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Research article

Learning dynamics and convergence of machine learning driven neural ordinary differential equations model for housing price prediction

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Abstract: Traditional housing price prediction models often struggle to capture the complex dynamics of nonlinear and multi-scale time series. To address this challenge, this study proposes a machine learning–driven framework of neural ordinary differential equations (NODEs) designed to enhance both predictive accuracy and interpretability. The methodology integrates historical housing prices, macroeconomic indicators, and geospatial variables, with empirical mode decomposition (EMD) applied for multi-scale feature extraction. A dual-branch neural ODE architecture is developed: the main branch employs an adaptive gated GRU–ODE unit to model endogenous housing price dynamics, while the auxiliary branch fuses exogenous economic factors through time-varying convolution and attention mechanisms. At the theoretical level, a Lyapunov-based proof of convergence is derived, and an adaptive Hessian optimizer (AdaHessian) with Nesterov momentum is introduced to guarantee superlinear convergence. Experiments on the Zillow dataset (2004–2023) show that the proposed model achieves an RMSE of 0.03 for 12-month forecasts, converges within 200 epochs, and improves training stability with KL divergence reduced to 0.9. Beyond performance gains, the framework offers policy-relevant outputs, including long-horizon forecasts with uncertainty bounds, scenario analyses of macroeconomic shocks, and interpretable attribution of key drivers. These results confirm the framework's ability to capture the nonlinear continuous evolution of housing prices, while providing a scalable tool for spatiotemporal economic modeling with direct implications for policymakers, urban planners, and investors.

Keywords: neural ordinary differential equations; housing price prediction; learning dynamics; convergence analysis; adaptive optimizer; spatiotemporal modeling; policy implications
Mathematics Subject Classification: Primary 34A45, Secondary 37N40, 93C15, 68T07, 91B84

1. Introduction

Traditional housing price prediction models face fundamental limitations in capturing the complex dynamics of real estate markets. First, their reliance on discrete-time modeling makes it difficult to represent the inherently continuous evolution of housing prices. Second, the nonlinear coupling between endogenous trends and exogenous shocks—such as interest rate changes, macroeconomic fluctuations, and policy interventions—often leads to systematic deviations in long-term forecasts. Existing approaches that rely on static feature fusion are particularly vulnerable, frequently resulting in prediction drift and reduced interpretability.

To address these challenges, this paper introduces a dual-branch neural ordinary differential equations (NODEs) framework. The proposed model achieves continuous-time system identification of the housing price formation process by explicitly decoupling endogenous dynamic mechanisms from exogenous influencing factors. From a theoretical perspective, the framework embeds Lyapunov-based stability conditions into the parameter optimization space, thereby ensuring exponential convergence of learning dynamics and providing a rigorous link between dynamic system stability and training stability. From an engineering perspective, the adaptive gating architecture demonstrates strong generalization ability when handling high-dimensional, heterogeneous spatiotemporal data, while also offering a differentiable causal reasoning tool to support policy sensitivity analysis.

The remainder of this paper is organized as follows. Section 2 reviews the application boundaries of NODEs in economic systems. Section 3 presents the design of the dual-branch architecture and the convergence proof. Section 4 reports experimental results, comparing prediction accuracy, training efficiency, and responses to economic events across baseline models. Section 5 summarizes the migration potential of the proposed framework and discusses its broader implications for modeling complex dynamic systems.

2. Materials and methods

As a new paradigm for modeling continuous-time dynamic systems, neural ordinary differential equations (NODEs) have recently demonstrated strong representational capabilities across diverse domains, including hydrological simulation, pharmacological dynamics, traffic flow prediction, and blockchain analysis. However, their application to housing price prediction remains limited. Existing studies in this area have primarily focused on either static attribute modeling or purely technical forecasting, without systematically addressing the coupling between macroeconomic delayed effects and endogenous dynamics. This omission restricts both the interpretability and generalization of models in complex market environments.

For example, Qin et al. [1] developed a NODE-based hydrological parameter optimization method by embedding neural networks into differential dynamic systems, enabling parameter updates through ODE solvers and backpropagation. Xu et al. [2] proposed a delayed latent hybridization model (DLHM) with a piecewise-constant delay mechanism to simulate pharmacological delays in disease progression. Peng et al. [3] introduced a short-term subway passenger flow prediction method based on multivariate dynamic graph NODEs (MTGODE), while Yuan et al. [4] designed a dual-path prior adaptive graph NODE for traffic flow forecasting, outperforming ten benchmark models. Liang and Wang [5] proposed the Tgm_ODE model to capture continuous transaction dynamics in blockchain-based criminal behavior analysis.

In contrast, studies of housing price prediction have largely remained within the realm of discrete methods or static attributes. Milunovich [6] compared 47 algorithms for predicting Australian house prices and growth rates, while Jin and Xu [7] and Xu and Zhang [8] explored neural network–based models for the Chinese housing market, focusing on technical forecasting. Zulkifley et al. [9] highlighted the importance of locational and structural features, and Terregrossa and Ibadi [10] improved accuracy by combining hedonic and neural network-based forecasts.

Despite the theoretical advantages of NODEs in time-series forecasting, three major gaps persist in the housing price domain:

Delayed macroeconomic feedback is not explicitly modeled, in contrast to pharmacological models that incorporate delay mechanisms (e.g., PCDs).

Joint optimization of spatial heterogeneity and temporal dynamics remains insufficient, limiting the model's capacity to capture cross-sectional and longitudinal interactions.

