Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(2): 2856–2870.
DOI:10.3934/math.2023149
Received: 31 August 2022
Revised: 26 October 2022
Accepted: 31 October 2022
Published: 10 November 2022

Research article

Approximation of fixed point of generalized non-expansive mapping via new faster iterative scheme in metric domain

Noor Muhammad[1], **Ali Asghar**[1], **Samina Irum**[1], **Ali Akgül**[2,3,*], **E. M. Khalil**[4] **and Mustafa Inc**[5,6,*]

[1] Department of Mathematics and Statistics, Institute of Southern Punjab, Multan, Pakistan
[2] Department of Mathematics, Art and Science faculty, Siirt University, 56100 Siirt, Turkey
[3] Near East University, Mathematics Research Center, Department of Mathematics, Near East Boulevard, PC 99138, Nicosia/Mersin, Turkey
[4] Department of Mathematics, College of Science, P. O. Box 11099, Taif University, Taif 21944, Saudi Arabia
[5] Department of Mathematics, Science Faculty Firat University, Elazig, Turkey
[6] Department of Medical Research China Medical University, Taichung, Taiwan, China

* **Correspondence:** Email:aliakgul@siirt.edu.tr, minc@firat.edu.tr.

Abstract: In this paper, we establish a new iterative process for approximation of fixed points for contraction mappings in closed, convex metric space. We conclude that our iterative method is more accurate and has very fast results from previous remarkable iteration methods like Picard-S, Thakur new, Vatan Two-step and K-iterative process for contraction. Stability of our iteration method and data dependent results for contraction mappings are exact, correspondingly on testing our iterative method is advanced. Finally, we prove enquiring results for some weak and strong convergence theorems of a sequence which is generated from a new iterative method, Suzuki generalized non-expansive mappings with condition (C) in uniform convexity of metric space. Our results are addition, enlargement over and above generalization for some well-known conclusions with literature for theory of fixed point.

Keywords: Suzuki generalized non expansive mapping; uniformly convex metric space; iteration process; weak convergence; strong convergence; condition (C)
Mathematics Subject Classification: 47H09, 47H10, 47J25

1. Introduction

Over the years, fixed point theory has been generalized in multi-directions by numerous mathematicians. For detail, we recommend these books [3] and [15] to readers. However, if the existence of a fixed point is guaranteed for some mapping then to find the value of that fixed point is not an easy task, that is why we use iterative methods for computing these. With the times, mathematicians played a considerable role in this field, and it's very hard to approach all of them by time. The very important and famous Banach contraction theorem uses Picard iterative method (we will denote "iterative process" by I.P throughout this paper) for approximation of fixed point. Few more important iterative methods are Mann [22], Ishikawa [12], Agarwal [2], Noor [18], Abbas [1], SP [19], S* [13], CR [7], Normal-S [23], Picard Mann [16], Picard-S [10], Thakur New [25], Vatan Two-step [14] and so on. The qualities like "Fastness and Stability" show the vital role of an I.P elevate to others. In [20], Rhoades proved for decreasing function the Mann iteration method converges quicker than compared to Ishikawa iteration method while Ishikawa iterative processes are better than compared to Mann iterative results for increasing function. Note that Mann I.P is not dependent on primary guess (for detail see [21]). [2], Agarwal et al. claimed that Agarwal I.P converges like Picard I.P also having better results as compared to Mann I.P for contraction mappings. In [1], Abbas et al. claimed that Abbas I.P converges quickly by comparing Agarwal I.P. In [6], Chugh et al. proved that CR I.P is equal to and having faster results comparing above-mentioned mathematicians having iterative processes of quasi-contractive operators in metric domain. Furthermore, mathematician [8] performed better and advanced results as compared to previous results. This is the beauty of this field. In this article, we introduce new iterative method also prove that our results are more stable and faster. Our new iterative process converges faster than Picard-S I.P and hence faster than others. In this paper some basic concepts and results are used. We also describe a brief summary of the existence of the iterative process. We also prove strong and weak fixed point convergence theorems for Suzuki generalized non-expansive mappings, which are generalizations of non-expansive and contraction mappings. Furthermore, we use convex metric space as an underlying space. We show that new iterative method has stability and faster convergence results relative to K-iterative process. We also prove some weak and strong convergence results for Suzuki generalized non-expansive mappings with respect to new iterative process satisfying condition (C).

1.1. Preliminaries

Assume that X be any non-empty set and $d : X \times X \to R$ be a function such that

$$\begin{aligned}(i)\, d(i, j) &\geq 0,\\ (ii)\, d(i, j) &= 0, i \iff j,\\ (iii)\, d(i, j) &= d(j, i),\\ (iv)\, d(i, k) &\leq d(i, j) + d(j, k) \forall\ i, j, k \in X.\end{aligned}$$

Then (X, d) is named as metric space. Let (X, d) be a metric space and $\{a_n\}$ be any sequence in X. Then $\{a_n\}$ converges to $a \in X$ if for any sequence in $\epsilon > 0$, there is a number $n_0 \in N$ such that, $d(a_n, a) \leq \epsilon$ for all n_0 If $\{a_n\}$ converges to a, then we can also write it as $lim_{n\to 0} a_n = a$. Suppose that (X, d) is a metric space and $\{a_n\}$ is any sequence with X. Then $\{a_n\}$ is a Cauchy sequence if for any $\epsilon > 0$, there is a number $n_0 \in N$ and $d\ (a_m, a_n) \leq \epsilon$ and $n_0 \leq m, n$. (X, d) is any metric space consider as complete when every Cauchy sequence in X be convergent. Let a metric space X known as Opial condition when every sequence $\{a_n\}$ in X, then condition $a_n \to a$ implies that

$$lim_{n\to\infty} \inf d(a_n, a) < lim_{n\to\infty} \inf d(a_n, b)\ ,\ \forall\ a,\ b \in X,$$

