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Research article

On the fractional-order mathematical model of COVID-19 with the effects of multiple non-pharmaceutical interventions

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Abstract: In this article, the Caputo fractional derivative operator of different orders $0 < \alpha \leq 1$ is applied to formulate the fractional-order model of the COVID-19 pandemic. The existence and boundedness of the solutions of the model are investigated by using the Gronwall-Bellman inequality. Further, the uniqueness of the model solutions is established by using the fixed-point theory. The Laplace Adomian decomposition method is used to obtain an approximate solution of the nonlinear system of fractional-order differential equations of the model with a different fractional-order α for every compartment in the model. Finally, graphical presentations are presented to show the effects of other fractional parameters α on the obtained approximate solutions.

Keywords: Adomian decomposition method; COVID-19; Uniqueness of the solutions; Arzelá-Ascoli theorem; Schauder's fixed-point theorem
Mathematics Subject Classification: 92B05, 92C10

1. Introduction

The outbreak of the coronavirus started in December 2019 in China [1]. It has been categorized as a pandemic by the World Health Organization. This new virus causes a severe acute respiratory illness, which can lead to death, that has been spread worldwide. This badly affects the general health system and the economic growth of the developing and underdeveloped countries. Because of the beginning of the first outbreaks, it is assessed that the worldwide disease burden due to COVID-19 amounts to over 91.2 million people infected globally and 1.9 million deaths worldwide [1, 2]. From the coronavirus across 215 countries has brought the whole word the series threat [3]. It has been confirmed that large number of huge reported COVID-19 cases tend to be observed as mild to moderate respiratory disorders with symptoms, like coughing, fever, and breathing difficulty. This virus transmits mostly through beads from the nose or mouth when an infected individual sneezes or coughs. When the uninfected one breathes the droplets from the air, they will be presented with the risk of getting the infection. As a result, the best technique to control the spread of the coronavirus is to keep it away from people. Thus, there have been research works to interpret the mechanism to prevent the spread of the coronavirus and examine the impacts of different drugs and non-drug mediation. Mathematical modeling is an importing technique for investigating the underlying dynamics of a virus disperse and developing a control technique when enough data about the infection is scant. Models have also been utilized to predict the infection elements and estimate the efficiency of the intervention strategies disease spreading [4–7]. In this way different models have been developed that focus on minimizing the spread of coronavirus [8–10], such as the isolation of victims, maintaining social distancing, face maskes, washing hands with soap, all official and unofficial gatherings like sports club, games matches, schools, colleges and universities. Mumbu et al. [10] and Verma et al. [11] proposed a model with seven compartments, which include the susceptible population, exposed population, infected population, asymptotic population and recovered population. This model shows the coronavirus infection India and South Korea countries.

Fractional order epidemic models are more helpful and realistic as a tool to evaluate the dynamics of a contagious disease than classical integer order models [12, 13]. Because classical mathematical models do not provide a high degree of accuracy for modeling these diseases, fractional differential equations were developed to handle such problems, which have many applications in applied fields related to optimization problems, production problems, artificial intelligence, robotics, cosmology, medical diagnoses, and many more [14, 15]. The fractional differential equation has been employed in the mathematical modelling of biological phenomena for many decades . Because mathematical models are effective tools for studying infectious diseases, recently, a lot of scientists have been investing mathematical models of the COVID-19 pandemic with fractional order derivatives; they have produced excellent results [16, 17].

Recently, Srivastav et al. [18] developed another compartmental model of the COVID-19 pandemic, particularly for the effects of face masks, hospitalization, and quarantine as follows:

$$\begin{cases} \dot{\mathbb{S}} = \delta - \beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_s - \beta^{'}(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \lambda\mathbb{S} \\ \dot{\mathbb{E}} = \beta 1(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a + \beta^{'}(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \sigma\mathbb{E} - \lambda\mathbb{E} \\ \dot{\mathbb{I}}_s = (1-a)\sigma\mathbb{E} + \rho_a\mathbb{I}_a - (\lambda_s+\lambda)\mathbb{I}_s - \phi_s\mathbb{I}_s \\ \dot{\mathbb{I}}_a = a\sigma\mathbb{E} - \tau_a\mathbb{I}_a - \rho_a\mathbb{I}_a - \lambda\mathbb{I}_a \\ \dot{\mathbb{H}} = \phi_s\mathbb{I}_s - (\tau_h+\lambda_h+\lambda)\mathbb{H} \\ \dot{\mathbb{Q}}_h = \tau_a\mathbb{I}_a - \rho_h\mathbb{Q}_h - \lambda\mathbb{Q} \\ \dot{\mathbb{R}} = \rho_h\mathbb{Q}_h + \tau_h\mathbb{H} - \lambda\mathbb{R} \end{cases} . \tag{1.1}$$

The transmission dynamics of Model (1.1) are represented in the Figure 1 flowchart.

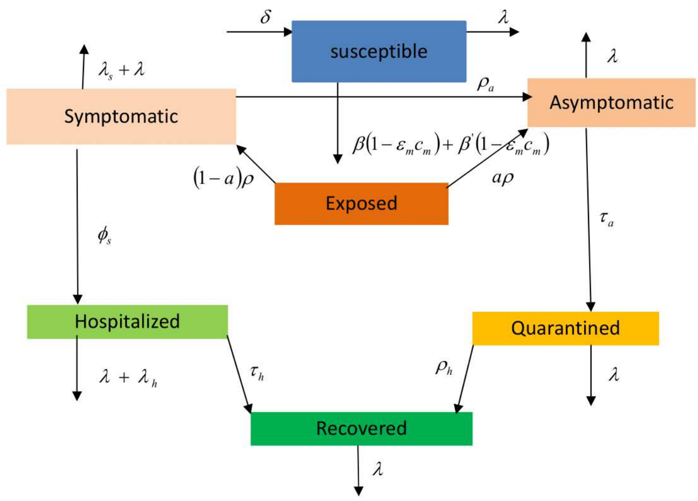

Figure 1. Flow diagram of the COVID-19 model (1.1).

In this model, they have divided the whole population $\mathbb{N}(t)$ into seven epidemiological groups: $\mathbb{S}(t)$ (susceptible population), $\mathbb{E}(t)$ (exposed population), $\mathbb{I}_s(t)$ (symptomatic population), $\mathbb{I}_a(t)$ (asymptomatic population), $\mathbb{H}(t)$ (hospitalized population), $\mathbb{Q}_h(t)$ (quarantined population) and $\mathbb{R}(t)$ (recovered population). They categorized the infectious peoples into two subgroups $\mathbb{I}_a$ and $\mathbb{I}_s$. They assumed that as quickly as symptoms appear, an $\mathbb{I}_a$ individual would join the $\mathbb{I}_s$ class. The biological interpretations of parameters are given in Table 1.

Table 1. Biological interpretations of the parameters of Model (1.1).

Parameters	Biological Interpretations
δ	Recruitment rate for the susceptible population
β'	Rate of infection of $\mathbb{S}$ among $\mathbb{I}_a$ population
β	Rate of infection of $\mathbb{S}$ among $\mathbb{I}_s$ population
ϵ_m	Effectiveness of face masks
e_m	Masks compliance
λ	Human mortality rate due to natural causes
σ	Rate of incubation
a	Fraction of those exposed who do not exhibit clinical symptoms
ρ_a	Rate of progression from $\mathbb{I}_a$ to $\mathbb{I}_s$ class
ρ_h	Recovery rate for $\mathbb{H}$ individuals
λ_s	Disease induced mortality rate for $\mathbb{I}_s$ class
τ_a	$\mathbb{Q}$ rate for $\mathbb{I}_a$ class
τ_h	Recovery rate for $\mathbb{R}$ class
λ_h	Disease induced mortality rate for $\mathbb{H}$ class
ϕ_s	Rate of $\mathbb{H}$ for $\mathbb{I}_s$ individuals

Their model focused on the effects of face masks, hospitalization of the $\mathbb{I}_a$ population and quarantine of $\mathbb{I}_s$ individuals on the spread of the coronavirus. Recently many researchers have proposed models of the COVID-19 pandemic based on the Caputo fractional derivative; they obtained some good results; for details see [19, 20]. Consequently, inspired by the previously mentioned work, here we study Model (1.1) under the conditions of using fractional-order derivatives. For $\alpha \in (0, 1]$,

$$\begin{cases} {}_0^c\mathscr{D}_t^\alpha[\mathbb{S}(t)] = \delta - \beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_s - \beta^{'}(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \lambda\mathbb{S} \\ {}_0^c\mathscr{D}_t^\alpha[\mathbb{E}(t)] = \beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a + \beta^{'}(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \sigma\mathbb{E} - \lambda\mathbb{E} \\ {}_0^c\mathscr{D}_t^\alpha[\mathbb{I}_s(t)] = (1-a)\sigma\mathbb{E} + \rho_a\mathbb{I}_a - (\lambda_s+\lambda)\mathbb{I}_s - \phi_s\mathbb{I}_s \\ {}_0^c\mathscr{D}_t^\alpha[\mathbb{I}_a(t)] = a\sigma\mathbb{E} - \tau_a\mathbb{I}_a - \rho_a\mathbb{I}_a - \lambda\mathbb{I} \\ {}_0^c\mathscr{D}_t^\alpha[\mathbb{H}(t)] = \phi_s\mathbb{I}_s - (\tau_h + \lambda_h + \lambda)\mathbb{H} \\ {}_0^c\mathscr{D}_t^\alpha[\mathbb{Q}_h(t)] = \tau_a\mathbb{I}_a - \rho_h\mathbb{Q}_h - \lambda\mathbb{Q} \\ {}_0^c\mathscr{D}_t^\alpha[\mathbb{R}(t)] = \rho_h\mathbb{Q}_h + \tau_h\mathbb{H} - \lambda\mathbb{R} \end{cases} . \tag{1.2}$$

We apply the following initial conditions:

$$\mathbb{S}(0) = \mathbb{S}_0 = \mathscr{K}_1,\ \mathbb{E}(0) = \mathbb{E}_0 = \mathscr{K}_2,\ \mathbb{I}_s(0) = \mathbb{I}_{s0} = \mathscr{K}_3,\ \mathbb{I}_a(0) = \mathbb{I}_{a0} = \mathscr{K}_4,$$
$$\mathbb{H}(0) = \mathbb{H}_0 = \mathscr{K}_5,\ \mathbb{Q}_h(0) = \mathbb{Q}_{h0} = \mathscr{K}_6,\ \mathbb{R}(0) = \mathbb{R}_0 = \mathscr{K}_7.$$

We believe that a good mathematical model will assist health authorities in taking preventative steps against the spread of the new coronavirus disease. Therefore, we first introduce a Caputo fractional-order model and then derive the existence and non-negativity of the Caputo fractional order model solutions by applying the fractional Gronwall-Bellman inequality using the Laplace transform method(LTM). The uniqueness of the solutions of the model is founded on the use of fixed-point theory. Then, we expand the Laplace Adomian decomposition method (LADM) for numerical simulations. The coupling of the Adomian decomposition method and LTM leads to a powerful technique known as Laplace Adomian decomposition. With the application of LTM, we transform a CFDE into algebraic equations. The nonlinear terms in the first two equations are decomposed in terms of Adonmain polynomials. We see that this iterative method works efficiently for a stochastic system as well as deterministic differential equation. More explicitly, we can also use it for a system of linear and nonlinear ordinary differential equations and partial differential equations of an integer and fractional order. In the above process, no perturbation is required. It is to be proved that the LADM is a better technique than the standard ADM method [21–24].

