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Research article

Stability analysis of COVID-19 outbreak using Caputo-Fabrizio fractional differential equation

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Abstract: The main aim of this paper is to construct a mathematical model for the spread of SARS-CoV-2 infection. We discuss the modified COVID-19 and change the model to fractional order form based on the Caputo-Fabrizio derivative. Also several definitions and theorems of fractional calculus, fuzzy theory and Laplace transform are illustrated. The existence and uniqueness of the solution of the model are proved based on the Banach's unique fixed point theory. Moreover Hyers-Ulam stability analysis is studied. The obtained results show the efficiency and accuracy of the model.

Keywords: Hyers-Ulam stability; variable Caputo-Fabrizio fractional derivative; Banach fixed point theorem; unit step function; existence and uniqueness
Mathematics Subject Classification: 26A33, 34A08, 65L07, 92D25, 34K20

1. Introduction

For the first time COVID-19 was diagnosed in Wuhan, China, in December 2019. The virus gene testing reveals that it is a beta coronavirus directly associated with SARS. A functional COVID-19 case is defined as a person who has exhibited clinical manifestations which are diagnostic of COVID-19 [1–4]. This infection is widespread and had an impact on the patients. Many researchers are investigating the effects of COVID-19 in a range of contexts. In the field, there are still some mathematical models that claim to represent the process of COVID-19 growth. COVID-19 is used by several studies as a feature extraction method (see [5–12, 30]).

A significant method for controlling the pandemic among the population is infectious model research. Also studding mathematical models of fractional orders, solving the models and analysis of results are among challenging problems. Non-singular kernels such as Caputo-Fabrizio (Short Denote: $\mathcal{CF}$) fractional derivatives had also been included in the fractional function formulation. A whole effect of memory is described by utilizing a non-singular kernel and other benefits achieved with the modern definition known as $\mathcal{CF}$ fractional derivative.

Fractional calculus arose from a question posed to Leibnitz by L'Hospital concerning his generalization of the meaning of the notation $\frac{d^n y}{dx^n}$ for the derivative of order n whn $n = \frac{1}{2}$. In his reply, dated September 30, 1695, Leibnitz wrote to L'Hospital, "This is an apparent paradox from which one-day useful consequences will be drawn".

Recent researches have focused generally on fractional differential equations, and there has been significant progress in this field. This concept has existed for a long time and is nearly as old as differential equations. Fractional order differential equations have recently proven to be a useful problem in many fields of science and engineering [31–33]. Indeed, electromagnetic technology has a lot of applications in electrochemistry, regulation, etc. In recent years, a lot of researchers have contributed to fractional derivative operators for various differential equations (see [34–39]). In 2021, Khan et al. were concerned with the existence of results and stability analysis for a nabla discrete ABC fractional COVID-19 [7, 14–16]. In the same year, Li et al. concerned a vigorous study of the fractional order COVID-19 model in ABC derivatives. In 2022, Khan et al. [43] studied a COVID-19 model in the fractal-fractional sense of operators for the existence of a solution in Ulam stability and related papers (see also [13, 17, 19, 20, 40–42, 44]).

The main objective of this study is to define the model variables that have the highest impact on early disease transmission when vaccination and medication are used. As far as we are aware, this paper offers the first comprehensive mathematical characterization of the qualitative dynamics of COVID-19 with an ineffective vaccine and treatment by the variable fractional derivative, which follows the $\mathcal{CF}$ fractional derivative. We also discussed and theoretically evaluate a mathematical model of the COVID-19 transmission mechanism, combining important dynamics of the illness with two important treatment measures: immunization of susceptible people and recovery/treatment of afflicted people. Along with more unique and original innovations for the intended COVID-19 system, applying Banach's fixed point idea, the existence and uniqueness of the solution are discussed to prove the Hyers-Ulam stability of the innovative COVID-19 model.

Most of them have mentioned that "the main advantage of this kind of operator is that the singular power-law kernel is now replaced by a non-singular kernel," which is easier to use in theoretical analysis, numerical calculations, and real-world applications. But we believe that the singular power-law kernel is very easy to use in the mentioned above calculations and applications.

2. Preliminaries

In this section, we present some basic definitions and lemmas in $\mathcal{CF}$ which used throughout the paper.

Definition 2.1. A fuzzy number Υ is defined on a set of real numbers which satisfies the following properties,

(i) Υ is convex, i.e., $\Upsilon[\beta a_1 + (1-\beta)a_2] \geq min[\Upsilon(a_1), \Upsilon(a_2)]$, for all $a_1, a_2 \in \mathbb{R}$ and $\beta \in [0,1]$,
(ii) Υ is normal i.e., there exists an $a \in \mathbb{R}$ such that $\Upsilon(a)$=1,
(iii) Υ is piecewise continuous.