with $b \neq a$. Consider X is metric space, moreover $I = [0,\ 1]$ any mapping like $W : X \times X \times I \to X$ be a structure of convex for X when $\forall$, $(a, b, \xi) \in X \times X \times I$ and $v \in X$ we have

$$d\ (v, W\ (a, b, \xi))\ \leq \zeta d\ (v, a) + (1 - \xi)\, d\ (v, b)\,,$$

then the metric space (X, d) mutually along with the convex structure W known as metric space and express as (X, d, W). A convex metric (X, d, W) is also known as strictly convex when one of the following condition is satisfied.
(i) For any $i, j \in X$ and $\alpha \in [0, 1]$ then $\exists$ unique $k \in X$ such that $d\,(k, i) = \alpha d\,(i, j)$ and $d\,(k, j) = (1 - \alpha)\, d\,(i, j)$.
(ii) For any $i, j, k \in X$ with $d\ (k, W\,(i, j, \alpha)) = d\,(i, k) = d\,(j, k)$ we have that $i = j$ for $\alpha \in (0, 1)$.

Let $\alpha : (0, 2] \to (0, 1]$ such that $Lim_{\epsilon \to 0}\alpha(\epsilon) = 0$ and $\alpha\,(2) = 1$ then the convex metric space (X, d, W) is also known as uniformly convex when any $r > 0$ and having $r \in (0, 2]$ $d\left(z, W\left(i, j, \frac{1}{2}\right)\right) \leq r\,(1 - \alpha)$ whenever $d\,(k, i) \leq r$ also $d\,(k, j) \leq r$ and $r \leq d\,(i, j)\ \epsilon$ for any $i, j, k \in X$. Let X be a non-empty set and τ be a collection of X such that
(I) $\kappa\,, X \in \tau$;
(II) Arbitrary union of numbers of τ is in τ;
(III) Finite intersection of numbers of τ is also belong to τ then τ is topology on X and then (x, τ) is called Topological space [4].

1.2. Geometry of convex metric spaces

The geometrical structure of the under discussion spaces perform a vital role in existence and approximation of the fixed points of many different nonlinear mappings. Therefore, in that part, we will highlight some important geometrical properties of the convexity of metric space [9].

$$\begin{aligned}
(i)\ W\ (i, j, \alpha) &= W\ (j, i\ (1 - \alpha))\ \forall\ i, j \in X, \alpha \in [0, 1]\,, \\
(ii)\ d\ (W\,(i, j, \alpha)\,, W\ (i, j, \beta)) &\leq (\alpha - \beta)\ d\ (i, j)\ \forall \in X, (\alpha, \beta) \in [0, 1]\,, \\
(iii)\ d\ (W\ (i, j, \alpha)\,, W\ (i, k, \alpha)) &\leq (1 - \alpha)\ d\ (j, k)\ \forall\ i, j, k \in X, \alpha \in [0, 1]\,, \\
(iv)\ d\ (W\ (i, j, \alpha)\,, W\ (k, l, \alpha)) &\leq (1 - \alpha)\, d\,(j, l) + \alpha d\ (i, k)\ \forall\ i, j, k, l \in X, \alpha \in [0, 1]\,.
\end{aligned}$$

Assume that K be non-empty subset of any metric space X. Any mapping $T : K \to K$ is known as contraction for $\exists$, $\theta \in (0, 1)$

$$d\,(Ti, Tj) \leq \theta d\,(i, j), \forall\ i, j \in K.$$

Let (X, d) is a non-empty subset of a metric space X. A mapping $T : C \to C$ is said to be generalized contraction if there exists $0 \leq h \leq 1$ for

$$d\,(Ti, Tj) \leq h \max[d\,(i, j), d(i, Ti), d\,(j, Tj), d\,(i, Tj) + d\,(j, Ti)], \forall i, j \in C.$$

Let C be a non-empty subset of a metric space X. A mapping $T : C \to C$ is said to be non expansive mapping if $(Ta, Tb) \leq d(a, b)$ for all $a, b \in C$. Let C be a non-empty subset of a metric space X. A mapping $T : K \to K$ is known as Suzuki generalized non expensive mapping when satisfy the criteria of condition (C) if $\forall\ i, j \in K$ we get

$$\frac{1}{2}d\,(i, j) \leq d\,(i, j) \Rightarrow d\,(Ti, Tj) \leq d\,(i, j).$$

1.3. Some basic results

Proposition 1.1. *[17] Suppose that K is non-empty subset of any metric space X and $T : K \to K$ is for every mapping. Then*
(i) If T be non expansive so T satisfy condition (C).
(ii) If T satisfy condition (C) and having fixed point, so T be quasi-nonexpansive mapping.
(iii) If T satisfy condition (C), so

$$d\,(i, Tj) \leq 3d\,(Ti, i) + d\,(i, j)\ \forall\ i, j \in K.$$

Lemma 1.1. *Assume that K is any non-empty subset for metric space X. Moreover, $T : K \to K$ is any mapping for Opial property. Assume T satisfy condition (C). If $\{i_n\}$ converges weakly to z also $lim_{n\to\infty} d(Ti_n, i_n) = 0$, then $Tk = k$. That is, $I - T$ is demiclosed at zero.*

Lemma 1.2. *Assume that K is any weakly compact convex subset for uniformly convexity of metric space X. Suppose that T is any mapping on K. Consider that T satisfy condition (C). So, T having a fixed point.*

Lemma 1.3. *Assume X be any uniformly convexity in metric space and $\{t_n\}$ is real sequence also providing $0 < u \leq t_n \leq v < 1$,for all $n \geq 1$. Moreover, assume that $\{i_n\}$ along with $\{j_n\}$ are two sequences for X*

$$lim_{n\to\infty} \sup i_n \leq r, \lim_{n\to\infty} \sup j_n \leq r,$$

and

$$\lim_{n\to\infty} \sup d\,(t_n i_n, (1 - t_n) j_n) = r,$$

and $r \geq 0$. So,

$$lim_{n\to\infty} d(i_n, j_n) = 0.$$

Suppose that G is any non-empty closed convex subset for any metric space X, and assume $\{i_n\}$ is any bounded sequence in X. When $i \in X$, we find that

$$r\,(i, \{i_n\}) = \lim_{n\to\infty} \sup d(i_n, i).$$

Then asymptotic radius for $\{i_n\}$ relative for G be providing as

$$r\,(G, \{i_n\}) = \inf\{r(i, \{i_n\}) : i \in G\},$$

and asymptotic center for $\{i_n\}$ relative for G be any set

$$B(G, \{i_n\}) = \{i \in G : r\,(i, \{i_n\}) = r\,(G, \{i_n\})\}.$$

This is called uniformly convex metric space, $B(G, \{i_n\})$ contain for fixed point.

