Mathematics

https://www.aimspress.com/journal/Math

AIMS Mathematics, 10(10): 24093–24114.
DOI: 10.3934/math.20251069
Received: 17 August 2025
Revised: 06 October 2025
Accepted: 15 October 2025
Published: 22 October 2025

Research article

Fixed-time synchronization of quaternion-valued memristive neural networks with time-varying delays and impulsive effect

Hong Zhou[1] **and Jie Zhou**[2,*]

[1] School of Public Basic, Jiangxi Modern Polytechnic College, Nanchang, China
[2] School of Mathematics and Statistics, Southwest University, Chongqing, China

* **Correspondence:** Email: zj3604240721@163.com.

Abstract: This study explored the fixed-time synchronization (FTS) problem for quaternion-valued memristive neural networks (QVMNNs) incorporating time-varying delays and impulsive disturbances. Initially, we established a model for QVMNNs incorporating these characteristics, namely time-varying delays, memristor behavior, impulsive effects, and quaternion-valued neural network properties. Given that QVMNNs do not obey the commutative law of multiplication, we decomposed them into four real-valued memristor neural networks for analysis. Second, due to the potential disruption of instantaneous impulsive disturbances to neural network synchronization, it was crucial to design an appropriate controller to manage these effects. Therefore, we devised a suitable feedback controller to effectively regulate the system. Next, we established the FTS conditions for QVMNNs using fixed-time stability theory combined with multiple inequality methods. Numerical experiments were performed to validate the theoretical results, demonstrating the effectiveness of our approach.

Keywords: fixed-time synchronization; quaternion-valued memristive neural networks; time-varying delays; impulsive effect
Mathematics Subject Classification: 92B20

1. Introduction

It was not until Chua's seminal work [1] that the memristor—now recognized as an indispensable circuit element—was formally introduced. The subsequent realization of a functional memristive device by HP researchers in 2008 [2] demonstrated no lack of potential, particularly in mimicking synaptic memory. Because of this characteristic, it can be used as a synapse to better modify the human brain [3]. Pattern recognition and data processing represent just two of the many research domains where memristor applications have been extensively studied [4].

Memristors are put in neural networks as the connection weight to form the memristive neural networks (MNNs) in recent years. They have attracted the attention of many scholars [5]. The majority of the existing studies have concentrated on real-valued memristor neural networks (RVMNNs) and complex-valued memristor neural networks (CVMNNs), with QVMNNs remaining largely unexplored in the literature [6]. Compared with RVMNNs and CVMNNs, QVMNNs are not satisfied with the commutative law of multiplication, as they have more complex dynamic behavior [7]. In [8], the authors established a novel QVMNN model to study FTS by designing a parameter independent controller. Reference [9] provided a comprehensive examination of exponential input-to-state stability in QVMNNs. Meanwhile, QVMNNs demonstrate significant practical utility across multiple domains including satellite attitude regulation, graphical processing, and low-light vision enhancement [10].

Neural network synchronization has emerged as a prominent research focus, with various synchronization schemes being actively studied, including complete, exponential, and projective synchronization [11]. While most synchronization schemes operate over an infinite time horizon, finite-time synchronization has more important application value in practice. The synchronization error in finite-time synchronization is primarily determined by the system's initial conditions [12]. Typically, larger initial values result in longer settling times. To address this limitation, Polyakov [13] introduced FTS, guaranteeing a bounded convergence time independent of initial conditions. In the domain of discontinuous neural networks, the issues of FTS and fixed-time stability have been thoroughly addressed in [14]. Until now, the synchronization schemes with impulsive control mainly focus on infinite time, and few results are about FTS [15].

In contemporary control literature, multiple sophisticated synchronization strategies have been effectively implemented, including but not limited to: sliding mode control [16], event-based control [17], adaptive control schemes [18], and robust control approaches [19]. Impulsive effect is an unstable factor of neural networks, and its sudden change may destroy the synchronization of the system. Unlike continuous control approaches, the impulsive control strategy operates discontinuously, requiring control signals only at discrete time instants while enabling instantaneous state adjustments in the slave system [20]. The impulsive control scheme significantly reduces control costs and simplifies implementation, since continuous state information is not required [21]. What remains crucially important is investigating how impulsive control affects synchronization in QVMNNs [22].

Most existing works on FTS focus on real-valued or complex-valued neural networks. However, research on QVMNNs, which offer a more compact and geometrically meaningful representation for multi-dimensional data, is still scarce. Recent studies on QVMNNs have advanced their application in high-dimensional data processing [10] and stability analysis under impulsive effects [23]. The non-commutativity of quaternion multiplication introduces significant challenges that cannot be directly addressed by methods designed for real or complex-valued systems. The physical motivation for studying QVMNNs lies in their superior capability to model and process high-dimensional data with inherent three- or four-dimensional structure—such as 3D rotation, color imagery (e.g., RGB), or complex signals—as a single, cohesive entity, which is inefficiently handled by real or complex-valued networks. The memristor component provides synaptic-like memory, enhancing biological plausibility and adaptive learning. FTS is critical for ensuring these networks achieve coordinated behavior within a guaranteed, predefined time, a strict requirement for time-sensitive applications like secure communications, networked robotic systems, and financial networks where convergence speed and predictability are paramount.

Motivated by existing research gaps, the present study examines FTS in QVMNNs subject to time-varying delays and impulsive disturbances. The key innovations include: (i) While previous studies often address memristive neural networks in real or complex domains, or ignore the combined effects of time-varying delays and impulsive disturbances, our work presents a unified quaternion-valued model that simultaneously incorporates time-varying delays, impulsive phenomena, and memristive characteristics. This integrated approach offers a more accurate representation of real-world systems, such as high-dimensional data processing and dynamic systems subject to sudden changes. To the best of our knowledge, this is one of the first attempts to address FTS under such a comprehensive framework. (ii) We have constructed a novel feedback controller specifically designed for the QVMNNs with time-varying delays and impulsive effects. The proposed controller is not a straightforward extension of real or complex-valued cases but carefully accounts for the non-commutativity of quaternion multiplication and the dynamic properties of memristors. This design enables efficient synchronization control without requiring excessive control gains. (iii) One of the most significant innovations lies in our theoretical analysis. By employing fixed-time stability theory and advanced inequality techniques (such as Lyapunov functions and matrix inequality methods), we have derived original and explicit upper bounds for the synchronization time. Importantly, these results are generally applicable not only to QVMNNs but also to RVMNNs and CVMNNs. This generalizability underscores the broader impact of our theoretical findings.

The remainder of this paper is structured as follows: Section 2 introduces necessary preliminaries and develops the mathematical framework. Section 3 provides the core theoretical contributions on QVMNNs' FTS. Section 4 presents computational experiments to verify the theoretical analysis, while Section 5 concludes the paper.

2. Preliminaries and model description

The following mathematical conventions are used throughout our analysis: R (real numbers), C (complex numbers), and Q (quaternions) represent their respective number systems. We define $C([-\rho, 0]; R^n)$ as the Banach space of continuous R^n-valued functions on $[-\rho, 0]$ with the norm $\| \Phi \| = \sup_{-\rho\leq\eta\leq 0} |\Phi(\eta)|$. The notation $co\{q_1, q_2\}$ stands for the convex closure of the set containing q_1 and q_2.

A quaternion, belonging to the hypercomplex number system, comprises one real component and three distinct imaginary components. For any quaternion $u \in Q$, it can be represented as

$$u = u^R + u^I i + u^J j + u^K k,$$

where $u^R, u^I, u^J, u^K \in R$, and the imaginary units i, j, k satisfy the Hamilton operation rule:

$$i^2 = j^2 = k^2 = -1,\ ij = -ji = k,\ jk = -kj = i,\ ki = -ik = j.$$

Remark 1. *Not obeying the commutative law is quaternion multiplication, which differs fundamentally from its traditional counterpart. For any $u, v \in Q$, $uv = vu$ cannot be guaranteed. Consequently, certain advantageous properties that hold in real and complex fields fail to extend to the quaternion domain, traditional methods cannot be directly applied to QVMNNs, so it is necessary to establish a new theory to deal with QVMNNs.*

The expression $\overline{u} = u^R - u^I i - u^J j - u^K k$ denotes the conjugate transpose of u, and its module is defined as

$$|u| = \sqrt{\overline{u}u} = \sqrt{(u^R)^2 + (u^I)^2 + (u^J)^2 + (u^K)^2}.$$

For any quaternions $v = v^R + v^I i + v^J j + v^K k$ and $w = w^R + w^I i + w^J j + w^K k$, their addition is defined as

$$v + w = v^R + w^R + (v^I + w^I)i + (v^J + w^J)j + (v^K + w^K)k.$$

According to the Hamilton operation rule, multiplication is defined as

$$\begin{aligned} vw =&(v^R w^R - v^I w^I - v^J w^J - v^K w^K) \\ &+ (v^R w^I + v^I w^R + v^J w^K - v^K w^J)i \\ &+ (v^R w^J + v^J w^R + v^K w^I - v^I w^K)j \\ &+ (v^R w^k + v^K w^R + v^I w^J - v^J w^I)k. \end{aligned}$$

In our analysis, the drive system is represented by a QVMNN model incorporating both time-varying delays and impulsive disturbances:

$$\begin{cases} \dot{x}_m(l) = - d_m x_m(l) + \sum\limits_{n=1}^{t} a_{mn}(x_m(l)) f_n(x_n(l)) \\ \qquad + \sum\limits_{n=1}^{t} b_{mn}(x_m(l)) g_n(x_n(l - \psi(l))) + I_m, l \geq 0, l \neq l_k, \\ \Delta x_m(l_k) = \mu x_m(l_k^-),\ l = l_k,\ k \in \mathbb{N}^+, \end{cases} \tag{2.1}$$

with $m = 1, 2, \ldots, t$, the state variable $x_m(l) \in \mathbb{Q}$ characterizes the m-th neuron's activity at time l, $f_m(\cdot)$ and $g_m(\cdot) \in \mathbb{Q}$ represent activation functions, $d_p > 0, \psi(l)$ is the time-varying delay, $a_{mn}(\cdot)$ and $b_{mn}(\cdot) \in \mathbb{Q}$ represent memristor-based weights, and $I_m \in \mathbb{Q}$ denotes the external input vector. $\{l_1, l_2, \ldots\}$ represents a series of monotonically increasing impulsive times. $\mu \in R$ is the strength of the impulses, and we suppose $x_m(l)$ is right continuous at $l = l_k$, $\Delta x_m(l_k) = x_m(l_k^+) - x_m(l_k^-)$.

Time-varying delays are incorporated into the model to reflect the more realistic scenario of signal transmission lags between neurons, which are often not constant but vary with time due to fluctuating network congestion or switching speeds. This makes our model a more accurate representation of practical hardware implementations. Impulsive disturbances are introduced to model sudden, abrupt state changes that can occur in a network. These represent a range of real-world phenomena, such as instantaneous external interference, sudden component failures, or the application of resetting control inputs at discrete instants.

