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*Research article*

# Periodic problem for non-instantaneous impulsive partial differential equations

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**Abstract:** We obtain a new maximum principle of the periodic solutions when the corresponding impulsive equation is linear. If the nonlinear is quasi-monotonicity, we study the existence of the minimal and maximal periodic mild solutions for impulsive partial differential equations by using the perturbation method, the monotone iterative technique and the method of upper and lower solution. We give an example in last part to illustrate the main theorem.

**Keywords:** non-instantaneous impulses; evolution equation; periodic mild solution; the monotone iterative technique; existence
**Mathematics Subject Classification:** 34A37, 34B77, 47D06, 43K13

## 1. Introduction

We investigate the existence of periodic mild solutions for semilinear non-instantaneous impulsive evolution equation in an ordered Banach space $E$

$$\begin{cases} u'(t) + Au(t) = f(t, u(t)), & t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, \\ u(t) = h_i(t, u(t)), & t \in (t_i, s_i], \quad i = 1, 2, \cdots, \end{cases} \tag{1.1}$$

where $0 = s_0 < t_1 \leq s_1 < t_2 \leq s_2 < \cdots < t_{m-1} \leq s_{m-1} < t_m \leq s_m < t_{m+1} \leq s_{m+1} < \cdots$, satisfying $s_m < \omega < t_{m+1}$ and $\lim\limits_{i\to\infty} s_i = \infty$, $\lim\limits_{i\to\infty} t_i = \infty$ are pre-fixed numbers, and $t_{nm+i} = n\omega + t_i$, $s_{nm+i} = n\omega + s_i, n \in \mathbb{N}^+$, $\omega > 0$ is a constant, let $J = [0, \omega]$; continuous function $f : [0, +\infty) \times E \to E$ is $\omega$-periodic about $t$. Functions $h_i \in C([t_i, s_i] \times E, E)$ and $h_{nm+i}(t + \omega, \cdot) = h_i(t + \omega, \cdot) = h_i(t, \cdot)$ for all $i = 1, 2, \cdots, m$; $A : D(A) \subset E \to E$ is a closed linear operator and $C_0$-semigroup $T(t)(t \geq 0)$ is generated by $-A$ in $E$.

We can find that the periodic problem (1.1) is equal to the following periodic value problem

$$
\begin{cases}
u'(t) + Au(t) = f(t, u(t)), \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\
u(t) = h_i(t, u(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m, \\
u(0) = u(\omega).
\end{cases} \tag{1.2}
$$

The theory of impulsive differential equations have paid more attention in numerous fields, such as physical, biological, economical, engineering background and so on, see [1, 4, 10, 12, 34, 35] and references therein. The instantaneous case has been deeply studied, see [3, 13, 16, 19, 27, 31], where more properties of the solutions of impulsive equations are considered.

The monotone iterative method are important mechanism. Abbas and Benchohra [2] investigated the existence of solutions for IVP of impulsive partial hyperbolic differential equations by employing the method of lower and upper solutions and the Schauder fixed point theorem. Li and Liu [26], Guo and Liu [17] studied impulsive integro-differential equations applying the monotone iterative method. In the papers [6,7], authors considered the nonlocal evolution equations with impulses by exploiting the monotone iterative method. For more monotone iterative method, we refer to the monographs [20–25] and references there in.

Recently, E. Hernandez and D. O'Regan [18] firstly studied new non-instantaneous impulsive evolution equations

$$
\begin{cases}
u'(t) = Au(t) + f(t, u(t)), \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\
u(t) = h_i(t, u(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m. \\
u(0) = u_0,
\end{cases}
$$

which have been used to describe gradual and continuous process such as the hemodynamic equilibrium of a person, the introduction of the drugs in the bloodstream , the consequent absorption for the body and so on. Authors investigated non-instantaneous impulsive fractional differential equations in [14, 29, 32]. In [8], Colao and Muglia considered bounded solutions of non-instantaneous impulsive differential equations with delay. Researchers in [30, 33] studied PBVP of nonlinear non-instantaneous impulsive volution equations.

However, the literature concerning the existence of periodic mild solutions to this problem is untreated by using the perturbation method and the monotone iterative technique. Inspired by the above literatures, this paper is to construct a new maximum principle for the $\omega$-periodic solutions of the corresponding linear equation with non instantaneous impulses. By using perturbation method and monotone iterative technique, we consider the existence of the minimal and maximal periodic solutions for Eq (1.1).

The organization of this paper as follows: some definitions and preliminary facts are recalled in next section, which will be used through this paper. First, we investigate the existence of periodic mild solution for linear non-instantaneous impulsive equation, which is significant for us to prove the key conclusion. Furthermore, for linear impulsive evolution equation corresponding to Eq (1.1), we established a new maximum principle. In Section 3, our major results on the periodic mild solutions of Eq (1.1) are proposed and proved. In Section 4, we presented an example to demonstrate our abstract results.

## 2. Preliminaries

Let $E$ be a Banach space, $C_0$-semigroup $T(t)$ $(t \geq 0)$ is generated by $-A$ in $E$, where $A : D(A) \subset E \to E$ be a closed linear operator. Denote finite number

$$N \equiv \sup_{t\in J} \|T(t)\|.$$

We can see more relevant the properties of the $C_0$-semigroup from the monographs [5, 28].

