



Research article

Double-quantized-based H_∞ tracking control of T-S fuzzy semi-Markovian jump systems with adaptive event-triggered

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Abstract: This paper investigates the issue of asynchronous H_∞ tracking control for nonlinear semi-Markovian jump systems (SMJSs) based on the T-S fuzzy model. Firstly, in order to improve the performance of network control systems (NCSs) and the efficiency of data transmission, this paper adopts a double quantization strategy which quantifies the input and output of the controllers. Secondly, for the purpose of reducing the burden of network communication, an adaptive event-triggered mechanism (AETM) is adopted. Thirdly, due to the influence of network-induced delay, the system mode information can not be transmitted to the controller synchronously, thus, a continuous-time hidden Markov model (HMM) is established to describe the asynchronous phenomenon between the system and the controller. Additionally, with the help of some improved Lyapunov-Krasovskii (L-K) functions with fuzzy basis, some sufficient criteria are derived to co-guarantee the state stability and the H_∞ performance for the closed-loop tracking control system. Finally, a numerical example and a practical example are given to verify the effectiveness of designed mentality.

Keywords: fuzzy semi-Markovian jump systems; asynchronous H_∞ control; adaptive event-triggered mechanism; double quantization; tracking control

Mathematics Subject Classification: 93C15, 93B36

1. Introduction

As we all know, fuzzy model is an extension of deterministic systems, which gives the input, output and state variables definitions on fuzzy sets. Recently, due to the effectiveness in handling the problems of safety control for nonlinear NCSs, fuzzy models have been applied in many research fields, such

as fault diagnosis systems, robot control systems and power systems. The ability of fuzzy systems to deal with uncertainty and nonlinear has also attracted much attention in network control, engineering, computer science, and other fields, such as [1–3]. The T-S fuzzy methods can approximate the nonlinear systems through a set of local linear systems and a set of IF-THEN rules. Based on the fuzzy systems, many methods of stability and synchronization analysis for linear systems can be extended to the study of nonlinear systems. This provides great convenience undoubtedly for the research of nonlinear NCSs.

Markov jump processes can describe the switch suddenly of system structure parameters due to external disturbance. The transition of the system among the various modes is determined by a set of Markov chains. Recently, SMJSs whose transition rates change with sojourn time have also been investigated. The semi-Markovian processes are similar to the Markov processes generally. However, compared with the Markov processes, the transition time and probability are decided by the time when the system reaches the current state. Recently, more and more researchers have investigated the SMJSs [4–6]. In [7], the stable and robust stability conditions for uncertain Markov transition neural networks (NNs) with partially unknown transition probabilities are derived. In [8], the problem of finite-time H_∞ fault tolerant control for the T-S SMJSs is investigated. In [9], the asynchronous control problem of a nonlinear MJS based on a method of fuzzy quantization is investigated. Moreover, the designed fuzzy sample controller and the system are asynchronous. The stochastic stability of T-S fuzzy MJSs is guaranteed based on the HMM. In [10], the problem of an elastic asynchronous H_∞ control for an uncertain slow sampling discrete-time Markov singular perturbation system (SPS) is investigated. A new conservative sampling controller is proposed to achieve the stability and the H_∞ performance of the closed-loop system. In [11], an asynchronous H_∞ filter based on an ETM for SMJSs at finite time intervals is designed. In [12], the resilient cooperative output regulation problem of a class of uncertain nonlinear multi-intelligent systems under denial-of-service attacks is studied. A distributed control scheme consisting of a resilient distributed observer and a distributed adaptive controller is proposed. The results show that the control scheme can solve the elastic cooperative output regulation problem for a related class of uncertain nonlinear mathematical models.

As is known to all, time-triggered mechanism and ETM are applied to decrease the network communication burden. Both two communication mechanisms have been introduced into the most NCSs. Based on the previous communication scheme, the signals of the system can be conveyed regularly. For ETM, the data signals of the system can be conveyed unless the given events occur. Meanwhile, ETM can bring more difficulties in constructing the controllers. The advantage over time triggered communication is that ETM can reduce more burden of network communication if it undergoes a little changes for the transmitted signals [13]. In order to make better use of network resources, some strategies on event-triggered communication are proposed for MJSs [14–16]. In [17], the guaranteed performance control for T-S fuzzy MJSs with time-varying delays and incompletely unknown transition probabilities is investigated. An ETM related to the system mode is considered at the feedback channel where the network-induced delay occurs randomly. In [18], the safety control problem of the nonlinear unmanned vehicle (UMV) systems is investigated. Sensor-to-controller and controller-to-actuator-side dynamic ETM and an observer-based sliding mode control (SMC) scheme are proposed to stabilize the system under DOS attacks.

Besides ETM, quantization protocol is another method to reduce the burden of network communication [19–21]. In [19, 20], the robust stability of unknown discrete linear systems with

the double quantization strategy is investigated. They also propose some sufficient conditions for closed-loop systems with norm bounded uncertainty and dual quantization. The latter takes into account the phenomenon of the data packets loss. In [21], the problem of quantitative feedback control for uncertain time varying systems based on a sliding mode observer (SMO) is investigated. By constructing a novel L-K function, a time delay-dependent SMO designed criteria is proposed so that the dynamical error system is practically stable.

In [22], the problem of a static output feedback robust H_∞ control for the discrete-time singular MJSs is studied. Moreover, the transition probability is incompletely known. In [23], an H_∞ tracking control method with adaptive dynamic programming (ADP) for the nonlinear continuous-time SMJSs is proposed. A neural network (NN) detector which includes the input and output of the system signals is designed. In [24], a tracking control problem of nonlinear SMJSs based on T-S fuzzy methods with the incompletely known transition probability is investigated. Furthermore, the designed controller does not fully control the mode that the system maintains at runtime. In this imperfect modal information scenario, a mechanism based on HMM is improved to simulate the defect of modal asynchrony. The main work of [25] is on investigating a class of state output tracking control problem for fuzzy SMJSs. Besides, the affect of both unpredictable external disturbances and uncertainties in the system are also considered. In [26], the location control based on intelligent event triggering in a networked unmanned vehicle (UMV) system based on hybrid attack is studied, where the UMV and the control station are connected through a communication network. The mean square exponential stability condition of the closed loop system and the design method of the controller based on the observer are achieved.

Based on the above analysis of many literatures, this paper investigates a quantitative output feedback control problem of fuzzy tracking control systems under the the influence of the uncertainty and external interference. The main contributions of this paper are summarized as the following:

1) An AETM based on the period data sampling is adopted to relieve the communication burden of the network. The triggered threshold in the AET condition can be adjusted dynamically through an adaptive threshold function $\epsilon(t)$, which overcomes the defect of the static ETM. Meanwhile, the size of $\epsilon(t)$ can be regulated by an equation related to its derivative $\dot{\epsilon}(t)$;

2) This paper employs a double quantization strategy, which quantizes the controller input and controller output, to compress the network signal. The uncertainty caused by double quantization will increase the difficulties of the research. Therefore, this paper applies two lemmas to deal with this uncertainty.

3) The asynchronous phenomenon is inevitable in realistic situation. In this paper, the time-delay caused by AETM will introduce the asynchronous phenomenon between the controller and the system. Thus, an HMM is established to deal with this problem. Moreover, some sufficient inequality conditions which can ensure the stochastic stability of the system and satisfy the prescribed H_∞ tracking performance are derived.

The remainder of this paper is arranged as follows. In Section 2, a controller with AETM and double quantization strategy is constructed for the continuous-time fuzzy SMJSs. In Section 3, some sufficient inequality conditions, which can guarantee the stochastic stability of the system with a prescribed H_∞ tracking performance, are derived. Two examples are shown in Section 4 to illustrate the effectiveness of proposed methods. Finally, some conclusions for this paper are summarized in Section 5.

Notations: \mathbb{R}^{n_v} represents the n_v dimension Euclidean space; $\mathbb{R}^{n \times m}$ represents the set of all $n \times m$

real matrices; $\mathfrak{P} > 0$ denotes that \mathfrak{P} is a positive definite matrix; $*$ in matrices represent the symmetric elements for a symmetric matrix; \mathbb{Z}^+ represents a set of non-negative integers; I and 0 denote unity matrix and zero matrix, respectively; \mathcal{Z}^T denotes the transpose of \mathcal{Z} ; $\text{sym}(\mathcal{AB})$ represents $\mathcal{AB} + \mathcal{B}^T \mathcal{A}^T$; $\text{diag}\{\cdot\}$ denotes a diagonal matrix; $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation in probability statistics.

2. Problem formulation

2.1. System dynamics

In consideration of a nonlinear fuzzy SMJS with Markov jump parameter ϕ_t . Let r be the number of the fuzzy rules. Then, the i th fuzzy rule can be described as follows:

Plant rule i : If $\eta_1(\varphi(t))$ is M_1^i , $\eta_2(\varphi(t))$ is M_2^i , \dots , and $\eta_a(\varphi(t))$ is M_a^i , then,

$$\begin{cases} \dot{\kappa}(t) = \mathcal{A}_i(\phi_t)\kappa(t) + \mathcal{B}_i(\phi_t)u(t) + \mathcal{D}_i(\phi_t)\omega(t), \\ z(t) = C(\phi_t)\kappa(t), \end{cases} \quad (2.1)$$

where $M_1^i, M_2^i, \dots, M_a^i$ denote the fuzzy sets of the rule i , $i \in \mathbb{I} = \{1, 2, \dots, r\}$. $\eta_\iota(t)$, $\iota \in \mathbb{A} = \{1, 2, \dots, a\}$ represent the premise variables. $\kappa(t) \in \mathbb{R}^{n_\nu}$ is the system state vector. And $u(t) \in \mathbb{R}^{n_u}$ is the input vector which will be designed in this paper. $\omega(t)$ denotes the external disturbance which belongs to $\mathcal{L}[0, +\infty]$. $\mathcal{A}_i(\phi_t)$, $\mathcal{B}_i(\phi_t)$, $C(\phi_t)$, $\mathcal{D}_i(\phi_t)$ are known real coefficient matrices which has appropriate dimensions. For notational brevity, these will be denoted by \mathcal{A}_{i_s} , \mathcal{B}_{i_s} , C_s , \mathcal{D}_{i_s} for $\phi_t = s$.

Through the method of fuzzy approach, denote $\phi_t = s$, the description of the system (2.1) can be modified as

$$\begin{cases} \dot{\kappa}(t) = \sum_{i=1}^r h_i(\eta(t))[\mathcal{A}_{i_s}\kappa(t) + \mathcal{B}_{i_s}u(t) + \mathcal{D}_{i_s}\omega(t)], \\ z(t) = C_s\kappa(t), \end{cases} \quad (2.2)$$

where $\eta(t) = [\eta_1(t) \ \eta_2(t) \ \dots \ \eta_a(t)]$ and

$$\begin{cases} h_i(\eta(t)) = \frac{\vartheta_i(\eta(t))}{\sum_{i=1}^r \vartheta_i(\eta(t))}, \\ \vartheta_i(\eta(t)) = \prod_{\iota=1}^a M_\iota^i(\eta_\iota(t)), \end{cases}$$

with $M_\iota^i(\eta_\iota(t))$ being the membership grade of $\eta_\iota(t)$ in M_ι^i . For $i \in \mathbb{I}$, $h_i(\eta(t))$ is the membership functions(MFs) of rule i which has been normalized. $h_i(\eta(t)) \geq 0$ and $\sum_{i=1}^r h_i(\eta(t)) = 1$. For the symbol simplification, h_i replaces $h_i(\eta(t))$ in the subsequent sections.