Definition 1.1. *[5] Let $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are two different fixed point iterative process sequences which converge to some fixed point p and $d\,(u_n, p) \leq a_n$ and $d\,(v_n, p) \leq b_n$ for all $n \geq 0$. If the sequences $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ converges to a and b respectively and $lim_{n\to\infty} \frac{d(p_n,p)}{d(q_n,q)} = 0$, then we say that $\{u_n\}_{n=0}^{\infty}$ converges faster than $\{v_n\}_{n=0}^{\infty}$ to p.*

Definition 1.2. *[11] Let* $\{t_n\}_{n=0}^{\infty}$ *is any aribitrary sequence for* K*. So, an iterative method* $i_{n+1} = f(T, i_n)$*, converge fixed point* F*, is considered as* $T-stable$ *may be stable with respect to* T*, When for* $\epsilon_n = d(t_{n+1}, f(T, t_n)), n = 0, 1, 2, 3, ...,$ *we get* $lim_{n\to\infty}\epsilon_n = 0 \iff \lim_{n\to\infty} t_n = F$*.*

Lemma 1.4. *[26] Let* $\{\lambda_n\}_{n=0}^{\infty}$ *and* $\{\mu_n\}_{n=0}^{\infty}$ *be non-negative real sequences satisfying the following inequality* $\lambda_{n+1} \leq (1-\xi_n)\lambda_n + \mu_n$*, where* $\xi_n \in (0, 1)$ $for\ all\ n \in N, \Sigma_{n=0}^{\infty} \xi_n = \infty$ *and* $\frac{\mu_n}{\xi_n} \to 0$ *as* $n \to \infty$*, then* $lim_{n\to\infty}\lambda_n = 0$*.*

Lemma 1.5. *[24] Let* $\{\psi_n\}_{n=0}^{\infty}$ *be non-negative real sequence and suppose that* $\exists$ *,* $n_0 \in N$ *,*$\forall$ $n \geq n_0$*, then the given inequality satisfies* $i_{n+1} \leq (1 - j_n)i_n + j_n\phi_n$*, when* $j_n \in (0, 1)$ $\forall$ $n \in N, \Sigma_{n=0}^{\infty} j_n = \infty$ *and* $\phi_n \geq 0$ $\forall$ $n \in N$*, So* $0 \leq lim_{n\to\infty} \sup i_n \leq lim_{n\to\infty} \sup \phi_n$*.*

1.4. Iterative processes

Overall in this portion we get $n \geq 0$, (α_n) and (β_n) are real sequences in $[0, 1]$, K be subset for metric space X and $T : K \to K$ be any mapping. Let the iterative sequence denoted by $\{u_n\}$ in this section. Gursoy and Karakaya (2014) set up new iterative method which is said to be "Picard-S iterative method" as follow:

$$\begin{aligned} u_0 &\in K, \\ w_n &= W(Tu_n, u_n, \beta_n), \\ v_n &= W(Tw_n, Tu_n, \alpha_n), \\ u_{n+1} &= Tv_n. \end{aligned} \tag{1}$$

The Picard-S iterative method may be utilized in the approximation of the fixed point for contraction mappings. Moreover, theysolved one mathematical example, which resulted in the Picard-S iterative method converging faster than others who have done outstanding work in this field. Afterward Karakaya et al. in 2015 set up an advance iterative process, we knew it by the name of a new two-step iterative method, they argued that the rate of convergence is better than Picard-S iterative process as follows:

$$\begin{aligned} u_0 &\in K, \\ v_n &= T(W(Tu_n, u_n, \beta_n)), \\ u_{n+1} &= T(W(Tv_n, v_n, \alpha_n)). \end{aligned} \tag{2}$$

Some time ago, Thakur et al. in 2016 defined a newly advanced iterative method to approximate of fixed points, which is called Thakur New iterative process:

$$\begin{aligned} u_0 &\in K, \\ w_n &= W(Tu_n, u_n, \beta_n), \\ v_n &= T(W(w_n, u_n, \alpha_n)), \\ u_{n+1} &= Tv_n. \end{aligned} \tag{3}$$

Lastly, Nawab Hussain, Kifayat Ullah and Muhammad Arshad established a new iterative method for approximation of fixed point of contraction mapping which is said to be "K iterative process" defined as:

$$u_0 \in K,$$

$$
\begin{aligned}
k_n &= W(Ti_n, i_n, \beta_n), \\
j_n &= T(W(Tj_n, Ti_n, \alpha_n)), \\
i_{n+1} &= Tj_n.
\end{aligned} \tag{4}
$$

With the solution of example, they concluded that K iterative method is converging faster than Vatan two-step iterative process, Picard, Mann, Ishikawa, Agarwal, Noor and Abbas iterative method by any class for mappings. By motivation above we propose a new iteration process. By definition of convexity of convex metric space $d\,(W(i, j, \alpha), k) \leq d\,(i, j) + (1-\alpha)\,d\,(j, k)$ iterative process

$$
\begin{aligned}
i &\in K, \\
k_n &= T[W(Ti_n, i_n, \beta_n], \\
j_n &= T[W(Tk_n, Ti_n, \alpha_n)], \\
i_{n+1} &= Tj_n.
\end{aligned} \tag{5}
$$

We will conclude our iteration process (5) is stable and having faster rate of convergence than others iteration processes.