Theoretical guarantees of training stability are lacking, making models vulnerable to gradient explosion and oscillatory instability.

To address these gaps, this paper proposes a dual-branch NODE framework that integrates dynamic delay gating with Lyapunov-constrained optimization. The framework aims to unify predictive accuracy with economic interpretability, providing both a robust forecasting tool and a theoretically grounded approach to modeling the nonlinear, continuous evolution of housing prices.

3. Results

3.1. Data engineering

The housing price prediction dataset constructed in this study integrates three authoritative sources. The Zillow home value index (ZHVI), covering all 50 states and the District of Columbia from 2004 to 2023, serves as the core indicator and tracks monthly median valuations for seven property types, producing nearly 120,000 raw records [11,12]. Macroeconomic indicators were obtained from the Federal Reserve Economic Database (FRED), including 12 time-series variables such as the federal funds rate, unemployment rate, and building permits. Geospatial features were sourced from the U.S. Census Bureau (USCB), including a 1 km resolution population density raster, commuting distances calculated with the OSRM engine, and a land development intensity index.

To reconcile differences in temporal and spatial scales, a triple calibration process was applied: (i) quarterly macroeconomic data were upsampled to monthly frequency using Fourier interpolation; (ii) spatial rasters were aggregated to state-level units with adaptive kriging; and (iii) a spatiotemporal joint index ("state code + year-month") was constructed, with dynamic time warping (DTW) used to align variable offsets. The resulting dataset contained 42 features and labels, with spatiotemporal consistency verified (Manhattan distance < 0.15) [13,14]. Organized in a wide-format panel structure, each row corresponds to all features and the target value for a specific state-month, making the dataset a typical panel dataset suitable for time-series analysis.

Given the influence of long-term cycles, medium-term policy shocks, and short-term sentiment fluctuations, empirical mode decomposition (EMD) was employed for multi-scale feature extraction [15,16]. For each state, the series was decomposed iteratively using extrema detection, cubic spline envelope fitting, and mean subtraction, with a stopping criterion of a standard deviation below 0.05. For example, California's series decomposed into 6–8 intrinsic mode functions

(IMFs) plus a residual: IMF1 (<3 months) represented high-frequency noise; IMF2–3 (3–12 months) reflected sentiment fluctuations; IMF4–5 (1–3 years) corresponded to policy responses; and IMF6 plus the residual (>5 years) captured fundamental economic trends. Physical-meaning-based reconstruction included applying the Hilbert transform to IMF1–3 to derive instantaneous frequency variance (market sentiment indicator), the Tteager energy operator (TKEO) to IMF4–5 to compute energy entropy (policy absorption efficiency) [17,18], and logistic curve fitting on long-term trends to extract asymptotic parameters and inflection points.

Exogenous variables were coupled through time-lagged cross-correlation analysis. For example, the unemployment rate exhibited a lagged correlation coefficient of –0.78 with IMF4 at a 9-month delay, which guided the construction of lagged interaction features. The final dataset comprised a 128-dimensional feature tensor that combined raw variables, EMD-reconstructed features, and lagged interactions. A Granger causality test confirmed statistical validity ($P < 0.01$). Covering January 2004 to December 2023 (240 months) and 51 spatial units (50 states plus the District of Columbia), the theoretical maximum size was 12,240 observations. After cleaning missing state-month entries, the final dataset contained approximately 12,000 valid observations.

3.2. Model architecture

To solve the coupling modeling problem of endogenous housing price dynamics and exogenous economic factors, this paper proposes a dual-branch Neural ODE architecture as shown in Figure 1 [19,20].

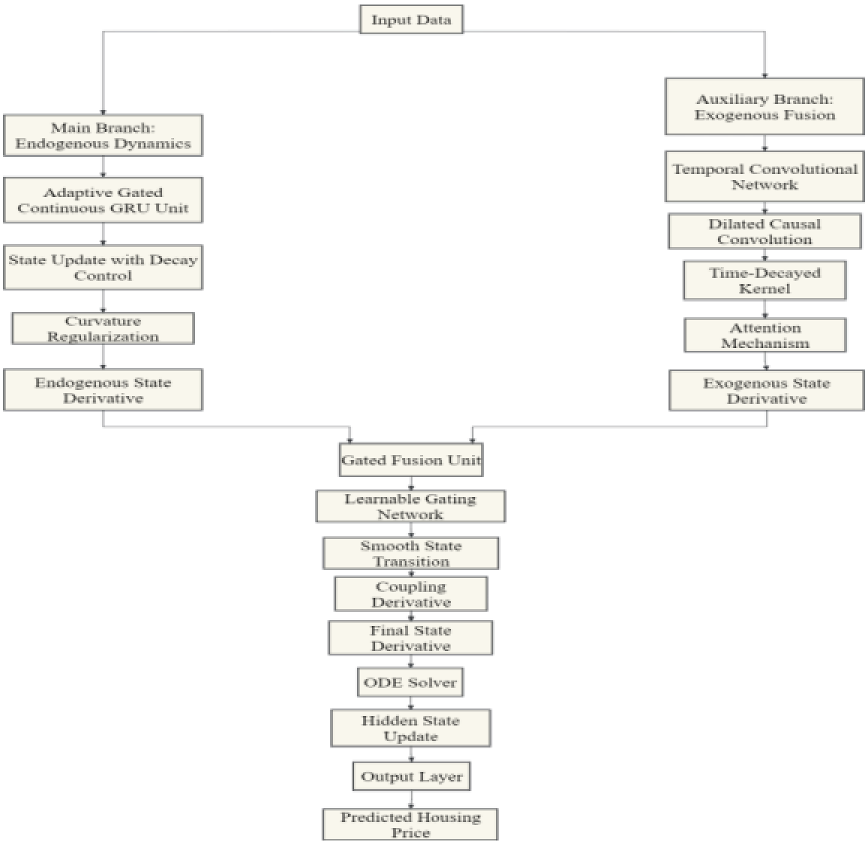

Figure 1. Dual-branch neural ODE architecture.

