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Research article

Cluster synchronization in finite/fixed time for semi-Markovian switching T-S fuzzy complex dynamical networks with discontinuous dynamic nodes

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Abstract: In this paper, cluster synchronization in finite/fixed time for semi-Markovian switching complex dynamical networks (CDNs) with discontinuous dynamic nodes is studied. Firstly, the global fixed-time convergence principle of nonlinear systems with semi-Markovian switching is developed. Secondly, the novel state-feedback controllers, which include discontinuous factors and integral terms, are designed to achieve the global stochastic finite/fixed-time cluster synchronization. In the framework of Filippov stochastic differential inclusion, the Lyapunov-Krasovskii functional approach, Takagi-Sugeno(T-S) fuzzy theory, stochastic analysis theory, and inequality analysis techniques are applied, and the global stochastic finite/fixed time synchronization conditions are proposed in the form of linear matrix inequalities (LMIs). Moreover, the upper bound of the stochastic settling time is explicitly proposed. In addition, the correlations among the obtained results are interpreted analytically. Finally, two numerical examples are given to illustrate the correctness of the theoretical results.

Keywords: complex dynamical networks; finite/fixed time cluster synchronization; semi-Markovian switching; T-S fuzzy; discontinuous system

1. Introduction

As we all know, complex dynamical networks (CDNs) generally are described by graphs, where the states are seen as nodes, and the communication information between nodes is denoted as an edge. In CDNs, each node has unique dynamic behavior, and the whole network can present different complex dynamics. In the past decades, CDNs have become a hot spot in various fields because they can represent multifarious real systems, such as biological networks, the World Wide Web, neural networks, genetic networks, ecosystems, social networks, biomolecular networks, and so forth [1–4]. CDNs consists of a large set of interconnected nodes, where each node is a basic unit with specific dynamic behavior, including stability, dissipativity, passivity and synchronization.

As a typical dynamic behavior of CDNs, synchronization is a fascinating phenomenon, which has important practical significance and broad application prospects. For example, after a great speech, the applause of the audience can change gradually from chaos to consensus. Therefore, the synchronization issue has attracted attention from many scholars in various fields: biomedicine, engineering technology, information communication and so on. Recently, there have many different synchronization patterns, such as phase synchronization [5], exponential synchronization [6], projective synchronization [7], finite-time synchronization [8], and fixed-time synchronization [9]. In these types of synchronization, all coupled nodes tend to present a common state as the network evolves, which is called complete synchronization. Nevertheless, complete synchronization of the entire network is not desirable or even possible. Therefore, cluster synchronization occurs in the application, that is, the nodes in the network are divided into several clusters, and the nodes in the same cluster are synchronized, while those nodes in different clusters are not synchronized. In recent years, a growing amount of attention has been paid to cluster synchronization in networks. In [10], cluster stochastic synchronization of complex networks was investigated, and a quantized controller was designed to realize the synchronization of CDNs within a settling time. The cluster synchronization problem for a class of CDNs with coupled time delays was discussed in [11].

It is worth noting that the above types of synchronization are asymptotic synchronization. However, compared with asymptotic synchronization, states can realize synchronization in finite time, in which the settling time is dependent on the initial states. Therefore, finite-time synchronization can only be utilized in a situation where the initial conditions are known. In comparison with finite-time synchronization, the stochastic settling time of fixed-time synchronization [12–16] is regardless of initial conditions. Recently, the finite-time and fixed-time cluster synchronization problems for CDNs have attracted increasing attention due to rapid convergence and better robustness to suppress uncertainties and disturbances. The problem of finite-time cluster synchronization for nonlinear CDNs with hybrid couplings based on aperiodically intermittent control was discussed in [17]. The authors in [18] studied the fixed-time cluster synchronization problem for a class of directed community networks with discontinuous nodes via periodically or aperiodically switching control. In [19], the cluster stochastic synchronization of CDNs via a fixed-time control scheme was discussed. The authors in [20] studied the fixed/preassigned-time cluster synchronization problem for multi-weighted CDNs with stochastic disturbances based on quantized adaptive pinning control.

As we all know, there are still a host of stochastic or unknown factors in the actual systems. Therefore, it is of great practical significance to study the dynamic properties of stochastic systems. Markovion jump systems are suitable for characterizing and modeling different types of systems with abrupt changes [21] and were extensively studied in many aspects, such as stability analysis, static output feedback controller design, and H_∞ filtering problems [22–28]. For singular Markovian jump systems, there are abundant conclusions: especially, the issue of static output feedback control was studied in [29–32]. Unfortunately, the sojourn time in the Markovian jump model used in [33] is subject to the exponential distribution with the memoryless property, which is hard to promise in many practical systems [34]. It is worth mentioning that the sojourn-time in a semi-Markovian switching process [35] can be supposed to obey other probability distributions, such as the Weibull distribution or the Gaussian distribution. Hence, the investigation of semi-Markovian switching CDNs is of great theoretical value and practical significance. In [36], finite-time H_∞ synchronization for CDNs with semi-Markovian jump topology was discussed. The event-triggered synchronization for semi-Markovian switching CDNs with hybrid couplings and time-varying delays was discussed in [37].

In reality, fuzzy logic has a close relationship with the synchronicity and complexity of CDNs. As an extremely important method proposed by Takagi and Sugeno, the fuzzy control approach provides a systematic method for studying the nonlinear systems by expressing a specific nonlinear system as a fuzzy sum of linear subsystems. For instance, in [38], fuzzy differential equations were used to describe the existing vague concepts of uncertainty components. The fuzzy logic theory [39] has been widely accepted as a simple and feasible method to deal with nonlinear systems. Hence, it is necessary to investigate the fuzzy CDNs. Among various fuzzy systems, one of the most important models is the Takagi-Sugeno (T-S) fuzzy system [40], which has been shown to approximate any smooth nonlinear system to any specified accuracy. In the past decade, T-S fuzzy systems have developed rapidly: fault detection, H_∞ control, sampling systems, networked control systems and so on (see [41–45] and references therein). Recently, the research of T-S fuzzy networks has become a hot research topic. In [46], the issue of reliable mixed H_∞ passive control for T-S fuzzy delayed networks based on a semi-Markovian jump model was concerned by using the LMI method. Synchronization and robust stability of T-S fuzzy networks with time-varying delay were discussed in [47] and [48]. In [49], global exponential synchronization of Takagi-Sugeno fuzzy CDNs with multiple time-varying delays and stochastic perturbations was studied via delayed impulsive distributed control. It is worth pointing out that, there is no relevant result about global stochastic finite/fixed-time cluster synchronization for discontinuous semi-Markovian switching T-S fuzzy CDNs currently.

Motivated by the aforementioned discussions, in this paper our objective is to investigate the global stochastic finite/fixed-time cluster synchronization for discontinuous semi-Markovian switching T-S fuzzy CDNs. By employing Filippov discontinuous theory, the Lyapunov stability theory, Lyapunov-Krasovskii functional approach and stochastic analysis techniques, the global finite/fixed-time cluster synchronization conditions are addressed in the form of LMIs. The innovations of this paper compared with the existing results are summarized below:

(1) It is the first time to investigating the global stochastic finite/fixed-time cluster synchronization for T-S fuzzy CDNs with discontinuous activations under semi-Markovian switching.

(2) A principle of the global stochastic stability in fixed time for the nonlinear system with semi-Markovian switching is developed; see Lemma 2.

(3) A fuzzy switching state-feedback discontinuous controller is designed to achieve the global finite/fixed time cluster synchronization.

(4) The stochastic finite/fixed-time cluster synchronization conditions are obtained in terms of LMIs.

(5) The upper bounds of the setting time of stochastic finite/fixed time cluster synchronization are explicitly evaluated.

The rest of this paper is organized as follows. In Section 2, some useful lemmas, definitions, and system models are provided. In Section 3, some criteria for the global stochastic finite/fixed-time cluster synchronization of T-S fuzzy semi-Markovian CDNs with discontinuous nodes are established, and the upper bound of stochastic settling time is explicitly proposed. In Section 4, two numerical simulations are provided to illustrate the effectiveness of the theoretical results. Finally, the conclusion is given in Section 5.

Table 1. Notations.

Symbol	Meaning
R and $\mathcal{N}$	Sets of real numbers and nonnegative integers
$R^{n\times m}$ and R^n	Set of $n \times m$ matrices and n-dimensional vectors
$Z_+, \bar{Z}_+$ and $\bar{R}_+$	$\{z \in Z : z > 0\}$, $\{z \in Z : z \geq 0\}$ and $\{z \in R : z > 0\}$
I_N	N-dimensional identify matrix
$\text{diag}\{\cdots\}$	Block diagonal matrix
$A > 0$ $(A < 0)$	Positive (Negative) definite matrix
$\lambda_{\max}(A)(\lambda_{\min}(A))$	Maximal (Minimal) eigenvalue of A
$A^T(A^{-1})$	Transpose(Inverse) of matrix A
$\|x\|_2$	Euclidean norm of x
$Pr\{\cdot\}$	Probability
$\mathcal{E}\cdot$	Mathematical expectation
$\otimes$	Kronecker product
$\mathcal{L}$	Infinitesimal operator

2. Materials and methods

2.1. Preliminaries

In this subsection, the fixed-time stochastic stability principle for the nonlinear semi-Markovian switching system is presented, and some useful definitions and lemmas are provided for the analysis of the main objective.

The three stochastic processes [35] are described as follows:

(1) Stochastic process $\{\rho_k\}_{k\in\bar{Z}_+}$ takes values in $\mathcal{N}$ in which ρ_k denotes the index of the system mode at the kth transition.

(2) Stochastic process $\{t_k\}_{t\in\bar{Z}_+}$ takes values in $\bar{R}_+$, in which t_k is the time at the kth transition. Moreover, $t_0 = 0$, t_k increases monotonically with k.

(3) Stochastic process $\{h_k\}_{t\in\bar{Z}_+}$ takes values in $\bar{R}_+$, in which $h_k = t_k - t_{k-1}$, refers to the sojourn time of mode r_{k-1} between the $(k-1)$th transition and kth transition, and $h_0 = 0$.

Then, we introduce the semi-Markovian process as follows:

Definition 1. *([35]) Stochastic process $\rho(t) = \rho_k$, $t \in [t_k, t_{k+1})$, is said to be a homogeneous semi-Markovian process if the following two conditions hold for $\imath, \jmath \in \mathcal{N}$, $t_0, t_1, \ldots, t_k \geq 0$:*

i) $\mathtt{Pr}\{\rho_{k+1} = \jmath, h_{k+1} \leq h \mid \rho_k, \ldots, \rho_0, t_k, \ldots, t_0\} = \mathtt{Pr}\{\rho_{k+1} = \jmath, h_{k+1} \leq h \mid \rho_k\}$,

ii) The probability $\mathtt{Pr}\{\rho_{k+1} = \jmath, h_{k+1} \leq h \mid \rho_k = \imath\}$ *is independent on k,*

hold, where h is sojourn time.

In this paper, the network model described by the continuous-time and discrete-state homogeneous semi-Markovian process with right continuous trajectories is established. Based on Definition 1, state $\rho(t)$ takes values in $\mathcal{N}$, and transition rate matrix $\Pi(h) = (\pi_{\imath\jmath}(h))_{\mathbb{N}\times\mathbb{N}}$ is characterized by

$$\mathtt{Pr}\{\rho_{k+1} = \jmath, h_{k+1} \leq h + \delta \mid \rho_k = \imath, h_{k+1} > h\} = \begin{cases} \pi_{\imath\jmath}(h)\delta + o(\delta), & \imath \neq \jmath, \\ 1 + \pi_{\imath\imath}(h)\delta + o(h), & \imath = \jmath \end{cases} \tag{2.1}$$

where $\delta > 0$, $\lim\limits_{\delta\to 0}\frac{o(\delta)}{\delta} = 0$, for $\imath \neq \jmath$ ($\imath, \jmath \in \mathcal{N}$), $\pi_{\imath\jmath}(h) \geq 0$ is the transition rate from mode $\imath$ at time t to mode $\jmath$ at time $t+\delta$, and $\pi_{\imath\imath}(h) = -\sum_{\jmath=1,\jmath\neq\imath}^{\mathbb{N}} \pi_{\imath\jmath}(h)$, for $\imath \in \mathcal{N}$.

Generally, the transition rate $\Pi_{\imath\jmath}(h)$ is bounded, i.e., $\underline{\pi}_{\imath\jmath} \leq \pi_{\imath\jmath}(h) \leq \bar{\pi}_{\imath\jmath}$, where $\underline{\pi}_{\imath\jmath}$ and $\bar{\pi}_{\imath\jmath}$ are positive constants. As a consequence, $\pi_{\imath\jmath}(h)$ can always be written as $\pi_{\imath\jmath}(h) = \pi_{\imath\jmath} + \triangle\pi_{\imath\jmath}$, where $\pi_{\imath\jmath}(h) = \frac{1}{2}(\bar{\pi}_{\imath\jmath} + \underline{\pi}_{\imath\jmath})$, $|\triangle\pi_{\imath\jmath}| \leq \lambda_{\imath\jmath}$ with $\lambda_{\imath\jmath} = \frac{1}{2}(\bar{\pi}_{\imath\jmath} - \underline{\pi}_{\imath\jmath})$.

Consider the following semi-Markovian switching system:

$$\dot{x}(t) = g(x(t), t, \rho(t)), x(0) = x_0, \rho(0) = \rho_0, \tag{2.2}$$

where $x(t) \in R^n$ is the state vector of the system, g: $R^n \times \bar{R}_+ \times \mathcal{N} \to R^n$ is a continuous nonlinear function, $\rho(t)$ is the continuous-time semi-Markovian process, and ρ_0 is the initial mode. Assume that, for any $x_0 \in R^n$, $\rho_0 \in \mathcal{N}$, there exists a global solution with the initial state x_0 and initial mode ρ_0, which is defined as $x(t, x_0, \rho_0)$ for system (2.2).

