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Research article

Fractional transportation problem under interval-valued Fermatean fuzzy sets

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Abstract: The concept of an interval-valued Fermatean fuzzy set (IVFFS), an extension of Fermatean fuzzy sets, is a more resilient and reliable tool for dealing with uncertain and incomplete data in practical applications. The purpose of this paper is to define a triangular interval-valued Fermatean fuzzy number (TIVFFN) and its arithmetic operations. Fractional transportation problems (FTPs) have important implications for cost reduction and service improvement in logistics and supply management. However, in practical problems, the parameters in the model are not precise due to some unpredictable factors, including diesel prices, road conditions, weather conditions and traffic conditions. Therefore, decision makers encounter uncertainty when estimating transportation costs and profits. To address these challenges, we consider a FTP with TIVFFN as its parameter and call it an interval-valued Fermatean fuzzy fractional transportation problem (IVFFFTP). A new method for solving this IVFFFTP is proposed without re-transforming the original problem into an equivalent crisp problem. Illustrative examples are discussed to evaluate the precision and accuracy of the proposed method. Finally, the results of the proposed method are compared with those of existing methods.

Keywords: interval-valued Fermatean fuzzy sets; fractional linear programming; fractional transportation problem
Mathematics Subject Classification: 90C32, 90C70

1. Introduction

To deal with imprecise information, Zadeh [1] proposed the concept of a fuzzy set (FS), which has been widely used in various fields. Inspired by the idea of the FS, Atanassov [2] proposed the concept of the intuitionistic fuzzy set (IFS), which is represented by the degree of membership (MD) and the degree of non-membership (NMD), with the restriction that the sum of these two degrees cannot exceed one. However, in some practical applications, the sum of MD and NMD may be greater than unity, but their sum of squares is limited to unity. To bridge this gap, Yager [3, 4] extended the scope of the IFS to Pythagorean fuzzy sets (PFS), where the sum of squares of MD and NMD is ≤ 1. In the manufacturing sector, Han and Rani [5] evaluated the barriers of block chain technology adoption in sustainable supply chain management in a Pythagorean fuzzy environment. Senapati and Yager [6–8] pioneered the theory of Fermatean fuzzy sets (FFSs), broadening the field of IFSs and PFSs, characterized by the MD and NMD of an element, and the sum of their cubes should be less than or equal to 1. Further discussions and different applications related to Fermatean fuzzy sets are also observed in [9–12].

In reality, the functions of uncertain information are becoming more and more diverse. Due to the ambiguity of information and the lack of experience and knowledge of decision experts (DEs), in many practical processes, decision experts often have difficulty describing their ideas adequately with clear numbers; however, they can be expressed with a $[0, 1]$ interval number representation. For more information, the concept of the FFS has been extended to the IVFFS [13–15]. The benefit of this extended theory is that it represents ambiguous data closer to DE expectations. It should be noted that when the upper and lower bounds of the interval value are the same, IVFFS becomes FFS. In terms of expressing information, IVFFS outperforms existing tools in describing human subjective intelligence.

A linear programming problem (LPP) is a problem concerned with determining the optimal value of a given linear function. The main goal of LPP is to obtain an optimal solution. Bellman and Zadeh [16] proposed the concept of a decision-making process in which goals and constraints are inherently ambiguous. Zimmerman [17] proposed the concept of a fuzzy LPP. Mahato et al. [18] demonstrated the effectiveness of defuzzification methods in redundant assignment problems with ambiguous values using a genetic algorithm. In 2015, Sahoo [19] identified the impact of defuzzification methods on solving fuzzy matrix games. Ali et al. [20] solved an intuitionistic fuzzy multi-objective LPP in a neutrosophic setting. Recently, some researchers [21–27] have developed different methods to evaluate LPPs in different environments. The physical distribution of products, sometimes referred to as the transportation problem (TP), is one of the most important and effective applications of quantitative analysis to solve business problems. Often, the goal is to reduce the cost of moving goods from one station to another so that each arrival area is met, and each transportation facility operates within its capacity. Transportation models provide a powerful framework to address this challenge. They guarantee efficient transportation of raw materials and timely supply of final products. Hitchcock [28] originally developed the basic structure of TP in 1941.

The FTP is a challenging task to optimize the ratio of one or more functions and has received extensive attention from both methodological and practical perspectives. In 1962, Charnes and Cooper [29] proposed the fractional programming problem (FPP). Pandey and Punnen [30] developed an algorithm for solving piecewise linear FPPs. The FTP, first proposed by Swarup [31] in 1966, is also significant in logistics and supply management, reducing costs and improving services. The related work on TPs and FTPs by different researchers is given in Table 1.

Table 1. Related work in TPs and FTPs.

Reference	Year	Significance Influence
Gupta et al. [32]	1993	Studied a paradox in linear FTP with mixed constraints
Monta [33]	2007	Defined some aspects of solving the linear FTP
Sivri et al. [34]	2011	Determined the solution of linear FTP
Joshi and Gupta [35]	2011	Developed the FTP with varying demands and supply
Kumar and Kaur [36]	2011	Gave applications of transportation techniques
Kaushal et al. [37]	2011	Provided an aspect of bilevel FTP
Guzel et al. [38]	2012	Provided a solution proposal for interval-valued TP
Ebrahimnejad [39]	2016	Proposed an improved approach to solve TP triangular fuzzy numbers
Liu [40]	2016	Introduced FTP in fuzzy environment
Ebrahimnejad [41]	2016	Developed a new method for solving TP with LR flat fuzzy numbers
Mohanaselvi and Ganesan [42]	2017	Proposed a new approach to solve linear fuzzy FTP
Safi and Ghasemi [43]	2017	Studied uncertainty of linear FTP
Bharati and Singh [44]	2018	Introduced TP using interval-valued intuitionistic fuzzy sets
Mahmoodirad et al. [45]	2019	Proposed a method to solve fully fuzzy TP
Kumar et al. [46]	2019	Proposed Pythagorean fuzzy approach to the TP
Bharati [47]	2019	Developed FTP with trapezoidal intuitionistic fuzzy numbers
Bharati [48]	2021	Introduced TP with interval-valued IFSs
Pratihar et al. [49]	2021	Developed modified Vogels approximation technique for TP
Veeramani et al. [50]	2021	Solved the multi-objective FTP using goal programming approach
Sahoo [51]	2021	Proposed TP based on new score function in Fermatean fuzzy environment
Sharma et al. [52]	2021	Developed fuzzy optimization approach for multi-objective aspirational level FTP
El Sayed & Abo-Sinna [53]	2021	Proposed a novel approach for solving fully intuitionistic fuzzy multi-objective FTP
Bas et al. [54]	2022	Presented a new method for solving linear FTP
Sing et al. [55]	2022	Introduced bilevel TP under neutrosophic environment

Due to the increasing complexity of several practical optimization problems, it is difficult for decision makers to give the values of parameters in a precise manner. Therefore, some studies on FS and IFS rankings have been published. Among several extensions of FSs, the concept of the interval-valued IFS (IVIFS) is an interesting one and a very valuable tool for modeling and decision-making in practical problems of uncertainty and hesitation. Thus, Mohanaselvi and Ganesan [42] solved the fuzzy FTP, and Bharati [44, 48] solved the interval-valued intuitionistic fuzzy TP. Inspired by these ideas, we develop a straightforward method for solving the FTP using TIVFFN and extend the work of Bharati [44, 48] to the FTP to check the optimality of the solution. However, IVFFS is a more general model than IVIFS and can handle higher levels of uncertainty than IVIFS. Our main contributions are as follows:

- Define TIVFFN and its arithmetic operations.
- Use TIVFFN to formulate FTP in a Fermatean fuzzy environment.
- Develop a new method for solving FTP with TIVFFN and extend the optimality criterion for TP given in [48] to FTP.
- Examples are provided to evaluate the precision and accuracy of the proposed method, and the obtained FTP cost is represented graphically.
- The solutions obtained by our proposed method are compared with those of existing methods.

The rest of the study is structured as follows: Section 2 gives the main definitions. In Section 3, FTP is formulated in an Interval-Valued Fermatean Fuzzy (IVFF) environment. Section 4 presents a solution method for IVFFFTP. Section 5 gives numerical examples. Section 6 compares the proposed method with existing methods. The conclusions are drawn in Section 7.

2. Preliminaries

In this section, we describe some basic definitions used throughout the study.

Definition 2.1. [6] Let Y be a universal set. A *Fermatean fuzzy set* $\tilde{A}^F$ on Y is defined to be a set of the form

$$\tilde{A}^F = \{\langle y, \mu_{\tilde{A}^F}(y), \nu_{\tilde{A}^F}(y)\rangle : y \in Y\},$$

where $\mu_{\tilde{A}^F} : Y \to [0,1]$, $\nu_{\tilde{A}^F} : Y \to [0,1]$, and

$$0 \leq (\mu_{\tilde{A}^F}(y))^3 + (\nu_{\tilde{A}^F}(y))^3 \leq 1,$$

for all $y \in Y$. The values $\mu_{\tilde{A}^F}(y)$ and $\nu_{\tilde{A}^F}(y)$ denote the membership degree and non-membership degree of the element y in the set $\tilde{A}^F$, respectively.