Let $PC(J, E) = \{u : J \to E \mid u(t)$ is continuous in $J'$, and left continuous at $t_i$, and $u(t_i^+)$ exists, $i = 1, 2, \cdots, m\}$. $K_{PC} = \{u \in PC(J, E)|u(t) \geq \theta, t \in J\}$ is the positive cone and " $\leq$ " is the partial order induced by $K_{PC}$, then $PC(J, E)$ is an order Banach space with the norm $\|\cdot\|_{PC} = \sup_{t\in J} \|u(t)\|$ and the partial order " $\leq$ ". $K_{PC}$ is normal with the same normal constant $N$. $[v, w] = \{u \in PC(J, E) \mid v \leq u \leq w\}$ is the order interval in $PC(J, E)$. In $E$, denote $[v(t), w(t)] = \{u(t) \in E \mid v(t) \leq u(t) \leq w(t), t \in J\}$. Let $J' = J \setminus \{t_1, t_2, \cdots, t_m\}$, $J'' = J \setminus \{0, t_1, t_2, \cdots, t_m\}$. We use $E_1$ to denote the Banach space $D(A)$ with the graph norm $\|\cdot\|_1 = \|\cdot\| + \|A\cdot\|$. We can find more relevant the properties of the partial and cone from the monographs [9, 15].

For the linear problem in $E$

$$\begin{cases} u'(t) + Au(t) = g(t), \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ u(t) = y_i(t), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m, \\ u(0) = x_0, \end{cases} \tag{2.1}$$

we have got the following conclusion.

**Lemma 2.1.** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be $C_0$-semigroup in Banach space $E$. For any $g \in PC(J, E)$, $y_i \in PC(J, E), i = 1, 2, \cdots, m$, problem (2.1) has a unique mild solution $u \in PC(J, E)$ given by*

$$u(t) = \begin{cases} T(t)x_0 + \int_0^t T(t-\tau)g(\tau)d\tau, \quad t \in [0, t_1]; \\ y_i(t), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m; \\ T(t-s_i)y_i(s_i) + \int_{s_i}^t T(t-\tau)g(\tau)d\tau, \quad t \in (s_i, t_{i+1}], \quad i = 1, 2, \cdots, m. \end{cases} \tag{2.2}$$

*Proof.* Let $t \in [0, t_1] = I$, problem (2.1) is equivalent to the initial value problem of linear evolution equation without impulse

$$\begin{cases} u'(t) + Au(t) = g(t), \quad t \in [0, t_1], \\ u(0) = x_0. \end{cases} \tag{2.3}$$

IVP (2.3) has a unique classical solution $u \in C^1(I, E) \cap C(I, E_1)$ expressed by

$$u(t) = T(t)x_0 + \int_0^t T(t-\tau)g(\tau)d\tau.$$

Let $t \in (t_i, s_i]$, then $u(t) = y_i(t), \quad i = 1, 2, \cdots, m.$

Let $t \in (s_i, t_{i+1}]$, problem (2.1) is changed into IVP of linear evolution equation

$$\begin{cases} u'(t) + Au(t) = g(t), \quad t \in (s_i, t_{i+1}], , i = 1, 2, \cdots, m, \\ u(s_i) = y_i(s_i). \end{cases} \tag{2.4}$$

Then (2.4) has a unique mild solution $u \in C([s_i, t_{i+1}], E)$ given by

$$u(t) = T(t - s_i)y_i(s_i) + \int_{s_i}^{t} T(t - \tau)g(\tau)d\tau.$$

Furthermore, after calculated, the function $u \in PC(J, E)$ defined by (2.2) is a mild solution of problem (2.1). Hence problem (2.1) has a unique mild solution $u \in PC(J, E)$ given by (2.2). This completes the proof.□

**Definition 2.1.**( [11]) *A function $u \in PC([0, \omega], E)$ is said to be a $\omega$-periodic PC-mild solution of the equation*

$$\begin{cases} u'(t) + Au(t) = g(t), \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, \\ u(t) = y_i(t), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots. \end{cases} \tag{2.5}$$

*if it is a PC-mild solution of Cauchy problem (2.1) corresponding to some $x_0$ and $u(t + \omega) = u(t)$ for $t \geq 0$.*

By Definition 2.1, if a function $u \in PC(J, E)$ defined by (2.2) is a solution of IVP (2.1), then

$$u(0) = x_0 = u(\omega), \tag{2.6}$$

namely

$$x_0 = T(\omega - s_m)y_m(s_m) + \int_{s_m}^{\omega} T(\omega - \tau)g(\tau)d\tau.$$

The solution of periodic boundary value problem

$$\begin{cases} u'(t) + Au(t) = g(t), \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ u(t) = y_i(t), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m, \\ u(0) = u(\omega). \end{cases} \tag{2.7}$$

can expressed by

$$u(t) = \begin{cases} T(t)[T(\omega - s_m)y_m(s_m) + \int_{s_m}^{\omega} T(\omega - \tau)g(\tau)d\tau] + \int_0^t T(t - \tau)g(\tau)d\tau, \quad t \in [0, t_1]; \\ y_i(t), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m; \\ T(t - s_i)y_i(s_i) + \int_{s_i}^{t} T(t - \tau)g(\tau)d\tau, \quad t \in (s_i, t_{i+1}], \quad i = 1, 2, \cdots, m. \end{cases} \tag{2.8}$$

Next, we show that the solution of the Eq (2.5) is a $\omega$-periodic.