2. Main results

We will prove the uniqueness and convergence of fixed points for contraction mapping generated by a new iterative process in convex metric space. Also, we will show that our advanced iterative process is stable and having faster convergence results than previously defined iterative processes.

2.1. Convergence analysis of new iterative process for contraction mapping

Theorem 2.1. *Suppose that K is any non-empty closed convex subset for a convex metric space X and $T : K \to K$ is a contraction mapping. Assume that $\{i_n\}_{n=0}^{\infty}$ is an iterative sequence generated from the real sequences $\{\alpha_n\}$ and $\{\beta_n\}$ in $[0, 1]$ satisfy $\Sigma_{n=0}^{\infty}\alpha_n\beta_n = \infty$. So, $\{i_n\}_{n=0}^{\infty}$ converge strongly to an unique fixed point for T.*

Proof. We will prove that $i_n \to l$ for $n \to \infty$ from (5) we get

$$
\begin{aligned}
d(i_n, l) &= d[T(W(Ti_n, i_n, \beta_n)), l] \\
&\leq \theta d\,[W(Ti_n, i_n, \beta_n), l] \\
&\leq \theta[\beta_n d\,(Ti_n, l) + (1-\beta_n)d(i_n, l)] \\
&\leq \theta[\beta_n \theta d\,(i_n, l) + (1-\beta_n)d(i_n, l)] \\
&\leq \theta[\theta\beta_n + (1-\beta_n)]d(i_n, l) \\
&\leq \theta[1-(1-\theta)\beta_n]d(i_n, l).
\end{aligned} \tag{6}
$$

Similarly,

$$
\begin{aligned}
d(j_n, l) &= d\,[T\,(W\,(Tk_n, Ti_n, \alpha_n^{\iota}), l] \\
&\leq \theta d[W(Tk_n, Ti_n, \alpha_n^{\iota}), l] \\
&\leq \theta[\alpha_n^{\iota} d\,(Tk_n, l) + (1-\alpha_n^{\iota})\ d\,(Ti_n, l)]
\end{aligned}
$$

$$
\begin{aligned}
&\leq \theta[\alpha_n^\iota \theta d\,(k_n, l) + (1-\alpha_n^\iota)\ \theta\, d(i_n, l)] \\
&\leq \theta[\alpha_n^\iota \theta^2 (1-(1-\theta)\beta_n) + (1-\alpha_n^\iota)\,\theta d\,(\alpha_n^\iota, l)] \\
&\leq \theta^2[(\alpha_n^\iota \theta (1-(1-\theta)\beta_n) + (1-\alpha_n^\iota))\ d\ (a_n^\iota, l)] \\
&\leq \theta^2[(\alpha_n^\iota \theta - \alpha_n^\iota \theta (1-\theta)\beta_n + 1 - \alpha_n^\iota) d(i_n, l)] \\
&\leq \theta^2[(1-(1-\theta)\alpha_n^\iota - (1-\theta)\alpha_n^\iota \beta_n \theta) d(i_n, l)] \\
&\leq \theta^2[(1-(1-\theta)\alpha_n^\iota (1+\beta_n\theta) d(i_n, l)]. \qquad (7)
\end{aligned}
$$

Hence

$$
\begin{aligned}
d(i_{n+1}, l) &= d(T j_n, l) \\
&\leq \theta d(j_n, l) \\
&\leq \theta^3[1-(1-\theta)\alpha_n^\iota (1+\beta_n\theta)] d(i_n, l). \qquad (8)
\end{aligned}
$$

Repetition of above processes gives the following inequalities

$$
\begin{aligned}
d(i_{n+1}, l) &\leq \theta^3[1-(1-\theta)\alpha_n (1+\beta_n\theta)] d(i_n, l), \\
d(i_n, l) &\leq \theta^3[1-(1-\theta)\alpha_{n-1} (1+\beta_{n-1}\theta)] d(i_{n-1}, l), \\
d(i_{n-1}, l) &\leq \theta^3[1-(1-\theta)\alpha_{n-2} (1+\beta_{n-2}\theta)] d(i, j), \\
d(i_1, l) &\leq \theta^3[1-(1-\theta)\alpha_0 (1+\beta_0\theta)] d(i_0, l), \qquad (9)
\end{aligned}
$$

from (9) we can easily get

$$
d(i_{n+1}, l) \leq d(i_0 - l)\theta^{3(n+1)} \Pi_{k=0}^{n} (1-(1-\theta)) i_k (1+\beta_k\theta)), \qquad (10)
$$

where$(1-(1-\theta))\alpha_k^\iota(1+\beta_k\theta) < 1$ the reason is that $\theta \in (0,1)$ and $\alpha_n^\iota \beta_n \in [0,1] \forall n \in N$, So we identify that $1 - i \leq \varrho^{-a} \forall i \in [0,1]$

$$
d(i_{n+1}, l) \leq d(i_0, l)\theta^{3(n+1)} \varrho^{-(1-\theta)} \Sigma_{k=0}^{n} \alpha_k (1+\beta_k\theta), \qquad (11)
$$

taking limit on both sides of (11) we get $lim_{n\to\infty} d(i_n, l) = 0$ i.e., $i_n \to l$ for $n \to \infty$ as required. □

Theorem 2.2. *Assume that K is any non-empty closed convex subset of metric space X and $T : K \to K$ is a contraction mapping. Suppose that $\{b_n\}_{n=0}^{\infty}$ is an iterative sequence generated by (5) having real sequence $\{\alpha_n^\iota\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=o}^{\infty}$ in $[0,1]$ satisfying $\Sigma_{n=0}^{\infty} \alpha_n^\iota \beta_n = \infty$. So, iterative method (5) are T-stable.*