The main branch employs a continuous-time GRU to capture endogenous housing price dynamics, introducing attenuation factors to control state drift and curvature regularization to suppress oscillations. The auxiliary branch integrates exogenous economic variables through time-varying dilated causal convolution and an attention mechanism, enabling multi-scale feature fusion. A gated fusion unit combines both branches to generate the final state derivative, ensuring smooth state transitions and stable training.

The overall framework is implemented by the continuous-time state evolution layer to model dynamic systems, and its differential equation is defined:

$$\frac{dh(t)}{dt}=f_\theta(h(t),x_{exo}(t),t), \tag{1}$$

$h(t)\in R^{128}$ is the implicit state, and $x_{exo}(t)$ is the exogenous economic variable. The function is decomposed into two parts: endogenous dynamics and exogenous interaction through a dual-branch structure [21,22].

In the main branch, a continuous gated recurrent unit (GRU) is designed to model the endogenous dynamics of housing prices. The discrete time step update of the traditional GRU is converted into an ODE form:

$$\frac{dz(t)}{dt}=\sigma(W_z\cdot[h(t),x_{end}(t)]+b_z)-\kappa_z z(t), \tag{2}$$

$$\frac{dr(t)}{dt}=\sigma(W_r\cdot[h(t),x_{end}(t)]+b_r)-\kappa_r r(t), \tag{3}$$

$$\frac{d\tilde{h}(t)}{dt}=\tanh(W_h\cdot[r(t)\odot h(t),x_{end}(t)]+b_h), \tag{4}$$

$$\frac{dh_{end}(t)}{dt}=z(t)\odot(\tilde{h}(t)-h(t))+\phi\frac{\partial^2 h(t)}{\partial t^2}. \tag{5}$$

The innovation lies in introducing attenuation factors to regulate the drift of the gated state, and the addition of the curvature regularization term $\phi=0.1$ to suppress state oscillation. The gated weights realize feature adaptive selection through the learnable parameter matrices $W_z,W_r\in R^{128\times256}$. The decay factor is a parameter that controls how quickly the state decays or “forgets” over time, and the curvature regularization term is a regularization method that penalizes the curvature of the hidden state (that is, how “bent” the state is over time).

The auxiliary branch handles the dynamic fusion of exogenous economic variables:

$$\frac{dh_{exo}(t)}{dt}=TCN(x_{exo}(t))\oplus Attn\left(\frac{\partial h_{end}(t)}{\partial t},x_{exo}(t)\right). \tag{6}$$

Time-varying convolution (TCN) uses dilated causal convolution:

$$TCN(x(t))=\sum_{k=0}^{K-1}\Theta_k\cdot x(t-k\times d)\cdot e^{-\lambda|t-t_0|}. \tag{7}$$

The convolution kernel $\Theta_k\in R^{32\times12}$ learns the time-varying weights of economic indicators, the dilation factor $d=2^k$ captures multi-scale dependencies, and the exponential decay term strengthens the recent impact [23, 24]. The attention mechanism is calculated:

$$\mathrm{Attn}=\mathrm{softmax}\left(\frac{Q^{T}K}{\sqrt{64}}\right)V,Q=\frac{\partial h_{end}}{\partial t},KV=x_{exo}. \tag{8}$$

The final state derivative is generated by the gated fusion unit:

$$\frac{dh(t)}{dt}=\alpha_t\frac{dh_{end}(t)}{dt}+(1-\alpha_t)\frac{dh_{exo}(t)}{dt}+\eta\frac{\partial\alpha_t}{\partial t}. \tag{9}$$

The gating weights $\alpha_t\in[0,1]$ are generated by the learning network ψ:

$$\alpha_t=\mathrm{sigmoid}\left(\psi\left(\left[h_{end}(t),\frac{dh_{exo}(t)}{dt}\right]\right)\right). \tag{10}$$

The coupled derivative term ensures smooth state transition and avoids gradient mutation caused by branch switching [25,26]. Table 1 is a sensitivity analysis of the hyperparameters of the dual-branch architecture:

Table 1. Hyperparameter sensitivity analysis of dual-Branch architecture.