Further, for all $y \in Y$,

$$\pi_{\tilde{A}^F}(y) = \sqrt[3]{1 - (\mu_{\tilde{A}^F}(y))^3 - (\nu_{\tilde{A}^F}(y))^3}$$

denotes the degree of hesitation for the element y in $\tilde{A}^F$.

Definition 2.2. [13, 15] Let Y be a universal set. An *interval-valued Fermatean fuzzy set* T on Y is mathematically defined as follows:

$$T = \{(y_i, [\lambda^-_{1_T}(y_i), \lambda^+_{1_T}(y_i)], [\lambda^-_{2_T}(y_i), \lambda^+_{2_T}(y_i)]) : y_i \in Y\},$$

where $0 \leq \lambda^-_{1_T}(y_i) \leq \lambda^+_{1_T}(y_i) \leq 1$, $0 \leq \lambda^-_{2_T}(y_i) \leq \lambda^+_{2_T}(y_i) \leq 1$, and $(\lambda^+_{1_T}(y_i))^3 + (\lambda^+_{2_T}(y_i))^3 \leq 1$. Here, $\mu_T(y_i) = [\lambda^-_{1_T}(y_i), \lambda^+_{1_T}(y_i)]$ and $\nu_T(y_i) = [\lambda^-_{2_T}(y_i), \lambda^+_{2_T}(y_i)]$ represent the interval-valued membership and non-membership degrees of $y_i \in Y$, respectively. The function $\pi_T(y_i)$ $=[\pi^-_T(y_i), \pi^+_T(y_i)]$ represents the IVFF-hesitancy index of y_i to T, where $\pi^-_T(y_i) = \sqrt[3]{1 - (\lambda^+_{1_T}(y_i))^3 + (\lambda^+_{2_T}(y_i))^3}$ and $\pi^+_T(y_i) = \sqrt[3]{1 - (\lambda^-_{1_T}(y_i))^3 + (\lambda^-_{2_T}(y_i))^3}$. For simplicity, an IVFFN is represented by $([\lambda^-_{1_T}(y_i), \lambda^+_{1_T}(y_i)], [\lambda^-_{2_T}(y_i), \lambda^+_{2_T}(y_i)])$, which fulfills $(\lambda^+_{1_T}(y_i))^3 + (\lambda^+_{2_T}(y_i))^3 \leq 1$.

Some special cases of the IVFFS are defined as follows:

(i) If $\lambda^-_{1_T}(y_i) = \lambda^+_{1_T}(y_i)$, and $\lambda^-_{2_T}(y_i) = \lambda^+_{2_T}(y_i)$, $y_i \in Y$, then an IVFFS reduces to FFS.
(ii) If $(\lambda^+_{1_T}(y_i)) + (\lambda^+_{2_T}(y_i)) \leq 1$, then an IVFFS reduces to an interval-valued IFS.
(iii) If $(\lambda^+_{1_T}(y_i))^2 + (\lambda^+_{2_T}(y_i))^2 \leq 1$, then an IVFFS reduces to an interval-valued PFS.

The graphical representation of an IVFFS is given in Figure 1.

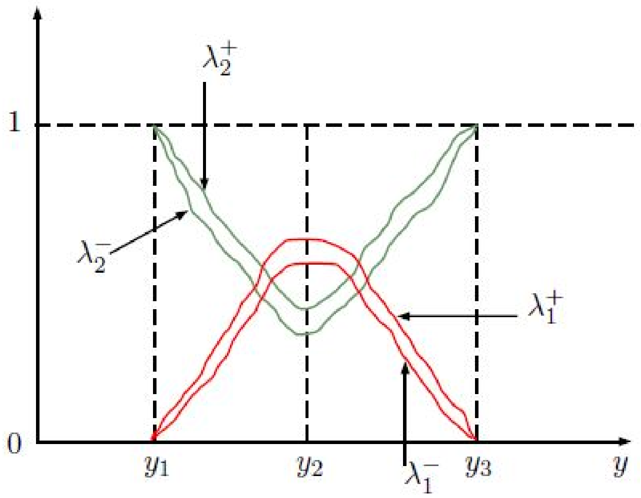

Figure 1. Interval-valued Fermatean fuzzy set.

Definition 2.3. Let Y be a universal set. A *triangular interval-valued Fermatean fuzzy number* $\tilde{A}^F$ on Y is represented as $\tilde{A}^F = \{(y_1, y_2, y_3), [\lambda_{1A}^-, \lambda_{1A}^+], [\lambda_{2A}^-, \lambda_{2A}^+]\}$, where $\lambda_{1A}^- : Y \to [0,1]$, $\lambda_{1A}^+ : Y \to [0,1]$ denote the minimum and maximum degrees of membership, and $\lambda_{2A}^- : Y \to [0,1]$, $\lambda_{2A}^+ : Y \to [0,1]$ denote the minimum and maximum degrees of non-membership, and these are:

$$\lambda_{1A}^-(y) = \begin{cases} \dfrac{(y-y_1)\lambda_1^-}{y_2-y_1}, & y_1 < y < y_2, \\ \lambda_1^-, & y = y_2, \\ \dfrac{(y_3-y)\lambda_1^-}{(y_3-y_2)}, & y_2 < y < y_3, \end{cases} \tag{2.1}$$

$$\lambda_{1\tilde{A}^F}^+(y) = \begin{cases} \dfrac{(y-y_1)\lambda_1^+}{(y_2-y_1)}, & y_1 < y < y_2, \\ \lambda_1^+, & y = y_2, \\ \dfrac{(y_3-y)\lambda_1^+}{(y_3-y_2)}, & y_2 < y < y_3, \end{cases} \tag{2.2}$$

$$\lambda_{2\tilde{A}^F}^-(y) = \begin{cases} 1-(1-\lambda_2^-)\dfrac{(y-y_1)}{(y_2-y_1)}, & y_1 < y < y_2, \\ \lambda_2^-, & y = y_2, \\ \lambda_2^- + (1-\lambda_2^-)\dfrac{(y-y_2)}{(y_3-y_2)}, & y_2 < y < y_3, \end{cases} \tag{2.3}$$

$$\lambda_{2\tilde{A}^F}^+(y) = \begin{cases} 1-(1-\lambda_2^+)\dfrac{(y-y_1)}{(y_2-y_1)}, & y_1 < y < y_2, \\ \lambda_2^+, & y = y_2, \\ \lambda_2^+ + (1-\lambda_2^+)\dfrac{(y-y_2)}{(y_3-y_2)}, & y_2 < y < y_3. \end{cases} \tag{2.4}$$

The graphical representation of TIVFFN is given in Figure 2.

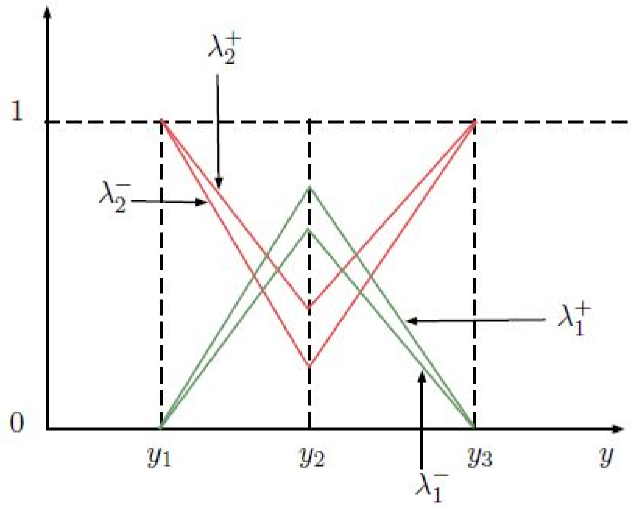

Figure 2. Triangular interval-valued Fermatean fuzzy number.

2.1. Arithmetic operations

Let $\tilde{A}^F = \{(x_1, x_2, x_3), [\lambda^-_{1A}, \lambda^+_{1A}], [\lambda^-_{2A}, \lambda^+_{2A}]\}$ and $\tilde{B}^F = \{(y_1, y_2, y_3), [\lambda^-_{1B}, \lambda^+_{1B}], [\lambda^-_{2B}, \lambda^+_{2B}]\}$ be two TIVFFNs. Then,

$$\begin{aligned}\tilde{A}^F \oplus \tilde{B}^F =&\{(x_1 + y_1, x_2 + y_2, x_3 + y_3),\\ &[\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})], [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\},\end{aligned} \tag{2.5}$$

$$\begin{aligned}\tilde{A}^F \ominus \tilde{B}^F =&\{(x_1 - y_3, x_2 - y_2, x_3 + y_1),\\ &[\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})], [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\},\end{aligned} \tag{2.6}$$

$$\tilde{A}^F \odot \tilde{B}^F = \begin{cases}\{(x_1y_1, x_2y_2, x_3y_3), [\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})],\\ [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\} \text{ if } x_1, y_1 \in \mathbb{R}^+,\\ \{(x_1y_3, x_2y_2, x_3y_3), [\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})],\\ [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\} \text{ if } x_1 < 0 \text{ and } y_1 > 0,\\ \{(x_1y_3, x_2y_2, x_3y_1), [\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})],\\ [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\} \text{ if } x_3 < 0 \text{ and } y_1 > 0,\end{cases} \tag{2.7}$$

$$\tilde{A}^F \oslash \tilde{B}^F = \begin{cases}\{(\frac{x_1}{y_3}, \frac{x_2}{y_2}, \frac{x_3}{y_1}), [\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})],\\ [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\} \text{ if } x_1, y_1 \in \mathbb{R}^+,\\ \{(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_1}), [\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})],\\ [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\} \text{ if } x_1 < 0 \text{ and } y_1 > 0,\\ \{(\frac{x_3}{y_1}, \frac{x_2}{y_2}, \frac{x_1}{y_3}), [\min(\lambda^-_{1A}, \lambda^-_{1B}), \min(\lambda^+_{1A}, \lambda^+_{1B})],\\ [\max(\lambda^-_{2A}, \lambda^-_{2B}), \max(\lambda^+_{2A}, \lambda^+_{2B})]\} \text{ if } x_3 < 0 \text{ and } y_1 > 0,\end{cases} \tag{2.8}$$

$$k\tilde{A}^F = \begin{cases}\{(kx_1, kx_2, kx_3), [\lambda^-_{1A}, \lambda^+_{1A}], [\lambda^-_{2A}, \lambda^+_{2A}]\} \text{ if } k > 0,\\ \{(kx_3, kx_2, kx_1), [\lambda^-_{1A}, \lambda^+_{1A}], [\lambda^-_{2A}, \lambda^+_{2A}]\} \text{ if } k < 0.\end{cases} \tag{2.9}$$