Proof. Assume $\{s_n\}_{n=0}^{\infty} \subset X$ is any aribitrary sequence in K. Suppose the give sequence generated (5) is a $b_{n+1} = f(T, a_n)$ converge to unique fixed point F. Moreover, $\varepsilon_n = d(s_{n+1}, f(T, s_n))$ we will conclude that $lim_{n\to\infty}\ \varepsilon_n = 0 \iff lim_{n\to\infty} s_n = F$. Let $\lim_{n\to\infty} \varepsilon_n = 0$ we get

$$
\begin{aligned}
(s_{n+1}, F) &\leq d(s_{n+1}, f\ (T, s_n)) + d(f(T, s_n), F) \\
&= \varepsilon_n + d((T b_n, F) \\
&\leq \varepsilon_n + \theta d((b_n, F) \\
&\leq \theta^3 (1-(1-\theta)\alpha_n^\iota (1+\beta_n\theta))\ d\ (sn, F) + \varepsilon_n.
\end{aligned}
$$

since $\theta \in (0,1)$ $\alpha^{\iota}_n, \beta_n \in [0,1] \forall n \in N$ and $\lim_{n\to\infty} \varepsilon_n = 0$ so the above inequality together with Lemma 1.4 leads to $\lim_{n\to\infty} d(s_n, F) = 0$. Hence $\lim_{n\to\infty} s_n = F$.

Conversely, let $\lim_{n\to\infty} s_n = F$ we have

$$\begin{aligned} \varepsilon_n &= d(s_{n+1}, f\,(T, s_n)) \\ &\leq d(s_{n+1}, p) + d(f(T, s_n), F) \\ &\leq d(s_{n+1}, p) + \theta^3(1 - (1-\theta)\alpha_n\,(1+\beta_n\theta))d(s_n, F). \end{aligned}$$

This implies that $lim_{n\to\infty}\varepsilon_n = 0$. □

Theorem 2.3. *Suppose that K be any non-empty and closed convex subset of a metric space X. Moreover, $T : K \to K$ is any contraction mapping having fixed point F. For given $u_0 = x_0 \in C$, let $\{u_n\}_{n=0}^{\infty}$ and $\{x_n\}_{n=0}^{\infty}$ are iteration sequences generated by (5) respectively, having real sequences$\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfy assumption (i) $\alpha \leq \alpha_n < 1$ and $\beta \leq \beta_n < 1$, for some $\alpha, \beta > 0$ as well as $\forall$ $n \in N$. So, $\{x_n\}_{n=0}^{\infty}$ converge to F faster than $\{u_n\}_{n=0}^{\infty}$.*

Proof. By (10) of Theorem 2.2 we get

$$d\,(x_{n+1}, F) \leq d(x_0, F)\theta^{3(n+1)}\Pi_{k=0}^{n}(1 - (1-\theta)\alpha_k(1+\beta_k\theta). \tag{12}$$

The following inequality is due to Definition 1.1 and (8) which is obtained from (5) also converging to unique fixed point F

$$d(u_{n+1}, F) \leq d(u_0, F)\theta^{2(n+1)}\Pi_{k=0}^{n}(1 - (1-\theta)\alpha_k(1+\beta_k\theta), \tag{13}$$

together with assumption (i) and (12) $\Longleftrightarrow$

$$\begin{aligned} d(x_{n+1}, F) &\leq d(x_0, F)\theta^{3(n+1)}\Pi_{k=0}^{n}(1 - (1-\theta)\alpha(1+\beta\theta)) \\ &= d(x_0, F)\theta^{3(n+1)}[1 - (1-\theta)\alpha(1+\beta\theta)]^{n+1}. \end{aligned} \tag{14}$$

Similarly, (13) together with assumption (i) leads to

$$d(u_{n+1}, F) = d(u_0, F)\theta^{2(n+1)}[1 - (1-\theta)\alpha(1+\beta\theta)]^{n+1}. \tag{15}$$

Define

$$\begin{aligned} a_n &= d(x_0, F)\theta^{3(n+1)}[1 - (1-\theta)\alpha(1+\beta\theta)]^{n+1}, \\ b_n &= d(u_0, F)\theta^{2(n+1)}[1 - (1-\theta))\alpha(1+\beta\theta)]^{n+1}. \end{aligned} \tag{16}$$

Then

$$\Psi_n = \frac{a_n}{b_n} = \theta^{n+1}.$$

Since

$$\lim_{n\to\infty} \frac{\Psi n+1}{\Psi n} = \lim_{n\to\infty} \frac{\theta^{n+2}}{\theta^{n+1}} = 1 > \theta.$$

Applying the ratio test

$$\Sigma_{n=0}^{\infty}\Psi_n < \infty.$$

From (16) we have

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \Psi_n = 0.$$

This is $\Longleftrightarrow$ $\{x_n\}_{n=0}^{\infty}$ having quicker result as compare to $\{u_n\}_{n=0}^{\infty}$. □

Now we are able to prove following data dependence results.

Theorem 2.4. *Assume that $\widetilde{T}$ is an approximate operator for a contraction mapping T. Consider, $\{i_n\}_{n=0}^{\infty}$ is an iteration sequence is generated by (5) of T and define the iteration sequence $\{\widetilde{j_n}\}_{n=0}^{\infty}$ which is given below*

$$\begin{aligned} \widetilde{i_0} &\in K, \\ \widetilde{k_n} &= \widetilde{T}[W\,(\widetilde{T\widetilde{i_n}}, \widetilde{i_n}, \gamma_n,)], \\ \widetilde{j_n} &= \widetilde{T}[W(\widetilde{T\widetilde{k_n}}, \widetilde{T\widetilde{i_n}}, \alpha_n,)], \\ \widetilde{i_{n+1}} &= \widetilde{T\widetilde{j_n}}. \end{aligned} \tag{17}$$

With real sequences $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying
(i) $0.5 \leq \alpha_n\gamma_n$, $\forall$, $n \in N$ and,
(ii) $\Sigma_{n=0}^{\infty}\alpha_n\gamma_n = \infty$ when $Tp = p$ also $\widetilde{T}\widetilde{p} = \widetilde{p}$ and $\lim_{n\to\infty}\widetilde{x_n} = \widetilde{p}$. Then we have

$$d(p, \widetilde{p}) \leq \frac{7\epsilon}{1-\theta}.$$

When $\epsilon > 0$ be any fixed number.