The rest of the paper is organized as follows: Section 1 is the introduction. Section 2 includes the basic definitions. The existence and non-negativity of the solution are discussed in Section 3. Section 4 is devoted to the uniqueness of the solution. The LAD method is applied to the soolution of the model in Section 5, and the numerical discussion is considered in Section 6. The last section is the conclusion of our work.

2. Preliminaries

Definition 2.1. *[25] The CFD of order $\gamma \in (\mathscr{K}-1, \mathscr{K}]$ of $\mathscr{U}(t)$ is defined as*

$${}_0^c\mathscr{D}_t^\gamma\{\mathscr{U}(t)\} = \frac{1}{\Gamma(\mathscr{K}-\gamma)}\int_0^t (t-\eta)^{\mathscr{K}-\gamma-1}\mathscr{U}^{(\mathscr{K})}(\eta)d\eta,$$

where $\mathscr{K} = [\gamma] + 1$ and $[\gamma]$ represents the integral parts of γ.

Definition 2.2. *Magin [26] The Riemann-Liouville fractional integration of order* $0 < \gamma \leq 1$ *of the continuous function* $\mathscr{U}$ *is defined as*

$$\mathscr{I}^{\gamma}(\mathscr{U}(t)) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-\eta)^{\gamma-1}\mathscr{U}(\eta)d\eta, \quad 0 < \eta < \infty.$$

Definition 2.3. *[27] The Laplace transform of* $\mathbb{G}(t)$*, as denoted by* $\mathbb{G}(\mathsf{S})$ *is defined as*

$$\mathbb{G}(\mathsf{S}) = \int_0^{\infty} e^{-\mathsf{S}t}\mathbb{G}(t)dt.$$

Definition 2.4. *Mittag-Leffler function (MLF) is defined as*

$$\mathscr{E}_{\alpha,\beta}(\mathscr{U}) = \sum_{\mathscr{K}=0}^{\infty} \frac{\mathscr{U}^{\mathscr{K}}}{\Gamma(\alpha\mathscr{K}+\beta)},$$

and

$$\mathscr{E}_{\alpha,1}(\mathscr{U}) = \sum_{\mathscr{K}=0}^{\infty} \frac{\mathscr{U}^{\mathscr{K}}}{\Gamma(\alpha\mathscr{K}+1)},$$

where α and β are greater than zero and the convergence of the MLF is described in [28, 29].

Lemma 2.1. *[30] The Laplace transform of the Caputo FDO is*

$$\mathcal{L}\{{}_0^c\mathscr{D}_t^{\gamma}(\mathscr{U}(t))\} = \mathsf{S}^{\mathscr{K}}\mathcal{L}\{\mathscr{U}(t)\} - \sum_{\mathscr{K}=0}^{\mathscr{N}-1} \mathsf{S}^{\gamma-\mathscr{K}-1}\mathscr{U}^{(\mathscr{K})}(t_0),$$

where $\mathscr{K} - 1 < \gamma < \mathscr{K}$, $\mathscr{K} = [\gamma] + 1$ *and* $[\gamma]$ *shows the integer part of* γ.

Lemma 2.2. *[31] The following result holds for Caputo FDO*

$$\mathscr{I}^{\gamma}[{}_0^c\mathscr{D}_t^{\gamma}\mathscr{U}](t) = \mathscr{U}(t) + \sum_{\mathscr{K}=0}^{\mathscr{N}-1} \frac{\mathscr{U}^{\mathscr{K}}(0)}{\Gamma\mathscr{K}+1}\mathscr{U}^{\mathscr{K}}$$

for any $\gamma > 0, \mathscr{K} \in \mathsf{N}$, *where* $\mathscr{K} = [\gamma] + 1$ *and* $[\gamma]$ *shows the integer part of* γ.

3. Main work

This section is dedicated to proving the positivity and boundedness of the solution of System (1.2). The size of each class of the population varies over time, and the whole population $\mathbb{N}(t)$ is given by

$$\mathbb{N} = \mathbb{S} + \mathbb{E} + \mathbb{I}_s + \mathbb{I}_a + \mathbb{H} + \mathbb{Q}_h + \mathbb{R}.$$

Applying Caputo fractional order derivatives and using the linearity property of the Caputo operator, we get

$$\begin{aligned}
{}_0^c\mathscr{D}_t^{\alpha}\{\mathbb{N}(t)\} =& {}_0^c\mathscr{D}_t^{\alpha}[\mathbb{S}(t)] + {}_0^c\mathscr{D}_t^{\alpha}[\mathbb{E}(t)] + {}_0^c\mathscr{D}_t^{\alpha}[\mathbb{I}_s(t)] + {}_0^c\mathscr{D}_t^{\alpha}[\mathbb{I}_a(t)] + {}_0^c\mathscr{D}_t^{\alpha}[\mathbb{H}(t)] \\
&+ {}_0^c\mathscr{D}_t^{\alpha}[\mathbb{Q}_h(t)] + {}_0^c\mathscr{D}_t^{\alpha}[\mathbb{R}(t)] \\
=& \delta - \lambda\mathbb{N}(t) - \lambda_s\mathbb{I}_s - \lambda_h\mathbb{H},
\end{aligned}$$

$$ {}_{0}^{c}\mathscr{D}_{t}^{\alpha}[\mathbb{N}(t)] - \lambda\mathbb{N}(t) \leq \delta. \tag{3.1} $$

Applying the LTM to solve the FGB inequality given by Eq (3.1) with initial conditions $\mathbb{N}(t_0) \geq 0$, we obtain

$$ \mathcal{L}\{{}_{0}^{c}\mathscr{D}_{t}^{\alpha}\mathbb{N}(t) - \lambda\mathbb{N}(t)\} \leq \mathcal{L}\{\delta\}, $$

$$ \mathcal{L}\{\mathbb{N}(t)\} \leq \frac{\delta}{\mathbb{S}(\lambda + \mathbb{S}^{\alpha})} + \sum_{\mathscr{K}=0}^{\mathscr{P}-1} \frac{\mathbb{S}^{\alpha-\mathscr{K}-1}\mathbb{N}^{\mathscr{K}}(t_0)}{(\mathbb{S}^{\alpha} + \lambda)}. \tag{3.2} $$

Splitting Eq (3.2) into partial fractions, we get

$$ \mathcal{L}\{\mathbb{N}(t)\} \leq \delta\left(\frac{1}{\mathbb{S}} - \frac{1}{\mathbb{S}\left(1 + \frac{\lambda}{\mathbb{S}^{\alpha}}\right)}\right) + \sum_{\mathscr{K}=0}^{\mathscr{P}-1} \frac{\mathbb{N}^{\mathscr{K}}(t_0)}{\mathbb{S}^{\mathscr{K}+1}\left(1 + \frac{\lambda}{\mathbb{S}^{\alpha}}\right)}. \tag{3.3} $$

By using binomial series expansion, we get

$$ \frac{1}{\left(1 + \frac{\lambda}{\mathbb{S}^{\alpha}}\right)} = \sum_{\mathscr{K}=0}^{\infty}\left(\frac{-\lambda}{\mathbb{S}^{\alpha}}\right)^{\mathscr{K}}. $$

Therefore, Eq (3.3) becomes

$$ \mathcal{L}\{\mathbb{N}(t)\} \leq \delta\left(\frac{1}{\mathbb{S}} - \sum_{\mathscr{K}=0}^{\infty}\frac{(-\lambda)^{\mathscr{K}}}{\mathbb{S}^{\mathscr{K}\alpha+1}}\right) + \sum_{\mathscr{P}=0}^{\mathscr{K}-1}\sum_{\mathscr{K}=0}^{\infty}\frac{(-\lambda)^{\mathscr{K}}\mathbb{N}^{\mathscr{K}}(t_0)}{\mathbb{S}^{\alpha\mathscr{K}+\mathscr{P}+1}}. \tag{3.4} $$

Applying the inverse Laplace transform to Eq (3.4) gives

$$ \begin{aligned} \mathbb{N}(t) &\leq \delta\left(1 - \sum_{\mathscr{K}=0}^{\infty}\frac{(-\lambda)^{\mathscr{K}}t^{\mathscr{K}\alpha}}{\Gamma(\mathscr{K}\alpha+1)}\right) + \sum_{\mathscr{P}=0}^{\mathscr{K}-1}\sum_{\mathscr{K}=0}^{\infty}\frac{(-\mu\lambda)\mathbb{N}^{\mathscr{P}}(t_0)t^{\mathscr{K}\alpha+\mathscr{P}}}{\Gamma(\mathscr{K}\alpha+\mathscr{P}+1)} \\ &\leq \delta\left(1 - \sum_{\mathscr{K}=0}^{\infty}\frac{(-\lambda t^{\alpha})^{\mathscr{K}}}{\Gamma(\mathscr{K}\alpha+1)}\right) + \sum_{\mathscr{P}=0}^{\mathscr{K}-1}\sum_{\mathscr{K}=0}^{\infty}\frac{(-\lambda t^{\alpha})^{\mathscr{K}}\mathbb{N}^{\mathscr{P}}(t_0)t^{\mathscr{P}}}{\Gamma(\mathscr{K}\alpha+\mathscr{P}+1)}. \end{aligned} $$

Now, we apply the definition of MLF

$$ \mathbb{N}(t) \leq \delta\left(1 - \sum_{\mathscr{K}=0}^{\infty}\frac{(-\lambda t^{\alpha})^{\mathscr{K}})}{\Gamma(\mathscr{K}\alpha+1)}\right) + \mathscr{E}_{\alpha,\mathscr{P}+1}(-\lambda t^{\alpha})t^{\mathscr{P}}\mathbb{N}^{\mathscr{P}}(t_0), \tag{3.5} $$

where Eq (3.5), $E_{\alpha,1}(-\lambda t^{\alpha})$ and $\mathscr{E}_{\alpha,\mathscr{K}+1}(-\lambda t^{\alpha})$ represent the series of entire function. Hence the solutions of Model (1.2) are bounded.