**Lemma 2.2.** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be $C_0$-semigroup in Banach space $E$. Suppose that the following conditions are satisfied:*

*(G1) $g \in C(J, E)$ is $\omega$-periodic in $t$, $g(t+\omega) = g(t)$, $t \geq 0$.*

*(Y1) $y_i \in C(J, E)$ is $\omega$-periodic in $t$, i.e. $y_{i+nm}(t+\omega) = y_i(t+\omega) = y_i(t)$, $n \in \mathbb{N}^+$, $t \geq 0$, $i = 1, 2, \cdots$.*

*Then Eq (2.5) has $\omega$-periodic mild solution.*

*Proof.* In $J$, the periodic problem (2.5) is equal to PBVP (2.7). We only prove the solution $u(t)$ expressed by (2.8) of PBVP(2.7) is periodic.

**Case 1.** For $t \in [0, t_1]$, i.e. $t + \omega \in [\omega, t_1 + \omega] = [\omega, t_{m+1}]$.

By the condition (G1) and (2.6), we have

$$\begin{aligned} u(t+\omega) &= T(t)u(\omega) + \int_{\omega}^{t+\omega} T(t+\omega-\tau)g(\tau)d\tau \\ &= T(t)u(\omega) + \int_0^t T(t-\tau)g(\tau+\omega)d\tau \\ &= u(t). \end{aligned}$$

**Case 2.** For $t \in (t_i, s_i]$, this implies $t + \omega \in (t_i + \omega, s_i + \omega] = (t_{m+i}, s_{m+i}], i = 1, 2, \cdots, m$.

By the assumption (Y1), we have

$$u(t+\omega) = y_{m+i}(t+\omega) = y_i(t) = u(t).$$

**Case 3.** For $t \in (s_i, t_{i+1}]$, this implies $t + \omega \in (s_i + \omega, t_{i+1} + \omega] = (s_{m+i}, t_{m+i+1}], i = 1, 2, \cdots, m$.

By the conditions (G1) and (Y1), we have

$$\begin{aligned} u(t+\omega) &= T(t+\omega-s_{m+i})y_{m+i}(s_{m+i}) + \int_{s_{m+i}}^{t+\omega} T(t+\omega-\tau)g(\tau)d\tau \\ &= T(t-s_i)y_i(s_i+\omega) + \int_{s_i+\omega}^{t+\omega} T(t+\omega-\tau)g(\tau)d\tau \\ &= u(t). \end{aligned}$$

Therefore, we can asset the solution $u(t)$ of PBVP(2.7) is periodic. $u(t)$ extended by $\omega$-periodic is the periodic mild solution of Eq (2.5).

The proof is completed.□

**Remark 2.1.** *In Lemma 2.2, let $T(t)$ $(t \geq 0)$ generated by $-A$ be a positive $C_0$-semigroup in an ordered Banach space $E$. For any $g \geq \theta$, and $y_i \geq \theta, i = 1, 2, \cdots, m$, then the mild solution of Eq (2.5) is a positive solution.*

Remark 2.1 implies the following maximum principle:

**Lemma 2.3.** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be a positive $C_0$-semigroup in an ordered Banach space $E$. If*

$$\begin{cases} u'(t) + Au(t) \geq \theta, \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ u(t) \geq \theta, \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m, \end{cases}$$

*then $u(t) \geq \theta$ for $t \geq 0$.*

## 3. Main results

In this section, we present and prove our major results. We state the definition of the lower and upper $\omega$-periodic solutions of PBVP(1.2).

**Definition 3.1.** If functions $v_0 \in PC(J, E) \cap C^1(J'', E) \cap C(J', E_1)$ satisfy

$$\begin{cases} v_0'(t) + Av_0(t) \leq f(t, v_0(t)), & t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ v_0(t) \leq h_i(t, v_0(t)), & t \in (t_i, s_i], \quad i = 1, 2, \cdots, m. \\ v_0(0) \leq v_0(\omega), \end{cases} \tag{3.1}$$

then $v_0$ is known as a lower $\omega$-periodic solution of problem (1.2); on the contrary , if all the inequalities of (3.1) are inverse, it is called an upper solution of problem (1.2).