Assumption 1. *Let $x_m = x_m^R + x_m^I i + x_m^J j + x_m^K k$, and the activation functions $f_n(x_n)$ and $g_n(x_n)$, being Lipschitz continuous, admit the following representation:*

$$f_n(x_n) = f_n^R(x_n^R) + f_n^I(x_n^I)i + f_n^J(x_n^J)j + f_n^K(x_n^K)k,$$

$$g_n(x_n) = g_n^R(g_n^R) + g_n^I(x_n^I)i + g_n^J(g_n^J)j + g_n(x_n^K)k.$$

Given any x_m^δ, y_m^δ, there are positive constants $\phi_m^\delta, \varphi_m^\delta$ for which the following holds:

$$|f_m^\delta(x_m^\delta) - f_m^\delta(y_m^\delta)| \leq \phi^\delta |x_m^\delta - y_m^\delta|,$$

$$|g_m^\delta(x_m^\delta) - g_m^\delta(y_m^\delta)| \leq \varphi^\delta |x_m^\delta - y_m^\delta|.$$

Remark 2. *Assumption 1 ensures both existence and uniqueness of the system solution. Notably, QVMNNs represent a generalized framework that encompasses RVMNNs and CVMNNs as specific instances, consequently exhibiting more sophisticated dynamical characteristics. For analytical tractability, we constrain the activation function such that each imaginary component depends solely on its corresponding dimension.*

Utilizing the properties of quaternion multiplication leads to the decomposition of system (2.1) into four components:

$$\begin{cases} \dot{x}_m^R(l) = -d_m x_m^R(l) + \sum_{n=1}^{t} a_{mn}^R(x_m^R(l)) f_n^R(x_n^R(l)) \\ \qquad - \sum_{n=1}^{t} a_{mn}^I(x_m^I(l)) f_n^I(x_n^I(l)) - \sum_{n=1}^{t} a_{mn}^J(x_m^J(l)) f_n^J(x_n^J(l)) \\ \qquad - \sum_{n=1}^{t} a_{mn}^K(x_m^K(l)) f_n^K(x_n^K(l)) + \sum_{n=1}^{t} b_{mn}^R(x_m^R(l)) g_n^R(x_n^R(l-\psi(l))) \\ \qquad - \sum_{n=1}^{t} b_{mn}^I(x_m^I(l)) g_n^I(x_n^I(l-\psi(l))) - \sum_{n=1}^{t} b_{mn}^J(x_m^J(l)) g_n^J(x_n^J(l-\psi(l))) \\ \qquad - \sum_{n=1}^{t} b_{mn}^K(x_m^K(l)) g_n^K(x_n^K(l-\psi(l))) + I_m^R, l \geq 0, l \neq l_k, \\ x_m^R(l_k) = (1+\mu) x_m^R(l_k^-), l = l_k, k \in \mathbb{N}^+. \end{cases} \tag{2.2}$$

In the same way, the other three parts can be easily obtained through calculation. The discontinuous characteristics of $a_{mn}(\cdot)$ and $b_{mn}(\cdot)$ necessitate employing Filippov's solution concept for system (2.1). Utilizing methodologies from both set-valued analysis and differential inclusion theory, we obtain the following representation:

$$\begin{cases} \dot{x}_m^R(l) \in -d_m x_m^R(l) + \sum_{n=1}^{t} \lambda(a_{mn}^R(x_m^R(l))) f_n^R(x_n^R(l)) \\ \qquad - \sum_{n=1}^{t} \lambda(a_{mn}^I(x_m^I(l))) f_n^I(x_n^I(l)) - \sum_{n=1}^{t} \lambda(a_{mn}^J(x_m^J(l))) f_n^J(x_n^J(l)) \\ \qquad - \sum_{n=1}^{t} \lambda(a_{mn}^K(x_m^K(l))) f_n^K(x_n^K(l)) + \sum_{n=1}^{t} \lambda(b_{mn}^R(x_m^R(l))) g_n^R(x_n^R(l-\psi(l))) \\ \qquad - \sum_{n=1}^{t} \lambda(b_{mn}^I(x_m^I(l))) g_n^I(x_n^I(l-\psi(l))) - \sum_{n=1}^{t} \lambda(b_{mn}^J(x_m^J(l))) g_n^J(x_n^J(l-\psi(l))) \\ \qquad - \sum_{n=1}^{t} \lambda(b_{mn}^K(x_m^K(l))) g_n^K(x_n^K(l-\psi(t))) + I_p^R, l \geq 0, l \neq l_k, \\ x_m^R(l_k) = (1+\mu) x_m^R(l_k^-), \ l = l_k, \ k \in \mathbb{N}^+. \end{cases}$$

In the same way, we can get the other three parts, which we will not describe here. The memristor weights are defined as:

$$\lambda(a_{mn}^R(x^R(l))) = \begin{cases} \acute{a}_{mn}^R, & |x^R(l)| < \Upsilon_m, \\ co\{\acute{a}_{mn}^R, \grave{a}_{mn}^R\}, & |x^R(l)| = \Upsilon_m, \\ \grave{a}_{mn}^R, & |x^R(l)| > \Upsilon_m, \end{cases} \quad \lambda(b_{mn}^R(x^R(l))) = \begin{cases} \acute{b}_{mn}^R, & |x^R(l)| \leq \Upsilon_m, \\ co\{\acute{b}_{mn}^R, \grave{b}_{mn}^R\}, & |x^R(l)| = \Upsilon_m, \\ \grave{b}_{mn}^R, & |x^R(l)| > \Upsilon_m, \end{cases}$$

$$\lambda(a_{mn}^I(x^I(l))) = \begin{cases} \acute{a}_{mn}^I, & |x^I(l)| \leq \Upsilon_m, \\ co\{\acute{a}_{mn}^I, \grave{a}_{mn}^I\}, & |x^I(l)| = \Upsilon_m, \\ \grave{a}_{mn}^I, & |x^I(l)| > \Upsilon_m, \end{cases} \quad \lambda(b_{mn}^I(x^I(l))) = \begin{cases} \acute{b}_{mn}^I, & |x^I(l)| \leq \Upsilon_m, \\ co\{\acute{b}_{mn}^I, \grave{b}_{mn}^I\}, & |x^I(l)| = \Upsilon_m, \\ \grave{b}_{mn}^I, & |x^I(l)| > \Upsilon_m, \end{cases}$$

$$\lambda(a_{mn}^J(x^J(l))) = \begin{cases} \acute{a}_{mn}^J, & |x^J(l)| \leq \Upsilon_m, \\ co\{\acute{a}_{mn}^J, \grave{a}_{mn}^J\}, & |x^J(l)| = \Upsilon_m, \\ \grave{a}_{mn}^J, & |x^J(l)| > \Upsilon_m, \end{cases} \quad \lambda(b_{mn}^J(x^J(l))) = \begin{cases} \acute{b}_{mn}^J, & |x^J(l)| \leq \Upsilon_m, \\ co\{\acute{b}_{mn}^J, \grave{b}_{mn}^J\}, & |x^J(l)| = \Upsilon_m, \\ \grave{b}_{mn}^J, & |x^J(l)| > \Upsilon_m, \end{cases}$$

$$\lambda(a_{mn}^K(x^K(l))) = \begin{cases} \acute{a}_{mn}^K, & |x^K(l)| \leq \Upsilon_m, \\ co\{\acute{a}_{mn}^K, \grave{a}_{mn}^K\}, & |x^K(l)| = \Upsilon_m, \\ \grave{a}_{mn}^K, & |x^K(l)| > \Upsilon_m, \end{cases} \quad \lambda(b_{mn}^K(x^K(l))) = \begin{cases} \acute{b}_{mn}^K, & |x^K(l)| \leq \Upsilon_m, \\ co\{\acute{b}_{mn}^K, \grave{b}_{mn}^K\}, & |x^K(l)| = \Upsilon_m, \\ \grave{b}_{mn}^K, & |x^K(l)| > \Upsilon_m, \end{cases}$$

where $\acute{a}_{mn}^\delta, \grave{a}_{mn}^\delta, \acute{b}_{mn}^\delta, \grave{b}_{mn}^\delta$ are the values of the corresponding memristors, $\Upsilon_m > 0$ is the threshold level, so we have $\bar{a}_{mn}^\delta(x_n^\delta(l)) \in \lambda(a_{mn}^\delta(x^\delta(l))),\ \bar{b}_{mn}^\delta(x_n^\delta(l)) \in \lambda(b_{mn}^\delta(x^\delta(l))),\ and\ \delta = R, I, J, K,\ mn = 1, 2, \ldots, t$ such that

$$\begin{cases} \dot{x}_m^R(l) = -d_m x_m^R(l) + \sum_{n=1}^t \bar{a}_{mn}^R(x_n^R(l)) f_n^R(x_n^R(l)) \\ \qquad - \sum_{n=1}^t \bar{a}_{mn}^I(x_n^I(l)) f_n^I(x_n^I(l)) - \sum_{n=1}^t \bar{a}_{mn}^J(x_n^J(l)) f_n^J(x_n^J(l)) \\ \qquad - \sum_{n=1}^t \bar{a}_{mn}^K(x_n^K(l)) f_n^K(x_n^K(l)) + \sum_{n=1}^t \bar{b}_{mn}^R(x_n^R(l)) g_n^R(x_n^R(l - \psi(l))) \\ \qquad - \sum_{n=1}^t \bar{b}_{mn}^I(x_n^I(l)) g_n^I(x_n^I(l - \psi(l))) - \sum_{n=1}^t \bar{b}_{mn}^J(x_n^J(l)) g_n^J(x_n^J(l - \psi(l))) \\ \qquad - \sum_{n=1}^t \bar{b}_{mn}^K(x_n^K(l)) g_n^K(x_n^K(l - \psi(l))) + I_m^R, l \geq 0, l \neq l_k, \\ x_m^R(l_k) = (1 + \mu) x_m^R(l_k^-),\ l = l_k,\ k \in \mathbb{N}^+. \end{cases} \tag{2.3}$$

In the same way, we can get the other three parts, which we will not describe here. The following is the corresponding slave system (2.4):

$$\begin{cases} \dot{y}_m(l) = - d_m y_m(l) + \sum_{n=1}^{t} a_{mn}(y_m(l)) f_n(y_n(l)) \\ \qquad + \sum_{n=1}^{t} b_{mn}(y_m(l)) g_n(y_n(l - \psi(l))) + I_m + \nu_m, l \geq 0, l \neq l_k, \\ y_m(l_k) = (1+\mu) y_m(l_k^-),\ l = l_k,\ k \in \mathbb{N}^+. \end{cases} \tag{2.4}$$