Let ϕ_ι , where $\iota = \{0, 1, 2, \dots\}$, denotes the ι th system mode switch. \hat{h}_ι is the residence time from the $(\iota - 1)$ th switch to the ι th switch. Let $\hat{h}_0 = 0$ and $\phi_t \in \mathcal{S} = \{1, 2, \dots, M\}$. $\{\phi_\iota, \hat{h}_\iota\}_{\iota \geq 0} \triangleq \{\phi_\iota, \hat{h}_\iota\}_{\iota \in \mathcal{S}_{\geq 1}}$ can be described as a semi-Markovian process. The matrix of the transition probability $\Lambda(\hat{h}) = [\lambda_{i_s j_s}(\hat{h})]_{M \times M}$ can be written as [5]

$$\begin{cases} Pr\{\phi_{\iota+1} = j_s, \hat{h}_{\iota+1} \leq \hat{h} + \theta | \phi_\iota = i_s, \hat{h}_{\iota+1} > \hat{h}\} = \lambda_{i_s j_s}(\hat{h})\theta + o(\theta), \quad i_s \neq j_s, \\ Pr\{\phi_{\iota+1} = j_s, \hat{h}_{\iota+1} > \hat{h} + \theta | \phi_\iota = i_s, \hat{h}_{\iota+1} > \hat{h}\} = 1 + \lambda_{i_s i_s}(\hat{h})\theta + o(\theta), \quad i_s = j_s, \end{cases} \quad (2.3)$$

where $\theta > 0$, $\lim_{\theta \rightarrow 0} (o(\theta)/\theta) = 0$, $\lambda_{i_s j_s}(h) \geq 0$ for $\forall i_s \neq j_s$, and $\lambda_{i_s i_s}(h) = -\sum_{j_s=1}^M \lambda_{i_s j_s}(h)$.

Remark 2.1. To avoid notational confusion, it is necessary to clarify that the i_s and j_s is a specific value of the system mode s . For example, the $\lambda_{i_s j_s}$ indicates the probability of transfer from modality i_s to modality j_s . The i_s and s represent the same concept. However, it should be noted that the i in i_s is not the same meaning as the fuzzy rule i previously mentioned in this article.

2.2. Reference model

In order to track the desired signal of the reference output, the following stable reference model is selected. The system output $z(t)$ in (2.2) tracks the reference signals $z_r(t)$:

$$\begin{cases} \dot{\varkappa}_r(t) = \mathcal{A}_r \varkappa_r(t) + \mathcal{B}_r r(t), \\ z_r(t) = \mathcal{C}_r \varkappa_r(t), \end{cases} \quad (2.4)$$

where $\varkappa_r(t)$, $z_r(t)$, $r(t)$ are the state vector, output vector, and the bounded reference input, respectively. \mathcal{A}_r , \mathcal{B}_r , \mathcal{C}_r are known real matrices which have appropriate dimensions.

Remark 2.2. *The system (2.4) is selected as the reference model of H_∞ tracking control. What's more, the reference system is known and stable. In the following research, by letting the system (2.2) track the system (2.4).*

2.3. AET mechanism

AETM is a strategy which is applied to save the communication cost and reduce the network bandwidth pressure. The measurement/control signal will be released when the pre-specified triggered condition are met. The AETM is based on the data sampling. We denote $\tau_0 < \tau_1 < \dots < \tau_m < \dots$ and $\eta = \tau_{m+1} - \tau_m$ as the sampling instants and the sampling period. Assume that $\tau_0 = 0$ and η is a constant. Let t_0, t_1, \dots, t_k denote the event triggered instants. Assume that $t_0 = 0$. For $t \in [\tau_m, \tau_{m+1})$, $m \in \mathbb{Z}_+$, the asynchronous fuzzy tracking controller related to AET can be constructed as

$$u(t) = \sum_{j=1}^r \hat{h}_j(\rho(t)) [K_j(\varphi_t) \varkappa(t_k) + F_j(\varphi_t) \varkappa_r(\tau_m)], \quad (2.5)$$

where

$$K_j(\varphi_t) = \sum_{j=1}^r \hat{h}_j(\rho(t)) K_j(\varphi_t), \quad \hat{h}_j(\rho(t)) = \frac{\mu_j(\rho(t))}{\sum_{j=1}^r \mu_j(\rho(t))}, \quad \mu_j(\rho(t)) = \prod_{i=1}^g \mu_j(\varphi_i)(\rho(k)),$$

and $\{\varphi_t\}_{t \in \mathbb{R}^+} \in \mathcal{M} = \{1, 2, \dots, S\}$. Due to the existence of the time delay and other external disturbance, the mode switching between the controller and the system is not synchronous at all time. Hence, a hidden process is introduced and its property is defined by

$$Pr\{\varphi_t = \mu | \phi_t = s\} = \varrho_{s\mu},$$

where for $\forall s \in \mathcal{S}$, $\mu \in \mathcal{M}$, $\varrho \in [0, 1]$, and $\sum_{\mu=1}^S \varrho_{s\mu} = 1$. In (2.5), $t_k = \max\{t | t \in \{t_d, d = 0, 1, \dots, \}\}$ denotes the latest event-triggered instants. $K_j(\varphi_t)$ and $F_j(\varphi_t)$ denote the control gain matrices. For notational brevity, we use $K_{j\mu}$ and $F_{j\mu}$ to denote $K_j(\varphi_t)$ and $F_j(\varphi_t)$.

The event trigger will work if $t = \tau_m$ ($m \in \mathbb{Z}_+$) and the condition (2.6) is violated:

$$\xi^T(\tau_m) \xi(\tau_m) \leq \epsilon(t) \zeta^T(\tau_m) \zeta(\tau_m), \quad (2.6)$$

where $\xi(\tau_m) = x(\tau_m) - x(t_k)$, $\zeta(\tau_m) = [x^T(\tau_m) \quad x_r^T(\tau_m)]^T$. $\epsilon(t)$ is a threshold which can be adjusted adaptively. According to [27], $\epsilon(t)$ can be adjusted by

$$\dot{\epsilon}(t) = \frac{1}{\epsilon(t)} \left[\frac{1}{\epsilon(t)} - \epsilon_0 \right] \xi^T(\tau_m) \xi(\tau_m), \quad (2.7)$$

where ϵ_0 is a given constant.

Remark 2.3. The conditional probability $q_{s\mu}$ in (2.5) is applied as a mechanism to measure the asynchronous degree. This will be considered in the design of the ultimate controller.

Remark 2.4. The AETM is more effective than traditional ETM in saving bandwidth resources. It can be obtained from (2.7) that the event trigger threshold can be adjusted dynamically. Moreover, if $\epsilon(0)$ is $\frac{1}{\epsilon_0}$, where $\epsilon(0)$ is the initial value of $\epsilon(t)$. In this case, $\dot{\epsilon}(t) = 0$ for $t \geq 0$, which applies that $\epsilon(t)$ is a constant. The AET condition is modified as the traditional ETM [28, 29].

2.4. Design of the controller with double quantization

Inspired by [30], this paper quantizes both the controller input and the controller output. Given a vector $b = [b_1 \ b_2 \ \cdots \ b_C]^T \in \mathbb{R}^n$, let $\hat{f}(b) = [\hat{f}_1(b_1) \ \hat{f}_2(b_2) \ \cdots \ \hat{f}_C(b_C)]^T$ be the quantizer of the controller input. For $c = 1, 2, \dots, C$, $\hat{f}_c(b_c)$ is described as

$$\hat{f}_c(b_c) = \begin{cases} u_l^c, & \frac{\hat{u}_l^c}{1+\theta_{f_c}} \leq b_c \leq \frac{\hat{u}_l^c}{1-\theta_{f_c}}, \quad b_c > 0, \\ 0, & b_c = 0, \\ -\hat{f}_c(-b_c), & b_c < 0, \end{cases}$$

where $\theta_{f_c} = (1 - \rho_{f_c}) / (1 + \rho_{f_c})$, $0 < \rho_{f_c} < 1$ is defined as the density of the double quantization. $\hat{u}_j^c \in \mathcal{U}_c$ is defined as the level of the double quantization, and $\mathcal{U}_c = \{\pm \hat{u}_j^c, \hat{u}_j^c = (\rho_{f_c})^j \hat{u}_0^c, j = \pm 1, \pm 2, \dots, \} \cup \{\pm \hat{u}_0^c\} \cup \{0\}$ with $\hat{u}_0^c > 0$.

For the quantization of the controller input, let $\Delta_{\hat{f}} = \text{diag}\{\Delta_{\hat{f}_1}, \Delta_{\hat{f}_2}, \dots, \Delta_{\hat{f}_C}\}$, $\Delta_{\hat{f}_c} \in [-\theta_{f_c}, \theta_{f_c}]$, for $c \in \mathcal{N} = \{1, 2, \dots, C\}$. Refer to [31], through the approach of the sector bound, $\hat{f}(b)$ is defined as

$$\hat{f}(b) = (I + \Delta_{\hat{f}})b. \quad (2.8)$$

For the quantization of the controller output, let $\Delta_{\hat{g}} = \text{diag}\{\Delta_{\hat{g}_1}, \Delta_{\hat{g}_2}, \dots, \Delta_{\hat{g}_C}\}$, $\Delta_{\hat{g}_c} \in [-\theta_{g_c}, \theta_{g_c}]$, $\hat{g}_c(b_c)$ is described in the same way as $\hat{f}_c(b_c)$:

$$\hat{g}(b) = (I + \Delta_{\hat{g}})b. \quad (2.9)$$

For simplicity, we set $\theta_{f_c} = \theta_f$ and $\theta_{g_c} = \theta_g$, $0 < \theta_f < 1$ and $0 < \theta_g < 1$. The following two inequalities are specified:

$$\Delta_{\hat{f}}^T \Delta_{\hat{f}} \leq \theta_f^2 I, \quad \Delta_{\hat{g}}^T \Delta_{\hat{g}} \leq \theta_g^2 I. \quad (2.10)$$

Through mentioned above quantization scheme, the tracking controller in (2.5) can be modified as

$$u(t) = \sum_{j=1}^r \hat{h}_j(\rho(t)) \hat{g}[K_j(\varphi_t) \hat{f}(\mathcal{X}(t_k)) + F_j(\varphi_t) \hat{f}(\mathcal{X}_r(\tau_m))]. \quad (2.11)$$

Combining (2.8), (2.9) and (2.11), we can rewrite the double quantization controller as

$$u(t) = \sum_{j=1}^r \hat{h}_j[(K_{j\mu} + \Delta_{r_1(t)})\mathcal{X}(t_k) + (F_{j\mu} + \Delta_{r_2(t)})\mathcal{X}_r(\tau_m)], \quad (2.12)$$

where

$$\begin{cases} \Delta_{r_1}(t) = \Delta_{\hat{g}} K_{j\mu} + K_{j\mu} \Delta_{\hat{f}} + \Delta_{\hat{g}} K_{j\mu} \Delta_{\hat{f}}, \\ \Delta_{r_2}(t) = \Delta_{\hat{g}} F_{j\mu} + F_{j\mu} \Delta_{\hat{f}} + \Delta_{\hat{g}} F_{j\mu} \Delta_{\hat{f}}. \end{cases} \quad (2.13)$$

Note that $K_j(\varphi_i)f(\mathcal{X}(t_k)) + F_j(\varphi_i)f(\mathcal{X}_r(\tau_m))$ is the output of the controller which varies with t . Hence, by the function of $g(\cdot)$ and $f(\cdot)$, $\Delta_{\hat{g}}$ and $\Delta_{\hat{f}}$ also change with t . Thus, we have

$$\Delta_{\hat{f}}^T(t) \Delta_{\hat{f}}(t) \leq \theta_f^2 I, \quad \Delta_{\hat{g}}^T(t) \Delta_{\hat{g}}(t) \leq \theta_g^2 I. \quad (2.14)$$

For simplicity, $\Delta_{\hat{f}}(t)$, $\Delta_{\hat{g}}(t)$, $\Delta_{r_1}(t)$ and $\Delta_{r_2}(t)$ are abbreviated as $\Delta_{\hat{f}}$, $\Delta_{\hat{g}}$, Δ_{r_1} and Δ_{r_2} , respectively. It is necessary to note

$$\begin{cases} \Delta_{1r_1} = \Delta_{\hat{g}} K_{j\mu}, & \Delta_{2r_1} = K_{j\mu} \Delta_{\hat{f}}, & \Delta_{3r_1} = \Delta_{\hat{g}} K_{j\mu} \Delta_{\hat{f}}, \\ \Delta_{1r_2} = \Delta_{\hat{g}} F_{j\mu}, & \Delta_{2r_2} = F_{j\mu} \Delta_{\hat{f}}, & \Delta_{3r_2} = \Delta_{\hat{g}} F_{j\mu} \Delta_{\hat{f}}. \end{cases} \quad (2.15)$$

Then,

$$\begin{cases} \Delta_{r_1} = \Delta_{1r_1} + \Delta_{2r_1} + \Delta_{3r_1}, \\ \Delta_{r_2} = \Delta_{1r_2} + \Delta_{2r_2} + \Delta_{3r_2}. \end{cases} \quad (2.16)$$

Remark 2.5. To quantify the data which need to be transmitted between the system and the controller, that is, to convert the form of transmitted data from continuous one to discrete one. At this time, less bandwidth resource is required for the data transmission. However, for most network quantization methods, they only consider the single quantization between the system and the controller [32, 33].