**Theorem 3.1.** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be a positive and compact $C_0$-semigroup in an ordered Banach space $E$, which positive cone $K$ is normal with the normal constant $N_0$. Assume that $v_0$ and $w_0$ with $v_0(t) \leq w_0(t)(t \in J)$ are lower and upper solutions of problem(1.2) and the following conditions are satisfied:*

*(F1) $f \in C(J \times E, E)$ is $\omega$-periodic about $t$, $f(t + \omega, u) = f(t, u)$, $t \geq 0$, and $\forall u \in E$.*

*(F2) There exists a constant $M \geq 0$ such that*

$$f(t, u) - f(t, v) \geq -M(u - v), \quad t \in J,$$

*for any $t \in J$, and $v_0(t) \leq v \leq u \leq w_0(t)$.*

*(H1) $h_i \in C(J, E)$ is $\omega$-periodic in $t$, i.e. $h_{i+nm}(t + \omega, u) = h_i(t + \omega, u) = h_i(t, u)$, $n \in \mathbb{N}^+$, $t \geq 0$, $\forall u \in E$, $i = 1, 2, \cdots$.*

*(H2) For $\forall t \in J, v_0(t) \leq v \leq u \leq w_0(t)$,*

$$h_i(t, u) \geq h_i(t, v), \; i = 1, 2, \cdots, m.$$

*(H3) $h_i \in C(J \times E, E)(i = 1, 2, \cdots, m)$ are compact operators.*

*Then PBVP (1.2) exist minimal and maximal $\omega$-periodic mild solutions $\underline{u}$ and $\overline{u}$ between $v_0$ and $w_0$, which can be obtained by iteration from $v_0$ and $w_0$.*

*Proof.* It is well know that $S(t) = e^{-Ct}T(t)$ generated by $-(A + CI)$ is a positive compact semigroup. Let $\overline{N} = \sup_{t \in J} \|S(t)\|$. Denote $D = [v_0, w_0]$. For $\forall g \in D$, we consider the following PBVP in $E$

$$\begin{cases} u'(t) + Au(t) + Mu(t) = f(t, g(t)) + Mg(t), & t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ u(t) = h_i(t, g(t)), & t \in (t_i, s_i], \quad i = 1, 2, \cdots, m, \\ u(0) = u(\omega). \end{cases} \tag{3.2}$$

From Lemma 2.2 and (2.8), PBVP (3.2) has periodic mild solution $u \in PC(J, E)$ given by

$$u(t) = \begin{cases} S(t)B_1(g) + \int_0^t S(t-\tau)(f(t, g(\tau)) + Mg(\tau))d\tau, \quad t \in [0, t_1]; \\ h_i(t, g(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m; \\ S(t-s_i)h_i(s_i, g(s_i)) + \int_{s_i}^t S(t-\tau)(f(t, g(\tau)) + Mg(\tau))d\tau, \\ t \in (s_i, t_{i+1}], \quad i = 1, 2, \cdots, m. \end{cases} \triangleq Q(g) \tag{3.3}$$

where

$$B_1(g) = S(\omega - s_m)h_m(s_m, g(s_m)) + \int_{s_m}^{\omega} S(\omega - \tau)(f(t, g(\tau)) + Mg(\tau))d\tau.$$

Since $f$ and $h_i$ are continuous, so $Q : D \to PC(J, E)$ is continuous. From Lemma 2.2, the $\omega$-periodic mild solutions of PBVP(1.2) are equivalent to the fixed points of operator $Q$. Now, we complete the proof by four steps.

**Step 1.** We show that $Q : D \to PC(J, E)$ is an increasing operator. For $\forall g_1, g_2 \in D$ and $g_1 \leq g_2$, from the conditions (F2) and (H2), we have

$$f(t, g_1(t)) + Mg_1(t) \leq f(t, g_2(t)) + Mg_2(t), t \in J. \tag{3.4}$$

and

$$h_i(t, g_1(t)) \leq h_i(t, g_2(t)), i = 1, 2, \cdots, m. \tag{3.5}$$

From $C_0$-semigroup $S(t)$ is positive, by (3.4) and (3.5), we have $B_1(g_1) \leq B_1(g_2)$.

**Case 1.** For $\forall g_1, g_2 \in D$ , $g_1 \leq g_2$ and for $t \in [0, t_1]$, we have

$$\begin{aligned} & S(t)B_1(g_1) + \int_0^t S(t-\tau)(f(\tau, g_1(\tau)) + Mg_1(\tau))d\tau \\ \leq \quad & S(t)B_1(g_2) + \int_0^t S(t-\tau)(f(\tau, g_2(\tau)) + Mg_2(\tau))d\tau. \end{aligned}$$

**Case 2.** For $\forall g_1, g_2 \in D$ , $g_1 \leq g_2$ and for $t \in (t_i, s_i], i = 1, 2, \cdots, m$, we have

$$h_i(t, g_1(t)) \leq h_i(t, g_2(t)), t \in (t_i, s_i], \quad i = 1, 2, \cdots, m.$$

**Case 3.** For $\forall g_1, g_2 \in D$ , $g_1 \leq g_2$ and for $t \in (s_i, t_{i+1}], i = 1, 2, \cdots, m$, we have

$$\begin{aligned} & S(t-s_i)h_i(s_i, g_1(s_i)) + \int_{s_i}^t S(t-\tau)(f(\tau, g_1(\tau)) + Mg_1(\tau))d\tau \\ \leq \quad & S(t-s_i)h_i(s_i, g_2(s_i)) + \int_{s_i}^t S(t-\tau)(f(\tau, g_2(\tau)) + Mg_2(\tau))d\tau. \end{aligned}$$

Therefore, $Q : D \to PC(J, E)$ is an increasing operator.

