Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(2): 2543–2567.
DOI: 10.3934/math.2022143
Received: 21 August 2021
Accepted: 14 October 2021
Published: 17 November 2021

Research article

Optimized distributed fusion filtering for singular systems with fading measurements and stochastic nonlinearity

Chen Wang[1], **Jun Hu**[1,2,*], **Hui Yu**[1] **and Dongyan Chen**[1]

[1] Department of Mathematics, Harbin University of Science and Technology, Harbin 150080, China
[2] School of Automation, Harbin University of Science and Technology, Harbin 150080, China

* **Correspondence:** Email: hujun2013@gmail.com.

Abstract: In this paper, the problem of optimized distributed fusion filtering is considered for a class of multi-sensor singular systems in the presence of fading measurements and stochastic nonlinearity. By utilizing the standard singular value decomposition, the multi-sensor stochastic singular systems are simplified to two reduced-order nonsingular subsystems (RONSs). The local filters (LFs) with corresponding error covariance matrices are proposed for RONSs via the innovation analysis approach. Then, on the basis of the matrix-weighted fusion estimation algorithm, the distributed fusion filters (DFFs) are designed for RONSs with multiple sensors in the linear minimum variance sense. Moreover, the DFFs are obtained by utilizing the state transformation for original singular systems. It can be observed that the DFFs have better accuracy in contrast with the LFs. Finally, an illustrate example is put forward to verify the feasibility of the proposed fusion filtering scheme.

Keywords: singular systems; distributed fusion filter; innovation analysis approach; singular value decomposition; multi-sensors
Mathematics Subject Classification: 93A14, 93E11

1. Introduction

In recent years, increasing research attention has been devoted to the filtering or state estimation problems for the singular systems. It should be noted that the singular systems have more extensive description forms and widespread application domains compared with the normal systems, such as the electronic networks, chemical industry, biomedical sciences, robots and economic systems, and so on [1–6]. For example, the estimation problem for singular systems with single sensor has been studied in [7, 8], including the filtering and smoothing problem. However, the estimation accuracy of the systems with single sensor would be affected by external interference and self-influence. Accordingly, the fusion estimation for singular systems with multiple sensors has long been one of the mainstream topics and a large number of literatures have been published to investigate the multi-sensor singular systems (MSSSs) [9, 10].

At present, great efforts have been made to discuss the estimation problem for MSSSs. Generally, most of the studies rely on the generalization of classical system theory. For example, the issues of reduced-order fusion estimation for MSSSs have been studied in [11, 12], where the singular systems have been converted into two subsystems with lower dimension than the original systems by means of the standard singular value decomposition methods [13, 14]. In [15], the full-order fusion estimation problem has also been investigated for stochastic singular systems by transforming the singular systems into nonsingular systems. Furthermore, there are mainly two methods to estimate the system state, including the Kalman filtering method and the modern time series analysis method. For example, in [16–19], by utilizing the projection theory (PT) in [20], the optimal linear estimators have been presented in the sense of linear minimum variance (LMV). In [21], the optimal state estimation problem has been investigated for finite-field networks with stochastic disturbances in the sense of minimum mean square error. In [22], the local optimal estimators for the singular systems have been given by means of the autoregressive moving average innovation model method. For the sake of improving the accuracy of estimation, it is quite common that we need to fuse all measurement data coming from all sensors by employing the information fusion methods. In general, the information fusion methods have been presented in [23], including the state fusion and measurement fusion. The schemes of state fusion have the distributed fusion [24–26] and the centralized fusion [27]. In particular, the weighted measurement fusion scheme [31] has fused and weighted directly all measurement information to a lower-dimensional fusion measurement, then an individual fusion measurement information has been used to receive the fusion estimator. In addition, the distributed fusion approach has the advantages of expanding the communication and storage space requirements of the fusion center, and the input data rates can be improved due to the parallel structure. Moreover, when one of the sensors fails, this structure is convenient for detection and isolation. In view of the above advantages of distributed fusion structures, the problem of distributed fusion filtering is investigated in this study.

