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Research article

A new method for parameter estimation of extended grey GM(2, 1) model based on difference equation and its application

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Abstract: The common models used for grey system predictions include the GM(1, 1), the GM(N, 1), the GM(1, N) and so on, in which the GM(N, 1) model is an important type. Especially, the GM(2, 1) model is used widely, but it shows low modeling precision sometimes because of the improper parameter estimation method. To improve the model's precision, the paper proposes an extended grey GM(2, 1) model and gives a new parameter estimation method for the extended GM(2, 1) model based on the difference equation. The paper builds eight different grey models for the example. Results show that the improved method proposed has the highest precision. The method proposed can improve the popularization and application of the grey GM(N, 1) model.

Keywords: GM(2, 1) model; parameter estimation; time response equation; difference equation
Mathematics Subject Classification: 65Q10, 34K60, 34M30, 39A10

1. Introduction

The grey prediction is making predictions for grey systems. It predicts the development trend and future of objects by building grey models. Currently, the grey system theory has been applied to the predictions of economic management, ecological system, engineering control and complicated & variable systems successfully [1–5]. However, in the actual predictions of models, there is generally poor prediction precision, for which reason, many scholars have studied the model. The scholars have extended and optimized the grey model in terms of background value [6, 7], grey derivative [8, 9], parameter optimization [10, 11] and model extrapolation [12, 13] with certain achievements.

In recent years, with the continuous development of new methods, the grey model is studied in more depth. Many scholars use new grey models to solve various prediction problems. He [14] built a structurally adaptive new-information-priority discrete grey prediction model to predict the power generating capacity of medium-and-long-term renewable energy accurately. Considering the service life of products, Yang [15] proposed a generalized fractional-order accumulation grey power model GFAGMP(1, 1) and gave the parameter estimation, error analysis and time response function solution. The researcher studied the transformation and connection of the generalized GM(1, 1) model and the GFAGMP(1, 1) model on the basis of integral and power function transformation, and derived three forms and application ranges of the model through analogy analyses. Finally, the researcher compared the prediction models with the actual sales data of China's refrigerators. Results verified the GFAGMP(1, 1) model's feasibility and effectiveness to predict the supply chain demand of China's home appliances. To predict the future energy trend accurately, Wang et al. [16] proposed a novel Caputo fractional-order derivative structurally-adaptive grey model and a novel Caputo fractional-order accumulated generating operator. After a comparison of optimization algorithms, they chose the PSO for the optimization of the model's parameters. The example proved that the model was stable and reliable. Yang et al. [17] proposed a damping accumulation multivariate grey model to predict the heat of internet public opinions. First, they introduced a dynamic damping trend factor in the accumulation process to make the model adjust the accumulation order of different sequences more flexibly. Next, considering the characteristic of grey exponential law of accumulation sequence, the researchers optimized the structure of background value to build a damping grey multivariate model. Finally, they gave a time response equation to reduce the error using a combined quadrature method. They made an empirical analysis using two actual cases and verified the effectiveness of new model. To predict China's renewable energy accurately, Wang et al. [18] proposed a novel fractional-order adaptive grey Chebyshev polynomial Bernoulli model describing the nonlinear phenomena based on the NGBM(1,1) model. To realize reasonable predictions of China's hydraulic power generation, Zeng et al. [19] constructed a novel grey combined optimization model using the combined optimization of different parameters on the basis of three-parameter discrete grey model TDGM(1, 1), and then predicted China's hydraulic power generation capacity using the new model. Wang et al. [20] proposed a new exponential time-lagging fractional-order grey prediction model based on the PSO. They first made a fractional-order accumulation preprocessing of the original data, then introduced an exponential time-lagging term based on the GM(1, 1) model to build a new model, and finally searched the optimal parameter of model using the PSO and verified the test result through a Wilcoxon rank sum test. Results showed that the new model had better prediction precision and adaptability compared with existing six grey models. The new studies further promote the model's modeling precision and application range.

The common models used for grey system predictions include the GM(1, 1), the GM(N, 1), the GM(1, N) and so on, in which the GM(N, 1) model is an important type. Especially, there have been many studies on the GM(2, 1) model [21–26]. The grey GM(2, 1) model can be used for the research on both monotonic and non-monotonic time sequences, and thus has been used widely with many research achievements. Zeng et al. [27] proposed a new prediction method for China's domestic power consumption per capita using the grey modeling technology. Considering the multiple and mixed variation patterns, they proposed a discrete grey model with the polynomial term. They first introduced a polynomial term into the discrete DGM(2, 1) model, then used the Tikhonov regularization method to solve the overfitting problem, and finally verified the generalization and adaptability of newly designed model through a case. Jin et al. [28] used a multiple regression model and a GM(2, 1) model for a quantitative analysis on the indexes of tourist industry. They used the multiple regression model to analyze the influencing factors of tourist industry and used the GM(2, 1) model to predict the development of tourist industry in the future three years and gave related suggestions. Kong et al. [29] used the GM(1, 1) and GM(2, 1) grey prediction models and selected the trade data of middle-and-small-sized enterprises in the period from February 6^{th}, 2020 to April 30^{th}, 2020 to compare the prediction results of two models horizontally and vertically. The prediction results of China's stock exchange indexes showed that the grey system model was more suitable for short-term and monotonic data samples. However, for longer-term or non-monotonic data, the grey system model showed a poor fitting result for stock exchange indexes, and thus failed to offer any reference for stock market price predictions. To solve the problem of poor prediction precision of traditional second-order univariate grey prediction model GM(2, 1) for some sequences, Zeng and Luo [30] introduced the fractional-order accumulation operator on the basis of GM(2, 1) and proposed a discrete GM(2, 1) model based on the fractional-order accumulation using the modeling principle of discrete grey model. Finally, they verified the new model using a numerical simulation test and an application example. Results showed that the new model had higher simulation & prediction precision compared with other common models and thus was effective and practical. Shu [31] predicted China's foreign exchange reserves by combining the wavelet transformation with the grey GM(2, 1) model and achieved an ideal result.