Thus,

$$ \{(\mathbb{S}(t), \mathbb{E}(t), \mathbb{I}_s(t), \mathbb{I}_a(t), \mathbb{H}(t), \mathbb{Q}_h(t), \mathbb{R}(t)) \in \mathscr{R}_{+}^{7}, 0 \leq \mathbb{N}(t) \leq \delta\left(1 - \mathscr{E}_{\alpha,1}(-\lambda t^{\alpha}) + \mathscr{E}_{\alpha,\mathscr{P}+1}(-\lambda t^{\alpha})t^{\mathscr{P}}N^{\mathscr{P}}(t_0)\right\}. $$

4. Existence and uniqueness

Using the fixed point theory, we will discuss the uniqueness of the solutions of the Caputo fractional-order Model (1.2). For this need, we will follow the following procedure.

Applying the initial conditions and the fractional integral to System (1.2) , we obtain

$$
\begin{aligned}
\mathbb{S}(t) =& \mathscr{K}_1 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \left[\delta - \beta\left(1-\epsilon_m c_m\right) \mathbb{S}\mathbb{I}_s - \beta^{'}\left(1-\epsilon_m c_m\right) \mathbb{S}\mathbb{I}_a - \lambda\mathbb{S}\right] d\eta, \\
\mathbb{E}(t) =& \mathscr{K}_2 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \left[\beta\left(1-\epsilon_m c_m\right) \mathbb{S}\mathbb{I}_s + \beta^{'}\left(1-\epsilon_m c_m\right) \mathbb{S}\mathbb{I}_a - \sigma\mathbb{E} - \lambda\mathbb{E}\right] d\eta, \\
\mathbb{I}_s(t) =& \mathscr{K}_3 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \left[(1-a)\,\sigma\mathbb{E} + \rho_a\mathbb{I}_a - (\lambda_s + \lambda) - \phi_s\mathbb{I}_s\right] d\eta, \\
\mathbb{I}_a(t) =& \mathscr{K}_4 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \left[a\sigma\mathbb{E} - \tau_a\mathbb{I}_a - \rho_a\mathbb{I}_a - \lambda\mathbb{I}\right] d\eta, \\
\mathbb{H}(t) =& \mathscr{K}_5 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \left[\phi_s\mathbb{I}_s - (\tau_h + \lambda_h + \lambda)\,\mathbb{H}\right] d\eta, \\
\mathbb{Q}_h(t) =& \mathscr{K}_6 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \left[\tau_a\mathbb{I}_a - \rho_h\mathbb{Q}_h - \lambda\mathbb{Q}\right] d\eta, \\
\mathbb{R}(t) =& \mathscr{K}_7 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \left[\rho_h\mathbb{Q}_h + \tau_h\mathbb{H}_h - \lambda\mathbb{R}\right] d\eta.
\end{aligned}
\tag{4.1}
$$

It is assumed that

$$
\mathscr{U}(t) = \begin{cases} \mathbb{S} \\ \mathbb{E} \\ \mathbb{I}_a \\ \mathbb{I}_s \\ \mathbb{H} \\ \mathbb{Q}_h \\ \mathbb{R} \end{cases}, \tag{4.2}
$$

$$
\mathscr{K} = \begin{cases} \mathscr{K}_1 \\ \mathscr{K}_2 \\ \mathscr{K}_3 \\ \mathscr{K}_4 \\ \mathscr{K}_5 \\ \mathscr{K}_6 \\ \mathscr{K}_7 \end{cases}. \tag{4.3}
$$

We define the functions under the integral signs in Eq (4.1) as

$$\mathcal{G}(t,\mathcal{U}(t))=\begin{cases}\mathcal{F}_1(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})=\delta-\beta(1-\epsilon_m c_m)\mathbb{S}\mathbb{I}_s-\beta^{'}(1-\epsilon_m c_m)\mathbb{S}\mathbb{I}_a-\lambda\mathbb{S}\\ \mathcal{F}_2(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})=\beta(1-\epsilon_m c_m)\mathbb{S}\mathbb{I}_s+\beta^{'}(1-\epsilon_m c_m)\mathbb{S}\mathbb{I}_a-\sigma\mathbb{E}-\lambda\mathbb{E}\\ \mathcal{F}_3(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})=(1-a)\sigma\mathbb{E}+\rho_a\mathbb{I}_a-(\lambda_s+\lambda)-\phi_s\mathbb{I}_s\\ \mathcal{F}_4(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})=a\sigma\mathbb{E}-\tau_a\mathbb{I}_a-\rho_a\mathbb{I}_a-\lambda\mathbb{I}\\ \mathcal{F}_5(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})=\phi_s\mathbb{I}_s-(\tau_h+\lambda_h+\lambda)\mathbb{H}\\ \mathcal{F}_6(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})=\tau_a\mathbb{I}_a-\rho_h\mathbb{Q}_h-\lambda\mathbb{Q}\\ \mathcal{F}_7(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})=\rho_h\mathbb{Q}_h+\tau_h\mathbb{H}_h-\lambda\mathbb{R}\end{cases}. \tag{4.4}$$

Substituting Eq (4.4) into Eq (4.1), we get

$$\begin{aligned}\mathbb{S}(t)&=\mathcal{K}_1+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{F}_1(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})\,d\eta,\\ \mathbb{E}(t)&=\mathcal{K}_2+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{F}_2(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})\,d\eta,\\ \mathbb{I}_s(t)&=\mathcal{K}_3+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{F}_3(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})\,d\eta,\\ \mathbb{I}_a(t)&=\mathcal{K}_4+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{F}_4(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})\,d\eta,\\ \mathbb{H}(t)&=\mathcal{K}_5+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{F}_5(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})\,d\eta,\\ \mathbb{Q}_h(t)&=\mathcal{K}_6+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{F}_6(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})\,d\eta,\\ \mathbb{R}(t)&=\mathcal{K}_7+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{F}_7(\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R})\,d\eta.\end{aligned} \tag{4.5}$$

So, System (4.5) becomes

$$\mathcal{U}(t)=\mathcal{K}+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{G}(\eta,\mathcal{U}(\eta))\,d\eta. \tag{4.6}$$

Consider a Banach space $\Theta=C[0,\mathcal{T}]xC[0,\mathcal{T}]$, with a norm

$$\|\mathbb{S},\mathbb{E},\mathbb{I}_s,\mathbb{I}_a,\mathbb{H},\mathbb{Q}_h,\mathbb{R}\|=\max_{t\in[0,\mathcal{T}]}|\mathbb{S}+\mathbb{E}+\mathbb{I}_s+\mathbb{I}_a+\mathbb{H}+\mathbb{Q}_h+\mathbb{R}|.$$

Let $\Psi:\Theta\to\Theta$ be a mapping defined as

$$\Psi[\mathcal{U}(t)]=\mathcal{K}+\frac{1}{\Gamma(\alpha)}\int_0^t(t-\eta)^{\alpha-1}\mathcal{G}(\eta,\mathcal{U}(\eta))\,d\eta. \tag{4.7}$$

Theorem 4.1. *System (4.4) has at least one solution if there exist constants $\mathcal{M}_{\mathcal{C}}>0$, and $\mathcal{N}_{\mathcal{C}}>0$, such that*

$$|\mathcal{G}(t,\mathcal{U}(t))|\leq\mathcal{M}_{\mathcal{C}}|\mathcal{U}(t)|+\mathcal{N}_{\mathcal{C}}. \tag{4.8}$$

Proof. To show that the operator $\varPsi$ is bounded on $\Omega : \{\mathscr{U} \in \Omega \,|\, \Lambda \geq \|\mathscr{U}\|\}$, where

$$\Lambda \geq \max_{t\in[0,\mathcal{T}]} \frac{\mathscr{K} + (\mathscr{M}_{\mathscr{C}}\mathcal{T}^{\alpha}/\varGamma(\alpha+1))}{\mathscr{K} - (\mathscr{N}_{\mathscr{C}}\mathcal{T}^{\alpha}/\varGamma(\alpha+1))}, \tag{4.9}$$

is a closed and convex subset of Θ. Now, take

$$\begin{aligned}
\|\varPsi(\mathscr{U})\| &= \max_{t\in[0,\mathcal{T}]} \left| \mathscr{K} + \frac{1}{\varGamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \mathscr{G}(\eta, \mathscr{U}(\eta))\, d\eta \right| \\
&\leq |\mathscr{K}| + \frac{1}{\varGamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} |\mathscr{G}(\eta, \mathscr{U}(\eta))|\, d\eta \\
&\leq |\mathscr{K}| + \frac{1}{\varGamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} |\mathscr{M}_{\mathscr{C}} |\mathscr{U}(t)| + \mathscr{N}_{\mathscr{C}}|\, d\eta \\
&\leq |\mathscr{K}| + |\mathscr{M}_{\mathscr{C}} |\mathscr{U}(t)| + \mathscr{N}_{\mathscr{C}}| \frac{\mathcal{T}^{\alpha}}{\varGamma(\alpha+1)} \leq \Lambda.
\end{aligned} \tag{4.10}$$

□

It means that $\mathscr{U} \in \varTheta \Rightarrow \varPsi(\varOmega) \subseteq \Omega$, which shows that d is bounded. Next, to show that $\varPsi$ is completely continuous, let $t_1 < t_2 \in [0, \mathscr{T}]$ and take

$$\begin{aligned}
\|\varPsi(\mathscr{U})(t_2) - \Psi(\mathscr{U})(t_1)\| &= \left| \frac{1}{\varGamma(\alpha)} \int_0^{t_2} (t_2-\eta)^{\alpha-1} \mathscr{G}(\eta, \mathscr{U}(\eta))\, d\eta \right. \\
&\quad \left. - \frac{1}{\varGamma(\alpha)} \int_0^{t_1} (t_1-\eta)^{\alpha-1} \mathscr{G}(\eta, \mathscr{U}(\eta))\, d\eta \right| \\
&\leq [t_1^{\alpha} - t_2^{\alpha}] \frac{|\mathscr{M}_{\mathscr{C}} |\mathscr{U}(t)| + \mathscr{N}_{\mathscr{C}}|}{\varGamma(\alpha+1)}.
\end{aligned} \tag{4.11}$$

This shows that $\|\Psi(\mathscr{U})(t_2) - \Psi(\mathscr{U})(t_1)\| \to 0$,as $t_2 \to t_1$. Hence, the operator Ψ is completely continuous according to the Arzelá-Ascoli theorem. Therefore, the given system, System (4.4) has at least one solution according to Schauder's fixed-point theorem.

Next, the fixed-point theorem of Banach was used to demonstrate that System (4.4) has a unique solution.

