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Research article

Event-triggered proportional consensus with input constraints in adversarial networks

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Abstract: This paper addresses the problem of proportional consensus for multi-agent systems (MASs) under input constraints in adversarial network environments. A dynamic event-triggered proportional consensus control algorithm based on distributed observers was proposed. An input constraint mechanism was introduced to model actuator limitations, reflecting practical applications and ensuring reliable agent operation. On this basis, a dynamic event-triggered scheme was designed to significantly reduce communication load, improve resource utilization, and enhance resilience against denial-of-service (DoS) attacks. Leveraging fully distributed observers, each agent achieved proportional consensus using only local information. Theoretical analysis based on Lyapunov stability theory and graph theory provided sufficient conditions for achieving proportional consensus under DoS attacks, offering rigorous support for the method's feasibility. Simulation results demonstrated that the proposed approach effectively achieves proportional consensus for MASs under dual constraints of input saturation and adversarial attacks, while avoiding Zeno behavior.

Keywords: proportional consensus; denial-of-service attack; input saturation; dynamic event-triggered; fully distributed observer
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1. Introduction

The primary goal of multi-agent cooperative control is to achieve coordinated group behavior. Among various coordination strategies, proportional consensus has emerged as a more general and flexible form compared to traditional complete consensus. Unlike conventional consensus, which enforces identical states for all agents, proportional consensus requires each agent's state to converge to equilibrium points maintaining predefined proportional relationships. This capability has practical significance in numerous scenarios. For instance, in satellite formation flying missions [1], satellites must maintain specific relative distances rather than identical absolute positions to ensure stable orbits. Similarly, in multi-unmanned vehicle cooperative systems [2], vehicles are required to maintain specific speed ratios or proportional formations to accomplish complex tasks such as cooperative transport or regional coverage. Proportional coordination thus enables sophisticated collaborative behaviors, including target envelopment, formation reconfiguration, and load distribution, extending the applicability of classical consensus control.

Proportional consensus introduces scaling coefficients as degrees of freedom, enhancing system flexibility and adaptability compared with traditional consensus. References [3–6] developed distributed observers for proportional tracking, ignoring communication sparsity and adversarial attacks. References [7, 8] improved communication efficiency via intermittent control but failed to address actuator saturation—leaving a gap for proportional consensus under dual constraints of attacks and input limits. Reference [9] established sufficient conditions for H_∞ group consensus under fixed, switching, and delayed topologies, providing a theoretical foundation for proportional coordination among subgroups.

Despite these advances, practical deployment of proportional consensus faces three critical challenges: limited communication resources, adversarial network attacks, and actuator input constraints. Event-triggered control (ETC) and multi-rate sampling have emerged as key approaches to alleviate communication load. Traditional static ETC, using fixed thresholds, is insufficient for dynamically changing states. Dynamic event-triggered mechanisms, as proposed in [10], adaptively adjust thresholds via internal state variables, improving communication efficiency. Subsequent works [11–15] have developed distributed and observer-based dynamic triggering schemes that reduce communication frequency while avoiding Zeno behavior. However, the application of these strategies to proportional consensus, especially under strong disturbances or attacks, remains underexplored.

Security of communication links is essential for proportional consensus. Denial-of-service (DoS) attacks [16] can disrupt information exchange, leading to estimation errors and control failures. Existing studies [17–21] focus mainly on complete consensus or mean convergence, offering limited insight into proportional consensus. Quantitative relationships between attack frequency, duration, and proportional convergence performance remain largely unaddressed.

Actuator constraints, such as input saturation, further complicate the control design. Saturation nonlinearity limits the amplitude of control signals, which may degrade the convergence performance or even lead to system instability. Although existing studies [22–24] have addressed the consensus problem under input saturation, these works often neglect communication constraints or rely on global information, thus limiting their practical applicability. It is worth noting that in the broader field of nonlinear control, advanced techniques such as command filtering [25], nonsingular predefined-time control [26], and fuzzy adaptive output feedback [27] have been developed to effectively handle state constraints and system uncertainties. However, the combined effects of communication attacks and input saturation on proportional consensus in multi-agent systems have not been thoroughly investigated.

On the other hand, recent advances in multi-agent network controllability—an essential prerequisite for achieving consensus—have provided new insights. Lu et al. [28] analyzed the controllability of discrete-time multi-agent systems via impulsive/switching systems, demonstrating that periodic topology switching can enhance controllability by breaking structural limitations. Long et al. [29] derived a controllability criterion for impulsive multi-agent systems, revealing that the number of control nodes and the impulse intervals jointly determine system performance. These findings underscore the importance of incorporating network dynamics, such as topology switching induced by DoS attacks, into the control design—an idea that this work extends to the problem of proportional consensus under both saturation and attack constraints.

In this context, achieving proportional consensus faces two major challenges: (1) the nonlinearity introduced by input saturation complicates the stability analysis, and (2) the intermittent communication interruptions caused by DoS attacks disrupt the cooperative control paradigm. To address these challenges, this paper proposes the following three innovative contributions:

1) A fully distributed event-triggered observer is designed. Unlike existing schemes [13, 15] that rely on semi-global topological information, the proposed observer utilizes sparse dynamic triggering signals to maintain state estimation during intermittent DoS attacks, significantly reducing communication overhead while ensuring estimation accuracy.
2) A low-gain feedback protocol with an adaptive anti-saturation compensator is developed. In contrast to conventional methods [22, 23] that require global Laplacian eigenvalues, this protocol dynamically adjusts control inputs using only local neighbor information to prevent actuator saturation, thereby eliminating the need for global topological knowledge.
3) Sufficient conditions for achieving proportional consensus under the dual constraints of DoS attacks and input saturation are established. Unlike prior studies [17–21] that only address complete consensus under DoS attacks, this work explicitly reveals the quantitative relationships among attack frequency/duration, saturation depth, and convergence rate, providing a rigorous theoretical foundation for system design.

Notations. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ the n-dimensional real vector space, and $\mathbb{R}^{n\times n}$ the set of $n \times n$ real matrices. $\mathbb{N}$ and $\mathbb{N}+$ represent the set of integers and positive integers, respectively. For a matrix A, A^T, A^{-1}, and $A > 0$ denote its transpose, inverse, and positive definiteness I_N is the N-dimensional identity matrix, and diag$\cdot$ is a diagonal matrix. sgn$(\cdot)$ is the sign function, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the minimum and maximum eigenvalues of a symmetric matrix A, $|\cdot|$ denotes the norm, and $\otimes$ is the Kronecker product.

2. Preliminaries

2.1. Graph theory

The communication topology among N agents is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \varepsilon)$, where $\mathcal{V} = v_1, v_2, \ldots, v_N$ denotes the set of nodes and $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of edges, with each edge defined by a pair of distinct nodes. If $(v_i, v_j) \in \varepsilon$, node v_i is called a neighbor of node v_j, and node v_j is referred to as an out-neighbor of node v_i. For graph $\mathcal{G}$, the adjacency matrix is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$, where $a_{ii} = 0$ and $a_{ij} = 1$ if $(v_j, v_i) \in \varepsilon$; otherwise, $a_{ij} = 0$. The Laplacian matrix associated with $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$, with $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. The degree of node i is defined as $d_i = \sum_{j\in N_i} a_{ij}$, where $N_i = j \in \mathcal{V} \mid (v_j, v_i) \in \varepsilon$ denotes the set of neighbors of node i. The corresponding degree matrix is $\mathrm{D} = \mathrm{diag} d_1, d_2, \ldots, d_N$.

2.2. Problem formulation

Consider N agents, and the dynamic model of each agent is as follows:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + B\delta_s(u_i(t)), \\ y_i(t) = Cx_i(t), \end{cases} \tag{2.1}$$

where A, B, C are constant matrices and $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, and $y_i(t) \in \mathbb{R}^p$ are the position state, input, and output, respectively. The saturation function $\delta_s(u_i(t)) = (\delta_s(u_{i1}(t)), \delta_s(u_{i2}(t)), \cdots, \delta_s(u_{im}(t)))$, and $\delta_s(u_{iw}(t)) = \text{sgn}(u_{iw}(t)) \min\{|u_{iw}(t)|, \varpi\}$ for $w \in \{1, 2, \cdots, m\}$, where ϖ is a positive constant.

Considering the case where the system state is unmeasurable, the following state observer is designed:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + H(\hat{x}_i(t) - x_i(t)), \\ \hat{y}_i(t) = C\hat{x}_i(t), \end{cases} \tag{2.2}$$

where H is an observer gain, $\hat{x}_i(t) \in \mathbb{R}^n$, and $\hat{y}_i(t) \in \mathbb{R}^p$ are the observed state and observed output, respectively.

According to the characteristics of attacks, the time axis is divided into two parts. Let $T_k = [t_{2k-1}, t_{2k})$ be the k-th attack interval. For the entire time period $[0, t)$, define $\Xi_a(0, t) = \bigcup_{k\in\mathbb{N}}[t_{2k-1}, t_{2k}) \cap [0, t)$ and $\Xi_s(0, t) = [0, t) \setminus \Xi_a(0, t)$ as the set intervals where attacks occur and do not occur, respectively.