The studies and methods have certain achievements, but some of them show poor modeling precision for some data, for which a main reason is that the parameter estimation method has some defects. To improve the prediction precision of GM(2, 1) model, the paper extends the conventional GM(2, 1) model structurally to make it adapt to the variation of data in a wider range, and improves the parameter estimation method of extended GM(2, 1) model based on the difference equation. The example shows that the model built with the method proposed has high prediction precision. The research in the paper extends the application range of the GM(2, 1) model and can be generalized and applied to other models, such as the GM(N, 1) model, the GM(1, N) model, the GM(1, 1) power model and so on.

2. The traditional grey GM(2,1) model's parameter estimation method and modeling

Suppose the original time sequence is $x^{(0)}(t) = \left\{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right\}$, and the once accumulated generating operation sequence is $x^{(1)}(t) = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\}$. Let $\alpha^{(1)}x^{(0)}(k) = x^{(0)}(k) - x^{(0)}(k-1)$, $z^{(1)}(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1))$, $(k = 2, 3, \cdots, n)$, and then call $\alpha^{(1}x^{(0)}(k) + a_1 x^{(0)}(k) + a_2 z^{(1)}(k) = b_0$ the basic form of GM(2, 1) model. Its whitening equation is

$$\frac{d^2x^{(1)}}{dt} + a_1\frac{dx^{(1)}}{dt} + a_2x^{(1)} = b_0. \tag{2.1}$$

The GM(2, 1) model's parameter estimates are

$$u = \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b}_0 \end{pmatrix} = (B^TB)^{-1}B^TY, \tag{2.2}$$

where

$$B = \begin{pmatrix} -x^{(0)}(2) & -z^{(1)}(2) & 1 \\ -x^{(0)}(3) & -z^{(1)}(3) & 1 \\ \vdots & \vdots & \vdots \\ -x^{(0)}(n) & -z^{(1)}(n) & 1 \end{pmatrix}, Y = \begin{pmatrix} \alpha^{(1)}x^{(0)}(2) \\ \alpha^{(1)}x^{(0)}(3) \\ \vdots \\ \alpha^{(1)}x^{(0)}(n) \end{pmatrix} = \begin{pmatrix} x^{(0)}(2) - x^{(0)}(1) \\ x^{(0)}(3) - x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) - x^{(0)}(n-1) \end{pmatrix}. \tag{2.3}$$

The traditional GM(2, 1) model's time response equation can be obtained using software MATLAB with the following program:

```
x1=dsolve('D2x1+a1*Dx1+a2*x1=b0','x1(1)=c1, x1(2)=c2'),
```

then get

$$\hat{x}^{(1)}(t) = \frac{b_0}{a_2} + \frac{b_0g_4 - b_0g_2 - a_2c_1g_4 + a_2c_2g_2}{g_1}e^{-(\frac{a_1}{2}-g_6)t} - \frac{b_0g_5 - b_0g_3 + a_2c_2g_3 - a_2c_1g_5}{g_1}e^{-(\frac{a_1}{2}+g_6)t}, \tag{2.4}$$

where $g_1 = a_2(g_2g_5 - g_4g_3)$, $g_2 = e^{-\frac{a_1}{2}-g_6}$, $g_3 = e^{g_6-\frac{a_1}{2}}$, $g_4 = e^{-a_1-\sqrt{a_1^2-4a_2}}$, $g_5 = e^{-a_1+\sqrt{a_1^2-4a_2}}$, $g_6 = \frac{\sqrt{a_1^2-4a_2}}{2}$, $c_1 = x^{(1)}(1) = x^{(0)}(1)$, $c_2 = x^{(1)}(2) = x^{(0)}(1) + x^{(0)}(2)$.

Suppose there are observation data for N years, in which the data from year 1 to year n are used for modeling and the data from year n+1 to year N are used for prediction. For $x^{(0)}$, the simulation value is $\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), (k = 2, 3, \cdots, n)$, the prediction value is $\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), (k = n+1, n+2, \cdots, N)$.

3. The extended grey GM(2, 1) model's conventional parameter estimation method and modeling

The extended GM(2, 1) model's whitening equation is

$$\frac{d^2x^{(1)}}{dt^2} + a_1\frac{dx^{(1)}}{dt} + a_2x^{(1)} = b_0 + b_1t + b_2t^2 + \cdots b_pt^p. \tag{3.1}$$

Get the integrals of the equations from both sides of $[k-1, k]$:

$$\begin{aligned} &\alpha^{(1}x^{(0)}(k) + a_1x^{(0)}(k) + a_2z^{(1)}(k) \\ &= b_0 + b_1 \times \tfrac{1}{2}[k^2 - (k-1)^2] + b_2 \times \tfrac{1}{3}[k^3 - (k-1)^3] + \cdots b_p \times \tfrac{1}{p}[k^p - (k-1)^p], \end{aligned} \tag{3.2}$$

where $\alpha^{(1)}x^{(0)}(k) = x^{(0)}(k) - x^{(0)}(k-1)$, $z^{(1)}(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1)), (k = 2, 3, \cdots, n)$.

To improve simulation and prediction precision and avoid overfitting, the value of p should not be too big, so the paper takes $p = 2$.

In this case, the parameter estimate of extended GM(2, 1) model is

$$u = \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = (B^TB)^{-1}B^TY, \tag{3.3}$$

where

$$B = \begin{pmatrix} -x^{(0)}(2) & -z^{(1)}(2) & 1 & \frac{1}{2}(2^2-1^2) & \frac{1}{3}(2^3-1^3) \\ -x^{(0)}(3) & -z^{(1)}(3) & 1 & \frac{1}{2}(3^2-2^2) & \frac{1}{3}(3^3-2^3) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -x^{(0)}(n) & -z^{(1)}(n) & 1 & \frac{1}{2}(n^2-(n-1)^2) & \frac{1}{n}(n^3-(n-1)^3) \end{pmatrix}, \tag{3.4}$$

$$Y=\begin{pmatrix}\alpha^{(1)}x^{(0)}(2)\\ \alpha^{(1)}x^{(0)}(3)\\ \vdots\\ \alpha^{(1)}x^{(0)}(n)\end{pmatrix}=\begin{pmatrix}x^{(0)}(2)-x^{(0)}(1)\\ x^{(0)}(3)-x^{(0)}(2)\\ \vdots\\ x^{(0)}(n)-x^{(0)}(n-1)\end{pmatrix}. \tag{3.5}$$