Definition 2. *([36]) For any $x_0 \in R^n$, $\rho_0 \in \mathcal{N}$, if there exists a stochastic function $T : R^n \to (0, +\infty)$, which is called the stochastic settling-time function, such that the solution $x(t, x_0, \rho_0)$ of system (2.2) satisfies*

$$\lim_{t\to T(x_0)} \mathcal{E}\{\| x(t, x_0, \rho_0) \|\} = 0,$$

when $t \geq \mathcal{E}\{T(x_0, \rho_0)\}$, $\| x(t, x_0, \rho_0) \| \equiv 0$, then system (2.2) is said to be globally stochastic stable in finite time.

Definition 3. *([36]) If system (2.2) is globally stochastic stable in finite time, and $\mathcal{E}\{T(x_0, \rho_0)\}$ is bounded, namely, $\exists T_{\max} > 0$ such that $\mathcal{E}\{T(x_0, \rho_0)\} \leq T_{\max}$ for $\forall x_0 \in R^n$, then system (2.2) is said to be globally stochastic stable in fixed time.*

In the present paper, we suppose that $V : \bar{R}_+ \times R^n \times \mathcal{N} \to R$ is a continuous and differential functional, $x(t) \in R^n$, and $\rho(t)$ is a continuous-time and discrete-state semi-Markovian process. Then, the infinitesimal operator of stochastic functional $V(t, x(t), \rho(t))$ is given by

$$\mathcal{L}V(t, x(t), \imath) = V_t(t, x(t), \imath) + V_x(t, x(t), \imath)\dot{x}(t) + \sum_{\jmath=1}^{\mathbb{N}} \pi_{\imath\jmath}(h)V(t, x(t), \jmath),$$

where $\imath, \jmath \in \mathcal{N}$, $V_t(t, x(t), \rho(t)) = \frac{\partial V(t,x(t),\rho(t))}{\partial t}$, $V_x(t, x(t), \rho(t)) = (\frac{\partial V(t,x(t),\rho(t))}{\partial x_{i1}}, \ldots, \frac{\partial V(t,x(t),\rho(t))}{\partial x_{in}})^\top$.

Lemma 1. *([41]) Let $V(t, x(t), \rho(t)) \in C^{2,1}(R_+ \times R^n \times \mathcal{N}; R_+)$ be positive definite and radially unbounded. If there exists a continuous function $\aleph : (0, +\infty) \to R$ for $v \in (0, +\infty)$, such that*

i) $\mathcal{L}V(t, x(t), \rho(t)) \leq -\aleph(V(t, x(t), \rho(t)))$,
ii) for any $0 \leq s < +\infty$, $\int_0^s \frac{1}{\aleph(v)} dv < +\infty$,
iii) for $v > 0$, $\dot{\aleph}(v) \geq 0$

hold, then system (2.2) is globally stochastic finite-time stable in probability. Moreover, the stochastic settling time T_ε satisfies

$$T_\varepsilon \leq \int_0^{V(0,x_0,\rho_0)} \frac{1}{\aleph(v)} dv.$$

In this paper, we consider $\aleph(v) = kv^{\mu} - \theta v$ *for all* $v \in (0, +\infty)$*, and* $\frac{\theta}{k}v^{1-\mu} < \mu$ *,where* $\mu \in (0, 1)$ *and* $k > 0$*,* $\theta > 0$*; then*

$$T_{\varepsilon} \leq \frac{\ln(1 - \frac{\theta}{k}V^{1-\mu}(0, x_0, \rho_0))}{\theta(\mu - 1)}.$$

Lemma 2. *Let* $x(t) = x(t, x_0, \rho_0)$ *be the solution of system (2.2) with initial value* $x_0 \in R^n\ \{0\}$ *and initial mode* $\rho_0 \in \mathcal{N}$*. If there exists a continuous stochastic functional* $V : R^n \times \mathcal{N} \to R_+$*, such that*

i) $V(x, \rho) > 0$*,* $x \neq 0$*, and* $V(0, \rho) = 0$*,*
ii)$V(x, \rho) \to +\infty$*, as*$\|x\| \to +\infty$*,*
iii)$\mathcal{L}V(x(t), \rho(t)) \leq -\alpha V^{\xi}(x(t), \rho(t)) - \beta V^{\eta}(x(t), \rho(t)) - c$

hold, then, system (2.2) is globally stochastic stable in fixed time, and the upper bound $T_{\max}$ *of the stochastic settling time can be calculated explicitly by*

$$\mathcal{E}\{T(x_0, \rho_0)\} \leq T_{\max} = \frac{1}{c}\left((\frac{c}{\alpha})^{\frac{1}{\xi}}\frac{\xi}{1-\xi} + (\frac{c}{\beta})^{\frac{1}{\eta}}\frac{\eta}{\eta - 1}\right),$$

where $\alpha, \beta > 0$ *and* $0 < \xi < 1$*,* $\eta > 1$*.*

Proof. The proof is divided into two cases.

Case 1: In this case, we prove that system (2.2) is globally stochastic stable in finite time.

Let $\phi(V) = \int_0^V \frac{1}{\mathfrak{T}(\theta)}d\theta$, where $\mathfrak{T}(\theta) = \alpha\theta^{\xi} + \beta\theta^{\eta} + c$. Obviously, $\phi(V) > 0$, and $\phi(V) = 0$ if and only if $V = 0$.

Set $T(x_0, \rho_0) = \phi(V(x_0, \rho_0))$. In the following, we claim that there exists $t_1 \in (0, \mathcal{E}\{T(x_0, \rho_0)\})$, such that $\mathcal{E}\{\| x(t, x_0, \rho_0) \|_2\} = 0$. Otherwise, $\mathcal{E}\{\| x(t, x_0, \rho_0) \|_2\} \neq 0$ on $(0, \mathcal{E}\{T(x_0, \rho_0)\})$.

By the formula $dV(x(t), \rho(t)) = \mathcal{L}V(x(t), \rho(t))dt$, it follows from condition iii) that

$$\begin{aligned}
&\mathcal{E}\{\phi(V(x(T(x_0, \rho_0)), \rho(T(x_0, \rho_0))))\} - \mathcal{E}\{\phi(V(x_0, \rho_0))\}\\
&= \mathcal{E}\left\{\int_{V(x_0,\rho_0)}^{V(x(T(x_0,\rho_0)),\rho(T(x_0,\rho_0))))} d\phi(V)\right\}\\
&= \mathcal{E}\left\{\int_{V(x_0,\rho_0)}^{V(x(T(x_0,\rho_0)),\rho(T(x_0,\rho_0))))} \frac{1}{\mathfrak{T}(V(x(t), \rho(t)))}dV(x(t), \rho(t))\right\}\\
&= \mathcal{E}\left\{\int_0^{T(x_0,\rho_0)} \frac{\mathcal{L}V(x(t), \rho(t))}{\mathfrak{T}(V(x(t), \rho(t)))}dt\right\}\\
&\leq -\mathcal{E}\left\{\int_0^{T(x_0,\rho_0)} dt\right\} = -\mathcal{E}\Big\{T(x_0, \rho_0)\Big\},
\end{aligned}$$

i.e,

$$\mathcal{E}\{\phi(V(x(T(x_0, \rho_0)), \rho(T(x_0, \rho_0))))\} \leq \mathcal{E}\{\phi(V(x_0, \rho_0))\} - \mathcal{E}\{T(x_0, \rho_0)\} = 0, \tag{2.3}$$

which yields that $\mathcal{E}\{\phi(V(x(T(x_0, \rho_0)), \rho(T(x_0, \rho_0))))\} = 0$ by the positive definiteness of $\phi(V)$. This leads to a contradiction.

Next, we prove that $\mathcal{E}\{\| x(t,x_0,\rho_0) \|_2\} = 0$ for all $t \geq t_1$. If it does not hold, then there exists $t_2 \geq t_1$, such that $\mathcal{E}\{\| x(t,x_0,\rho_0) \|_2\} \neq 0$. Let

$$t_3 = \sup\{t \in [t_1,t_2) : \mathcal{E}\{\| x(t,x_0,\rho_0) \|_2\} = 0\}.$$

Obviously, $t_1 < t_3 < t_2$, $\mathcal{E}\{\| x(t,x_0,\rho_0) \|_2\} = 0$, and $\mathcal{E}\{\| x(t,x_0,\rho_0) \|_2\} \neq 0$ for any $t \in (t_3,t_2]$. Analogous to the proof of (2.3), we can get

$$\mathcal{E}\{\phi(V(x(T(x(t_2),\rho(t_2)))))\} \leq -(t_2 - t_3) < 0.$$

It contradicts with the non-negativity of ϕ. Therefore, for all $t > t_1$, $\mathcal{E}\{\| x(t,x_0,\rho_0) \|_2\} = 0$. This implies that system (2.2) is globally stochastic stable in finite time.

Case 2: In this case, we show that $\mathcal{E}\{T(x_0,\rho_0)\}$ is bounded.

$$\begin{aligned}
\mathcal{E}\{T(x_0,\rho_0)\} =&\mathcal{E}\{\phi(V(x_0,\rho_0))\} = \mathcal{E}\left\{\int_0^{V(x_0,\rho_0)} \frac{1}{\alpha\theta^{\xi}+\beta\theta^{\eta}+c}d\theta\right\} \leq \mathcal{E}\left\{\int_0^{+\infty} \frac{1}{\alpha\theta^{\xi}+\beta\theta^{\eta}+c}d\theta\right\}\\
\leq&\left\{\int_0^{s_1} \frac{1}{\alpha\theta^{\xi}+\beta\theta^{\eta}+c}d\theta\right\} + \left\{\int_{s_1}^{s_2} \frac{1}{\alpha\theta^{\xi}+\beta\theta^{\eta}+c}d\theta\right\} + \left\{\int_{s_2}^{+\infty} \frac{1}{\alpha\theta^{\xi}+\beta\theta^{\eta}+c}d\theta\right\}\\
\leq&\int_0^{s_1} \frac{1}{\alpha\theta^{\xi}}d\theta + \int_{s_1}^{s_2} \frac{1}{c}d\theta + \int_{s_2}^{+\infty} \frac{1}{\beta\theta^{\eta}}d\theta\\
=&\frac{1}{\alpha(1-\xi)}s_1^{1-\xi} + \frac{s_2-s_1}{c} + \frac{1}{\beta(\eta-1)}s_2^{1-\eta},
\end{aligned}$$

where s_1, s_2 are arbitrary positive numbers. This derives that $\mathcal{E}\{T(x_0,\rho_0)\}$ is bounded for any $x_0 \in R^n$ and $\rho_0 \in \mathcal{N}$.

On the basis of Cases 1 and 2, we can conclude that system (2.2) is globally stochastic stable in fixed time.

In the following, we develop an accurate estimation for $\mathcal{E}\{T(x_0,\rho_0)\}$. To do so, set

$$g(s_1,s_2) = \frac{1}{\alpha(1-\xi)}s_1^{1-\xi} + \frac{s_2-s_1}{c} + \frac{1}{\beta(\eta-1)}s_2^{1-\eta};$$

then

$$\begin{cases} g_{s_1}(s_1,s_2) = \frac{1}{\alpha}s_1^{-\xi} - \frac{1}{c} = 0,\\ g_{s_2}(s_1,s_2) = \frac{1}{c} - \frac{1}{\beta}s_2^{-\eta} = 0, s_1, s_2 > 0, \end{cases}$$

and we obtain that the stationary point $(s_1,s_2) = ((\frac{c}{\alpha})^{\frac{1}{\xi}}, (\frac{c}{\beta})^{\frac{1}{\eta}})$, which shows that $g(s_1,s_2)$ reaches its minimum value $g_{\min}$,

$$g_{\min} = \frac{1}{c}((\frac{c}{\alpha})^{\frac{1}{\xi}}\frac{\xi}{1-\xi} + (\frac{c}{\beta})^{\frac{1}{\eta}}\frac{\eta}{\eta-1}).$$

Thus, $\mathcal{E}\{T(x_0,\rho_0)\} \leq g_{\min}$. The proof is complete.

Remark 1. *It should be noted that, fixed-time stability problem of systems was widely studied [50–52]. However, there is no result with respect to the settling-time in the published literature, which* *can be derived by Lyapunov functional $V(t)$ satisfying $\mathcal{L}V(x(t)) \leq -\alpha V^{\xi}(x(t)) - \beta V^{\eta}(x(t)) - c$. In this paper, on the basis of the conditions of Lemma 2, global fixed-time stability is discussed with respect to nonlinear systems (2.2) with stochastic switching, and the upper bound of the settling time is proposed. Furthermore, since Lemma 2 is independent of a specific stochastic process in reality, the semi-Markovian process $\rho(t)$ in system (2.2) can be related to any stochastic process.*

2.2. System model description

In this paper, we consider a class of CDNs with semi-Markovian switching and time-varying delay, which can be described by

$$\begin{aligned}\dot{x}_i(t) =& -A(\rho(t))x_i(t) + B(\rho(t))f_i(x_i(t)) + c_1\sum_{j=1}^{N} d(\rho(t))_{ij}\Gamma_1(\rho(t))x_j(t)\\&+ c_2\sum_{j=1}^{N} q(\rho(t))_{ij}\Gamma_2(\rho(t))x_j(t-\tau(t)) + u_i(t),\end{aligned}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\top} \in R^n$ is the state vector of the ith node, $\rho(t)$ is a semi-Markovian switching process, and $f_i : R \times R^n \times R^n \to R^n$ is a nonlinear vector-valued function. $A(\rho(t)) = \mathrm{diag}\{a_1(\rho(t)), a_2(\rho(t)), \ldots, a_n(\rho(t))\}$ is a diagonal matrix with positive entries $a_i(\rho(t))$, $B(\rho(t)) \in R^{n\times n}$, $\tau(t)$ represents time-varying delay in CDNs, and c_1 and c_2 are coupling strengths. $\Gamma_1(\rho(t)) = \mathrm{diag}\{\delta_1^1(\rho(t)), \delta_1^2(\rho(t)), \ldots, \delta_1^n(\rho(t))\}$ and $\Gamma_2(\rho(t)) = \mathrm{diag}\{\delta_2^1(\rho(t)), \delta_2^2(\rho(t)), \ldots, \delta_2^n(\rho(t))\}$ represent the inner-coupling matrices among the clusters, respectively. $D(\rho(t)) = (d_{ij}(\rho(t)))_{N\times N}$ and $Q(\rho(t)) = (q_{ij}(\rho(t)))_{N\times N}$ are non-delayed and time-varying delayed out-coupling configuration matrices that stand for the topological structure; $u_i(t)$ is the control input.