Definition 2.4. Let $\tilde{A}^F = \{(x_1, x_2, x_3), [\lambda^-_{1A}, \lambda^+_{1A}], [\lambda^-_{2A}, \lambda^+_{2A}]\}$ be a TIVFFN. Then, its ranking is defined as

$$\Re(\tilde{A}^F) = \frac{(x_1 + 2x_2 + x_3)(\lambda^-_{1A} + \lambda^+_{1A} + 2 - \lambda^-_{2A} - \lambda^+_{2A})}{16}.$$

Let $\tilde{A}^F$ and $\tilde{B}^F$ be two TIVFFNs. Then,

1) If $\Re(\tilde{A}^F) > \Re(\tilde{B}^F)$, then $\tilde{A}^F \succ \tilde{B}^F$.
2) If $\Re(\tilde{A}^F) < \Re(\tilde{B}^F)$, then $\tilde{A}^F \prec \tilde{B}^F$.
3) If $\Re(\tilde{A}^F) = \Re(\tilde{B}^F)$, then $\tilde{A}^F \approx \tilde{B}^F$.

Definition 2.5. Two TIVFFNs $\tilde{A}^F = \{(x_1, x_2, x_3), [\lambda^-_{1A}, \lambda^+_{1A}], [\lambda^-_{2A}, \lambda^+_{2A}]\}$ and $\tilde{B}^F = \{(y_1, y_2, y_3), [\lambda^-_{1B}, \lambda^+_{1B}], [\lambda^-_{2B}, \lambda^+_{2B}]\}$ are said to be equal (that is, $\tilde{A}^F = \tilde{B}^F$) if and only if $x_1 = y_1$, $x_2 = y_2$, $x_3 = y_3$, $\lambda^-_{1A} = \lambda^-_{1B}$, $\lambda^+_{1A} = \lambda^+_{1B}$, $\lambda^-_{2A} = \lambda^-_{2B}$, and $\lambda^+_{2A} = \lambda^+_{2B}$.

3. Linear interval-valued Fermatean fuzzy fractional transportation model

The linear IVFFFTP is designed for the transportation of varying quantities of a single homogeneous product from multiple origins to different destinations, while keeping overall IVFF fractional transportation costs to a minimum. Suppose there are m sources from which items must be delivered to n destinations. Let $\hat{C} = (\hat{c}_{ij})_{m\times n}$ be the IVFF cost matrix, where $\hat{c}_{ij}$ denotes the IVFF cost to transport the product from the source i to destination j. Let $\hat{P} = (\hat{p}_{ij})_{m\times n}$ denote the IVFF profit matrix, and $\hat{p}_{ij}$ denotes the obtained IVFF profit, if a commodity is moved from source i to destination j. Let $\hat{x}_{ij}$ denote unknown number of items from source i to destination j. Let $\hat{c}_0$ and $\hat{p}_0$ denote the specified fixed IVFF costs and profits. Then, the mathematical model of the linear IVFFFTP is

$$\begin{aligned}
\mathrm{Min}\hat{Q}(x) \approx & \frac{\sum_{i=1}^m \sum_{j=1}^n \{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\lambda^-_{1c}, \lambda^+_{1c}], [\lambda^-_{2c}, \lambda^+_{2c}]\} x_{ij} + \hat{c}_0}{\sum_{i=1}^m \sum_{j=1}^n \{(p_1^{ij}, p_2^{ij}, p_3^{ij}), [\lambda^-_{1p}, \lambda^+_{1p}], [\lambda^-_{2p}, \lambda^+_{2p}]\} x_{ij} + \hat{p}_0} \\
& \sum_{j=1}^n x_{ij} \le b_i \\
& \sum_{i=1}^m x_{ij} \ge a_j \\
& a_j \approx b_i \\
& x_{ij} \ge 0, \quad \forall i, j.
\end{aligned} \tag{3.1}$$

As we are evaluating the fraction of a linear IVFF function, it is possible that the denominator for some $\hat{x}_{ij}$ is equal to zero. To prevent this problem, we suppose that $\sum_{i=1}^m \sum_{j=1}^n \{(p_1^{ij}, p_2^{ij}, p_3^{ij}), [\lambda^-_{1p}, \lambda^+_{1p}], [\lambda^-_{2p}, \lambda^+_{2p}]\} x_{ij} + \hat{p}_0 \neq \hat{0}$.

Remark. The main objective for expressing FFNs in practical problems with TIVFFNs is that they are easy to use and interpret. Furthermore, these numbers can handle higher levels of uncertainty and accurately represent the situation. Therefore, we propose a new method to model and solve the IVFFFTP in which all parameters are denoted as TIVFFNs.

4. Methodology

In this section, we develop a new method to find the IVFF optimal solution of the linear IVFFFTP. Proceed as follows.

(i) Represents the given linear IVFFFTP in tabular form (see Table 2).

Table 2. Linear IVFFFTP.

	Destination 1	Destination 2	...	Destination k	Supply
Source 1	$\hat{c}_{11}$	$\hat{c}_{12}$	...	$\hat{c}_{1n}$	$\hat{a}_1$
	$\hat{p}_{11}$	$\hat{p}_{12}$	...	$\hat{p}_{1n}$	
Source 2	$\hat{c}_{21}$	$\hat{c}_{22}$	...	$\hat{c}_{2n}$	$\hat{a}_2$
	$\hat{p}_{21}$	$\hat{p}_{22}$	...	$\hat{p}_{2n}$	
...	...	...	...	...	...
Source m	$\hat{c}_{m1}$	$\hat{c}_{m2}$	...	$\hat{c}_{mn}$	$\hat{a}_m$
	$\hat{p}_{m1}$	$\hat{p}_{m2}$	...	$\hat{p}_{mn}$	
Demands	$\hat{b}_1$	$\hat{b}_2$	...	$\hat{b}_n$	

(ii) Find the difference between the two lowest IVFF costs $\hat{c}_{ij}$ for each row and column. Similarly, find the difference between the two lowest IVFF profits $\hat{p}_{ij}$ for each row and column.

(iii) Determines the sum of the differences between $\hat{c}_{ij}$ and $\hat{p}_{ij}$ for all rows and columns.

(iv) Indicate the row or column with the largest sum of differences compared to all other rows and columns.

(v) Suppose the j^{th} column has the largest sum of differences. Find the line with the smallest $\dfrac{\hat{c}_{ij}}{\hat{p}_{ij}}$ and make the largest allocation in it.

(vi) This process is repeated until all goods in the source have been moved.

(vii) Compute $\hat{u}_i^{'}, \hat{u}_i^{''}$ and $\hat{v}_j^{'}, \hat{v}_j^{''}$ for the allocated cells using $\hat{c}_{ij} = \hat{u}_i^{'} + \hat{v}_j^{'}$ and $\hat{p}_{ij} = \hat{u}_i^{''} + \hat{v}_j^{''}$ by setting a multiplier to zero and connecting it to the row or column of the transportation table with the most allocated cells. Here, $\hat{u}_1^{'}, \hat{u}_2^{'}, \dots \hat{u}_m^{'}$ and $\hat{u}_1^{''}, \hat{u}_2^{''}, \dots \hat{u}_m^{''}$ are the multipliers for IVFF cost and IVFF profit to the m constraints, and $\hat{v}_1^{'}, \hat{v}_2^{'}, \dots \hat{v}_n^{'}$ and $\hat{v}_1^{''}, \hat{v}_2^{''}, \dots \hat{v}_n^{''}$ are the multipliers for IVFF cost and IVFF profit to the n constraints.

(viii) Compute $\triangle_{ij}^{'} = \hat{u}_i^{'} + \hat{v}_j^{'} - \hat{c}_{ij}$ and $\triangle_{ij}^{''} = \hat{u}_i^{''} + \hat{v}_j^{''} - \hat{p}_{ij}$, and then the optimal criterion is given by $\triangle_{ij} \leq \hat{0}$, where $\triangle_{ij} = \hat{Q}(x)\triangle_{ij}^{'} - \triangle_{ij}^{''}$ $\forall i, j$ for unassigned units of the IVFF fractional transportation table.