In practical networked systems, sensor aging or imperfect communication channels usually lead to fading measurements, which can lead to various distortions and information constraints [28–30, 32]. Generally, the measurement signals may fade in a probabilistic way [33, 34]. Obviously, the missing measurements may be considered as a special shape of fading measurements. In [35], a random variable obeying the Bernoulli distribution has been introduced to characterize the phenomenon of missing measurements. The missing measurements considered in [36] have different forms in [35], where the missing measurements have been modeled as a diagonal matrix that can be extensively applied to describe the multi-channel systems. In [37], the recursive filter has been designed for a stochastic system subject to multiple fading measurements, random parameter matrix and stochastic nonlinearity, where a diagonal matrix has been used to describe the phenomenon of multi-channel fading measurements. According to the methods in [35–37], a set of Bernoulli distributed random variables has been used to model the missing/fading measurements. However, it should be noted that only the case of single sensor has been considered. Recently, in [38], the issue of multi-sensor information fusion state estimation has been considered for random uncertain systems in the presence of unknown measurement disturbance and missing measurements, but it is only applicable for handling the nonsingular case. So far, the filtering problem for multi-sensor systems subject to fading measurements has not gained adequate attention, not to mention the singular systems with multiple sensors.

In addition, the nonlinearity inevitably exists in many practical systems, such as the telecommunication, neural networks and economics systems [39, 40]. If not properly handled, the nonlinearity would influence the performance of the systems. Hence, a great number of results have been given to handle the nonlinear systems. For example, the problem of prescribed finite-time H_∞ has been investigated in [41] for nonlinear singular systems with and without actuator saturation. In order to achieve prescribed finite-time stable for the closed-loop system, where the saturation nonlinearity has been decomposed into a linear form consisting of the control signal and its constant constraints. In addition, as a special kind of nonlinearity, the stochastic nonlinearity has received considerable attention. In [42], the issue of estimation with regard to the multi-step delays (MSDs) and packet dropouts (PDs) has been investigated for nonlinear stochastic systems. Then, the optimal linear estimators have been obtained by employing the innovation analysis approach (IAA). In [43], a distributed fusion filter (DFF), on the basis of the distributed matrix-weighted fusion algorithm (MWFA), has been proposed for stochastic nonlinear systems in the presence of MSDs and PDs, where the stochastic nonlinear effects come from the state equation and measurement equation. Unfortunately, up to now, very little research effort has been made on the fusion filtering problem in the presence of stochastic nonlinearity catering for multi-sensor circumstances, not to mention the case where multiple sensors may undergo the fading measurements phenomenon in singular systems.

Motivated by the above discussion, in this paper, we aim to deal with the fusion filtering problem for singular systems with fading measurements and stochastic nonlinearity. The problem seems to be significant owing to the following substantial challenges: (1) how to characterize the phenomenon of fading measurements with multiple channels; (2) how to select an appropriate way to convert the original singular systems into nonsingular systems; (3) how to obtain the local filters (LFs) and corresponding estimation error cross-covariances dependent on fading probability; (4) how to develop an effective distributed fusion algorithm so as to improve the accuracy of LFs. Accordingly, the main work of this paper is summarized as follows: (1) the fusion filtering issue is, for the first time, addressed for MSSSs in the presence of both fading measurements and stochastic nonlinearity; (2) the proposed LFs are optimal and unbiased in the sense of LMV; (3) the presented DFF weighted by matrices, which has the property of robustness and flexibility due to the parallel structure, has better precision than the LFs.

Notation In this paper, I_s represents the unit matrix with dimension s. The superscript T represents the transpose of a matrix. $\mathrm{tr}(A)$ is used to describe the trace of matrix A. $\perp$ denotes orthogonality. $\mathrm{diag}\{A_1, A_2, \cdots, A_N\}$ stands for a diagonal matrix with diagonal element A_i. δ_{sh} represents the Kronecker delta function. $\odot$ is the Hadamard product.