Remark 2.1. The saturation function $\delta_s(u_i(t))$ in the multi-agent dynamic model simulates the power limitation of the actuator. When the input $u_i(t)$ exceeds the threshold ϖ, the output is limited within the threshold range, avoiding control failure due to actuator overload and making the model more consistent with actual hardware characteristics.

The following are some general assumptions and useful lemmas.

Assumption 2.1 (Attack Duration [30].) There exist positive constants N_1 and τ_a such that the following inequality holds:

$$\Xi_a(t_0, t) \leq N_1 + \frac{t}{\tau_a}. \tag{2.3}$$

Assumption 2.2 (Attack Frequency [30].) Define $D(t_0, t)$ as the total number of DoS attacks occurring in $[t_0, t)$. Then there exist constants $N_2 > 0$ and $\tau_b > 0$ such that

$$D(t_0, t) \leq N_2 + \frac{t}{\tau_b}. \tag{2.4}$$

Remark 2.2. Assumption 2.1 limits the total duration of DoS attacks, ensuring that attacks will not last indefinitely, which provides a premise for subsequently deriving the stability conditions of the system under attacks, i.e., the attack intensity must be within a tolerable range. Assumption 2.2 constrains the number of DoS attacks per unit time, avoiding excessively frequent attacks that make the system unable to recover, which is an important condition to ensure that the system can eventually achieve consensus. Assumptions 2.1 and 2.2 constrain the energy and frequency of admissible DoS attacks, ensuring they are not catastrophic. These are system-level assumptions. To mitigate such attacks, each agent requires a local mechanism to detect communication outages. We define a piecewise function $\varphi(t)$ to represent the local attack status.

For the convenience of subsequent analysis, the following piecewise function is given to represent the time when attacks occur and do not occur:

$$\varphi(t) = \begin{cases} 0, & t \in \Xi_s(\tau, t), \\ 1, & t \in \Xi_a(\tau, t). \end{cases} \tag{2.5}$$

Assumption 2.3 [31]. (A, B) is stabilizable.
Assumption 2.4 [32]. (A, C) is detectable.
Assumption 2.5 [33]. The communication topology graph G is undirected and connected.
Lemma 2.1 [34]. Under Assumption 2.3, by solving the algebraic Riccati equation:

$$A^T P(\varsigma) + P(\varsigma)A - P(\varsigma)BR^{-1}B^T P(\varsigma) + \varsigma I = 0, \tag{2.6}$$

a unique positive definite solution $P(\varsigma)$ is obtained, where $\varsigma \in (0, 1]$ is a low-gain parameter.
Lemma 2.2 [35]. Under Assumption 2.4, by solving the algebraic Riccati equation:

$$AQ + QA^T - QC^T CQ + I = 0, \tag{2.7}$$

a unique positive definite solution Q is obtained.

The objective of *proportional consensus* is for the states of agents to converge to values that preserve predetermined proportional relationships, rather than converging to a common value. This is mathematically defined as:

$$\lim_{t\to\infty}(\mu_i x_i(t) - \mu_j x_j(t)) = 0, \quad \forall i, j \in \{1, 2, \ldots, N\},$$

where $\mu_i > 0$ is the scaling factor for agent i. This capability is crucial in applications such as formation control, where agents must maintain relative positions or speeds, and load distribution, where tasks must be allocated proportionally to agent capabilities.

Definition 2.1. *(Semi-global proportional consensus with actuator saturation)* A multi-agent system with actuator saturation achieves semi-global proportional consensus if and only if for any large and bounded initial state set $\chi \subset \mathbb{R}^n$, the following holds:

$$\lim_{t\to\infty} \|\mu_i x_i(t) - \mu_j x_j(t)\| = 0 \quad \text{and} \quad \|u_i(t)\|_\infty \leq \varpi, \quad \forall i, j,$$

where $\mu_i > 0$ is a predefined scaling constant.

3. Results

3.1. Controller and observer design

In this section, the fully distributed consensus problem of MAS (2.1) with event-triggered communication, actuator saturation, and unmeasurable states under DoS attacks is solved. The detailed design scheme analysis is as follows.

Define the observation measurement error of the i-th agent as:

$$e_i(t) = \hat{x}_i(t_k^i) - \hat{x}_i(t), \quad t \in [t_k^i, t_{k+1}^i), \tag{3.1}$$

where t_k^i denotes the k-th event triggering time of the i-th agent.

Considering the situation where the communication channel is subjected to DoS attacks, to mitigate the impact of attacks on system stability, a reasonable and effective compensation mechanism is designed as follows:

$$\hat{x}_j(t_{k'}^j) = (1 - \varphi(t))\hat{x}_j(t_{k'}^j) + \varphi(t)\hat{x}_j(\tilde{t}_{k'}^j),$$

where $t_{k'}^j$ denotes the k'-th triggering time of the j-th agent, and $\tilde{t}_{k'}^j$ denotes the latest time when agent j successfully transmitted information to its neighbor agents before the system was attacked.

The fully distributed adaptive controller based on the observer is designed as:

$$u_i(t) = (1 - \varphi(t))\, u_{i,s} + \varphi(t)\, u_{i,a}, \tag{3.2}$$

$$\dot{c}_{ij} = (1 - \varphi(t))\, \dot{c}_{ij}^s(t) + \varphi(t)\, \dot{c}_{ij}^a(t), \tag{3.3}$$

where

$$\begin{aligned}
u_{i,s}(t) &= K \sum_{j=1}^{N} c_{ij}(t) a_{ij}(\hat{x}_i(t_k^i) - \frac{\mu_j}{\mu_i}\hat{x}_j(t_{k'}^j)),\\
u_{i,a}(t) &= K \sum_{j=1}^{N} c_{ij}(t)(\hat{x}_i(t_k^i) - \frac{\mu_j}{\mu_i}\hat{x}_j(\tilde{t}_{k'}^j)),\\
\dot{c}_{ij}^s &= k_{ij}[-m_{ij}c_{ij}(t) + a_{ij}(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j))],\\
\dot{c}_{ij}^a &= k_{ij}[-m_{ij}c_{ij}(t) + (\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))],
\end{aligned}$$

where K and Γ are feedback gain matrices; since the topology graph is undirected, there are positive constants $k_{ij} = k_{ji} > 0$ and $c_{ij}(t) = c_{ji}(t) > 0$.

The event triggering function of agent i is:

$$f_i(t) = (1 - \varphi(t))\, f_{i,s}(t) + \varphi(t)\, f_{i,a}(t), \tag{3.4}$$

where

$$\begin{aligned}
f_{i,s}(t) = \theta_i\Bigg[&\sum_{j=1}^{N}\left(\left(\frac{1}{\delta_2} - 1\right) + \gamma c_{ij}(t)\right) a_{ij}\mu_i^2 e_i^T(t)\Gamma e_i(t)\\
&- \frac{1}{4}\sum_{j=1}^{N} a_{ij}\left(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j)\right)^T \Gamma\left(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j)\right)\Bigg] - \eta_i^s(t),\\
f_{i,a}(t) = \theta_i\Bigg[&\sum_{j=1}^{N}\left(\left(\frac{1}{\delta_2} - 1\right) + \gamma c_{ij}(t)\right) \mu_i^2 e_i^T(t)\Gamma e_i(t)\\
&- \frac{1}{4}\sum_{j=1}^{N} \left(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)\right)^T \Gamma\left(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)\right)\Bigg] - \eta_i^a(t),
\end{aligned}$$

and θ_i, γ, $0 < \delta_2 < 1$ are positive constants to be determined; η_i^s, η_i^a are internal dynamic variables, and their time derivatives satisfy:

$$\begin{aligned}\dot{\eta}_i^s(t) =& -\alpha_i\eta_i^s(t) + \beta_i[\frac{1}{4}\sum_{j=1}^{N} a_{ij}(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j))^T\Gamma\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j))] \\ &- \sum_{j=1}^{N}((\frac{1}{\delta_2} - 1) + \gamma c_{ij}(t))a_{ij}\mu_i^2 e_i^T(t)\Gamma e_i(t)],\end{aligned} \tag{3.5}$$

$$\begin{aligned}\dot{\eta}_i^a(t) =& -\alpha_i\eta_i^a(t) + \beta_i[\frac{1}{4}\sum_{j=1}^{N}(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))] \\ &- \sum_{j=1}^{N}((\frac{1}{\delta_2} - 1) + \gamma c_{ij}(t))\mu_i^2 e_i^T(t)\Gamma e_i(t)],\end{aligned} \tag{3.6}$$

where $\alpha_i > 0$ and $0 < \beta_i < 1$ are positive constants.