Theorem 4.2. *System (4.4) has a unique solution if there exists a constant* $C_{\mathscr{C}} > 0$, *such that for each* $\mathscr{U}_1(t), \mathscr{U}_2(t) \in \Theta$, *such that*

$$|\mathscr{G}(t, \mathscr{U}_1(t)) - \mathscr{G}(t, \mathscr{U}_2(t))| \leq C_{\mathscr{C}} |\mathscr{U}_1(t) - \mathscr{U}_2(t)|. \tag{4.12}$$

Proof. Let $\mathscr{U}_1(t), \mathscr{U}_2(t) \in \Theta$. Take □

$$\begin{aligned}
|\Psi(\mathscr{U}_1) - \Psi(\mathscr{U}_2)| &= \max_{t\in[0,\mathcal{T}]} \left| \frac{1}{\varGamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \mathscr{G}(\eta, \mathscr{U}_1(\eta))\, d\eta - \frac{1}{\varGamma(\alpha)} \int_0^t (t-\eta)^{\alpha-1} \mathscr{G}(\eta, \mathscr{U}_2(\eta))\, d\eta \right| \\
&\leq \frac{\mathcal{T}^{\alpha}}{\varGamma(\alpha+1)} C_{\mathscr{C}} |\mathscr{U}_1 - \mathscr{U}_2|.
\end{aligned}$$

Hence, Ψ is the contraction. System (4.4) has a unique solution based on the Banach contraction theorem.

5. The Laplace Adomian decomposition method

Here, we discuss the use of the Laplace Adomian algorithm for the nonlinear and linear fractional differential equations. The nonlinear terms in this model are $\mathbb{S}\mathbb{I}_s$ and $\mathbb{S}\mathbb{I}_a$. Further, $\delta,\ \beta,\ \beta',\ \varepsilon_m,\ e_m,\ \sigma,\ a,\ \rho_a,\ \phi_s,\ \tau_a,\ \tau_h,\ \rho_h,\ \lambda,\ \lambda_s$, and λ_h are known constants.

Applying the Laplace transform to Model (1.2), we get

$$\begin{aligned}
\mathcal{L}\{{}_0^c\mathscr{D}_t^\alpha[\mathbb{S}(t)]\} &= \mathcal{L}\left\{\delta - \beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_s - \beta'(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \lambda\mathbb{S}\right\},\\
\mathcal{L}\{{}_0^c\mathscr{D}_t^\alpha[\mathbb{E}(t)]\} &= \mathcal{L}\left\{\beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a + \beta'(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \sigma\mathbb{E} - \lambda\mathbb{E}\right\},\\
\mathcal{L}\{{}_0^c\mathscr{D}_t^\alpha[\mathbb{I}_s(t)]\} &= \mathcal{L}\{(1-a)\sigma\mathbb{E} + \rho_a\mathbb{I}_a - (\lambda_s+\lambda)\mathbb{I}_s - \phi_s\mathbb{I}_s\},\\
\mathcal{L}\{{}_0^c\mathscr{D}_t^\alpha[\mathbb{I}_a(t)]\} &= \mathcal{L}\{a\sigma\mathbb{E} - \tau_a\mathbb{I}_a - \rho_a\mathbb{I}_a - \lambda\mathbb{I}_a\},\\
\mathcal{L}\{{}_0^c\mathscr{D}_t^\alpha[\mathbb{H}(t)]\} &= \mathcal{L}\{\phi_s\mathbb{I}_s - (\tau_h+\lambda_h+\lambda)\mathbb{H}\},\\
\mathcal{L}\{{}_0^c\mathscr{D}_t^\alpha[\mathbb{Q}_h(t)]\} &= \mathcal{L}\{\tau_a\mathbb{I}_a - \rho_h\mathbb{Q}_h - \lambda\mathbb{Q}_h\},\\
\mathcal{L}\{{}_0^c\mathscr{D}_t^\alpha[\mathbb{R}(t)]\} &= \mathcal{L}\{\rho_h\mathbb{Q}_h + \tau_h\mathbb{H} - \lambda\mathbb{R}\}.
\end{aligned} \tag{5.1}$$

Using the differentiation property of the LT and given initial conditions given in Eq (5.1), we get

$$\begin{cases}
\mathsf{S}^\alpha\mathcal{L}\{\mathbb{S}(t)\} - \mathsf{S}^{\alpha-1}\mathbb{S}(0) = \mathcal{L}\left\{\delta - \beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_s - \beta'(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \lambda\mathbb{S}\right\}\\
\mathsf{S}^\alpha\mathcal{L}\{\mathbb{E}(t)\} - \mathsf{S}^{\alpha-1}\mathbb{E}(0) = \mathcal{L}\left\{\beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a + \beta'(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \sigma\mathbb{E} - \lambda\mathbb{E}\right\}\\
\mathsf{S}^\alpha\mathcal{L}\{\mathbb{I}_s(t)\} - \mathsf{S}^{\alpha-1}\mathbb{I}_s(0) = \mathcal{L}\{(1-a)\sigma\mathbb{E} + \rho_a\mathbb{I}_a - (\lambda_s+\lambda)\mathbb{I}_s - \phi_s\mathbb{I}_s\}\\
\mathsf{S}^\alpha\mathcal{L}\{\mathbb{I}_a(t)\} - \mathsf{S}^{\alpha-1}\mathbb{I}_a(0) = \mathcal{L}\{a\sigma\mathbb{E} - \tau_a\mathbb{I}_a - \rho_a\mathbb{I}_a - \lambda\mathbb{I}_a\}\\
\mathsf{S}^\alpha\mathcal{L}\{\mathbb{H}(t)\} - \mathsf{S}^{\alpha-1}\mathbb{H}(0) = \mathcal{L}\{\phi_s\mathbb{I}_s - (\tau_h+\lambda_h+\lambda)\mathbb{H}\}\\
\mathsf{S}^\alpha\mathcal{L}\{\mathbb{Q}(t)\} - \mathsf{S}^{\alpha-1}\mathbb{Q}_h(0) = \mathcal{L}\{\tau_a\mathbb{I}_a - \rho_h\mathbb{Q}_h - \lambda\mathbb{Q}_h\}\\
\mathsf{S}^\alpha\mathcal{L}\{\mathbb{R}(t)\} - \mathsf{S}^{\alpha-1}\mathbb{R}(0) = \mathcal{L}\{\rho_h\mathbb{Q}_h + \tau_h\mathbb{H} - \lambda\mathbb{R}\}
\end{cases}. \tag{5.2}$$

Now, applying the inverse Laplace transform to System (5.2) and the initial conditions, we get

$$\begin{cases}
\mathbb{S}(t) = \mathscr{K}_1 + \mathcal{L}^{-1}\left[\frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\left\{\delta - \beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_s - \beta'(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \lambda\mathbb{S}\right\}\right]\\
\mathbb{E}(t) = \mathscr{K}_2 + \mathcal{L}^{-1}\left[\frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\left\{\beta(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a + \beta'(1-\varepsilon_m e_m)\mathbb{S}\mathbb{I}_a - \sigma\mathbb{E} - \lambda\mathbb{E}\right\}\right]\\
\mathbb{I}_s(t) = \mathscr{K}_3 + \mathcal{L}^{-1}\left[\frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{(1-a)\sigma\mathbb{E} + \rho_a\mathbb{I}_a - (\lambda_s+\lambda)\mathbb{I}_s - \phi_s\mathbb{I}_s\}\right]\\
\mathbb{I}_a(t) = \mathscr{K}_4 + \mathcal{L}^{-1}\left[\frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{a\sigma\mathbb{E} - \tau_a\mathbb{I}_a - \rho_a\mathbb{I}_a - \lambda\mathbb{I}_a\}\right]\\
\mathbb{H}(t) = \mathscr{K}_5 + \mathcal{L}^{-1}\left[\frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{\phi_s\mathbb{I}_s - (\tau_h+\lambda_h+\lambda)\mathbb{H}\}\right]\\
\mathbb{Q}(t) = \mathscr{K}_6 + \mathcal{L}^{-1}\left[\frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{\tau_a\mathbb{I}_a - \rho_h\mathbb{Q}_h - \lambda\mathbb{Q}_h\}\right]\\
\mathbb{R}(t) = \mathscr{K}_7 + \mathcal{L}^{-1}\left[\frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{\rho_h Q_h + \tau_h H - \lambda R\}\right]
\end{cases}. \tag{5.3}$$

Now, we assume that the solution of $\mathbb{S}(t), \mathbb{E}(t), \mathbb{I}_s(t), \mathbb{I}_a(t), \mathbb{H}(t), \mathbb{Q}_h(t)$ and $\mathbb{R}(t)$ in the form of infinite series is defined as

$$\begin{cases} \mathbb{S}(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{S}_{\mathscr{P}}(t) \\ \mathbb{E}(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{E}_{\mathscr{P}}(t) \\ \mathbb{I}_s(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{I}_{s,\mathscr{P}}(t) \\ \mathbb{I}_a(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{I}_{a,\mathscr{P}}(t) \\ \mathbb{H}(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{H}_{\mathscr{P}}(t) \\ \mathbb{Q}_h(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{Q}_{h,\mathscr{P}} \\ \mathbb{R}(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{R}_{\mathscr{P}}(t) \end{cases} . \tag{5.4}$$

The nonlinear terms of the fractional order model are $\mathbb{S}\mathbb{I}_s$ and $\mathbb{S}\mathbb{I}_a$. The nonlinear terms are decomposed by using an Adomian polynomial, as follow:

$$\mathbb{S}(t)\mathbb{I}_s(t) = \sum_{\mathscr{P}=0}^{\infty} \mathbb{C}_{\mathscr{P}}, \mathbb{S}\mathbb{I}_a(t) = \sum_{\mathscr{P}=0}^{\infty} B_{\mathscr{P}}, \tag{5.5}$$

where $\mathbb{C}_{\mathscr{P}}$ and $\mathbb{B}_{\mathscr{P}}$ are called Adomian polynomials, and they depend upon $\mathbb{S}_0\mathbb{I}_{s0}, \mathbb{S}_1\mathbb{I}_{s1}, ...\mathbb{S}_m\mathbb{I}_{sm}$ and $\mathbb{S}_0\mathbb{I}_{a0}, \mathbb{S}_1\mathbb{I}_{a1}, ...\mathbb{S}_m\mathbb{I}_{am}$, respectively. We can calculate the polynomials by using the following formula:

$$\begin{aligned} \mathbb{C}_{\mathscr{P}} &= \frac{1}{\Gamma(\mathscr{P}+1)} \frac{d^{\mathscr{P}}}{dt^{\mathscr{P}}} \left[\sum_{\mathscr{K}=0}^{\mathscr{P}} \lambda^{\mathscr{K}} \mathbb{S}_{\mathscr{K}} \sum_{\mathscr{K}=0}^{\mathscr{P}} \lambda^{\mathscr{K}} \mathbb{I}_{s,\mathscr{K}} \right] \downharpoonleft_{\lambda=0,} \\ \mathbb{B}_{\mathscr{P}} &= \frac{1}{\Gamma(\mathscr{P}+1)} \frac{d^{\mathscr{P}}}{dt^{\mathscr{P}}} \left[\sum_{\mathscr{K}=0}^{\mathscr{P}} \lambda^{\mathscr{K}} \mathbb{S}_{\mathscr{K}} \sum_{\mathscr{K}=0}^{\mathscr{P}} \lambda^{\mathscr{K}} \mathbb{I}_{a,\mathscr{K}} \right] \downharpoonleft_{\lambda=0.} \end{aligned}$$