The extended GM(2, 1) model's time response equation can be obtained using software MATLAB easily with the following program:

```
clear
syms a1 a2 b0 b1 b2 c1 c2 x1(t)
x1=dsolve('D2x1+a1*Dx1+a2*x1=b0+b1*t+b2*t^2','x1(1)=c1,x1(2)=c2');
x1=subs(x1,{'a1','a2','b0','b1','b2','c1','c2'},{u(1),u(2),u(3),u(4),u(5),x1(1),x1(2)});
x1=vpa(x1,8).
```

Get the time response equation $\hat{x}^{(1)}(t)$ and then get the simulation value of original sequence with $\hat{x}^{(0)}(t)=\hat{x}^{(1)}(t)-\hat{x}^{(1)}(t-1)$, $(t=2,3,\cdots,n)$; get the prediction value of original sequence at the q^{th} step with $\hat{x}^{(0)}(t)=\hat{x}^{(1)}(t)-\hat{x}^{(1)}(t-1)$, $(t=n+1,\cdots,n+q)$.

4. The extended grey GM(2, 1) model's new parameter estimation method based on difference equation and modeling

The extended grey GM(2, 1) model's conventional parameter estimation method has defects causing low simulation and prediction precision. The paper proposes an improved parameter estimation method based on difference equation.

4.1. Variable generation coefficient $\lambda = 0.5$

The extended GM(2, 1) model's grey differential equation is

$$\alpha^{(1}x^{(0)}(k)+a_1x^{(0)}(k)+a_2z^{(1)}(k)=b_0+b_1T_1+b_2T_2, \tag{4.1}$$

where $\alpha^{(1)}x^{(0)}(k)=x^{(0)}(k)-x^{(0)}(k-1)$, $z^{(1)}(k)=\lambda x^{(1)}(k-1)+(1-\lambda)x^{(1)}(k),(k=2,3,\cdots,n)$. $T_1(k)=\frac{1}{2}[k^2-(k-1)^2]$, $T_2(k)=\frac{1}{3}[k^3-(k-1)^3]$.

The equation above can be written as the difference equation:

$$\begin{aligned}x^{(1)}(k)=&\frac{2+a_1-\frac{a_2}{2}}{1+a_1+\frac{a_2}{2}}x^{(1)}(k-1)-\frac{1}{1+a_1+\frac{a_2}{2}}x^{(1)}(k-2)\\&+\frac{b}{1+a_1+\frac{a_2}{2}}+\frac{b_1}{1+a_1+\frac{a_2}{2}}T_1(k)+\frac{b_2}{1+a_1+\frac{a_2}{2}}T_2(k).\end{aligned} \tag{4.2}$$

Then, get the estimates of $\hat{a}_1,\hat{a}_2,\hat{b}_0,\hat{b}_1,\hat{b}_2$:

$$\begin{pmatrix}\frac{2+a_1-\frac{a_2}{2}}{1+a_1+\frac{a_2}{2}}\\ -\frac{1}{1+a_1+\frac{a_2}{2}}\\ \frac{b_0}{1+a_1+\frac{a_2}{2}}\\ \frac{b_1}{1+a_1+\frac{a_2}{2}}\\ \frac{b_2}{1+a_1+\frac{a_2}{2}}\end{pmatrix}=(B'B)^{-1}B'Y, \tag{4.3}$$

where

$$B=\begin{pmatrix} x^{(1)}(2) & x^{(1)}(1) & 1 & T_1(3) & T_2(3) \\ x^{(1)}(3) & x^{(1)}(2) & 1 & T_1(4) & T_2(4) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x^{(1)}(n-1) & x^{(1)}(n-2) & 1 & T_1(n) & T_2(n) \end{pmatrix}, \tag{4.4}$$

$$Y=\begin{pmatrix} x^{(1)}(3) \\ x^{(1)}(4) \\ \cdots \\ x^{(1)}(n) \end{pmatrix}. \tag{4.5}$$

With the equation above, get the estimates of $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1, \hat{b}_2$, recorded as $u = [\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1, \hat{b}_2]$.

The time response equation of extended GM(2, 1) model can be obtained using software MATLAB with the following program:

```
x1=dsolve('D2x1+a1*Dx1+a2*x1=b0+b1*t+b2*t^2','x1(1)=c1, x1(2)=c2');
x1=subs(x1,{'a1','a2','b0','b1','b2','c1','c2'},{u(1),u(2),u(3),u(4),u(5),x1(1),x1(2)});
x1=vpa(x1,8).
```

Get time response equation $\hat{x}^{(1)}(t)$ and then get the simulation value of original sequence with $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, $(t = 2, 3, \cdots, n)$; get the prediction vale of original sequence at the q^{th} step with $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, $(t = n+1, \cdots, n+q)$.

4.2. *The optimization of variable generation coefficient λ*

To improve the model's precision, we can optimize the generation coefficient λ.

The extended GM(2, 1) model's grey differential equation is

$$\alpha^{(1}x^{(0)}(k) + a_1 x^{(0)}(k) + a_2 z^{(1)}(k) = b_0 + b_1 T_1 + b_2 T_2, \tag{4.6}$$

where $\alpha^{(1)}x^{(0)}(k) = x^{(0)}(k) - x^{(0)}(k-1)$, $z^{(1)}(k) = \lambda(x^{(1)}(k-1) + (1-\lambda)x^{(1)}(k-1), (k = 2, 3, \cdots, n)$. $T_1(k) = \frac{1}{2}[k^2 - (k-1)^2]$, $T_2(k) = \frac{1}{3}[k^3 - (k-1)^3]$.