5. Numerical examples

Example 5.1. [56] A firm owns three electric power stations that fulfill the needs of four communities. Each power station may generate the following numbers of megawatt-hours (MWh).

	Station 1	Station 2	Station 3
Supply(MWh)	40	45	35

The peak electricity demands of these communities are as follows (in MWh).

	Community 1	Community 2	Community 3	Community 4
Demand(MWh)	35	35	25	25

Transportation costs and profits vary based on various uncontrollable factors, such as weather, traffic, and gasoline prices. The values of cost and profit do not handle the situation properly; to overcome this, we use the TIVFFNs as parameters. The IVFF cost of transporting 1 MWh of electricity from a power station to a community is determined by the distance traveled (see Table 3), and the IVFF profit earned by the firm for each 1 MWh of electricity delivered is shown in Table 4.

Table 3. Transportation costs in dollars.

	Station 1	Station 2	Station 3
Community 1	{(7,8,9),[0.3,0.5],[0.2,0.4]}	{(10,12,15),[0.4,0.5],[0.1,0.3]}	{(12,14,16),[0.4,0.7],[0.1,0.3]}
Community 2	{(4,6,7),[0.5,0.6],[0.2,0.3]}	{(10,12,13),[0.5,0.8],[0.4,0.7]}	{(12,13,14),[0.2,0.3],[0.15,0.95]}
Community 3	{(14,15,16),[0.5,0.7],[0.2,0.4]}	{(10,13,17),[0.6,0.8],[0.2,0.6]}	{(13,16,20),[0.2,0.7],[0.6,0.8]}
Community 4	{(6,9,10),[0.4,0.45],[0.25,0.4]}	{(5,7,9),[0.5,0.6],[0.3,0.4]}	{(9,11,14),[0.7,0.8],[0.3,0.4]}

Table 4. Profit of firm in dollars.

	Station 1	Station 2	Station 3
Community 1	{(3,5,6),[0.2,0.3],[0.3,0.35]}	{(3,5,6),[0.8,0.9],[0.5,0.6]}	{(6,8,10),[0.3,0.5],[0.2,0.3]}
Community 2	{(3,4,5),[0.4,0.5],[0.1,0.4]}	{(1,2,3),[0.6,0.7],[0.2,0.8]}	{(3,4,5),[0.75,0.85],[0.65,0.7]}
Community 3	{(4,6,8),[0.3,0.4],[0.2,0.5]}	{(2,3,6),[0.4,0.7],[0.5,0.6]}	{(4,6,7),[0.5,0.7],[0.4,0.6]}
Community 4	{(2,3,8),[0.35,0.7],[0.2,0.5]}	{(3,4,6),[0.4,0.7],[0.1,0.4]}	{(3,4,6),[0.3,0.6],[0.4,0.5]}

Let

$$\begin{aligned} P(x) = {} & \{(7,8,9),[0.3,0.5],[0.2,0.4]\}x_{11} + \{(4,6,7),[0.5,0.6],[0.2,0.3]\}x_{12} + \{(14,15,16),[0.5,0.7],\\ & [0.2,0.4]\}x_{13} + \{(6,9,10),[0.4,0.45],[0.25,0.4]\}x_{14} + \{(10,12,15),[0.4,0.5],[0.1,0.3]\}x_{21}\\ & + \{(10,12,13),[0.5,0.8],[0.4,0.7]\}x_{22} + \{(10,13,17),[0.6,0.8],[0.2,0.6]\}x_{23} + \{(5,7,9),\\ & [0.5,0.6],[0.3,0.4]\}x_{24} + \{(12,14,16),[0.4,0.7],[0.1,0.3]\}x_{31} + \{(12,13,14),[0.2,0.3],\\ & [0.15,0.95]\} + \{(13,16,20),[0.2,0.7],[0.6,0.8]\}x_{33} + \{(9,11,14),[0.7,0.8],[0.3,0.4]\}x_{34},\\ R(x) = {} & \{(3,5,6),[0.2,0.3],[0.3,0.35]\}x_{11} + \{(3,4,5),[0.4,0.5],[0.1,0.4]\}x_{12} + \{(4,6,8),[0.3,0.4],\\ & [0.2,0.5]\}x_{13} + \{(2,3,8),[0.35,0.7],[0.2,0.5]\}x_{14} + \{(3,5,6),[0.8,0.9],[0.5,0.6]\}x_{21} + \{(1,2,3),\\ & [0.6,0.7],[0.2,0.8]\}x_{22} + \{(2,3,6),[0.4,0.7],[0.5,0.6]\}x_{23} + \{(3,4,6),[0.4,0.7],[0.1,0.4]\}x_{24}\\ & + \{(6,8,10),[0.3,0.5],[0.2,0.3]\}x_{31} + \{(3,4,5),[0.75,0.85],[0.65,0.7]\}x_{32} + \{(4,6,7),[0.5,0.7],\\ & [0.4,0.6]\}x_{33} + \{(3,4,6),[0.3,0.6],[0.4,0.5]\}x_{34}. \end{aligned}$$

Then, the mathematical model of the linear IVFFFTP is

$$\text{Min } \frac{P(x)}{R(x)}$$

$$\text{subject to} \sum_{j=1}^{4} x_{1j} \leq 40, \quad \sum_{j=1}^{4} x_{2j} \leq 45, \quad \sum_{j=1}^{4} x_{3j} \leq 35,$$
$$\sum_{i=1}^{3} x_{i1} \geq 35, \quad \sum_{i=1}^{3} x_{i2} \geq 35, \quad \sum_{i=1}^{3} x_{i3} \geq 25, \quad \sum_{i=1}^{3} x_{i4} \geq 25,$$
$$x_{ij} \geq 0, \quad i = 1, 2, 3, \quad j = 1, 2, 3, 4. \tag{5.1}$$

We represent the given linear IVFFFTP in Table 5.

Table 5. Data of Example 5.1.

{(7,8,9),[0.3,0.5],[0.2,0.4]} {(3,5,6),[0.2,0.3],[0.3,0.35]}	{(4,6,7),[0.5,0.6],[0.2,0.3]} {(3,4,5),[0.4,0.5],[0.1,0.4]}	{(14,15,16),[0.5,0.7],[0.2,0.4]} {(4,6,8),[0.3,0.4],[0.2,0.5]}	{(6,9,10),[0.4,0.45],[0.25,0.4]} {(2,3,8),[0.35,0.7],[0.2,0.5]}	40
{(10,12,15),[0.4,0.5],[0.1,0.3]} {(3,5,6),[0.8,0.9],[0.5,0.6]}	{(10,12,13),[0.5,0.8],[0.4,0.7]} {(1,2,3),[0.6,0.7],[0.2,0.8]}	{(10,13,17),[0.6,0.8],[0.2,0.6]} {(2,3,6),[0.4,0.7],[0.5,0.6]}	{(5,7,9),[0.5,0.6],[0.3,0.4]} {(3,4,6),[0.4,0.7],[0.1,0.4]}	45
{(12,14,16),[0.4,0.7],[0.1,0.3]} {(6,8,10),[0.3,0.5],[0.2,0.3]}	{(12,13,14),[0.2,0.3],[0.15,0.95]} {(3,4,5),[0.75,0.85],[0.65,0.7]}	{(13,16,20),[0.2,0.7],[0.6,0.8]}} {(4,6,7),[0.5,0.7],[0.4,0.6]}	{(9,11,14),[0.7,0.8],[0.3,0.4]} {(3,4,6),[0.3,0.6],[0.4,0.5]}	35
35	35	25	25	

By applying the proposed method, the initial interval-valued Fermatean fuzzy basic feasible solution (FFBFS) is given in Table 6.

Table 6. Initial interval-valued FFBFS of problem 5.1.

{(7,8,9),[0.3,0.5],[0.2,0.4]} $\boxed{5}$ {(3,5,6),[0.2,0.3],[0.3,0.35]}	{(4,6,7),[0.5,0.6],[0.2,0.3]} $\boxed{35}$ {(3,4,5),[0.4,0.5],[0.1,0.4]}	{(14,15,16),[0.5,0.7],[0.2,0.4]} {(4,6,8),[0.3,0.4],[0.2,0.5]}	{(6,9,10),[0.4,0.45],[0.25,0.4]} {(2,3,8),[0.35,0.7],[0.2,0.5]}	40
{(10,12,15),[0.4,0.5],[0.1,0.3]} {(3,5,6),[0.8,0.9],[0.5,0.6]}	{(10,12,13),[0.5,0.8],[0.4,0.7]} {(1,2,3),[0.6,0.7],[0.2,0.8]}	{(10,13,17),[0.6,0.8],[0.2,0.6]} $\boxed{20}$ {(2,3,6),[0.4,0.7],[0.5,0.6]}	{(5,7,9),[0.5,0.6],[0.3,0.4]} $\boxed{25}$ {(3,4,6),[0.4,0.7],[0.1,0.4]}	45
{(12,14,16),[0.4,0.7],[0.1,0.3]} $\boxed{30}$ {(6,8,10),[0.3,0.5],[0.2,0.3]}	{(12,13,14),[0.2,0.3],[0.15,0.95]} {(3,4,5),[0.75,0.85],[0.65,0.7]}	{(13,16,20),[0.2,0.7],[0.6,0.8]}} $\boxed{5}$ {(4,6,7),[0.5,0.7],[0.4,0.6]}	{(9,11,14),[0.7,0.8],[0.3,0.4]} {(3,4,6),[0.3,0.6],[0.4,0.5]}	35
35	35	25	25	