Let $\nu_p \in \mathbb{Q}$ denote the feedback controller, which enables us to reformulate system (2.1) as:

$$\begin{cases} \dot{y}_m^R(l) = - d_m y_m^R(l) + \sum_{n=1}^{t} \tilde{a}_{mn}^R(y_n^R(l)) f_n^R(y_n^R(l)) \\ \qquad - \sum_{n=1}^{t} \tilde{a}_{mn}^I(y_n^I(l)) f_n^I(y_n^I(l)) - \sum_{n=1}^{t} \tilde{a}_{mn}^J(y_n^J(l)) f_n^J(y_n^J(l)) \\ \qquad - \sum_{n=1}^{t} \tilde{a}_{mn}^K(y_n^K(l)) f_n^K(y_n^K(l)) + \sum_{n=1}^{t} \tilde{b}_{mn}^R(y_n^R(l)) g_n^R(y_n^R(l - \psi(l))) \\ \qquad - \sum_{n=1}^{t} \tilde{b}_{mn}^I(y_n^I(l)) g_n^I(y_n^I(l - \psi(l))) - \sum_{n=1}^{l} \tilde{b}_{pq}^J(y_q^J(l)) g_q^J(y_q^J(l - \psi(l))) \\ \qquad - \sum_{n=1}^{t} \tilde{b}_{mn}^K(y_n^K(l)) g_n^K(y_n^K(l - \psi(l))) + I_m^R + \nu_m^R,\ l \geq 0,\ l \neq l_k. \end{cases} \tag{2.5}$$

We only provided the derivation of the real part, and similarly, the other three parts can be obtained. Let $e_p(t) = y_p(t) - x_p(t)$ represent the synchronization error, from which we derive the error system in the following form:

$$\begin{cases} \dot{e}_m(l) = - d_m e_m(l) + \sum_{n=1}^{t} \hat{a}_{mn}(e_n(l)) f_n(e_n(l)) \\ \qquad + \sum_{n=1}^{t} \hat{b}_{mn}(e_n(l)) g_n(e_n(l - \psi(l))) + I_m + \nu_m, l \geq 0, l \neq l_k, \\ e_m(l_k) = (1+\mu) e_m(l_k^-),\ l = l_k,\ k \in \mathbb{N}^+, \end{cases} \tag{2.6}$$

where $\hat{a}_{mn}(e(l))) \in \lambda(a_{mn}(e(l)))$, $\hat{b}_{mn}(e(l))) \in \lambda(b_{mn}(e(l)))$,

$$\begin{aligned} \hat{a}_{mn}(e(l))) f_n(e_n(l)) &= \tilde{a}_{mn}(y_n(l)) f_n(y_n(l)) - \bar{a}_{mn}(x_n(l)) f_n(x_n(l)), \\ \hat{b}_{mn}(e(l))) g_n(e_n(l - \psi(l))) &= \tilde{b}_{mn}(y_n(l)) g_n(y_n(l - \psi(l))) - \bar{b}_{mn}(x_n(l)) g_n(x_n(l - \psi(l))). \end{aligned}$$

Let

$$e_m(l) = e_m^R(l) + e_m^I(l) i + e_m^J(l) j + e_m^K(l) k.$$

Following the preceding analysis, error system (2.6) decomposes into four distinct components. We only provided the derivation of the real part, and similarly, the other three parts can be obtained:

$$
\begin{cases}
\dot{e}_m^R(l) = -d_m e_m^R(l) + \sum_{n=1}^{t} \hat{a}_{mn}^R(e_n^R(l)) f_n^R(e_n^R(l)) \\
\qquad - \sum_{n=1}^{t} \hat{a}_{mn}^I(e_n^I(l)) f_n^I(e_n^I(l)) - \sum_{n=1}^{t} \hat{a}_{mn}^J(e_n^J(l)) f_n^J(e_n^J(l)) \\
\qquad - \sum_{n=1}^{t} \hat{a}_{mn}^K(e_n^K(l)) f_n^K(e_n^K(l)) + \sum_{n=1}^{t} \hat{b}_{mn}^R(e_n^R(l)) g_n^R(e_n^R(l-\psi(l))) \\
\qquad - \sum_{n=1}^{t} \hat{b}_{mn}^I(e_n^I(l)) g_n^I(e_n^I(l-\psi(l))) - \sum_{n=1}^{t} \hat{b}_{mn}^J(e_n^J(l)) g_n^J(e_n^J(l-\psi(l))) \\
\qquad - \sum_{n=1}^{t} \hat{b}_{mn}^K(e_n^K(l)) g_n^K(e_n^K(l-\psi(l))) + v_m^R, \ l \geq 0, \ \neq l_k, \\
e_p^R(l_k) = (1+\mu) e_m^R(l_k^-), \ l = l_k, \ k \in \mathbb{N}^+.
\end{cases}
\tag{2.7}
$$

Assumption 2. *Boundedness is assumed for the activation functions $f_m(\cdot)$ and $g_m(\cdot)$. More precisely, given any $p \in n$, the system admits positive real bounds M_m^δ and K_m^δ with the property that:*

$$|f_m^\delta(x_m^\delta)| \leq M_m^\delta, |g_m^\delta(x_m^\delta)| \leq K_m^\delta,$$

where $\delta = R, I, J, K$.

Assumption 3. *The impulse sequence $\{l_k, l \in \mathbb{N}^+\}$ belong to*

$$D(\psi_{min}, \psi_{max}) \doteq \{l_0 < l_1 < \cdots < l_k < \ldots, \lim_{k\to+\infty} l_k = +\infty, \psi_{min} \leq l_k - l_{k-1} \leq \psi_{max}\}.$$

In the time interval (a, b), we use $\omega(a, b)$ to express the number of impulsive jumps. So, $\{l_k, k \in \mathbb{N}^+\} \in D(\psi_{min}, \psi_{max})$, where $\frac{b-a}{\psi_{max}} - 1 \leq \omega(a, b) \leq \frac{b-a}{\psi_{min}}$.

Lemma 1. *[13] Suppose a continuous radically unbounded function $V(l) : \mathbb{R}^t \to \mathbb{R}^+ \cup \{0\}$ satisfies*
(1) $V(l) = 0 \to l = 0$,
(2) $\dot{V}(l) \leq -\alpha V^b(l) - \beta V^d(l)$, *where $\alpha, \beta > 0, b > 1, 0 < d < 1$.*

The analysis demonstrates that FTS is attained between the master-slave systems, $V(l) = 0, l \geq L_1$, and the settling time is given by

$$L_1 = \frac{1}{\alpha(b-1)} + \frac{1}{\beta(1-d)}.$$

Lemma 2. *[24] If $x_1, x_2, \ldots, x_t \geq 0, 0 < d \leq 1, b > 1$, then*

$$\sum_{m=1}^{t} x_m^d \geq (\sum_{m=1}^{t} x_m)^d, \quad \sum_{m=1}^{t} x_m^b \geq t^{1-b}(\sum_{m=1}^{t} x_m)^b.$$

Lemma 3. *[25] Let $0 \leq V(l) \leq \epsilon(l)$, and if there is an $L > 0$, for $l > L$, $\epsilon(L) = 0$. For any $l \geq L$, $V(l) \equiv 0$ is always satisfied. From the zero-solution stability of $\epsilon(l)$, the fixed-time stability of $V(l)$ follows.*

3. Main results

To realize FTS of the error system (2.6), we construct the feedback controller as follows:

$$\begin{cases} \nu_m^R = -N_m^R e_m^R(l) - sgn(e_m^R(l))(r_m^R + s_1^R|e_m^R(l)|^{\beta_1^R} + s_2^R|e_m^R(l)|^{\beta_2^R}) - \sigma_m^R sgn(e_m^R(l))|e_m^R(l-\psi_p(l))|, \\ \nu_m^I = -N_m^I e_m^I(l) - sgn(e_m^I(l))(r_m^I + s_1^I|e_m^I(l)|^{\beta_1^I} + s_2^I|e_m^I(l)|^{\beta_2^I}) - \sigma_m^I sgn(e_m^I(l))|e_m^I(l-\psi_p(l))|, \\ \nu_m^J = -N_m^J e_m^J(l) - sgn(e_m^J(l))(r_m^J + s_1^J|e_m^J(l)|^{\beta_1^J} + s_2^J|e_m^J(l)|^{\beta_2^J}) - \sigma_m^J sgn(e_m^J(l))|e_m^J(l-\psi_p(l))|, \\ \nu_m^K = -N_m^R e_m^K(l) - sgn(e_m^K(l))(r_m^K + s_1^K|e_m^K(l)|^{\beta_1^K} + s_2^K|e_m^K(l)|^{\beta_2^K}) - \sigma_m^K sgn(e_m^K(l))|e_m^K(l-\psi_p(l))|, \end{cases} \tag{3.1}$$

where $\nu_m^R, \nu_m^I, \nu_m^J, \nu_m^K$ represent the corresponding feedback controllers, $N_m^R, N_m^I, N_m^J, N_m^K$, $r_m^R, r_m^I, r_m^J, r_m^K$, $\sigma_m^R, \sigma_m^I, \sigma_m^J, \sigma_m^K$ are positive constants, $s_1^R, s_1^I, s_1^J, s_1^K, s_2^R, s_2^I, s_2^J, s_2^K$ are changeable constants, and $\beta_1^R, \beta_1^I, \beta_1^J, \beta_1^K > 1, 0 < \beta_2^R, \beta_2^I, \beta_2^J, \beta_2^K < 1$.

The following new concepts will be used later:

$$L(\bar{s}_1, \bar{s}_2) = \frac{\psi_{max}}{(1-\beta_1)\ln\vartheta}\ln[1 - \frac{\vartheta^{1-\beta_1}\ln\vartheta}{\bar{s}_1\psi_{max}}] + \frac{\psi_{max}}{(1-\beta_2)\ln\vartheta}\ln[\frac{\bar{s}_2\psi_{min}}{\bar{s}_2\psi_{min} - \vartheta^{\beta_2-1}\ln\vartheta}],$$

$$\hat{L}(\bar{s}_1, \bar{s}_2) = \frac{1}{\bar{s}_1(\beta_1 - 1)} + \frac{1}{\bar{s}_2(1-\beta_2)}, \ \bar{s}_1 = s_1 n^{(1-\beta_1)}, \ \bar{s}_2 = s_2,$$

where

$$s_1 = \min\{s_1^R, s_1^I, s_1^J, s_1^K\}, \quad s_2 = \min\{s_2^R, s_2^I, s_2^J, s_2^K\},$$
$$\beta_1 = \min\{\beta_1^R, \beta_1^I, \beta_1^J, \beta_1^K\}, \ \ \beta_2 = \max\{\beta_2^R, \ \beta_2^I, \ \beta_2^J, \beta_2^K\}.$$

Theorem 1. *Based on the fulfillment of Assumptions 1–3, provided that these parameters fulfill the subsequent conditions:*

$$\begin{cases} d_m \geq -N_m^\delta + \sum_{n=1}^{t} \phi_n^\delta(\breve{a_{mn}}^R + \breve{a_{mn}}^I + \breve{a_{mn}}^J + \breve{a_{mn}}^K)), \\ \sigma_m^\delta \geq \sum_{n=1}^{t} \varphi_n^\delta(\breve{b_{mn}}^R + \breve{b_{mn}}^I + \breve{b_{mn}}^J + \breve{b_{mn}}^K), \\ r_m^\delta \geq \xi_m, \delta = R, I, J, K. \end{cases} \tag{3.2}$$

Under the index $m \in 1, 2, ..., l$, *the master-slave systems* (2.1)–(2.4) *attain FTS with different convergence times:* $L(\bar{s}_1, \bar{s}_2)$ *for* $0 < \vartheta < 1$ *and* $\hat{L}(\bar{s}_1, \bar{s}_2)$ *when* $\vartheta = 1$.