Substituting Eqs (5.4) and (5.5) into Eq (5.3), we get

$$\begin{cases} \mathcal{L}\{\mathbb{S}_1(t)\} = \frac{k_1}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\left\{\delta - \beta(1-\varepsilon_m e_m)\mathbb{C}_0 - \beta'(1-\varepsilon_m e_m)\mathbb{B}_0 - \lambda\mathbb{S}\right\} \\ \mathcal{L}\{\mathbb{E}_1(t)\} = \frac{k_2}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\left\{\beta(1-\varepsilon_m e_m)\mathbb{C}_0 + \beta'(1-\varepsilon_m e_m)\mathbb{B}_0 - \sigma E - \lambda\mathbb{E}_0\right\} \\ \mathcal{L}\{\mathbb{I}_{s1}(t)\} = \frac{k_3}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{(1-a)\sigma\mathbb{E}_0 + \rho_a\mathbb{I}_{a0} - (\lambda_s+\lambda)\mathbb{I}_s - \phi_s\mathbb{I}_{s0}\} \\ \mathcal{L}\{\mathbb{I}_{a1}(t)\} = \frac{k_4}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{a\sigma\mathbb{E}_0 - \tau_a\mathbb{I}_{a0} - \rho_a\mathbb{I}_{a0} - \lambda\mathbb{I}_{a0}\} \\ \mathcal{L}\{\mathbb{H}_1(t)\} = \frac{k_5}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{\phi_s\mathbb{I}_{s0} - (\tau_h+\lambda_h+\lambda)\mathbb{H}_0\} \\ \mathcal{L}\{\mathbb{Q}_{h1}(t)\} = \frac{k_6}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{\tau_a\mathbb{I}_{a0} - \rho_h\mathbb{Q}_{h0} - \lambda\mathbb{Q}_{h0}\} \\ \mathcal{L}\{\mathbb{R}_1(t)\} = \frac{k_7}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{\rho_h\mathbb{Q}_{h0} + \tau_h\mathbb{H}_0 - \lambda\mathbb{R}_0\} \end{cases} . \tag{5.6}$$

The subsequent terms are

$$\begin{cases} \mathcal{L}\{\mathbb{S}_2(t)\} = \frac{\mathscr{K}_1}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\left\{\delta - \beta(1-\varepsilon_m e_m)\mathbb{C}_1 - \beta'(1-\varepsilon_m e_m)\mathbb{B}_1 - \lambda\mathbb{S}_1\right\} \\ \mathcal{L}\{\mathbb{E}_2(t)\} = \frac{\mathscr{K}_2}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\left\{\beta(1-\varepsilon_m e_m)\mathbb{C}_1 + \beta'(1-\varepsilon_m e_m)\mathbb{B}_1 - \sigma\mathbb{E}_1 - \lambda\mathbb{E}_1\right\} \\ \mathcal{L}\{\mathbb{I}_{s2}(t)\} = \frac{\mathscr{K}_3}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{(1-a)\sigma\mathbb{E}_1 + \rho_a\mathbb{I}_{a1} - (\lambda_s+\lambda)\mathbb{I}_{s1} - \phi_s\mathbb{I}_{s1}\} \\ \mathcal{L}\{\mathbb{I}_{a2}(t)\} = \frac{\mathscr{K}_4}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{a\sigma\mathbb{E}_1 - \tau_a\mathbb{I}_{a1} - \rho_a\mathbb{I}_{a1} - \lambda\mathbb{I}_{a1}\} \\ \mathcal{L}\{\mathbb{H}_2(t)\} = \frac{\mathscr{K}_5}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{\phi_s\mathbb{I}_{s1} - (\tau_h+\lambda_h+\lambda)\mathbb{H}_1\} \\ \mathcal{L}\{\mathbb{Q}_{h2}(t)\} = \frac{\mathscr{K}_6}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{\tau_a\mathbb{I}_{a1} - \alpha_h\mathbb{Q}_{h1} - \lambda\mathbb{Q}_{h1}\} \\ \mathcal{L}\{\mathbb{R}_2(t)\} = \frac{\mathscr{K}_7}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}} \mathcal{L}\{\rho_h\mathbb{Q}_{h1} + \tau_h\mathbb{H}_1 - \lambda\mathbb{R}_1\} \end{cases} . \tag{5.7}$$

$$\begin{cases}\mathcal{L}\{\mathbb{S}_{n+1}(t)\} = \frac{\mathcal{K}_1}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\left\{\delta - \beta(1-\varepsilon_m e_m)\mathbb{C}_n - \beta^{'}(1-\varepsilon_m e_m)\mathbb{B}_n - \lambda\mathbb{S}_n\right\}\\ \mathcal{L}\{\mathbb{E}_{n+1}(t)\} = \frac{\mathcal{K}_2}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\left\{\beta(1-\varepsilon_m e_m)\mathbb{C}_n + \beta^{'}(1-\varepsilon_m e_m)\mathbb{B}_n - \sigma\mathbb{E}_n - \lambda\mathbb{E}_n\right\}\\ \mathcal{L}\{\mathbb{I}_{s,n+1}(t)\} = \frac{\mathcal{K}_3}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{(1-a)\sigma\mathbb{E}_n + \rho_a\mathbb{I}_{an} - (\lambda_s+\lambda)\mathbb{I}_{s,n} - \phi_s\mathbb{I}_{sn}\}\\ \mathcal{L}\{\mathbb{I}_{a,n+1}(t)\} = \frac{\mathcal{K}_4}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{a\sigma\mathbb{E}_n - \tau_a\mathbb{I}_{an} - \rho_a\mathbb{I}_{an} - \lambda\mathbb{I}_{an}\}\\ \mathcal{L}\{\mathbb{H}_{n+1}(t)\} = \frac{\mathcal{K}_5}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{\phi_s\mathbb{I}_{sn} - (\tau_h+\lambda_h+\lambda)\mathbb{H}_n\}\\ \mathcal{L}\{\mathbb{Q}_{h,n+1}(t)\} = \frac{\mathcal{K}_6}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{\tau_a\mathbb{I}_{an} - \rho_h\mathbb{Q}_{hn} - \lambda\mathbb{Q}_{hn}\}\\ \mathcal{L}\{\mathbb{R}_{n+1}(t)\} = \frac{\mathcal{K}_7}{\mathsf{S}} + \frac{1}{\mathsf{S}^{\alpha-1}}\mathcal{L}\{\rho_h\mathbb{Q}_{hn} + \tau_h\mathbb{H}_n - \lambda\mathbb{R}\}\end{cases}. \tag{5.8}$$

By applying the initial conditions, we obtained $\mathbb{C}_0 = \mathbb{S}_0\mathbb{I}_{s0} = \mathcal{K}_1\mathcal{K}_3$, $\mathbb{B}_0 = \mathbb{S}_0\mathbb{I}_{ao} = \mathcal{K}_1\mathcal{K}_4$.

Then, applying the inverse Laplace transform to Eq (5.8), we obtain the values for Eq (5.8) recursively.

$$\begin{aligned}
&\mathbb{S}_1(t) = \mathcal{K}_1 + \left\{\delta - \beta(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_3 - \beta^{'}(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_4 - \lambda\mathcal{K}_1\right\}\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{E}_1(t) = \mathcal{K}_2 + \left\{\beta(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_3 + \beta^{'}(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_4 - \sigma\mathcal{K}_2 - \lambda\mathcal{K}_2\right\}\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{I}_{s,1}(t) = \mathcal{K}_3 + \{(1-a)\sigma\mathcal{K}_2 + \rho_a\mathcal{K}_4 - (\lambda_s+\lambda)\mathcal{K}_3 - \phi_s\mathcal{K}_4\}\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{I}_{a,1}(t) = \mathcal{K}_4 + \{a\sigma\mathcal{K}_2 - \tau_a\mathcal{K}_4 - \rho_a\mathcal{K}_4 - \lambda\mathcal{K}_4\}\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{H}_1(t) = \mathcal{K}_5 + \{\phi_s\mathcal{K}_3 - (\tau_h+\lambda_h+\lambda)\mathcal{K}_5\}\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{Q}_{h,1}(t) = \mathcal{K}_6 + \{\tau_a\mathcal{K}_4 - \rho_h\mathcal{K}_6 - \lambda\mathcal{K}_6\}\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{R}_1(t) = \mathcal{K}_7 + \{\rho_h\mathcal{K}_5 + \tau_h\mathcal{K}_5 - \lambda\mathcal{K}_7\}\frac{t^\alpha}{\Gamma(\alpha+1)}.
\end{aligned} \tag{5.9}$$