Hence, the initial interval-valued FFBFS to the given linear IVFFFTP is obtained as $x_{11} = 5$, $x_{12} = 35$, $x_{23} = 20$, $x_{24} = 25$, $x_{31} = 30$, $x_{33} = 5$ and

$$\hat{Q}(x) = \frac{\{(925, 1185, 1435), [0.1, 0.3], [0.6, 0.8]\}}{\{(435, 595, 810), [0.2, 0.3], [0.5, 0.6]\}} = \{(1.14, 1.99, 3.3), [0.1, 0.3], [0.6, 0.8]\}.$$

Put $\hat{u}_1^{'} = 0$, and we get
$\hat{u}_2^{'} = \{(-7, 3, 13), [0.2, 0.5], [0.6, 0.8]\}$,
$\hat{u}_3^{'} = \{(3, 6, 9), [0.3, 0.5], [0.2, 0.4]\}$,
$\hat{v}_1^{'} = \{(7, 8, 9), [0.3, 0.5], [0.2, 0.4]\}$,
$\hat{v}_2^{'} = \{(4, 6, 7), [0.5, 0.6], [0.2, 0.3]\}$,
$\hat{v}_3^{'} = \{(4, 10, 17), [0.2, 0.5], [0.6, 0.8]\}$,
$\hat{v}_4^{'} = \{(-6, 4, 11), [0.2, 0.3], [0.5, 0.6]\}$.
Similarly, put $\{\hat{u}_1^{''} = 0\}$, and then
$\hat{u}_2^{''} = \{(-5, 0, 9), [0.2, 0.3], [0.5, 0.6]\}$,
$\hat{u}_3^{''} = \{(0, 3, 7), [0.2, 0.3], [0.3, 0.35]\}$,

$\hat{v}_1^{''} = \{(3,5,6),[0.2,0.3],[0.3,0.35]\}$,
$\hat{v}_2^{''} = \{(3,4,5),[0.4,0.5],[0.1,0.4]\}$,
$\hat{v}_3^{''} = \{(-3,3,7),[0.2,0.3],[0.4,0.6]\}$,
$\hat{v}_4^{''} = \{(-6,4,11),[0.2,0.3],[0.5,0.6]\}$.
The net evaluation corresponding to all non-basic cells is

$$\begin{aligned}
\triangle_{13}^{'} &= \hat{u}_1^{'} + \hat{v}_3^{'} - \hat{c}_{13} \\
&= 0 + \{(4,10,17),[0.2,0.5],[0.6,0.8]\} - \{(14,15,16),[0.5,0.7],[0.2,0.4]\} \\
&= \{(-12,1,3),[.2,0.5],[0.6,0.8]\}, \\
\triangle_{13}^{''} &= \hat{u}_1^{''} + \hat{v}_3^{''} - \hat{p}_{13} \\
&= 0 + \{(-3,3,7),[0.2,0.3],[0.4,0.6]\} - \{(4,6,8),[0.3,0.4],[0.2,0.5]\} \\
&= \{(-11,-3,11),[0.2,0.3],[0.4,0.6]\}, \\
\hat{Q}(x)\triangle_{13}^{'} - \triangle_{13}^{''} &= \{(1.14,1.99,3.30),[0.1,0.3],[0.6,0.8]\}\{(-12,1,3),[.2,0.5],[0.6,0.8]\} \\
&- \{(-11,-3,11),[0.2,0.3][0.4,0.6]\} \\
&= \{(-50.6,4.99,20.9),[0.1,0.3],[0.6,0.8]\} < 0.
\end{aligned}$$

$$\begin{aligned}
\triangle_{14}^{'} &= \hat{u}_1^{'} + \hat{v}_4^{'} - \hat{c}_{14} \\
&= 0 + \{(-6,4,11),[0.2,0.3],[0.5,0.6]\} - \{(6,9,10),[0.4,0.45],[0.25,0.4]\} \\
&= \{(-16,-5,5),[0.2,0.3],[0.5,0.6]\}, \\
\triangle_{14}^{''} &= \hat{u}_1^{''} + \hat{v}_4^{''} - \hat{p}_{14} \\
&= 0 + \{(-6,4,11),[0.2,0.3],[0.5,0.6]\} - \{(2,3,8),[0.35,0.7],[0.2,0.5]\} \\
&= \{(-14,1,9),[0.2,0.3],[0.5,0.6]\}, \\
\hat{Q}(x)\triangle_{14}^{'} - \triangle_{14}^{''} &= \{(1.14,1.99,3.30),[0.1,0.3],[0.6,0.8]\}\{(-16,-5,5),[0.2,0.3],[0.5,0.6]\} \\
&- \{(-14,1,9),[0.2,0.3],[0.5,0.6]\} \\
&= \{(-61.8,-10.95,30.5),[0.1,0.3],[0.6,0.8]\} < 0.
\end{aligned}$$

$$\begin{aligned}
\triangle_{21}^{'} &= \hat{u}_2^{'} + \hat{v}_1^{'} - \hat{c}_{21} \\
&= \{(-7,3,13),[0.2,0.5],[0.6,0.8]\} + \{(7,8,9),[0.3,0.5],[0.2,0.4]\} \\
&- \{(10,11,15),[0.4,0.5],[0.1,0.3]\} \\
&= \{(-15,-1,12),[0.2,0.5],[0.6,0.8]\}, \\
\triangle_{21}^{''} &= \hat{u}_2^{''} + \hat{v}_1^{''} - \hat{p}_{21} \\
&= \{(-5,0,9),[0.2,0.3],[0.5,0.6]\} + \{(3,5,6),[0.2,0.3],[0.3,0.35]\} \\
&- \{(3,5,6),[0.8,0.9],[0.5,0.6]\} \\
&= \{(-8,0,12),[0.2,0.3],[0.5,0.6]\} \\
\hat{Q}(x)\triangle_{21}^{'} - \triangle_{21}^{''} &= \{(1.14,1.99,3.30),[0.1,0.3],[0.6,0.8]\}\{(-15,-1,12),[0.2,0.5],[0.6,0.8]\} \\
&- \{(-8,0,12),[0.2,0.3],[0.5,0.6]\} \\
&= \{(-48.672,-1.094,40.337),[0.2,0.3],[0.6,0.8]\} < 0.
\end{aligned}$$

$$
\begin{aligned}
\triangle_{22}^{'} =&\hat{u}_2^{'} + \hat{v}_2^{'} - \hat{c}_{22}\\
&= \{(-7,3,13),[0.2,0.5],[0.6,0.8]\} + \{(4,6,7),[0.5,0.6],[0.2,0.3]\}\\
&- \{(10,12,13),[0.5,0.8],[0.4,0.7]\}\\
&= \{(-16,-3,10),[0.2,0.5],[0.6,0.8]\},\\
\triangle_{22}^{''} &= \hat{u}_2^{''} + \hat{v}_2^{''} - \hat{p}_{22}\\
&= \{(-5,0,9),[0.2,0.3],[0.5,0.6]\} + \{(3,4,5),[0.4,0.5],[0.1,0.4]\}\\
&- \{(1,2,3),[0.6,0.7],[0.2,0.8]\}\\
&= \{(-5,2,13),[0.2,0.3],[0.5,0.8]\},\\
\hat{Q}(x)\triangle_{22}^{'} - \triangle_{22}^{''} &= \{(1.14,1.99,3.30),[0.1,0.3],[0.6,0.8]\}\{(-16,-3,10),[0.2,0.5],[0.6,0.8]\}\\
&- \{(-5,2,13),[0.2,0.3],[0.5,0.8]\}\\
&= \{(-65.8,-7.97,35),[0.1,0.3],[0.6,0.8]\} < 0.
\end{aligned}
$$

$$
\begin{aligned}
\triangle_{32}^{'} &= \hat{u}_3^{'} + \hat{v}_2^{'} - \hat{c}_{32}\\
&= \{(3,6,9),[0.3,0.5],[0.2,0.4]\} + \{(4,6,7),[0.5,0.6],[0.2,0.3]\}\\
&- \{(12,13,14),[0.2,0.3],[0.15,0.95]\}\\
&= \{(-7,-1,4),[0.2,0.3],[0.2,0.95]\},\\
\triangle_{32}^{''} &= \hat{u}_3^{''} + \hat{v}_2^{''} - \hat{p}_{32}\\
&= \{(0,3,7),[0.2,0.3],[0.3,0.35]\} + \{(3,4,5),[0.4,0.5],[0.1,0.4]\}\\
&- \{(3,4,5),[0.75,0.85],[0.65,0.7]\}\\
&= \{(-2,3,9),[0.3,0.35],[0.65,0.7]\},\\
\hat{Q}(x)\triangle_{32}^{'} - \triangle_{32}^{''} &= \{(1.14,1.99,3.30),[0.1,0.3],[0.6,0.8]\}\{(-7,-1,4),[0.2,0.3],[0.2,0.95]\}\\
&- \{(-2,3,9),[0.3,0.35],[0.65,0.7]\}\\
&= \{(-32.1,-4.99,15.2),[0.1,0.3],[0.65,0.8]\} < 0.
\end{aligned}
$$