Proof. In our stability analysis, the Lyapunov function [26] was

$$V(l) = V_1(l) + V_2(l) + V_3(l) + V_4(l),$$

$$V_1(l) = \sum_{m=1}^{t} |e_m^R(l)|, V_2(l) = \sum_{m=1}^{t} |e_m^I(l)|,$$

$$V_3(l) = \sum_{m=1}^{t} |e_m^J(l)|, V_4(l) = \sum_{m=1}^{t} |e_m^K(l)|,$$

$$
\begin{aligned}
\dot{V}_1(l) &= \sum_{m=1}^{t} sgn(e_m^R(l))\dot{e_m^R}(l) \\
&= \sum_{m=1}^{t} sgn(e_m^R(l))(-d_m e_m^R(l) + \sum_{n=1}^{t} \hat{a}_{mn}^R(e_n^R(l)) f_n^R(e_n^R(l)) \\
&\quad - \sum_{n=1}^{t} \hat{a}_{mn}^I(e_n^I(l)) f_n^I(e_n^I(l)) - \sum_{n=1}^{t} \hat{a}_{mn}^J(e_n^J(l)) f_n^J(e_n^J(l)) \\
&\quad - \sum_{n=1}^{t} \hat{a}_{mn}^K(e_n^K(l)) f_n^K(e_n^K(l)) + \sum_{n=1}^{t} \hat{b}_{mn}^R(e_n^R(l)) g_n^R(e_n^R(l-\psi(l))) \\
&\quad - \sum_{n=1}^{t} \hat{b}_{mn}^I(e_n^I(l)) g_n^I(e_n^I(l-\psi(l))) - \sum_{n=1}^{t} \hat{b}_{mn}^J(e_n^J(l)) g_n^J(e_n^J(l-\psi(l))) \\
&\quad - \sum_{n=1}^{t} \hat{b}_{mn}^K(e_n^K(l)) g_n^K(e_n^K(l-\psi(l))) - N_m^R e_m^R(l) - sgn(e_m^R(l))(r_m^R \\
&\quad + s_1^R|e_m^R(l)|^{\beta_1^R} + s_2^R|e_m^R(l)|^{\beta_2^R}) - \sigma_m^R sgn(e_m^R(l))|e_m^R(l-\psi_p(l))| \\
&\le \sum_{m=1}^{t} -d_m|e_m^R(l)| + \sum_{m=1}^{t}\sum_{n=1}^{t}(|\hat{a}_{mn}^R(e_n^R(l)) f_n^R(e_n^R)| - |\hat{a}_{mn}^I(e_n^I(l)) f_n^I(e_n^I(l))| \\
&\quad - |\hat{a}_{mn}^J(e_n^J(l)) f_n^J(e_n^J(l))| - |\hat{a}_{mn}^K(e_n^K(l)) f_n^K(e_n^K(l))| \\
&\quad + |\hat{b}_{mn}^R(e_n^R(l)) g_n^R(e_n^R(l-\psi(l)))| - |\hat{b}_{mn}^I(e_n^I(l)) g_n^I(e_n^I(l-\psi(l)))| \\
&\quad - |\hat{b}_{mn}^J(e_n^J(l)) g_n^J(e_n^J(l-\psi(l)))| - |\hat{b}_{mn}^K(e_n^K(l)) g_n^K(e_n^K(l-\psi(l)))|) \\
&\quad - \sum_{m=1}^{t} N_m^R|e_m^R(l)| - \sum_{m=1}^{t} r_m^R - \sum_{m=1}^{t} s_1^R|e_m^R(l)|^{\beta_1^R} \\
&\quad - \sum_{m=1}^{t} s_2^R|e_m^R(l)|^{\beta_2^R} - \sum_{m=1}^{t} \sigma_m^R sgn(e_m^R(l))|e_m^R(l-\psi_p(l))|.
\end{aligned}
\tag{3.3}
$$

In light of both Assumption 1 and the findings in [25],

$$|\hat{a}_{mn}^\delta(e_m^\delta(l)) f_n^\delta(e_n^\delta(l))| \le \breve{a}_{mn}^\delta \phi_n^\delta |e_m^\delta(t)| + M_n^\delta |\acute{a}_{mn}^\delta - \grave{a}_{mn}^\delta|.$$

$$|\hat{b}_{mn}^\delta(e_m^\delta(l)) g_n^\delta(e_n^\delta(l-\psi(l)))| \le \breve{b}_{mn}^\delta \varphi_n^\delta |e_m^\delta(l-\psi(l))| + L_n^\delta |\acute{b}_{mn}^\delta - \grave{b}_{mn}^\delta|.$$

Let

$$\xi_m = \sum_{n=1}^{t}(4\overline{M_n}\cdot\overline{A_{mn}} + 4\overline{L_n}\cdot\overline{B_{mn}}),$$

where

$$
\begin{aligned}
\overline{M_n} &= \max\{M_n^R, M_n^I, M_n^J, M_n^K\},\ \ \overline{L_n} = \max\{L_n^R, L_n^I, L_n^J, L_n^K\}, \\
\overline{A_{mn}} &= \max\{|\acute{a}_{mn}^R - \grave{a}_{mn}^R|, |\acute{a}_{mn}^I - \grave{a}_{mn}^I|, |\acute{a}_{mn}^J - \grave{a}_{mn}^J|, |\acute{a}_{mn}^K - \grave{a}_{mn}^K|\}, \\
\overline{B_{mn}} &= \max\{|\acute{b}_{mn}^R - \grave{b}_{mn}^R|, |\acute{b}_{mn}^I - \grave{b}_{mn}^I|, |\acute{b}_{mn}^J - \grave{b}_{mn}^J|, |\acute{b}_{mn}^K - \grave{b}_{mn}^K|\},
\end{aligned}
$$

so

$$
\begin{aligned}
\dot{V}_1(l) \leq & \sum_{m=1}^{n}(-d_m - N_m^R)|e_m^R(l)| + \sum_{m=1}^{t}\sum_{n=1}^{t}(\breve{a}_{mn}^R\phi_n^R|e_m^R(l)| - \breve{a}_{mn}^I\phi_n^I|e_m^I(l)| \\
& - \breve{a}_{mn}^J\phi_n^J|e_m^J(l)| - \breve{a}_{mn}^K\phi_n^K|e_m^K(l)| + \breve{b}_{mn}^R\varphi_n^R|e_m^R(l-\psi(l))| \\
& - \breve{b}_{mn}^I\varphi_n^I|e_m^I(l-\psi(l))| - \breve{b}_{mn}^J\varphi_n^J|e_m^J(l-\psi(l))| - \breve{b}_{mn}^K\varphi_n^K|e_m^K(l-\psi(l))| \\
& + \sum_{n=1}^{t}(M_n^R|\acute{a}_{mn}^R - \grave{a}_{mn}^R| - M_n^I|\acute{a}_{mn}^I - \grave{a}_{mn}^I| - M_n^J|\acute{a}_{mn}^J - \grave{a}_{mn}^J| - M_n^K|\acute{a}_{mn}^K - \grave{a}_{mn}^K| \\
& + L_n^R|\acute{b}_{mn}^R - \grave{b}_{mn}^R| - L_n^I|\acute{b}_{mn}^I - \grave{b}_{mn}^I| - L_n^J|\acute{b}_{mn}^J - \grave{b}_{mn}^J| - L_n^K|\acute{b}_{mn}^K - \grave{b}_{mn}^K|) \\
& - \sum_{m=1}^{t} r_m^R - \sum_{m=1}^{t} s_1^R|e_m^R(l)|^{\beta_1^R} - \sum_{m=1}^{t} s_2^R|e_m^R(l)|^{\beta_2^R} - \sum_{m=1}^{t}\sigma_m^R sgn(e_m^R(l))|e_m^R(l-\psi(l))| \\
\leq & \sum_{m=1}^{t}(-d_m - N_m^R + \sum_{n=1}^{t}\breve{a}_{mn}^R\phi_n^R)|e_m^R(l)| + \sum_{m=1}^{t}(\sum_{n=1}^{t}\breve{b}_{mn}^R\varphi_n^R - \sigma_m^R)|e_m^R(l-\psi(l))| \\
& + \sum_{m=1}^{t}\sum_{n=1}^{t}(\breve{a}_{mn}^I\phi_n^I|e_m^I(l)| + \breve{a}_{mn}^J\phi_n^J|e_m^J(l)| + \breve{a}_{mn}^K\phi_n^K|e_m^K(l)|) \\
& + \sum_{m=1}^{t}\sum_{n=1}^{t}(\breve{b}_{mn}^I\varphi_n^I|e_m^I(l-\psi(l))| + \breve{b}_{mn}^J\varphi_n^J|e_m^J(l-\psi(l))| + \breve{b}_{mn}^K\varphi_n^K|e_m^K(l-\psi(l))|) \\
& + \sum_{m=1}^{t}(\xi_m - r_m^R) - \sum_{m=1}^{t} s_1^R|e_m^R(l)|^{\beta_1^R} - \sum_{m=1}^{t} s_2^R|e_m^R(l)|^{\beta_2^R}. \qquad (3.4)
\end{aligned}
$$

Similarly, the other three parts can be obtained. From the preceding inequality, it follows that

$$
\begin{aligned}
\dot{V}(l) \leq & \sum_{m=1}^{t}(-d_m - N_m^R + \sum_{n=1}^{t}\phi_n^R(\breve{a}_{mn}^R + \breve{a}_{mn}^I + \breve{a}_{mn}^J + \breve{a}_{mn}^K))|e_m^R(l)| \\
& + \sum_{m=1}^{t}(-d_m - N_p^I + \sum_{n=1}^{t}\phi_n^I(\breve{a}_{mn}^R + \breve{a}_{mn}^I + \breve{a}_{mn}^J + \breve{a}_{mn}^K))|e_m^I(l)| \\
& + \sum_{m=1}^{t}(-d_m - N_p^J + \sum_{n=1}^{t}\phi_n^J(\breve{a}_{mn}^R + \breve{a}_{mn}^I + \breve{a}_{mn}^J + \breve{a}_{mn}^K))|e_m^J(l)| \\
& + \sum_{m=1}^{t}(-d_m - N_p^K + \sum_{n=1}^{t}\phi_n^K(\breve{a}_{mn}^R + \breve{a}_{mn}^I + \breve{a}_{mn}^J + \breve{a}_{mn}^K))|e_m^K(l)| \\
& + \sum_{m=1}^{t}(\sum_{n=1}^{t}\varphi_q^R(\breve{b}_{mn}^R + \breve{b}_{mn}^I + \breve{b}_{mn}^J + \breve{b}_{mn}^K) - \sigma_m^R)|e_m^R(l-\psi(l))| \\
& + \sum_{m=1}^{t}(\sum_{n=1}^{t}\varphi_q^I(\breve{b}_{mn}^R + \breve{b}_{mn}^I + \breve{b}_{mn}^J + \breve{b}_{mn}^K) - \sigma_m^I)|e_m^I(l-\psi(l))| \\
& + \sum_{m=1}^{t}(\sum_{n=1}^{t}\varphi_n^J(\breve{b}_{mn}^R + \breve{b}_{mn}^I + \breve{b}_{mn}^J + \breve{b}_{mn}^K) - \sigma_m^J)|e_m^J(l-\psi(l))|
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{m=1}^{t}(\sum_{n=1}^{t}\varphi_n^K(\breve{b}_{mn}^R+\breve{b}_{mn}^I+\breve{b}_{mn}^J+\breve{b}_{mn}^K)-\sigma_m^K)|e_m^K(l-\psi(l))|\\
&+\sum_{m=1}^{t}\{4\xi_m-(r_m^R+r_m^I+r_m^J+r_m^K))\}\\
&-\sum_{m=1}^{t}(s_1^R|e_m^R(l)|^{\beta_1^R}+s_1^I|e_m^I(l)|^{\beta_1^I}+s_1^J|e_m^J(l)|^{\beta_1^J}+s_1^K|e_m^K(l)|^{\beta_1^K})\\
&-\sum_{m=1}^{t}(s_2^R|e_m^R(l)|^{\beta_2^R}+s_2^I|e_m^I(l)|^{\beta_2^I}+s_2^J|e_m^J(l)|^{\beta_2^J}+s_2^K|e_m^K(l)|^{\beta_2^K}).
\end{aligned}
\tag{3.5}
$$