In this paper, the double quantization between the system and the controller is considered. Therefore, compared with the existing work, this paper adopts the method of double quantization in [34] to design the tracking controller of (2.12) to save more bandwidth resources. Here, double quantization refers to the quantization of the input and output of the controller, that is, from system to controller and from controller to system both require quantization. The advantage of quantizing the input and output of the controller respectively is that it can reduce the communication burden more efficiently. However, there are also some disadvantages, such as this strategy will introduce more uncertainty in the final system. At the same time, it will add more difficulties to the derivation of the paper. Since the states of the original system and the reference system are not identical, two control gain matrices are introduced to fully account for the differences between the two systems. It is necessary to note that the quantization controller contains two different control gain matrices $K_{j\mu}$ and $F_{j\mu}$ corresponding with the system (2.2) and reference system (2.4), respectively.

2.5. Tracking error dynamic system

Denote the tracking error $e(t) = z(t) - z_r(t)$ and let $\varepsilon(t) = t - \tau_m$. Then, the following augment tracking error dynamic system can be obtained:

$$\begin{cases} \dot{\zeta}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) \hat{h}_j(\rho(t)) [\bar{A}_{is} \zeta(t) + \bar{B}_{ijs\mu} \zeta(t - \varepsilon(t)) - \bar{E}_{ijs\mu} \xi(t - \varepsilon(t)) + \bar{D}_{is} \bar{\omega}(t)], \\ e(t) = \bar{C}_s \zeta(t), \end{cases} \quad (2.17)$$

where $i, j \in \mathbb{I}$ and

$$\zeta(t) = [\mathcal{X}^T(t) \quad \mathcal{X}_r^T(t)]^T, \quad \bar{\omega}(t) = [\omega^T(t) \quad r^T(t)]^T, \quad \mathcal{X}(t_k) = \mathcal{X}(\tau_m) - \xi(\tau_m),$$

$$\bar{A}_{is} = \begin{bmatrix} \mathcal{A}_{is} & 0 \\ 0 & \mathcal{A}_r \end{bmatrix}, \quad \bar{B}_{ijs\mu} = \begin{bmatrix} \mathcal{B}_{is}(K_{j\mu} + \Delta_{r1}) & \mathcal{B}_{is}(F_{j\mu} + \Delta_{r2}) \\ 0 & 0 \end{bmatrix},$$

$$\bar{E}_{ijs\mu} = \begin{bmatrix} \mathcal{B}_{is}(K_{j\mu} + \Delta_{r1}) \\ 0 \end{bmatrix}, \quad \bar{D}_{is} = \begin{bmatrix} \mathcal{D}_{is} & 0 \\ 0 & \mathcal{B}_r \end{bmatrix}, \quad \bar{C}_s = [C_s \quad -C_r].$$

Figure 1 shows the framework of NCS with double quantizers and the AET. As shown in Figure 1, in consideration of the limited capacity of the communication channels and in order to decrease the rate of the data transmission, two quantizers will quantize the signals of the state and control before they are transmitted to the network medium. To this end, the following definitions and some related lemmas are necessary to list here.

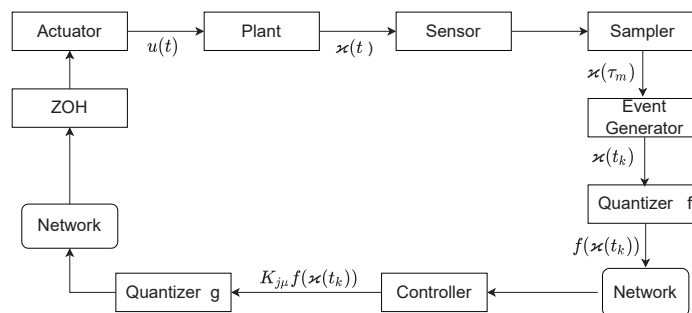


Figure 1. The framework of NCS with double quantizers and the AET mechanism.

Definition 2.1. The system (2.17) will achieve a stochastic stability, if for $\forall \hat{t}$, $\bar{\omega}(\hat{t}) \equiv 0$ and $\zeta(0) \in \mathbb{R}^n$, the following inequality holds:

$$\mathbb{E}\left\{ \int_{\hat{t}=0}^{\infty} \|\zeta(\hat{t})\|^2 | \zeta(0), \lambda_0 \right\} < \infty. \quad (2.18)$$

Definition 2.2. For the dynamic tracking error system (2.17), there is a definition for the H_∞ norm:

$$\|\Phi\|_\infty = \sup \left\{ \frac{\|e(\hat{t})\|_2}{\|\bar{\omega}(\hat{t})\|_2}; \|\bar{\omega}(\hat{t})\|_2 \neq 0 \right\}.$$

Under the controller (2.12) and the zero initial condition, if system (2.17) satisfies the condition in the Definition 2.1, and $\|\Phi\|_\infty < \lambda$, such that the following inequality holds:

$$\int_0^\infty e^T(\hat{t})e(\hat{t}) d\hat{t} < \int_0^\infty \lambda^2 \bar{\omega}^T(\hat{t})\bar{\omega}(\hat{t}) d\hat{t}, \quad (2.19)$$

then, it is reasonable to believe that the system (2.17) achieves a stochastic stability and satisfies the given H_∞ output tracking performance index λ .

Lemma 2.1. There exists matrices $\mathcal{R} > 0$, \mathcal{X} is a matrix which has appropriate dimensions, the following inequality can be established under all the circumstance:

$$-\mathcal{X}^T \mathcal{R}^{-1} \mathcal{X} \leq -\mathcal{X}^T - \mathcal{X} + \mathcal{R}.$$

Lemma 2.2. For $\forall \nu > 0$, there exists matrices $\tilde{\Upsilon}_1$ and $\tilde{\Upsilon}_2$ the following inequality can be established:

$$\tilde{\Upsilon}_1^T \tilde{\Upsilon}_2 + \tilde{\Upsilon}_2^T \tilde{\Upsilon}_1 \leq \nu \tilde{\Upsilon}_1^T \tilde{\Upsilon}_1 + \nu^{-1} \tilde{\Upsilon}_2^T \tilde{\Upsilon}_2.$$

Lemma 2.3. For $N_1 \geq 2n$, $\mathfrak{N} \in \mathbb{R}^{n \times n}$ is a semi-positive matrix, and $\mathfrak{N}_1 \in \mathbb{R}^{N_1 \times N_2}$ is a symmetric matrix which is described as

$$\mathfrak{N} = \begin{bmatrix} \mathfrak{N}_1 & 0 & 0 & \mathfrak{N}_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathfrak{N}_1 & 0 & 0 & \mathfrak{N}_1 \end{bmatrix}.$$

Then, we can get $\mathfrak{N} \geq 0$.

Remark 2.6. It is necessary to note that the researches about the stochastic stability for the fuzzy SMJSs mostly focus on the field of discrete-time. This paper extends the tracking control problem of T-S fuzzy MJSs to the continuous-time domain compared with the extant researches [35, 36].

3. Results

This section will firstly analyze the stability and H_∞ tracking performance of the augment system (2.17) with the AETM and the double quantization strategy.

Theorem 3.1. For some given scalars $\rho_i, \rho_j, \lambda > 0$, the control gain matrices $K_{j\mu}$ and some other matrices $P_{i_s} > 0$, $Q > 0$, $W > 0$, $Z_i = Z_i^T$, $Z_j = Z_j^T$ with appropriate dimensions, the system (2.17) can achieve the defined stochastic stability and satisfy the prescribed H_∞ tracking performance λ , if $\hat{h}_j - \rho_j h_j \geq 0$, such that for $\forall i_s, j_s \in \mathcal{S}$, $\forall \mu \in \mathcal{M}$. Meanwhile, $\forall i, j \in \mathbb{I}$, the following inequalities conditions hold:

$$\bar{\Theta}_{ijj_s} = \hat{\Omega} + \hat{\Upsilon} < 0, \quad (3.1)$$

$$\bar{\Theta}_{ijj_s} - Z_i < 0, \quad (3.2)$$

$$\rho_i \bar{\Theta}_{ijj_s} + (1 - \rho_i) Z_i < 0, \quad (3.3)$$

$$\rho_i \bar{\Theta}_{ijj_s} + \rho_j \bar{\Theta}_{ijj_s} + (1 - \rho_i) Z_j + (1 - \rho_j) Z_i < 0, \quad i < j, \quad (3.4)$$

where

$$\hat{\Omega} = \begin{bmatrix} \text{sym}(P_{i_s} \bar{A}_{i_s}) + \sum_{j_s \in \mathcal{S}} \bar{\lambda}_{i_s j_s} P_{j_s} + Q + \bar{C}_s^T \bar{C}_s & P_{i_s} \bar{B}_{ij_s \mu} & 0 & -P_{i_s} \bar{E}_{ij_s \mu} & 0 & \bar{A}_{i_s}^T \\ * & I & 0 & 0 & 0 & \bar{B}_{ij_s \mu}^T \\ * & * & -Q & 0 & 0 & 0 \\ * & * & * & -\epsilon_0 I & 0 & -\bar{E}_{ij_s \mu}^T \\ * & * & * & * & -\lambda^2 I & 0 \\ * & * & * & * & * & -\frac{1}{\eta} W^{-1} \end{bmatrix},$$

$$\hat{\Upsilon} = -\frac{1}{\eta} \hat{F}^T \tilde{W} \hat{F}, \quad \tilde{W} = \begin{bmatrix} W & 0 \\ 0 & W \end{bmatrix}, \quad \hat{F} = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 \\ 0 & I & -I & 0 & 0 & 0 \end{bmatrix}, \quad \bar{\lambda}_{i_s j_s} = \int_0^\infty \lambda_{i_s j_s}(\hat{h}) g(\hat{h}) d\hat{h},$$

where $g(\hat{h})$ is the probability density function (PDF) of the residence time \hat{h} for the system mode i_s .

Proof. Suppose $\zeta_t(y) = \zeta(t + y)$ for $\eta \leq y \leq 0$. For $t \in [\tau_m, \tau_{m+1})$, $m \in \mathbb{Z}_+$, the following L-K functions are selected as

$$\mathcal{V}(\zeta_t, s, t) = \sum_{k=1}^4 \mathcal{V}_k(\zeta_t, s, t), \quad (3.5)$$

where

$$\begin{aligned}\mathcal{V}_1(\zeta_t, s, t) &= \zeta^T(t)P_{sh}\zeta(t), \\ \mathcal{V}_2(\zeta_t, s, t) &= \int_{-\eta}^0 \zeta^T(y)Q\zeta(y) dy, \\ \mathcal{V}_3(\zeta_t, s, t) &= \int_{-\eta}^0 \int_{\nu}^0 \zeta_t^T(y)W\zeta_t(y) dyd\nu, \\ \mathcal{V}_4(\zeta_t, s, t) &= \frac{1}{2}\epsilon^2(t),\end{aligned}$$

where $P_{sh} = \sum_{i=1}^r h_i P_{si}$. For a given small scalar θ which is not the same as θ in θ_g and θ_f . Let $\gamma(t) = [\zeta^T(t) \ \zeta^T(t - \epsilon(t)) \ \zeta^T(t - \eta) \ \xi^T(t - \epsilon(t))]^T$. The following equalities hold:

$$\zeta(t + y) = \zeta(t) + \theta\dot{\zeta}(t) + o(\theta) = [I + \theta\bar{A}_{sh} \ \theta\bar{B}_{sh\mu} \ 0 \ -\theta\bar{E}_{hs\mu}]\gamma(t) + o(\theta), \quad (3.6)$$

$$\lim_{\theta \rightarrow 0} \frac{G_{i_s}(\hat{h} + \theta) - G_{i_s}(\hat{h})}{\theta(1 - G_{i_s}(\hat{h}))} = \lambda_i(\hat{h}), \quad (3.7)$$

$$\lim_{\theta \rightarrow 0} \frac{1 - G_{i_s}(\hat{h} + \theta)}{1 - G_{i_s}(\hat{h})} = 1, \quad (3.8)$$

where $\bar{A}_{sh} = \sum_{i=1}^r h_i \bar{A}_{is}$, $\bar{B}_{sh\mu} = \sum_{i=1}^r \sum_{j=1}^r h_i \hat{h}_j \bar{B}_{ijs\mu}$, $\bar{E}_{hs\mu} = \sum_{i=1}^r \sum_{j=1}^r h_i \hat{h}_j \bar{E}_{ijs\mu}$. $G_{i_s}(\hat{h})$ represents the cumulative distribution function (CDF) of the residence time h for the mode i_s , and $q_{i_s j_s}$ represents the probability that the mode of the system changes from i_s to j_s , i.e., $q_{i_s j_s} = P_r\{\phi_{t+1} = j_s | \phi_t = i_s\}$ and $\sum_{\substack{j_s=1 \\ j_s \neq i_s}}^M q_{i_s j_s} = 1$. $\lambda_{i_s}(\hat{h})$ is the transition rate of the system mode i_s . $\lambda_{i_s j_s}(\hat{h}) = q_{i_s j_s} \lambda_{i_s}(\hat{h})$ for $i_s \neq j_s$,

$$\lambda_{i_s i_s}(\hat{h}) = - \sum_{\substack{j_s=1 \\ j_s \neq i_s}}^M \lambda_{i_s j_s}(\hat{h}).$$