$$
\begin{aligned}
\triangle_{34}^{'} &= \hat{u}_3^{'} + \hat{v}_4^{'} - \hat{c}_{34}\\
&= \{(3,6,9),[0.3,0.5],[0.2,0.4]\} + \{(-6,4,11),[0.2,0.3],[0.5,0.6]\}\\
&- \{(9,11,14),[0.7,0.8],[0.3,0.4]\}\\
&= \{(-17,-1,11),[0.2,0.3],[0.5,0.6]\},\\
\triangle_{34}^{''} &= \hat{u}_3^{''} + \hat{v}_4^{''} - \hat{p}_{34}\\
&= \{(0,3,7),[0.2,0.3],[0.3,0.35]\} + \{(-6,4,11),[0.2,0.3],[0.5,0.6]\}\\
&- \{(3,4,6),[0.3,0.6],[0.4,0.5]\}\\
&= \{(-12,3,15),[0.2,0.3],[0.5,0.6]\},\\
\hat{Q}(x)\triangle_{34}^{'} - \triangle_{34}^{''} &= \{(1.14,1.99,3.30),[0.1,0.3],[0.6,0.8]\}\{(-17,-1,11),[0.2,0.3],[0.5,0.6]\}\\
&- \{(-12,3,15),[0.2,0.3],[0.5,0.6]\}\\
&= \{(-71.1,-4.99,48.3),[0.1,0.3],[0.6,0.8]\} < 0.
\end{aligned}
$$

Finally, we obtain $\hat{Q}(x)\triangle_{ij}^{'} - \triangle_{ij}^{''} < 0$ for all non-basic cells. Therefore, $x_{11} = 5,\ x_{12} = 35,\ x_{23} =$

20, $x_{24} = 25$, $x_{31} = 30$, $x_{33} = 5$ are optimal solutions, and the IVFF optimal value is

$$\hat{Q}(x) = \frac{\{(925, 1185, 1435), [0.1, 0.3], [0.6, 0.8]\}}{\{(345, 535, 720), [0.2, 0.3], [0.5, 0.6]\}} = \{(1.14, 1.99, 3.30), [0.1, 0.3], [0.6, 0.8]\}.$$

The graphical representation of the IVFF optimal value is given in Figure 3.

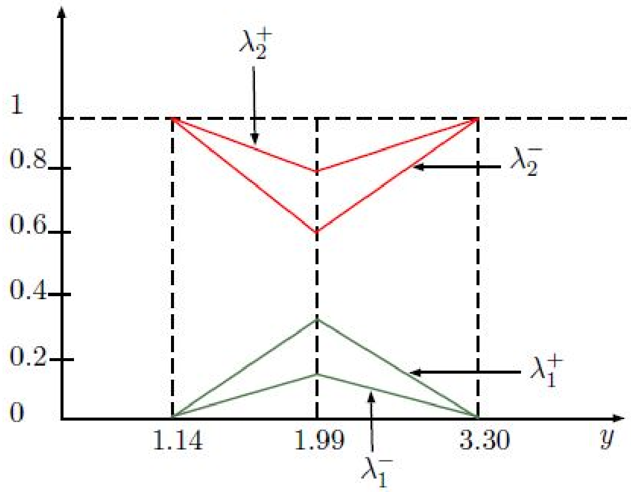

Figure 3. IVFF optimal value.

Example 5.2. [56] A car firm wants to transfer cars from three different supply stations (W1, W2, and W3) to three different sales locations (M1, M2, and M3). There are 30, 30 and 10 cars available at the supply points, respectively. The three sales locations have demands for 25, 25 and 20 cars, respectively. The profit and cost per car shipped from supply to sales locations are shown in the Tables 7 and 8, respectively.

Table 7. Cost per car.

	M_1	M_2	M_3
W_1	$\{(8, 10, 11), [0.3, 0.7], [0.5, 0.8]\}$	$\{(5, 7, 9), [0.2, 0.4], [0.5, 0.8]\}$	$\{(10, 12, 13), [0.3, 0.6], [0.1, 0.2]\}$
W_2	$\{(7, 10, 12), [0.3, 0.6], [0.4, 0.8]\}$	$\{(5, 8, 9), [0.3, 0.5], [0.2, 0.7]\}$	$\{(4, 8, 12), [0.1, 0.2], [0.3, 0.7]\}$
W_3	$\{(20, 23, 25), [0.4, 0.5], [0.3, 0.4]\}$	$\{(12, 14, 16), [0.1, 0.2], [0.6, 0.7]\}$	$\{(13, 15, 17), [0.3, 0.5], [0.8, 0.9]\}$

Table 8. Profit per car.

	M_1	M_2	M_3
W_1	$\{(4, 7, 8), [0.5, 0.6], [0.4, 0.8]\}$	$\{(4, 5, 10), [0.3, 0.7], [0.4, 0.6]\}$	$\{(7, 10, 14), [0.1, 0.4], [0.3, 0.7]\}$
W_2	$\{(3, 4, 9), [0.1, 0.5], [0.4, 0.8]\}$	$\{(1, 2, 3), [0.1, 0.4], [0.4, 0.9]\}$	$\{(13, 15, 17), [0.1, 0.4], [0.2, 0.5]\}$
W_3	$\{(5, 6, 7), [0.4, 0.6], [0.1, 0.3]\}$	$\{(1, 2, 4), [0.3, 0.5], [0.8, 0.9]\}$	$\{(4, 5, 6), [0.2, 0.3], [0.8, 0.9]\}$

Let

$$P(x) = \{(8, 10, 11), [0.3, 0.7], [0.5, 0.8]\}x_{11} + \{(5, 7, 9), [0.2, 0.4], [0.5, 0.8]\}x_{12} + \{(10, 12, 13), [0.3, 0.6], [0.1, 0.2]\}x_{13} + \{(7, 10, 12), [0.3, 0.6], [0.4, 0.8]\}x_{21} + \{(5, 8, 9), [0.3, 0.5], [0.2, 0.7]\}x_{22} + \{(4, 8, 12),$$

$[0.1, 0.2], [0.3, 0.7]\}x_{23}\{(20, 23, 25), [0.4, 0.5], [0.3, 0.4]\}x_{31} + \{(12, 14, 16), [0.1, 0.2], [0.6, 0.7]\}x_{32} + \{(13, 15, 17), [0.3, 0.5], [0.8, 0.9]\}x_{33},$

$R(x) = \{(4, 7, 8), [0.5, 0.6], [0.4, 0.8]\}x_{11} + \{(4, 5, 10), [0.3, 0.7], [0.4, 0.6]\}x_{12} + \{(7, 10, 14), [0.1, 0.4], [0.3, 0.7]\}x_{13} + \{(3, 4, 9), [0.1, 0.5], [0.4, 0.8]\}x_{21} + \{(1, 2, 3), [0.1, 0.4], [0.4, 0.9]\}x_{22} + \{(13, 15, 17), [0.1, 0.4], [0.2, 0.5]\}x_{23} + \{(5, 6, 7), [0.4, 0.6], [0.1, 0.3]\}x_{31} + \{(1, 2, 4), [0.3, 0.5], [0.8, 0.9]\}x_{32} + \{(4, 5, 6), [0.2, 0.3], [0.8, 0.9]\}x_{33}.$

Then, the mathematical model of the linear IVFFFTP is

$$\begin{aligned} &\text{Min } \frac{P(x)}{R(x)} \\ &\text{subject to } \sum_{j=1}^{3} x_{1j} \leq 30, \quad \sum_{j=1}^{3} x_{2j} \leq 30, \quad \sum_{j=1}^{3} x_{3j} \leq 10, \\ &\sum_{i=1}^{3} x_{i1} \geq 25, \quad \sum_{i=1}^{3} x_{i2} \geq 25, \quad \sum_{i=1}^{3} x_{i3} \geq 20, \\ &x_{ij} \geq 0, \quad i = 1, 2, 3, \quad j = 1, 2, 3. \end{aligned} \tag{5.2}$$

We represent the given linear IVFFFTP in Table 9.

Table 9. Data of Example 5.2.

{(8,10,11),[0.3,0.7],[0.5,0.8]} {(4,7,8),[0.5,0.6],[0.4,0.8]}	{(5,7,9),[0.2,0.4],[0.5,0.8]} {(4,5,10),[0.3,0.7],[0.4,0.6]}	{(10,12,13),[0.3,0.6],[0.1,0.2]} {(7,10,14),[0.1,0.4],[0.3,0.7]}	30
{(7,10,12),[0.3,0.6],[0.4,0.8]} {(3,4,9),[0.1,0.5],[0.4,0.8]}	{(5,8,9),[0.3,0.5],[0.2,0.7]} {(1,2,3),[0.1,0.4],[0.4,0.9]}	{(4,8,12),[0.1,0.2],[0.3,0.7]} {(13,15,17),[0.1,0.4],[0.2,0.5]}	30
{(20, 23, 25), [0.4, 0.5], [0.3, 0.4]} {(5, 6, 7), [0.4, 0.6], [0.1, 0.3]}	{(12, 14, 16), [0.1, 0.2], [0.6, 0.7]} {(1, 2, 4), [0.3, 0.5], [0.8, 0.9]}	{(13, 15, 17), [0.3, 0.5], [0.8, 0.9]} {(4, 5, 6), [0.2, 0.3], [0.8, 0.9]}	10
25	25	20	

By applying the proposed method, the initial interval-valued FFBFS is given in Table 10.