Following Lemma 2,

$$
\begin{aligned}
\dot{V}(l)\leq&-\sum_{m=1}^{t}(s_1^R|e_m^R(l)|^{\beta_1^R}+s_1^I|e_m^I(l)|^{\beta_1^I}+s_1^J|e_m^J(l)|^{\beta_1^J}+s_1^K|e_m^K(l)|^{\beta_1^K})\\
&-\sum_{m=1}^{t}(s_2^R|e_m^R(l)|^{\beta_2^R}+s_2^I|e_m^I(l)|^{\beta_2^I}+s_2^J|e_m^J(l)|^{\beta_2^J}+s_2^K|e_m^K(l)|^{\beta_2^K}),\\
\leq&-s_1n^{1-\beta_1}V^{\beta_1}(l)-s_2V^{\beta_2}(l),
\end{aligned}
\tag{3.6}
$$

where

$$
\begin{gathered}
s_1=\min\{s_1^R,s_1^I,s_1^J,s_1^K\},\quad s_2=\min\{s_2^R,s_2^I,s_2^J,s_2^K\},\\
1<\beta_1=\min\{\beta_1^R,\beta_1^I,\beta_1^J,\beta_1^K\},\quad 0<\beta_2=\max\{\beta_2^R,\beta_2^I,\beta_2^J,\beta_2^K\}<1,
\end{gathered}
$$

when $l=l_k, k\in\mathbb{N}^+$. According to (3.2),

$$
V(l_k)=\sum_{m=1}^{l}(1+\mu)e_m(l_k^-)\leq|1+\mu|V(l^-)=\vartheta V(l^-),
\tag{3.7}
$$

where $\vartheta=|1+\mu|$, and take $0<\vartheta\leq 1$. For further analysis, auxiliary impulsive differential equations are considered as follows:

$$
\begin{cases}
\dot{\epsilon}(l)=\begin{cases}-\overline{s_1}\epsilon^{\beta_1}(l),\ \epsilon>1,\ l\neq l_k,\\ -\overline{s_2}\epsilon^{\beta_2}(l),\ 0\leq\epsilon<1,\ l\neq l_k,\end{cases}\\
\epsilon(l_k)=\vartheta\epsilon(l_k^-),\ l=l_k,\\
\epsilon(0)=\sum_{p=1}^{n}|e_p(0)|.
\end{cases}
\tag{3.8}
$$

By (3.6)–(3.8), $0\leq V(l)\leq\epsilon(l)$ can be obtained. Therefore, if there is an $L>0$, for $l>L,\epsilon(L)=0$, and for any $l\geq L, V(l)\equiv 0$ is always satisfied. Stability analysis reveals that the zero-solution stability of (3.8) directly implies the fixed-time stability property of (3.6). When $\epsilon\geq 1$, let $\gamma=\epsilon^{1-\beta_1}$, and we can get

$$
\begin{cases}
\dot{\gamma}(l)=\overline{s_1}(\beta_1-1),\ l\neq l_k,\ 0<\gamma\leq 1,\\
\gamma(l_k)=\overline{\vartheta}\gamma(l_k^-),\ l=l_k,\\
\gamma(0)=\epsilon_0^{1-\beta_1},
\end{cases}
\tag{3.9}
$$

where $\overline{\vartheta} = \vartheta^{1-\beta_1}$, *when* $\epsilon \to 1^+$ *is equivalent to* $\gamma \to 1^-$.

When $0 \le \epsilon < 1$, *let* $\gamma = \epsilon^{1-\beta_2}$, *and we can get*

$$\begin{cases} \dot{\gamma}(l) =\overline{s_2}(1-\beta_2),\ l \neq l_k,\ 0 \le \gamma < 1, \\ \gamma(l_k) =\widetilde{\vartheta}\gamma(l_k^-),\ l = l_k, \\ \gamma(0) =1, \end{cases} \tag{3.10}$$

where $\widetilde{\vartheta} = \vartheta^{1-\beta_2}$, *when* $\epsilon \to 0^+$ *is equivalent to* $\gamma \to 0^+$. *If Eq (3.8) can achieve fixed-time stability, it can be equivalent to satisfying the following two conditions:*

(a) *In a fixed time* L_1, *the solution of (3.9) approaches 1.*

(b) *In a fixed time* L_2, *the solution of (3.10) tends from 1 to 0.*

In other words, for any given initial value $\epsilon(0)$, $\epsilon(l) \to 0$ *is satisfied in a fixed time* $L_1 + L_2$.

Next, it is divided into two cases to prove.

When $0 < \vartheta < 1$. *So we can get* $\overline{\vartheta} > 1$ *and* $0 < \widetilde{\vartheta} < 1$. *Consider the first condition, when* $\epsilon(l_k^-) \ge 1, \epsilon(l_k) \ge 1$. *According to Eq (3.9), we can get*

$$\gamma(l) = \overline{\vartheta}^{\omega(0,l)}\gamma(0) + \overline{s_1}(\beta_1 - 1)\int_0^l \overline{\vartheta}^{\omega(s,l)}ds, \tag{3.11}$$

where $\overline{\vartheta}^{\frac{l-s}{\psi_{max}}-1} \le \overline{\vartheta}^{\omega(s,l)} \le \overline{\vartheta}^{\frac{l-s}{\psi_{min}}}, \gamma(0) = \overline{\vartheta}^{\omega(0,0)}\gamma(0) < 1$, *and* $\lim_{l\to\infty}\gamma(l) = \infty$. *So there is an* L_1 *that makes* $\lim_{l\to L_1}\gamma(l) = 1$, *and satisfies* $0 < \gamma(l) < 1$ *under* $0 < l < L_1$. *Therefore, the following equation is obtained:*

$$\overline{\vartheta}^{\omega(0,l)}\gamma(0) + \overline{s_1}(\beta_1 - 1)\int_0^l \overline{\vartheta}^{\omega(s,l)}ds = 1,$$

$$\overline{s_1}(\beta_1 - 1)\int_0^l \overline{\vartheta}^{\omega(s,l)}ds \le 1.$$

According to $\overline{\vartheta}^{\frac{l-s}{\psi_{max}}-1} \le \overline{\vartheta}^{\omega(s,l)} \le \overline{\vartheta}^{\frac{l-s}{\psi_{min}}}$,

$$\int_0^l \overline{\vartheta}^{\frac{l-s}{\psi_{max}}}ds \le \frac{\overline{\vartheta}}{\overline{s_1}(\beta_1 - 1)}.$$

Finally, we get the inequality

$$l \le \frac{\psi_{max}}{\ln\overline{\vartheta}}\ln[1 + \frac{\overline{\vartheta}\ln\overline{\vartheta}}{\overline{s_1}(\beta_1 - 1)\psi_{max}}].$$

Take $\overline{\vartheta} = \vartheta^{1-\beta_1}$ *back to the above inequality:*

$$L_1 = \frac{\psi_{max}}{(1-\beta_1)\ln\vartheta}\ln[1 - \frac{\vartheta^{(1-\beta_1)}\ln\vartheta}{\overline{s_1}\psi_{max}}]. \tag{3.12}$$

The fixed time L_2 *of (3.10) is calculated by a similar method:*

$$\gamma(l) = \widetilde{\vartheta}^{\omega(0,l)}\gamma(0) - \overline{s_2}(1-\beta_2)\int_0^l \widetilde{\vartheta}^{\omega(s,l)}ds, \tag{3.13}$$

where $0 < \widetilde{\vartheta} < 1$*, and because* L_2 *needs to satisfy* $\lim_{t\to L_2}\gamma(l) = 0,\ \gamma(0) = 1,$

$$\begin{aligned}\gamma(l) \leq & \widetilde{\vartheta}^{\frac{l}{\psi_{max}}-1} - \overline{s_2}(1-\beta_2)\int_0^l \widetilde{\vartheta}^{\frac{l-s}{\psi_{min}}} ds,\\ = & \widetilde{\vartheta}^{\frac{l}{\psi_{max}}-1} - \overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}}\widetilde{\vartheta}^{\frac{l}{\psi_{min}}} + \overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}}\\ \leq & [\widetilde{\vartheta}^{-1} - \overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}}]\widetilde{\vartheta}^{\frac{l}{\psi_{max}}} + \overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}}.\end{aligned}$$

Let

$$S(l) = [\widetilde{\vartheta}^{-1} - \overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}}]\widetilde{\vartheta}^{\frac{l}{\psi_{max}}} + \overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}},$$

so $S(0) = \widetilde{\vartheta}^{-1} > 0, S(+\infty) = \overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}} < 0$*, and* $\dot{S}(l) < 0$*. Therefore, there is only one* L_2 *that makes* $S(L) = 0$*:*

$$L_2 = \frac{\psi_{max}}{\ln\widetilde{\vartheta}}\ln[\frac{\overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}}}{\overline{s_2}(1-\beta_2)\frac{\psi_{min}}{\ln\widetilde{\vartheta}} - \widetilde{\vartheta}^{-1}}].$$

Take $\widetilde{\vartheta} = \vartheta^{1-\beta_2}$ *back to the above inequality:*

$$L_2 = \frac{\psi_{max}}{(1-\beta_2)\ln\vartheta}\ln[\frac{\overline{s_2}\psi_{min}}{\overline{s_2}\psi_{min} - \vartheta^{\beta_2-1}\ln\vartheta}]. \tag{3.14}$$

Next consider another condition $\epsilon(l_k^-) \geq 1, \epsilon(l_k) < 1$*. The time from* $\epsilon(l) > 1$ *to 1 is significantly less than* L_1*, and the time interval of* $\epsilon(l)$ *from 1 to 0 is significantly less than* L_2*. Therefore, the error system (2.6) has fixed-time stability at* $L(\overline{s_1}, \overline{s_2}) = L_1 + L_2$*. In other word, when* $0 < \vartheta < 1$*, at a fixed time* $L(\overline{s_1}, \overline{s_2}) = L_1 + L_2$*, system (2.4) synchronizes to (2.1).*

When $\vartheta = 1$*,* $\overline{\vartheta} = \widetilde{\vartheta} = 1$*, and according to Eq (3.11),*

$$\gamma(t) = \gamma(0) + \overline{s_1}(\beta_1 - 1)\int_0^l ds = \gamma(0) + \overline{s_1}(\beta_1 - 1)l, \tag{3.15}$$

so

$$\overline{s_1}(\beta_1 - 1)l \leq 1, l \leq \frac{1}{\overline{s_1}(\beta_1 - 1)} = \widetilde{L_1}.$$

Since $\gamma(0) = 1, \gamma(L_2) = 0$*, and* $\widetilde{\vartheta} = 1$*, according to (3.12), we can get*

$$\gamma(l) = 1 - \overline{s_2}(1-\beta_2)\int_0^l ds,\ \ \gamma(\widetilde{L_2}) = 1 - \overline{s_2}(1-\beta_2)\int_0^{\widetilde{L_2}} ds = 0,\ \ \widetilde{L_2} = \frac{1}{\overline{s_2}(1-\beta_2)}.$$

Therefore, when $\vartheta = 1$*, at a fixed time* $\widetilde{L}(\overline{s_1}, \overline{s_2}) = \widetilde{L_1} + \widetilde{L_2}$*, system (3.2) synchronizes to (2.1).*

4. Numerical simulation

Numerical experiments are conducted to validate the effectiveness of the developed approaches.