The triggering mechanism of agent i is:

$$t_{k+1}^i = inf\left\{t > t_k^i | f_i(t) \geq 0\right\}, k = 1, 2, \cdots. \tag{3.7}$$

Define the tracking error of agent i as $\Psi_i = \frac{1}{N}\sum_{j=1}^{N}(\mu_i x_i - \mu_j x_j)$. From (2.1), we can obtain:

$$\dot{\Psi}_i = A\Psi_i + \mu_i B\delta_s(u_i) - \frac{1}{N}B\sum_{j=1}^{N}\mu_j\delta_s(u_j). \tag{3.8}$$

Let $\hat{\Psi}_i = \frac{1}{N}(\mu_i\hat{x}_i - \mu_j\hat{x}_j)$. Combining with $\hat{x}_i$ in (2.2), we can obtain:

$$\dot{\hat{\Psi}}_i = A\hat{\Psi}_i - HC(\Psi_i - \hat{\Psi}_i) + \mu_i Bu_i - \frac{1}{N}B\sum_{j=1}^{N}\mu_j u_j. \tag{3.9}$$

Since $a_{ij} = a_{ji}$ and $c_{ij} = c_{ji}$, we can obtain:

$$\sum_{i=1}^{N}\mu_i u_i = \sum_{j=1}^{N}\mu_j u_j = \mu_i K\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)a_{ij}\left(\hat{x}_i(t_k^i) - \frac{\mu_j}{\mu_i}\hat{x}_j(t_{k'}^j)\right) = 0. \tag{3.10}$$

Substituting (3.11) into (3.10), we can obtain:

$$\dot{\hat{\Psi}}_i = A\hat{\Psi}_i - HC(\Psi_i - \hat{\Psi}_i) + B\mu_i u_i. \tag{3.11}$$

Let $\xi_i = \hat{\Psi}_i - \Psi_i$. Combining with Eqs (3.8) and (3.11), we can obtain:

$$\dot{\xi}_i = (A + HC)\xi_i + B\mu_i u_i - B\left(\mu_i\delta_s(u_i) - \frac{1}{N}\sum_{j=1}^{N}\mu_j\delta_s(u_j)\right). \tag{3.12}$$

3.2. Stability proof

Theorem 3.1. Under Assumptions 2.1–2.5, Lemmas 2.1 and 2.2, and the event-triggering condition (3.7), consider the multi-agent system (2.1) with observer (2.2) and controller (3.2) under DoS attacks. If we set $H = -QC^T$, $K = -B^T P$, and select the relevant positive parameters $\theta_i, \gamma, \alpha_i, \beta_i \in (0, 1)$, k_{ij}, and m_{ij}, we then choose

$$a_1 = \min\left\{\frac{\varsigma - \delta_1}{\lambda_{\max}(P)}, \frac{\rho}{\lambda_{\max}(\tilde{Q})}, k_{ij}m_{ij}, \alpha_i - \frac{1-\beta_i}{\theta_i}\right\}, \quad a_2 > 0, \quad z > 1,$$

$$\hat{a} = a_1 - \frac{2\ln z}{\tau_b} - \frac{a_1 + a_2}{\tau_a} > 0,$$

where $\alpha_i - \frac{1-\beta_i}{\theta_i} > 0$, $\rho > 0$, and $\varsigma - \delta_1 > 0$. Then, there exists a parameter $\bar{\varsigma} \in (0, 1]$ such that for any $\varsigma \in (0, \bar{\varsigma})$, with arbitrarily large but bounded initial states $\chi \in \mathbb{R}^n$ and $\chi_1 \in \mathbb{R}$, the dynamic event-triggered fully distributed observer-based scaled consensus is achieved under DoS attacks with actuator saturation.

Proof. Select the Lyapunov function as follows:

$$V(t) = V_1(t) + V_2(t), \tag{3.13}$$

where

$$V_1(t) = \sum_{i=1}^{N} \hat{\Psi}_i^T P \hat{\Psi}_i + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \frac{(c_{ij} - k)^2}{4k_{ij}},$$

$$V_2(t) = \sum_{i=1}^{N} k\eta_i(t) + \sigma \sum_{i=1}^{N} \xi_i^T \tilde{Q} \xi_i,$$

matrix $P > 0$, $\tilde{Q} > 0$, σ is a positive constant, and k is a positive constant to be designed. According to Eqs (3.5) and (3.8), we know that

$$\dot{\eta}_i(t) \geq -\left(\alpha_i + \frac{\beta_i}{\theta_i}\right)\eta_i(t) > 0, \quad \forall t \geq 0,$$

and further,

$$\eta_i(t) \geq \eta_i(0) e^{-\left(\alpha_i + \frac{\beta_i}{\theta_i}\right)t} > 0, \quad \forall t \geq 0.$$

Then, $V(t) > 0$.

First,we will discuss the actuator saturation. There exists a constant Δ such that for any $c_{ij}(0) \in \chi_1$ and $\hat{\Psi}_i(0), \xi_i(0) \in \chi$, we have

$$\sup\left\{V\left(\hat{\Psi}_i(0), \xi_i(0), c_{ij}(0), 0\right)\right\} \leq \Delta + 1. \tag{3.14}$$

Define the set $L_V(\Delta + 1)$ as

$$L_V(\Delta + 1) = \left\{\hat{\Psi}_i, \xi_i, c_{ij} : V\left(\hat{\Psi}_i, \xi_i, c_{ij}, t\right) \leq \Delta + 1\right\}. \tag{3.15}$$

Obviously, when $\hat{\Psi}_i, \xi_i, c_{ij} \in L_V(\Delta + 1)$, $\hat{\Psi}_i$, ξ_i, and c_{ij} are bounded. Since $\hat{\Psi}_i = \mu_i \hat{x}_i - \frac{1}{N}\sum_{j=1}^{N} \mu_j \hat{x}_j$ is bounded, $\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_k^j)$ is bounded within a finite time, and then the control input $u_i(t)$ is also bounded. Under Lemma 2.1, $\lim_{\varsigma\to 0} P(\varsigma) = 0$. Therefore, for any $\hat{\Psi}_i, \xi_i, c_{ij} \in L_V(\Delta + 1)$, there exists a constant $\bar{\varsigma}$ such that when $\varsigma \in (0, \bar{\varsigma})$, $\|u_i\| \leq \varpi$.

Therefore, when $0 \leq t < +\infty$, substituting (3.11) into (3.13), we can obtain:

$$\dot{\xi}_i = (A + HC)\xi_i + B\mu_i u_i - B\mu_i u_i + \frac{1}{N}\sum_{j=1}^{N}\mu_j u_j = (A + HC)\xi_i. \tag{3.16}$$

Next, according to the characteristics of attacks, the system stability analysis is divided into two cases: DoS attack and no attack.

Case 1: When $t \in \Xi_s(\tau, t)$ and $\varphi(t) = 0$, agents maintain normal communication. For this case, the stability proof is given below.

According to Eqs (3.2), (3.3), and (3.11), the derivative of $V_1(t)$ is obtained as:

$$\begin{aligned}
\dot{V}_1(t) = {} & 2\sum_{i=1}^{N}\hat{\Psi}_i^T P\dot{\hat{\Psi}}_i + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}-k)}{2k_{ij}}\dot{c_{ij}}(t) \\
= {} & 2\sum_{i=1}^{N}\hat{\Psi}_i^T PA\hat{\Psi}_i + 2\sum_{i=1}^{N}\hat{\Psi}_i^T PHC\xi_i + \sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\frac{c_{ij}(t)-k}{2}(-m_{ij}c_{ij}(t)) \\
& + 2\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}\hat{\Psi}_i^T PBK(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j)) \\
& + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{ij}a_{ij}(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j)) \\
& - \frac{k}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j)).
\end{aligned} \tag{3.17}$$

The third term on the right-hand side of the above equation can be bounded as:

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\frac{c_{ij}(t)-k}{2}(-m_{ij}c_{ij}(t)) \\
&\leq \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2 - \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2.
\end{aligned} \tag{3.18}$$

Since $c_{ij} = c_{ji}$ and $a_{ij} = a_{ji}$, and according to $K = -B^T P$ and $\Gamma = PBB^T P$, the fourth term on the right-hand side of the equation can be transformed into:

$$\begin{aligned}
&2\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}\hat{\Psi}_i^T PBK(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j)) \\
&= -\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}(\hat{\Psi}_i - \hat{\Psi}_j)^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(t_{k'}^j)).
\end{aligned} \tag{3.19}$$

Since $\hat{\Psi}_i - \hat{\Psi}_j = \mu_i \hat{x}_i - \mu_j \hat{x}_j$ and $e_i = \hat{x}_i(t_k^i) - \hat{x}_i(t)$, we can obtain:

$$\begin{aligned}
& (\mu_i e_i - \mu_j e_j)^T \Gamma (\mu_i e_i - \mu_j e_j) \\
= & [(\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)) - (\hat{\Psi}_i - \hat{\Psi}_j)]^T \Gamma \\
& [(\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)) - (\hat{\Psi}_i - \hat{\Psi}_j)] \\
= & (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j))^T \Gamma (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)) \\
& + (\hat{\Psi}_i - \hat{\Psi}_j)^T \Gamma (\hat{\Psi}_i - \hat{\Psi}_j) - 2(\hat{\Psi}_i - \hat{\Psi}_j)^T \Gamma (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)).
\end{aligned} \tag{3.20}$$