Table 10. Initial interval-valued FFBFS of problem 5.2.

{(8,10,11),[0.3,0.7],[0.5,0.8]} $\boxed{15}$ {(4,7,8),[0.5,0.6],[0.4,0.8]}	{(5,7,9),[0.2,0.4],[0.5,0.8]} $\boxed{15}$ {(4,5,10),[0.3,0.7],[0.4,0.6]}	{(10,12,13),[0.3,0.6],[0.1,0.2]} {(7,10,14),[0.1,0.4],[0.3,0.7]}	30
{(7,10,12),[0.3,0.6],[0.4,0.8]} $\boxed{10}$ {(3,4,9),[0.1,0.5],[0.4,0.8]}	{(5,8,9),[0.3,0.5],[0.2,0.7]} {(1,2,3),[0.1,0.4],[0.4,0.9]}	{(4,8,12),[0.1,0.2],[0.3,0.7]} $\boxed{20}$ {(13,15,17),[0.1,0.4],[0.2,0.5]}	30
{(20, 23, 25), [0.4, 0.5], [0.3, 0.4]} {(5, 6, 7), [0.4, 0.6], [0.1, 0.3]}	{(12, 14, 16), [0.1, 0.2], [0.6, 0.7]} $\boxed{10}$ {(1, 2, 4), [0.3, 0.5], [0.8, 0.9]}	{(13, 15, 17), [0.3, 0.5], [0.8, 0.9]} {(4, 5, 6), [0.2, 0.3], [0.8, 0.9]}	10
25	25	20	

Hence, the initial interval-valued FFBFS to the given linear IVFFFTP is obtained as $x_{11} = 5, \ x_{12} =$

35, $x_{23} = 20$, $x_{24} = 25$, $x_{31} = 30$, $x_{33} = 5$ and

$$\hat{Q}(x) = \frac{\{(545, 755, 930), [0.1, 0.2], [0.6, 0.8]\}}{\{(420, 540, 740), [0.1, 0.4], [0.8, 0.9]\}} = \{(0.74, 1.39, 2.21), [0.1, 0.2], [0.8, 0.9]\}.$$

Put $\hat{u}_1^{'} = 0$, and we get
$\hat{u}_2^{'} = \{(-4, 0, 4), [0.3, 0.6], [0.5, 0.8]\}$,
$\hat{u}_3^{'} = \{(3, 7, 11), [0.1, 0.2], [0.6, 0.8]\}$,
$\hat{v}_1^{'} = \{(8, 10, 11), [0.3, 0.7], [0.5, 0.8]\}$,
$\hat{v}_2^{'} = \{(5, 7, 9), [0.2, 0.4], [0.5, 0.8]\}$,
$\hat{v}_3^{'} = \{(0, 8, 16), [0.1, 0.2], [0.5, 0.8]\}$.
Similarly, put $\hat{u}_1^{''} = 0$, and then
$\hat{u}_2^{''} = \{(-5, -3, 5), [0.1, 0.5], [0.4, 0.8]\}$,
$\hat{u}_3^{''} = \{(-9, -3, 0), [0.1, 0.4], [0.4, 0.8]\}$,
$\hat{v}_1^{''} = \{(4, 7, 8), [0.5, 0.6], [0.4, 0.8]\}$,
$\hat{v}_2^{''} = \{(4, 5, 10), [0.3, 0.7], [0.4, 0.6]\}$,
$\hat{v}_3^{''} = \{(8, 18, 22), [0.1, 0.4], [0.4, 0.8]\}$.
The net evaluation corresponding to all non-basic cells is

$$\begin{aligned}
\triangle_{13}^{'} &= \hat{u}_1^{'} + \hat{v}_3^{'} - \hat{c}_{13} \\
&= 0 + \{(0, 8, 16), [0.1, 0.2], [0.5, 0.8]\} - \{(10, 12, 13), [0.3, 0.6], [0.1, 0.2]\} \\
&= \{(-13, -4, 6), [0.1, 0.2], [0.5, 0.8]\}, \\
\triangle_{13}^{''} &= \hat{u}_1^{''} + \hat{v}_3^{''} - \hat{p}_{13} \\
&= 0 + \{(8, 18, 22), [0.1, 0.4], [0.4, 0.8]\} - \{(7, 10, 14), [0.1, 0.4], [0.3, 0.7]\} \\
&= \{(-6, 8, 15), [0.1, 0.4], [0.4, 0.8]\}, \\
\hat{Q}(x)\triangle_{13}^{'} - \triangle_{13}^{''} &= \{(0.74, 1.39, 2.21), [0.1, 0.2], [0.8, 0.9]\}\{(-13, -4, 6), [0.1, 0.2], [0.5, 0.8]\} \\
&\quad - \{(-6, 8, 15), [0.1, 0.4], [0.4, 0.8]\} \\
&= \{(-43.73, -13.56, 19.26), [0.1, 0.2], [0.8, 0.9]\} < 0.
\end{aligned}$$

$$\begin{aligned}
\triangle_{22}^{'} &= \hat{u}_2^{'} + \hat{v}_2^{'} - \hat{c}_{22} \\
&= \{(-4, 0, 4), [0.3, 0.6], [0.5, 0.8]\} + \{(5, 7, 9), [0.2, 0.4], [0.5, 0.8]\} \\
&\quad - \{(5, 8, 9), [0.3, 0.5], [0.2, 0.7]\} \\
&= \{(-8, -1, 3), [0.2, 0.4], [0.5, 0.8]\}, \\
\triangle_{22}^{''} &= \hat{u}_2^{''} + \hat{v}_2^{''} - \hat{p}_{22} \\
&= \{(-5, -3, 5), [0.1, 0.5], [0.4, 0.8]\} + \{(4, 5, 10), [0.3, 0.7], [0.4, 0.6]\} \\
&\quad - \{(1, 2, 3), [0.1, 0.4], [0.4, 0.9]\} \\
&= \{(-4, 0, 14), [0.1, 0.4], [0.4, 0.9]\}, \\
\hat{Q}(x)\triangle_{22}^{'} - \triangle_{22}^{''} &= \{(0.74, 1.39, 2.21), [0.1, 0.2], [0.8, 0.9]\}\{(-8, -1, 3), [0.2, 0.4], [0.5, 0.8]\} \\
&\quad - \{(-4, 0, 14), [0.1, 0.4], [0.4, 0.9]\} \\
&= \{(-31.68, -1.39, 10.63), [0.1, 0.2], [0.8, 0.9]\} < 0.
\end{aligned}$$

$$
\begin{aligned}
\triangle_{31}^{'} &= \hat{u}_3^{'} + \hat{v}_1^{'} - \hat{c}_{31}\\
&= \{(3,7,11),[0.1,0.2],[0.6,0.8]\} + \{(8,10,11),[0.3,0.7],[0.5,0.8]\}\\
&- \{(20,23,25),[0.4,0.5],[0.3,0.4]\}\\
&= \{(-14,-6,2),[0.1,0.2],[0.6,0.8]\},\\
\triangle_{31}^{''} &= \hat{u}_3^{''} + \hat{v}_1^{''} - \hat{p}_{31}\\
&= \{(-9,-3,0),[0.3,0.5],[0.8,0.9]\} + \{(4,7,8),[0.5,0.6],[0.4,0.8]\}\\
&- \{(5,6,7),[0.4,0.6],[0.1,0.3]\}\\
&= \{(-12,-2,3),[0.3,0.5],[0.8,0.9]\},\\
\hat{Q}(x)\triangle_{13}^{'} - \triangle_{13}^{''} &= \{(0.74,1.39,2.21),[0.1,0.2],[0.8,0.9]\}\{(-14,-6,2),[0.1,0.2],[0.6,0.8]\}\\
&- \{(-12,-2,3),[0.3,0.5],[0.8,0.9]\}\\
&= \{(-33.94,-6.34,16.42),[0.1,0.2],[0.8,0.9]\} < 0.
\end{aligned}
$$

$$
\begin{aligned}
\triangle_{33}^{'} &= \hat{u}_3^{'} + \hat{v}_3^{'} - \hat{c}_{33}\\
&= \{(3,7,11),[0.1,0.2],[0.6,0.8]\} + \{(0,8,16),[0.1,0.2],[0.5,0.8]\}\\
&- \{(13,15,17),[0.3,0.5],[0.8,0.9]\}\\
&= \{(-14,0,14),[0.1,0.2],[0.8,0.9]\},\\
\triangle_{33}^{''} &= \hat{u}_3^{''} + \hat{v}_3^{''} - \hat{p}_{33}\\
&= \{(-9,-3,0),[0.3,0.5],[0.8,0.9]\} + \{(8,18,22),[0.1,0.4],[0.4,0.8]\}\\
&- \{(4,5,6),[0.2,0.3],[0.8,0.9]\}\\
&= \{(-7,10,18),[0.2,0.3],[0.8,0.9]\},\\
\hat{Q}(x)\triangle_{13}^{'} - \triangle_{13}^{''} &= \{(0.74,1.39,2.21),[0.1,0.2],[0.8,0.9]\}\{(-14,0,14),[0.1,0.2],[0.8,0.9]\}\\
&- \{(-7,10,18),[0.2,0.3],[0.8,0.9]\}\\
&= \{(-48.94,-10,37.94),[0.1,0.2],[0.8,0.9]\} < 0.
\end{aligned}
$$

Finally, we obtain $\hat{Q}(x)\triangle_{ij}^{'} - \triangle_{ij}^{''} < 0$ for all non-basic cells. Therefore, $x_{11} = 15,\ x_{12} = 15,\ x_{21} = 10,\ x_{23} = 20$, $x_{32} = 10$ are optimal solutions, and the IVFF optimal value is

$$\hat{Q}(x) = \frac{\{(350,535,700),[0.1,0.2],[0.5,0.8]\}}{\{(425,520,765),[0.1,0.4],[0.4,0.8]\}} = \{(0.74,1.39,2.21),[0.1,0.2],[0.8,0.9]\}.$$

The graphical representation of the IVFF optimal value is given in Figure 4.