Refer to [5, 29], for the semi-Markov process $\{\zeta_t, s, t \geq 0\}$ in this paper. Denote \mathcal{J} to be the weak infinitesimal operator. For $s = i_s \in \mathcal{S}$, the derivative of $\mathcal{V}_1(\zeta_t, s, t)$ along the trajectories of the system (2.17) can be calculated by \mathcal{L} :

$$\begin{aligned}\mathcal{J}\mathcal{V}_1(\zeta_t, s, t) &= \lim_{\theta \rightarrow 0} \frac{1}{\theta} \left[\mathcal{E} \left\{ \sum_{\substack{j_s=1 \\ j_s \neq i_s}}^M P_r\{\phi_{t+1} = j_s, \hat{h}_{t+1} \geq \hat{h} + \theta | \phi_t = i_s, \hat{h}_{t+1} > \hat{h}\} \zeta^T(t + \theta) P_{j_s} \zeta(t + \theta) \right. \right. \\ &\quad \left. \left. + P_r\{\phi_{t+1} = i_s, \hat{h}_{t+1} > \hat{h} + \theta | \phi_t = i_s, \hat{h}_{t+1} > \hat{h}\} \zeta^T(t + \theta) P_{i_s} \zeta(t + \theta) \right\} - \zeta^T(t) P_{i_s} \zeta(t) \right] \\ &= \lim_{\theta \rightarrow 0} \frac{1}{\theta} \left[\mathcal{E} \left\{ \sum_{\substack{j_s=1 \\ j_s \neq i_s}}^M \frac{q_{i_s j_s} (G_{i_s}(\hat{h} + \theta) - G_{i_s}(\hat{h}))}{1 - G_{i_s}(\hat{h})} \zeta^T(t + \theta) P_{j_s} \zeta(t + \theta) \right. \right. \\ &\quad \left. \left. + \frac{1 - G_{i_s}(\hat{h} + \theta)}{1 - G_{i_s}(\hat{h})} \zeta^T(t + \theta) \zeta(t + \theta) \right\} - \zeta^T(t) P_{i_s} \zeta(t) \right] \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{\mu=1}^n h_i \hat{h}_j Q_{s\mu} \gamma^T(t) \left(X_{i_s} + \mathcal{F}_1^T \sum_{j_s \in \mathcal{S}} \bar{\lambda}_{i_s j_s} P_{j_s} \mathcal{F}_1 \right) \gamma(t),\end{aligned} \quad (3.9)$$

where $\mathcal{F}_1 = [I, 0, 0, 0]$ and

$$X_{i_s} = \begin{bmatrix} \text{sym}(P_{i_s} \bar{A}_{sh}) & P_{i_s} \bar{B}_{sh\mu} & 0 & -P_{i_s} \bar{E}_{sh\mu} \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix}.$$

Take the derivative of $\mathcal{V}_2(\zeta_t, s, t)$ as follows:

$$\mathcal{J}\mathcal{V}_2(\zeta_t, s, t) = \zeta^T(t)Q\zeta(t) - \zeta^T(t-\eta)Q\zeta(t-\eta) = \gamma(t)(\mathcal{F}_1^T Q \mathcal{F}_1 - \mathcal{F}_3^T Q \mathcal{F}_3)\gamma(t), \quad (3.10)$$

where $\mathcal{F}_3 = [0, 0, I, 0]$.

Then, we take the derivative of $\mathcal{V}_3(\zeta_t, s, t)$ as follows:

$$\mathcal{J}\mathcal{V}_3(\zeta_t, s, t) = \sum_{i=1}^r \sum_{j=1}^r \sum_{\mu=1}^n h_i \hat{h}_j \varrho_{s\mu} \gamma^T(t) \left[\eta Y_{i_s}^T W Y_{i_s} - \int_{t-\eta}^t \zeta^T(y) W \dot{\zeta}(y) dy \right] \gamma(t), \quad (3.11)$$

where $Y_{i_s} = [\bar{A}_{sh} \quad \bar{B}_{sh\mu} \quad 0 \quad -\bar{E}_{sh\mu}]$. By the Jensen-inequality in [37], the second term in the right of (3.11) is modified as

$$\begin{aligned} - \int_{t-\eta}^t \zeta^T(y) W \dot{\zeta}(y) dy &= - \int_{t-\eta}^{t-\varepsilon(t)} \zeta^T(y) W \dot{\zeta}(y) dy - \int_{t-\varepsilon(t)}^t \zeta^T(y) W \dot{\zeta}(y) dy \\ &\leq - \frac{\eta-\varepsilon(t)}{\eta} \int_{t-\eta}^{t-\varepsilon(t)} \zeta^T(y) W \dot{\zeta}(y) dy - \frac{\varepsilon(t)}{\eta} \int_{t-\varepsilon(t)}^t \zeta^T(y) W \dot{\zeta}(y) dy \\ &\leq - \frac{1}{\eta} \gamma^T(t) (\mathcal{F}_2 - \mathcal{F}_3)^T W (\mathcal{F}_2 - \mathcal{F}_3) \gamma(t) - \frac{1}{\eta} \gamma^T(t) (\mathcal{F}_1 - \mathcal{F}_2)^T W (\mathcal{F}_1 - \mathcal{F}_2) \gamma(t) \\ &= - \gamma^T(t) \frac{1}{\eta} \mathcal{F}^T \tilde{W} \mathcal{F} \gamma(t), \end{aligned} \quad (3.12)$$

where $\mathcal{F}_2 = [0, I, 0, 0]$ and

$$\mathcal{F} = \begin{bmatrix} I & -I & 0 & 0 \\ 0 & I & -I & 0 \end{bmatrix}, \quad \tilde{W} = \begin{bmatrix} W & 0 \\ 0 & W \end{bmatrix}.$$

Hence, from (3.11) and (3.12), it follows that

$$\mathcal{J}\mathcal{V}_3(\zeta_t, s, t) \leq \sum_{i=1}^r \sum_{j=1}^r \sum_{\mu=1}^n h_i \hat{h}_j \varrho_{s\mu} \gamma^T(t) \left(\eta Y_{i_s}^T W Y_{i_s} - \frac{1}{\eta} \mathcal{F}^T \tilde{W} \mathcal{F} \right) \gamma(t). \quad (3.13)$$

Then, considering the event-triggered condition in (2.6) and (2.7), the derivative of $\mathcal{V}_4(\zeta_t, s, t)$ is calculated as follows:

$$\begin{aligned} \mathcal{J}\mathcal{V}_4(\zeta_t, s, t) &= \varepsilon(t) \dot{\varepsilon}(t) = \frac{\xi^T(t-\varepsilon(t)) \xi(t-\varepsilon(t))}{\varepsilon(t)} - \varepsilon_0 \xi^T(t-\varepsilon(t)) \xi(t-\varepsilon(t)) \\ &\leq \xi^T(t-\varepsilon(t)) \xi(t-\varepsilon(t)) - \varepsilon_0 \xi^T(t-\varepsilon(t)) \xi(t-\varepsilon(t)) \\ &= \gamma^T(t) (\tilde{I}_1 - \tilde{I}_2) \gamma(t), \end{aligned} \quad (3.14)$$

where

$$\tilde{I}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \tilde{I}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \epsilon_0 I \end{bmatrix}.$$

Combining (3.9), (3.10), (3.13) and (3.14) yields

$$\mathcal{J}\mathcal{V}(\zeta_t, s, t) \leq \sum_{i=1}^r \sum_{j=1}^r \sum_{\mu=1}^n h_i \hat{h}_j \varrho_{s\mu} \gamma^T(t) \Theta_{ij_s} \gamma(t), \quad (3.15)$$

where $\Theta_{ij_s} = \Omega + \Upsilon$, $\Upsilon = -\frac{1}{\eta} \mathcal{F}^T \tilde{W} \mathcal{F}$ and

$$\Omega = \begin{bmatrix} \Omega_{11} & P_{i_s} \bar{B}_{ijs\mu} & 0 & -P_{i_s} \bar{E}_{ijs\mu} & \bar{A}_{i_s}^T \\ * & I & 0 & 0 & \bar{B}_{ijs\mu}^T \\ * & * & -Q & 0 & 0 \\ * & * & * & -\epsilon_0 I & -\bar{E}_{ijs\mu}^T \\ * & * & * & * & -\frac{1}{\eta} W^{-1} \end{bmatrix},$$

$$\Omega_{11} = \text{sym}(P_{i_s} \bar{A}_{i_s}) + \sum_{j \in \mathcal{S}} \bar{\lambda}_{i_s, j_s} P_{j_s} + Q.$$

Then, for (3.1), through the Schur complement, $\Theta_{ij_s} < 0$ can be deduced, i.e., $\mathcal{J}\mathcal{V}(\zeta_t, s, t) < 0$. Through the Dynkin's formula, if $\mathcal{J}\mathcal{V}(\zeta_t, s, t) < 0$, for a cut-off time $\mathcal{T} > 0$ and a constant $\epsilon > 0$, the following inequality will hold:

$$\mathbb{E}\{\mathcal{V}(\zeta(\mathcal{T}), s, \mathcal{T})\} - \mathcal{V}(\zeta_0, \phi_0) < -\epsilon \mathbb{E}\left\{\int_0^{\mathcal{T}} \|\zeta(t)\|^2 dt\right\}.$$

Then, let $\mathcal{T} \rightarrow \infty$, it has

$$\mathbb{E}\left\{\int_0^{\mathcal{T}} \|\zeta(t)\|^2 dt\right\} < \frac{1}{\epsilon} \mathcal{V}(\zeta_0, \phi_0, 0) < \infty.$$

From Definition 2.1, it can be obtained that the system (2.17) have achieved a stochastic stability. Furthermore, the H_∞ tracking performance under the condition of external disturbance $\bar{\omega}(t) \neq 0$ is considered. From (3.15), we can obtain

$$\begin{aligned} \mathfrak{J} &= \int_0^\infty \mathbb{E}\left\{e^T(t)e(t) - \lambda^2 \bar{\omega}^T(t)\bar{\omega}(t)\right\} dt \\ &\leq \int_0^\infty \mathbb{E}\left\{e^T(t)e(t) - \lambda^2 \bar{\omega}^T(t)\bar{\omega}(t)\right\} dt - \mathcal{V}(\zeta_0, \phi_0) + \mathbb{E}\left\{\mathcal{V}(\zeta(\infty), \phi(\infty))\right\} \\ &= \int_0^\infty \mathbb{E}\left\{e^T(t)e(t) - \lambda^2 \bar{\omega}^T(t)\bar{\omega}(t) + \mathcal{L}\mathcal{V}(\zeta_t, s, t)\right\} dt \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{\mu=1}^n h_i \hat{h}_j \varrho_{s\mu} \int_0^\infty \bar{\gamma}^T(t) \bar{\Theta}_{ij_s} \bar{\gamma}(t) dt, \end{aligned} \quad (3.16)$$

where $\bar{\gamma}(t) = [\gamma^T(t) \quad \bar{\omega}^T(t)]^T$ and $\bar{\Theta}_{ijis} = \hat{\Omega} + \hat{\Upsilon}$.