Substituting Eq (3.20) into Eq (3.19), we get:

$$\begin{aligned}
& 2 \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} a_{ij} \hat{\Psi}_i^T PBK (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)) \\
= & -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} a_{ij} (\hat{\Psi}_i - \hat{\Psi}_j)^T \Gamma (\hat{\Psi}_i - \hat{\Psi}_j) \\
& -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} a_{ij} (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j))^T \Gamma (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)) \\
& +\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} a_{ij} (\mu_i e_i - \mu_j e_j)^T \Gamma (\mu_i e_i - \mu_j e_j).
\end{aligned} \tag{3.21}$$

According to $\hat{\Psi}_i - \hat{\Psi}_j = \mu_i \hat{x}_i - \mu_j \hat{x}_j$ and $e_i = \hat{x}_i(t_k^i) - \hat{x}_i(t)$, half of the sixth term on the right-hand side of Eq (3.17) is transformed into:

$$\begin{aligned}
& -\frac{k}{4} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} a_{ij} (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j))^T \Gamma (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)) \\
= & -\frac{k}{4} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} a_{ij} (\mu_i e_i - \mu_j e_j)^T \Gamma (\mu_i e_i - \mu_j e_j) \\
& -\frac{k}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} a_{ij} (\mu_i e_i - \mu_j e_j)^T \Gamma (\hat{\Psi}_i - \hat{\Psi}_j) \\
& -\frac{k}{4} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} a_{ij} (\hat{\Psi}_i - \hat{\Psi}_j)^T \Gamma (\hat{\Psi}_i - \hat{\Psi}_j).
\end{aligned} \tag{3.22}$$

According to the Young's inequality, Eq (3.22) can be transformed into:

$$\begin{aligned}
& -\frac{k}{4} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} a_{ij} (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j))^T \Gamma (\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j)) \\
\leq \quad & \frac{k}{4} (\frac{1}{\delta_2} - 1) \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} a_{ij} (\mu_i e_i - \mu_j e_j)^T \Gamma (\mu_i e_i - \mu_j e_j)
\end{aligned}$$

$$-\frac{k}{4}(1-\delta_2)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}(\hat{\Psi}_i-\hat{\Psi}_j)^T\Gamma(\hat{\Psi}_i-\hat{\Psi}_j). \tag{3.23}$$

Using Young's inequality, and substituting Eqs (3.18), (3.21), and (3.23) into Eq (3.17), we can obtain:

$$\begin{aligned}\dot{V}_1(t)\leq\ & 2\sum_{i=1}^{N}\hat{\Psi}_i^TPA\hat{\Psi}_i+\delta_1\sum_{i=1}^{N}\hat{\Psi}_i^T\hat{\Psi}_i+\frac{1}{\delta_1}\sum_{i=1}^{N}\|PHC\|_2^2\xi_i^T\xi_i\\ &-\frac{k}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(t_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(t_{k'}^j))\\ &+2\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}a_{ij}\mu_i^2e_i^T\Gamma e_i+k(\frac{1}{\delta_2}-1)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}\mu_i^2e_i^T\Gamma e_i\\ &-\frac{k}{4}(1-\delta_2)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}a_{ij}(\hat{\Psi}_i-\hat{\Psi}_j)^T\Gamma(\hat{\Psi}_i-\hat{\Psi}_j)\\ &+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2.\end{aligned} \tag{3.24}$$

According to Eqs (3.5) and (3.16), the derivative of $V_2(t)$ is obtained as:

$$\begin{aligned}\dot{V}_2(t)=&\sum_{i=1}^{N}k\dot{\eta}_i^s(t)+2\sigma\sum_{i=1}^{N}\xi_i^T(t)\tilde{Q}\xi_i(t)\\ =&-\sum_{i=1}^{N}k\alpha_i\eta_i^s(t)-\sum_{i=1}^{N}k\beta_i[\sum_{j=1}^{N}((\frac{1}{\delta_2}-1)+\gamma c_{ij})a_{ij}\mu_i^2e_i^T\Gamma e_i\\ &-\frac{1}{4}\sum_{j=1}^{N}a_{ij}(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(t_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(t_{k'}^j))]\\ &+2\sigma\sum_{i=1}^{N}\xi_i^T(t)\tilde{Q}(A+HC)\xi_i(t).\end{aligned} \tag{3.25}$$

Combining Eqs (3.24) and (3.25), we can obtain $\dot{V}(t)$ as:

$$\begin{aligned}\dot{V}(t)=\ &\dot{V}_1(t)+\dot{V}_2(t)\\ \leq\ &\hat{\Psi}^T(I\otimes(PA+A^TP)+\delta_1I-\frac{k}{2}(1-\delta_2)L\otimes\Gamma)\hat{\Psi}+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2\\ &+\sum_{i=1}^{N}\xi_i^T(\frac{1}{\delta_1}\|PHC\|_2^2I+\sigma(\tilde{Q}(A+HC)+(A+HC)^T\tilde{Q}))\xi_i\\ &-k(1-\beta_i)[\frac{1}{4}\sum_{j=1}^{N}a_{ij}(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(t_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(t_{k'}^j))\end{aligned}$$

$$
\begin{aligned}
&-\sum_{j=1}^{N}((\frac{1}{\delta_2}-1)+\gamma c_{ij})a_{ij}\mu_i^2 e_i^T\Gamma e_i]-\sum_{i=1}^{N}k\alpha_i\eta_i^s(t)-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2\\
\leq\ &\hat{\Psi}^T(I\otimes(PA+A^TP)+\delta_1 I-\frac{k}{2}(1-\delta_2)L\otimes\Gamma)\hat{\Psi}\\
&+\sum_{i=1}^{N}\xi_i^T(\frac{1}{\delta_1}\|PHC\|_2^2 I+\sigma(\tilde{Q}(A+HC)+(A+HC)^T\tilde{Q}))\xi_i\\
&-k\sum_{i=1}^{N}(\alpha_i-\frac{1-\beta_i}{\theta_i})\eta_i^s(t)+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2,
\end{aligned}
\tag{3.26}
$$

where $k\geq\max\left\{\frac{2}{\gamma},\frac{4}{\lambda_2(L)}\right\}$.

According to Lemma 2.1, we can obtain $PA+A^TP-\Gamma=-\varsigma I$. According to Lemma 2.2, it is known that there exist constants $\rho>0,\sigma>0$, and a positive definite matrix $\tilde{Q}$ such that $\frac{1}{\delta_1}\|PHC\|_2^2 I+\sigma(\tilde{Q}(A+HC)+(A+HC)^T\tilde{Q})=-\rho I$. Therefore, Eq (3.26) can be written as:

$$
\begin{aligned}
\dot{V}(t)\leq&-(\varsigma-\delta_1)\hat{\Psi}^T\hat{\Psi}-\rho\xi^T\xi-k\sum_{i=1}^{N}(\alpha_i-\frac{1-\beta_i}{\theta_i})\eta_i(t)-\sum_{i=1}^{N}k\alpha_i\eta_i^s(t)\\
&-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2,
\end{aligned}
\tag{3.27}
$$

where $M=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2$.

Combining Eqs (3.13) and (3.27), from the conditions of Theorem 3.1, we can obtain:

$$
\begin{aligned}
\dot{V}(t)<&-a_1V(t)+(a_1\lambda_{max}(P)-(\sigma-\delta_1))\sum_{i=1}^{N}\hat{\Psi}_i^T\hat{\Psi}_i+(a_1\lambda_{max}(\tilde{Q})-\rho)\sum_{i=1}^{N}\xi_i^T\xi_i\\
&+\frac{1}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\frac{a_1}{k_{ij}}-m_{ij})(c_{ij}-k)^2+[a_1-(\alpha_i-\frac{1-\beta_i}{\theta_i})]\sum_{i=1}^{N}k\eta_i^s(t)\\
<&-a_1V(t)+M.
\end{aligned}
\tag{3.28}
$$

Case 2: When $t\in\Xi_a(\tau,t)$ and $\varphi(t)=1$, the communication channels between agents are attacked, and agents cannot communicate normally. For this case, the stability proof is given below.