Definition 2.2. Let $x, y \in \mathbb{M}$ and $\beta \in \mathbb{R}$, then the α-cut set of fuzzy number is closed and bounded interval $x = (\underline{x}(\alpha), \overline{x}(\alpha))$, $y = (\underline{y}(\alpha), \overline{y}(\alpha))$ and $\beta > 0$. Then the operation on fuzzy numbers,

(i) **Addition:** $x \oplus y = (\underline{x}(\alpha) + \underline{y}(\alpha), \overline{x}(\alpha) + \overline{y}(\alpha))$.
(ii) **Subraction:** $x \ominus y = (\underline{x}(\alpha) - \underline{y}(\alpha), \overline{x}(\alpha) - \overline{y}(\alpha))$.
(iii) **Scalar multiplication:**

$$k.v = \begin{cases} (\beta\underline{x}, \beta\overline{x}), & \lambda \geq 0, \\ (\beta\underline{x}, \beta\overline{x}), & \lambda < 0, \end{cases} \tag{2.1}$$

if $\beta = -1$ then $\beta \odot x = -x$.

Definition 2.3. Consider mapping $\Gamma : M \times M \to \mathbb{R}$ and $\Gamma(x,y) = sup_{\zeta \in [0,1]} max\{|\underline{x}(\zeta) - \underline{y}(\zeta)\ |, |\overline{x}(\zeta) - \overline{y}(\zeta)\ |\}$, be the Hausdorff distance between fuzzy number. where, $[x]^{\zeta} = [\underline{x}(\zeta), \overline{x}(\zeta)]$ and $[y]^{\zeta} = [\underline{y}(\zeta), \overline{y}(\zeta)]$.
Then Γ is a metric in M and following criteria.
(i) $\Gamma(x+z, y+z) = \Gamma(x,y)$, for all $x, y, z \in M$,
(ii) $\Gamma(kx, ky) = |k|\Gamma(x,y)$, for all $k \in \mathbb{R}, x, y, z \in M$,
(iii) $\Gamma(x+y, z+H) \leq \Gamma(x,z) + \Gamma(y,H)$, for all $x, y, z, H \in M$.
Then (Γ, M) is complete metric space.

Definition 2.4. Let $\mathbb{A}$ and $\mathbb{B}$ be two fuzzy sets with $C \subseteq \mathbb{R}$ and consider a two variable function $M : \mathbb{A} \times \mathbb{B} \to C$. Let $\mu_{\mathbb{A}}(a), \mu_{\mathbb{B}}(b)$ and $\mu_C(c)$ be their associate member function,

(i) **In addition:** Let $C = M(a,b) = a+b$. Then $C = \{c | c = a+b; a \in \mathbb{A}, b \in \mathbb{B}\}$ and $\mu_C(c) = \vee_{c=a+b} = \{\mu_{\mathbb{A}}(a) \wedge \mu_{\mathbb{B}}(b)\}$.
(ii) **In α-cut notation:**
$(C)_\alpha = M[(\mathbb{A})_\alpha, (\mathbb{B})_\alpha]$, then $(c)_\alpha = (\mathbb{A})_\alpha + (\mathbb{B})_\alpha$.
For real numbers S_1 and S_2, $S_1 \wedge S_2 = min\{S_1, S_2\}$ and $S_1 \vee S_2 = max\{S_1, S_2\}$.

Definition 2.5. The Laplace transform of fuzzy valued function $f(t)$ is defined as

$$F(S) = L[f(t)] = \int_0^\infty e^{-st} f(t) = \lim_{h \to \infty} \int_0^h e^{-st} f(t) dt.$$

Definition 2.6. The unit step function which is called Heavyside's unit function is defined as

$$\mu(t) = \begin{cases} 0 & if \quad t < 0, \\ 1 & if \quad t \geq 0. \end{cases}$$

Definition 2.7. The variable caputo derivative of function $f(t) \in [0,1)$ is defined as

$$ {}_{0}^{CF}D_t^{\varphi(t)} f(t) = \frac{1}{\Gamma(1-\varphi(t))} \int_0^t \frac{1}{(t-x)^{\varphi(t)}} f^{'}(x)dx. \tag{2.2}$$

Definition 2.8. For $f(t) \in H^1([0,T])$, and $\varphi(t) \in [0,1)$, the $\varphi(t)th$-order variable of $C\mathcal{F}$ derivative of $f(t)$ in the Caputo sense is

$$ {}_{0}^{CF}D_t^{\varphi(t)} f(t) = \frac{\mathbb{M}(\varphi(t))}{1-\varphi(t)} \int_0^t e^{\left(\frac{-\varphi(t)(t-a)}{1-\varphi(t)}\right)} f^{'}(x)dx, \tag{2.3}$$

where $H^1([0,T])$ is Hilbert space, the value $\mathbb{M}(\varphi(t))$ is a normalising function in Eq (2.3), i.e., $\mathbb{M}(0) = \mathbb{M}(1) = 1$, then

$$ {}_{0}^{CF}D_t^{\varphi(t)} G = 0, \tag{2.4}$$

which G is constant.