Divide N nodes into Ω clusters, i.e, $\{1, 2, \ldots, N\} = \mathbb{K}_1 \cup \mathbb{K}_2 \cup \ldots \cup \mathbb{K}_{\Omega}$, $\mathbb{K}_{\omega} \bigcap \mathbb{K}_h = \emptyset$, where $h, \omega \in \{1, 2, \ldots, \Omega\}$. For convenience, set $\mathbb{K}_1 = \{1, 2, \ldots, \nu_1\}$, $\mathbb{K}_2 = \{\nu_1 + 1, \nu_1 + 2, \ldots, \nu_2\}$, $\mathbb{K}_{\Omega} = \{\nu_{\Omega-1} + 1, \nu_{\Omega-1} + 2, \ldots, \nu_{\Omega}\}$, and $\nu_0 = 0$, $\nu_{\Omega} = N$.

Then, the outer coupling matrix D can be characterized by the following block form:

$$D(\rho(t)) = \begin{pmatrix} D_{11}(\rho(t)) & D_{12}(\rho(t)) & \ldots & D_{1\Omega}(\rho(t)) \\ D_{21}(\rho(t)) & D_{22}(\rho(t)) & \ldots & D_{2\Omega}(\rho(t)) \\ \vdots & \vdots & \ddots & \vdots \\ D_{\Omega 1}(\rho(t)) & D_{\Omega 2}(\rho(t)) & \ldots & D_{\Omega\Omega}(\rho(t)) \end{pmatrix},$$

where each diagonal block $D_{\omega\omega} = (d_{ij}(\rho(t)))_{(\nu_{\omega}-\nu_{\omega-1})\times(\nu_{\omega}-\nu_{\omega-1})}$ represents the interactions in the community $\mathbb{K}_{\omega}$. Here, for $i, j \in \mathbb{K}_{\omega}$, $i \neq j$, $d_{ij}(\rho(t)) > 0$ are satisfied, $d_{ii}(\rho(t)) = -\sum_{j=\nu_{\omega-1}+1}^{\nu_{\omega}} d_{ij}(\rho(t))$, and each non-diagonal block $D_{\omega h} = d_{ij}(\rho(t)))_{(\nu_{\omega}-\nu_{\omega-1})\times(\nu_h-\nu_{h-1})}$ represents the interactions between the communities $\mathbb{K}_{\omega}$ and $\mathbb{K}_h$, which are satisfied $\sum_{j=\nu_{h-1}+1}^{\nu_h} d_{ij}(\rho(t)) = 0$, $i \in \mathbb{K}_{\omega}$, $j \in \mathbb{K}_h$, $h \neq \omega$. Similarly, $Q(\rho(t))$ has the same properties as $D(\rho(t))$.

(A_1) For $i = 1, 2, \ldots, N$, $f_i : R^n \to R^n$ is continuous except on a countable set of isolated points σ_k, each of which has a finite left limit $f_i(\sigma_k^-)$ and right limit $f_i(\sigma_k^+)$, respectively. Moreover, f_i has at most a finite number of jump discontinuous points in every compact interval of R.

Under the assumption (A_1), for the ωth cluster $(1 \leq \omega \leq \Omega)$, we define $f_{\nu_{\omega-1}+1} = f_{\nu_{\omega-1}+2} = \ldots = f_{\nu_{\omega}} = f_{\omega}$; f_{ω} is undefined at the points where f_{ω} is discontinuous, and $\bar{co}[f_{\omega}(x)] =$

$(\bar{co}[f_{\omega 1}(x_{i1})], \bar{co}[f_{\omega 2}(x_{i2})], \ldots, \bar{co}[f_{\omega n}(x_{in})])$, where $\bar{co}[*]$ denotes the closure of the convex hull of set $*$. It follows that $\bar{co}[f_{\omega l}] = [\min\{f_{\omega l}(x_{il}^-), f_{\omega l}(x_{il}^+)\}, \max\{f_{\omega l}(x_{il}^-), f_{\omega}(x_{il}^+)\}]$, $i \in \mathbb{K}_\omega$.

(A_2) for each $i = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n$, there exist positive constants $L_{\omega l\varepsilon}$, $z_{\omega l}$, $\varepsilon = 1, 2, \ldots, n$, such that

$$| f_{\omega l}(x(t)) - f_{\omega l}(y(t)) | \leq \sum_{\varepsilon=1}^{n} L_{\omega l\varepsilon} \mid x_\varepsilon(t) - y_\varepsilon(t) \mid + z_{\omega l}. \tag{2.4}$$

(A_3) $\tau(t)$ is a bounded and continuously differentiable function, meeting $0 < \dot{\tau}(t) < \vartheta \leq 1$ and $0 \leq \tau(t) \leq \tau$.

Definition 4. *A function* $x : [0, T) \to R^n$, $T \in (0, +\infty)$, *is a Filippov solution of CDNs on* $[-\tau, T)$ *if:*
(i) x(t) is continuous on $[-\tau, T]$ *and absolutely continuous on* $[0, T)$,
(ii) there is $\phi_\omega(t) \in \bar{co}[f_\omega(x_i(t))]$, *which is a measurable function, such that*
$\phi(t) = (\phi_{\omega 1}(t), \phi_{\omega 2}(t), \ldots, \phi_{\omega n}(t))^\top$*:* $[-\tau, T) \to R^n$, *for a.e.* $t \in [0, T)$,

$$\begin{aligned}\dot{x}_i(t) =& - A(\rho(t))x_i(t) + B(\rho(t))\phi_\omega(x_i(t)) + c_1 \sum_{j=1}^{N} d(\rho(t))_{ij}\Gamma_1(\rho(t))x_j(t)\\ &+ c_2 \sum_{j=1}^{N} q(\rho(t))_{ij}\Gamma_2(\rho(t))x_j(t - \tau(t)) + u_i(t),\end{aligned} \tag{2.5}$$

where $i \in \mathbb{K}_\omega$, ϕ_ω *satisfying system (2.5) is called an output solution associated with the state* $x_i(t)$.

Let s_ω, $\omega = 1, 2, \ldots, \Omega$, denotes the target trajectories defined by

$$\dot{s}_\omega(t) = -A(\rho(t))s_\omega(t) + B(\rho(t))\phi_\omega(s_\omega(t)), \tag{2.6}$$

which may be equilibrium points, periodic orbits or even chaotic attractors.

Analogous to Definition 4, the solution of system (2.6) in the Filippov sense can be given as follows:

Definition 5. *A function* $s(t) : [0, T) \to R^n$, $T \in (0, +\infty)$, *is defined as a solution of CDNs on* $[0, T)$ *if*
(i) s(t) is absolutely continuous on $[0, T)$,
(ii) given $\bar{\phi}(t) \in \bar{co}[f_\omega(s_\omega(t))]$, *there exists a measurable function* $\bar{\phi}(t) = (\bar{\phi}_1(t), \bar{\phi}_2(t), \ldots, \bar{\phi}_n(t))^\top$ $:[0, T) \to R^n$, *such that, for almost all* $t \in [0, T)$,

$$\dot{s}_i(t) = - A(\rho(t))s_\omega(t) + B(\rho(t))\bar{\phi}_\omega(s_\omega(t)). \tag{2.7}$$

Define $e_i(t) = x_i(t) - s_\omega(t)$, $i \in \mathbb{K}_\omega$, as the synchronization errors. Then, taking (2.5) with (2.7), in the Filippov sense, the error dynamic system can be written as

$$\begin{aligned}\dot{e}_i(t) =& - A(\rho(t))e_i(t) + B(\rho(t))\hat{\phi}_\omega(e_i(t)) + c_1 \sum_{j=1}^{N} d(\rho(t))_{ij}\Gamma_1(\rho(t))e_j(t)\\ &+ c_2 \sum_{j=1}^{N} q(\rho(t))_{ij}\Gamma_2(\rho(t))e_j(t - \tau(t)) + u_i(t),\end{aligned} \tag{2.8}$$

where $i \in \mathbb{K}_\omega$, $\hat{\phi}_\omega(t) = \phi_\omega(t) - \bar{\phi}_\omega(t)$, $\phi_\omega(t) \in \bar{co}[f_\omega(x_i(t))]$, $\bar{\phi}_\omega(t) \in \bar{co}[f_\omega(s_\omega(t))]$.

For simplicity, we denote $A(\rho(t))$, $B(\rho(t))$, $\Gamma_1(\rho(t))$, $\Gamma_2(\rho(t))$, $D(\rho(t))$, $Q(\rho(t))$ by A_ρ, B_ρ, Γ^1_ρ, Γ^2_ρ, D_ρ, Q_ρ for $\rho(t) \in \mathcal{N}$. Then, system (2.8) can be rewritten as

$$\dot{e}_i(t) = -A_\rho e_i(t) + B_\rho \hat{\phi}_\omega(e_i(t)) + c_1 \sum_{j=1}^{N} d_{\rho,ij}\Gamma^1_\rho e_j(t) + c_2 \sum_{j=1}^{N} q_{\rho,ij}\Gamma^2_\rho e_j(t-\tau(t)) + u_i(t), \quad i \in \mathbb{K}_\omega. \tag{2.9}$$

Moreover, a T-S fuzzy model can be described by a set of fuzzy IF-THEN rules that characterize local relations of a nonlinear system in the state space. The **l**-th rule for semi-Markovian switching CDNs in (2.9) is represented as

Fuzzy rule l: **IF** $\emptyset_1$ is M_{l1}, $\emptyset_2$ is M_{l2},...,$\emptyset_s$ is M_{ls},

THEN:

$$\dot{e}_i(t) = -A^l_\rho e_i(t) + B^l_\rho \hat{\phi}_\omega(e_i(t)) + c_1 \sum_{j=1}^{N} d^l_{\rho,ij}\Gamma^1_\rho e_j(t) + c_2 \sum_{j=1}^{N} q^l_{\rho,ij}\Gamma^2_\rho e_j(t-\tau(t)) + u_i(t), \quad i \in \mathbb{K}_\omega, \tag{2.10}$$

where $l = 1, 2, \ldots, m$, where m is the number of IF-THEN rules. The premise variables $\emptyset_1, \emptyset_2, \ldots \emptyset_s$ are proper state variables, and $M_{lp}(p = 1, 2, \ldots, s)$ is the fuzzy set that is characterized by the membership function. Using the singleton fuzzifier, product fuzzy inference, and a weighted average defuzzifier, the final output of the T-S fuzzy system is inferred as follows:

$$\dot{e}_i(t) = \sum_{l=1}^{m} h_l(\emptyset(t))[-A^l_\rho e_i(t) + B^l_\rho \hat{\phi}_\omega(e_i(t)) + c_1 \sum_{j=1}^{N} d^l_{\rho,ij}\Gamma^1_\rho e_j(t) + c_2 \sum_{j=1}^{N} q^l_{\rho,ij}\Gamma^2_\rho e_j(t-\tau(t))] + u_i(t), \quad i \in \mathbb{K}_\omega, \tag{2.11}$$

where $\emptyset(t) = (\emptyset_1(t), \emptyset_2(t), \ldots, \emptyset_s(t))^\top$, $h_l(\emptyset(t)) = \frac{w_l(\emptyset(t))}{\sum_{l=1}^m w_l(\emptyset(t))}$, $w_l(\emptyset(t)) = \prod_{p=1}^{s} M_{lp}(\emptyset_p(t))$, $w_l(\emptyset(t)) \geq 0$, and $\sum_{l=1}^{m} w_l(\emptyset(t)) \geq 0$. It is clear that

$$\sum_{l=1}^{m} h_l(\emptyset(t)) = 1, h_l(\emptyset(t)) \geq 0,$$

for all $t \in R^+$, where $h_l(\emptyset(t))$ can be regarded as the normalized weight of the IF-THEN rules. In this paper, we will denote $h_l(\emptyset(t)) = h_l$ for simplicity.

In order to obtain the main results in this paper, the following lemmas are given.

Lemma 3. *([8]) For any vector x, $y \in R^n$, scalar $\delta > 0$ and positive definite matrix $Q \in R^{n\times n}$,*

$$2x^\top y \leq \delta x^\top Q x + \delta^{-1} y^\top Q^{-1} y.$$

Lemma 4. *([50]) Let $\varphi_1, \varphi_2, \ldots, \varphi_n \geq 0$, $0 < p \leq 1$, and $q > 1$. Then,*

$$\sum_{i=1}^{n} \varphi_i^p \geq (\sum_{i=1}^{n} \varphi_i)^p, \sum_{i=1}^{n} \varphi_i^q \geq n^{1-q}(\sum_{i=1}^{n} \varphi_i)^q.$$

Lemma 5. *([51]) Let $\psi_1, \psi_2, \ldots, \psi_n \geq 0$, $0 < p < q$. Then,*

$$(\sum_{i=1}^{n} \psi_i^p)^{\frac{1}{p}} \geq (\sum_{i=1}^{n} \psi_i^q)^{\frac{1}{q}}.$$

Lemma 6. *([53]) (Schur complement) Given constant matrices Ξ_1, Ξ_2 and Ξ_3, where $\Xi_1 = \Xi_1^\top$ and $\Xi_2 > 0$,*

$$\Xi_1 + \Xi_3^\top \Xi_2^{-1} \Xi_3 < 0,$$

if and only if

$$\begin{pmatrix} \Xi_1 & \Xi_3^\top \\ \Xi_3 & -\Xi_2 \end{pmatrix} < 0.$$

3. Main results

3.1. Stochastic finite-time cluster synchronization for semi-Markovian switching T-S fuzzy CDNs with discontinuous dynamic nodes

In this section, by designing state-feedback controllers with the discontinuous terms, we consider the global stochastic cluster synchronization in finite time for semi-Markovian switching T-S fuzzy CDNs under the case $\tau(t) = \tau$. The global stochastic finite-time cluster synchronization conditions are addressed in the form of LMIs.