According to Eqs (3.2), (3.3), and (3.11), the derivative of $V_1(t)$ is obtained as:

$$
\begin{aligned}
\dot{V}_1(t)=\ &2\sum_{i=1}^{N}\hat{\Psi}_i^TP\dot{\hat{\Psi}}_i+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}(t)-k)^2}{2k_{ij}}\dot{c}_{ij}(t)\\
=\ &2\sum_{i=1}^{N}\hat{\Psi}_i^TP(A\hat{\Psi}_i+HC\xi_i+BK\sum_{j=1}^{N}c_{ij}(t)(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j)))\\
&+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}(t)-k)}{2k_{ij}}k_{ij}(-m_{ij}c_{ij}(t)+(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j))
\end{aligned}
$$

$$
\begin{aligned}
=\ & 2\sum_{i=1}^{N}\hat{\Psi}_i^T PA\hat{\Psi}_i + 2\sum_{i=1}^{N}\hat{\Psi}_i^T PHC\xi_i + 2\sum_{i=1}^{N}\hat{\Psi}_i^T PBK\sum_{j=1}^{N}c_{ij}(t)(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)) \\
& + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} c_{ij}(t)(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)) \\
& - \frac{k}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} (\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)) \\
& + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \frac{(c_{ij}(t) - k)}{2}(-m_{ij}c_{ij}(t)).
\end{aligned} \tag{3.29}
$$

Since $c_{ij}(t) = c_{ji}(t)$, and according to $K = -B^T P$ and $\Gamma = PBB^T P$, the third term on the right-hand side of the equation can be transformed into:

$$
\begin{aligned}
& 2\sum_{i=1}^{N}\hat{\Psi}_i^T PBK\sum_{j=1}^{N}c_{ij}(t)(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)) \\
=& -\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)(\hat{\Psi}_i - \hat{\Psi}_j)^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)).
\end{aligned} \tag{3.30}
$$

From Eq (3.21) above, we can obtain:

$$
\begin{aligned}
& 2\sum_{i=1}^{N}\hat{\Psi}_i^T PBK\sum_{j=1}^{N}c_{ij}(t)(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)) \\
=& -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)(\hat{\Psi}_i - \hat{\Psi}_j)^T\Gamma(\hat{\Psi}_i - \hat{\Psi}_j) \\
& -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)) \\
& +\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}(t)(\mu_i e_i - \mu_j e_j)^T\Gamma(\mu_i e_i - \mu_j e_j).
\end{aligned} \tag{3.31}
$$

According to $\hat{\Psi}_i - \hat{\Psi}_j = \mu_i\hat{x}_i - \mu_j\hat{x}_j$ and $e_i = \hat{x}_i(t_k^i) - \hat{x}_i(t)$, half of the sixth term on the right-hand side of Eq (3.29) is transformed into:

$$
\begin{aligned}
& -\frac{k}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)) \\
=& -\frac{k}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\mu_i e_i - \mu_j e_j)^T\Gamma(\mu_i e_i - \mu_j e_j) \\
& -\frac{k}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\mu_i e_i - \mu_j e_j)^T\Gamma(\hat{\Psi}_i - \hat{\Psi}_j)
\end{aligned}
$$

$$-\frac{k}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\hat{\Psi}_i-\hat{\Psi}_j)^T\Gamma(\hat{\Psi}_i-\hat{\Psi}_j). \tag{3.32}$$

According to Young's inequality, Eq (3.32) can be transformed into:

$$\begin{aligned}
&-\frac{k}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j))\\
\leq\ &\frac{k}{4}(\frac{1}{\delta_2}-1)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\mu_ie_i-\mu_je_j)^T\Gamma(\mu_ie_i-\mu_je_j)\\
&-\frac{k}{4}(1-\delta_2)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\hat{\Psi}_i-\hat{\Psi}_j)^T\Gamma(\hat{\Psi}_i-\hat{\Psi}_j).
\end{aligned} \tag{3.33}$$

The second and fourth terms on the right-hand side of Eq (3.29) are processed in the same way as Eqs (3.18) and (3.24). Then, substituting Eqs (3.31) and (3.33) into Eq (3.29), we can obtain:

$$\begin{aligned}
\dot{V}_1(t)\leq\ &2\sum_{i=1}^{N}\hat{\Psi}_i^TPA\hat{\Psi}_i+\delta_1\sum_{i=1}^{N}\hat{\Psi}_i^T\hat{\Psi}_i+\frac{1}{\delta_1}\sum_{i=1}^{N}\|PHC\|_2^2\xi_i^T\xi_i\\
&-\frac{k}{4}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j))\\
&+2\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}\mu_i^2e_i^T\Gamma e_i+k(\frac{1}{\delta_2}-1)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\mu_i^2e_i^T\Gamma e_i\\
&-\frac{k}{4}(1-\delta_2)\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(\hat{\Psi}_i-\hat{\Psi}_j)^T\Gamma(\hat{\Psi}_i-\hat{\Psi}_j)\\
&+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2.
\end{aligned} \tag{3.34}$$

According to Eqs (3.5) and (3.16), the derivative of $V_2(t)$ is obtained as:

$$\begin{aligned}
\dot{V}_2(t)=&\sum_{i=1}^{N}k\dot{\eta}_i^a(t)+2\sigma\sum_{i=1}^{N}\xi_i^T(t)\tilde{Q}\xi_i(t)\\
=&-\sum_{i=1}^{N}k\alpha_i\eta_i^a(t)-\sum_{i=1}^{N}k\beta_i[\sum_{j=1}^{N}((\frac{1}{\delta_2}-1)+\gamma c_{ij})\mu_i^2e_i^T\Gamma e_i\\
&-\frac{1}{4}\sum_{j=1}^{N}(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j)^T\Gamma(\mu_i\hat{x}_i(t_k^i)-\mu_j\hat{x}_j(\tilde{t}_{k'}^j)]\\
&+2\sigma\sum_{i=1}^{N}\xi_i^T(t)\tilde{Q}(A+HC)\xi_i(t).
\end{aligned} \tag{3.35}$$

Combining Eqs (3.34) and (3.35), we can obtain $\dot{V}(t)$ as:

$$\begin{aligned}\dot{V}(t) = &\quad \dot{V}_1(t) + \dot{V}_2(t)\\
\leq &\quad \hat{\Psi}^T(I \otimes (PA + A^T P) + \delta_1 I - \frac{k}{2}(1-\delta_2)L \otimes \Gamma)\hat{\Psi} + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2\\
&+ \sum_{i=1}^{N}\xi_i^T(\frac{1}{\delta_1}\|PHC\|_2^2 I + \sigma(\tilde{Q}(A+HC) + (A+HC)^T\tilde{Q}))\xi_i\\
&- k(1-\beta_i)[\frac{1}{4}\sum_{j=1}^{N}(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j))^T\Gamma(\mu_i\hat{x}_i(t_k^i) - \mu_j\hat{x}_j(\tilde{t}_{k'}^j)\\
&- \sum_{j=1}^{N}((\frac{1}{\delta_2}-1) + \gamma c_{ij})\mu_i^2 e_i^T\Gamma e_i] - \sum_{i=1}^{N}k\alpha_i\eta_i^a(t) - \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2\\
\leq &\quad \hat{\Psi}^T(I \otimes (PA + A^T P) + \delta_1 I - \frac{k}{2}(1-\delta_2)L \otimes \Gamma)\hat{\Psi}\\
&+ \sum_{i=1}^{N}\xi_i^T(\frac{1}{\delta_1}\|PHC\|_2^2 I + \sigma(\tilde{Q}(A+HC) + (A+HC)^T\tilde{Q}))\xi_i\\
&- k\sum_{i=1}^{N}(\alpha_i - \frac{1-\beta_i}{\theta_i})\eta_i^a(t) + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2 - \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}(c_{ij}(t)-k)^2, \qquad (3.36)\end{aligned}$$

where $k \geq \max\left\{\frac{2}{\gamma}, \frac{4}{\lambda_2(L)}\right\}$.

According to Lemmas 2.1 and 2.2, and the conditions of Theorem 3.1, the above equation can be bounded as:

$$\begin{aligned}\dot{V}(t) <& -(\varsigma - \delta_1)\hat{\Psi}^T\hat{\Psi} + M\\
<& \quad a_2 V(t) + M, \end{aligned} \qquad (3.37)$$

where $M = \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{m_{ij}}{4}k^2$ and $a_2 > 0$.

Based on the above analysis, by selecting a sufficiently large parameter $k > \max\left\{1, \frac{2}{\gamma}, \frac{4}{\lambda_2(L)}\right\}$, and combining Eqs (3.28) and (3.37), we obtain:

$$\dot{V}(t) < \begin{cases} -a_1 V(t) + M, & t \in \Xi_s(\tau, t),\\ a_2(t)V(t) + M, & t \in \Xi_a(\tau, t).\end{cases} \qquad (3.38)$$

To facilitate subsequent analysis, we represent the time intervals $t \in \Xi_s(\tau, t)$ and $t \in \Xi_a(\tau, t)$ as $t \in [t_{2k}, t_{2k+1})$ and $t \in [t_{2k+1}, t_{2k+2})$. Additionally, the parameter a^{Δ} where $\Delta = 1$ or $\Delta = 2$ is expressed as the following piecewise function:

$$a(t) = \begin{cases} -a_1, & t \in [t_{2k}, t_{2k+1}),\\ a_2, & t \in [t_{2k+1}, t_{2k+2}).\end{cases} \qquad (3.39)$$