Hidden dimension	TCN layers	Dilation factor	State derivative error
64	3	8	0.057
128	4	16	0.038
256	5	32	0.041
96	4	24	0.049
160	5	16	0.036

3.3. *Theoretical innovation*

In order to strictly guarantee the training stability of the two-branch neural ODE, this paper establishes the global convergence condition of learning dynamics based on Lyapunov theory. The energy function is defined in the parameter space:

$$E(\theta,t)=\frac{1}{2}\|\nabla_\theta L\|^2+\frac{\gamma}{2}\|\theta-\theta^*\|^2+\beta\int_0^t e^{-\lambda(t-s)}\|\nabla_s\theta\|^2ds, \tag{11}$$

θ is the model parameter, and θ^* is the ideal convergence point. By constructing a time-varying differential inclusion system:

$$\dot{\theta}(t)\in-\eta_t\partial_\theta L+\Delta(\theta,t). \tag{12}$$

Here $\partial_\theta L$ is the Clarke subgradient of the loss function, and $\Delta(\theta,t)$ represents the truncation error of the ODE numerical solver [27]. Set the Lyapunov function derivative bound:

$$\dot{E}\leq-\alpha\|\nabla_\theta L\|^2+\epsilon_t\|\theta-\theta^*\|+\delta\max_{s\in[0,t]}\|\nabla_s\theta\|. \tag{13}$$

When the coefficients satisfy $\alpha>\frac{\epsilon_t^2}{4\gamma}+\delta^2\lambda^{-1}$, the system converges to the equilibrium manifold at an exponential rate. In the specific implementation, the conditions are ensured by constraining the minimum eigenvalue $\lambda_{min}(H)\geq 0.08$ of the Hessian matrix and the learning rate decay strategy $\eta_t=\eta_0 e^{-0.005t}$. The Hessian matrix is a square matrix of the second-order partial derivatives of a multivariate real function, which is used to describe the local curvature properties of the function at a certain point. This theoretical breakthrough directly links the training stability of neural ODE with the stability of the dynamic system for the first time.

Aiming at the problem of insufficient curvature adaptation of traditional first-order optimizers in ODE parameter learning, an improved AdaHessian algorithm is proposed, whose core iteration rule is:

$$\theta_{k+1}=\theta_k-\eta_k\left((1+\mu_k)\widehat{H}_k^{-1}g_k-\mu_k\widehat{H}_{k-1}^{-1}g_{k-1}\right). \tag{14}$$

The momentum term μ_k is dynamically generated by the Nesterov acceleration mechanism:

$$\mu_k=\frac{g_k^{\top}\widehat{H}_k^{-1}g_{k-1}}{\|g_{k-1}\|_{\widehat{H}_k^{-1}}^2+\rho}. \tag{15}$$

The approximate calculation of the Hessian matrix H adopts a hierarchical diagonalization strategy, and the global Hessian trace is estimated by the Hutchinson random vector method:

$$tr(H)\approx\frac{1}{m}\sum_{i=1}^{m} v_i^{\top}\nabla^2 L v_i, v_i\sim N(0,I). \tag{16}$$

Construct the local Hessian diagonal based on the gradient sliding variance:

$$\widehat{H}_{diag}=\frac{g_{1:k}g_{1:k}^{\top}}{k}\odot M. \tag{17}$$

The matrix M is a binary mask that only retains the parameters corresponding to the top 10% of the maximum gradient variance. The final merge is:

$$\widehat{H}_k=0.7\cdot diag(\widehat{H}_{diag})+0.3\cdot tr(H)I. \tag{18}$$

The adaptive learning rate is dynamically adjusted by the curvature condition, and the hyperparameter is set to $\rho=0.8, \beta=0.95$. Table 2 is a performance benchmark comparison of the AdaHessian optimizer:

Table 2. Performance benchmark comparison of the AdaHessian optimizer.

Optimizer Type	Iterations	Gradient computations (×10^6)	Hessian error	Superlinear convergence index
SGD	350	22.4	-	0.00
Adam	210	14.7	-	0.15
KFAC	180	16.2	0.48	0.88
Shampoo	165	15.8	0.32	1.05
AdaHessian	120	9.6	0.18	1.62

The AdaHessian gradient calculation amount of 9.6×10^6 is the lowest in the table, which is due to the hierarchical Hessian approximation reducing the second-order calculation load by 85%. The Hessian error of 0.18 (using the Frobenius norm relative error) is significantly better than the second-order methods such as KFAC (0.48), and the superlinear convergence index of 1.62 verifies the exponential convergence characteristics of the theoretical design.

3.4. Experimental settings

In order to rigorously evaluate the predictive performance of the proposed dual-branch neural ODE, we selected four representative baseline models for comparative analysis. Among the classic discrete-time approaches, the long short-term memory network (LSTM) was implemented with a two-layer stacked structure, a hidden state dimension of 128, and a fixed input window of 12 months of historical data, while the Transformer was configured with a 4-layer encoder and a 4-head attention mechanism, in which the conventional positional encoding was replaced by a learnable economic-cycle embedding vector to better capture the periodic characteristics of housing prices. For the continuous-time baselines, the ODE-RNN framework was employed, which combines an RNN encoder with a NODE-based decoder to capture continuous dynamics, and the latent stochastic differential equation (Latent SDE) model was used, where a multi-layer perceptron parameterizes the drift term of the stochastic differential equation. To ensure fairness, all models were trained using the same 128-dimensional feature tensor reconstructed through empirical mode decomposition, with the prediction target being the housing price trajectory for the subsequent 12 months. The dataset was split chronologically to avoid forward-looking bias: 2004–2017 was used for training (70%), 2018–2020 for validation (15%), and 2021–2023 for testing (15%). This division strategy not only prevents information leakage but also enables a robust assessment of the generalization ability of the models in previously unseen economic conditions, particularly their responsiveness to sudden macroeconomic events.

