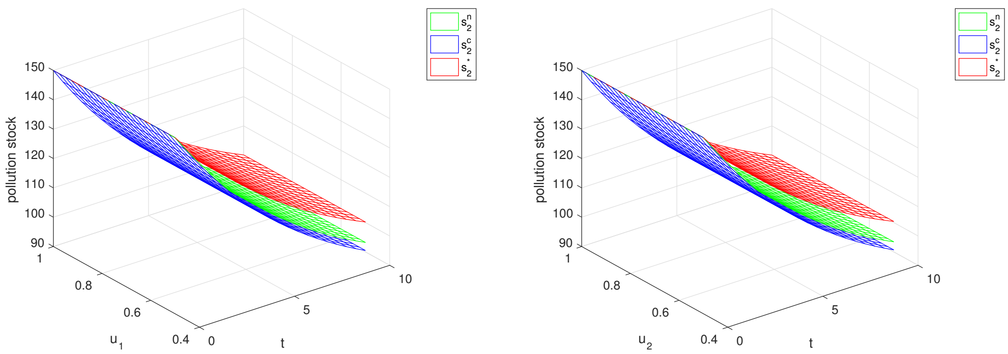


Figure 10. The effects of u_i on Sichuan Province's pollution stocks under the ecological compensation mechanism.

A comprehensive analysis of Figures 8–10 reveals that ecological compensation, achieved through the cross-regional reallocation of financial resources and technological capabilities, differentiates the value of learning rates and "discounts" them into investment and pollution stock. Downstream regions, propelled by elevated learning rates, assume a heightened responsibility for emissions reduction, while upstream regions leverage technology spillovers to sustain emissions reduction performance with reduced investment. This development ultimately gives rise to a novel basin-wide collaborative model, one that is distinguished by a shift in which downstream regions assume a greater investment role, while upstream regions adopt a more proactive stance in their learning and adaptation processes.

6. Conclusions and managerial insights

6.1. Conclusions

Comparative analysis of three game scenarios yields four key findings regarding transboundary pollution management in river basins:

(i) Coordination reduces regional emissions. Under both noncooperative and cooperative scenarios, Sichuan's optimal emissions exceed Chongqing's. Introducing ecological compensation further reduces Chongqing's emissions below cooperative levels but increases Sichuan's emissions relative to cooperation. Higher net income correlates with higher emissions, whereas higher environmental damage costs reduce emissions. Consequently, declining emissions lower regional pollution stocks.

(ii) Without ecological compensation, cooperative abatement investments exceed noncooperative levels. After implementation, Chongqing's investment increases while Sichuan's decreases. This occurs because downstream Chongqing compensates upstream Sichuan through investments.

(iii) Pre-compensation, cooperation yields significantly lower pollution stocks than noncooperation. Post-implementation, Chongqing's stocks follow this pattern, but Sichuan's stocks increase relative to other models.

(iv) Cooperation increases total net income compared to noncooperation. Introducing ecological compensation in noncooperative games further boosts regional net income beyond cooperative levels. Given policy, informational, and technological constraints, noncooperative strategies prevail in practice. Thus, ecological compensation standards are vital for balancing regional interests during transboundary pollution events.

6.2. Managerial insights

First, the incorporation of "learning speed" into the compensation rules is imperative. The central government is poised to assume a leadership role in the establishment of a green fund for the upper reaches of the Yangtze River. The calculation of compensation amounts will undergo a shift in methodology. The current practice of basing compensation solely on emissions volume will be replaced by a new approach. The new approach will incorporate the knowledge accumulation rate (u value) of Chongqing and Sichuan as a dynamic coefficient. This coefficient will serve to allocate rewards to regions based on their rate of learning. Regions that demonstrate faster rates of learning will receive higher compensation, while those with slower learning rates will receive lower compensation. This financial incentive is expected to catalyze ongoing technological enhancements.

Second, the facilitation of technological dissemination is imperative. The establishment of an interprovincial "pollution control technology marketplace" is imperative, utilizing blockchain technology to meticulously record each technology transfer and emission reduction outcome. The additional environmental taxes collected in the downstream region of Chongqing will be directly returned to the upstream region of Sichuan, contingent on the volume of technology shared. This establishes a closed-loop system in which "downstream invests more, upstream learns more," thereby reducing transaction costs caused by information asymmetry.

Finally, the introduction of a "learning lever" into the market is imperative. Within the existing carbon emissions trading system, the introduction of tradable "learning credits" is recommended. Enterprises would earn credits for each certified emissions reduction technology upgrade, which would be bundled with emissions quotas for trading. The determination of credit prices is based on market forces, thereby generating novel revenue streams for enterprises and incentivizing local governments to proactively enhance regional learning rates. This approach fosters the establishment of a "dual-drive" model of government compensation and market incentives.

6.3. Research limitations

Despite its theoretical contributions and policy implications, the present study is not without its limitations. These limitations include the following: model simplification (the use of a symmetric two-region framework, which fails to reflect the complex multi-regional interactions and asymmetric economic conditions in real-world river basins), static learning rates (learning rates are exogenous and constant), and the failure to consider policy implementation costs (such as transaction costs and administrative burdens). Future research could explore directions such as multi-regional expansion, incorporating random and uncertain factors, establishing endogenous learning mechanisms, and integrating behavioral and policy factors to refine the theoretical framework and enhance its practicality.

Author contributions

Zuliang Lu: Writing-review & editing, Supervision, Funding acquisition; Mingsong Li: Writing-original draft, Software, Methodology; Longzhou Cao: Funding acquisition, Methodology, Investigation; Junman Li: Software, Formal analysis; Zhihui Cao: Formal analysis, Conceptualization; Zhuran Xiang: Validation, Software. All authors have read and approved the final version of the manuscript for publication.

Use of Generative-AI tools declaration

The authors declare that they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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