2. Problem formulation and preliminaries

In this paper, we consider the following class of nonlinear singular systems with N sensors:

$$\mathcal{M}\mathcal{X}_{s+1} = \Phi\mathcal{X}_s + f(\mathcal{X}_s, \xi_s) + \Upsilon\varpi_s, \tag{2.1}$$

$$\mathcal{Y}_{i,s} = \Omega_{i,s}\mathcal{H}_i\mathcal{X}_s + \vartheta_{i,s}, i = 1, 2, \cdots, N \tag{2.2}$$

where $\mathcal{X}_s \in \mathbb{R}^n$ represents the state vector of the system, $\mathcal{Y}_{i,s} \in \mathbb{R}^{m_i}$ $(i = 1, 2, \cdots, N)$ denote the measurement outputs, $\varpi_s \in \mathbb{R}^r$ is the zero-mean process noise, $\vartheta_{i,s} \in \mathbb{R}^{m_i}$ $(i = 1, 2, \cdots, N)$ are the zero-mean measurement noises. The subscript i represents the ith sensor. $\Omega_{i,s} = \text{diag}\{\alpha_{i1,s}, \cdots, \alpha_{im_i,s}\}$ $(i = 1, 2, \cdots, N)$ characterize the phenomenon of fading measurements with multiple channels, where $\alpha_{ik,s}$ $(k = 1, 2, \cdots, m_i)$ are m_i random variables, which are uncorrelated in s as well as k. The random variable $\alpha_{ik,s}$ represents the fading situation of the kth measurement channel, and the probability density function $p_{ik,s}(l)$ is on the interval [0,1] with known mathematical expectation $\mu_{ik,s}$ and variance $\sigma^2_{ik,s}$. The function $f(\mathcal{X}_s, \xi_s)$ is the stochastic nonlinearity, where ξ_s is a zero-mean Gaussian white noise. $\mathcal{M}$, Φ, Υ and $\mathcal{H}_i$ are constant matrices with proper dimensions.

To begin, the following assumptions are introduced.

Assumption 1. *$\mathcal{M}$ is a singular square matrix, i.e.,* $\text{rank}(\mathcal{M}) = n_1 < n$.

Assumption 2. *System (2.1) is regular, i.e.,* $\det(\kappa\mathcal{M} - \Phi) \neq 0$*, where κ is an arbitrary complex number.*

Assumption 3. $\text{rank}(\Phi) \geq n_2$ *and* $n_1 + n_2 = n$.

Assumption 4. *ϖ_s and $\vartheta_{i,s}$ $(i = 1, 2, \cdots, N)$ have the following statistical properties:*

$$\mathbb{E}\left\{\begin{bmatrix} \varpi_s \\ \vartheta_{i,s} \end{bmatrix}\begin{bmatrix} \varpi_h^T & \vartheta_{i,h}^T \end{bmatrix}\right\} = \begin{bmatrix} Q_{\varpi,s} & 0 \\ 0 & Q_{\vartheta_i,s} \end{bmatrix}\delta_{sh},$$

$$\mathbb{E}\left\{\vartheta_{i,s}\vartheta_{j,h}^T\right\} = 0, (i \neq j, \forall s, h),$$

where δ_{sh} is Kronecker function, that is to say

$$\delta_{sh} = \begin{cases} 1 & s = h \\ 0 & s \neq h \end{cases}.$$

Assumption 5. *The initial state $\mathcal{X}_0$, ϖ_s, $\vartheta_{i,s}$, ξ_s and $\alpha_{ik,s}$ $(i = 1, 2, \cdots, N; k = 1, 2, \cdots, m_i)$ are mutually independent, moreover,* $\mathbb{E}\{\mathcal{X}_0\} = \pi_0$ *and* $\mathbb{E}\left\{(\mathcal{X}_0 - \pi_0)(\mathcal{X}_0 - \pi_0)^T\right\} = P_0$.

Assumption 6. *The function $f(\mathcal{X}_s, \xi_s)$ satisfies*

$$\mathbb{E}\{f(\mathcal{X}_s, \xi_s)|\mathcal{X}_s\} = 0,\ \mathbb{E}\left\{f(\mathcal{X}_s, \xi_s)f^T(\mathcal{X}_h, \xi_h)|\mathcal{X}_s\right\} = \sum_{l=1}^{m} \Pi_l \mathcal{X}_s^T \Gamma_l \mathcal{X}_s \delta_{sh},$$

where m is a known positive integer, Π_l and Γ_l $(l = 1, 2, \cdots, m)$ are given matrices with appropriate dimensions.