Definition 2.9. The variable fractional integral of non-singular kernel type is stated as follows

$$ {}_{0}^{CF}J_t^{\varphi(t)} f(t) = \frac{(1-\varphi(t))}{\mathbb{M}(\varphi(t))} f(t) + \frac{\varphi(t)}{\mathbb{M}(\varphi(t))} \int_0^t f(x)dx, \tag{2.5}$$

where $0 < \varphi(t) \leq 1$.

Definition 2.10. Suppose $\gamma \in \mathbb{R}$ is a solution of equation with non-negative initial condition. For $\gamma(0) = \gamma_0$ and there is $\Phi[t, \gamma(t)] \in N^F([0,v] \times \mathbb{R})$ for $t \in [0,v]$ then we get

$$ \Psi[t,\gamma(t)] = \frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))} \odot \Phi[t,\gamma(t)] \oplus \frac{\Xi(t)}{\mathbb{M}(\Xi(t))} \int_0^t \Phi(\aleph, \gamma(\aleph)) d\aleph, \tag{2.6}$$

with

$$ \gamma(t) = \gamma_0 + {}_{0}^{CF}J_t^{\Xi(t)}[\Phi(t,\gamma(t))]. \tag{2.7}$$

Definition 2.11. Let $\Xi^* = (S_1 \wedge S_2)_{t\in[0,v]}\{\Xi(t), t \in [0,v]\}$ and $\Xi^{**} = (S_1 \vee S_2)_{t\in[0,v]}\{\Xi(t), t \in [0,v]\}$ be the minimum and maximum values of the variable fractional order $\Xi(t)$ on $[0,v]$ then we get

$$ ||\Phi(t,\gamma_1(t)) - \Phi(t,\gamma_2(t))|| \leq \iota ||\gamma_1 - \gamma_2||. $$

Definition 2.12. Let $\epsilon > 0$ and $\gamma \in \mathbb{R}$ be the solution of the following inequality. Then the equation is Hyers-Ulam stable if

$$ ||{}_{0}^{CF}D_t^{\Xi(t)}\gamma(t) - \Phi(t,\gamma(t))|| \leq \epsilon, \tag{2.8}$$

for $t \in [0,v]$ and there is unique solution $\gamma_2(t)$ with as $J_k > 0$ that it follows

$$ ||\gamma(t) - \gamma_2(t)|| \leq J_k\epsilon;\ t \in [0,v] \tag{2.9}$$

is said to be Hyers-Ulam-stable.

Definition 2.13. Let $\epsilon > 0$ and $\gamma \in \mathbb{R}$ be the solution of the following inequality. Then the equation is Hyers Ulam Rassias-stable.

$$||{}_0^{CF}D_t^{\Xi(t)}\gamma(t) - \Phi(t, \gamma(t))|| \leq \Omega(t)\epsilon, \tag{2.10}$$

for $t \in [0, v]$ and there is a unique solution $\gamma_2(t)$ with $J_k > 0$ that it follows

$$||\gamma(t) - \gamma_2(t)|| \leq J_k\Omega(t)\epsilon; \; t \in [0, v] \tag{2.11}$$

is said to be Hyers-Ulam-Rassias stable.

Lemma 2.14. *Suppose $f(t) \in C([0, T])$, then the fractional differential equation with variable $\mathcal{CF}$ is*

$${}_0^{CF}D_t^{\varphi(t)}f(t) = \varrho(t), \; t \in [0, T], \;\; o < \varphi(t) \leq 1, \tag{2.12}$$

$f(0) = f_0, \varrho_0 \in \mathbb{R}$ and then

$$f(t) = \varrho_0 + \frac{(1 - \varphi(t))}{\mathbb{M}(\varphi(t))}\varrho(t) + \frac{\varphi(t)}{\mathbb{M}(\varphi(t))}\int_0^t \varrho(x)dx. \tag{2.13}$$

Theorem 2.15. *(Arzela-Ascoli theorem [21]): Let X be a compact metric space. Let $C(X, R)$ be given the sup norm metric. Then a set $\mathbb{M}(C, X)$ is compact iff $\mathbb{M}$ is bounded, closed and equicontinuous.*