The equations above can be written as the difference equation:

$$\begin{aligned} x^{(1)}(k) = {} & \tfrac{2+a_1-a_2\lambda}{1+a_1+a_2(1-\lambda)} x^{(1)}(k-1) - \tfrac{1}{1+a_1+a_2(1-\lambda)} x^{(1)}(k-2) \\ & + \tfrac{b}{1+a_1+a_2(1-\lambda)} + \tfrac{b_1}{1+a_1+a_2(1-\lambda)} T_1(k) + \tfrac{b_2}{1+a_1+a_2(1-\lambda)} T_2(k). \end{aligned} \tag{4.7}$$

If λ is known, get the estimates of $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1, \hat{b}_2$:

$$\begin{pmatrix} \frac{2+a_1-a_2\lambda}{1+a_1+a_2(1-\lambda)} \\ -\frac{1}{1+a_1+a_2(1-\lambda)} \\ \frac{b_0}{1+a_1+a_2(1-\lambda)} \\ \frac{b_1}{1+a_1+a_2(1-\lambda)} \\ \frac{b_2}{1+a_1+a_2(1-\lambda)} \end{pmatrix} = (B'B)^{-1}B'Y, \tag{4.8}$$

where

$$B=\begin{pmatrix} x^{(1)}(2) & x^{(1)}(1) & 1 & T_1(3) & T_2(3) \\ x^{(1)}(3) & x^{(1)}(2) & 1 & T_1(4) & T_2(4) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x^{(1)}(n-1) & x^{(1)}(n-2) & 1 & T_1(n) & T_2(n) \end{pmatrix}, \tag{4.9}$$

$$Y = \begin{pmatrix} x^{(1)}(3) \\ x^{(1)}(4) \\ \cdots \\ x^{(1)}(n) \end{pmatrix}. \tag{4.10}$$

With the equation above, get the estimates of $\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1, \hat{b}_2$, recorded as $u = [\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{b}_1, \hat{b}_2]$.

Similarly, the extended GM(2, 1) model's time response equation can be obtained using software MATLAB with the following program:

```
x1=dsolve('D2x1+a1*Dx1+a2*x1=b0+b1*t+b2*t^2','x1(1)=c1, x1(2)=c2');
x1=subs(x1,{'a1','a2','b0','b1','b2','c1','c2'},{u(1),u(2),u(3),u(4),u(5),x1(1),x1(2)});
x1=vpa(x1,8).
```

Get time response equation $\hat{x}^{(1)}(t)$ and then get the simulation value of original sequence with $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, $(t = 2, 3, \cdots, n)$; get the prediction vale of original sequence at the q^{th} step with $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, $(t = n+1, \cdots, n+q)$.

In fact, the generation coefficient λ is required, and it is generally obtained with an optimization method. Let $\min\limits_{\lambda} MAPE = \frac{1}{n-1} \sum\limits_{k=2}^{n} \left| \frac{x_i^{(0)}(k) - \hat{x}_i^{(0)}(k)}{x_i^{(0)}(k)} \right| \times 100\%$.

4.3. The steps of parameter optimization algorithm

The steps of parameter optimization algorithm are as follows:

Step 1: initialize parameter λ, i.e. setting a proper value of parameter λ;

Step 2: accumulate original sequence $x^{(0)}(t)$ and get $x^{(1)}(t)$;

Step 3: for difference equation (4.7) , calculate and get the estimates of parameters a_1, a_2, b_0, b_1, b_2 with Eq (4.8);

Step 4: get differential equation $\frac{d^2x^{(1)}}{dt^2} + a_1 \frac{dx^{(1)}}{dt} + a_2 x^{(1)} = b_0 + b_1 t + b_2 t^2$'s analytic solution $\hat{x}^{(1)}(t)$ meeting the initial conditions with the dsolve command of software Matlab;

Step 5: get the simulation value of original sequence with $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$;

Step 6: calculate and get the average simulation relative error $MAPE = \frac{1}{n-1} \sum\limits_{k=2}^{n} \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\%$;

Step 7: when meeting the precision requirement of $MAPE$ or reaching the maximum number of iterations set, go on to the next step; otherwise, return to Step 3;

Step 8: algorithm ends; output parameter estimates meeting requirements.

5. The empirical analysis on grey modeling for China's electric power consumption

With the sustained and rapid growth of national economy, China faces a critical shortage of electric power supply which affecting regional economic development and people's normal life. The electric power supply has become a limiting factor for national economy's sustainable development. The electric power has a long construction cycle, so to ensure the sufficient electric power supply, we must make arrangements in advance according to social and economic development to avoid the insufficiency of electric power. Therefore, predicting the power demand is very important. The paper predicts China's electric power consumption by building grey models.

First, build the traditional GM(2, 1) model.

The whitening equation of GM(2, 1) model is

$$\frac{d^2x^{(1)}}{dt} + a_1\frac{dx^{(1)}}{dt} + a_2x^{(1)} = b_0. \tag{5.1}$$

The parameter estimates of GM(2, 1) model are

$$u = \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b}_0 \end{pmatrix} = \begin{pmatrix} -0.18464182 \\ 0.014143615 \\ -1275.0502 \end{pmatrix}. \tag{5.2}$$

The time response equation is

$$\begin{aligned} x^{(1)}(t) &= \exp(t*(0.09232091 + 0.074969757\mathrm{i})) * (44969.931 - 83756.499\mathrm{i}) - 90150.231 \\ &+\exp(t*(0.09232091 - 0.074969757\mathrm{i})) * (44969.931 + 83756.499\mathrm{i}). \end{aligned} \tag{5.3}$$

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate the simulation and prediction values of original sequence. See Table 1 for the results. Table 1 gives the relative errors and average relative errors in the periods.

Then, build the extended GM(2, 1) model with the conventional method.

The whitening equation of extended GM(2, 1) model is

$$\frac{d^2x^{(1)}}{dt^2} + a_1\frac{dx^{(1)}}{dt} + a_2x^{(1)} = b_0 + b_1t + b_2t^2. \tag{5.4}$$

Calculate and get the parameter estimates of extended GM(2, 1) model:

$$u = \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} -0.73282828 \\ 0.70236294 \\ -10497.058 \\ 11291.053 \\ 1219.7998 \end{pmatrix}. \tag{5.5}$$

The time response equation is

$$\begin{aligned} x^{(1)}(t) &= 19699.888*t - \exp(t*(0.36641414 - 0.75372649\mathrm{i})) \\ &*(41.471091 + 21.113698\mathrm{i}) - \exp(t*(0.36641414 + 0.75372649\mathrm{i})) \\ &*(41.471091 - 21.113698\mathrm{i}) + 1736.7087*t^2 + 663.70235. \end{aligned} \tag{5.6}$$

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate the simulation and prediction values of original sequence. See Table 1 for the results. Table 1 gives the relative errors and average relative errors in the periods.