Example 1. *The dynamical behavior of our two-dimensional QVMNN drive system can be characterized as:*

$$\begin{cases} \dot{x}_m(l) = -d_m x_m(l) + \displaystyle\sum_{n=1}^{t} a_{mn}(x_m(l)) f_n(x_n(t)) \\ \qquad\qquad + \displaystyle\sum_{n=1}^{t} b_{mn}(x_m(l)) g_n(x_n(l-\psi(l))) + I_m,\ l \geq 0,\ l \neq l_k, \\ x_m(l_k) = (1+\mu) x_m(l_k^-),\ l = l_k,\ k \in \mathbb{N}^+,\ m = 1, 2, \end{cases} \tag{4.1}$$

where $d_1 = d_2 = 1$.

$$a_{11}^R(x^R(l)) = \begin{cases} -1.8, & |x^R(l)| \leq 1.5, \\ -1.5, & |x^R(l)| > 1.5, \end{cases} \quad a_{12}^R(x^R(l)) = \begin{cases} 2.8, & |x^R(l)| \leq 1.5, \\ 2.5, & |x^R(l)| > 1.5, \end{cases}$$

$$a_{21}^R(x^R(l)) = \begin{cases} 1.2, & |x^R(l)| \leq 1.5, \\ 0.8, & |x^R(l)| > 1.5, \end{cases} \quad a_{22}^R(x^R(l)) = \begin{cases} -1.5, & |x^R(l)| \leq 1.5, \\ -1.3, & |x^R(l)| > 1.5, \end{cases}$$

$$a_{11}^I(x^I(l)) = \begin{cases} -2.3, & |x^I(l)| \leq 1.5, \\ -2.1, & |x^I(l)| > 1.5, \end{cases} \quad a_{12}^I(x^I(l)) = \begin{cases} 2.5, & |x^I(l)| \leq 1.5, \\ 2.2, & |x^I(l)| > 1.5, \end{cases}$$

$$a_{21}^I(x^I(l)) = \begin{cases} 1.3, & |x^I(l)| \leq 1.5, \\ 1.1, & |x^I(l)| > 1.5, \end{cases} \quad a_{22}^I(x^I(l)) = \begin{cases} -1.0, & |x^I(l)| \leq 1.5, \\ -0.8, & |x^I(l)| > 1.5, \end{cases}$$

$$a_{11}^J(x^J(l)) = \begin{cases} -1.7, & |x^J(l)| \leq 1.5, \\ -1.4, & |x^J(l)| > 1.5, \end{cases} \quad a_{12}^J(x^J(l)) = \begin{cases} 2.6, & |x^J(l)| \leq 1.5, \\ 2.3, & |x^J(l)| > 1.5, \end{cases}$$

$$a_{21}^J(x^J(l)) = \begin{cases} 1.4, & |x^J(l)| \leq 1.5, \\ 0.9, & |x^J(l)| > 1.5, \end{cases} \quad a_{22}^J(x^J(l)) = \begin{cases} -1.2, & |x^J(l)| \leq 1.5, \\ -1, & |x^J(l)| > 1.5, \end{cases}$$

$$a_{11}^K(x^K(l)) = \begin{cases} -2.2, & |x^K(l)| \leq 1.5, \\ -2, & |x^K(l)| > 1.5, \end{cases} \quad a_{12}^K(x^K(l)) = \begin{cases} 2.7, & |x^K(l)| \leq 1.5, \\ 2.4, & |x^K(l)| > 1.5, \end{cases}$$

$$a_{21}^K(x^K(l)) = \begin{cases} 1.2, & |x^K(l)| \leq 1.5, \\ 1, & |x^K(l)| > 1.5, \end{cases} \quad a_{22}^K(x^K(l)) = \begin{cases} -1.2, & |x^K(l)| \leq 1.5, \\ -1, & |x^K(l)| > 1.5, \end{cases}$$

$$b_{11}^R(x^R(l)) = \begin{cases} -3.8, & |x^R(l)| \leq 1.5, \\ -3.5, & |x^R(l)| > 1.5, \end{cases} \quad b_{12}^R(x^R(l)) = \begin{cases} 0.5, & |x^R(l)| \leq 1.5, \\ 0.4, & |x^R(l)| > 1.5, \end{cases}$$

$$b_{21}^{R}(x^{R}(l)) = \begin{cases} 0.1, & |x^{R}(l)| \le 1.5, \\ 0.3, & |x^{R}(l)| > 1, \end{cases} \quad b_{22}^{R}(x^{R}(l)) = \begin{cases} -3.5, & |x^{R}(l)| \le 1.5, \\ -3, & |x^{R}(l)| > 1.5, \end{cases}$$

$$b_{11}^{I}(x^{I}(l)) = \begin{cases} -3.2, & |x^{I}(l)| \le 1.5, \\ -3, & |x^{I}(l)| > 1.5, \end{cases} \quad b_{12}^{I}(x^{I}(l)) = \begin{cases} 0.6, & |x^{I}(l)| \le 1.5, \\ 0.3, & |x^{I}(l)| > 1.5, \end{cases}$$

$$b_{21}^{I}(x^{I}(l)) = \begin{cases} 0.5, & |x^{I}(l)| \le 1.5, \\ 0.2, & |x^{I}(l)| > 1.5, \end{cases} \quad b_{22}^{I}(x^{I}(l)) = \begin{cases} -3.8, & |x^{I}(l)| \le 1.5, \\ -3.5, & |x^{I}(l)| > 1.5, \end{cases}$$

$$b_{11}^{J}(x^{J}(l)) = \begin{cases} -3.7, & |x^{J}(l)| \le 1.5, \\ -3.4, & |x^{J}(l)| > 1.5, \end{cases} \quad b_{12}^{J}(x^{J}(l)) = \begin{cases} 0.6, & |x^{J}(l)| \le 1.5, \\ 0.3, & |x^{J}(l)| > 1.5, \end{cases}$$

$$b_{21}^{J}(x^{J}(l)) = \begin{cases} 0.4, & |x^{J}(l)| \le 1.5, \\ 0.9, & |x^{J}(l)| > 1.5, \end{cases} \quad b_{22}^{J}(x^{J}(l)) = \begin{cases} -3.2, & |x^{J}(l)| \le 1.5, \\ -3.0, & |x^{J}(l)| > 1.5, \end{cases}$$

$$b_{11}^{K}(x^{K}(l)) = \begin{cases} -3.2, & |x^{K}(l)| \le 1.5, \\ -3.0, & |x^{K}(l)| > 1.5, \end{cases} \quad b_{12}^{K}(x^{K}(l)) = \begin{cases} 2.7, & |x^{K}(l)| \le 1.5, \\ 2.4, & |x^{K}(l)| > 1.5, \end{cases}$$

$$b_{21}^{K}(x^{K}(l)) = \begin{cases} 0.2, & |x^{K}(l)| \le 1.5, \\ 0.1, & |x^{K}(l)| > 1.5, \end{cases} \quad b_{22}^{K}(x^{K}(l)) = \begin{cases} -3.2, & |x^{K}(l)| \le 1.5, \\ -3.1, & |x^{K}(l)| > 1.5. \end{cases}$$

The response system is

$$\begin{cases} \dot{y}_m(l) = -d_m y_m(t) + \sum_{n=1}^{t} a_{mn}(y_m(l)) f_n(y_n(l)) \\ \qquad\qquad + \sum_{n=1}^{t} b_{mn}(y_m(l)) g_n(y_n(l-\psi(l))) + I_m + \nu_m, \ l \ge 0, \ l \ne l_k, \\ y_m(l_k) = (1+\mu) y_m(l_k^-), \ l = l_k, \ k \in \mathbb{N}^+, \end{cases} \tag{4.2}$$

where the delay is $\psi(l) = \frac{e^l}{1+e^l}$,

$$f_m(x_m(l)) = tanh(x_m^R(l)) + tanh(x_m^I(l))i + tanh(x_m^J(l))j + tanh(x_m^K(l))k$$

and

$$g_m(x_m(l)) = \frac{1}{2}(|x_m^R(l)+1| - |x_m^R(l)-1|) + \frac{1}{2}(|x_m^I(l)+1| - |x_m^I(l)-1|)i$$

$$+\frac{1}{2}(|x_m^J(l)+1|-|x_m^J(l)-1|)j+\frac{1}{2}(|x_m^K(l)+1|-|x_m^K(l)-1|)k$$

are taken as activation functions. The parameters of controller v_m *are* $N_1^\delta = 2, N_2^\delta = 1, r_1^\delta = 3, r_2^\delta = 3.6, s_1^\delta = 3, s_2^\delta = 2, \beta_1^\delta = 1.5, \beta_2^\delta = 0.5, \sigma_1^\delta = 3.1, \sigma_2^\delta = 3.2$, *where* $\delta = R, I, J, K$. *Therefore, it is easy to get* $\phi^\delta = \varphi^\delta = M_m^\delta = L_m^\delta, m = 1, 2, \delta = R, I, J, K$. *The corresponding initial values of the master system and the slave system are* $x_1(l) = (0.5 + 0.2i + 0.6j + 0.3k), x_2(l) = (0.75 + 0.35i + 0.9j + 0.5k), y_1(l) = (-0.5 + 0.8i - 0.7j + 0.9k), y_2(l) = (-0.8 + 1i - 0.85j + 0.95k)$ *for* $l \in [-1, 0]$.

Figure 1 depicts an impulsive signal, where $\psi_{min} = 0.01, \psi_{max} = 0.04$. When $\vartheta = 1 + \mu = 0.9$, from the trajectories of the error in Figures 2 and 3, the analytical results demonstrate that synchronization between master system (4.1) and slave system (4.2) is unattainable in the absence of control intervention. Figures 4 and 5 illustrate the dynamical behavior of these systems when feedback controller (3.1) is implemented. Through simulation, it can achieve FTS, where the settling time $L(\bar{s}_1, \bar{s}_2) \approx 2.3$. In the same way, when $\vartheta = 1 + \mu = 1.0$, the synchronization cannot be realized without the controller according to Figures 6 and 7. In addition, FTS can be achieved with feedback controller (3.1) according to Figures 8 and 9, where the settling time $\hat{L}(\bar{s}_1, \bar{s}_2) \approx 1.9$.