In order to reduce the conservativeness and make full use of the MFs information, we introduce the slack matrices Z_i . Due to $\sum_{i=1}^r h_i - \sum_{j=1}^r \hat{h}_j = 0$. Then, according to [38–41], combining (3.2)–(3.4) with the system (2.17) and considering $\hat{h}_j - \rho_j h_j \geq 0$, we can get

$$\sum_{i=1}^r \sum_{j=1}^r h_i (h_j - \hat{h}_j) Z_i = \sum_{i=1}^r h_i \left(\sum_{j=1}^r h_j - \sum_{j=1}^r \hat{h}_j \right) Z_i = 0.$$

Then, combining the above formula, we can get

$$\begin{aligned} \bar{\Theta}_{hi_s} &= \sum_{i=1}^r \sum_{j=1}^r h_i \hat{h}_j \bar{\Theta}_{ijis} \\ &= \sum_{i=1}^r \sum_{j=1}^r h_i \hat{h}_j \bar{\Theta}_{ijis} + \sum_{i=1}^r \sum_{j=1}^r h_i (h_j - \hat{h}_j + \rho_j h_j - \rho_j \hat{h}_j) Z_i \\ &= \sum_{i=1}^r h_i^2 (\rho_j \bar{\Theta}_{ijis} + (1 - \rho_j) Z_i) + \sum_{i=1}^r \sum_{j=1}^r h_i (\hat{h}_j - \rho_j h_j) (\bar{\Theta}_{ijis} - Z_i) \\ &\quad + \sum_{i=1}^{r-1} \sum_{j=1}^r h_i h_j (\rho_j \bar{\Theta}_{ijis} + \rho_i \bar{\Theta}_{ijis} + (1 - \rho_j) Z_i + (1 - \rho_i) Z_j) < 0. \end{aligned}$$

Hence, the condition (3.1) in Theorem 3.1 shows that $\mathbb{J} < 0$. i.e.,

$$\int_0^{\infty} e^T(t) e(t) dt < \int_0^{\infty} \lambda^2 \bar{\omega}^T(t) \bar{\omega}(t) dt.$$

From Definition 2.2, it is obvious that the system (2.17) achieves a stochastic stable state and satisfies the prescribed H_{∞} tracking performance λ .

Thus, the proof is completed.

Remark 3.1. In consideration of reducing the conservatism, referring to [30]. The difference from [30] is that we select the fuzzy Lyapunov matrix $P_{sh} = \sum_{i=1}^r h_i P_{si}$ as the modal dependency matrix in this paper. This paper studies the stability of the system and H_{∞} tracking performance, but [30] studies the stability of multi-agent system. For the purpose of making full use of the AET conditions, the quadratic integral term $\int_{-\eta}^0 \int_y^0 \zeta_t^T(y) W \zeta_t(y) dy dv$ related to $\zeta(t - \varepsilon(t))$ and $\frac{1}{2} \varepsilon^2(t)$ related to AET threshold $\varepsilon(t)$ are all considered in Theorem 3.1, which can decrease the conservatism of the system.

In Theorem 3.1, some sufficient criteria which can ensure the stochastic stability with $\bar{\omega}(k) = 0$ and achieve a given H_{∞} tracking performance for the system (2.17). However, based on the analysis results in Theorem 3.1, it is difficult for us to parameterize the controller gains directly because of the existence of nonlinear terms. For the purpose of using the method of LMIs to solve the problem, the results of the controller construction are summarized in the following achievements.

The matrices in Theorem 3.2 are mainly some equivalent transformations of the final matrix of Theorem 3.1 such as contract transformation and Schur complementary, etc. The matrix separation technique is also used to separate out some uncertain terms. Meanwhile, two lemmas given in the paper are also applied to perform the corresponding deflation without changing the properties of the original matrix. Finally, we can achieve Theorem 3.2.

Theorem 3.2. For some given scalars $\rho, \eta, \gamma, \beta, \rho_i, \rho_j, \lambda > 0, a_1 > 0, a_2 > 0$ and $a_3 > 0$ and controller gain matrices $K_{j\mu}$, the system (2.17) can achieve a stochastic stability and satisfy the prescribed H_∞ tracking performance, and for some other matrices $H_{i_s} > 0, \hat{Q} > 0, \hat{W} > 0, W_{i_s} > 0, \hat{K}_{j\mu}$ and \hat{Z}_i with appropriate dimensions such that, for $\forall i_s \in \mathcal{S}, \forall \mu \in \mathcal{M}, \forall i, j \in \mathbb{I}$, the following inequalities hold:

$$\hat{\Theta}_{ij_i s} - \hat{Z}_i < 0, \quad (3.17)$$

$$\rho_i \hat{\Theta}_{ij_i s} + (1 - \rho_i) \hat{Z}_i < 0, \quad (3.18)$$

$$\rho_i \hat{\Theta}_{ij_i s} + \rho_j \hat{\Theta}_{ij_j s} + (1 - \rho_i) \hat{Z}_j + (1 - \rho_j) \hat{Z}_i < 0, \quad i < j, \quad (3.19)$$

$$\rho H_{i_s} + \rho H_{i_s}^T - \rho^2 I_n \geq W_{i_s}, \quad (3.20)$$

$$\begin{bmatrix} \gamma W_{i_s} & \hat{K}_{j\mu} \\ * & I \end{bmatrix} \geq 0, \quad \begin{bmatrix} \beta W_{i_s} & \hat{F}_{j\mu} \\ * & I \end{bmatrix} \geq 0, \quad (3.21)$$

$$\hat{\Theta}_{ij_i s} = \begin{bmatrix} \tilde{\Omega}^{(11)} + \tilde{\Upsilon} + \mathbb{B}_r & G_{i_s}^T & G_{i_s}^T & G_{i_s}^T \\ * & -a_1 I & 0 & 0 \\ * & * & -a_2 I & 0 \\ * & * & * & -a_3 I \end{bmatrix} < 0, \quad (3.22)$$

$$\tilde{\Omega}^{(11)} = \begin{bmatrix} \tilde{\Omega}_{11}^{(11)} & 0 & \mathcal{B}_{i_s} \hat{K}_{j\mu} & \mathcal{B}_{i_s} \hat{F}_{j\mu} & 0 & -\mathcal{B}_{i_s} \hat{K}_{j\mu} & 0 & H_{i_s}^T \mathcal{A}_{i_s}^T & 0 & H_{i_s}^T & 0 & H_{i_s}^T \mathcal{C}_s^T & \tilde{\Omega}_{113}^{(11)} \\ * & \tilde{\Omega}_{22}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 & H_{i_s}^T \mathcal{A}_r^T & 0 & 0 & -H_{i_s}^T \mathcal{C}_r^T & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & \hat{K}_{j\mu}^T \mathcal{B}_{i_s}^T & 0 & 0 & H_{i_s}^T & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & \hat{F}_{j\mu}^T \mathcal{B}_{i_s}^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Omega}_{55}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\Omega}_{66}^{(11)} & 0 & -\hat{K}_{j\mu}^T \mathcal{B}_{i_s}^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \tilde{\Omega}_{77}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{1}{\eta} \hat{W} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{1}{\eta} \hat{W} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\hat{Q} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1313}^{(11)} \end{bmatrix},$$

$$\tilde{\Omega}_{11}^{(11)} = \text{sym}(\mathcal{A}_{i_s} H_{i_s}) + \bar{\lambda}_{i_s} H_{i_s}^T, \quad \tilde{\Omega}_{22}^{(11)} = \text{sym}(\mathcal{A}_r H_{i_s}) + \bar{\lambda}_{i_s} H_{i_s}^T,$$

$$\tilde{\Omega}_{55}^{(11)} = -H_{i_s}^T - H_{i_s} + \hat{Q}, \quad \tilde{\Omega}_{66}^{(11)} = -H_{i_s}^T - H_{i_s} + \frac{1}{\epsilon_0} I, \quad \tilde{\Omega}_{77}^{(11)} = -H_{i_s}^T - H_{i_s} + \frac{1}{\lambda^2} I,$$

$$\tilde{\Omega}_{113}^{(11)} = [\sqrt{\lambda_{i_s 1}} H_{i_s}^T, \sqrt{\lambda_{i_s 2}} H_{i_s}^T, \dots, \sqrt{\lambda_{i_s(i_s-1)}} H_{i_s}^T, \sqrt{\lambda_{i_s(i_s+1)}} H_{i_s}^T, \dots, \sqrt{\lambda_{i_s M}} H_{i_s}^T],$$

$$\tilde{\Omega}_{1313}^{(11)} = -\text{diag}\{H_1, H_2, \dots, H_{i_s-1}, H_{i_s+1}, \dots, H_M\},$$

$$\tilde{Y} = \frac{1}{\eta} \mathbb{F}^T \begin{bmatrix} -\text{sym}(H_{i_s}) + \hat{W} & 0 \\ 0 & -\text{sym}(H_{i_s}) + \hat{W} \end{bmatrix} \mathbb{F}, \quad \mathbb{F} = \begin{bmatrix} I & -I & 0 & 0 & 0 \cdots 0 \\ 0 & I & -I & \underbrace{0 & 0 \cdots 0}_{10} \end{bmatrix},$$

$$G_{i_s} = \begin{bmatrix} 0 & 0 & H_{i_s} & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & H_{i_s} & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & H_{i_s} & \underbrace{0 & 0 \cdots 0}_7 \end{bmatrix}, \quad \mathbb{B}_r = a_1 \mathbb{B}_{1r} + a_2 \mathbb{B}_{2r} + a_3 \mathbb{B}_{3r},$$

$$\mathbb{B}_{1r} = \begin{bmatrix} \mathbb{B}_{1r}^{11} & 0 & 0 \cdots 0 & \mathbb{B}_{1r}^{18} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \mathbb{B}_{1r}^{81} & 0 & 0 \cdots 0 & \mathbb{B}_{1r}^{88} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \underbrace{0 \cdots 0}_6 & 0 & 0 & \underbrace{0 \cdots 0}_5 \end{bmatrix}, \quad \mathbb{B}_{2r} = \begin{bmatrix} \mathbb{B}_{2r}^{11} & 0 & 0 \cdots 0 & \mathbb{B}_{2r}^{18} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \mathbb{B}_{2r}^{81} & 0 & 0 \cdots 0 & \mathbb{B}_{2r}^{88} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \underbrace{0 \cdots 0}_6 & 0 & 0 & \underbrace{0 \cdots 0}_5 \end{bmatrix},$$

$$\mathbb{B}_{3r} = \begin{bmatrix} \mathbb{B}_{3r}^{11} & 0 & 0 \cdots 0 & \mathbb{B}_{3r}^{18} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \mathbb{B}_{3r}^{81} & 0 & 0 \cdots 0 & \mathbb{B}_{3r}^{88} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \underbrace{0 \cdots 0}_6 & 0 & 0 & \underbrace{0 \cdots 0}_5 \end{bmatrix}, \quad \begin{cases} \mathbb{B}_{1r}^{11} = \mathbb{B}_{1r}^{18} = \mathbb{B}_{1r}^{81} = \mathbb{B}_{1r}^{88} = (2\gamma + \beta)\alpha_{i_s}\theta_g^2 I, \\ \mathbb{B}_{2r}^{11} = \mathbb{B}_{2r}^{18} = \mathbb{B}_{2r}^{81} = \mathbb{B}_{2r}^{88} = (2\gamma + \beta)\alpha_{i_s}\theta_f^2 I, \\ \mathbb{B}_{3r}^{11} = \mathbb{B}_{3r}^{18} = \mathbb{B}_{3r}^{81} = \mathbb{B}_{3r}^{88} = (2\gamma + \beta)\alpha_{i_s}\theta_g^2\theta_f^2 I, \end{cases}$$

where θ_g and θ_f are defined in (2.14), α_{i_s} is the largest eigenvalue of $\mathcal{B}_{i_s}\mathcal{B}_{i_s}^T$. Then, the tracking control gains $K_{j\mu}$ and $F_{j\mu}$ for $i_s \in \mathcal{S}$, is given by $K_{j\mu} = \hat{K}_{j\mu}H_{i_s}^{-1}$ and $F_{j\mu} = \hat{F}_{j\mu}H_{i_s}^{-1}$.