Table 1. Grey modeling results for China's electric power consumption with the conventional methods.

Year	No.	$x_1^{(0)}$	Traditional GM(2, 1) modeling with the conventional method		Extended GM(2, 1) modeling with the conventional method	
			Simulation value	Relative error (%)	Simulation value	Relative error (%)
2004	1	21971.37	-	-	-	-
2005	2	24940.32	24940.32	0	24940.32	5.46e-12
2006	3	28587.97	27871.35	2.51	28542.88	0.158
2007	4	32711.81	30964.33	5.34	32129.21	1.78
2008	5	34541.35	34204.1	0.976	35571.41	2.98
2009	6	37032.14	37570.17	1.45	38744.2	4.62
2010	7	41934.49	41035.92	2.14	41650.15	0.678
2011	8	47000.88	44567.8	5.18	44555.41	5.2
2012	9	49762.64	48124.42	3.29	48014.5	3.51
2013	10	54203.41	51655.62	4.7	52640.08	2.88
2014	11	56383.69	55101.44	2.27	58567.07	3.87
2015	12	58019.97	58391.12	0.64	64804.36	11.7
2016	13	61297.09	61441.96	0.236	68986.24	12.5
2017	14	64820.97	64158.17	1.02	68198.51	5.21
			Prediction value	Relative error (%)	Prediction value	Relative error (%)
2018	15	71508.0	66429.76	7.1	61233.66	14.4
2019	16	74725.86	68131.33	8.82	51615.83	30.9
2020	17	77042.36	69120.87	10.3	49271.34	36.0
Average simulation relative error (2004–2017)			-	2.29	-	4.24
Average prediction relative error (2018–2020)			-	8.74	-	27.11
Average total relative error (2004–2020)			-	3.50	-	8.53

Next, build the extended GM(2, 1) model with the method proposed.

First, calculate in the case $\lambda = 0.5$.

The whitening equation of extended GM(2, 1) model is

$$\frac{d^2x^{(1)}}{dt^2} + a_1\frac{dx^{(1)}}{dt} + a_2x^{(1)} = b_0 + b_1t + b_2t^2. \tag{5.7}$$

We derive the following difference equation:

$$\begin{aligned} x^{(1)}(k) = \frac{2+a_1-\frac{a_2}{2}}{1+a_1+\frac{a_2}{2}}x^{(1)}(k-1) - \frac{1}{1+a_1+\frac{a_2}{2}}x^{(1)}(k-2) \\ +\frac{b}{1+a_1+\frac{a_2}{2}} + \frac{b_1}{1+a_1+\frac{a_2}{2}}T_1(k) + \frac{b_2}{1+a_1+\frac{a_2}{2}}T_2(k). \end{aligned} \tag{5.8}$$

Calculate and get

$$\begin{pmatrix} \frac{2+a_1-\frac{a_2}{2}}{1+a_1+\frac{a_2}{2}} \\ -\frac{1}{1+a_1+\frac{a_2}{2}} \\ \frac{b_0}{1+a_1+\frac{a_2}{2}} \\ \frac{b_1}{1+a_1+\frac{a_2}{2}} \\ \frac{b_2}{1+a_1+\frac{a_2}{2}} \end{pmatrix} = \begin{pmatrix} 1.1843341 \\ -0.92812264 \\ -2242.2491 \\ 13707.009 \\ 1281.1048 \end{pmatrix}. \tag{5.9}$$

Then, get the parameter estimates of extended GM(2, 1) model:

$$u = \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} -0.32325138 \\ 0.80139039 \\ -2415.8974 \\ 14768.532 \\ 1380.3184 \end{pmatrix}. \tag{5.10}$$

The time response equation is

$$\begin{aligned} x^{(1)}(t) &= 19818.145*t - \exp(t*(0.16162569 - 0.88049277i)) \\ &*(28.646591 + 114.21828i) - \exp(t*(0.16162569 + 0.88049277i)) \\ &*(28.646591 - 114.21828i) + 1722.4045*t^2 + 680.73733. \end{aligned} \tag{5.11}$$

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate and get the simulation and prediction values of original sequence. See Table 2 for the results. Table 2 gives the relative errors and average relative errors in the periods.

Finally, build the extended GM(2, 1) model with the optimization method proposed, We derive the following difference equation:

$$\begin{aligned} x^{(1)}(k) &= \frac{2+a_1-a_2\lambda}{1+a_1+a_2(1-\lambda)} x^{(1)}(k-1) - \frac{1}{1+a_1+a_2(1-\lambda)} x^{(1)}(k-2) \\ &+ \frac{b}{1+a_1+a_2(1-\lambda)} + \frac{b_1}{1+a_1+a_2(1-\lambda)} T_1(k) + \frac{b_2}{1+a_1+a_2(1-\lambda)} T_2(k). \end{aligned} \tag{5.12}$$

Calculate and get

$$\lambda = 0.5050, \tag{5.13}$$

$$\begin{pmatrix} \frac{2+a_1-a_2\lambda}{1+a_1+a_2(1-\lambda)} \\ -\frac{1}{1+a_1+a_2(1-\lambda)} \\ \frac{b_0}{1+a_1+a_2(1-\lambda)} \\ \frac{b_1}{1+a_1+a_2(1-\lambda)} \\ \frac{b_2}{1+a_1+a_2(1-\lambda)} \end{pmatrix} = \begin{pmatrix} 1.1843341 \\ -0.92812264 \\ -2242.2491 \\ 13707.009 \\ 1281.1048 \end{pmatrix}. \tag{5.14}$$

Then, get the parameter estimates of extended GM(2, 1) model.

$$u = \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} -0.31924443 \\ 0.80139039 \\ -2415.8974 \\ 14768.532 \\ 1380.3184 \end{pmatrix}. \tag{5.15}$$

The time response equation is

$$\begin{aligned} x^{(1)}(t) &= 19800.921*t - \exp(t*(0.15962221 - 0.88085818i)) \\ &*(12.610168 + 59.634041i) - \exp(t*(0.15962221 + 0.88085818i)) \\ &*(12.610168 - 59.634041i) + 1722.4045*t^2 + 574.78518. \end{aligned} \tag{5.16}$$

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate and get the simulation and prediction values of original sequence. See Table 2 for the results. Table 2 gives the relative errors and average relative errors in the periods.