Then, system (2.11) can achieve global stochastic finite time stability under the following controller:

$$\begin{aligned} u_i^1(t) = &- K_\rho e_i(t) - H_\rho sign(e_i(t))\lambda_{\min}^2(P_\rho) - \frac{1}{2}\eta_\rho \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}^2(P_\rho)} sign(e_i(t)) \mid e_i(t) \mid^\alpha \\ &- \frac{1}{2}\eta_\rho \Big(\int_{t-\tau}^t e_i^\top(s) M_\rho e_i(s) ds\Big)^{\frac{1+\alpha}{2}} \frac{e_i(t)}{\|e_i(t)\|^2}, \end{aligned}$$

where $0 < \alpha < 1$, $K_\rho = \text{diag}\{k_{\rho 1}, k_{\rho 2}, \ldots, k_{\rho n}\}$, $H_\rho = \text{diag}\{h_{\rho 1}, h_{\rho 2}, \ldots, h_{\rho n}\}$ and $M_\rho = \text{diag}\{m_{\rho 1}, m_{\rho 2}, \ldots, m_{\rho n}\}$ are the controller gain matrices, $k_{\rho\epsilon}$, $h_{\rho\epsilon}$ and $m_{\rho\epsilon}$ $\epsilon = 1, 2, \ldots, n$ are non-negative constants to be designed, and η_ρ is a tunable constant.

Note that controller $u_i^1(t)$ is discontinuous, which is a special case of assumption (A_1). Then, there exists a measurable function $Sign(e_i(t)) \in \bar{co}[sign(e_i(t))]$ such that

$$\begin{aligned} \xi_i^1(t) = &- K_\rho e_i(t) - H_\rho Sign(e_i(t)) - \frac{1}{2}\eta_\rho \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}^2(P_\rho)} Sign(e_i(t)) \mid e_i(t) \mid^\alpha \\ &- \frac{1}{2}\eta_\rho \Big(\int_{t-\tau}^t e_i^\top(s) M_\rho e_i(s) ds\Big)^{\frac{1+\alpha}{2}} \frac{e_i(t)}{\|e_i(t)\|^2}, \end{aligned}$$

where $\xi_i^1(t) \in \bar{co}[u_i^1(t)]$,

$$\bar{co}[sign(e_i(t))] = \begin{cases} 1, & e_i(t) > 0, \\ [-1,1], & e_i(t) = 0, \\ -1, & e_i(t) < 0. \end{cases} \tag{3.1}$$

The designed controller described by the T-S fuzzy model is composed of a set of fuzzy rules.

Fuzzy rule l: **IF** $\emptyset_1$ is M_{l1}, $\emptyset_2$ is M_{l2},. . .,$\emptyset_s$ is M_{ls}, **THEN**

$$\begin{aligned}\xi_i^1(t)=&-K_\rho^l e_i(t)-H_\rho^l Sign(e_i(t))-\frac{1}{2}\eta_\rho\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}^2(P_\rho)}Sign(e_i(t))\mid e_i(t)\mid^\alpha\\&-\frac{1}{2}\eta_\rho(\int_{t-\tau}^t e_i^\top(s)M_\rho e_i(s)ds)^{\frac{1+\alpha}{2}}\frac{e_i(t)}{\|e_i(t)\|^2}.\end{aligned}\tag{3.2}$$

The state-feedback controller is deduced as

$$\begin{aligned}\xi_i^1(t)=\sum_{l=1}^m h_l\Big[&-K_\rho^l e_i(t)-H_\rho^l Sign(e_i(t))-\frac{1}{2}\eta_\rho\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}^2(P_\rho)}Sign(e_i(t))\mid e_i(t)\mid^\alpha\\&-\frac{1}{2}\eta_\rho(\int_{t-\tau}^t e_i^\top(s)M_\rho e_i(s)ds)^{\frac{1+\alpha}{2}}\frac{e_i(t)}{\|e_i(t)\|^2}\Big].\end{aligned}\tag{3.3}$$

Theorem 1. *Suppose that assumptions A_1 and A_2 are satisfied. For any $\rho\in\mathcal{N}$, if there exist positive definite matrices P_ρ, M_ρ, O_ρ, H_ρ and positive scalars ζ, η_ρ, such that the following LMIs hold,*

$$\begin{aligned}\left(\left(Z\otimes\frac{\mid P_\rho B_\rho^l\mid+\mid P_\rho B_\rho^l\mid^\top}{2}\right)-I_N\otimes H_\rho^l\right)<0,\\ \sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(I_N\otimes M_k)-\eta_\rho(I_N\otimes M_\rho)<0,\end{aligned}\tag{3.4}$$

$$\Phi=\begin{pmatrix}\Phi_{11}&0&\Phi_{13}\\ *&\Phi_{22}&0\\ *&*&\Phi_{33}\end{pmatrix}<0,\tag{3.5}$$

then system (2.11) can achieve global stochastic stability in finite time. Moreover, the upper bound of the stochastic settling time is given by

$$T_\epsilon=\frac{2\ln(1-\frac{b}{a}V^{\frac{1-\alpha}{2}}(0,x_0,\rho_0))}{a(1-\alpha)},\tag{3.6}$$

where $\Phi_{11}=\sum_{k=1,k\neq\rho}^{\mathbb{N}}\left(\pi_{\rho k}(I_N\otimes P_k)+\frac{\lambda_{\rho k}^2}{4}(I_N\otimes O_{\rho k})\right)+\pi_{\rho\rho}(I_N\otimes P_\rho)-(I_N\otimes P_rA_\rho^l)+(L\otimes\frac{|P_\rho B_\rho^l|+|P_\rho B_\rho^l|^\top}{2})+c_1(D_\rho^l\otimes P_\rho\Gamma_\rho^1)+c_2\frac{1}{2}\zeta(Q_\rho^l\otimes P_\rho\Gamma_\rho^2)(Q_\rho^l\otimes P_\rho\Gamma_\rho^2)^\top-(I_N\otimes K_\rho^l)$, $\Phi_{22}=\frac{c_2}{\zeta}(I_N\otimes I_n)-(I_N\otimes M_\rho)$, $\Phi_{13}=\{I_N\otimes(P_\rho-P_1),\dots,I_N\otimes(P_\rho-P_{\rho-1}),I_N\otimes(P_\rho-P_{\rho+1}),\dots,I_N\otimes(P_\rho-P_{\mathbb{N}})\}$, $\Phi_{33}=\{I_N\otimes O_{\rho,1},\dots,I_N\otimes O_{\rho,\rho-1},I_N\otimes O_{\rho,\rho+1},\dots,I_N\otimes O_{\rho,\mathbb{N}}\}$, $Z=diag\{\underbrace{z_1,\dots,z_1}_{\nu_1},\dots,\underbrace{z_\Omega,\dots,z_\Omega}_{\nu_\Omega-\nu_{\Omega-1}}\}$, $L=diag\{\underbrace{L_1,\dots,L_1}_{\nu_1},\dots,\underbrace{L_\Omega,\dots,L_\Omega}_{\nu_\Omega-\nu_{\Omega-1}}\}$, $a=\min\{\eta_\rho\lambda_{\max}^{-\frac{1+\alpha}{2}}(P_\rho),\eta_\rho\lambda_{\min}(P_\rho)\}$, $b=\{\eta_\rho,\frac{\lambda_{\max}(M_\rho)}{\lambda_{\min}(P_\rho)}\}$.

Proof. Consider the following stochastic Lyapunov-Krasovskii functional:

$$V(t,e(t),\rho)=\sum_{i=1}^N e_i^\top(t)P_\rho e_i(t)+\sum_{i=1}^N\int_{t-\tau}^t e_i^\top(s)M_\rho e_i(s)ds.\tag{3.7}$$

Calculating $\mathcal{L}V(t,e(t),\rho)$ along the trajectory of the error system (2.11), we have

$$\begin{aligned}\mathcal{L}V(t,e(t),\rho)=&\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}e_i^\top(t)P_\rho\Big(\sum_{l=1}^{m}h_l\Big[-A_\rho^l e_i(t)+B_\rho^l\hat{\phi}_\omega(t)+c_1\sum_{j=1}^{N}d_{\rho,ij}^l\Gamma_\rho^1 e_j(t)\\&+c_2\sum_{j=1}^{N}q_{\rho,ij}^l\Gamma_\rho^1 e_j(t-\tau)\Big]+\xi_i^1\Big)+\sum_{i=1}^{N}e_i^\top(t)(\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(h)P_k)e_i(t)\\&+\sum_{i=1}^{N}\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(h)\int_{t-\tau}^{t}e_i^\top(s)M_k e_i(s)ds+\sum_{i=1}^{N}e_i^\top(t)M_\rho e_i(t)-\sum_{i=1}^{N}e_i^\top(t-\tau)M_\rho e_i(t-\tau).\end{aligned}\tag{3.8}$$

Substituting (3.3) into (3.8), we obtain that

$$\begin{aligned}\mathcal{L}V(t,e(t),\rho)=&\sum_{l=1}^{m}h_l\Big[\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}-2e_i^\top(t)P_\rho A_\rho^l e_i(t)+\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}e_i^\top(t)P_\rho B_\rho^l\hat{\phi}_\omega(t)\\&+c_1\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\sum_{j=1}^{N}e_i^\top(t)d_{\rho,ij}^l\Gamma_\rho^1 e_j(t)+c_2\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\sum_{j=1}^{N}e_i^\top(t)q_{\rho,ij}^l\Gamma_\rho^2 e_j(t-\tau)\\&-\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}e_i^\top(t)K_\rho^l P_\rho e_i(t)-\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}H_\rho^l Sign(e_i(t))e_i(t)-\eta_\rho\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}^2(P_\rho)}\sum_{i=1}^{N}e_i^\top(t)P_\rho\mid e_i(t)\mid^\alpha\\&-\sum_{i=1}^{N}\eta_\rho P_\rho(\int_{t-\tau}^{t}e_i^\top(s)M_\rho e_i(s)ds)^{\frac{1+\alpha}{2}}\Big]+\sum_{i=1}^{N}e_i^\top(t)M_\rho e_i(t)+\sum_{i=1}^{N}e_i^\top(t)(\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(h)P_\rho)e_i(t)\\&+\sum_{i=1}^{N}\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(h)\int_{t-\tau}^{t}e_i^\top(s)M_k e_i(s)ds-\sum_{i=1}^{N}e_i^\top(t-\tau)M_\rho e_i(t-\tau).\end{aligned}\tag{3.9}$$

Employing Lemma 3, there is a positive constant ζ satisfying

$$\begin{aligned}&2c_2\sum_{m=1}^{v}\sum_{i\in C_m}\sum_{j=1}^{N}e_i^\top(t)q_{\rho,ij}^l\Gamma_r^2 e_j(t-\tau)=2c_2e^\top(t)(Q_\rho^l\otimes P_\rho\Gamma_\rho^2)e(t-\tau)\\&\leq\frac{c_2}{\zeta}e^\top(t-\tau)(I_N\otimes I_n e(t-\tau))+c_2\zeta e^\top(t)[Q_\rho^l\otimes P_\rho\Gamma_\rho^2][Q_\rho^l\otimes P_\rho\Gamma_\rho^2]^\top e(t).\end{aligned}\tag{3.10}$$

In addition, by means of assumptions (A_1) and (A_2), we have

$$\begin{aligned}&\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}e_i^\top(t)P_\rho B_\rho^l\hat{\phi}_\omega(t)=\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\sum_{\varepsilon=1}^{n}\sum_{l=1}^{n}\mid e_{il}^\top(t)\mid\mid P_\rho B_\rho^l\mid\hat{\phi}_{\omega\varepsilon}(t)\\&\leq\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\mid e_i^\top(t)\mid P_\rho B_\rho^l\mid L_\omega\mid e_i(t)\mid+\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\mid P_\rho B_\rho^l\mid\mid e_i^\top(t)\mid z_\omega\end{aligned}$$

$$
\begin{aligned}
&\leq \sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\left(\frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2}\right)L_\omega e_i^\top(t)e_i(t) + \sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega} z_\omega \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2} \, |e_i^\top(t)| \\
&= \left(L\otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2}\right)e_i^\top(t)e_i(t) + \left(Z\otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2}\right)|e_i^\top(t)|,
\end{aligned}
\tag{3.11}
$$

where $Z = \mathrm{diag}\{\underbrace{z_1,\ldots,z_1}_{\nu_1},\ldots,\underbrace{z_\Omega,\ldots,z_\Omega}_{\nu_\Omega-\nu_{\Omega-1}}\}$, $L = \mathrm{diag}\{\underbrace{L_1,\ldots,L_1}_{\nu_1},\ldots,\underbrace{L_\Omega,\ldots,L_\Omega}_{\nu_\Omega-\nu_{\Omega-1}}\}$.