Proof. See from (5) and (17) that

$$\begin{aligned} d(k_n, \widetilde{k_n}) &= d\,[T(W\,(Ti_n, i_n, \gamma_n), T(W\,(\widetilde{T\widetilde{i_n}}, \widetilde{i_n}, \gamma_n))] \\ &\leq d\,[T(W\,(Ti_n, i_n, \gamma_n), T\,(W\,(\widetilde{T\widetilde{i_n}}, \widetilde{i_n}, \gamma_n))] \\ &\quad + d\,[T\,(W\,(\widetilde{T\widetilde{i_n}}, \widetilde{i_n}, \gamma_n)), \widetilde{T}\,(W\,(\widetilde{T\widetilde{i_n}}, \widetilde{i_n}, \gamma_n))] \\ &\leq \theta[(1-(1-\theta)\,\gamma_n)d(i_n, \widetilde{i_n}) + \gamma_n\theta] + \epsilon. \end{aligned} \tag{18}$$

Using (18), we get

$$\begin{aligned} d(j_n, \widetilde{j_n}) &= d\,[T(W(Tk_n, Ti_n, \alpha_n), \widetilde{T}\,(W\,(\widetilde{T\widetilde{i_n}}, \widetilde{T\widetilde{k_n}}, \alpha_n))] \\ &\leq \theta^2[1-(1-\theta)\alpha_n(1+\theta\gamma_n)]d(i_n, \widetilde{i_n}) + \theta\epsilon(1+\alpha_n\gamma_n\theta) + \epsilon. \end{aligned} \tag{19}$$

By using (19), we get

$$\begin{aligned} d(i_{n+1}, \widetilde{i_{n+1}}) &= d(Tj_n, \widetilde{T\widetilde{j_n}}) \\ &\leq \theta d(j_n, \widetilde{j_n}) + \epsilon \\ &\leq [1-(1-\theta)\alpha_n(1+\theta\gamma_n)]d\,(i_n, \widetilde{i_n}) \\ &\quad + \alpha_n\gamma_n\theta\epsilon + 3\,(1-\alpha_n\gamma_n + \alpha_n\gamma_n)\,\epsilon, \end{aligned} \tag{20}$$

by assumption (i) we get

$$\begin{aligned} 1-\alpha_n\gamma_n &\leq \alpha_n\gamma_n, \\ d(i_{n+1}, \widetilde{i_{n+1}}) &\leq [1-(1-\theta)\,\alpha_n(1+\theta\gamma_n)]d(i_n, \widetilde{i_n}) \\ &\quad + \alpha_n\gamma_n\,(1-\theta)\,\frac{7\epsilon}{1-\theta}. \end{aligned} \tag{21}$$

Let $\Psi_n = d(i_n, \widetilde{i_n})$, $\phi_n = \alpha_n \gamma_n (1-\theta)$, $\phi_n = \frac{7\epsilon}{1-\theta}$ then from Lemma 1.5 together with (20) we get

$$0 \leq \lim_{n\to\infty} \sup d(i_n, \widetilde{i_n}) \leq \lim_{n\to\infty} \sup \frac{7\epsilon}{1-\theta}. \tag{22}$$

Since by Theorem 2.1 we have $lim_{n\to\infty} i_n = p$ and assumption we get the results $lim_{n\to\infty} \widetilde{i_n} = \widetilde{p}$ apply all of these simultaneously with (22) we have

$$d(p, \widetilde{p}) \leq \frac{7\epsilon}{1-\theta}.$$

As required. □

2.2. Convergence result for suzuki generalized non expansive mapping for condition (C)

In this section, we prove some weak and strong convergence theorems for a sequence generated by a new iteration process for Suzuki type generalized non-expansive mappings with condition (C) with uniformly convex metric space.

Lemma 2.1. *Assume that K is non-empty and closed convex subset of metric space X. Moreover, consider $T : K \to K$ is mapping which satisfy the condition (C) for $F(T) \neq 0$. To arbitrary chosen $a_0 \in K$, consider the sequence $\{a_n\}$ is generated by (5), so, $lim_{n\to\infty} d(a_n, s)$ exists on any $s \in F(T)$.*

Proof. Suppose that $s \in F(T)$ also $c \in K$. Since T satisfies condition (C)

$$\frac{1}{2} d\,(s, Ts) = 0 \leq d(s, c) \Leftrightarrow d(Ts, Tc) \leq d(s, c),$$

so by using Proposition 1.1(ii) we have the result as

$$\begin{aligned}
d(c_n, s) &= d(T[W\,(Ta_n, a_n, \beta_n)), s] \\
&\leq d[W(Ta_n, a_n, \beta_n), s] \\
&\leq \gamma_n d(Ta_n, s) + (1-\gamma_n) d(a_n, s) \\
&\leq \gamma_n d(a_n, s) + (1-\gamma_n) d(a_n, s) \\
&\leq \gamma_n d(a_n, s) + d(a_n, s) - \gamma_n d(a_n, s) \\
&\leq d(a_n, s),
\end{aligned} \tag{23}$$

by using (23) we get

$$\begin{aligned}
d(b_n, q) &= d\,[T\,(W\,(Tc_n, Ta_n, \alpha_n^{\iota})), s] \\
&\leq d\,(W\;(Tc_n, Ta_n, \alpha_n^{\iota}), s) \\
&\leq \alpha_n^{\iota} d\,(Tc_n, s) + (1-\alpha_n^{\iota}) d(Ta_n, s) \\
&\leq \alpha_n^{\iota} d(c_n, s) + (1-\alpha_n^{\iota}) d(a_n, s) \\
&\leq \alpha_n^{\iota} d(a_n, s) + d(a_n, s) - \alpha_n^{\iota} d\,(a_n, s) \\
&= d(a_n, s).
\end{aligned} \tag{24}$$