6. Comparative analysis

Now, if we solve Example 5.2 by maximum profit and Vogel's approximation methods [56] (with respect to cost), then the initial interval-valued FFBFSs are given in Tables 11 and 12 , respectively.

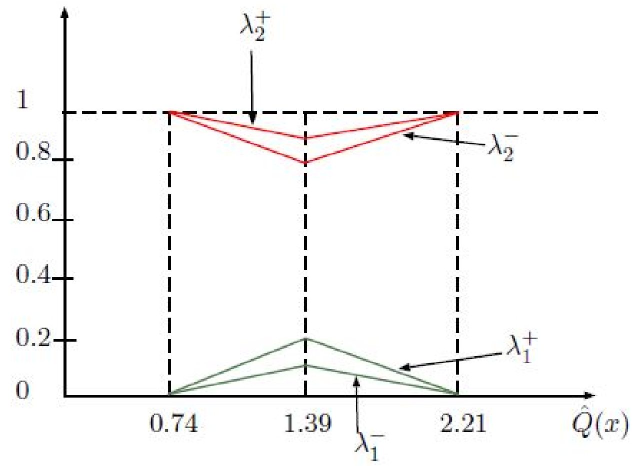

Figure 4. IVFF optimal value.

Table 11. Solution by maximum profit method.

{(8,10,11),[0.3,0.7],[0.5,0.8]} $\boxed{5}$ {(4,7,8),[0.5,0.6],[0.4,0.8]}	{(5,7,9),[0.2,0.4],[0.5,0.8]} $\boxed{25}$ {(4,5,10),[0.3,0.7],[0.4,0.6]}	{(10,12,13),[0.3,0.6],[0.1,0.2]} {(7,10,14),[0.1,0.4],[0.3,0.7]}	30
{(7,10,12),[0.3,0.6],[0.4,0.8]} $\boxed{10}$ {(3,4,9),[0.1,0.5],[0.4,0.8]}	{(5,8,9),[0.3,0.5],[0.2,0.7]} {(1,2,3),[0.1,0.4],[0.4,0.9]}	{(4,8,12),[0.1,0.2],[0.3,0.7]} $\boxed{20}$ {(13,15,17),[0.1,0.4],[0.2,0.5]}	30
{(20, 23, 25), [0.4, 0.5], [0.3, 0.4]} $\boxed{10}$ {(5, 6, 7), [0.4, 0.6], [0.1, 0.3]}	{(12, 14, 16), [0.1, 0.2], [0.6, 0.7]} {(1, 2, 4), [0.3, 0.5], [0.8, 0.9]}	{(13, 15, 17), [0.3, 0.5], [0.8, 0.9]} {(4, 5, 6), [0.2, 0.3], [0.8, 0.9]}	10
25	25	20	

So, the initial interval-valued FFBFS to the given linear IVFFFTP using the maximum profit method is obtained as $x_{11} = 5,\ x_{12} = 25,\ x_{21} = 10,\ x_{23} = 20,\ x_{31} = 10$ and

$$\hat{Q}(x) = \frac{\{(515, 715, 890), [0.1, 0.2], [0.5, 0.8]\}}{\{(610, 730, 980), [0.1, 0.4], [0.4, 0.8]\}} = \{(0.53, 0.98, 1.46), [0.1, 0.2], [0.5, 0.8]\}.$$

Table 12. Solution by Vogel's approximation method.

{(8,10,11),[0.3,0.7],[0.5,0.8]} $\boxed{5}$ {(4,7,8),[0.5,0.6],[0.4,0.8]}	{(5,7,9),[0.2,0.4],[0.5,0.8]} $\boxed{25}$ {(4,5,10),[0.3,0.7],[0.4,0.6]}	{(10,12,13),[0.3,0.6],[0.1,0.2]} {(7,10,14),[0.1,0.4],[0.3,0.7]}	30
{(7,10,12),[0.3,0.6],[0.4,0.8]} $\boxed{20}$ {(3,4,9),[0.1,0.5],[0.4,0.8]}	{(5,8,9),[0.3,0.5],[0.2,0.7]} {(1,2,3),[0.1,0.4],[0.4,0.9]}	{(4,8,12),[0.1,0.2],[0.3,0.7]} $\boxed{10}$ {(13,15,17),[0.1,0.4],[0.2,0.5]}	30
{(20, 23, 25), [0.4, 0.5], [0.3, 0.4]} {(5, 6, 7), [0.4, 0.6], [0.1, 0.3]}	{(12, 14, 16), [0.1, 0.2], [0.6, 0.7]} {(1, 2, 4), [0.3, 0.5], [0.8, 0.9]}	{(13, 15, 17), [0.3, 0.5], [0.8, 0.9]} $\boxed{10}$ {(4, 5, 6), [0.2, 0.3], [0.8, 0.9]}	10
25	25	20	

Therefore, the initial interval-valued FFBFS to the given linear IVFFFTP using Vogel's approximation method (with respect to cost) is obtained as $x_{11} = 5,\ x_{12} = 25,\ x_{21} = 20,\ x_{23} = 10,\ x_{33} = 10$, and

$$\hat{Q}(x) = \frac{\{(475, 655, 810)\}, [0.1, 0.2], [0.8, 0.9]}{\{(350, 440, 700), [0.1, 0.3], [0.8, 0.9]\}} = \{(0.68, 1.49, 2.31), [0.1, 0.2], [0.8, 0.9]\}.$$

Comparison of the proposed method with the maximum profit and Vogel's approximation methods is given in Table 13. It is clear that the proposed method gives better results than the maximum profit and Vogel's approximation methods.

Table 13. Comparison of the solutions for Example 5.2.

	Maximum profit method [56]	Vogel's method [56] (with respect to cost)	Proposed method
X^*	(5,25,0,10,0,20,10,0,0)	(5,25,0,20,0,10,0,0,10)	(15,15,0,10,0,20,0,10,0)
$\hat{Q}(x)$	{(0.53,0.98,1.46),[0.1,0.2],[0.5,0.8]}	{(0.68,1.49,2.31),[0.1,0.2],[0.8,0.9]}	{(0.74,1.39,2.21),[0.1,0.2],[0.8,0.9]}
Ranking	0.25	0.22	0.21

The above table shows that our proposed method for determining the interval-valued Fermatean fuzzy optimal cost of the IVFFFTP is preferable compared to the maximum profit and Vogel methods [56]. Also, the maximum profit method simply maximizes the profit, i.e., maximizes the denominator of the FTP, without considering the cost of the FTP (i.e., the numerator of the FTP). In this way, Vogel's method also determines the interval-valued Fermatean fuzzy optimal cost of the IVFFFTP. On the other hand, our proposed method determines the interval-valued Fermatean fuzzy optimal cost of the IVFFFTP by considering the cost and profit of the IVFFFTP. Therefore, our proposed method is preferable compared to these methods.

7. Conclusions

The FTP is designed for the movement of varying quantities of a single homogeneous item from multiple origins to multiple destinations while minimizing overall transportation costs. In this manuscript, we have defined the TIVFFN and formulated its arithmetic operations. We have proposed a straightforward method for solving a linear IVFFFTP. By applying the aforementioned IVFF algorithm and ordering, we have obtained the IVFF optimal solution for a given linear IVFFFTP without re-transforming the original problem into a classical one. We have provided numerical examples to demonstrate the performance and superiority of the proposed method. Furthermore, we have compared our proposed method with maximal profit and Vogel's methods [56]. One of the main advantages of the proposed method is the minimum cost of the IVFFFTP is obtained, compared to the maximum profit and Vogel methods [56]. However, this approach is only valuable if demand and supply are given as clear values. In the future, we will develop an LP-type method for determining the optimal solution of the FTP in the Fermatean fuzzy environment. Furthermore, we would like to point out that the proposed method cannot be used to determine the Fermatean fuzzy optimal solution for an unbalanced IVFFFTP. Therefore, further research to extend the proposed method to address these shortcomings is an interesting way for future research. We will report significant results from these ongoing projects in the near future. Furthermore, we plan to extend our research work to address the multi-objective fractional transport problem.

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Conflict of interest

The authors declare no conflict of interest.

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