For $\rho, k \in \mathcal{N}$, considering $\pi_{\rho k} = \pi_{\rho k} + \triangle\pi_{\rho k}$, $\triangle\pi_{\rho\rho} = \sum_{k=1,k\neq\rho}^{\mathbb{N}} \triangle\pi_{\rho k}$ and employing Lemma 6, the following inequality holds:

$$
\begin{aligned}
\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(h)P_k &= \sum_{k=1}^{\mathbb{N}}\pi_{\rho k}P_k + \sum_{k=1,k\neq\rho}^{\mathbb{N}} \triangle\pi_{\rho k}P_k + \triangle\pi_{\rho\rho}P_\rho \\
&= \sum_{k=1}^{\mathbb{N}}\pi_{\rho k}P_k + \sum_{k=1,k\neq\rho}^{\mathbb{N}} \triangle\pi_{\rho k}(P_k - P_\rho) \\
&= \sum_{k=1}^{\mathbb{N}}\pi_{\rho k}P_k + \sum_{k=1,k\neq\rho}^{\mathbb{N}} [\frac{1}{2}\triangle\pi_{\rho k}(P_k - P_\rho) + \frac{1}{2}\triangle\pi_{\rho k}(P_k - P_\rho)] \\
&\leq \sum_{k=1}^{\mathbb{N}}\pi_{\rho k}P_k + \sum_{k=1,k\neq\rho}^{\mathbb{N}} [\frac{\lambda_{\rho k}^2}{4}O_{\rho k} + (P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho)].
\end{aligned}
\tag{3.12}
$$

For simplicity, let $e(t) = [e_1^\top(t), e_2^\top(t), \ldots, e_N^\top(t)]^\top$; by using the Kronecker product and substituting (3.10)-(3.12) into (3.9), it holds that

$$
\begin{aligned}
\mathcal{L}V(t,e(t),\rho) \leq \sum_{l=1}^{m} h_l \Bigg[& e^\top(t)\Bigg(\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(I_N\otimes P_k) + \sum_{k=1,k\neq\rho}^{\mathbb{N}}\Big[\frac{\lambda_{\rho k}^2}{4}(I_N\otimes O_{\rho k}) + (I_N\otimes(P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho))\Big]\Bigg)e(t) \\
& - e^\top(t)(I_N\otimes P_\rho A_\rho^l)e(t) + c_1 e^\top(t)(D_\rho^l\otimes P_\rho\Gamma_\rho^2)e(t) + \frac{c_2}{2\zeta}e^\top(t-\tau)(I_N\otimes I_n)e(t-\tau) \\
& + (L\otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2})e^\top(t)e(t) + (Z\otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2})\,|e(t)| \\
& + \frac{c_2\zeta}{2}e^\top(t)[Q_\rho^l\otimes P_\rho\Gamma_\rho^2][Q_\rho^l\otimes P_\rho\Gamma_\rho^2]^\top e(t) - e^\top(t)(I_N\otimes P_\rho K_\rho^l)e(t) - (I_N\otimes H_\rho^l)\,|e(t)| \\
& - \sum_{i=1}^{N}\eta_\rho\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)}\,|e_i(t)|^\alpha |e_i(t)| - \eta_\rho\lambda_{\min}(P_\rho)\sum_{i=1}^{N}\Big(\int_{t-\tau}^{t} e_i^\top(t)M_\rho e_i(t)\Big)^{\frac{1+\alpha}{2}}\Bigg] \\
& + e^\top(t)(I_N\otimes M_\rho)e(t) + \sum_{i=1}^{N}\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}\int_{t-\tau}^{t} e_i^\top(s)M_k e_i(s)ds - e^\top(t-\tau)(I_N\otimes M_\rho)e(t-\tau) \\
& + \eta_\rho\sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^\top(t)M_\rho e_i(t) - \eta_\rho\sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^\top(t)M_\rho e_i(t).
\end{aligned}
\tag{3.13}
$$

By virtue of (3.4), (3.13) is rewritten as

$$\begin{aligned}\mathcal{L}V(t,e(t),\rho) \leq & \mathbb{E}\Phi\mathbb{E} - \eta_\rho \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} \mid e_i(t) \mid^{\alpha} \mid e_i(t) \mid \ -\lambda_{\min}(P_\rho) \sum_{i=1}^{N} \eta_\rho \left(\int_{t-\tau}^{t} e_i^\top(s) M_\rho e_i(s) ds \right)^{\frac{1+\alpha}{2}} \\ & + \eta_\rho \sum_{i=1}^{N} \int_{t-\tau}^{t} e_i^\top(s) M_\rho e_i(s) ds + \frac{\lambda_{\min}(M_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} e_i^\top(t) P_\rho e_i(t),\end{aligned} \tag{3.14}$$

where $\mathbb{E} = [e(t), e(t-\tau)]^\top$. $\Phi = \begin{pmatrix} \hat{\Phi}_{11} & 0 \\ * & \Phi_{22} \end{pmatrix}$, in which, $\hat{\Phi}_{11} = \sum_{k=1,k\neq\rho}^{\mathbb{N}} \left(\pi_{\rho k}(I_N \otimes P_k) + \frac{\lambda_{\rho k}^2}{4}(I_N \otimes O_{\rho k}) \right) + \pi_{\rho\rho}(I_N \otimes P_\rho) + \sum_{k=1,k\neq\rho}^{\mathbb{N}} I_N \otimes ((P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho)) - (I_N \otimes P_\rho A_\rho^l) + (L \otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2}) + c_1(D_\rho^l \otimes P_\rho \Gamma_\rho^1) + \frac{c_2\zeta}{2}(Q_\rho^l \otimes P_\rho \Gamma_\rho^2)(Q_\rho^l \otimes P_\rho \Gamma_\rho^2)^\top - (I_N \otimes K_\rho^l)$, $\Phi_{22} = \frac{c_2}{\zeta}(I_N \otimes I_n) - (I_N \otimes M_\rho)$.

It should be noted that there exist nonlinear terms $(P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho)$ in matrix Φ. As we know, it's difficult to solve matrix inequalities with nonlinear terms. To this end, we denote $\Phi_{11} = [\sum_{k=1,k\neq\rho}^{\mathbb{N}} (\pi_{\rho k}(I_N \otimes P_k) + \frac{\lambda_{\rho k}^2}{4}(I_N \otimes O_{\rho k})) + \pi_{\rho\rho}(I_N \otimes P_\rho) - (I_N \otimes P_\rho A_\rho^l) + \left(L \otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2} \right) + c_1(D_\rho^l \otimes P_\rho \Gamma_\rho^1) + \frac{c_2\zeta}{2}(Q_\rho^l \otimes P_\rho \Gamma_\rho^2)(Q_\rho^l \otimes P_\rho \Gamma_\rho^2)^\top - (I_N \otimes K_\rho^l)]$. It is easy to see that, $\hat{\Phi}_{11} = \Phi_{11} + \sum_{k=1,k\neq\rho}^{\mathbb{N}}(I_N \otimes \left((P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho) \right)$. By Lemma 6 and (3.12), we get

$$\hat{\Phi} = \begin{pmatrix} \hat{\Phi}_{11} & 0 \\ * & \Phi_{22} \end{pmatrix} + \mathrm{diag}\{ \sum_{k=1,k\neq\rho}^{m} I_N \otimes ((P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho)) \} = \begin{pmatrix} \Phi_{11} & 0 & \Phi_{13} \\ * & \Phi_{22} & 0 \\ * & * & \Phi_{33} \end{pmatrix} < 0,$$

where $\Phi_{13} = \{I_N \otimes (P_\rho - P_1), \ldots, I_N \otimes (P_\rho - P_{\rho-1}), I_N \otimes (P_\rho - P_{\rho+1}), \ldots, I_N \otimes (P_\rho - P_{\mathbb{N}})\}$, $\Phi_{33} = \{I_N \otimes O_{\rho 1}, \ldots, I_N \otimes O_{\rho\rho-1}, I_N \otimes O_{\rho\rho+1}, \ldots, I_N \otimes O_{\rho\mathbb{N}}\}$.

According to Lemma 4, we have

$$\begin{aligned} & (\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} \mid e_i(t) \mid^{\alpha} \mid e_i(t) \mid) = \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} \mid e_{ij}(t) \mid^{\alpha+1}, \\ & (\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} \mid e_{ij}(t) \mid^{\alpha+1})^{\frac{1}{1+\alpha}} \geq (\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} \mid e_{ij}(t) \mid^{2})^{\frac{1}{2}}, \\ & (\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} \mid e_{ij}(t) \mid^{\alpha+1}) \geq (\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} \mid e_{ij}(t) \mid^{2})^{\frac{1+\alpha}{2}} \geq \lambda_{\min}^{-\frac{1+\alpha}{2}}(P_\rho)(\sum_{i=1}^{N} e_i(t) P_\rho e_i(t))^{\frac{1+\alpha}{2}}. \end{aligned} \tag{3.15}$$

Combining with (3.14) and (3.15), it follows that

$$\begin{aligned}\mathcal{L}V(t,e(t),\rho) \leq & -\eta_\rho \lambda_{\min}^{-\frac{1+\alpha}{2}}(P_\rho) \left(\sum_{i}^{N} e_i(t) P_\rho e_i(t) \right)^{\frac{1+\alpha}{2}} - \lambda_{\min}(P_\rho)\eta_\rho \sum_{i=1}^{N} \left(\int_{t-\tau}^{t} e_i^\top(s) M_\rho e_i(s) ds \right)^{\frac{1+\alpha}{2}} \\ & + \eta_\rho \sum_{i=1}^{N} \int_{t-\tau}^{t} e_i^\top(s) M_\rho e_i(s) ds \ + \frac{\lambda_{\max}(M_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^{N} e_i^\top(t) P_\rho e_i(t)\end{aligned}$$

$$
\begin{aligned}
&\leq -\eta_\rho \lambda_{\min}^{-\frac{1+\alpha}{2}}(P_\rho)\left(\sum_{i=1}^{N} e_i(t)P_\rho e_i(t)\right)^{\frac{1+\alpha}{2}} - \eta_\rho \lambda_{\min}(P_\rho)\left(\sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^\top(s)M_\rho e_i(s)ds\right)^{\frac{1+\alpha}{2}} \\
&\quad + \eta_\rho \sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^\top(s)M_r e_i(s)ds + \frac{\lambda_{\max}(M_\rho)}{\lambda_{\min}(P_\rho)}\sum_{i=1}^{N} e_i^\top(t)P_\rho e_i(t) \\
&\leq -a\left(\sum_{i=1}^{N} e_i(t)P_\rho e_i(t) + \sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^\top(s)M_\rho e_i(s)ds\right)^{\frac{1+\alpha}{2}} \\
&\quad + b\left(\sum_{i=1}^{N} e_i^\top(t)P_\rho e_i(t) + \sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^\top(s)M_\rho e_i(s)ds\right) \\
&\leq -aV^{\frac{1+\alpha}{2}}(t, e(t), \rho) + bV(t, e(t), \rho),
\end{aligned} \tag{3.16}
$$

where $a = \min\{\eta_\rho \lambda_{\max}^{-\frac{1+\alpha}{2}}(P_\rho), \eta_\rho \lambda_{\min}(P_\rho)\}$, $b = \{\eta_\rho, \frac{\lambda_{\max}(M_\rho)}{\lambda_{\min}(P_\rho)}\}$.

On the basis of Lemma 1, system (2.11) is globally stochastic finite-time stable. This means that system (2.5) and (2.7) can achieve global stochastic finite-time synchronization, and the settling time is estimated by

$$
T_\epsilon = \frac{2\ln(1 - \frac{b}{a}V^{\frac{1-\alpha}{2}}(0, x_0, \rho_0))}{a(\alpha - 1)}. \tag{3.17}
$$

This completes the proof.

Remark 2. *It is seen from (3.17) that the stochastic cluster synchronization can be achieved in finite time. However, the settling time depends on the initial value. This implies that, when the initial value is unknown, the synchronization results in finite time have certain limitations.*

3.2. Stochastic fixed-time cluster synchronization for semi-Markovian switching T-S fuzzy CDNs with discontinuous dynamic nodes

In this subsection, the global stochastic fixed-time synchronization conditions for the considered network systems (2.11) are achieved. To this end, the control law is designed as follows:

$$
u_i^2 = \begin{cases} -H_\rho sign(e_i(t)) - k_\rho^1 \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} sign(e_i(t)) \mid e_i(t) \mid^\lambda - k_\rho^2 \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}^2(P_\rho)} sign(e_i(t)) \mid e_i(t) \mid^\mu \\ -\varsigma_\rho \frac{e_i(t)}{\|e_i(t)\|^2} - (\int_{t-\tau(t)}^{t} e_i^\top(s)W_\rho e_i(s)ds)\frac{e_i(t)}{\|e_i(t)\|^2} - \varpi_\rho(\int_{t-\tau(t)}^{t} e_i^\top(s)M_\rho e_i(s)ds)^{\frac{\mu+1}{2}}\frac{e_i(t)}{\|e_i(t)\|^2} \\ -\varrho_\rho(\int_{t-\tau(t)}^{t} e_i^\top(s)M_\rho e_i(s)ds)^{\frac{\lambda+1}{2}}\frac{e_i(t)}{\|e_i(t)\|^2}, & e_i(t) \neq 0, \\ 0, & e_i(t) = 0, \end{cases} \tag{3.18}
$$

where $0 < \alpha < 1$, $H_\rho = \mathrm{diag}\{h_{\rho 1}, h_{\rho 2}, \ldots, h_{\rho n}\}$ is the controller gain matrix, $h_{\rho\epsilon}$, $\epsilon = 1, 2, \ldots, n$ are nonnegative constants, P_ρ,W_ρ and M_ρ are positive definite matrices, and k_ρ^1, k_ρ^2, ς_ρ and ϱ_ρ are tunable constants.

Based on assumption (A_1) and the IF-THEN rules, we redesign the T-S fuzzy state feedback controller with the discontinuity. Similar to (3.3), we get

$$\xi_i^2 = \begin{cases} \sum_{l=1}^m h_l\Big[-H_\rho^l Sign(e_i(t)) - \varsigma_\rho \frac{e_i(t)}{\|e_i(t)\|^2} - k_\rho^1 \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} Sign(e_i(t)) \mid e_i(t) \mid^\lambda \\ -k_\rho^2 \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}^2(P_\rho)} Sign(e_i(t)) \mid e_i(t) \mid^\mu - \left(\int_{t-\tau(t)}^t e_i^\top(s) W_\rho e_i(s) ds\right) \frac{e_i(t)}{\|e_i(t)\|^2} \\ -\varpi_\rho (\int_{t-\tau(t)}^t e_i^\top(s) M_\rho e_i(s) ds)^{\frac{\mu+1}{2}} \frac{e_i(t)}{\|e_i(t)\|^2} - \varrho_\rho \left(\int_{t-\tau(t)}^t e_i^\top(s) M_\rho e_i(s) ds\right)^{\frac{\lambda+1}{2}} \frac{e_i(t)}{\|e_i(t)\|^2}\Big], & e_i(t) \neq 0, \\ 0, & e_i(t) = 0. \end{cases} \tag{3.19}$$

Next, we will establish a set of sufficient conditions for system (2.11) to realize global stochastic stability in fixed time in the presence of the designed controller (3.19).