Using the designed Lyapunov function (3.13) and parameter $z > 1$, we have:

$$V(t_{2k}) = \sum_{i=1}^{N}\hat{\Psi}_i^T(t_{2k})P\hat{\Psi}_i(t_{2k}) + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}(t_{2k})-k)^2}{4k_{ij}}$$

$$
\begin{aligned}
& + \sum_{i=1}^{N} k\eta_i(t_{2k}) + \sigma \sum_{i=1}^{N} \xi_i^T(t_{2k})\tilde{Q}\xi_i(t_{2k}) \\
< \quad & zV(t_{2k}^-), \qquad (3.40)\\
V(t_{2k+1}) = & \sum_{i=1}^{N} \hat{\Psi}_i^T(t_{2k+1})P\hat{\Psi}_i(t_{2k+1}) + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \frac{(c_{ij}(t_{2k+1}) - k)^2}{4k_{ij}} \\
& + \sum_{i=1}^{N} k\eta_i(t_{2k+1}) + \sigma \sum_{i=1}^{N} \xi_i^T(t_{2k+1})\tilde{Q}\xi_i(t_{2k+1}) \\
< \quad & zV(t_{2k+1}^-). \qquad (3.41)
\end{aligned}
$$

Applying the comparison principle to solve the differential inequality (3.39), we obtain:

$$
V(t) < \begin{cases} e^{a(t_{2k})(t-t_{2k})}V(t_{2k}) + \int_{t_{2k}}^{t} Me^{a(t_{2k})(t-\tau)}d\tau, & t \in [t_{2k}, t_{2k+1}), \\ e^{a(t_{2k+1})(t-t_{2k+1})}V(t_{2k+1}) + \int_{t_{2k=1}}^{t} Me^{a(t_{2k+1})(t-\tau)}d\tau, & t \in [t_{2k+1}, t_{2k+2}). \end{cases} \qquad (3.42)
$$

For $t \in [t_{2k}, t_{2k+1})$, applying a recursive method to the first inequality in (3.42) yields:

$$
\begin{aligned}
V(t) < \quad & z^{2D(0,t)}e^{-a_1\Xi_s(\tau,t)+a_2\Xi_a(\tau,t)}V(0) \\
& + z^{2D(0,t)} \int_{0}^{t_1} e^{W_{2k}(t,2)+a(0)(t_1-\tau)}Md\tau \\
& + z^{2D(0,t)} \int_{t_1}^{t_2} e^{W_{2k}(t,3)+a(t_1)(t_2-\tau)}Md\tau \\
& + \cdots \\
& + z^2 \int_{t_{2k-2}}^{t_{2k-1}} e^{W_{2k}(t,2k)+a(t_{2k-1})(t_{2k}-\tau)}Md\tau \\
& + z^2 \int_{t_{2k-1}}^{t_{2k}} e^{W_{2k}(t,2k)+a(t_{2k-1})(t_{2k}-\tau)}Md\tau \\
& + \int_{t_{2k}}^{t} e^{a(t_{2k})(t-\tau)}Md\tau, \qquad (3.43)
\end{aligned}
$$

where $W_{2k}(t,u) = a(t_{2k})(t - t_{2k}) + \sum_{r=u}^{2k} a(t_{r-1})(t_r - t_{r-1})$.

For $t \in [t_{2k+1}, t_{2k+2})$, applying a recursive method to the second inequality in (3.42) yields:

$$
\begin{aligned}
V(t) < \quad & z^{2D(0,t)-1}e^{-a_1\Xi_s(\tau,t)+a_2\Xi_a(\tau,t)}V(0) \\
& + z^{2D(0,t)-1} \int_{0}^{t_1} e^{W_{2k+1}(t,2)+a(0)(t_1-\tau)}Md\tau \\
& + z^{2D(0,t)-1} \int_{t_1}^{t_2} e^{W_{2k+1}(t,3)+a(t_1)(t_2-\tau)}Md\tau \\
& + \cdots \\
& + z^2 \int_{t_{2k}}^{t_{2k+1}} e^{W_{2k+1}(t,2k+2)+a(t_{2k})(t_{2k+1}-\tau)}Md\tau
\end{aligned}
$$

$$+z^2\int_{t_{2k+1}}^{t} e^{a(t_{2k+1})(t-\tau)}Md\tau, \tag{3.44}$$

where $W_{2k+1}(t,u)=a(t_{2k+1})(t-t_{2k+1})+\sum_{r=u}^{2k+1}a(t_{r-1})(t_r-t_{r-1})$.

According to Assumptions 2.1 and 2.2, Eqs (3.43) and (3.44) can be unified as:

$$V(t)<z^{2D(0,t)}e^{-a_1\Xi_s(\tau,t)+a_2\Xi_a(\tau,t)}V(0)+M\int_0^t z^{2D(\tau,t)}e^{-a_1\Xi_s(0,t)+a_2\Xi_a(0,t)}d\tau, \tag{3.45}$$

where $z^{2D(0,t)}\le z^{2N_2+\frac{2t}{\tau_b}}=z^{2N_2}e^{\frac{2\ln z}{\tau_b}t}$, and

$$\begin{aligned}&-a_1\Xi_s(0,t)+a_2\Xi_a(0,t)\\=&-a_1(t-0-\Xi_a(0,t))+a_2\Xi_a(0,t)\\\le&-a_1t+(a_1+a_2)\times(N_1+\frac{t}{\tau_a}).\end{aligned} \tag{3.46}$$

Using $\hat{a}=a_1-\frac{2\ln z}{\tau_b}-\frac{a_1+a_2}{\tau_a}$ from Theorem 3.1 and simplifying Eq (3.45):

$$V(t)<\nu e^{-\hat{a}t}V(0)+\frac{\nu\tau_b}{2\ln z}M, \tag{3.47}$$

where $\nu=z^{2N_2}e^{(a_1+a_2)N_a}$ and $\hat{a}=a_1-(2\ln z/\tau_b)-[(a_1+a_2)/\tau_a]>0$. This demonstrates that $V(t)$ is bounded.

3.3. Exclusion of Zeno behavior

This subsection provides a feasibility proof for the designed event-triggered mechanism.

From Eq (3.28), we have:

$$V(t)<(V(0)-\frac{M}{a_1})e^{-a_1t}+\frac{M}{a_1}, \tag{3.48}$$

which indicates that $V(t)$ has an upper bound defined as $S=\left\{\hat{\Psi}_i,c_{ij}(t),\eta_i(t),\xi_i(t)\mid V<\frac{M}{a_1}\right\}$. Using Eq (3.13), we obtain:

$$\begin{aligned}V(t)\ge\quad&\lambda_{\min}(P)\left\|\hat{\Psi}(t)\right\|^2+\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{(c_{ij}(t)-k)^2}{4k_{ij}}\\&+\sum_{i=1}^{N}k\eta_i(t)+\sigma\lambda_{\min}(\tilde{Q})\left\|\xi(t)\right\|^2,\end{aligned} \tag{3.49}$$

demonstrating that $\hat{\Psi}_i,c_{ij}(t),\eta_i(t),\xi_i(t)$ are bounded for all $t\in[0,\infty)$. Consequently, $\hat{x}_i(t),x_i(t)$, and $\hat{x}_i(t_k^i)$ are bounded.

Theorem 3.2. Under the event-triggered condition (3.7), MASs (2.2) exhibit no Zeno behavior.

Proof. First, consider $t\in\Xi_s(\tau,t)$. Combining Eqs (2.2), (3.1), and (3.2):

$$\frac{\mathrm{d}\left\|e_i(t)\right\|}{\mathrm{d}t}=\quad\frac{e_i^T}{\left\|e_i(t)\right\|}\dot{e}_i\le\left\|\dot{e}_i(t)\right\|$$

$$\leq \|A\| \|e_i(t)\| + \frac{1}{\mu_i} \sum_{j=1}^{N} c_{ij}(t) a_{ij} \|BK\| \left\| \mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j) \right\| + \|HC\| \|\hat{e}_i\|$$
$$\leq \|A\| \|e_i(t)\| + \pi_i^s, \tag{3.50}$$

where $\hat{e}_i = x_i - \hat{x}_i$, $\pi_i^s = \max \frac{1}{\mu_i} \sum_{j=1}^{N} c_{ij}(t) a_{ij} \|BK\| \left\| \mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j) \right\| + \|HC\| \|\hat{e}_i\|$, and $c_{ij}(t) \leq \bar{c}$.

Consider a non-negative continuous function $\phi_s(t) : [0, \infty) \to \mathbb{R}_{\geq 0}$:

$$\dot{\phi}_s = \|A\| \phi_s + \pi_i^s, \quad \phi_s(0) = \left\| e_i(t_k^i) \right\| = 0. \tag{3.51}$$

According to $\|e_i(t)\| \leq \phi_s(t - t_k^i)$, we have

$$\phi_s(t) = \frac{\bar{c}\pi_i^s}{\|A\|} (e^{\|A\| t} - 1), \tag{3.52}$$

where $\phi_s(t)$ is the solution to Eq (3.51).