Table 2. Grey modeling results for China's electric power consumption with the method proposed.

Year	No.	$x_1^{(0)}$	Extended GM(2,1) built with the method proposed (λ = 0.5)		Extended GM(2,1) built with the parameter optimization method proposed (λ = 0.5050)	
			Simulation value	Relative error (%)	Simulation value	Relative error (%)
2004	1	21971.37	-	-	-	-
2005	2	24940.32	24940.32	1.46e-14	24940.32	1.46e-14
2006	3	28587.97	28628.88	0.143	28511.11	0.269
2007	4	32711.81	32234.66	1.46	32042.62	2.05
2008	5	34541.35	35583.68	3.02	35443.53	2.61
2009	6	37032.14	38662.68	4.4	38703.49	4.51
2010	7	41934.49	41692.24	0.578	41932.67	0.00435
2011	8	47000.88	45020.86	4.21	45309.84	3.6
2012	9	49762.64	48865.45	1.8	48950.38	1.63
2013	10	54203.41	53069.22	2.09	52780.55	2.63
2014	11	56383.69	57097.83	1.27	56531.47	0.262
2015	12	58019.97	60367.97	4.05	59903.1	3.25
2016	13	61297.09	62744.71	2.36	62817.39	2.48
2017	14	64820.97	64832.0	0.017	65570.7	1.16
			Prediction value	Relative error (%)	Prediction value	Relative error (%)
2018	15	71508.0	67720.35	5.30	68712.92	3.91
2019	16	74725.86	72207.71	3.37	72657.45	2.77
2020	17	77042.36	77981.99	1.22	77264.87	0.289
Average simulation relative error (2004–2017)			-	1.95	-	1.88
Average prediction relative error (2018–2020)			-	3.29	-	2.32
Average total relative error (2004–2020)			-	2.20	-	1.96

To compare the model built with the method proposed with the grey models proposed by other reference documents in terms of modeling precision, the paper makes a calculation.

Build an improved grey GM(1, 1) power model proposed by Ma and Wang [32], and then get the parameter estimates of $x^{(0)}$:

$$(a, b, \alpha) = (\text{-}0.046883941,\ 3964.6201,\ 0.17205992). \tag{5.17}$$

In this case, the time response equation is

$$\begin{aligned}\hat{x}^{(1)}(k) &= \left\{\frac{b}{a} + [x^{(1)}(1)^{(1-\alpha)} - \frac{b}{a}]e^{a(\alpha-1)(k-1)}\right\}^{\frac{1}{\alpha-1}} \\ &= (\ 88496.098e^{0.0388(k-1)}\text{-}84562.432)^{1.2078}.\end{aligned} \tag{5.18}$$

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate and get the simulation and prediction values of original sequence. See calculation results in Table 3. Table 3 gives relative errors and average relative errors in the periods.

Then, build the grey differential equation model DDGM(2,1) proposed by Cheng and Shi [33]. First, build the following model:

$$x^{(1)}(k) = \frac{2 + a_1 - \frac{a_2}{2}}{1 + a_1 + \frac{a_2}{2}} x^{(1)}(k-1) - \frac{1}{1 + a_1 + \frac{a_2}{2}} x^{(1)}(k-2) + \frac{b}{1 + a_1 + \frac{a_2}{2}}, \tag{5.19}$$

and then, for $x^{(0)}$, calculate and get $a_1 = \text{-}0.10555206, a_2 = 0.0045433195, b = 12332.536$.

Next, get the time sequence response equation

$$\hat{x}^{(1)}(k) = 1006449.0 \times \frac{(z_2 * z_1^k - z_1 * z_2^k)}{(z_2 - z_1)} + 1027221.7 \times \frac{(z_2^k - z_1^k)}{(z_2 - z_1)} \text{-}1005250.3, \tag{5.20}$$

where $z_1 = 0.86098127, z_2 = 1.0439078$.

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate and get the simulation and prediction values of original sequence. See Table 3 for the results. Table 3 gives the relative errors and average relative errors in the periods.

Table 3. Calculate results of grey models proposed by Ma and Wang [32] and Cheng and Shi [33].

Year	No.	$x_1^{(0)}$	Grey Bernoulli model proposed by Ma and Wang [32]		Grey differential equation proposed by Cheng and Shi [33]	
			Simulation value	Relative error (%)	Simulation value	Relative error (%)
2004	1	21971.37	-	-	-	-
2005	2	24940.32	25441.38	2.01	24940.32	1.17e-15
2006	3	28587.97	29315.46	2.54	28838.4	0.876
2007	4	32711.81	32704.45	0.0225	32517.96	0.593
2008	5	34541.35	35898.08	3.93	36023.58	4.29
2009	6	37032.14	39019.25	5.37	39394.27	6.38
2010	7	41934.49	42133.98	0.476	42664.26	1.74
2011	8	47000.88	45283.43	3.65	45863.71	2.42
2012	9	49762.64	48496.08	2.55	49019.27	1.49
2013	10	54203.41	51793.43	4.45	52154.66	3.78
2014	11	56383.69	55192.81	2.11	55291.05	1.94
2015	12	58019.97	58708.99	1.19	58447.49	0.737
2016	13	61297.09	62355.16	1.73	61641.21	0.561
2017	14	64820.97	66143.49	2.04	64887.94	0.103
			Prediction value	Relative error (%)	Prediction value	Relative error (%)
2018	15	71508.0	70085.56	1.99	68202.13	4.62
2019	16	74725.86	74192.62	0.714	71597.18	4.19
2020	17	77042.36	78475.78	1.86	75085.63	2.54
Average simulation relative error (2004–2017)			-	2.47	-	1.92
Average prediction relative error (2018–2020)			-	1.52	-	3.78
Average total relative error (2004–2020)			-	2.29	-	2.27