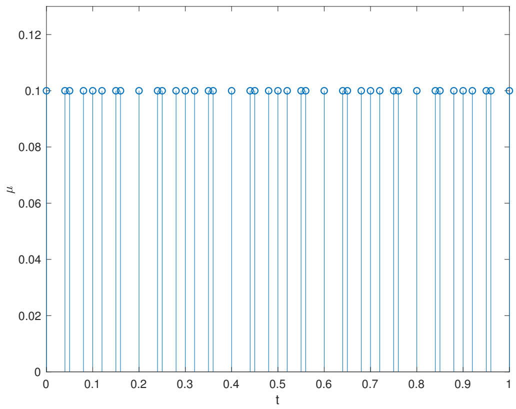

Figure 1. Impulsive sequence.

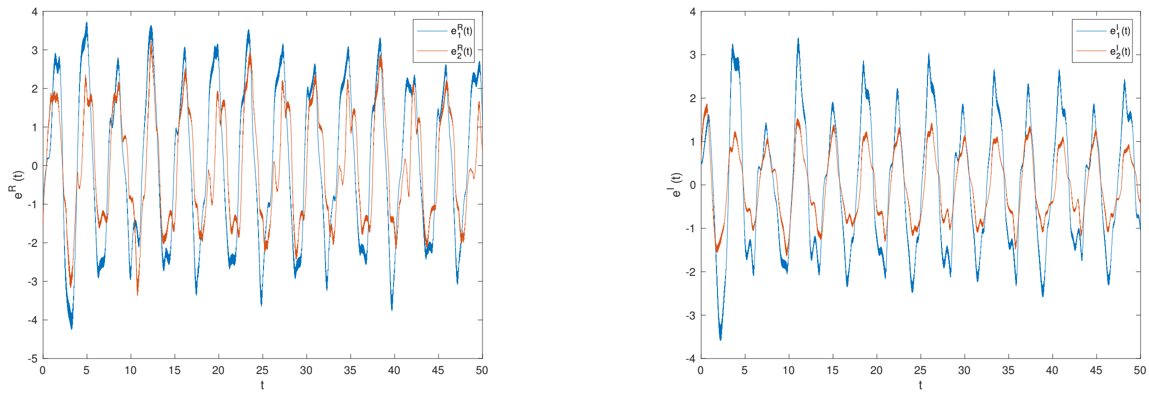

Figure 2. State trajectories of error system $e_1^R(l), e_2^R(l), e_1^I(l), e_2^I(l)$ without control inputs when $\vartheta = 0.9$.

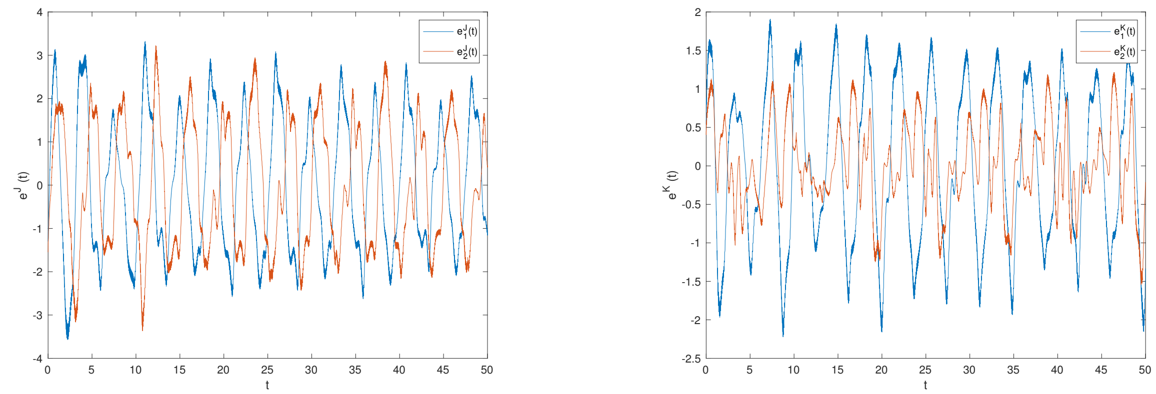

Figure 3. State trajectories of error system $e_1^J(l), e_2^J(l), e_1^K(l), e_2^K(l)$ without control inputs when $\vartheta = 0.9$.

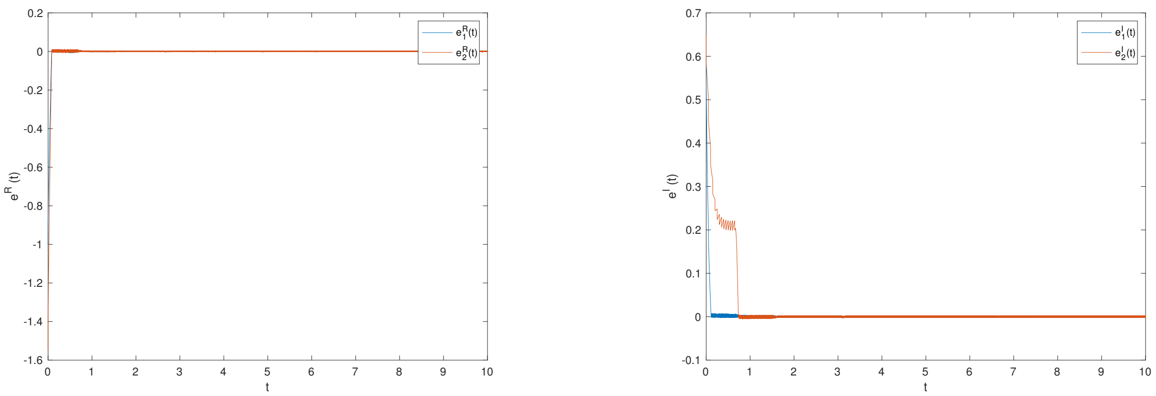

Figure 4. State trajectories of error system $e_1^R(l), e_2^R(l), e_1^I(l), e_2^I(l)$ under controller (3.1) when $\vartheta = 0.9$.

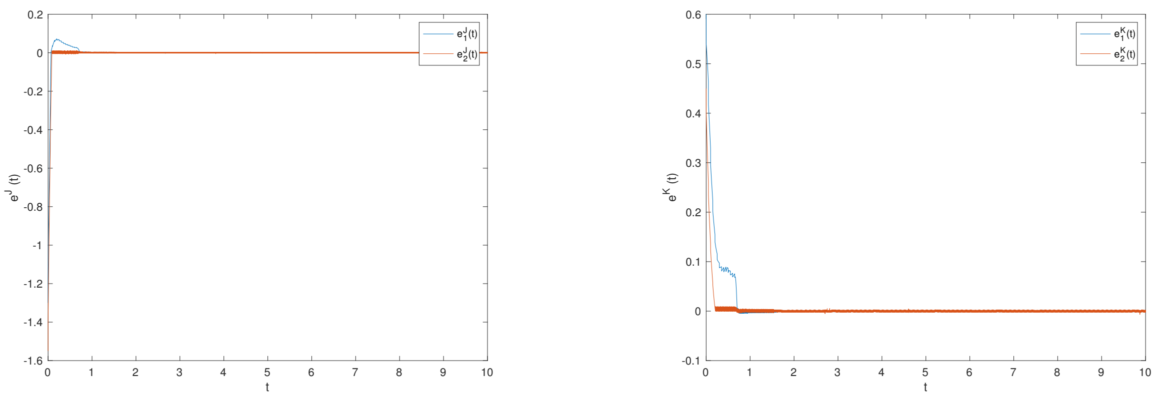

Figure 5. State trajectories of error system $e_1^J(l), e_2^J(l), e_1^K(l), e_2^K(l)$ under controller (3.1) when $\vartheta = 0.9$.

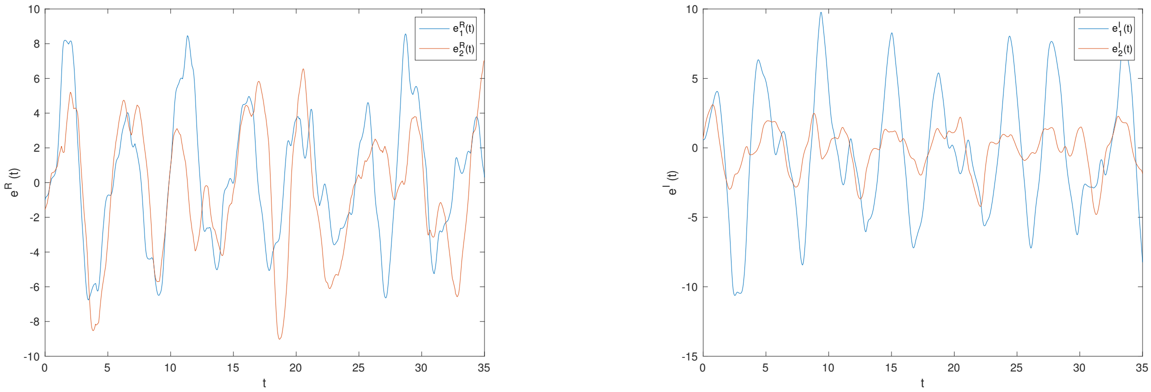

Figure 6. State trajectories of error system $e_1^R(l), e_2^R(l), e_1^I(l), e_2^I(l)$ without control inputs when $\vartheta = 1.0$.

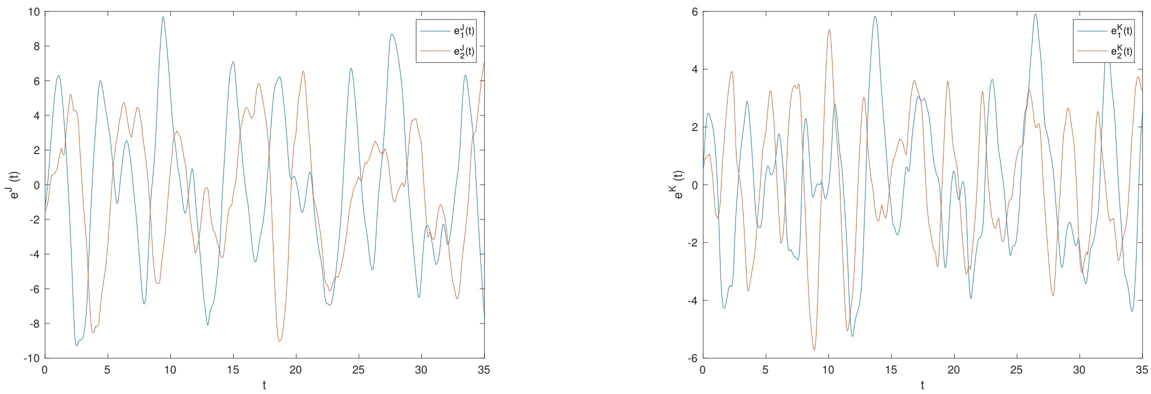

Figure 7. State trajectories of error system $e_1^J(l), e_2^J(l), e_1^K(l), e_2^K(l)$ without control inputs when $\vartheta = 1.0$.