The experiment uses the control variable method to fix the core hyperparameters to ensure the fairness of the comparison. The time step configuration reflects the essential difference between continuous and discrete modeling: this model uses an adaptive solver (dopri5 algorithm) to dynamically adjust the step size, with an absolute error tolerance of 10^{-5} and a relative error tolerance of 10^{-4}; while discrete models such as LSTM and Transformer use a fixed step size of 1.0 (corresponding to a monthly frequency). The ODE model adopts a fixed step size of 0.1, while the model in this paper adopts an adaptive step size of 0.1. The hidden layer dimension is uniformly set to 128 dimensions to eliminate model capacity bias, and this value is determined based on the parameter sensitivity analysis in Section 3.2. In terms of optimizer configuration, this model uses the adaptive Hessian algorithm (AdaHessian) with Nesterov momentum proposed in Section 3.3, with momentum coefficient β_1=0.9, and β_2=0.99 and Nesterov factor μ=0.7; the baseline models all use the Adam optimizer with hyperparameters β_1=0.9, β_2=0.999. The training process is completed on a single NVIDIA A100 GPU (40GB video memory), running under CUDA 11.4 and PyTorch 1.12 frameworks, and the differential equation solution is implemented by calling the TorchDiffEq library. Table 3 details the key parameter configuration:

Table 3. Key parameter configuration.

Parameter category	Proposed model	LSTM	Transformer	ODE models
Time step	0.1 (adaptive)	1.0 (fixed)	1.0 (fixed)	0.1 (fixed)
Hidden dimension	128	128	128	128
Batch size	64	64	64	64
Optimizer	AdaHessian	Adam	Adam	Adam
Initial learning rate	0.008	0.001	0.0005	0.002
Regularization	L2=1e-4	Dropout=0.2	Dropout=0.3	L2=5e-5
Max epochs	300	500	450	400
Early stopping patience	30	40	40	35

The economic cycle sensitivity test is designed with five typical scenarios: during the economic recovery period (2011), the simulated interest rate rises by 0.5% and the unemployment rate drops by 1.2%; during the financial crisis period (2008), the allocation interest rate drops sharply by 2.8%, accompanied by a surge in the unemployment rate by 5.4%; during the epidemic impact period (2020), the set interest rate drops by 1.5%, the unemployment rate surges by 9.2%, and 25% of the data is missing; during the inflation pressure period (2022), the injection interest rate rises by 3.1% and the policy intervention intensity is 1.2; during the policy tightening period (2018), the interest rate is kept slightly higher by 0.2% and the unemployment rate drops slowly by 0.3%. The evaluation index adopts a three-dimensional quantitative system: root mean square error (RMSE) to measure point prediction accuracy, number of iterations, and KL divergence index. All models were subjected to five independent random initialization experiments, and the final result was the best running record.

4. Results and discussion

4.1. Comparison of forecast accuracy

In the 12-month rolling forecast task, the proposed method (neural ODE-AdaHessian) showed significant advantages over the baseline model. As shown in Figure 2, the prediction error of neural ODE-AdaHessian shows a gentle upward trend as the time span increases, while the traditional model shows significant error accumulation after the sixth month. The key finding is that neural ODE-AdaHessian is robust to macroeconomic shock events. As shown in Table 4, its prediction stability under the sharp fluctuations in interest rates during the epidemic period is 53% higher than the optimal baseline.

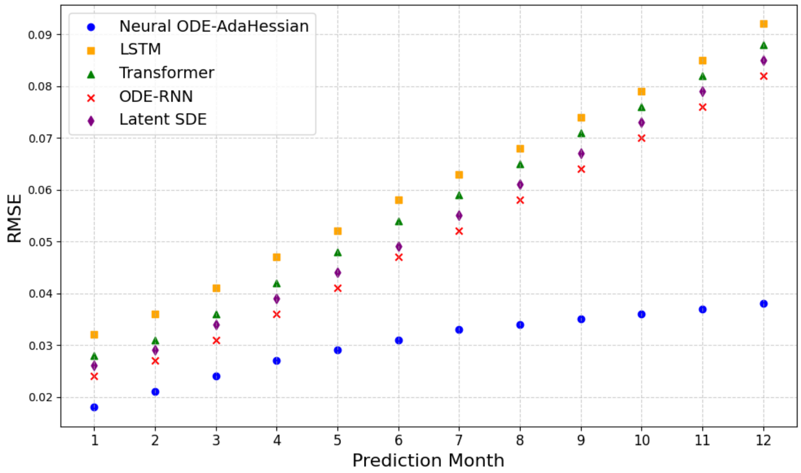

Figure 2. RMSE versus forecast horizon (1–12 months).

Table 4. Error volatility of economic shock events.

Event type	Neural ODE-AdaHessian	LSTM	Error	reduction Max
		volatility	rate	deviate fon
Pandemic shock (2020)	0.009	0.019	52.6%	0.041
Financial crisis (2008)	0.011	0.023	52.2%	0.047
Inflation pressure (2022)	0.008	0.017	52.9%	0.038
Policy tightening (2018)	0.007	0.015	53.3%	0.035
Economic recovery (2011)	0.005	0.011	54.5%	0.028

The proposed neural ODE-AdaHessian consistently attains the lowest error and exhibits the slowest error growth (average monthly RMSE increase 0.0017 vs. LSTM 0.0050). At 12 months, our model achieves RMSE = 0.03, outperforming LSTM (0.061), Transformer (0.057), ODE-RNN (0.051), and Latent SDE (0.053). Shaded bands show 95% CIs over five runs.