Proof. Let $H_{i_s} = P_{i_s}^{-1}$. Define $\Lambda = \text{diag}\{H_{i_s}, H_{i_s}, H_{i_s}, H_{i_s}, H_{i_s}, I\}$. Then, pre- and post-multiplying (3.1) with Λ^T and Λ , respectively, it can be obtained that

$$\Lambda^T \bar{\Theta}_{ijj_s} \Lambda = \Lambda^T \hat{\Omega} \Lambda + \Lambda^T \hat{Y} \Lambda < 0. \tag{3.23}$$

Let $\tilde{\Omega} = \Lambda^T \hat{\Omega} \Lambda$ and $\tilde{Y} = \Lambda^T \hat{Y} \Lambda$. Thus,

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & \bar{B}_{ijjs\mu} H_{i_s} & 0 & -\bar{E}_{ijjs\mu} H_{i_s} & 0 & H_{i_s}^T \bar{A}_{i_s}^T \\ * & H_{i_s}^T H_{i_s} & 0 & 0 & 0 & H_{i_s}^T \bar{B}_{ijjs\mu}^T \\ * & * & -H_{i_s}^T Q H_{i_s} & 0 & 0 & 0 \\ * & * & * & -\epsilon_0 H_{i_s}^T H_{i_s} & 0 & -H_{i_s}^T \bar{E}_{ijjs\mu}^T \\ * & * & * & * & -\lambda^2 H_{i_s}^T H_{i_s} & 0 \\ * & * & * & * & * & -\frac{1}{\eta} W^{-1} \end{bmatrix}$$

$$\tilde{\Omega}_{11} = \text{sym}(\bar{A}_{i_s} H_{i_s}) + \sum_{\substack{j_s \in \mathcal{S} \\ j_s \neq i_s}} \bar{\lambda}_{i_s j_s} H_{i_s}^T H_{j_s}^{-1} H_{i_s} + \bar{\lambda}_{i_s i_s} H_{i_s}^T + H_{i_s}^T Q H_{i_s} + H_{i_s}^T \bar{C}_s^T \bar{C}_s H_{i_s}. \tag{3.24}$$

Based on Lemma 2.1, we can obtain

$$\begin{aligned}\tilde{\Upsilon} &= -\frac{1}{\eta}\hat{F}^T \begin{bmatrix} H_{i_s}^T & 0 \\ 0 & H_{i_s}^T \end{bmatrix} \begin{bmatrix} W & 0 \\ 0 & W \end{bmatrix} \begin{bmatrix} H_{i_s} & 0 \\ 0 & H_{i_s} \end{bmatrix} \hat{F} \\ &\leq \frac{1}{\eta}\hat{F}^T \begin{bmatrix} -\text{sym}(H_{i_s}) + W^{-1} & 0 \\ 0 & -\text{sym}(H_{i_s}) + W^{-1} \end{bmatrix} \hat{F}\end{aligned}\quad (3.25)$$

and

$$\begin{cases} -H_{i_s}^T Q H_{i_s} \leq -H_{i_s}^T - H_{i_s} + Q^{-1}, \\ -\epsilon_0 H_{i_s}^T H_{i_s} = -H_{i_s}^T (\epsilon_0 I) H_{i_s} \leq -H_{i_s}^T - H_{i_s} + \frac{1}{\epsilon_0} I, \\ -\lambda^2 H_{i_s}^T H_{i_s} = -H_{i_s}^T (\lambda^2 I) H_{i_s} \leq -H_{i_s}^T - H_{i_s} + \frac{1}{\lambda^2} I. \end{cases}\quad (3.26)$$

Integrating (3.24)–(3.26) into (3.23) and using the Schur complement for (3.23). Thus, (3.1) holds if the following condition holds:

$$\tilde{\Omega}^{(1)} + \tilde{\Upsilon} < 0, \quad (3.27)$$

where

$$\tilde{\Omega}^{(1)} = \begin{bmatrix} \tilde{\Omega}_{11}^{(11)} & 0 & \Phi_1 & \Phi_2 & 0 & -\Phi_1 & 0 & H_{i_s}^T \mathcal{A}_{i_s}^T & 0 & H_{i_s}^T & 0 & H_{i_s}^T C_s^T & \tilde{\Omega}_{113}^{(11)} \\ * & \tilde{\Omega}_{22}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 & H_{i_s}^T \mathcal{A}_r^T & 0 & 0 & -H_{i_s}^T C_r^T & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & \Phi_1^T & 0 & 0 & H_{i_s}^T & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & \Phi_2^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Omega}_{55}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\Omega}_{66}^{(11)} & 0 & -\Phi_1^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \tilde{\Omega}_{77}^{(11)} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{1}{\eta}\hat{W} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{1}{\eta}\hat{W} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\hat{Q} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \tilde{\Omega}_{1313}^{(11)} \end{bmatrix},$$

where

$$\begin{aligned}\Phi_1 &= \mathcal{B}_{i_s}(\hat{K}_{j\mu} + \Delta_{r_1} H_{i_s}), \quad \Phi_2 = \mathcal{B}_{i_s}(\hat{F}_{j\mu} + \Delta_{r_2} H_{i_s}), \\ \hat{K}_{j\mu} &= K_{j\mu} H_{i_s}, \quad \hat{F}_{j\mu} = F_{j\mu} H_{i_s}, \quad \hat{Q} = Q^{-1}, \quad \hat{W} = W^{-1}.\end{aligned}$$

Considering (2.15), (2.16) and (3.27), $\tilde{\Omega}^{(1)}$ can be rewritten as

$$\tilde{\Omega}^{(1)} = \tilde{\Omega}^{(11)} + \text{sym}\{B_{1r} G_{i_s} + B_{2r} G_{i_s} + B_{3r} G_{i_s}\}, \quad (3.28)$$

where

$$B_{1r} = \begin{bmatrix} \mathcal{B}_{i_s} \Delta_{1r1} & \mathcal{B}_{i_s} \Delta_{1r2} & \mathcal{B}_{i_s} \Delta_{1r1} \\ I_0^1 & I_0^1 & I_0^1 \\ \mathcal{B}_{i_s} \Delta_{1r1} & \mathcal{B}_{i_s} \Delta_{1r2} & \mathcal{B}_{i_s} \Delta_{1r1} \\ I_0^2 & I_0^2 & I_0^2 \end{bmatrix}, \quad B_{2r} = \begin{bmatrix} \mathcal{B}_{i_s} \Delta_{2r1} & \mathcal{B}_{i_s} \Delta_{2r2} & \mathcal{B}_{i_s} \Delta_{2r1} \\ I_0^1 & I_0^1 & I_0^1 \\ \mathcal{B}_{i_s} \Delta_{2r1} & \mathcal{B}_{i_s} \Delta_{2r2} & \mathcal{B}_{i_s} \Delta_{2r1} \\ I_0^2 & I_0^2 & I_0^2 \end{bmatrix},$$

$$B_{3r} = \begin{bmatrix} \mathcal{B}_{is\Delta_{3r1}} & \mathcal{B}_{is\Delta_{3r2}} & \mathcal{B}_{is\Delta_{3r1}} \\ I_0^1 & I_0^1 & I_0^1 \\ \mathcal{B}_{is\Delta_{3r1}} & \mathcal{B}_{is\Delta_{3r2}} & \mathcal{B}_{is\Delta_{3r1}} \\ I_0^2 & I_0^2 & I_0^2 \end{bmatrix}, I_0^1 = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T, I_0^2 = [0 \ 0 \ 0 \ 0 \ 0]^T.$$

From Lemma 2.2, we have

$$\begin{aligned} & \text{sym}\{B_{1r}G_{is} + B_{2r}G_{is} + B_{3r}G_{is}\} \\ & \leq a_1 B_{1r}^T B_{1r} + a_2 B_{2r}^T B_{2r} + a_3 B_{3r}^T B_{3r} + a_1^{-1} G_{is}^T G_{is} + a_2^{-1} G_{is}^T G_{is} + a_3^{-1} G_{is}^T G_{is} \\ & = \mathbb{B}_r^*, \end{aligned} \quad (3.29)$$

where $0 < a_1, a_2, a_3 \in \mathbb{R}$ and

$$B_{1r}^T B_{1r} = \begin{bmatrix} B_{1r}^{11} & I_0^{1T} & B_{1r}^{18} & I_0^{2T} \\ I_0^1 & I_0^3 & I_0^1 & I_0^{3T} \\ B_{1r}^{81} & I_0^{1T} & B_{1r}^{88} & I_0^{2T} \\ I_0^2 & I_0^3 & I_0^2 & I_0^{3T} \end{bmatrix}, B_{2r}^T B_{2r} = \begin{bmatrix} B_{2r}^{11} & I_0^{1T} & B_{2r}^{18} & I_0^{2T} \\ I_0^1 & I_0^3 & I_0^1 & I_0^{3T} \\ B_{2r}^{81} & I_0^{1T} & B_{2r}^{88} & I_0^{2T} \\ I_0^2 & I_0^3 & I_0^2 & I_0^{3T} \end{bmatrix}, B_{3r}^T B_{3r} = \begin{bmatrix} B_{3r}^{11} & I_0^{1T} & B_{3r}^{18} & I_0^{2T} \\ I_0^1 & I_0^3 & I_0^1 & I_0^{3T} \\ B_{3r}^{81} & I_0^{1T} & B_{3r}^{88} & I_0^{2T} \\ I_0^2 & I_0^3 & I_0^2 & I_0^{3T} \end{bmatrix},$$

$$I_0^3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{cases} B_{1r}^{11} = B_{1r}^{18} = B_{1r}^{81} = B_{1r}^{88} = 2(\mathcal{B}_{is\Delta_{1r1}})(\mathcal{B}_{is\Delta_{1r1}})^T + (\mathcal{B}_{is\Delta_{1r2}})(\mathcal{B}_{is\Delta_{1r2}})^T, \\ B_{2r}^{11} = B_{2r}^{18} = B_{2r}^{81} = B_{2r}^{88} = 2(\mathcal{B}_{is\Delta_{2r1}})(\mathcal{B}_{is\Delta_{2r1}})^T + (\mathcal{B}_{is\Delta_{2r2}})(\mathcal{B}_{is\Delta_{2r2}})^T, \\ B_{3r}^{11} = B_{3r}^{18} = B_{3r}^{81} = B_{3r}^{88} = 2(\mathcal{B}_{is\Delta_{3r1}})(\mathcal{B}_{is\Delta_{3r1}})^T + (\mathcal{B}_{is\Delta_{3r2}})(\mathcal{B}_{is\Delta_{3r2}})^T. \end{cases}$$

Then, considering (3.28) and (3.29), we can obtain that (3.27) holds if

$$\tilde{\Omega}^{(11)} + \tilde{\Upsilon} + \mathbb{B}_r^* < 0. \quad (3.30)$$

By Lemma 2.1, it is obvious that

$$H_{is}^T H_{is} = \rho H_{is}^T (\frac{1}{\rho^2} I) \rho H_{is} \geq \rho H_{is}^T + \rho H_{is} - \rho^2 I. \quad (3.31)$$

Applying Schur complement to (3.21), we have

$$-\gamma W_{is} + \hat{K}_{j\mu}^T \hat{K}_{j\mu} < 0. \quad (3.32)$$

Considering (3.20), (3.31), (3.32) and $\hat{K}_{j\mu} = K_{j\mu} H_{is}$, it can be achieved that

$$H_{is}^T H_{is} \geq \frac{1}{\gamma} H_{is}^T K_{j\mu}^T K_{j\mu} H_{is}. \quad (3.33)$$

Pre- and post-multiplying (3.33) with H_{is}^{-1} yields $K_{j\mu}^T K_{j\mu} \leq \gamma I$. Since $K_{j\mu}^T K_{j\mu}$ and $K_{j\mu} K_{j\mu}^T$ have same nonzero eigenvalues, thus $K_{j\mu} K_{j\mu}^T \leq \gamma I$.