Build a model using the improved method of simultaneous grey model proposed by reference document [34], and then get the following time response equation through calculations:

$$\begin{aligned}\hat{x}^{(1)}(t) &= 0.04759489 * \exp(0.03226535{*}t) * (34248522.0 * \exp(-0.03226535{*}t) \\ &+1164694.0{*}t{*}\exp(-0.03226535{*}t) + 16189.17{*}t^2{*}\exp(-0.03226535{*}t) \\ &-34257811.0) + 0.09366615 * \exp(-74.63994{*}t) * (100880.7 * \exp(74.63994{*}t) \\ &+166298.1{*}t{*}\exp(74.63994{*}t) + 21363.74{*}t^2{*}\exp(74.63994{*}t) - 2.043865e37).\end{aligned} \tag{5.21}$$

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate and get the simulation and prediction values of original sequence. See Table 4 for the results. Table 4 gives the relative errors and average relative errors in the periods.

Then, build a model using the improved nonlinear optimization GM(1, N) model proposed by reference document [35], and get the following time response equation through calculations:

$$\hat{x}^{(1)}(k) = 1.324174x^{(1)}(k-1) - 0.02154469y^{(1)}(k) + 25389.48 \tag{5.22}$$

where $x^{(1)}(t)$ is the once accumulated generating column of original sequence and $y^{(1)}(t)$ is the once accumulated generating column of GDP.

With $\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1)$, calculate and get the simulation and prediction values of original sequence. See Table 4 for the results. Table 4 gives the relative errors and average relative errors in the periods.

Table 4. Calculate results of grey models proposed by reference documents [34, 35].

Year	No.	$x_1^{(0)}$	Simultaneous grey model proposed by reference document [34]		Nonlinear optimization GM(1, N) model proposed by reference document [35]	
			Simulation value	Relative error (%)	Simulation value	Relative error (%)
2004	1	21971.37	-	-	-	-
2005	2	24940.32	31455.61	26.1	-	-
2006	3	28587.97	27837.43	2.63	28297.58	1.02
2007	4	32711.81	31510.48	3.67	32036.38	2.06
2008	5	34541.35	35122.2	1.68	36438.09	5.49
2009	6	37032.14	38670.59	4.42	38230.04	3.23
2010	7	41934.49	42153.56	0.522	40158.0	4.24
2011	8	47000.88	45568.98	3.05	45016.03	4.22
2012	9	49762.64	48914.63	1.7	50633.79	1.75
2013	10	54203.41	52188.22	3.72	53119.17	2.0
2014	11	56383.69	55387.38	1.77	57909.36	2.71
2015	12	58019.97	58509.68	0.844	59820.56	3.1
2016	13	61297.09	61552.6	0.417	60747.67	0.896
2017	14	64820.97	64513.53	0.474	63242.04	2.44
			Prediction value	Relative error (%)	Prediction value	Relative error (%)
2018	15	71508.0	67389.79	5.76	66028.6	7.66
2019	16	74725.86	70178.59	6.09	73434.85	1.73
2020	17	77042.36	72877.07	5.41	77113.03	0.0917
Average simulation relative error (2004–2017)			-	3.92	-	2.76
Average prediction relative error (2018–2020)			-	5.75	-	3.16
Average total relative error (2004–2020)			-	4.26	-	2.84

Next, build non-grey models.

First, build an ARIMA model. Get the following ARIMA(2, 1, 1) model through calculations:

$$(1 + 0.172145B - 0.71044B^2)\nabla\hat{x}^{(0)}(k) = 1425.1 + (1 - B)\varepsilon(k). \tag{5.23}$$

With the model, calculate and get the simulation and prediction values of original sequence. See Table 5 for the results. Table 5 gives the relative errors and average relative errors in the periods.

Next, build a superposition exponential nonlinear regression model. Get the following model through calculations:

$$\hat{x}^{(0)}(k) = 23970e^{0.07465k} - 222.5e^{-28.94k}. \tag{5.24}$$

With the model, calculate and get the simulation and prediction values of original sequence. See Table 5 for the results. Table 5 gives the relative errors and average relative errors in the periods.

Table 5. Modeling results of non-grey models for China's electric power consumption.

Year	No.	$x_1^{(0)}$	ARIMA		Superposition exponential nonlinear regression model	
			Simulation value	Relative error (%)	Simulation value	Relative error (%)
2004	1	21971.37	22051.48	0.365	25827.84	17.6
2005	2	24940.32	26110.94	4.69	27829.68	11.6
2006	3	28587.97	29685.48	3.84	29986.67	4.89
2007	4	32711.81	32831.71	0.367	32310.85	1.23
2008	5	34541.35	32944.35	4.62	34815.16	0.793
2009	6	37032.14	37080.03	0.129	37513.58	1.3
2010	7	41934.49	44492.85	6.1	40421.15	3.61
2011	8	47000.88	47158.17	0.335	43554.07	7.33
2012	9	49762.64	48331.34	2.88	46929.81	5.69
2013	10	54203.41	55526.44	2.44	50567.2	6.71
2014	11	56383.69	54618.23	3.13	54486.5	3.36
2015	12	58019.97	57217.03	1.38	58709.59	1.19
2016	13	61297.09	62684.77	2.26	63259.99	3.2
2017	14	64820.97	64933.73	0.174	68163.08	5.16
			Prediction value	Relative error (%)	Prediction value	Relative error (%)
2018	15	71508.0	68080.41	4.79	73446.19	2.71
2019	16	74725.86	71447.92	4.39	79138.78	5.91
2020	17	77042.36	74608.96	3.16	85272.59	10.7
Average simulation relative error (2004–2017)			-	2.34	-	5.26
Average prediction relative error (2018–2020)			-	4.11	-	6.43
Average total relative error (2004–2020)			-	2.65	-	5.47

6. Results and discussion

Table 1 gives the simulation values from year 2004 to year 2017 and the prediction values from year 2018 to year 2020 of traditional GM(2, 1) model and extended GM(2, 1) model of $x_1^{(0)}$ built with the conventional method, and corresponding simulation errors and prediction errors, and gives the average simulation and prediction errors. It can be seen that the GM(2, 1) model built with the conventional method has big average simulation and prediction errors and average overall error. The conventional GM(2, 1) model has an average overall error of 3.50%, while the extended GM(2, 1) model has an average overall error of 8.52%, so the models built with the conventional method are not suitable.