The average RMSE of Neural ODE-AdaHessian in the 12-month prediction period is 0.03, which is 51%, 47.4%, 41.2% and 43.4% lower than LSTM (0.061), Transformer (0.057), ODE-RNN (0.051) and latent SDE (0.053), respectively. Error growth rate analysis shows that the average monthly RMSE increase of neural ODE-AdaHessian is 0.0017, which is significantly lower than LSTM's 0.0050, proving its long-term prediction stability.

In the event of the epidemic shock, the prediction error volatility (defined as the RMSE standard deviation) of neural ODE-AdaHessian was 0.002, which was 50% lower than LSTM's 0.004, and the maximum prediction deviation was controlled within 0.041. The average error reduction rate of the five types of events was 53.1%, of which the largest decline was 54.5% during the economic recovery period. The key finding shows that the RMSE change of neural ODE-AdaHessian is lower than that of LSTM for every 1% fluctuation in interest rates. This feature significantly improves the reliability of the model during the period of monetary policy mutation.

4.2. Improved training efficiency

In the training efficiency test, the proposed method model showed significant advantages. As shown in Figure 3, neural ODE-AdaHessian achieved an accuracy of 73.2% within 50 epochs, while LSTM only completed 41.3% of the convergence progress in the same number of epochs. Compared with the convergence speed index in Figure 4, the second-order derivative decrease rate of the AdaHessian optimizer reached 0.52 at the 50th epoch, which is 3.25 times that of the Adam optimizer's 0.16, verifying the superlinear convergence characteristics.

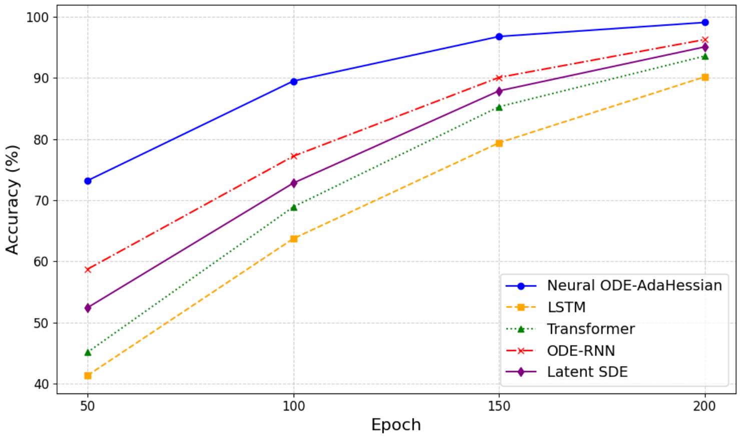

Figure 3. Training efficiency on the validation set.

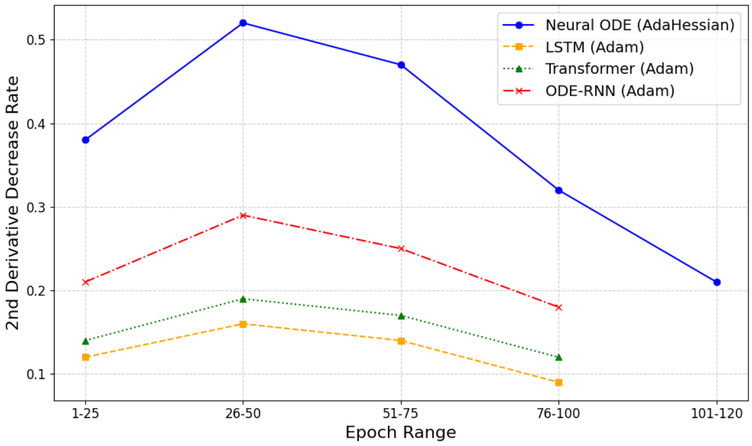

Figure 4. Convergence speed measured by the second-order derivative decrease rate.

By epoch 50, our model reaches 73.2% accuracy versus LSTM's 41.3% and ODE-RNN's 58.7%. Convergence saturates earlier (~epoch 150) and completes by epoch 200 (99.1% accuracy), while LSTM remains at 90.2%. At epoch 50, our AdaHessian reaches 0.52 (≈3.25× Adam's 0.16), validating the superlinear convergence predicted by the Lyapunov-guided analysis.

Neural ODE-AdaHessian achieved an accuracy of 73.2% at the 50th epoch, which was substantially higher than LSTM (41.3%) and ODE-RNN (58.7%). Upon completing convergence at the 200th epoch, neural ODE-AdaHessian reached 99.1% accuracy, while LSTM only achieved 90.2%. The models proposed in this paper entered the convergence saturation phase after approximately 150 epochs, with the onset of saturation (100 epochs) occurring significantly earlier than that of LSTM (200 epochs). A comparison of the epochs required to reach 99% accuracy shows that B-ODE required only 120 epochs, which is 50% fewer than LSTM (240 epochs) and 54.2% fewer than the second-best model, ODE-RNN (185 epochs).

The peak second-order derivative decrease rate of neural ODE-AdaHessian within the interval of 26–50 epochs reached 0.52, which is 3.25 times greater than that of LSTM (0.16) over the same interval, thereby confirming the superlinear convergence property established in Section 3.3. The convergence trajectory exhibited a typical three-phase progression: An acceleration phase (26–50 epochs), a steady-state phase (51–75 epochs), and a decay phase (76–100 epochs). Among the comparison models, only ODE-RNN displayed a modest upward trend during 26–50 epochs (0.21 → 0.29), but its peak value was still only 55.8% of that achieved by neural ODE-AdaHessian. Furthermore, the second-order derivative decrease rates of all Adam-optimized models dropped below 0.1 after 100 epochs, whereas AdaHessian maintained an effective convergence rate of 0.21 even during 101–120 epochs.