Based on those and (2.15), we can obtain

$$(\mathcal{B}_{is\Delta_{1r1}})(\mathcal{B}_{is\Delta_{1r1}})^T = \mathcal{B}_{is\Delta_{1r1}} \Delta_{1r1}^T \mathcal{B}_{is\Delta_{1r1}}^T \leq \mathcal{B}_{is\Delta_{1r1}} \gamma \Delta_{1r1} \Delta_{1r1}^T \mathcal{B}_{is\Delta_{1r1}}^T \leq \alpha_{is} \theta_g^2 \gamma I, \quad (3.34)$$

where α_{is} denotes the largest eigenvalue of $\mathcal{B}_{is}\mathcal{B}_{is}^T$.

Using similar methods, we can get

$$(\mathcal{B}_{is\Delta_{1r2}})(\mathcal{B}_{is\Delta_{1r2}})^T \leq \alpha_{is}\theta_g^2\beta I.$$

Similarly, we can also get

$$\begin{cases} (\mathcal{B}_{is\Delta_{2r1}})(\mathcal{B}_{is\Delta_{2r1}})^T \leq \alpha_{is}\theta_f^2\gamma I, & (\mathcal{B}_{is\Delta_{2r2}})(\mathcal{B}_{is\Delta_{2r2}})^T \leq \alpha_{is}\theta_f^2\beta I, \\ (\mathcal{B}_{is\Delta_{3r1}})(\mathcal{B}_{is\Delta_{3r1}})^T \leq \alpha_{is}\theta_g^2\theta_f^2\gamma I, & (\mathcal{B}_{is\Delta_{3r2}})(\mathcal{B}_{is\Delta_{3r2}})^T \leq \alpha_{is}\theta_g^2\theta_f^2\beta I. \end{cases}$$

Hence, we can see that $B_{1r}^T B_{1r} \leq \mathbb{B}_{1r}$, $B_{2r}^T B_{2r} \leq \mathbb{B}_{2r}$ and $B_{3r}^T B_{3r} \leq \mathbb{B}_{3r}$. Therefore, (3.30) holds if the following holds:

$$\tilde{\Omega}^{(11)} + \tilde{\Upsilon} + a_1\mathbb{B}_{1r} + a_2\mathbb{B}_{2r} + a_3\mathbb{B}_{3r} + (a_1^{-1} + a_2^{-1} + a_3^{-1})G_{i_s}^T G_{i_s} < 0. \quad (3.35)$$

By the Schur complement, (3.35) is equivalent (3.22). In summary, (3.1)–(3.4) hold if (3.17)–(3.22) hold. So far, the proof is completed.

4. Examples

In order to verify the effectiveness of the theoretical results, a numerical example and a single-link flexible joint manipulator system are selected in this section.

Example 1. In the numerical example, three modes of the system (2.2) are specified. The corresponding three-dimensional transition probability matrix with a semi-Markovian process is chosen as

$$\tilde{\Lambda}(\hat{h}) = \begin{bmatrix} -2\hat{h} & -1.5\hat{h} & 1.2\hat{h} \\ 1.5\hat{h}^2 & -3\hat{h}^2 & -0.5\hat{h} \\ 2.5\hat{h}^2 & -2.5\hat{h}^2 & -0.5\hat{h} \end{bmatrix}.$$

Through relevant calculations, the mathematical expectation of $\tilde{\Lambda}(\hat{h})$ is

$$\mathfrak{E}\{\tilde{\Lambda}(\hat{h})\} = \begin{bmatrix} -1.725 & 0.088 & 0.076 \\ 1.354 & -2.708 & 0.985 \\ 1.345 & -0.976 & 1.213 \end{bmatrix}.$$

Consider a two-rules T-S fuzzy MJSs of the form (2.1):

Plant rule 1: If $\varkappa_1(t)$ is $-m_{\phi_r}$, then,

$$\begin{cases} \dot{\varkappa}(t) = \mathcal{A}_{1,\phi_r}\varkappa(t) + \mathcal{B}_{1,\phi_r}u(t) + \mathcal{D}_{1,\phi_r}\omega(t), \\ z(t) = \mathcal{C}_{\phi_r}\varkappa(t). \end{cases} \quad (4.1)$$

Plant rule 2: If $\varkappa_2(t)$ is m_{ϕ_r} , then,

$$\begin{cases} \dot{\varkappa}(t) = \mathcal{A}_{2,\phi_r}\varkappa(t) + \mathcal{B}_{2,\phi_r}u(t) + \mathcal{D}_{2,\phi_r}\omega(t), \\ z(t) = \mathcal{C}_{\phi_r}\varkappa(t), \end{cases} \quad (4.2)$$

with its parameters selected as

$$\begin{aligned} \mathcal{A}_{11} &= \begin{bmatrix} -4.5 & 5 \\ 5 & 5.5 \end{bmatrix}, \mathcal{A}_{21} = \begin{bmatrix} -9.5 & 4.3 \\ 3 & -3.5 \end{bmatrix}, \mathcal{A}_{12} = \begin{bmatrix} -4.5 & 6 \\ 5.5 & -6.2 \end{bmatrix}, \mathcal{A}_{22} = \begin{bmatrix} -2.5 & 2.3 \\ 1 & -3.6 \end{bmatrix}, \\ \mathcal{A}_{13} &= \begin{bmatrix} -4.5 & 5 \\ 5 & 5.5 \end{bmatrix}, \mathcal{A}_{23} = \begin{bmatrix} -9.5 & 4.3 \\ 3 & -3.5 \end{bmatrix}, \mathcal{B}_{11} = \mathcal{B}_{12} = \mathcal{B}_{13} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathcal{B}_{21} = \mathcal{B}_{22} = \mathcal{B}_{23} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \\ C_1 = C_2 = C_3 &= [1 \ 0], \mathcal{D}_{11} = \mathcal{D}_{12} = \mathcal{D}_{13} = \begin{bmatrix} -1.3 \\ 1.5 \end{bmatrix}, \mathcal{D}_{21} = \mathcal{D}_{22} = \mathcal{D}_{23} = \begin{bmatrix} -1.1 \\ 1.3 \end{bmatrix}. \end{aligned}$$

According to the reference system (2.4), assume a reference model with its parameters:

$$\mathcal{A}_r = \begin{bmatrix} -5 & 0 \\ 0 & -1.5 \end{bmatrix}, \mathcal{B}_r = \begin{bmatrix} 0.5 \\ -0.1 \end{bmatrix}, C_r = [1 \ 2].$$

The parameters of AET condition (2.6) are selected as $\epsilon_0 = 0.1$. The parameters of quantization are chosen as $\hat{u}_0^c = 2, \theta_{f_c} = (1 - \rho_{f_c})/(1 + \rho_{f_c}) = 1.0001, \theta_{g_c} = (1 - \rho_{g_c})/(1 + \rho_{g_c}) = 1.00015$ where $c \in C = \{1, 2, \dots, C\}$. In Theorem 3.2, let $\gamma = 1.5, \beta = 0.5$. An expected H_∞ performance $\lambda^* = 0.0052$ can be achieved with the results of the controller gains as follows:

$$\begin{aligned} K_{11} &= [-13.612 \ -2.687], K_{12} = [-19.911 \ -3.182], K_{13} = [-17.268 \ -4.484], \\ K_{21} &= [-1.612 \ -0.323], K_{22} = [-2.613 \ -0.484], K_{23} = [-4.623 \ -0.581], \\ F_{11} &= [16.638 \ 30.941], F_{12} = [19.877 \ 23.487], F_{13} = [14.163 \ 3.481], \\ F_{21} &= [1.897 \ 3.783], F_{22} = [2.415 \ 4.692], F_{23} = [1.073 \ 3.488]. \end{aligned}$$

Let the reference input $r(t) = \sin(t)$ and the external disturbances $w(t) = 0.5 \exp(-t)$. Figure 2 shows the mode evolution of the system and the controller. The state trajectories of two-rules without a controller are depicted in Figure 3. We can see that the system is unable to achieve a stochastic stability. When the designed controllers (2.12) are added to the system, it is obvious that the system is stable stochastically and the corresponding trajectories are shown in the Figure 4. Figure 5 shows that the output-tracking trajectories of the $z(t)$ and $z_r(t)$. It can be seen that the system tracks the reference system perfectly with the controller. Figure 6 depicts that the trajectories of the tracking error with the designed controllers. The value of the tracking error reaches a stochastic stability around the 15th second. Figure 7 is the state trajectory of $\zeta(t)$ which also remains stable after the 15th second. Figure 8 demonstrates the AET release instants and intervals. From Figure 5, it can be found that the number of the triggered times is 132. By simple calculation, it implies that there has 10.1% of the sampled data require to be transmitted to the controller. Additionally, it can be calculated that the maximal value of the release interval is 2.109 and the average value of the release interval is 1.046. Next, in order to improve the performance of the network system, it is necessary to analyze that how two quantization parameters θ_f and θ_g affect the optimal H_∞ tracking performance index λ^* . Through corresponding numerical calculation and simulation, the related results are presented in Table 1.

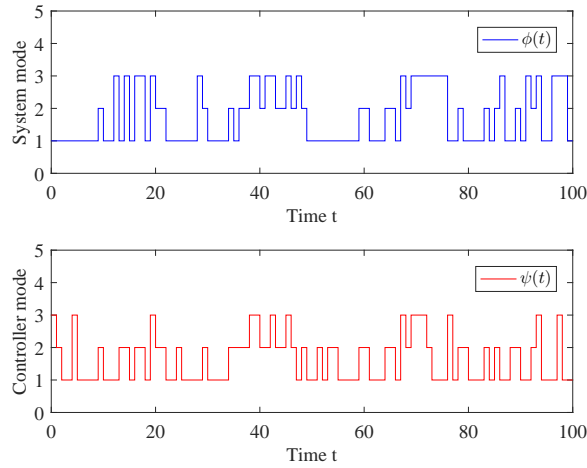


Figure 2. The mode evolution of the system (2.2) and the controller (2.12).

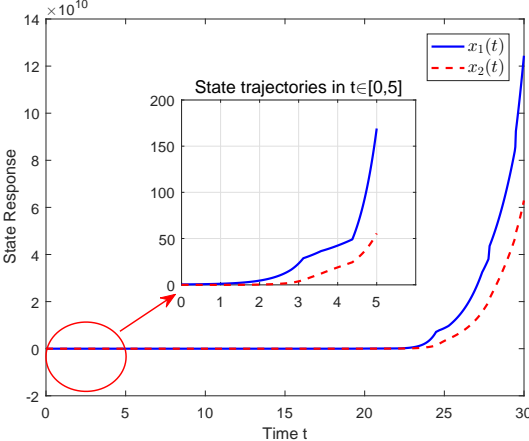


Figure 3. The state trajectories of the system (2.2) without a controller.

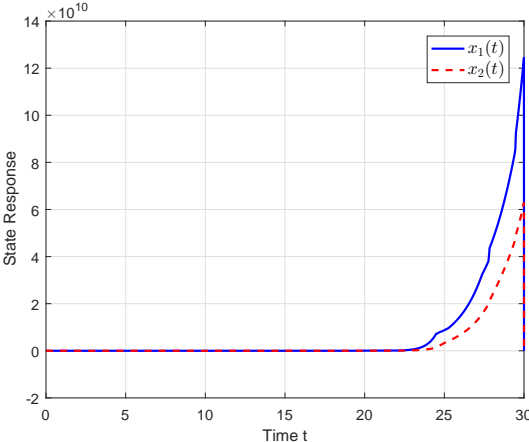


Figure 4. The state trajectories of the system (2.2) with the controllers.

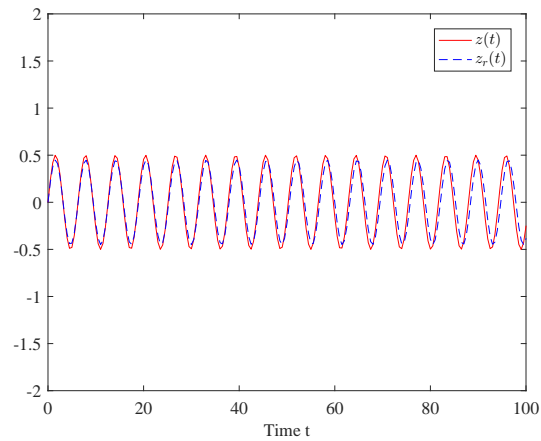


Figure 5. The output trajectories of the system (2.2) and the system (2.4) with the controllers.