**Step 2.** We show $v_0 \leq Q(v_0)$, $Q(w_0) \leq w_0$.

Let

$$\begin{cases} v_0'(t) + Av_0(t) + Mv_0(t) = \widetilde{g}(t) \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ v_0(t) = \widetilde{h}_i(t), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m. \\ v_0(0) = v_0(\omega), \end{cases} \tag{3.6}$$

by the definition of $v_0$, we have

$$\begin{cases} \widetilde{g}(t) \le f(t, v_0(t)) + Mv_0(t), \quad t \in (s_i, t_{i+1}], i = 0, 1, 2, \cdots, m, \\ \widetilde{h}_i(t) \le h_i(t, v_0(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m. \end{cases} \tag{3.7}$$

By Lemma 2.2, (3.6) and (3.7), we have

$$v_0(t) = \begin{cases} S(t)B_2(\widetilde{g}) + \int_0^t S(t-\tau)\widetilde{g}(\tau)d\tau, \quad t \in [0, t_1]; \\ \widetilde{h}_i(t), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m; \\ S(t - s_i)\widetilde{h}_i(s_i) + \int_{s_i}^t S(t-\tau)\widetilde{g}(\tau)d\tau, t \in (s_i, t_{i+1}], \quad i = 1, 2, \cdots, m. \end{cases}$$

where

$$B_2(\widetilde{g}) = S(\omega - s_m)\widetilde{h}_m(s_m) + \int_{s_m}^{\omega} S(\omega - \tau)\widetilde{g}(\tau)d\tau.$$

By (3.7), we obtain

$$B_2(\widetilde{g}) \le S(\omega - s_m)h_m(s_m, v_0(s_m)) + \int_{s_m}^{\omega} S(\omega - \tau)(f(\tau, v_0(\tau)) + Mv_0(\tau))d\tau = B_1(v_0).$$

Particularly, $v_0(0) = B_2(\widetilde{g})$. We divide our proof into three cases.

**Case 1.** For $t \in [0, t_1]$, we have

$$\begin{aligned} & S(t)B_2(\widetilde{g}) + \int_0^t S(t-\tau)\widetilde{g}(\tau)d\tau \\ \le \quad & S(t)B_1(v_0) + \int_0^t S(t-\tau)(f(\tau, v_0(\tau)) + Mv_0(\tau))d\tau, t \in [0, t_1]. \end{aligned}$$

**Case 2.** For $t \in (t_i, s_i], i = 1, 2, \cdots, m$, by (3.7), we have

$$\widetilde{h}_i(t) \le h_i(t, v_0(t)), t \in (t_i, s_i], \quad i = 1, 2, \cdots, m.$$

**Case 3.** For $t \in (s_i, t_{i+1}], i = 1, 2, \cdots, m$, combing case 2 with (3.7), we have

$$\begin{aligned} & S(t - s_i)\widetilde{h}_i(s_i) + \int_{s_i}^t S(t-\tau)\widetilde{g}(\tau)d\tau \\ \le \quad & S(t - s_i)h_i(s_i, v_0(s_i)) + \int_{s_i}^t S(t-\tau)(f(\tau, v_0(\tau)) + Mv_0(\tau))d\tau. \end{aligned}$$

Hence, $v_0(t) \le Q(v_0)(t)$. Analogously, we also prove that $Q(w_0)(t) \le w_0(t)$. Therefore, $Q : [v_0, w_0] \to [v_0, w_0]$ is a continuous and increase operator.

**Step 3.** The operator $Q$ exist fixed points on interval $[v_0, w_0]$.

Denote two sequences $\{v_n\}$ and $\{w_n\}$ by

$$v_n = Q(v_{n-1}), \quad w_n = Q(w_{n-1}), \quad n = 1, 2, \cdots \tag{3.8}$$

Since the operator $Q$ is monotonous, we have

$$v_0 \le v_1 \le v_2 \le \cdots \le v_n \le \cdots \le w_n \le \cdots \le w_2 \le w_1 \le w_0. \tag{3.9}$$

Next, we prove that $\{v_n\}$ and $\{w_n\}$ are convergent in $J$. Let $G = \{v_n \mid n \in \mathbb{N}\}$, $G_0 = \{v_{n-1} \mid n \in \mathbb{N}\}$, then $G_0 = \{v_0\} \cup G$ and $G = Q(G_0)$. For any $v_{n-1} \in G_0$, let

$$\begin{aligned}
(Q_1 v_{n-1})(t) = &\ S(t)B_1(v_{n-1}) + \int_0^t S(t-\tau)(f(\tau, v_{n-1}(\tau)) + M v_{n-1}(\tau))d\tau, \quad t \in [0, t_1];\\
(Q_2 v_{n-1})(t) = &\ h_i(t, v_{n-1}(t)), \quad t \in (t_i, s_i], \quad i = 1, 2, \cdots, m;\\
(Q_3 v_{n-1})(t) = &\ S(t-s_i)h_i(s_i, v_{n-1}(s_i)) + \int_{s_i}^t S(t-\tau)(f(\tau, v_{n-1}(\tau)) + M v_{n-1}(\tau))d\tau,\\
&\ t \in (s_i, t_{i+1}], \quad i = 1, 2, \cdots, m.
\end{aligned}$$