Similarly, we get the following series

$$\begin{aligned}
\mathbb{S}_2(t) = {} & \mathcal{K}_1 + \Bigg\{\delta - (\beta(1-\varepsilon_m e_m)\left(\mathcal{K}_3 + ((1-a)\sigma\mathcal{K}_2 + \rho_a\mathcal{K}_4 - (\lambda_s+\lambda)\mathcal{K}_3 - \phi_s\mathcal{K}_4)\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\\
& -\beta^{'}(1-\varepsilon_m e_m)\left(\mathcal{K}_4 + (a\sigma\mathcal{K}_2 - \tau_a\mathcal{K}_4 - \rho_a\mathcal{K}_4 - \lambda\mathcal{K}_4)\frac{t^\alpha}{\Gamma(\alpha+1)}\right) - \lambda)\,(\mathcal{K}_1 + (\delta - \beta(1-\varepsilon_m e_m)\,\mathcal{K}_1\mathcal{K}_3\\
& -\beta^{'}(1-\varepsilon_m e_m\Big)\,\mathcal{K}_1\mathcal{K}_4 - \lambda\mathcal{K}_1\Big)\frac{t^\alpha}{\Gamma(\alpha+1)})\Bigg\}\frac{t^\alpha}{\Gamma(\alpha+1)}.\\
\mathbb{E}_2(t) = {} & \mathcal{K}_2 + \Bigg\{\beta(1-\varepsilon_m e_m)\left(\mathcal{K}_3 + ((1-a)\sigma\mathcal{K}_2 + \rho_a\mathcal{K}_4 - (\lambda_s+\lambda)\mathcal{K}_3 - \phi_s\mathcal{K}_4)\frac{t^\alpha}{\Gamma(\alpha+1)}\right) + \beta^{'}(1-\varepsilon_m e_m)\\
& \left(\mathcal{K}_4 + (a\sigma\mathcal{K}_2 - \tau_a\mathcal{K}_4 - \rho_a\mathcal{K}_4 - \lambda\mathcal{K}_4)\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\Big(\mathcal{K}_1 + \Big(\delta - \beta(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_3 - \beta^{'}(1-\epsilon_m e_m)\mathcal{K}_1\mathcal{K}_4 - \lambda\mathcal{K}_1\Big)\\
& \frac{t^\alpha}{\Gamma(\alpha+1)}\Big) - (\sigma+\lambda)\left(\mathcal{K}_2 + \Big(\beta(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_3 + \beta^{'}(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_4 - \sigma\mathcal{K}_2 - \lambda\mathcal{K}_2\Big)\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\Bigg\}\frac{t^\alpha}{\Gamma(\alpha+1)}.\\
\mathbb{I}_{s2}(t) = {} & \mathcal{K}_3 + \Bigg\{(1-a)\sigma\left(\mathcal{K}_2 + \Big(\beta(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_3 + \beta^{'}(1-\varepsilon_m e_m)\mathcal{K}_1\mathcal{K}_4 - \sigma\mathcal{K}_2 - \lambda\mathcal{K}_2\Big)\frac{t^\alpha}{\Gamma(\alpha+1)}\right)\\
& +\rho_a(\mathcal{K}_4 + (a\sigma\mathcal{K}_2 - \tau_a\mathcal{K}_4 - \rho_a\mathcal{K}_4 - \lambda\mathcal{K}_4)\frac{t^\alpha}{\Gamma(\alpha+1)}\Big) - (\lambda_s+\lambda+\phi_s)\,(\mathcal{K}_3 + \{(1-a)\sigma\mathcal{K}_2 + \rho_a\mathcal{K}_4
\end{aligned}$$

$$
\begin{aligned}
&\quad -(\lambda_s+\lambda)\mathscr{K}_3-\phi_s\mathscr{K}_4)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)}.\\
&\mathbb{I}_{a2}(t)=\mathscr{K}_4+\bigg\{a\sigma\bigg(\mathscr{K}_2+\Big(\beta(1-\varepsilon_m e_m)\mathscr{K}_1\mathscr{K}_3+\beta^{'}(1-\varepsilon_m e_m)\mathscr{K}_1\mathscr{K}_4-\sigma\mathscr{K}_2-\lambda\mathscr{K}_2\Big)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)-(\tau_a+\rho_a\\
&\quad +\lambda\bigg(\mathscr{K}_4+(a\sigma\mathscr{K}_2-\tau_a\mathscr{K}_4-\rho_a\mathscr{K}_4-\lambda\mathscr{K}_4)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)}.\\
&\mathbb{H}_2(t)=\mathscr{K}_5+\bigg\{\phi_s\bigg(\mathscr{K}_3+((1-a)\sigma\mathscr{K}_2+\rho_a\mathscr{K}_4-(\lambda_s+\lambda)\mathscr{K}_3-\phi_s\mathscr{K}_4)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)-(\tau_h+\lambda_h+\lambda)\\
&\quad \bigg(\mathscr{K}_5+(\phi_s\mathscr{K}_3-(\tau_h+\lambda_h+\lambda)\mathscr{K}_5)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)}.\\
&\mathbb{Q}_{h2}(t)=\mathscr{K}_6+\bigg\{\tau_a\bigg(\mathscr{K}_4+(a\sigma\mathscr{K}_2-\tau_a\mathscr{K}_4-\rho_a\mathscr{K}_4-\lambda\mathscr{K}_4)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)-(\rho_h+\lambda)\,(\mathscr{K}_6+(\tau_a\mathscr{K}_4-\rho_h\mathscr{K}_6\\
&\quad -\lambda\mathscr{K}_6)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)}.\\
&\mathbb{R}_2(t)=\mathscr{K}_7+\bigg\{\rho_h+\tau_h)\bigg(\mathscr{K}_5+(\phi_s\mathscr{K}_3-(\tau_h+\lambda_h+\lambda)\mathscr{K}_5)\,\frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)-\lambda\,(\mathscr{K}_7+(\rho_h\mathscr{K}_5+\tau_h\mathscr{K}_5+\lambda\mathscr{K}_7)\\
&\quad \frac{t^\alpha}{\Gamma(\alpha+1)}\bigg)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)}.
\end{aligned}
$$

$$
\begin{aligned}
&\mathbb{S}_n(t)=\sum_{n=0}^{\infty}\mathbb{S}_{n+1}(t)=\mathscr{K}_1+\bigg\{\delta-\beta(1-\varepsilon_m e_m)\sum_{n=0}^{\infty}(\mathbb{S}_n\mathbb{I}_{an})-\beta^{'}(1-\varepsilon_m e_m)\sum_{n=0}^{\infty}(\mathbb{S}_n\mathbb{I}_{sn})\ -\lambda\sum_{n=0}^{\infty}\mathbb{E}_n\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{E}_n(t)=\sum_{n=0}^{\infty}\mathbb{E}_{n+1}(t)=\mathscr{K}_2+\bigg\{\beta(1-\varepsilon_m e_m)\sum_{n=0}^{\infty}(\mathbb{S}_n\mathbb{I}_{an})+\beta^{'}(1-\varepsilon_m e_m)\sum_{n=0}^{\infty}(\mathbb{S}_n\mathbb{I}_{sn})\ -\sum_{n=0}^{\infty}\mathbb{E}_n(t)(\sigma+\lambda)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{I}_{ns}(t)=\sum_{n=0}^{\infty}\mathbb{I}_{s(n+1)}(t)=\mathscr{K}_3+\bigg\{(1-a)\sigma\sum_{n=0}^{\infty}\mathbb{E}_n(t)+\rho_a\sum_{n=0}^{\infty}\mathbb{I}_{an}(t)-(\lambda_s+\lambda)\sum_{n=0}^{\infty}\mathbb{I}_{sn}(t)\ -\phi_s\sum_{n=0}^{\infty}\mathbb{I}_{an}(t)\bigg\},\\
&\mathbb{I}_{na}(t)=\sum_{n=0}^{\infty}\mathbb{I}_{a(n+1)}(t)=\mathscr{K}_4+\bigg\{a\sigma\sum_{n=0}^{\infty}\mathbb{E}_n(t)-(\tau_a+\rho_a+\lambda)\sum_{n=0}^{\infty}\mathbb{I}_{an}\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{H}_n(t)=\sum_{n=0}^{\infty}\mathbb{H}_{n+1}(t)=\mathscr{K}_5+\bigg\{\phi_s\sum_{n=0}^{\infty}\mathbb{I}_{sn}-(\tau_h+\lambda_h+\lambda)\sum_{n=0}^{\infty}\mathbb{H}_n(t)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{Q}_{nh}(t)=\sum_{n=0}^{\infty}\mathbb{Q}_{h(n+1)}(t)=\mathscr{K}_6+\bigg\{\tau_a\sum_{n=0}^{\infty}\mathbb{I}_{an}(t)-\sum_{n=0}^{\infty}\mathbb{Q}_{hn}(t)\,(\rho_h+\lambda)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)},\\
&\mathbb{R}_n(t)=\sum_{n=0}^{\infty}\mathbb{R}_{n+1}(t)=\mathscr{K}_7+\bigg\{\rho_h\sum_{n=0}^{\infty}\mathbb{Q}_{hn}(t)+\tau_h\sum_{n=0}^{\infty}\mathbb{H}_n(t)-\lambda\sum_{n=0}^{\infty}\mathbb{R}_n(t)\bigg\}\,\frac{t^\alpha}{\Gamma(\alpha+1)}.
\end{aligned}
\tag{5.10}
$$

We noted that the result obtained via LADM is the same as the ADM [32, 33].

6. Numerical simulation

Now, we considered a graphical representation of the obtained approximate solutions given by Eq (5.10). Here, we calculated only three terms for the required approximate solutions of System (1.2). The values of the parameters have either been taken from suitable sources or were assumed. The importance of these parameters is given in Table 2.

Table 2. Parameters and their values.

Name	Parameters values	References
δ	0.23	Assumed
β'	0.000516	Assumed
λ_s	0.0062	Assumed
ϕ_s	0.06	[34]
ρ_a	0.07	[34]
ε_m	0.5	[35]
e_m	0.2	[36]
σ	0.28	[37]
a	0.66	[37]
λ_h	0.0045	Assumed
λ	0.000428	Demographic
τ_h	1/14	[38, 39]

In Figures 2–8, we present the approximate solutions of the Caputo fractional-order model given by Model (1.2) by using the LADM corresponding to different fractional-order values of $\alpha \in (0, 1]$. Figure 2 demonstrates that the susceptible population rapidly decreases when the value of α decreases. The graph in Figure 3 shows the exposed class when the value of α goes down, the rate of increase slowly reduces. The graph in Figure 5 shows that the number of asymptomatic populations decreases for different values of α. Therefore, it can be seen in Figure 8 that the people are recover very quickly with different fractional values of α. We also notice that the combined effects of face masks and the hospitalization of asymptomatic individuals on the asymptomatic populations which consequently decreases very rapidly Figure (5). The plot shows that the asymptomatic population disappears completely in a shorter time if the face masks are used together with the hospitalization of confirmed infected people.

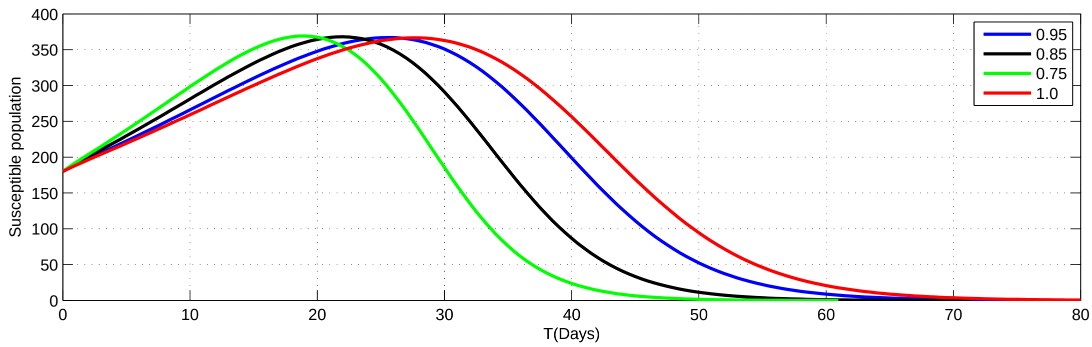

Figure 2. Graphical representation of the approximate solutions for the susceptible class for four different fractional orders.