Theorem 2. *Suppose the assumptions* $(A_1) - (A_3)$ *hold. If there exist scalars* $k_\rho^1 > 0$, $k_\rho^2 > 0$, $\varsigma_\rho > 0$, $\varrho_\rho > 0$ *and* $n \times n$ *real matrices* H_ρ, P_ρ, W_ρ,O_ρ, $M_\rho > 0$, $\rho \in \{1, 2, \ldots, \mathbb{N}\}$, *such that the following LMIs holds,*

$$\Delta = \begin{pmatrix} \Delta_{11} & 0 & \Delta_{13} \\ * & \Delta_{22} & 0 \\ * & * & \Delta_{33} \end{pmatrix} < 0, \tag{3.20}$$

$$\begin{aligned} &\sum_{k=1}^{\mathbb{N}} \pi_{\rho k}(h)(I_N \otimes P_\rho) - (I_N \otimes P_\rho W_\rho) < 0, \\ &\left(Z \otimes \frac{\mid P_\rho B_\rho^l \mid + \mid P_\rho B_\rho^l \mid^\top}{2}\right) - (I_N \otimes M_\rho H_\rho^l) < 0, \end{aligned} \tag{3.21}$$

then system (2.11) can achieve global stochastic stability in fixed time. Moreover, the upper bound of the stochastic settling time is given by

$$\mathcal{E}\{T(x_0, \rho_0)\} \le T_{\max} = \frac{1}{c}\left[\left(\frac{c}{\gamma}\right)^{\frac{2}{u+1}} \frac{u+1}{1-u} + \left(\frac{c}{\beta}\right)^{\frac{2}{\lambda+1}} \frac{\lambda+1}{\lambda-1}\right], \tag{3.22}$$

where $\Omega_{11} = \sum_{k=1,k\neq\rho}^{\mathbb{N}} \left(\pi_{\rho k}(I_N \otimes P_k) + \frac{\lambda_{\rho k}^2}{4}(I_N \otimes O_{\rho k})\right) + \pi_{\rho\rho}(I_N \otimes P_\rho) - (I_N \otimes P_\rho A_\rho^l) + (L \otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2}) + \frac{1}{2}c_2\zeta(Q_\rho^l \otimes P_\rho \Gamma_\rho^2)(Q_\rho^l \otimes P_\rho \Gamma_\rho^2)^\top + c_1(D_\rho^l \otimes P_\rho \Gamma_\rho^1)$, $\Omega_{22} = \frac{c_2}{2\zeta}(I_N \otimes I_n) - (1-\vartheta)(I_N \otimes M_r)$, $\Omega_{13} = [I_N \otimes (P_\rho - P_1), \ldots, I_N \otimes (P_\rho - P_{\rho-1}), I_N \otimes (P_\rho - P_{\rho+1}), \ldots, I_N \otimes (P_\rho - P_{\mathbb{N}})]$, $\Omega_{33} = diag\{I_N \otimes O_{\rho 1}, \ldots, I_N \otimes O_{\rho(\rho-1)}, I_N \otimes O_{\rho(\rho+1)}, \ldots, I_N \otimes O_{\rho\mathbb{N}}\}$, $\gamma = \min\{k_\rho^1 \lambda_{\min}^{-\frac{1+\mu}{2}}(P_\rho), \lambda_{\min}(P_\rho)\varpi_\rho\}$, $\beta = \min[2k_2^\rho(N\lambda_{\max})^{\frac{1-\lambda}{2}}, \lambda_{\min}(M_\rho)\varrho_\rho N^{\frac{1-\lambda}{2}}]$, $c = \varsigma_\rho \lambda_{\min}(I_N \otimes P_\rho)$. *Z and L are the same as in Theorem 1.*

Proof. Consider the following Lyapunov-Krasovskii functional:

$$V(t, e(t), \rho) = \sum_{i=1}^N e_i^\top(t) P_\rho e_i(t) + \sum_{i=1}^N \int_{t-\tau(t)}^t e_i^\top(s) M_\rho e_i(s) ds. \tag{3.23}$$

By assumption (A_3), calculating $\mathcal{L}V(t,e(t),\rho)$ along the trajectory of the error system (2.11) gives

$$\mathcal{L}V(t,e(t),\rho)=\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\Bigg(e_i^\top(t)P_\rho\sum_{l=1}^{m}h_l\Big\{-A_\rho^l e_i(t)+B_\rho^l\hat{\phi}_\omega(t)+c_1\sum_{j=1}^{N}d_{\rho,ij}^l\Gamma_\rho^1 e_j(t)+c_2\sum_{j=1}^{N}q_{\rho,ij}^l\Gamma_\rho^2 e_j(t-\tau(t))\Big\}$$
$$+\xi_i^2\Bigg)+\sum_{i=1}^{N}e_i^\top(t)(\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(h)P_\rho)e_i(t)-\sum_{i=1}^{N}(1-\vartheta)e_i^\top(t-\tau(t))M_\rho e_i(t-\tau(t))$$
$$+\sum_{i=1}^{N}e_i^\top(t)M_\rho e_i(t)+\sum_{k=1}^{\mathbb{N}}\sum_{i=1}^{N}\pi_{\rho k}(h)\int_{t-\tau(t)}^{t}e_i^\top(s)M_\rho e_i(s)ds. \tag{3.24}$$

Substituting (3.19) into the above inequality, we can get

$$\mathcal{L}V(t,e(t),\rho)=\sum_{l=1}^{m}h_l\Bigg[-\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}e_i^\top(t)P_\rho A_\rho^l e_i(t)+\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}e_i^\top(t)P_\rho B_\rho^l\hat{\phi}_\omega(t)+c_1\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\sum_{j=1}^{N}e_i^\top(t)d_{\rho,ij}^l P_\rho\Gamma_\rho^1 e_j(t)$$
$$+c_2\sum_{\omega=1}^{\Omega}\sum_{i\in\mathbb{K}_\omega}\sum_{j=1}^{N}e_i^\top(t)q_{\rho,ij}^l P_\rho\Gamma_\rho^2 e_j(t-\tau(t))-\sum_{i=1}^{N}e_i^\top(t)P_\rho H_\rho^l Sign(e_i(t))$$
$$-k_\rho^1\sum_{i=1}^{N}e_i^\top(t)\lambda_{\max}(P_\rho)Sign(e_i(t))\mid e_i(t)\mid^\lambda-k_\rho^2\sum_{i=1}^{N}e_i^\top(t)\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)}Sign(e_i(t))\mid e_i(t)\mid^\mu$$
$$-\varsigma_\rho\sum_{i=1}^{N}e_i^\top(t)P_\rho\frac{e_i(t)}{\|e_i(t)\|^2}+\sum_{i=1}^{N}e_i^\top(t)M_\rho e_i(t)-\sum_{i=1}^{N}P_\rho\left(\int_{t-\tau(t)}^{t}e_i^\top(s)W_\rho e_i(s)ds\right)$$
$$-\varrho_\rho\sum_{i=1}^{N}P_\rho\left(\int_{t-\tau(t)}^{t}e_i^\top(s)M_\rho e_i(s)ds\right)^{\frac{\lambda+1}{2}}\Bigg]+\sum_{i=1}^{N}e_i^\top(t)\left(\sum_{k=1}^{\mathbb{N}}\pi_{\rho k}(h)P_\rho\right)e_i(t)$$
$$-\sum_{i=1}^{N}(1-\vartheta)e_i^\top(t-\tau(t))M_\rho e_i(t-\tau(t))+\sum_{k=1}^{\mathbb{N}}\sum_{i=1}^{N}\pi_{\rho k}(h)\int_{t-\tau(t)}^{t}e_i^\top(s)M_\rho e_i(s)ds. \tag{3.25}$$

By using the Kronecker product and substituting (3.10)-(3.12) into (3.25), we get

$$\mathcal{L}V(t,e(t),\rho)=\sum_{l=1}^{m}h_l\Bigg[-e^\top(t)(I_N\otimes P_\rho A_\rho^l)e(t)+\left(L\otimes\frac{\mid P_\rho B_\rho^l\mid+\mid P_\rho B_\rho^l\mid^\top}{2}\right)e^\top(t)e(t)$$
$$+\left(Z\otimes\frac{\mid P_\rho B_\rho^l\mid+\mid P_\rho B_\rho^l\mid^\top}{2}\right)\mid e(t)\mid+c_1e^\top(t)(D_\rho^l\otimes P_\rho\Gamma_\rho^1)e(t)$$
$$+\frac{c_2\zeta}{2}e^\top(t)(Q_\rho^l\otimes P_\rho\Gamma_\rho^2)(Q_\rho^l\otimes P_\rho\Gamma_\rho^2)^\top e(t)+\frac{c_2}{2\zeta}e^\top(t-\tau(t))(I_N\otimes I_n)e(t-\tau(t))$$
$$-(I_N\otimes P_rH_\rho^l)\mid e_i(t)\mid-\varsigma_\rho(I_N\otimes P_\rho)-k_\rho^1\sum_{i=1}^{N}\lambda_{\max}(P_\rho)\mid e_i(t)\mid^{\lambda+1}$$
$$-k_\rho^2\sum_{i=1}^{N}\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)}\mid e_i(t)\mid^{\mu+1}-(I_N\otimes P_\rho)\left(\int_{t-\tau(t)}^{t}e^\top(s)(I_N\otimes W_\rho)e(s)ds\right)$$

$$
\begin{aligned}
&- \varrho_\rho e^\top(t)(I_N \otimes P_\rho)\Big(\int_{t-\tau(t)}^t e^\top(s)(I_N \otimes M_\rho)e(s)ds\Big)^{\frac{\lambda+1}{2}} \Big] + e^\top(t)\Big(\sum_{k=1}^{\mathbb{N}} \pi_{\rho k}(I_N \otimes P_k) \\
&+ \sum_{k=1,k\neq\rho}^{\mathbb{N}} \Big[\frac{\lambda_{\rho k}^2}{4}(I_N \otimes O_{\rho k}) + (I_N \otimes (P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho))\Big]\Big) e(t) + e^\top(t)(I_N \otimes M_\rho)e(t) \qquad (3.26) \\
&- (1-\vartheta)e^\top(t-\tau(t))(I_N \otimes M_\rho)e(t-\tau(t)) + \sum_{k=1}^{\mathbb{N}} \pi_{\rho k}(h) \int_{t-\tau(t)}^t e_i^\top(s)(I_N \otimes M_\rho)e(s)ds.
\end{aligned}
$$

In the light of (3.21), (3.26) is rewritten as

$$
\begin{aligned}
\mathcal{L}V(t, e(t), \rho) \leq & \mathbb{E}\Delta\mathbb{E} - 2\varsigma_\rho(I_N \otimes P_\rho) - k_\rho^1 \sum_{i=1}^N e_i^\top(t)\lambda_{\max}(P_\rho) \mid e_i(t) \mid^{\lambda+1} \\
& - k_\rho^2 \sum_{i=1}^N e_i^\top(t)\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \mid e_i(t) \mid^{\mu+1} - \varrho_\rho(I_N \otimes P_\rho)\Big(\int_{t-\tau(t)}^t e^\top(s)(I_N \otimes M_\rho)e(s)ds\Big)^{\frac{\lambda+1}{2}},
\end{aligned} \qquad (3.27)
$$

where $\mathbb{E} = [e(t), e(t-\tau)]^\top$, and $\Delta = \begin{pmatrix} \bar{\Delta}_{11} & 0 \\ * & \Delta_{22} \end{pmatrix}$, where $\bar{\Delta}_{11} = \sum_{l=1}^m h_l\Big[\sum_{k=1,k\neq\rho}^{\mathbb{N}}(\pi_{\rho k}(I_N \otimes P_k) + \frac{\lambda_{\rho k}^2}{4}(I_N \otimes O_{\rho k})) + \pi_{\rho\rho}(I_N \otimes P_\rho) + \sum_{k=1,k\neq\rho}^{\mathbb{N}} I_N \otimes ((P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho)) - (I_N \otimes P_\rho A_\rho^l) + (L \otimes \frac{|P_\rho B_\rho^l| + |P_\rho B_\rho^l|^\top}{2}) + \frac{c_2\zeta}{2}(Q_\rho^l \otimes P_\rho\Gamma_\rho^2)(Q_\rho^l \otimes P_r\Gamma_\rho^2)^\top + c_1(D_\rho^l \otimes P_r\Gamma_\rho^1)\Big]$, $\Delta_{22} = \frac{c_2}{2\zeta}(I_N \otimes I_n) - (I_N \otimes M_\rho)$.

For the nonlinear terms $(P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho)$, the method in this part is similar to that in Theorem 1, so we can get

$$
\Delta = \begin{pmatrix} \bar{\Delta}_{11} & 0 \\ * & \Delta_{22} \end{pmatrix} + \mathrm{diag}\{ \sum_{k=1.k\neq\rho} (I_N \otimes (P_k - P_\rho)O_{\rho k}^{-1}(P_k - P_\rho))\} = \begin{pmatrix} \Delta_{11} & 0 & \Delta_{13} \\ * & \Delta_{22} & 0 \\ * & * & \Delta_{33} \end{pmatrix} < 0,
$$

where $\Omega_{13} = [I_N \otimes (P_\rho - P_1), \ldots, I_N \otimes (P_\rho - P_{\rho-1}), I_N \otimes (P_\rho - P_{\rho+1}), \ldots, I_N \otimes (P_\rho - P_{\mathbb{N}})]$, $\Omega_{33} = [I_N \otimes O_{\rho 1}, \ldots, I_N \otimes O_{\rho(\rho-1)}, I_N \otimes O_{\rho(\rho+1)}, \ldots, I_N \otimes O_{\rho\mathbb{N}}]$.