By the assumption (F2), it follows that

$$f(t, v_0(t)) + M v_0(t) \le f(t, v_{n-1}(t)) + M v_{n-1}(t) \le f(t, w_0(t)) + M w_0(t).$$

Since image sets of $f(t, v_0(t))$ and $f(t, w_0(t))$ are compact sets in $E$ by the continuity of $f(t, v_0(t))$ and $f(t, w_0(t))$ in compact set $[0, \omega]$, furthermore image sets are bounded. Additionally, since the cone $K$ is normal in $E$, we have $\exists C_1 > 0$, $\forall v_{n-1} \in G_0$,

$$\begin{aligned}
&\ \|f(t, v_{n-1}(t)) + M v_{n-1}(t)\|\\
\le &\ \|f(t, v_0(t)) + M v_0(t)\| + N_0 \|f(t, w_0(t)) + M w_0(t) - f(t, v_0(t)) - M v_0(t)\|\\
\le &\ C_1.
\end{aligned}$$

From the condition (H2), we get that

$$h_i(t, v_0(t)) \le h_i(t, v_{n-1}(t)) \le h_i(t, w_0(t)), i = 1, 2, \cdots, m.$$

Since the cone $K$ is normal in $E$, for $i = 1, 2, \cdots, m$, we have $\exists C_2 > 0$, $\forall v_{n-1} \in G_0$,

$$\begin{aligned}
&\ \|h_i(t, v_{n-1}(t))\|\\
\le &\ \|h_i(t, v_0(t))\| + N_0 \|h_i(t, w_0(t)) - h_i(t, v_0(t))\|\\
\le &\ C_2.
\end{aligned}$$

We divide our proof into three cases.

**Case 1.** In $[0, t_1]$, for $\forall \epsilon > 0$ and $t, t-\epsilon \in [0, t_1]$, let

$$(Q_1^\epsilon v_{n-1})(t) := S(t)B_1(v_{n-1}) + \int_0^{t-\epsilon} S(t-\tau)(f(\tau, v_{n-1}(\tau)) + M v_{n-1}(\tau))d\tau,$$

then

$$
\begin{aligned}
& \|(Q_1 v_{n-1})(t)) - (Q_1^{\epsilon} v_{n-1})(t)\| \\
= & \ \| \int_0^t S(t-\tau)(f(\tau, v_{n-1}(\tau)) + M v_{n-1}(\tau)) d\tau \\
& - \int_0^{t-\epsilon} S(t-s)\tau)(f(\tau, v_{n-1}(\tau)) + M v_{n-1}(\tau)) d\tau\| \\
\leq & \int_{t-\epsilon}^t \|S(t-\tau)\| \|f(\tau, v_{n-1}(\tau)) + M v_{n-1}(\tau)\| d\tau \\
\leq & \ \overline{N} C_1 \epsilon.
\end{aligned}
$$

Therefore, applying the definition of the total boundedness, $\{(Q_1 v_{n-1})(t) \mid v_{n-1} \in G_0\}$ is precompact in $E$.

**Case 2.** In $t \in (t_i, s_i], i = 1, 2, \cdots, m$, from the condition (H3), the set $\{(Q_2 v_{n-1})(t) \mid v_{n-1} \in G_0\}$ is precompact in $E$.

**Case 3.** In interval $t \in (s_i, t_{i+1}], i = 1, 2, \cdots, m$, similar to the case 1, $\{(Q_3 v_{n-1})(t) \mid v_{n-1} \in G_0\}$ is precompact in $E$.

Hence, $\{v_n(t)\} = \{Q(v_{n-1})(t) \mid v_{n-1} \in G_0\}$ is precompact in $E$ for $t \in J$, combining the normality of $K$ with the monotonicity of $\{v_n\}$, we easily prove that $\{v_n(t)\}$ is convergent. Let $\{v_n(t)\} \to \underline{u}(t)$ in $t \in J$. Similarly, we prove that $\{w_n(t)\} \to \overline{u}(t)$ in $t \in J$.

Evidently $\{v_n(t)\}, \{w_n(t)\} \in PC(J, E)$, so $\underline{u}(t)$ and $\overline{u}(t)$ is bounded integrable in $J$. Since for any $t \in J$, $v_n(t) = Q(v_{n-1})(t)$, $w_n(t) = Q(w_{n-1})(t)$, letting $n \to \infty$, by the Lebesgue dominated convergence theorem, we have $\underline{u}(t) = Q(\underline{u})(t)$, $\overline{u}(t) = Q(\overline{u})(t)$ and $\underline{u}(t), \overline{u}(t) \in PC(J, E)$. Combining this with monotonicity (3.9), we have $v_0(t) \leq \underline{u}(t) \leq \overline{u}(t) \leq w_0(t)$.