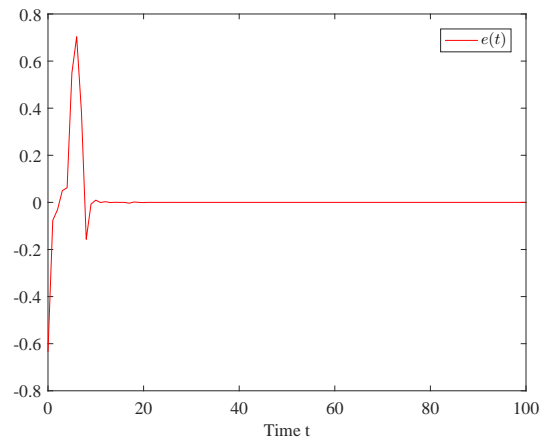


Figure 6. The tracking error of the system (2.2) and the system (2.4) with the controllers.

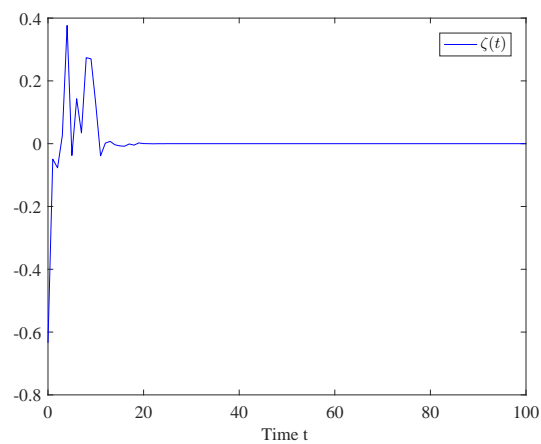


Figure 7. The state trajectories of $\zeta(t)$ in the system (2.17) with the controllers.

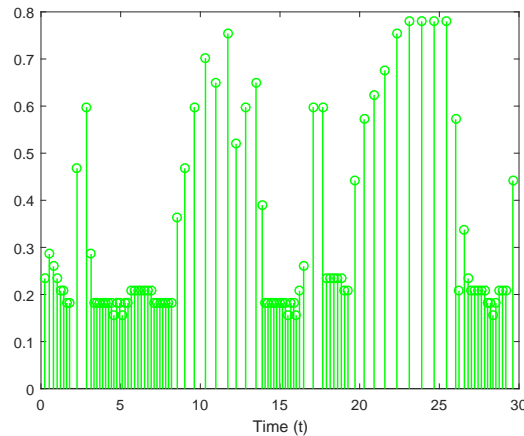


Figure 8. The release instants and release intervals of the event generator.

Table 1. Optimal tracking performance λ^* for different θ_f and θ_g .

θ_f	θ_g	λ^*
0.01	0.001	0.0523
0.02	0.001	0.0589
0.05	0.001	0.0834
0.08	0.001	0.0978
0.1	0.001	0.3241
0.01	0.002	0.0849
0.01	0.005	0.1787
0.01	0.008	0.2903
0.01	0.01	0.3890

Due to the limited space of the paper, the data in Table 1 only shows some representative simulation results, and the following conclusions can be obtained through the remaining large number of simulation results. From Table 1, it can be obtained that with the value of output quantization parameter unchanged, the input quantization parameter value increases, λ^* also increases. Correspondingly, when the value of input quantization parameter remains unchanged, the output quantization parameter value increases, λ^* also increases. This shows that the smaller both the quantization parameters are, the more superior tracking performance which the original system has.

Example 2. Borrowed from [24], an example of a single-link flexible joint manipulator is selected. The dynamic equation of the system is

$$\begin{cases} \dot{\chi}_1(\tilde{t}) = \chi_3(\tilde{t}), \\ \dot{\chi}_2(\tilde{t}) = \chi_4(\tilde{t}), \\ \dot{\chi}_3(\tilde{t}) = \frac{L_s}{M_h} \chi_2(\tilde{t}) - \frac{L_m^2 L_g^2}{R_m M_h} \chi_3(\tilde{t}) + \frac{L_m L_g}{R_m M_h} u(\tilde{t}), \\ \dot{\chi}_4(\tilde{t}) = -\frac{L_s}{M_h} \chi_2(\tilde{t}) + \frac{L_m^2 L_g^2}{R_m M_h} \chi_3(\tilde{t}) - \frac{L_m L_g}{R_m M_h} u(\tilde{t}) - \frac{L_s}{M_h} \chi_2(\tilde{t}) + \frac{m_l g h}{M_l} \sin(\chi_1(\tilde{t}) + \chi_2(\tilde{t})), \end{cases} \quad (4.3)$$

where $\chi_1(\tilde{t})$ is the angular position of the arm, $\chi_2(\tilde{t})$ is the angular displacement, $\chi_3(\tilde{t})$ is the angular

velocity of the motor, $\kappa_4(\tilde{t})$ is the angular velocity. Moreover, $\kappa_1(\tilde{t}) + \kappa_2(\tilde{t}) \in [-\pi/2, \pi/2]$, Thus, $y_1(\kappa_1(\tilde{t})) = \frac{\sin(\kappa_1(\tilde{t}) + \kappa_2(\tilde{t}))}{\kappa_1(\tilde{t}) + \kappa_2(\tilde{t})} \in [y_{1min}, y_{1max}] = [0, 1]$, some related parameters are listed in Table 2.

Table 2. The parameters of the system (4.3).

Parameters	Symbol	Value
Elastic hardness	L_s	1.23[N/m]
Hub inertia	$M_h \nu$	0.005[Kg·m ²]
Connecting rod mass	m_l	0.504[Kg]
Gravitational constant	g	-9.8[N/m]
Height of C.M	h	0.08[m]
Motor const	L_m	0.00676[N/rad/s]
Load inertia	M_l	0.0065[Kg·m ²]
Gears ratio	L_g	99 for mode one

The MFs are denoted as $h_1(\kappa_1(\tilde{t})) = \frac{y_1(\kappa_1(\tilde{t})) - y_{1max}}{y_{1min} - y_{1max}}$ and $h_2(\kappa_1(\tilde{t})) = 1 - h_1(\kappa_1(\tilde{t}))$. The manipulator is considered as a 2-rule fuzzy MJS of system (2.1). Its parameters are selected as

$$\mathcal{A}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{L_s}{M_h} & -\frac{L_m^2 L_g^2}{R_m M_h} & 0 \\ y_{1min} \frac{m_l g h}{M_l} & a_1 & \frac{L_m^2 L_g^2}{R_m M_h} & 0 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{L_s}{M_h} & -\frac{L_m^2 L_g^2}{R_m M_h} & 0 \\ y_{1max} \frac{m_l g h}{M_l} & a_2 & \frac{L_m^2 L_g^2}{R_m M_h} & 0 \end{bmatrix},$$

$$\mathcal{B}_1 = \begin{bmatrix} 0 \\ 0 \\ \frac{L_m L_g}{R_m M_h} \\ -\frac{L_m L_g}{R_m M_h} \end{bmatrix}, \quad \mathcal{B}_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{L_m L_g}{R_m M_h} \\ -\frac{L_m L_g}{R_m M_h} \end{bmatrix}, \quad \mathcal{C}_1 = \mathcal{C}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \mathcal{D}_1 = \mathcal{D}_2 = \begin{bmatrix} -0.4 \\ 0.5 \\ 0.1 \\ 0.2 \end{bmatrix},$$

with

$$\begin{cases} a_1 = -\left(\frac{L_s}{M_h} + \frac{L_s}{M_l}\right) + y_{1min} \frac{m_l g h}{M_l}, \\ a_2 = -\left(\frac{L_s}{M_h} + \frac{L_s}{M_l}\right) + y_{1max} \frac{m_l g h}{M_l}. \end{cases}$$

Then, the reference model is selected as follows:

$$\mathcal{A}_r = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4123.234 & 134.789 & -2742.753 & 39.349 \\ 4234.172 & -427.776 & 2671.075 & -39.467 \end{bmatrix}, \quad \mathcal{B}_r = \begin{bmatrix} 0 \\ 0 \\ \frac{L_m L_g}{R_m M_h} \\ -\frac{L_m L_g}{R_m M_h} \end{bmatrix}.$$

By setting the initial value $\kappa(0) = [2 \ 1 \ 1 \ 0]^T$, $\kappa_r(0) = [0.1 \ 1 \ 1 \ 0]^T$, the desired control gains with $\lambda_{min} = 1.2456 > \frac{\|e(\tilde{t})\|_2}{\|\bar{w}(\tilde{t})\|_2} = 0.0627$. The corresponding state and error trajectories are shown in Figures 9–11. It is obtained that the above system can also reaches the stochastic stability under some prescribed conditions. Meanwhile, the manipulator system can also track the desired reference system well under the given H_∞ tracking performance index λ_{min} .

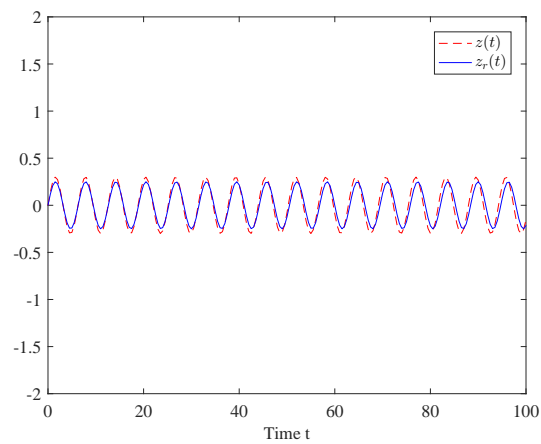


Figure 9. The output-tracking trajectories in Example 2.

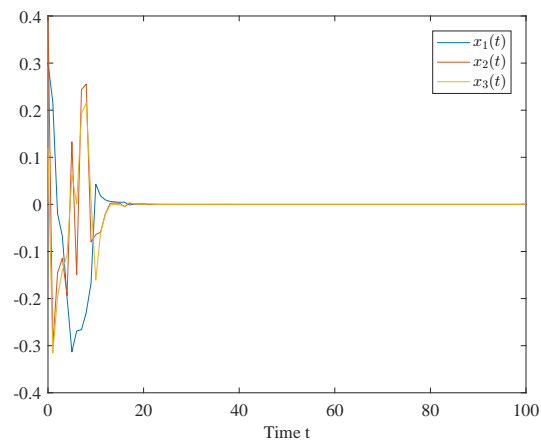


Figure 10. The state trajectories of the system (4.3) in Example 2.

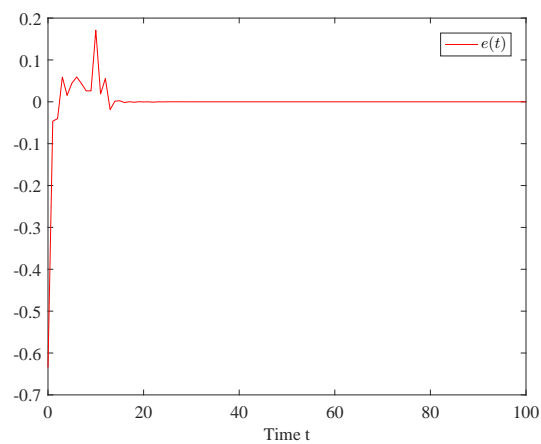


Figure 11. The tracking error in Example 2.

5. Conclusions

In this paper, a class of H_∞ tracking control problem for continuous-time fuzzy SMJSs is investigated. The AETM and double quantization strategy for the controller input and the controller output are adopted to compress network communication signals and improve the transmission efficiency. Based on the improved L-K functions and LMI techniques, some sufficient criteria which can enable the system achieve a stochastic stability and satisfy the H_∞ tracking performance of the tracking control system are derived. At the same time, a quantitative tracking feedback controller based on fuzzy basis and mode dependence is constructed. Finally, two examples are given to demonstrate the effectiveness and rationality of the above method. In the future work, it is hoped that the results can be extended to other fuzzy SMJSs with the time-varying delay, deception attacks, DOS attacks, and so on. Of course, other quantized strategies are also worth our further research.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 11661028, the Natural Science Foundation of Guangxi under Grant 2020GXNSFAA159141, Guangxi Philosophy and Social Science Programming Project (2022) under Grant 22BTJ001.

Conflict of interest

The authors declare that they have no conflicts of interest concerning the publication of this manuscript.

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