In view of Lemma 4 and Lemma 5, it follows that

$$
\begin{aligned}
&\Big(\sum_{i=1}^N \frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \mid e_i(t) \mid^{\mu+1}\Big)^{\frac{1}{\mu+1}} \geq \Big(\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \mid e_i(t) \mid^2\Big)^{\frac{1}{2}}, \\
&\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^N \mid e_i(t) \mid^{\mu+1} \geq \Big(\frac{\lambda_{\max}(P_\rho)}{\lambda_{\min}(P_\rho)} \sum_{i=1}^N \mid e_i(t) \mid^2\Big)^{\frac{1+\mu}{2}} \geq \lambda_{\min}^{-\frac{1+\mu}{2}}(P_\rho)\Big(\sum_{i=1}^N e_i^\top(t)P_\rho e_i(t)\Big)^{\frac{1+\mu}{2}}, \qquad (3.28) \\
&\varpi_\rho \sum_{i=1}^N P_\rho\Big(\int_{t-\tau(t)}^t e_i^\top(s)M_\rho e_i(s)ds\Big)^{\frac{\mu+1}{2}} \geq \lambda_{\min}(P_\rho)\varpi_\rho\Big(\sum_{i=1}^N \int_{t-\tau(t)}^t e_i^\top(s)M_\rho e_i(s)ds\Big)^{\frac{\mu+1}{2}}.
\end{aligned}
$$

Moreover,

$$\sum_{i=1}^{N} e_i^{\top}(t)\lambda_{\max}(P_\rho) \mid e_i(t) \mid^{\lambda+1} = \lambda_{\max}(P_\rho) \sum_{i=1}^{N} \left(e_i^{\top}(t)e_i(t)\right)^{\frac{1+\lambda}{2}}$$

$$\geq \lambda_{\max}(P_\rho)^{\frac{1-\lambda}{2}} \sum_{i=1}^{N} \left(e_i^{\top}(t)P_\rho e_i(t)\right)^{\frac{1+\lambda}{2}} \geq (N\lambda_{\max}(P_\rho))^{\frac{1-\lambda}{2}} \left(\sum_{i=1}^{N} e_i^{\top}(t)P_r e_i(t)\right)^{\frac{1+\lambda}{2}}, \tag{3.29}$$

$$\varrho_\rho \sum_{i=1}^{N} P_\rho \left(\int_{t-\tau(t)}^{t} e_i^{\top}(s)M_\rho e_i(s)ds\right)^{\frac{\lambda+1}{2}} \geq \lambda_{\min}(P_\rho)\varrho_\rho N^{\frac{1-\lambda}{2}} \left(\sum_{i=1}^{N} \int_{t-\tau(t)}^{t} e_i^{\top}(s)M_\rho e_i(s)ds\right)^{\frac{\lambda+1}{2}}.$$

Applying (3.29) and Lemma 4, it follows that

$$\begin{aligned} &k_\rho^1 \lambda_{\min}^{-\frac{1+\mu}{2}}(P_\rho)\left(\sum_{i=1}^{N} e_i^{\top}(t)P_\rho e_i(t)\right)^{\frac{1+\mu}{2}} + \lambda_{\min}(P_\rho)\varpi_\rho \left(\sum_{i=1}^{N} \int_{t-\tau(t)}^{t} e_i^{\top}(s)M_\rho e_i(s)ds\right)^{\frac{1+u}{2}} \\ \geq &\gamma \left[(\sum_{i=1}^{N} e_i^{\top}(t)P_\rho e_i(t)) + (\sum_{i=1}^{N} (\int_{t-\tau(t)}^{t} e_i^{\top}(s)M_\rho e_i(s)ds)) \right]^{\frac{1+u}{2}}, \end{aligned} \tag{3.30}$$

where $\gamma = \min\{k_\rho^1 \lambda_{\min}^{-\frac{1+\mu}{2}}(P_\rho), \lambda_{\min}(P_\rho)\varpi_\rho\}$.

Similarly, based on (3.30) and Lemma 4, we can derive

$$\begin{aligned} &k_\rho^2 (N\lambda_{\max}(P_\rho))^{\frac{1-\lambda}{2}} \left(\sum_{i=1}^{N} e_i^{\top}(t)P_\rho e_i(t)\right)^{\frac{1+\lambda}{2}} + \lambda_{\min}(P_\rho)\varrho_r N^{\frac{1-\lambda}{2}} \left(\sum_{i=1}^{N} \int_{t-\tau(t)}^{t} e_i^{\top}(s)M_\rho e_i(s)ds\right)^{\frac{\lambda+1}{2}} \\ \geq &\beta \left[\sum_{i=1}^{N} e_i^{\top}(t)M_\rho e_i(t) + \sum_{i=1}^{N} \int_{t-\tau(t)}^{t} e_i^{\top}(s)P_\rho e_i(s)ds \right]^{\frac{\lambda+1}{2}}, \end{aligned} \tag{3.31}$$

where $\beta = \min[2^{\frac{3-\lambda}{2}} k_2^\rho (N\lambda_{\max})^{\frac{1-\lambda}{2}}, \lambda_{\min}(M_\rho)\varrho_\rho (2N)^{\frac{1-\lambda}{2}}]$.

Then, according to (3.21), (3.22), (3.30) and (3.31), we can obtain that

$$\mathcal{L}V(t, e(t), \rho) \leq -\gamma V^{\frac{u+1}{2}} - \beta V^{\frac{\lambda+1}{2}} - c, \tag{3.32}$$

where $c = \varsigma_\rho \lambda_{\min}(I_N \otimes P_\rho)$.

On the basis of Definition 2 and Lemma 2, system (2.11) can achieve global stochastic stability in fixed time under controller (3.19). Moreover,

$$T_{\max} = \frac{1}{c}[(\frac{c}{\gamma})^{\frac{2}{u+1}} \frac{u+1}{1-u} + (\frac{c}{\beta})^{\frac{2}{\lambda+1}} \frac{\lambda+1}{\lambda-1}]. \tag{3.33}$$

According to Lemma 2, the fixed-time cluster synchronization is finally realized. The proof is completed.

Remark 3. *It can be seen that the settling time of fixed-time synchronization does not depends on the initial value. Compared with the finite time, fixed-time synchronization is more practical when the initial value is arbitrarily selected.*

4. Simulation results

In this section, we provide two examples to illustrate the correctness of the obtained theoretical results. Consider the T-S fuzzy semi-Markovian switching CDNs with 5 nodes, where the nodes are divided into two groups, $\mathbb{K}_1 = \{x_1, x_2, x_3\}$ and $\mathbb{K}_2 = \{x_4, x_5\}$, and have two modes. The dynamical equations are described by

$$\dot{x}_i(t) = -A_\rho x_i(t) + B_\rho f_\omega(x_i(t)) + c_1 \sum_{j=1}^{5} d_{\rho,ij}\Gamma_\rho^1 x_j(t) + c_2 \sum_{j=1}^{5} q_{\rho,ij}\Gamma_\rho^2 x_j(t-\tau(t)) + u_i(t), \quad i \in \mathbb{K}_\omega, \quad (4.1)$$

where $\omega = 1, 2$, $r = 1, 2$.

• Mode 1

Fuzzy rule 1: **IF** $\emptyset_1$ is 0, **THEN**:

$$\dot{x}_i(t) = -A_1^1 x_i(t) + B_1^1 f_\omega(x_i(t)) + c_1 \sum_{j=1}^{5} d_{1,ij}^1\Gamma_1^1 x_j(t) + c_2 \sum_{j=1}^{5} q_{1,ij}^1\Gamma_1^2 x_j(t-\tau(t)) + u_i(t), \quad i \in \mathbb{K}_\omega.$$

Fuzzy rule 2: **IF** $\emptyset_1$ is 1, **THEN**:

$$\dot{x}_i(t) = -A_1^2 x_i(t) + B_1^2 f_\omega(x_i(t)) + c_1 \sum_{j=1}^{5} d_{1,ij}^2\Gamma_1^1 x_j(t) + c_2 \sum_{j=1}^{5} q_{1,ij}^2\Gamma_1^2 x_j(t-\tau(t)) + u_i(t), \quad i \in \mathbb{K}_\omega.$$

• Mode 2

Fuzzy rule 1: **IF** $\emptyset_2$ is 0, **THEN**:

$$\dot{x}_i(t) = -A_2^1 x_i(t) + B_2^1 f_\omega(x_i(t)) + c_1 \sum_{j=1}^{5} d_{2,ij}^1\Gamma_2^1 x_j(t) + c_2 \sum_{j=1}^{5} q_{2,ij}^1\Gamma_2^2 x_j(t-\tau(t)) + u_i(t), \quad i \in \mathbb{K}_\omega.$$

Fuzzy rule 2: **IF** $\emptyset_2$ is 1, **THEN**:

$$\dot{x}_i(t) = -A_2^2 x_i(t) + B_2^2 f_\omega(x_i(t)) + c_1 \sum_{j=1}^{5} d_{2,ij}^2\Gamma_2^1 x_j(t) + c_2 \sum_{j=1}^{5} q_{2,ij}^2\Gamma_2^2 x_j(t-\tau(t)) + u_i(t), \quad i \in \mathbb{K}_\omega.$$

Example 1. *In this example, the effectiveness of Theorem 1 is verified. Consider the three-dimensional T-S fuzzy semi-Markovian switching CDNs, where each node is described by Chua's circuit. Set* $\tau(t) = \tau = 1$*; the system parameters are given as follows:* $c_1 = 1.065$, $c_2 = 1.09545$,

$$A_1^1 = A_1^2 = A_2^1 = A_2^2 = \begin{bmatrix} -13.199 & -13.989 & 0 \\ 1 & -1 & 1 \\ 0 & -13.797 & -1.199 \end{bmatrix},$$

$$B_1^1 = B_1^2 = B_2^1 = B_2^2 = \begin{bmatrix} 6.8546639 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_1^1 = \begin{bmatrix} 0.995 & 0 & 0 \\ 0 & 0.974 & 0 \\ 0 & 0 & 1.405 \end{bmatrix}, \Gamma_2^1 = \begin{bmatrix} 0.655 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.0086 \end{bmatrix},$$

$$D_1^1 = D_2^1 = \begin{bmatrix} -7 & 1 & 2 & 2 & 2 \\ 1 & -5 & 4 & 1 & -1 \\ 1 & 4 & -5 & 1 & -1 \\ -2 & 1 & 1 & -2 & 2 \\ 2 & -1 & -1 & 2 & -2 \end{bmatrix}, D_1^2 = D_2^2 = \begin{bmatrix} -3 & 2 & 1 & -1 & 1 \\ 2 & -6 & 4 & 2 & -2 \\ 1 & 4 & -5 & -1 & 1 \\ -1 & 2 & -1 & -3 & 3 \\ 1 & -2 & 1 & 3 & -3 \end{bmatrix},$$

$$Q_1^1 = Q_2^1 = \begin{bmatrix} -5 & 1 & 4 & 1 & -1 \\ 1 & -4 & 2 & 1 & 1 \\ 4 & -10 & 6 & 0 & 0 \\ 1 & -1 & 0 & -3 & 3 \\ -1 & 1 & 0 & 3 & -3 \end{bmatrix}, Q_1^2 = Q_2^2 = \begin{bmatrix} -5 & 2 & 3 & 2 & -2 \\ 2 & -6 & 4 & -1 & 1 \\ 3 & 4 & -7 & -1 & 1 \\ 2 & -1 & -1 & -5 & 5 \\ -2 & 1 & 1 & 5 & -5 \end{bmatrix}.$$

Let $f(x_i(t)) = -0.5379x_i(t) + 0.5(-1.577 + 0.5379)(|x_i(t) + 1| - |x_i(t) - 1|)$. *It is easy to check that assumptions* A_1 *and* A_2 *hold, and* $L_{11} = 0.05$, $L_{12} = 0.075$, $L_{21} = 0.3$, $L_{22} = 0.46$, $z_{11} = z_{12} = z_{21} = z_{22} = 0$.

Design the fuzzy weighting function: $\lambda_1(\phi(t)) = \cos^2(t)$ *and* $\lambda_2(\phi(t)) = \sin^2(t)$.

The transition rates of the semi-Markovian switching system in this mode are given.

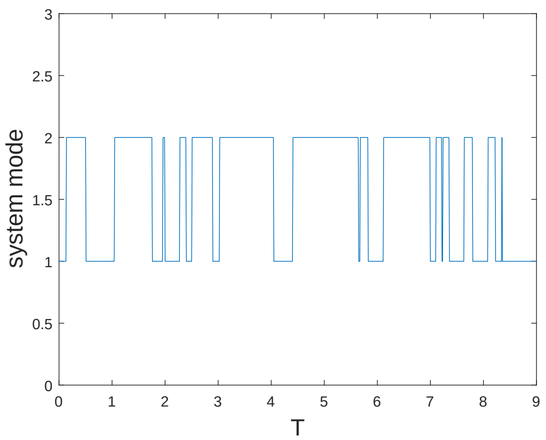

Figure 1. The semi-Markovian jumping switching signal $\rho(t)$.

For mode 1: $\pi_{11}(h) \in (-4.26, -3.98), \pi_{12}(h) \in (3.98, 4.26)$. *For mode 2:* $\pi_{21}(h) \in (-6.17, -5.89), \pi_{22}(h) \in (5.89, 6.17)$. *Then, we can get the parameters* $\pi_{\rho k}, \lambda_{\rho k}$, *where* $\rho, k \in \mathcal{N} = \{1, 2\}$, $\pi_{11} = -3.95, \pi_{12} = 3.95, \lambda_{11} = \lambda_{12} = 0.96$, $\pi_{21} = 6.1, \pi_{22} = -6.1, \lambda_{21} = \lambda_{22} = 0.07$.