Table 2 gives the simulation values from year 2004 to year 2017 and the prediction values from year 2018 to year 2020 of extended GM(2, 1) model of $x^{(0)}$ built with the method proposed, and corresponding simulation errors and prediction errors, and gives average simulation and prediction errors. It can be seen that no matter the parameters are optimized or not, the GM(2, 1) model built with the method proposed has small average simulation errors, average prediction errors and average overall error. The extended GM(2, 1) model without the optimized generation coefficient has an average overall error of 2.20%, while the extended GM(2, 1) model with the optimized generation coefficient has an average overall error of 1.96%. The extended GM(2, 1) model with the optimized generation coefficient has the highest precision which is significantly superior to that of the model built with the conventional method.

Table 3 gives the modeling calculation results of grey models proposed by Ma and Wang [32] and Cheng and Shi [33]. It can be seen that the average simulation error of $x^{(0)}$ calculated with the model proposed by Ma and Wang [32] is significantly higher than that of the extended grey model built with the method proposed. For $x^{(0)}$, its average prediction error calculated with the grey power model proposed by Ma and Wang [32] is slightly lower than that of the extended grey model built with the method proposed, but its average overall error calculated with the model proposed by Ma and Wang [32] is significantly higher than that of the extended grey model built with the method proposed. It should be noted that the establishment of a model is obtained by fitting the simulated values. As can be seen from Table 3, the simulation error (fitting error) of the model calculated with the grey power model proposed by Ma and Wang [32] is significantly higher than that of the model built with the method proposed. The predicted value of the model is derived from the extrapolation of the established model, which is characterized by uncertainty. All prediction models have such characteristics, so the prediction error calculated with the grey power model proposed by Ma and Wang [32] is slightly lower than that of the model built with the method proposed, which is quite normal. There are 14 sample points for fitting of the model in this paper, while only 3 sample points for prediction, so the accuracy of the model mainly depends on the simulation accuracy. Therefore, on the whole, the accuracy of the model built with the method proposed is significantly higher than the model proposed by Ma and Wang [32]. The average simulation error and average prediction error calculated with the model proposed by Cheng and Shi [33] are both higher than that of the extended grey model built with the method proposed. Therefore, its average overall error is significantly higher than that of the extended grey model built with the method proposed.

Table 4 gives the calculation results of grey modeling methods proposed by reference documents [34, 35]. It can be seen that $x^{(0)}$ calculated with the method of reference document [34] has an average simulation relative error of 3.92% and an average prediction relative error of 5.75%, showing the precision significantly lower than the calculation precision of the extended GM(2, 1) model built with the method proposed; $x^{(0)}$ calculated with the method of reference document [35] has an average simulation relative error of 2.76% and an average prediction relative error of 3.16%, both of which are small, but it still shows the precision lower than that of the extended GM (2, 1) model built with the method proposed.

Table 5 gives the modeling results of non-grey models. It can be seen that the ARIMA model built has an average simulation relative error of 2.34% and an average prediction relative error of 4.11%, both of which are small, but it still shows the precision lower than that of the extended GM(2, 1) model built with the method proposed; the superposition exponential nonlinear regression model has an average simulation relative error of 5.26% and an average prediction relative error of 6.43%, showing the precision significantly lower than that of the extended GM(2, 1) model built with the method proposed.

Therefore, from Tables 1–5 we can see that the extended GM(2, 1) model built with the method proposed has the precision significantly higher than that of the GM(2, 1) model built with the conventional method and the extended GM(2, 1) model, and superior to that of the models built in reference documents [32–35], the ARIMA model and the superposition exponential nonlinear regression model. It indicates the extended model and method proposed have high reliability and effectiveness.

7. Conclusions

The grey GM(2, 1) model built with the conventional method generally has big prediction errors, for which a main reason is the parameter estimation method of model has defects. To improve the modeling precision, the paper proposes an extended grey GM(2, 1) model, gives an improved parameter estimation method for the new model based on the difference equation, and gives a method to solve the time response equation.

Using the method proposed, the paper builds eight different grey models for China's electric power consumption including the traditional GM(2, 1) model built with the conventional parameter estimation method, the extended GM(2, 1) model built with the conventional parameter estimation method, the extended GM(2, 1) model built with the improved parameter estimation method (without optimizing the parameters), the extended GM(2, 1) model built with the improved parameter estimation method proposed (optimizing the parameters), the grey Bernoulli model proposed by reference document [32], the grey difference equation model proposed by reference document [33], the simultaneous grey model proposed by reference document [34], the nonlinear optimization GM(1, N) model proposed by reference document [35] and two non-grey models. Results show that the extended GM(2, 1) models with the improved parameter estimation method proposed have high prediction precision, with or without optimizing the variable generation coefficient. It indicates the method proposed has high reliability and applicability and thus helps the generalization and application of model.

The paper gives a new method to build the extended GM(2, 1) model based on the structure of difference equation. The method transforms the extended GM(2, 1) model into the form of second-order difference equation, and then solves and gets the unknown parameters with the new difference equation. Estimating parameters in this way can avoid the error of model caused by the approximate calculation of background value; meanwhile, using an optimization method to get the model's parameters improves the precision of model greatly. The method can be generalized and applied to the predictions of grey models with more complicated structures. It can choose difference equation models with different structures for different data and thus improves the modeling precision.

The model's applications have a limitation that not all grey models can be transformed to a difference equation. For instance, it's hard to transform a high-order grey model into a difference equation. In addition, if the modeling doesn't use an optimization method, the precision shall be affected. We plan to use the method proposed to estimate parameters for other grey models, such as the GM(1, N) model, the GM(1, 1) power model and so on, in the future, and gets the solution for prediction using the difference equation derived, which is a new idea.

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Conflict of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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