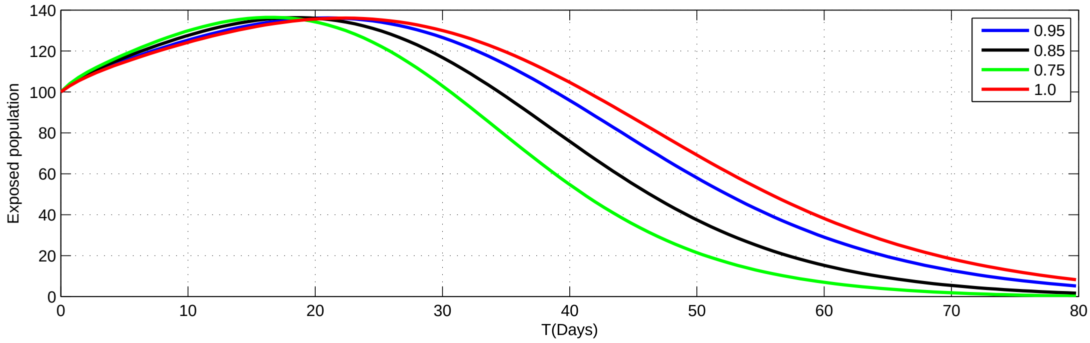

Figure 3. Graphical representation of the approximate solutions for the exposed class for four different fractional orders.

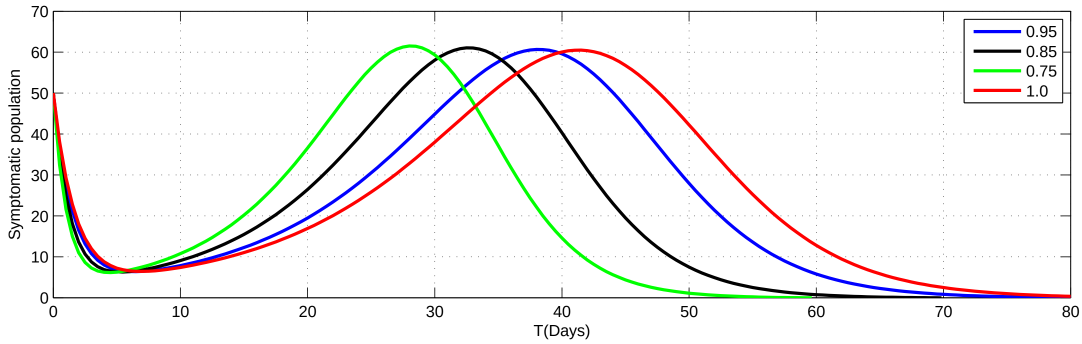

Figure 4. Graphical representation of the approximate solutions for the symptomatic class for four different fractional orders.

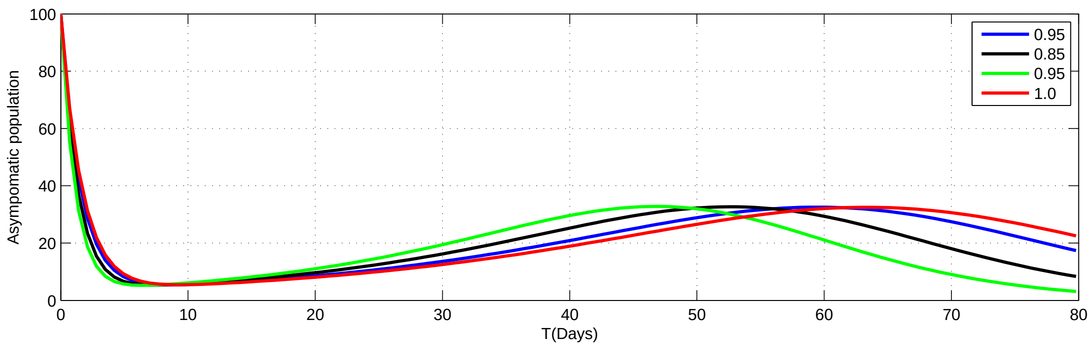

Figure 5. Graphical representation of the approximate solutions for the symptomatic class for four different fractional orders.

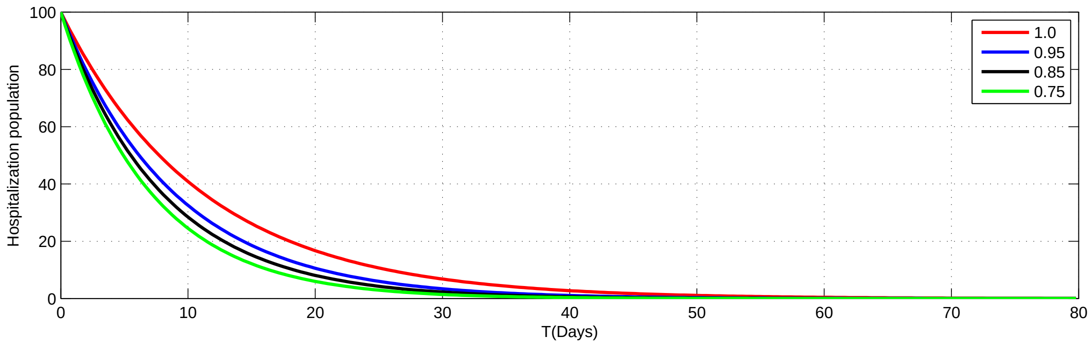

Figure 6. Graphical representation of the approximate solutions for the hospitalized class for four different fractional orders.

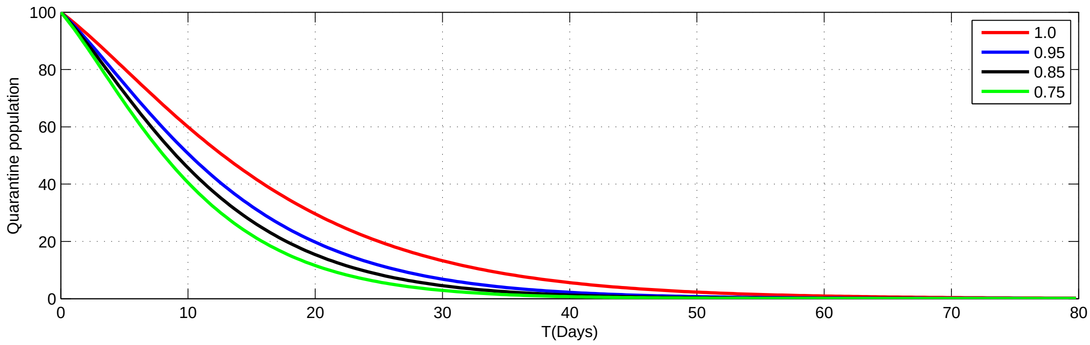

Figure 7. Graphical representation of the approximate solutions for the quarantine class for four different fractional orders.

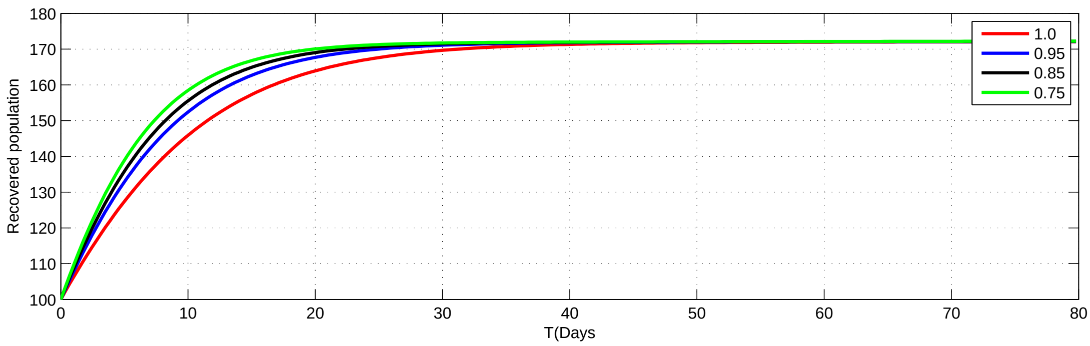

Figure 8. Graphical representation of the approximate solutions for the recovered class for four different fractional orders.

Figure 9 shows all the seven compartments of Model (1.2) when the integer order $\alpha = 1$, and considering the data from Table 2. As compered to traditional derivatives of integer order, fractional order derivatives give more accuracy.

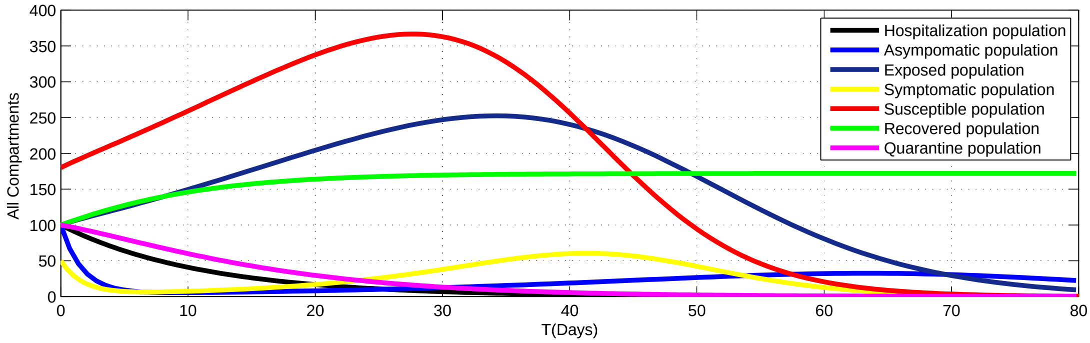

Figure 9. Plot showing the compartments of the fractional model given by Model (1.2) for a integer order of $\alpha = 1$ for the first four terms of the iterative series.

7. Conclusions

This study investigated the caputo fractional-order model of the COVID-19 pandemic, wherein we modeled the effects of quarantine, hospitalization, and face mask use on the COVID-19 pandemic. The existence and non-negativity of the solutions of Model (1.2) have been demonstrated by solving the FGB inequality using the LTM. We proved the uniqueness of the solution of the Caputo fractional order model. We found that the LADM is a powerful technique capable of heading linear and nonlinear Caputo fractional differential equations. This method produces the same solution as the ADM with the correct choice of initial approximation. From these examples, we can see that this method is considered effective for solving many Caputo fractional-order models. Finally, we derived the graphical results of the obtained approximate solutions presented in Eq (5.10) with different values of the fractional-order $\alpha \in (0, 1]$.

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Conflict of interest

The authors declare that they have no competing interests.