Take the controller gain parameters as $\eta_1 = 20.42$, $\eta_2 = 0.53$,

$$K_1^1 = \begin{bmatrix} 15.879 & 0 & 0 \\ 0 & 19.826 & 0 \\ 0 & 0 & 21.577 \end{bmatrix}, K_1^2 = \begin{bmatrix} 15.879 & 0 & 0 \\ 0 & 19.826 & 0 \\ 0 & 0 & 21.577 \end{bmatrix},$$

$$K_2^1=\begin{bmatrix}15.879&0&0\\0&19.826&0\\0&0&21.577\end{bmatrix},K_2^2=\begin{bmatrix}15.879&0&0\\0&19.826&0\\0&0&21.577\end{bmatrix},$$

$$H_1^1=\begin{bmatrix}10.97&0&0\\0&18.7988&0\\0&0&20.8\end{bmatrix},H_1^2=\begin{bmatrix}19.5479&0&0\\0&12.355&0\\0&0&25.577\end{bmatrix},$$

$$H_2^1=\begin{bmatrix}17.5879&0&0\\0&19.9826&0\\0&0&22.3577\end{bmatrix},H_2^2=\begin{bmatrix}13.2879&0&0\\0&17.6826&0\\0&0&27.577\end{bmatrix},$$

$$M_1=\begin{bmatrix}15.99059&0&0\\0&15.99059&0\\0&0&15.99059\end{bmatrix},M_2=\begin{bmatrix}28.959&0&0\\0&28.959&0\\0&0&28.959\end{bmatrix}.$$

By using the MATLAB tools, it is illustrated that the conditions of Theorem 1 are satisfied, and $T=7.55s$. Meanwhile, the graph of semi-Markovian switch signals is displayed in Figure. 1. The synchronization for nodes in the same group with their target trajectories is described in Figure. 2.

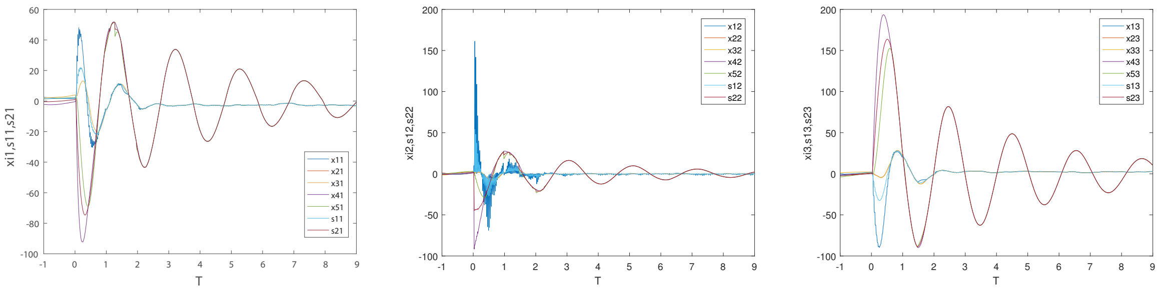

Figure 2. Time evolution of states $x_{i1},x_{i2},x_{i3},i=1,2,3,4,5$, and target trajectories of the two clusters $s_{1j},s_{2j},j=1,2,3$.

Example 2. *In this example, the effectiveness of Theorem 2 is checked. For model (4.1), we consider that the nodes are two-dimensional.*

Set

$$A_1^1=\begin{bmatrix}0.674&0\\0&0.912\end{bmatrix},A_1^2=\begin{bmatrix}1.465&0\\0&0.913\end{bmatrix},A_2^1=\begin{bmatrix}2.117&0\\0&0.898\end{bmatrix},A_2^2=\begin{bmatrix}2.419&0\\0&1.064\end{bmatrix},$$

$$B_1^1=\begin{bmatrix}3.086&1.214\\1.171&1.197\end{bmatrix},B_1^2=\begin{bmatrix}2.287&1.109\\1.215&2.124\end{bmatrix},B_2^1=\begin{bmatrix}1.798&0.815\\1.017&1.803\end{bmatrix},B_2^2=\begin{bmatrix}3.521&1.214\\1.027&2.612\end{bmatrix},$$

$$D_1^1=D_2^1=\begin{bmatrix}-4&2&2&-1&1\\2&-3&1&-1&1\\2&1&-3&2&-2\\-1&-1&2&-2&2\\1&2&-2&2&-2\end{bmatrix},D_1^2=D_2^2=\begin{bmatrix}-4&3&1&-2&2\\3&-5&2&-1&1\\1&2&-3&3&-3\\-2&-1&3&-4&4\\2&1&-3&4&-4\end{bmatrix},$$

$$Q_1^1 = Q_2^1 = \begin{bmatrix} -7 & 6 & 1 & -2 & 2 \\ 6 & -8 & 2 & 4 & -4 \\ 1 & 2 & -3 & -2 & 2 \\ -2 & 4 & -2 & -5 & 5 \\ 2 & -4 & 2 & 5 & -5 \end{bmatrix}, Q_1^2 = Q_2^2 = \begin{bmatrix} -4 & 4 & 0 & 5 & -5 \\ 4 & -5 & 1 & -2 & 2 \\ 0 & 1 & -1 & -3 & 3 \\ 5 & -2 & -3 & -7 & 7 \\ -5 & 2 & 3 & 7 & -7 \end{bmatrix}.$$

The membership functions are defined as follows:

$$h_1(\emptyset(t)) = \frac{1 + \sin^2(t)}{2}, h_2(\emptyset(t)) = \frac{\cos^2(t)}{2}.$$

The system parameters are given as :$c_1 = 0.38$, $c_2 = 0.45$, $\Gamma_1^1 = \Gamma_2^1 = 0.75I_2$, $\Gamma_1^2 = \Gamma_2^2 = 0.55I_2$; $f_{11} = 0.02(x_i(t)) + 0.3sign(x_i(t))$, $f_{12} = 0.14(x_i(t)) + 0.42sign(x_i(t))$, $i = 1, 2, 3$; $f_{21} = 0.17(x_i(t)) + 0.38sign(x_i(t))$, $f_{22} = 0.21(x_i(t)) + 0.46sign(x_i(t))$, $i = 4, 5$. *It is easy to verify that* $L_{11} = 0.02$, $L_{12} = 0.014$, $z_{11} = 0.6$, $z_{12} = 0.84$, $L_{21} = 0.17$, $L_{22} = 0.21$, $z_{21} = 0.76$, $z_{22} = 0.62$. *Take* $\tau(t) = 0.65 + 0.35\sin(t-1)$. *It is easy to check that* $\vartheta = 0.35$, $\tau_1 = 0.3$, $\tau_2 = 1$.

The transition rates with respect to the semi-Markovian process are given as follows:

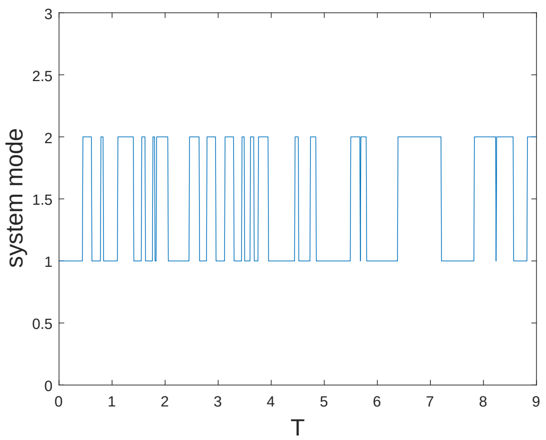

Figure 3. The semi-Markovian jumping switching signal $\rho(t)$.

For mode 1:

$$\pi_{11}(h) \in (-3.16, -2.47), \pi_{12}(h) \in (2.47, 3.16).$$

For mode 2:

$$\pi_{21}(h) \in (1.76, 2.54), \pi_{22}(h) \in (-2.54, -1.76).$$

Accordingly,

$$\pi_{11} = -3.07, \pi_{12} = 3.07, \pi_{21} = 1.88, \pi_{22} = -1.88,$$
$$\lambda_{11} = \lambda_{12} = 0.69, \lambda_{21} = \lambda_{22} = 0.78.$$

Take the controller gain parameters as $k_1^1 = k_1^2 = 0.95$, $k_2^1 = k_2^2 = 0.86$, $\varsigma_1 = \varsigma_2 = 10.8$, $\varpi_1 = \varpi_2 = 10.4$, $\varrho_1 = \varrho_2$*=12.8,*λ*=2.1,* μ*=0.5,*

$$H_1^1 = \begin{bmatrix} 3.415 & 0 \\ 0 & 2.115 \end{bmatrix}, H_2^1 = \begin{bmatrix} 1.928 & 0 \\ 0 & 2.107 \end{bmatrix}, H_1^2 = \begin{bmatrix} 3.601 & 0 \\ 0 & 2.191 \end{bmatrix}, H_2^2 = \begin{bmatrix} 2.121 & 0 \\ 0 & 1.587 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 3.554 & -0.427 \\ -0.427 & 1.921 \end{bmatrix}, W_2 = \begin{bmatrix} 2.121 & -0.848 \\ -0.848 & 0.941 \end{bmatrix}, M_1 = \begin{bmatrix} 3.214 & -0.248 \\ -0.248 & 1.901 \end{bmatrix}, M_2 = \begin{bmatrix} 1.121 & -0.454 \\ -0.454 & 1.817 \end{bmatrix}.$$

By solving LMIs (3.20)-(3.21),we can obtain that

$$P_1 = \begin{bmatrix} 3.685 & 0.713 \\ 0.546 & 6.914 \end{bmatrix}, P_2 = \begin{bmatrix} 4.218 & 0.215 \\ 0.525 & 5.529 \end{bmatrix}, O_1 = \begin{bmatrix} 3.016 & 0.278 \\ 0.146 & 1.901 \end{bmatrix}, O_2 = \begin{bmatrix} 2.215 & 0.554 \\ 0.713 & 1.871 \end{bmatrix}.$$

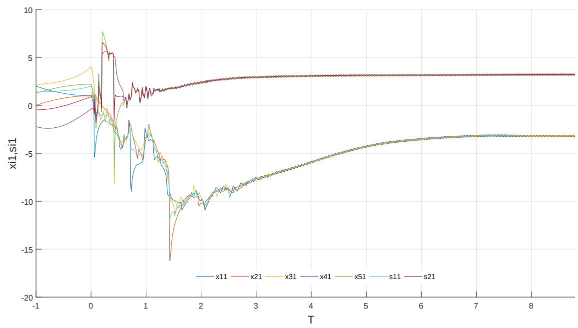

Figure 4. Time evolution of states $x_{i1}, i = 1, 2, 3, 4, 5,$ and target trajectories $s_{i1}, i = 1, 2,$ of the two clusters.

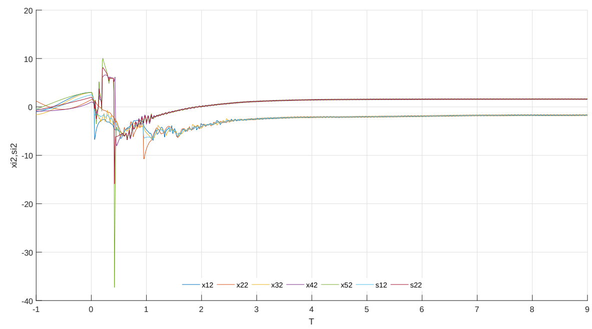

Figure 5. Time evolution of states $x_{i2}, i = 1, 2, 3, 4, 5,$ and target trajectories $s_{i2}, i = 1, 2,$ of the two clusters.

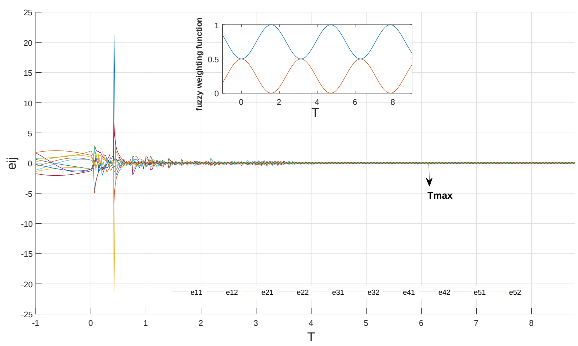

Figure 6. Time evolutions of $e_{ij}, i = 1, 2, 3, 4, 5, j = 1, 2$.

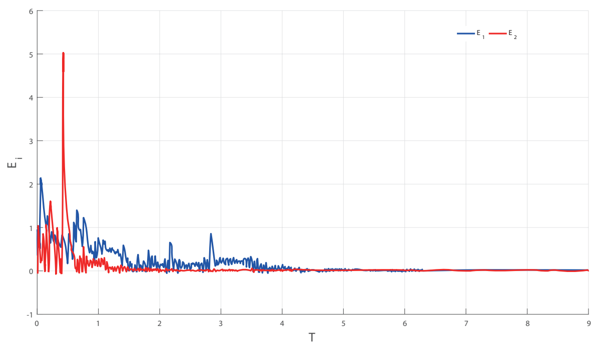

Figure 7. Time evolutions of E_i.

Based on the above parameters, it is easy to verify that the conditions of Theorem 2 hold. The semi-Markovian switching T-S fuzzy CDNs with the above parameters can achieve global stochastic fixed-time cluster synchronization. Based on (3.22), we can obtain that $T_{max} = 6.1226s$.

In addition, the semi-Markovian process is presented in Figure 3. The state trajectories of nodes in each cluster are depicted in Figures 4–7. displays this intra-cluster synchronization behavior. We can easily see that the state trajectories of nodes in each cluster can reach globally stochastic fixed-time synchronization. However, the synchronization goal cannot be achieved between different clusters.

5. Conclusions

In this paper, we have investigated global stochastic cluster synchronization in finite/fixed time for T-S fuzzy CDNs with semi-Markovian switching topologies and discontinuous activations. A new lemma about global stochastic stability in fixed time for the nonlinear system with semi-Markovian switching was developed. In addition, some novel T-S fuzzy state-feedback controllers were designed, which involve double integral terms and discontinuous factors, to achieve the global stochastic finite/fixed-time cluster synchronization objective. The global stochastic cluster synchronization conditions have been addressed in the form of LMIs. Furthermore, the upper bound of the settling time, which depends on the control gains and the chosen controller parameters, has been explicitly calculated. Finally, numerical examples have been provided to illustrate the effectiveness and less conservativeness of the theoretical results.

In the future, the following research works are considered:

1) stochastic cluster synchronization in pre-assigned time for T-S fuzzy discontinuous CDNs with semi-Markovian switching,

2) stochastic synchronization in finite time for T-S fuzzy fractional CDNs with Markovian switching.

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Conflict of interest

The authors declare there is no conflict of interests.

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