3. Classical COVID-19 model

In this paper, we cover the COVID-19 model which was chosen recently [22], as well as the parameters and covariants used in the model. Assume a demographic with homogeneous mixing, which signifies any living thing that has a fair probability of colliding with itself. But using a causal physically based proposed model to categorize effectively evaluation, the current population is segmented up into various infections and disease asserts related to individual overall health at any time $'t'$ with $S(t)$ is susceptible, $V(t)$ is vaccinated, $E(t)$ is exposed, $I(t)$ is symptoms in patients highly infections people, $A(t)$ is infected latent, $H(t)$ is seriously injured, $R(t)$ is regained.

$$\begin{aligned}
dS/dt &= [1 - p]\pi + \eta R - (\beta_S + \mu + v)S, \\
dV/dt &= p\pi + vS - (\beta_V + \mu)V, \\
dE/dt &= \beta_S S + \beta_V V - (\sigma + \mu)E, \\
dI/dt &= \sigma\psi E + \lambda(1 - \phi)A - (\gamma + \mu + \delta)I, \\
dA/dt &= \sigma(1 - \psi)E - (\lambda + \mu)A, \\
dH/dt &= \gamma(1 - k)I - (\tau + \mu + \delta)H, \\
dR/dt &= \gamma kI + \lambda\phi A + \tau H - (\eta + \mu)R,
\end{aligned} \tag{3.1}$$

with initial conditions,

$$\begin{aligned}
S(t_0) &= S_0, \; V(t_0) = V_0, \; E(t_0) = E_0, \\
I(t_0) &= I_0, \; A(t_0) = A_0, \; H(t_0) = H_0, \\
R(t_0) &= R_0.
\end{aligned} \tag{3.2}$$

Susceptible people are infected after having close contact with symptomatic, asymptomatic, and patient people. We assume that disease transmission from asymptomatic to immunocompromised patients is lower than from symptomatic and hospitalized people whereas for a brief significant period, the COVID-19 pandemic which began in December 2019, is still ongoing. Consider σ escape rate from the exposed class, where a minority of the individuals is infected while the other $(1-\psi)$ becomes asymptotic. The asymptomatic class has an abandonment rate of λ. Natural death at the rate of μ. Even these improving diagnoses and traveling to the undiagnosed class at the rate of $(1-\phi)$ and the fraction ϕ struggling to recover normally from the pathogens and traveling to the obtained class R are all influences that limit the number of patients. Departure from the sick class γ with a percentage $(1-k)$ of one being detained and other healing spontaneous.

People get attracted through into society at the pace of π with a portion of p being immunized and the rest $(1-p)$ exposed. A few are vaccines at a much higher rate than the other v. The infected persons can also become introduced to the illness $b(1-\epsilon)$ if the antibiotic is imperfect. Just a remotely effective treatment has already begun to promote a pure product to the SARS-COV-2 virus. The variable reflects the new vaccine's success.

Non-pharmacological use such as self-quarantine of positive samples, isolation, face shields, wash hands, and social distancing, but the most limiting lockdowns, closures, or restricted openings of shops and schools were depended on and proceeded to be broadly applied before the access of pharma initiatives such as diagnosis and immunization.

4. The fuzzy variable fractional differential equation COVID-19 model

This chapter discusses the COVID-19 model with variable $\mathcal{CF}$ fractional derivative and the fuzzy approach

$$\begin{aligned}
{}_{0}^{CF}D_{t}^{\Xi(t)}S(t) &= (1-p)\pi-\eta R+(\beta_S+\mu+v)S,\\
{}_{0}^{CF}D_{t}^{\Xi(t)}V(t) &= p\pi+vS-(\beta_V+\mu)V,\\
{}_{0}^{CF}D_{t}^{\Xi(t)}E(t) &= \beta_S S+\beta_V V-(\sigma+\mu)E,\\
{}_{0}^{CF}D_{t}^{\Xi(t)}I(t) &= \sigma\psi E+\lambda(1-\phi)A-(\gamma+\mu+\delta)I,\\
{}_{0}^{CF}D_{t}^{\Xi(t)}A(t) &= \sigma(1-\psi)E-(\lambda+\mu)A,\\
{}_{0}^{CF}D_{t}^{\Xi(t)}H(t) &= \gamma(1-k)I-(\tau+\mu+\sigma)H,\\
{}_{0}^{CF}D_{t}^{\Xi(t)}R(t) &= \gamma kI+\lambda\phi A+\tau H-(\eta+\mu)R.
\end{aligned} \tag{4.1}$$

Here Ξ (t) denotes indicates for the variable $\mathcal{CF}$ fractional order and the initial conditions accompanying,

$$\begin{aligned}
S(t_0) &= S_0 \geq 0,\\
V(t_0) &= V_0 \geq 0,\\
E(t_0) &= E_0 \geq 0,\\
I(t_0) &= I_0 \geq 0,\\
A(t_0) &= A_0 \geq 0,\\
H(t_0) &= H_0 \geq 0,\\
R(t_0) &= R_0 \geq 0.
\end{aligned} \tag{4.2}$$