References

1. M. Garcia, N. Lipskiy, J. Tyson, R. Watkins, E. Stein, T. Kinley, Centers for disease control and prevention 2019 novel coronavirus disease (COVID-19) information management: Addressing national health-care and public health needs for standardized data definitions and codified vocabulary for data exchange, *J. Am. Med. Inf. Assoc.*, **27** (2020), 1476–1487. https://doi.org/10.1093/jamia/ocaa141

2. *Report 9–Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand*, MRC Centre for Global Infectious Disease Analysis COVID-19, 2020. Available from: `https://www.imperial.ac.uk/mrc-global-infectious-disease-analysis/covid-19/report-9-impact-of-npis-on-covid-19/`.
3. P. Roy, K. R. Upadhyay, J. Caur, Modeling Zika transmission dynamics: Prevention and control, *J. Biol. Syst.*, **28** (2020), 719–749. https://doi.org/10.1142/S021833902050014X
4. S. Dilshad, N. Singh, M. Atif, A. Hanif, N. Yaqub, W. A. Farooq, et al., Automated image classification of chest X-rays of COVID-19 using deep transfer learning, *Results Phys.*, **28** (2021), 104529. https://doi.org/10.1016/j.rinp.2021.104529
5. M. Nicola, Z. Alsafi, C. Sohrabi, A. Kerwan, A. Al-Jabir, C. Iosifidis, et al., The socio-economic implications of the coronavirus pandemic (COVID-19): A review, *Int. J. Surg.*, **78** (2020), 185–193. https://doi.org/10.1016/j.ijsu.2020.04.018
6. I. A. Bashir, B. A. Nasidi, Fractional order model for the role of mild cases in the transmission of COVID-19, *Chaos Solition. Fract.*, **142** (2021), 110374–110383. https://doi.org/10.1016/j.chaos.2020.110374
7. S. Ahmada, A. Ullaha, Q. Mdallal, H. Khan, K. Shaha, A. Khan, Fractional order mathematical modeling of COVID-19 transmission, *Chaos Solition. Fract.*, **139** (2020), 110256–110263. https://doi.org/10.1016/j.chaos.2020.110256
8. T. Chen, J. Rui, Q. Wang, Z. Zhao, J. Cui, L. Yin, A mathematical model for simulating the phase-based transmissibility of a novel coronavirus, *Infect. Dis. Poverty*, **9** (2020), 24. https://doi.org/10.1186/s40249-020-00640-3
9. K. Nisar, S. Ahmad, A. Ullah, K. Shah, H. Alrabaiahc, M. Arfana, Mathematical analysis of SIRD model of COVID-19 with Caputo fractional derivative based on real data, *Results Phys.*, **21** (2021), 103772–103780. https://doi.org/10.1016/j.rinp.2020.103772
10. A. J. Mumbu, A. K. Hugo, Mathematical modelling on COVID-19 transmission impacts with preventive measures: A case study of Tanzania, *Adv. Differ. Equ.*, **14** (2020), 748–766. https://doi.org/10.1080/17513758.2020.1823494
11. R. Verma, S. P. Tiwari, R. Upadhyay, Transmission dynamics of epidemic spread and outbreak of Ebola in West Africa: Fuzzy modeling and simulation, *J. Biol. Dyn.*, **60** (2019), 637–671. https://doi.org/10.1007/s12190-018-01231-0
12. M. A. Khan, S. Ullah, K. O. Okosun, K. Shah, A fractional order pine wilt disease model with Caputo-Fabrizio derivative, *Adv. Differ. Equ.*, **2018** (2018), 410. https://doi.org/10.1186/s13662-018-1868-4
13. I. Area, H. Batarfi, J. Losada, J. J. Nieto, W. Shammakh, Á. Torres, On a fractional order Ebola epidemic model, *Adv. Differ. Equ.*, **2015** (2015), 278. https://doi.org/10.1186/s13662-015-0613-5
14. M. Dulău, A. Gligor, T. M. Dulău, Fractional order controllers versus integer order controllers, *Procedia Eng.*, **181** (2017), 538–545. https://doi.org/10.1016/j.proeng.2017.02.431
15. D. Sain, B. M. Mohan, A simple approach to mathematical modelling of integer order and fractional order fuzzy PID controllers using one-dimensional input space and their experimental realization, *J. Franklin Inst.*, **358** (2021), 3726–3756. https://doi.org/10.1016/j.jfranklin.2021.03.010

16. M. Higazy, Novel fractional order SIDARTHE mathematical model of COVID-19 pandemic, *Chaos Solition. Fract.*, **138** (2020), 110007. https://doi.org/10.1016/j.chaos.2020.110007
17. N. H. Tuan, H. Mohammadi, S. Rezapour, A mathematical model for COVID-19 transmission by using the Caputo fractional derivative, *Chaos Solition. Fract.*, **140** (2020), 110107. https://doi.org/10.1016/j.chaos.2020.110107
18. A. Srivastav, P. Tiwari, P. Srivastava, M. Ghosh, Y. Kang, A mathematical model for the impacts of face mask, hospitalization and quarantine on the dynamics of COVID-19 in India: Deterministic vs. stochastic, *Math. Biosci. Eng.*, **18** (2021), 182–213. https://doi.org/10.3934/mbe.2021010
19. K. Shah , T. A. jawad, I. Mahariq, F. Jarad, Qualitative analysis of a mathematical model in the time of COVID-19, *BioMed Res. Int.*, **2020** (2020), 5098598. https://doi.org/10.1155/2020/5098598
20. Z. Zhang, A. Zeb, O. F. Egbelowo, V. S. Erturk, Dynamics of a fractional order mathematical model for COVID-19 epidemic, *Adv. Differ. Equ.*, **2020** (2020), 420. https://doi.org/10.1186/s13662-020-02873-w
21. H. Jafari, C. M. Khalique, M. Nazari, Application of the Laplace decomposition method for solving linear and nonlinear fractional diffusion–wave equations, *Appl. Math. Lett.*, **24** (2011), 1799–1805. https://doi.org/10.1016/j.aml.2011.04.037
22. F. Haq, K. Shah, G. Rahman, M. Shahzada, Numerical solution of fractional order smoking model via Laplace Adomian decomposition method, *Alex. Eng. J.*, **57** (2018), 1061–1069. https://doi.org/10.1016/j.aej.2017.02.015
23. M. Z. Mohamed, T. M. Elzaki, Comparison between the Laplace decomposition method and Adomian decomposition in time-space fractional nonlinear fractional differential equations, *Appl. Math.*, **9** (2018), 84309. https://doi.org/10.4236/am.2018.94032
24. M. De la Sen, Positivity and stability of the solutions of Caputo fractional linear time-invariant systems of any order with internal point delays, *Abstr. Appl. Anal.*, **2011** (2011), 161246. https://doi.org/10.1155/2011/161246
25. S. Bushnaq, T. Saeed, D. F. M. Torres, A. Zeb, Control of COVID-19 dynamics through a fractional-order model, *Alex. Eng. J.*, **60** (2021), 3587–3592. https://doi.org/10.1016/j.aej.2021.02.022
26. R. Magin, Fractional calculus in bioengineering, *Crit. Rev. Biomed. Eng.*, **32** (2006), 1–104. https://doi.org/10.1615/critrevbiomedeng.v32.i1.10
27. *Introduction to Differential Equations*, The Hong Kong University of Science and Technology, 2003. Available from: `https://hostnezt.com/cssfiles/appliedmaths/Introduction20to20Differential20Equations20By20Jeffrey20R.0Chasnov.pdf`.
28. I. Podlubny, A. Chechkin, T. Skovranek, Y. Q. Chen, M. V. Jara, Matrix approach to discrete fractional calculus II: Partial fractional differential equations, *J. Comput. Phys.*, **228** (1019), 3137–3153. https://doi.org/10.1016/j.jcp.2009.01.014
29. J. Sabatier, O. P. Agrawal, J. A. Tenreiro Machado, *Advances in fractional calculus*, Theoretical Developments and Applications in Physics and Engineering, 4 Eds., Berlin: Springer, 2007. https://doi.org/10.1007/978-1-4020-6042-7
30. M. S. Abdo, K. Shah, H. A. Wahash, S. K. Panchal, On a comprehensive model of the novel coronavirus (COVID-19) under Mittag-Leffler derivative, *Chaos Solition. Fract.*, **135** (2020), 109867. https://doi.org/10.1016/j.chaos.2020.109867

31. S. Kumar, A. Yildirim, Y. Khan, L. Wei, A fractional model of the diffusion equation and its analytical solution using Laplace transform, *Sci. Iran.*, **19** (2012), 1117–1123. https://doi.org/10.1016/j.scient.2012.06.016
32. H. Khan, R. Shah, P. Kumam, D. Baleanu, M. Arif, A two-step Laplace decomposition method for solving nonlinear partial differential equations, *Int. J. Phys. Sci.*, **6** (2011), 4102–4109. https://doi.org/10.5897/IJPS11.146
33. J. H. He, Variational iteration method–a kind of non-linear analytical technique: Some examples, *Int. J. Non-Linear Mech.*, **34** (1999), 699–708. https://doi.org/10.1016/S0020-7462(98)00048-1
34. D. Aldila, M. Z. Ndii, B. M. Samiadji, Optimal control on COVID-19 eradication program in Indonesia under the effect of community awareness, *Math. Biosci. Eng.*, **17** (2020), 6355–6389. http://dx.doi.org/10.3934/mbe.2020335
35. A. Davies, K. Thompson, K. Giri, G. Kafatos, J. Walker, A. Bennett, Testing the efficacy of homemade masks: Would they protect in an influenza pandemic, *Disaster Med. Public Health Prep.*, **7** (2013), 413–418. https://doi.org/10.1017/dmp.2013.43
36. C. J. Noakes, P. Sleigh, Mathematical models for assessing the role of airflow on the risk of airborne infection in hospital wards, *J. R. Soc. Interface*, **6** (2009), S791–S800. https://doi.org/10.1098/rsif.2009.0305.focus
37. M. A. Khan, A. Atangana, Modeling the dynamics of novel coronavirus (2019-nCov) with fractional derivative, *Alex. Eng. J.*, **59** (2020), 2379–2389. https://doi.org/10.1016/j.aej.2020.02.033
38. F. Zhou, T. Yu, R. Du, G. Fan, Y. Liu, Z. Liu, et al., Clinical course and risk factors for mortality of adult inpatients with COVID-19 in Wuhan, China: A retrospective cohort study, *Lancet*, **395** (2020), 1054–1062. https://doi.org/10.1016/S0140-6736(20)30566-3
39. B. Tang, X. Wang, Q. Li, N. L. Bragazzi, S. Tang, Y. Xiao, et al., Estimation of the transmission risk of the 2019-nCoV and its implication for public health interventions, *J. Clin. Med.*, **9** (2020), 462. https://doi.org/10.3390/jcm9020462