Same way using (24) we attain

$$d(u_{n+1}, s) = d(Tv_n, s) \leq d(v_n, s) \leq d(a_n, s) \Rightarrow d(a_n, s) \tag{25}$$

be bounded and decreasing $\forall\ s \in F\,(T)$. Therefor, $\lim_{n\to\infty} d(a_n, s)$ exist as required. □

Theorem 2.5. *Assume that K is any non-empty closed convex subset for a uniformly convex metric space X, and consider that $T : K \to K$ is a mapping which satisfy the condition (C). For arbitrary chosen $i_0 \in C$, suppose that the sequence $\{i_n\}$ is generated by (5) $\forall\ n \geq 1$, and $\{\alpha_n\}$ and $\{\beta_n\}$ be sequence for some real numbers in $[i, j]$ and having few points i, j along with $0 < i \leq j < 1$. So, $F(T) \neq \theta \iff \{i_n\}$ is bounded and $lim_{n\to\infty} d(Ti_n, i_n) = 0$.*

Proof. Suppose that $F(T) \neq \phi$ also consider that $q \in F(T)$. Then, from Lemma 2.1, $lim_{n\to\infty} d(i_n, q)$ exists and $\{i_n\}$ are bounded.

$$lim_{n\to\infty} d(i_n, q) = r. \tag{26}$$

By (23) and (26), we get

$$\lim_{n\to\infty} \sup d(k_n, q) \leq \lim_{n\to\infty} \sup d(i_n, q) = r, \tag{27}$$

by Proposition 1.1(ii)

$$\lim_{n\to\infty} \sup d(Ti_n, q) \leq \lim_{n\to\infty} \sup d(i_n, q) = r, \tag{28}$$

in other way

$$\begin{aligned} d(i_{n+1}, q) &= d(Tj_n, q) \\ &\leq d(j_n, q) \\ &= d\left[T\ (W\ (Tk_n, Ti_n, \alpha_n)), q\right] \\ &\leq (1-\alpha_n) d(Ti_n, q) + \alpha_n d(Tk_n, q) \\ &\leq (1-\alpha_n)(i_n q) + \alpha_n d(k_n, q) \\ &\leq (i_n, q) - \alpha_n d(i_n, q) + \alpha_n d(k_n, q) \end{aligned}$$

this implies

$$\begin{aligned} \frac{d\ (i_{n+1}, q) - d\ (i_n, q)}{\alpha_n} &\leq d(k_n, q) - d(i_n, q) \\ d(i_{n+1}, q) - d(i_n, q) &\leq \frac{d(i_{n+1}, q) - d(i_n, q)}{\alpha_n} \leq d(k_n, q) - d(i_n, q) \\ &\implies d(i_{n+1}, q) \leq d(k_n, q) \end{aligned}$$

therefore

$$r \leq \lim_{n\to\infty} \inf d(k_n, q), \tag{29}$$

from (27) and (29) we get

$$\begin{aligned} r &= d(k_n, q) \\ &= \lim_{n\to\infty} d(T\ (W\ (Ti_n, i_n, \beta_n)), q) \\ &= \lim_{n\to\infty} d(\ W(\ Ti_n, i_n, \beta_n)), q) \\ &\leq \lim_{n\to\infty} [(\beta_n d\ (Ti_n - q) + (1-\beta_n))(i_n, q)], \end{aligned} \tag{30}$$

from (26), (28) and (30) together we have $\lim_{n\to\infty} d(Ti_n, i_n) = 0$.Conversely, suppose $\{i_n\}$ be bounded

$$\lim_{n\to\infty} d(Ti_n, i_n) = 0.$$

Consider $q \in (c, \{i_n\})$ we get

$$\begin{aligned} r(Tq, \{i_n\}) &= \lim_{n\to\infty} \sup d(i_n, Tq) \\ &\leq \lim_{n\to\infty} \sup[3d(i_n - Tq + d(i_n, q)] \\ &\leq \lim_{n\to\infty} \sup d(i_n, q) \\ &= r(q, \{i_n\}) \\ \Longrightarrow Tq &\in A(K, \{i_n\}). \end{aligned}$$

So X be a uniformly convex, $A(K, \{i_n\})$ be a singleton, so we get the results $Tq = q$, $F(T) \neq \phi$. Hence proved the theorem. □

Theorem 2.6. *Suppose that K is any non-empty closed convex subset for a uniformly convexity for metric space X, also having opial property, consider that $T : K \to K$ is any mapping which satisfies the condition (C). By arbitrary chosen $a_0 \in C$, consider that a sequence $\{a_n\}$ is generated from (5) $\forall\ n \geq 1$, $\{\delta_n\}$ also $\{\eta_n\}$ are two different sequences having real numbers with $[i, j]$ by some i, j along with $0 < i \leq j < 1$ such that $F(T) \neq \phi$. So, $\{i_n\}$ converges weakly to any fixed point for T.*