Take a look at the right side of the proposal model in Eq (4.1). Now let us continue with the form $\Phi_j[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)] = \Phi_j(*)$, $j = 1, 2, ...7$.

$$\begin{aligned}
\Phi_1(*) &= {}_0^{CF} D_t^{\Xi(t)} S(t) = (1-p)\pi - \eta R + (\beta_S + \mu + v)S, \\
\Phi_2(*) &= {}_0^{CF} D_t^{\Xi(t)} V(t) = p\pi + vS - (\beta_V + \mu)V, \\
\Phi_3(*) &= {}_0^{CF} D_t^{\Xi(t)} E(t) = \beta_S S + \beta_V V - (\sigma + \mu)E, \\
\Phi_4(*) &= {}_0^{CF} D_t^{\Xi(t)} I(t) = \sigma\psi E + \lambda(1-\phi)A - (\gamma + \mu + \delta)I, \\
\Phi_5(*) &= {}_0^{CF} D_t^{\Xi(t)} A(t) = \sigma(1-\psi)E - (\lambda + \mu)A, \\
\Phi_6(*) &= {}_0^{CF} D_t^{\Xi(t)} H(t) = \gamma(1-k)I - (\tau + \mu + \sigma)H, \\
\Phi_7(*) &= {}_0^{CF} D_t^{\Xi(t)} R(t) = \gamma k I + \lambda\phi A + \tau H - (\eta + \mu)R.
\end{aligned} \tag{4.3}$$

Here $\Phi_j(*)$, $j = 1, 2...7$ are fuzzy functions for $\Xi(t) \in (0, 1]$. Then Eq (4.1) modified as,

$$\gamma(t) = \gamma(0) + {}_0^{CF} J_t^{\Xi(t)}[\Phi(t, \gamma(t))], \tag{4.4}$$

$\Xi(t) \in (0, 1]$. Here, by using initial condition ${}_0^{CF} J_t^{\Xi(t)}$, we transform Eq (4.2) into integral equations used to solve the problem. We convert Eq (4.1) into the integral equations as shown below,

$$\begin{aligned}
{}_0^{CF} D_t^{\Xi(t)}(S(t)) &= \Phi_1(*), \\
{}_0^{CF} D_t^{\Xi(t)}(V(t)) &= \Phi_2(*), \\
{}_0^{CF} D_t^{\Xi(t)}(E(t)) &= \Phi_3(*), \\
{}_0^{CF} D_t^{\Xi(t)}(I(t)) &= \Phi_4(*), \\
{}_0^{CF} D_t^{\Xi(t)}(A(t)) &= \Phi_5(*), \\
{}_0^{CF} D_t^{\Xi(t)}(H(t)) &= \Phi_6(*), \\
{}_0^{CF} D_t^{\Xi(t)}(R(t)) &= \Phi_7(*).
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
S(t) &= S_0 + {}_0^{CF} J_t^{\Xi(t)}(\Phi_1(*)), \\
V(t) &= V_0 + {}_0^{CF} J_t^{\Xi(t)}(\Phi_2(*)), \\
E(t) &= E_0 + {}_0^{CF} J_t^{\Xi(t)}(\Phi_3(*)), \\
I(t) &= I_0 + {}_0^{CF} J_t^{\Xi(t)}(\Phi_4(*)), \\
A(t) &= A_0 + {}_0^{CF} J_t^{\Xi(t)}(\Phi_5(*)), \\
H(t) &= H_0 + {}_0^{CF} J_t^{\Xi(t)}(\Phi_6(*)), \\
R(t) &= R_0 + {}_0^{CF} J_t^{\Xi(t)}(\Phi_7(*)).
\end{aligned} \tag{4.6}$$

Again, we modify Eq (4.6) into,

$\gamma(t)$	$\gamma(0)$	$\Phi(t, \gamma(t))$
$S(t)$	S_0	$\Phi_1[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)]$
$V(t)$	V_0	$\Phi_2[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)]$
$E(t)$	E_0	$\Phi_3[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)]$
$I(t)$	I_0	$\Phi_4[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)]$
$A(t)$	A_0	$\Phi_5[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)]$
$H(t)$	H_0	$\Phi_6[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)]$
$R(t)$	R_0	$\Phi_7[t, S(t), V(t), E(t), I(t), A(t), H(t), R(t)]$