Assuming $\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_i(t_{k'}^j) \leq \iota$, the triggering threshold from condition (3.7) $f_i(t) \leq 0$ gives:

$$\|e_i(t)\|^2 \leq \frac{\frac{\eta_i(t)}{\theta_i} + \frac{1}{4} d_i \iota^2 \|\Gamma\|}{((\frac{1}{\delta_2} + 1) + \gamma\bar{c}) d_i \mu_i^2 \|\Gamma\|}. \tag{3.53}$$

By the event-triggering mechanism definition, the minimum triggering interval τ_k^i between consecutive events t_k^i and t_{k+1}^i for agent v_i is found by combining the measurement error bound (3.52) and event-triggering boundary condition (3.53):

$$\frac{(\bar{c}\pi_i^s)^2}{\|A\|^2} (e^{\|A\| \tau_{k_i}^i} - 1) \geq \frac{\frac{\eta_i(t)}{\theta_i} + \frac{1}{4} d_i \iota^2 \|\Gamma\|}{((\frac{1}{\delta_2} + 1) + \gamma\bar{c}) d_i \mu_i^2 \|\Gamma\|}, \tag{3.54}$$

yielding the minimum triggering interval τ_k^i:

$$\tau_{k,s}^i > \frac{1}{\|A\|} \ln (1 + \frac{\|A\|}{\bar{c}\pi_i^s} \sqrt{\frac{\frac{\eta_i(t)}{\theta_i} + \frac{1}{4} d_i \iota^2 \|\Gamma\|}{((\frac{1}{\delta_2} + 1) + \gamma\bar{c}) d_i \mu_i^2 \|\Gamma\|}}) > 0. \tag{3.55}$$

Next, consider $t \in \Xi_a(\tau, t)$. Combining Eqs (2.2), (3.1), and (3.2):

$$\frac{\mathrm{d}\,\|e_i(t)\|}{\mathrm{d}t} = \frac{e_i^T}{\|e_i(t)\|} \dot{e}_i \leq \|\dot{e}_i(t)\|$$
$$\leq \|A\| \|e_i(t)\| + \frac{1}{\mu_i} \sum_{j=1}^{N} c_{ij}(t) a_{ij} \|BK\| \left\| \mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j) \right\| + \|HC\| \|\hat{e}_i\|$$
$$\leq \|A\| \|e_i(t)\| + \pi_i^a, \tag{3.56}$$

where $\pi_i^a = \max \frac{1}{\mu_i} \sum_{j=1}^{N} c_{ij}(t) \|BK\| \left\| \mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_j(t_{k'}^j) \right\| + \|HC\| \|\hat{e}_i\|$.

Consider a non-negative continuous function $\phi_a(t) : [0, \infty) \to \mathbb{R}_{\geq 0}$:

$$\dot{\phi}_a = \|A\| \phi_a + \pi_i^a, \phi_{sa}(0) = \left\| e_i(t_k^i) \right\| = 0. \tag{3.57}$$

According to $\|e_i(t)\| \leq \phi_a(t - t_k^i)$, we have

$$\phi_a(t) = \frac{\bar{c}\pi_a^s}{\|A\|}(e^{\|A\|t} - 1), \tag{3.58}$$

where $\phi_a(t)$ is the solution to Eq (3.57).

Assuming $\mu_i \hat{x}_i(t_k^i) - \mu_j \hat{x}_i(\tilde{t}_{k'}^j) \leq \vartheta$, the triggering threshold from condition (3.7) $f_i(t) \leq 0$ gives:

$$\|e_i(t)\|^2 \leq \frac{\frac{\eta_i(t)}{\theta_i} + \frac{1}{4}\vartheta^2 \|\Gamma\|}{((\frac{1}{\delta_2} + 1) + \gamma\bar{c})\mu_i^2 \|\Gamma\|}. \tag{3.59}$$

Combining (3.58) and (3.59):

$$\frac{(\bar{c}\pi_i^s)^2}{\|A\|^2}(e^{\|A\|\tau_{k_i}^i} - 1) \geq \frac{\frac{\eta_i(t)}{\theta_i} + \frac{1}{4}\vartheta^2 \|\Gamma\|}{((\frac{1}{\delta_2} + 1) + \gamma\bar{c})\mu_i^2 \|\Gamma\|}, \tag{3.60}$$

yielding the minimum triggering interval τ_k^i:

$$\tau_{k,a}^i > \frac{1}{\|A\|} \ln(1 + \frac{\|A\|}{\bar{c}\pi_i^s} \sqrt{\frac{\frac{\eta_i(t)}{\theta_i} + \frac{1}{4}\vartheta^2 \|\Gamma\|}{((\frac{1}{\delta_2} + 1) + \gamma\bar{c})\mu_i^2 \|\Gamma\|}}) > 0, \tag{3.61}$$

confirming the absence of Zeno behavior.

4. Simulations and comparative experiments

4.1. Simulation verification

To validate the effectiveness of the proposed algorithm under the dual constraints of denial-of-service (DoS) attacks and input saturation, a multi-agent system (MAS) consisting of six agents is constructed. The undirected and connected communication topology of the MAS is illustrated in Figure 1.

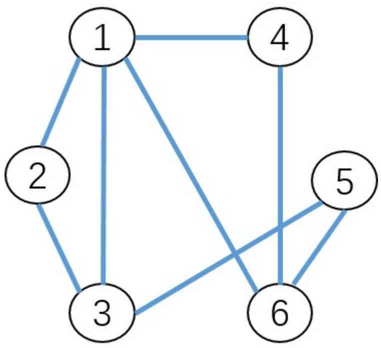


Figure 1. Communication topology of the multi-agent system.

The system matrices are given as

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The controller gain $K = [-0.2236, -0.7051]$ and observer gain matrix

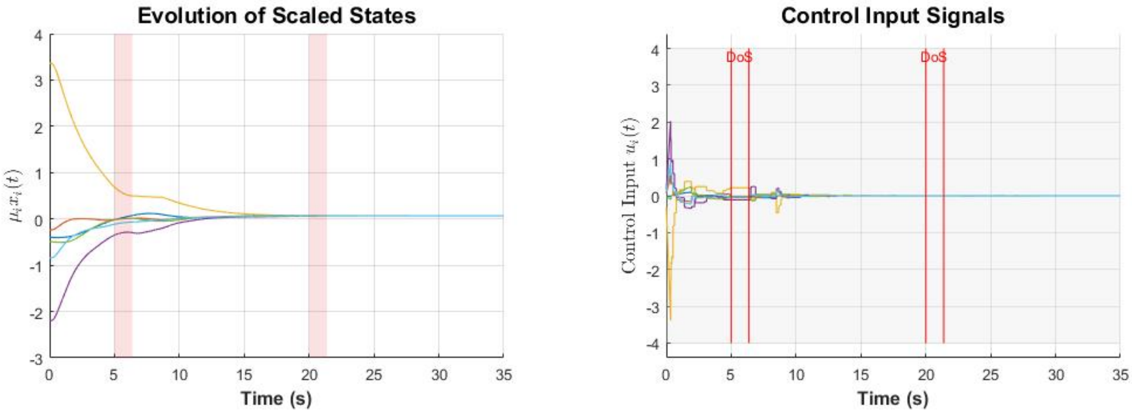
$$H = \begin{bmatrix} -1.7321 \\ -1.0000 \end{bmatrix}$$

are obtained by solving the algebraic Riccati equations in (2.6) and (2.7).

The initial states of the agents are set as $x_1(0) = [-0.4, 0]^T$, $x_2(0) = [-0.2, 0]^T$, $x_3(0) = [4.2, 0]^T$, $x_4(0) = [-2.0, 0]^T$, $x_5(0) = [-0.55, 0]^T$, $x_6(0) = [-0.65, 0]^T$.

The initial coupling weights are $c_{12}(0) = c_{23}(0) = c_{34}(0) = c_{45}(0) = c_{56}(0) = c_{61}(0) = 0.1$. The scaling factors are chosen as $\mu_1 = 1.0$, $\mu_2 = 1.2$, $\mu_3 = 0.8$, $\mu_4 = 1.1$, $\mu_5 = 0.9$, and $\mu_6 = 1.3$. The parameters of the dynamic event-triggering mechanism are set to $\alpha_i = 0.95, \beta_i = 0.5, \theta_i = 0.6, \gamma_i = 0.7$, $\delta_1 = 0.02$, and $\delta_2 = 0.9$. Other system parameters are chosen as $\varsigma = 0.05$, $\rho = 0.08$, $z = 1.05$, and $m_{ij} = 0.0005$. This yields $a_1 = 0.004$, and $a_2 = 0.01$ is selected.

The DoS attack parameters are chosen as $N_1 = 1$, $\tau_a = 20$, $N_2 = 1$, and $\tau_b = 30$, resulting in $\hat{a} = 0.00005 > 0$. The attack duration is less than 2.75 s, and the attack frequency is less than 2.17. Two attack intervals are imposed: [5.00 s, 6.37 s] and [20.00 s, 21.38 s]. The corresponding simulation results are shown in Figures 2 and 3.