**Step 4.** we prove that $\underline{u}(t)$ and $\overline{u}(t)$ are the minimal and maximal fixed points of $Q$ in $[v_0, w_0]$, respectively. In fact, for any $u^* \in [v_0, w_0], Q(u^*) = u^*$, we have $v_0 \leq u^* \leq w_0$ and $v_1 = Q(v_0) \leq Q(u^*) = u^* \leq Q(w_0) = w_1$. Continuing such progress, we get $v_n \leq u^* \leq w_n$. Letting $n \to \infty$, we get $\underline{u}(t) \leq u^* \leq \overline{u}(t)$. Therefor, $\underline{u}(t)$ and $\overline{u}(t)$ between $v_0$ and $w_0$ are the minimal and maximal $\omega$-periodic mild solutions of PBVP (1.2), which can be obtained by iteration from $v_0$ and $w_0$, respectively. □

Replacing the condition (H3), we can get the following result.

**Theorem 3.2.** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be a positive and compact $C_0$-semigroup in an ordered Banach space $E$, which positive cone $K$ is normal with the normal constant $N_0$. Assume that problem(1.2) has lower and upper solutions $v_0$ and $w_0$ with $v_0(t) \leq w_0(t)(t \in J)$. Suppose that conditions (F1), (F2), (H1), (H2) and the following condition are satisfied:*

*(H3′) For any increasing or decreasing monotonic sequence $\{x_n\} \subset [v_0, w_0]$, $\{h_i(\cdot, x_n)\}(i = 1, 2, \cdots, m)$ are precompact in $E$.*

*Then PBVP (1.2) exist minimal and maximal $\omega$-periodic mild solutions $\underline{u}$ and $\overline{u}$ between $v_0$ and $w_0$, which can be obtained by iteration from $v_0$ and $w_0$.*

Now, when we cancel the assumption of existence of the lower and upper solutions of problem (1.2), we consider the existence of solutions for problem (1.2). For this purpose, we need the following

conditions:
(H4) $\exists b \geq 0$, $g \in PC(J,E), g \geq \theta, y_i(s_i) \in D(A), y_i \geq \theta, i = 1,2,\cdots,m$, such that

$$f(t,x) \leq bx + g(t), \quad h_i(t,x) \leq y_i(t), \quad x \geq 0;$$

$$bx - g(t) \leq f(t,x), \quad -y_i(t) \leq g_i(t,x), \quad x \leq 0.$$

**Theorem 3.3.** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be a positive and compact $C_0$-semigroup in an ordered Banach space $E$, which positive cone $K$ is normal with the normal constant $N_0$. Suppose that the conditions (F1), (F2), (H1), (H2), (H3) and (H4) satisfied. Then PBVP (1.2) exist minimal and maximal $\omega$-periodic mild solutions, which can be obtained by monotone iterative procedure.*

*Proof.* For $g(t) \geq \theta, y_i(t) \geq \theta$, we discuss the linear non-instantaneous impulsive evolution equation in $E$

$$\begin{cases} u'(t) + Au(t) - bu(t) = g(t), \quad t \in (s_i, t_{i+1}], i = 0,1,2,\cdots,m, \\ u(t) = y_i(t), \quad t \in (t_i, s_i], \quad i = 1,2,\cdots,m. \\ u(0) = u(\omega), \end{cases} \tag{3.10}$$

It is well know that $S(t) = e^{bt}T(t)(t \geq 0)$ generated by $-(A - bI)$ is a positive compact $C_0$-semigroup in $E$. By Lemma2.2 and assumption (H4), the problem (3.10) exist positive solution $u^* \geq \theta$. Let $v_0 = -u^*, w_0 = u^*$, from the assumptions (H1)–(H3) and (H4), we obtain

$$\begin{cases} v_0'(t) + Av_0(t) = bv_0(t) - g(t) \leq f(t, v_0(t)), \quad t \in (s_i, t_{i+1}], i = 0,1,2,\cdots,m, \\ v_0(t) = -y_i(t) \leq h_i(t, v_0(t)), \quad t \in (t_i, s_i], \quad i = 1,2,\cdots,m. \\ v_0(0) \leq v_0(\omega), \end{cases}$$

and

$$\begin{cases} w_0'(t) + Aw_0(t) = bw_0(t) + g(t) \geq f(t, w_0(t)), \quad t \in (s_i, t_{i+1}], i = 0,1,2,\cdots,m, \\ w_0(t) = y_i(t) \geq h_i(t, w_0(t)), \quad t \in (t_i, s_i], \quad i = 1,2,\cdots,m. \\ w_0(0) \geq w_0(\omega), \end{cases}$$

So, it is indicated that the problem (1.2) has a lower solution $v_0$ and an upper solution $w_0$. Hence, conclusion follow from Theorem 3.1. Then the proof is complete.□

When the positive cone is regular, we obtain the following conclusion of existence of PBVP (1.2).