Theorem 4.1. *If we assume that* $\gamma \in \mathbb{R}$*, then the Eq (4.6) of the solution is*

$$ {}_0^{CF}D_t^{\Xi(t)}[\gamma(t))] = \Phi(\aleph, \gamma(\aleph)), t \in [0, v], 0 < \Xi(t) \le 1, \tag{4.7}$$

$\gamma(0) = \gamma_0, \gamma_0 \in \mathbb{R}$ *with,*

$$\gamma(t) = \gamma(0) +{}_0^{CF} J_t^{\Xi(t)}[\Phi(t, \gamma(t))]. \tag{4.8}$$

Proof. By using the initial conditions (4.2), Theorem 4.1 can be proved [17, 18]. □

Theorem 4.2. *The operator (2.6) satisfies the Banach fixed point theory with the hypothesis if and only if*

$$\iota\left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)} + \frac{\Xi^* v}{\mathbb{M}(\Xi^*)}\right] < 1. \tag{4.9}$$

Proof. Assume that $\gamma_1, \gamma_2 \in \mathbb{R}$

$$\begin{aligned}
\|\Psi\gamma_1 - \Psi\gamma_2\| &\le \|\Psi_1\gamma_1 - \Psi_1\gamma_2\| + \|\Psi_2\gamma_1 - \Psi_2\gamma_2\| \\
&\le \left\|\left[\frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))} \odot \Phi[t, \gamma_1(t)] \oplus \frac{\Xi(t)}{\mathbb{M}(\Xi(t))} \int_0^t \Phi(\aleph, \gamma_1(\aleph)) d\aleph - \right.\right. \\
&\left.\left. \frac{1-\Xi(t)}{\mathbb{M}(\Xi(t))} \odot \Phi[t, \gamma_2(t)] \oplus \frac{\Xi(t)}{\mathbb{M}(\Xi(t))} \Phi(\aleph, \gamma_2(\aleph)) d\aleph\right]\right\|, \\
&\le \left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)}\iota + \frac{\Xi^*}{\mathbb{M}(\Xi^*)}\iota v\right] (S_1 \wedge S_2)_{t\in[0,v]} |\gamma_1 - \gamma_2| \\
&\le \iota\left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)} + \frac{\Xi^*}{\mathbb{M}(\Xi^*)} v\right] \|\gamma_1 - \gamma_2\|.
\end{aligned}$$

The operator Φ in Eq (2.6) satisfying the Banach fixed point theorem, depending on the inequality (4.9), and using Eqs (4.4) and (4.8) satisfies

$$\gamma_q(t) = \frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))} \odot \Phi[t, \gamma_q(t)] \oplus \frac{\Xi(t)}{\mathbb{M}(\Xi(t))} \int_0^t \Phi(\aleph, \gamma_q(\aleph)) d\aleph. \tag{4.10}$$

□

Theorem 4.3. *The following Eqs (4.2)–(4.6) with $\mathcal{CF}$ in fuzzy variable fractional integral equation (FVFIE) has unique solution*

$$\iota\left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)} + \frac{\Xi^* v}{\mathbb{M}(\Xi^*)}\right] < 1. \tag{4.11}$$

Proof. Assume that there is another solution for the $\mathcal{CF}$ in FVFIE (4.2)–(4.6), which is solution of $(\gamma_1(t))$.

$$\|\gamma_1(t)-\gamma_2(t)\| \leq \left\|\left[\frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))}\odot\Phi[t,\gamma_1(t)]\oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\int_0^t \Phi(\aleph,\gamma_1(\aleph))d\aleph - \right.\right.$$
$$\left.\left.\frac{1-\Xi(t)}{\mathbb{M}(\Xi(t))}\odot\Phi[t,\gamma_2(t)]\oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\Phi(\aleph,\gamma_2(\aleph))d\aleph\right]\right\|,$$
$$\leq \frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))}\odot|\Phi[t,\gamma_1(t)-\Phi[t,\gamma_2(t)]|\oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\left|\int_0^t \Phi[t,\gamma_1(t)-\Phi[t,\gamma_2(t)]\right|$$
$$\leq \iota\left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)}+\frac{\Xi^* v}{\mathbb{M}(\Xi^*)}\right]\|\gamma_1(t)-\gamma_2(t)\|.$$

Additionally, this indicates that

$$\iota\left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)}+\frac{\Xi^* v}{\mathbb{M}(\Xi^*)}\right]\|\gamma_1(t)-\gamma_2(t)\| \geq 0. \tag{4.12}$$