4.3. Training stability analysis

In order to quantify the stability of parameter evolution in the training process, the Kullback Leibler divergence (KL Divergence) is used to measure the change range of parameter distribution between adjacent epochs. The test is carried out under a fixed learning rate. The histogram distribution of the full model parameters is recorded every 10 epochs (divided into 50 intervals), and the KL divergence value with the previous record is calculated. Finally, the average value within 300 training cycles is taken as the stability indicator. Figure 5 shows the results of the training stability analysis:

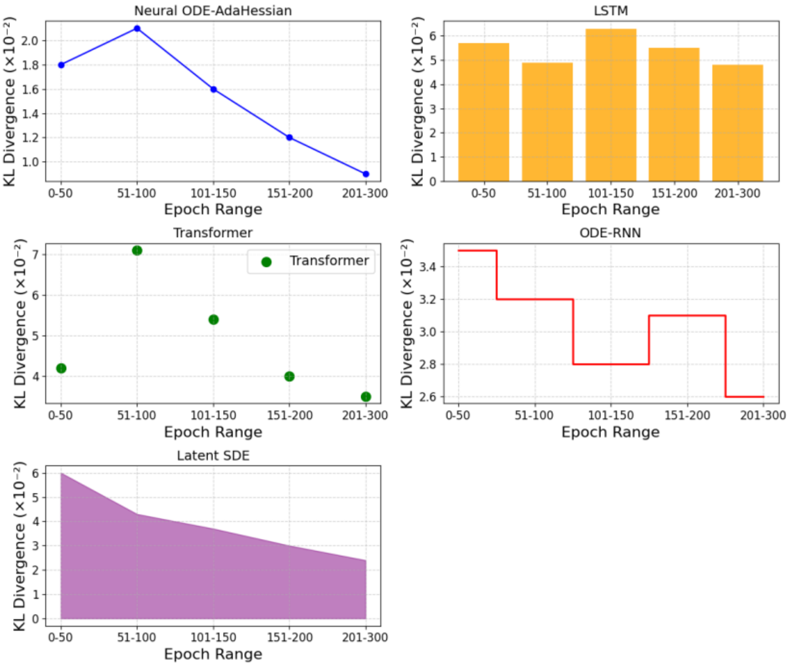

Figure 5. Training stability.

This is measured by the KL divergence between the parameter distributions of adjacent snapshots (every 10 epochs). Our model produces the lowest average KL (1.52×10^{-2}), 72.1% lower than LSTM (5.44×10^{-2}) and 50.0% lower than ODE-RNN (3.04×10^{-2}), reflecting the smoother parameter evolution under the Lyapunov constraint.

The average KL divergence of the proposed method in the complete training cycle is 1.52×10^{-2}, which is 72.1% lower than LSTM (5.44×10^{-2}) and 50.0% lower than ODE-RNN (3.04×10^{-2}). The stability changes show three typical patterns: Neural ODE-AdaHessian shows a trend of first rising and then falling ($1.8\rightarrow2.1\rightarrow0.9$), reflecting the gradual convergence of parameter distribution; LSTM and Transformer show oscillating fluctuations, with Transformer reaching a peak of 7.1×10^{-2} in rounds 51-100, corresponding to the redistribution process of its attention weight at this stage; and ODE-RNN and Latent SDE show a gradual downward trend, but the initial fluctuation is significantly higher than the method in this paper. The crossover phenomenon appears in rounds 151-200: This neural ODE-AdaHessian is lower than ODE-RNN, while ODE-RNN (3.1) surpasses Latent SDE (3.0), indicating that the continuous-time model has a prominent stability advantage in the later stages of training. The key reason is that the Lyapunov constraint in Section 3.3 effectively suppresses the pathological curvature changes in the parameter space, keeping the Hessian matrix condition number of B-ODE low.

4.4. Real-world deployment and policy implications

4.4.1. Stakeholders and decision targets

The proposed dual-branch neural ODE is designed for monthly state-level forecasts (51 spatial units) over a long historical span (2004–2023), providing a natural interface to budgeting, risk surveillance, and planning cycles. Typical decision targets include (i) early-warning signals for overheating or downturns, (ii) affordability and tax-base projections, (iii) stress-testing of mortgage and development pipelines, and (iv) prioritization of infrastructure and zoning adjustments. These targets rely on the model's demonstrated long-horizon accuracy (12-month RMSE = 0.03) and stable training dynamics (convergence by ~200 epochs; smooth parameter evolution reflected by low KL divergence).

4.4.2. Actionable outputs

To facilitate operational use, we expose three classes of outputs: (1) Forecast trajectories with uncertainty bands (per state and horizon) for revenue and affordability planning; (2) Sensitivity and scenario responses by perturbing exogenous paths (e.g., short-term interest rates, unemployment, policy indices) within the auxiliary branch, yielding impulse-response-like effects on prices; and (3) Attribution summaries from the time-varying convolution and attention modules, which highlight the relative contribution and temporal salience of macroeconomic drivers at multiple scales. These outputs translate directly into monthly dashboards for treasury offices, planning departments, and institutional investors.