**Corollary 3.4.** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be a positive $C_0$-semigroup in an ordered Banach space $E$, which positive cone $K$ is regular. Assume that $v_0$ and $w_0$ with $v_0(t) \leq w_0(t)(t \in J)$ are lower and upper solutions of problem (1.2) and the conditions (F1), (F2), (H1), (H2) and (H3) are satisfied. Then PBVP (1.2) exist minimal and maximal $\omega$-periodic mild solutions $\underline{u}$ and $\overline{u}$ between $v_0$ and $w_0$, which can be obtained by iteration from $v_0$ and $w_0$.*

*Proof.* We show that $Q : [v_0, w_0] \to [v_0, w_0]$ is a continuous and increase operator by Theorem 3.1. Similarly, in $[v_0, w_0]$, we define the two sequences $\{v_n(t)\}$ and $\{w_n(t)\}$ by (3.8). Since conditions

(F2), (H2) and (H3) are satisfied, so sequences $\{v_n(t)\}$ and $\{w_n(t)\}$ are ordered-monotonic and ordered-bounded in $E$.

Any ordered-monotonic and ordered-bounded sequence in $E$ is convergent while the cone $K$ is regular. Using the similar method of Theorem 3.1, we can prove that $\underline{u}(t)$ and $\overline{u}(t)$ are the minimal and maximal $\omega$-periodic mild solutions of the problem (1.2) between $v_0$ and $w_0$, which can be obtained by iteration from $v_0$ and $w_0$, respectively. □

**Corollary 3.5** *Let $T(t)$ $(t \geq 0)$ generated by $-A$ be a positive $C_0$-semigroup in an ordered and weakly sequentially complete Banach space $E$, which positive cone $K$ is normal with the normal constant $N_0$. Assume that $v_0$ and $w_0$ with $v_0(t) \leq w_0(t)(t \in J)$ are lower and upper solutions of problem (1.2) and the conditions (F1), (F2), (H1), (H2) and (H3) are satisfied.*

*Then PBVP (1.2) exist minimal and maximal $\omega$-periodic mild solutions $\underline{u}$ and $\overline{u}$ between $v_0$ and $w_0$, which can be obtained by iteration from $v_0$ and $w_0$.*

*Proof.* We know that the normal cone $K$ is regular in an ordered and weakly sequentially complete Banach space. □

## 4. Application

We make an example in this section to illustrate the main theorem.

**Example 4.1** Let integer $n \geq 1, \Omega \subset \mathbb{R}^n$ is a bounded domain with a sufficiently smooth boundary $\partial\Omega$. We consider the following parabolic partial differential equation with non-instantaneous impulses:

$$\begin{cases} \frac{\partial}{\partial t}w(x,t) - \nabla^2 w(x,t) = \frac{1}{3}sin(w(x,t)), & x \in \Omega, \quad t \in [0, \frac{\pi}{2}] \bigcup (\pi, 2\pi], \\ w(x,t) = \frac{|w(x,t)|}{(1+|w(x,t)|)e^t}, & x \in \Omega, \quad t \in (\frac{\pi}{2}, \pi], \\ w \mid_{\partial\Omega} = 0, \\ w(x,0) = w(x,2\pi), & x \in \Omega, \end{cases} \tag{4.1}$$

where $\nabla^2$ is the Laplace operator, $J = [0, 2\pi]$, $s_0 = 0, t_1 = \frac{\pi}{2}, s_1 = \pi, t_2 = 2\pi = \omega$.

Let $E = L^2(\Omega)$, $K = \{u \in L^2(\Omega) \mid u(x) \geq 0 \;\; a.e. \;\; x \in \Omega\}$, and we define the operator $A$ as follows:

$$D(A) = H^2(\Omega) \cap H_0^1(\Omega), \; Au = -\nabla^2 u.$$

Then we know that $E$ is a Banach space, $K$ is a regular cone of $E$, and $-A$ generates a positive and compact analytic $C_0$-semigroup $T(t)(t \geq 0)$ in $E$ (see [6,7,28]).

Denote $u(t) = w(\cdot, t), f(t, u(t)) = \frac{1}{3}sin(w(x,t)), h_i(t, u(t)) = \frac{|w(x,t)|}{(1+|w(x,t)|)e^t}$, then the impulsive parabolic partial differential equation (4.1) can be abstracted into the form of PBVP (1.2).

**Theorem 4.2** Let the first eigenvalue of operator $-\nabla^2 u$ be $\lambda_1$ under zero boundary conditions and $\varphi_1(x)$ be the corresponding positive eigenvector. Then the impulsive parabolic partial differential equation (4.1) has minimal and maximal mild solutions.

*Proof.* It is easy to prove that $v_0 \equiv 0$ and $w_0 \equiv \varphi_1$ are lower and upper solutions of the Eq (4.1) respectively. We can easily verify that conditions (F1), (F2) are satisfied with $\frac{1}{3} < M < 1$ and the conditions (H1), (H2) and (H3) are satisfied too. Therefore, by Theorem 3.1, we have that PBVP (4.1) has minimal and maximal mild solutions. Then the proof is complete.□

## 5. Conclusions

In this paper, when the nonlinear of the non instantaneous impulsive evolution equation is quasi-monotonicity, we have considered the existence of the minimal and maximal $\omega$-periodic mild solutions by combining perturbation method and monotone iterative technique. The main result (Theorem 3.1) is new.

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**Conflict of interest**

The authors declare that they have no competing interests.

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