The Eqs (4.11) and (4.12) is relates $\|\gamma_1(t)-\gamma_2(t)\| = 0$, then

$$\|\gamma_2(t), \frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))}\odot\Phi[t,\gamma_2(t)]\oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\int_0^t \Phi(\aleph,\gamma_2(\aleph))d\aleph\|, \tag{4.13}$$

there exists $\gamma(t)$ which satisfies,

$$\gamma(t) = \frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))}\odot\Phi[t,\gamma(t)]\oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\int_0^t \Phi(\aleph,\gamma(\aleph))d\aleph, \tag{4.14}$$

i.e., $\gamma_1(t) = \gamma_2(t)$. The $\mathcal{CF}$ in FVFIE (4.2)–(4.6) is a unique solution. As a result $\mathcal{CF}$ with fuzzy variable fractional differential equation COVID-19 model has a unique solution. □

5. Hyers-Ulam stability analysis of the COVID-19 model

Theorem 5.1. *The solution of the Eqs (4.2)–(4.6) is Ulam-Hyers stable with $\mathcal{CF}$ with FVFIE, under the Theorem 4.3.*

$$\iota\left[\frac{1-\Xi^*}{\mathbb{M}(\Xi^*)}+\frac{\Xi^* v}{\mathbb{M}(\Xi^*)}\right] < 1. \tag{5.1}$$

Proof. By $\mathcal{CF}$ with FVFIE (4.2)–(4.6) has a unique solution from Theorem 4.3 and relates with (4.14)

$$\begin{aligned}
|\gamma(t)-\gamma_2(t)| \leq & \left\|\left[\frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))}\odot\Phi[t,\gamma(t)]\oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\int_0^t\Phi(\aleph,\gamma(\aleph))d\aleph - \right.\right.\\
&\left.\left.\frac{1-\Xi(t)}{\mathbb{M}(\Xi(t))}\odot\Phi[t,\gamma_2(t)]\oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\Phi(\aleph,\gamma_2(\aleph))d\aleph\right]\right\|\\
\leq & \frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))}\odot|\Phi(t,\gamma(t))-\Phi(t,\gamma_2(t))|\\
& \oplus\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\left[\int_0^t|\Phi(t,\gamma(t))-\Phi(t,\gamma_2(t))|\right]\\
\leq & \iota\frac{(1-\Xi(t))}{\mathbb{M}(\Xi(t))}\odot|\gamma(t)-\gamma_2(t)|\oplus\iota\frac{\Xi(t)}{\mathbb{M}(\Xi(t))}\left[\int_0^t|\gamma(t)-\gamma_2(t)|\right]\\
\leq & \iota\left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)}+\frac{\Xi^*\nu}{\mathbb{M}(\Xi^*)}\right]\|\gamma(t))-\gamma_2(t)\|\\
|\gamma(t)-\gamma_2(t)| \leq & J_k\epsilon.
\end{aligned}\tag{5.2}$$

Where $\epsilon=\iota\left[\frac{(1-\Xi^*)}{\mathbb{M}(\Xi^*)}+\frac{\Xi^*\nu}{\mathbb{M}(\Xi^*)}\right]$ *and* $J_k=\|\gamma(t)-\gamma_2(t)\|$. From Eq (5.2) is said to be Hyers-Ulam stable with $\mathcal{CF}$ in FVFIE (4.2) – (4.6) in COVID-19 model and the $\mathcal{CF}$ with fuzzy variable fractional differential equation COVID-19 model has Hyers-Ulam stable. □

Numerical simulations and discussion

Numerical simulations are performed to demonstrate the theoretical findings. Table 1 shows the model input variables for the numerical simulations as well as their source and description. We examined the effect of the vaccine and treatment of COVID-19 spreading [30]. Due to natural and seasonal changes, potential measurement flaws, and the fact that mathematical models are representations of complex biological systems, some model parameter values are not always known with certainty. We determine the relative significance of model parameters to disease transmission and graphically display how sensitive the functional reproduction number is to model parameters to show how changes in model parameter values affect R. Timely rating of prevention strategies and other hyperparameters based on their impact may ideally potentially enable policy and outcome prioritize public health intervention measures to be implemented, focusing attention on the most effective interventions. Similarly, the treatment rate and the effective contact rate should be kept as low as possible to limit COVID-19 spread in the environment. Now we discuss the obtained numerical outcomes of the governing method. Let us assume that the initial condition is taken as $S=50$, $V=20$, $E=30$, $I=100$, $A=45$, $H=200$, $R=300$ with parameters given in Table 1.

Table 1. Parameter and bilogical interpretations.