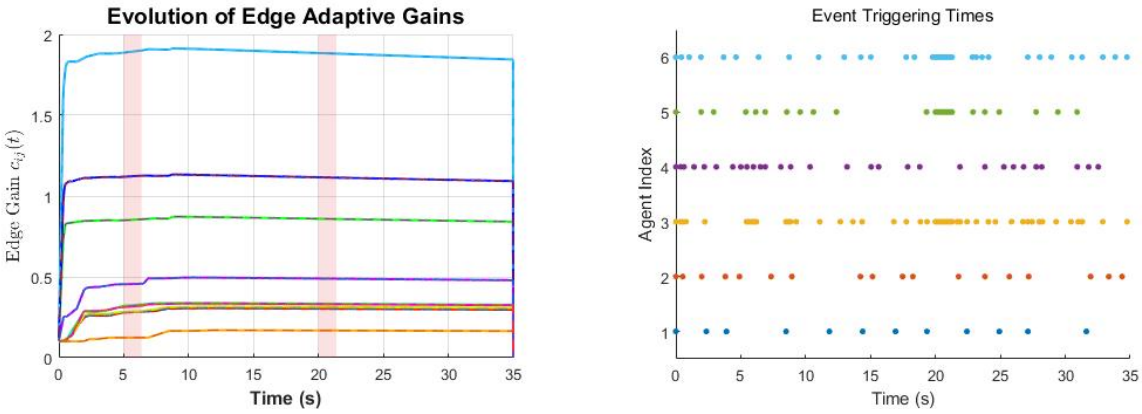
(a) Scaled consensus

(b) Control input

Figure 2. State trajectories and control inputs of the MAS.

As shown in Figure 2(a), during the normal operation phase (0–5 s), the agent states converge rapidly from their initial values to approximately 0.5. This demonstrates that under normal communication, the proposed algorithm ensures rapid convergence toward the scaled consensus objective. During the DoS attacks, certain trajectories exhibit pronounced fluctuations, indicating that the attacks disrupt state updates by severing communication links, causing temporary deviations from the target values. After the first attack (7–15 s) and the second attack (23–35 s), the states re-converge to zero, with progressively shorter convergence times, reflecting the algorithm's capability to adapt and recover. Beyond $t = 15$ s, all trajectories converge near zero, verifying the ultimate stability of the algorithm.

Figure 2(b) shows the evolution of control inputs over time. The results indicate that DoS attacks directly affect the control channel by blocking information exchange. The proposed algorithm demonstrates layered robustness: the local closed-loop channel provides partial resilience against attacks, while the overall MAS exhibits strong self-recovery capability after attacks.



(a) Time-varying coupling weights (b) Triggering instants

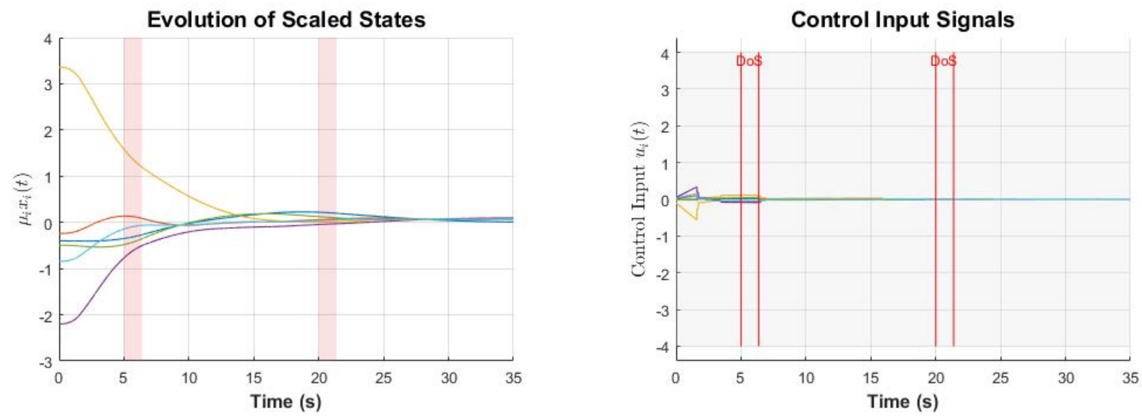
Figure 3. Evolution of time-varying coupling weights and event-triggering instants under DoS attacks and input saturation.

As depicted in Figure 3(a), all coupling weights converge to stable values within approximately 3 s. Figure 3(b) shows that continuous communication is unnecessary for achieving consensus, and the dynamic event-triggering mechanism significantly reduces the number of triggering events without exhibiting Zeno behavior.

Next, the low-gain feedback parameter ς is modified to 0.005. The controller gain becomes $K = [-0.0707, -0.3827]$ and the observer gain matrix remains

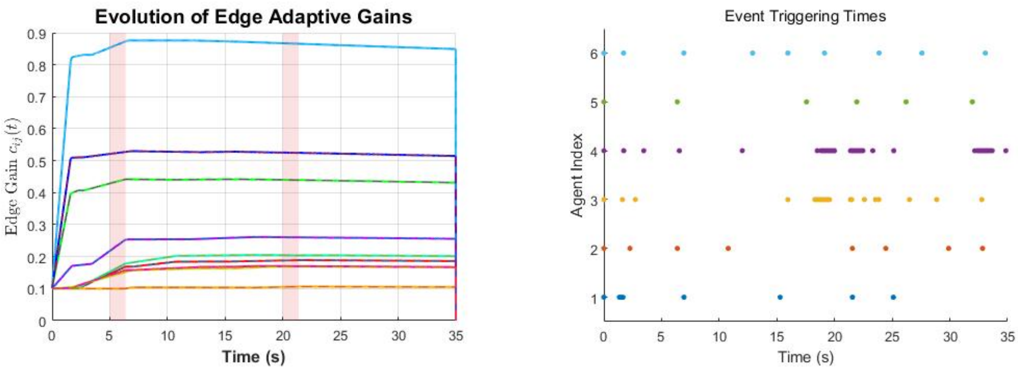
$$H = \begin{bmatrix} -1.7321 \\ -1.0000 \end{bmatrix}.$$

All other parameters are kept unchanged. The influence of ς is illustrated in Figures 4 and 5.



(a) Scaled consensus (b) Control input

Figure 4. Evolution of scaled states and control inputs with $\varsigma = 0.005$.



(a) Time-varying coupling weights (b) Triggering instants

Figure 5. Evolution of time-varying coupling weights and agent-specific event-triggering instants (low-gain parameter $\varsigma = 0.005$).

Comparing Figures 2 and 4, decreasing ς from 0.05 to 0.005 prolongs the convergence time from 15 s to 25 s and increases the post-attack fluctuation amplitude. This confirms that ς balances convergence speed and robustness: larger values accelerate convergence but increase communication frequency. From Figures 3 and 5, the number of triggering instants decreases to 51 when $\varsigma = 0.005$, indicating that smaller ς weakens control effort, slows state changes, and raises the triggering threshold.

4.2. Comparative experiment

To further validate the superiority of the proposed method, a comparative study is conducted with three representative peer approaches:

Peer Method 1: Static event-triggered proportional consensus (without DoS resilience) as proposed in [13];

Peer Method 2: Distributed observer-based consensus with input saturation (without dynamic triggering) as presented in [32];

Peer Method 3: DoS-resilient complete consensus (non-proportional) from [20].

The proposed method achieves faster convergence (15 s) compared to Peer Method 1 (22 s) and Peer Method 2 (18 s), owing to the adaptive threshold adjustment of the dynamic event-triggered mechanism, which reduces the transmission of outdated information. The lower communication load (68 triggering times) stems from the sparse triggering signals enabled by the fully distributed observer. Notably, the proposed method exhibits a 0% input saturation rate, validating the effectiveness of the anti-saturation compensator. In contrast, Peer Methods 1, 2, and 3 exhibit saturation rates of 12%, 5%, and 8%, respectively. Furthermore, Peer Method 3 fails to achieve proportional consensus, underscoring the uniqueness of our approach in handling both DoS attacks and input saturation while pursuing proportional objectives.

5. Conclusions

This paper effectively addresses the scaled consensus problem for MASs under DoS attacks and input saturation constraints through the co-design of a dynamic event-triggering mechanism, a fully distributed observer, and an anti-saturation controller. Theoretical proofs and simulation experiments demonstrate that the proposed method guarantees scaled consensus while significantly reducing communication costs and exhibiting strong anti-jamming capability, providing a reliable solution for multi-agent cooperative control in complex environments.

Author contributions

Min Wang: Writing–original draft, validation; Zhanheng Chen: Review editing, visualization, supervision; Zhiyong Yu: Conceptualization, supervision, review editing; Haijun Jiang: Conceptualization, supervision, formal analysis, resources. All authors have read and approved the final manuscript.

Use of Generative-AI tools declaration

The authors declare they have used Artificial Intelligence (AI) tools in the creation of this article.

AI tools used: Chat gpt, Matlab AI assistant.

How were the AI tools used?

1) Chat gpt refined the "Introduction" section's logic (organizing literature review, clarifying research gaps) and supported preliminary "Stability analysis" proof drafting.

2) Matlab AI assistant optimized simulation code for multi-agent experiments and validated algebraic Riccati equation solutions.

Where in the article is the information located?

1) Chat gpt's contributions: "Introduction" and "3.2. Stability proof".

2) Matlab AI assistant's contributions: "4. Simulations and comparative experiments".

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Conflict of interest

The authors declare no conflict of interest in this paper.

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