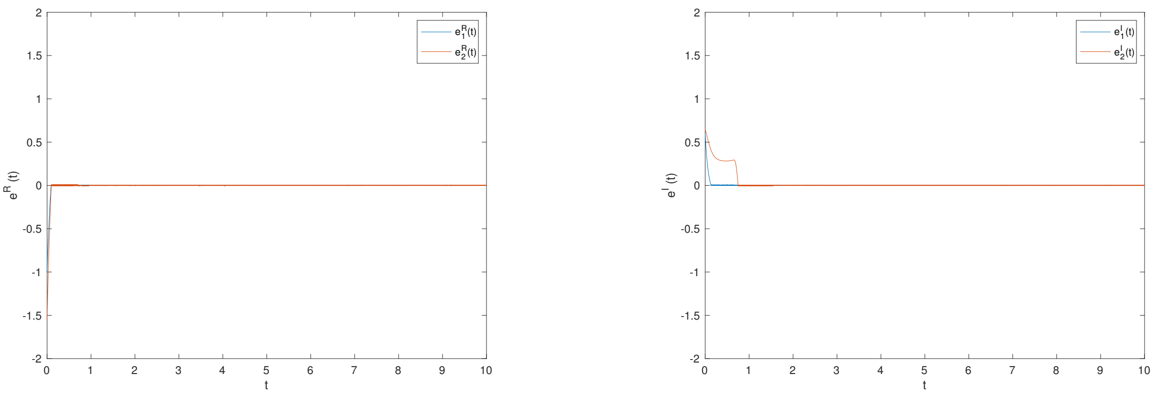

Figure 8. State trajectories of error system $e_1^R(l), e_2^R(l), e_1^I(l), e_2^I(l)$ under controller (3.1) when $\vartheta = 1.0$.

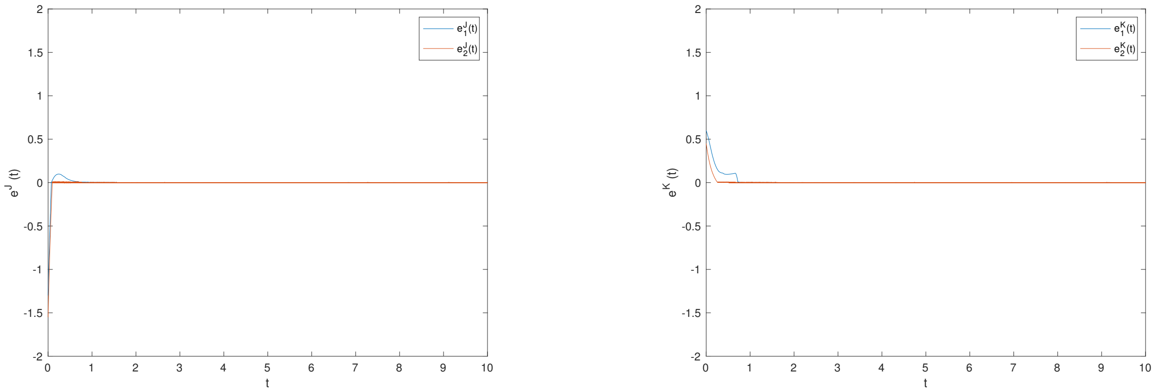

Figure 9. State trajectories of error system $e_1^J(l), e_2^J(l), e_1^K(l), e_2^K(l)$ under controller (3.1) when $\vartheta = 1.0$.

5. Conclusions

The paper presents sufficient FTS conditions for impulsive QVMNNs with time-varying delays, supported by numerical verification. Beyond impulsive effects, we note that random disturbances and parameter discrepancies may significantly impact system behavior, motivating our planned investigation of these factors.

Author contributions

Hong Zhou: Conceptualization, methodology, modeling, validation, resources, draft preparation, writing–review and editing, project administration; Jie Zhou: Validation, resources, writing–review and editing, supervision, project administration. Both authors reviewed the results and approved the final version of the manuscript.

Use of Generative-AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Acknowledgments

The authors thank the anonymous editors and reviewers for their earnest and professional work.

Conflict of interest

All authors declare no conflicts of interest.

References

1. L. Chua, Memristor-the missing circuit element, *IEEE Trans. Circuit Theory*, **18** (1971), 507–519. https://doi.org/10.1109/TCT.1971.1083337
2. D. B. Strukov, G. S. Snider, D. R. Stewart, R. S. William, The missing memristor found, *Nature*, **453** (2008), 80–83. https://doi.org/10.1038/nature06932
3. F. Corinto, A. Ascoli, M. Gilli, Nonlinear dynamics of memristor oscillators, *IEEE Trans. Circuits Syst. I*, **58** (2011), 1323–1336. https://doi.org/10.1109/TCSI.2010.2097731
4. J. Huang, C. Li, X. He, Stabilization of a memristor-based chaotic system by intermittent control and fuzzy processing, *Int. J. Control Autom. Syst.*, **11** (2013), 643–647. https://doi.org/10.1007/s12555-012-9323-x
5. R. Li, J. Cao, Stability analysis of reaction-diffusion uncertain memristive neural networks with time-varying delays and leakage term, *Appl. Math. Comput.*, **278** (2016), 54–69. https://doi.org/10.1016/j.amc.2016.01.016
6. Z. Meng, Z. Xiang, Stability analysis of stochastic memristor-based recurrent neural networks with mixed time-varying delays, *Neural Comput. Appl.*, **28** (2017), 1787–1799. https://doi.org/10.1007/s00521-015-2146-y
7. R. Li, X. Gao, J. Cao, Exponential synchronization of stochastic memristive neural networks with time-varying delays, *Neural Proces. Lett.*, **50** (2019), 459–475. https://doi.org/10.1007/s11063-019-09989-5
8. R. Wei, J. Cao, Fixed-time synchronization of quaternion-valued memristive neural networks with time delays, *Neural Networks*, **113** (2019), 1–10. https://doi.org/10.1016/j.neunet.2019.01.014
9. X. Qi, H. Bao, J. Cao, Exponential input-to-state stability of quaternion-valued neural networks with time delay, *Appl. Math. Comput.*, **358** (2019), 382–393. https://doi.org/10.1016/j.amc.2019.04.045
10. T. Isokawa, T. Kusakabe, N. Matsui, F. Peper, Quaternion neural network and its application, In: V. Palade, R. J. Howlett, L. Jain, *Knowledge-based intelligent information and engineering systems*, KES 2003, Lecture Notes in Computer Science, Springer, **2774** (2003), 318–324. https://doi.org/10.1007/978-3-540-45226-3_44
11. X. Yang, C. Huang, J. Cao, An LMI approach for exponential synchronization of switched stochastic competitive neural networks with mixed delays, *Neural Comput. Appl.*, **21** (2012), 2033–2047. https://doi.org/10.1007/s00521-011-0626-2
12. X. Yang, J. Cao, J. Liang, Exponential synchronization of memristive neural networks with delays: Interval matrix method, *IEEE Trans. Neural Networks Learn. Syst.*, **28** (2017), 1878–1888. https://doi.org/10.1109/TNNLS.2016.2561298
13. A. Polyakov, Nonlinear feedback design for fixed-time stabilization of linear control systems, *IEEE Trans. Autom. Control*, **57** (2012), 2106–2110. https://doi.org/10.1109/TAC.2011.2179869
14. C. Hu, J. Yu, Z. Chen, H. Jiang, T. Huang, Fixed-time stability of dynamical systems and fixed-time synchronization of coupled discontinuous neural networks, *Neural Networks*, **89** (2017), 74–83. https://doi.org/10.1016/j.neunet.2017.02.001

15. C. Aouiti, E. A. Assali, F. Chérif, A. Zeglaoui, Fixed-time synchronization of competitive neural networks with proportional delays and impulsive effect, *Neural Comput. Appl.*, **32** (2020), 13245–13254. https://doi.org/10.1007/s00521-019-04654-3
16. L. Zhao, J. Yu, C. Lin, H. Yu, Distributed adaptive fixed-time consensus tracking for second-order multi-agent systems using modified terminal sliding mode, *Appl. Math. Comput.*, **312** (2017), 23–35. https://doi.org/10.1016/j.amc.2017.05.049
17. Z. G. Wu, Y. Xu, Y. J. Pan, H. Su, Y. Tang, Event-triggered control for consensus problem in multi-agent systems with quantized relative state measurements and external disturbance, *IEEE Trans. Circuits Syst. I*, **65** (2018), 2232–2242. https://doi.org/10.1109/TCSI.2017.2777504
18. J. Zhou, T. Chen, L. Xiang, Robust synchronization of delayed neural networks based on adaptive control and parameters identification, *Chaos Soliton. Fract.*, **27** (2006), 905–913. https://doi.org/10.1016/j.chaos.2005.04.022
19. W. Yu, J. Cao, Robust control of uncertain stochastic recurrent neural networks with time-varying delay, *Neural Process. Lett.*, **26** (2007), 101–119. https://doi.org/10.1007/s11063-007-9045-x
20. X. Ding, J. Cao, A. Alsaedi, F. E. Alsaadi, T. Hayat, Robust fixed-time synchronization for uncertain complex-valued neural networks with discontinuous activation functions, *Neural Networks*, **90** (2017), 42–55. https://doi.org/10.1016/j.neunet.2017.03.006
21. X. Li, J. Fang, H. Li, Master-slave exponential synchronization of delayed complex-valued memristor-based neural networks via impulsive control, *Neural Networks*, **93** (2017), 165–175. https://doi.org/10.1016/j.neunet.2017.05.008
22. L. Zhang, Y. Yang, X. Xu, Synchronization analysis for fractional order memristive Cohen–Grossberg neural networks with state feedback and impulsive control, *Phys. A*, **506** (2018), 644–660. https://doi.org/10.1016/j.physa.2018.04.088
23. J. Zhu, J. Sun, Stability of quaternion-valued impulsive delay difference systems and its application to neural networks, *Neurocomputing*, **284** (2018), 63–69. https://doi.org/10.1016/j.neucom.2018.01.018
24. A. Polyakov, D. Efimov, W. Perruquetti, Robust stabilization of MIMO systems in finite/fixed time, *Int. J. Robust Nonlinear Control*, **26** (2016), 69–90. https://doi.org/10.1002/rnc.3297
25. Y. Zhang, J. Zhuang, Y. Xia, Y. Bai, J. Cao, L. Gu, Fixed-time synchronization of the impulsive memristor-based neural networks, *Commun. Nonlinear Sci. Numer. Simulat.*, **77** (2019), 40–53. https://doi.org/10.1016/j.cnsns.2019.04.021
26. X. Yang, J. Lam, D. W. C. Ho, Z. Feng, Fixed-time synchronization of complex networks with impulsive effects via nonchattering control, *IEEE Trans. Autom. Control*, **62** (2017), 5511–5521. https://doi.org/10.1109/TAC.2017.2691303