Parameter	Descriptions	Value	Reference
π	People are getting hired at a quick speed into the population.	$\frac{10000}{59\times365}$	[23, 24]
p	Average fraction of those that were inoculated from recruited.	0.0001	Assumed
v	Inoculation completion.	0.4	Assumed
μ	Risk of mortality due to the extreme weather event.	$\dfrac{1}{59\times365}$	[23, 24]
δ	Risk of morality illness.	0.018	[25, 26]
ϵ	Vaccinated people got serious illness.	0.8	Assumed
σ	A vulnerable groups departure rate.	0.13	[27]
γ	Departure rate in the highly contagious group.	0.0833	[26]
k	A portion of patients that heal gradually.	0.05	[28]
ψ	Primary infection in a low minority those who are vaccinated.	0.7	[24]
b	Amount of secure connection.	1.12	[26]
τ	People who are discharged get a great recovery.	0.0701	[25]
ϕ	The fraction of undiagnosed patients who recovery.	0.14	[27]
λ	A present for all who exist of their sick grade.	0.13978	[24, 26]
η	A ratio for which people have lost an antibodies.	0.011	[29]

Figure 1 shows the time series for the susceptibility, vaccination, exposure, and infectious categories in the mathematical model. Because vaccines may have limits in some situations. When vaccinations are easily obtainable the number of persons at risk gradually reduces as more people receive the vaccine (the number of vaccinations of class v grows at the start of implementation). Even though vaccine availability is high, vaccine efficiency is low, and is larger than unity, the disease may not be cured. The statistical conclusion is that high vaccine efficiency and immunization coverage significantly lower the number of community-acquired secondary infections Figure 2.

Early identifications of stability and mechanics that have a higher impact on medical transmission are critical for informing policy decisions about which parameters to focus on for data gathering or disease mitigation. The time series of the model system is shown for the susceptible, immunized, exposed, and infected groups, because vaccinations may not be effective in all circumstances in Figure 3. When vaccinations are widely accessible, we observe that as increasing numbers of people receive the vaccine (the number of class V vaccine recipients grows at the start of implementation), the number of susceptible individuals rapidly drops. In our proposed model performance is well because the number of recovered people slightly increases after vaccination. Hence, the fractional derivative model proves to be a model easier to use for theoretical and numerical calculations and real word applications.

Corollary 5.2. *The solution of the system Eqs (4.2)–(4.6) is Ulam-Hyers-Rassias stable with $\mathcal{CF}$ with FVFIE, under the Theorem 4.3.*

$$\iota\left[\frac{(1-(\Xi)^{*})}{\mathbb{M}((\Xi)^{*})}+\frac{(\Xi)^{*}v}{\mathbb{M}((\Xi)^{*})}\right]<1. \tag{5.3}$$

By this Eq (4.14) we can prove that Hyers-Ulam-Rassias stability.

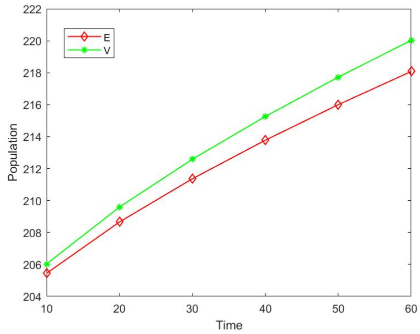

Figure 1. Dynamics of exposed E with control and V vaccinated class.

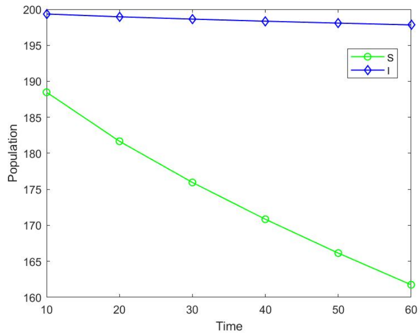

Figure 2. Dynamics of susceptible class S and infected with control I.

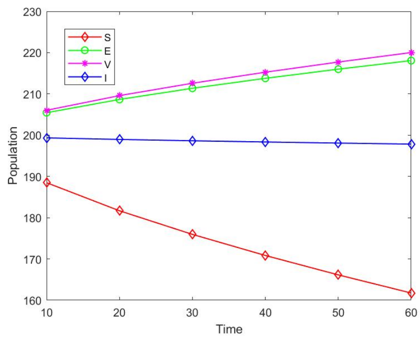

Figure 3. Profile of S, E, V and I with saturation.

6. Conclusions

The mathematical modeling tends towards the stability analysis of COVID-19, we developed a deterministic model of the COVID-19 transmission dynamics using an inadequate vaccination. The model is theoretically examined for both its effective and intrinsic reproduction frequencies to be computed. The findings of the paper explain how the COVID-19 system was generalized with fuzzy variable differential equation and unit step function carried out under $\mathcal{CF}$ fractional derivative. Using

Banach's fixed point theory, we analyzed new existence and unique conditions for the COVID-19 system model. The investigation to prove Hyers-Ulam stability for the COVID-19 system is carried out. Future studies will investigate how COVID-19 patterns are affected by different treatment methods and objectives.

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Conflic of interest

All authors declare no conflicts of interest in this paper.

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