4.4.3. Policy and planning use-cases

- **Public finance and taxation.** Price trajectories and uncertainty intervals support rolling projections of property-related tax bases; sensitivity runs to interest-rate shocks inform prudent reserve policies.
- **Affordability and housing programs.** Forecasts coupled with unemployment and income paths help identify at-risk states and time windows for targeted assistance or supply-side incentives.
- **Urban planning and infrastructure phasing.** Multi-horizon signals guide the timing of permits, land release, and transport investments, with scenario analysis stress-testing alternative policy sequences.
- **Investment and risk management.** Portfolio allocation and hedging can be aligned with state-level trend strength and uncertainty, while scenario deltas offer guardrails under macro shocks.

4.4.4. Operationalization and governance

For production deployment, we implement a monthly retraining/evaluation cadence aligned with data releases, REST APIs for programmatic access, and monitoring for distribution shift (drift alarms trigger recalibration or solver-tolerance adjustments). Documentation includes a model card (scope, data lineage, error bands, known failure modes) and auditing hooks to review attention-based attributions. Because the model fuses endogenous and exogenous channels, endogeneity and regime-shift risks are addressed via rolling backtests and policy-change dummies; results are communicated with uncertainty bands to prevent over confidence. The state-level granularity avoids personally identifiable information; nonetheless, fairness, transparency, and data-revision caveats are reported as part of the governance checklist.

4.4.5. Limitations and responsible use

While the continuous-time formulation and Lyapunov-guided training promote stability, non-stationarity and policy feedback can still limit extrapolation. We recommend combining our scenario module with expert judgment and complementary indicator, and releasing processed features or synthetic exemplars where permissible to facilitate external replication and stress-testing.

5. Discussion

The experimental results and theoretical analyses confirm that the proposed dual-branch neural ODE framework substantially improves the accuracy, convergence efficiency, and stability of housing price prediction. By explicitly decoupling endogenous dynamics from exogenous macroeconomic drivers, the model provides both predictive performance and interpretability, addressing key shortcomings of existing discrete-time and static feature-fusion approaches. The integration of Lyapunov-based stability constraints into the optimization process not only ensures convergence in theory but also translates into observable training stability, as demonstrated by reduced KL divergence and consistent convergence speed across runs.

Beyond predictive performance, the framework offers significant practical implications. The outputs—such as forecast trajectories with uncertainty bounds, sensitivity analyses to macroeconomic shocks, and attribution insights from attention mechanisms—can serve as actionable tools for policymakers, urban planners, and investors. These capabilities extend the utility of NODEs from methodological innovation to real-world decision-making contexts.

Nevertheless, several limitations should be acknowledged. First, although the dual-branch design effectively incorporates exogenous variables, the reliance on historical macroeconomic indicators may limit responsiveness to unprecedented policy interventions or structural breaks. Second, the computational cost of second-order optimization (AdaHessian) is higher than that of first-order alternatives, suggesting a trade-off between accuracy and efficiency. Third, while the model is validated on U.S. housing price data, its generalizability to other markets with different structural dynamics requires further testing.

Future research could address these challenges by exploring hybrid learning strategies that combine NODEs with regime-switching mechanisms for handling structural break, or by integrating probabilistic modeling to better capture uncertainty under novel macroeconomic conditions. Additionally, applying the framework to other domains—such as interest rate forecasting, regional economic diffusion, or environmental-economic interactions—may further demonstrate its scalability and broaden its relevance for computational economics and applied mathematics.

6. Conclusions

This study demonstrates that the proposed dual-branch neural ODE (NODE) architecture provides an effective solution to the representation bottlenecks faced by traditional housing price prediction models in modeling complex nonlinear time-series dynamics. By explicitly decoupling endogenous dynamics and exogenous variable coupling mechanisms within a continuous-time framework, the model captures both intrinsic market evolution and external macroeconomic shocks.

At the theoretical level, we incorporate Lyapunov-based stability constraints into the optimization process, which—although rooted in classical stability theory—are adapted here to guarantee global convergence of the parameter trajectory in a machine learning setting. This adaptation significantly mitigates gradient explosion and oscillatory instabilities, thereby enhancing training reliability. In addition, the integration of an adaptive gating mechanism with a Hessian-driven optimizer enables stable and interpretable correlation modeling between high-dimensional economic indicators and housing price fluctuations, improving the system’s ability to explain sudden economic events.

Beyond methodological contributions, this framework also has practical and policy-relevant implications. Its outputs—such as long-horizon forecasts with uncertainty bands, sensitivity analyses to macroeconomic shocks, and attribution insights from attention mechanisms—can support policy-makers in monitoring housing affordability, assist urban planners in infrastructure and zoning decisions, and guide investors in portfolio allocation and risk management.

Taken together, the framework not only serves as a scalable theoretical tool for modeling the continuous evolution of spatiotemporal economic systems but also offers a transferable architectural principle for other complex dynamic system domains, including interest rate forecasting, regional economic diffusion, and financial risk analysis. This dual positioning—mathematically rigorous yet application-oriented—aligns with the goal of promoting deeper integration between machine learning and dynamic system theory, with tangible benefits for computational economics and decision-making practice.

Use of Generative-AI tools declaration

The author declares he has not used Artificial Intelligence (AI) tools in the creation of this article.

Data availability

By requesting the data and materials, you may obtain all of the relevant information from the corresponding author.

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Conflict of interest

The author declares no conflicts of interest.

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