Proof. As $F(T) \neq \phi$, so from Theorem 2.5 it is obvious that $\{a_n\}$ is not only bounded and $lim_{n\to\infty} d(Ta_n, a_n) = 0$. As X be uniformly convex so by reflexive, from Eberlin's theorem $\exists$ a subsequence $\{a_{nj}\}$ of $\{a_n\}$ which converges weakly to some points $q_1 \in X$. So C is closed and convex, by Mazur's theorem $q_1 \in C$ and using Lemma 2.1, $q_1 \in F(T)$. At once, we prove the $\{a_n\}$ converges weakly by q_1. Actually, if this is false, so there may be have a subsequence $\{a_{nk}\}$ for $\{a_n\}$, $\{a_{nk}\}$ converges weakly for $q_2 \in C$ also $q_2 \neq q_1$. From Lemma 1.1, $q_2 \in F(T)$. So $lim_{n\to\infty} d(a_n, p)$ exists $\forall\ p \in F(T)$. From Theorem 2.5 and by Opial's property, we get the results

$$\begin{aligned} \lim_{n\to\infty} \inf d(a_n, q_1) &= \lim_{j\to\infty} \inf d(a_{nj}, q_1) \\ &< \lim_{j\to\infty} \inf d(a_{nj}, q_2) \\ &= \lim_{n\to\infty} \inf d(a_n, q_2) \\ &= \lim_{k\to\infty} \inf d(a_{nk}, q_2) \\ &< \lim_{k\to\infty} \inf d(a_{nk}, q_1) \\ &= \lim_{n\to\infty} \inf d(a_n, q_1), \end{aligned}$$

which is contradiction. So $q_1 = q_2$. $\Longrightarrow \{a_n\}$ converges weakly to a fixed point for T. □

Theorem 2.7. *Suppose that K is a non-empty compact closed convex subset for a uniformly convex metric space X, also consider $T : K \to K$ is mapping which satisfies the condition (C). By arbitrary chosen $m_0 \in K$, consider the sequence $\{m_n\}$ is generated from (5) $\forall$, $n \geq 1$, also $\{\delta_n\}$ and $\{\eta_n\}$ are two sequences of real numbers in $[i, j]$ by some i, j having condition $0 < i \leq j < 1$. Therefor, $\{m_n\}$ converges strongly to fixed point for T.*

Proof. From Lemma 1.2, take $F(T) \neq \phi$ also using Theorem 2.5 we obtained the results $lim_{n\to\infty} d(Tm_n, m_n) = 0$. Then K is compact, so $\exists$ any subsequence $\{m_{nk}\}$ for $\{m_n\}$ and $\{m_{nk}\}$ converges strongly to q and $q \in K$. By Proposition 1.1(iii), we get

$$d(m_{nk} - Tq) \leq 3d(Tm_{nk}, m_{nk}) + d(m_{nk}, q), \forall n \geq 1.$$

Assume that $k \to \infty$, also we obtained $Tq = q$, and i.e., $q \in F(T)$. Since, from Lemma 2.1, $lim_{n\to\infty} d(m_n, q)$ hold for every $q \in F(T)$, then m_n converge strongly to q. Senter and Dotson established a notation for a mappings which satisfy the condition (I) . A mapping $T : K \to K$ is knwon as to satisfy condition (I), if $\exists$ an increasing function $f : [0, \infty) \to [0, \infty)$ along with $f(0) = 0$ and $f(r) > 0\ \forall\ r > 0$ and $d(m, Tm) \geq f(d(m, F(T)))\ \forall, m \in K$, also $d(m, F(T)) = \inf_q \in F(T) d(m, q)$. □

Theorem 2.8. *Suppose that K is any non-empty closed convex subset for uniformly convex metric space X, also consider that $T : K \to K$ be any mapping which satisfy condition (C). By arbitrary chosen $i_0 \in K$, and consider that sequence $\{i_n\}$ is generated by (5) $\forall\ n \geq 1$, ,$\{\alpha_n\}$ and $\{\beta_n\}$ are two different sequences having real numbers along with $[l, m]$ for some l, m with $0 < l \leq m < 1$ such that $G(T) \neq \phi$. If T satisfy condition (I), so $\{i_n\}$ converges strongly to fixed point T.*

Proof. From Lemma 2.1, we obtained the $lim_{n\to\infty} d(i_n, q)$ holds $\forall\ q \in G(T)$ and $lim_{n\to\infty} d(i_n, G(T))$ hold. Assume that $lim_{n\to\infty} d(i_n, q) = r$ for $0 \leq r$ if $r = 0$ so we attain following results. Suppose that $0 < r$, by Proposition 1.1 and condition (I),

$$f(d(i_n, G(T))) \leq d(Ti_n, i_n). \tag{31}$$

So $G(T) \neq \phi$, so from Theorem 2.6, we get

$$\lim_{n\to\infty} d(Ti_n, i_n) = 0. \tag{32}$$

So (31) implies that

$$\lim_{n\to\infty} f(d(i_n, G(T))) = 0.$$

So f is increasing function, so from (32), we get

$$\lim_{n\to\infty} d(i_n, G(T)) = 0.$$

So, we get the subsequence $\{i_{nk}\}$ of $\{i_n\}$ and a sequence $\{j_k\} \subset G(T)$

$$d(i_{nk}, j_k) < \frac{1}{2^k},$$

for all $k \in N$, so using Lemma 2.1, from (25) we get

$$\begin{aligned}
d(i_{nk+1}, j_k) &\leq d(i_{nk}, j_k) < \frac{1}{2^k} \\
d(j_{k+1}, j_k) &\leq d(j_{k+1}, i_{k+1}) + d\,(i_{k+1}, j_k) \\
&\leq \frac{1}{2^{k+1}} + \frac{1}{2^k}, \\
\frac{1}{2^{k-1}} &\to 0 \text{ as } k \to \infty.
\end{aligned}$$

It is proved that $\{j_k\}$ are Cauchy sequence in $G(T)$ and it also converges to any point q. Since $G(T)$ be closed, so, $q \in G(T)$ also $\{i_{nk}\}$ converges strongly to p. So $lim_{n\to\infty} d(i_n, q)$ exists, we have $i_n \to q \in G(T)$. Hence proved. □

Acknowledgments

This study was supported by the Taif University Researchers Supporting Project (No. TURSP 2020/17), Taif University, Taif, Saudi Arabia.

Conflict of interest

The authors declare no conflict of interest.

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