Next, the key idea is to transform the original singular systems into two reduced-order nonsingular subsystems (RONSs) by using the singular value decomposition method. Then, the LFs for each sensor are presented in the sense of LMV via the PT. In the fusion center, the DFFs are presented for RONSs by utilizing the MWFA. Finally, we get the DFF for original singular systems.

For the convenience of subsequent developments, we introduce the following Lemma.

Lemma 1. *[44] Let $\mathcal{A} = \left[a_{ij}\right]_{p\times p}$ be a real matrix and $Q = \text{diag}\left\{\varrho_1, \varrho_2, \cdots, \varrho_p\right\}$ be a diagonal random matrix. Then*

$$\mathbb{E}\left\{Q\mathcal{A}Q^T\right\} = \begin{bmatrix} \mathbb{E}\left\{\varrho_1^2\right\} & \mathbb{E}\{\varrho_1\varrho_2\} & \cdots & \mathbb{E}\left\{\varrho_1\varrho_p\right\} \\ \mathbb{E}\{\varrho_2\varrho_1\} & \mathbb{E}\left\{\varrho_2^2\right\} & \cdots & \mathbb{E}\left\{\varrho_2\varrho_p\right\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\left\{\varrho_p\varrho_1\right\} & \mathbb{E}\left\{\varrho_p\varrho_2\right\} & \cdots & \mathbb{E}\left\{\varrho_p^2\right\} \end{bmatrix} \odot \mathcal{A},$$

where $\odot$ is the Hadamard product.

3. Main results

In this section, our goal is to provide an innovative scheme to deal with the fusion filtering problem for singular systems with fading measurements and stochastic nonlinearity, that is to say, we need to find the DFF $\hat{\mathcal{X}}_{0,s|s}$ on the basis of the measurements $(\mathcal{Y}_{i,s}, \cdots, \mathcal{Y}_{i,0})$ $(i = 1, \cdots, N)$.

For systems (2.1) and (2.2) under Assumptions 1-3, according to [14], there exists nonsingular matrices U and R such that

$$U\mathcal{M}R = \begin{bmatrix} \mathcal{M}_1 & 0 \\ \mathcal{M}_2 & 0 \end{bmatrix}, U\Phi R = \begin{bmatrix} \Phi_1 & 0 \\ \Phi_2 & \Phi_3 \end{bmatrix}, U\Upsilon = \begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \end{bmatrix}, \mathcal{H}_i R = \begin{bmatrix} \mathcal{H}_i^{(1)} & \mathcal{H}_i^{(2)} \end{bmatrix}, U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix},$$

where $\mathcal{M}_1 \in \mathbb{R}^{n_1\times n_1}$ is nonsingular lower-triangular, $\Phi_1 \in \mathbb{R}^{n_1\times n_1}$ is quasi-lower-triangular, $\Phi_3 \in \mathbb{R}^{n_2\times n_2}$ is nonsingular lower-triangular, other matrix blocks have corresponding dimensions. By using the transformation $\mathcal{X}_s = R\begin{bmatrix}\mathcal{X}_{1,s}^T & \mathcal{X}_{2,s}^T\end{bmatrix}^T$ with $\mathcal{X}_{1,s} \in \mathbb{R}^{n_1}$ and $\mathcal{X}_{2,s} \in \mathbb{R}^{n_2}$, the singular systems (2.1) and (2.2) are converted into the following systems:

$$\begin{aligned} \mathcal{X}_{1,s+1} &= \bar{\Phi}\mathcal{X}_{1,s} + \bar{U}f(\mathcal{X}_s, \xi_s) + \bar{\Gamma}\varpi_s, && (3.1)\\ \mathcal{X}_{2,s} &= B\mathcal{X}_{1,s} + Cf(\mathcal{X}_s, \xi_s) + D\varpi_s, && (3.2)\\ \mathcal{Y}_{i,s} &= \Omega_{i,s}\bar{H}_i\mathcal{X}_{1,s} + \Omega_{i,s}E_i f(\mathcal{X}_s, \xi_s) + \eta_{i,s}, \quad i = 1, 2, \cdots, N && (3.3) \end{aligned}$$

with

$$\begin{aligned} \bar{\Phi} &= \mathcal{M}_1^{-1}\Phi_1,\ \bar{U} = \mathcal{M}_1^{-1}U_1,\ \bar{\Gamma} = \mathcal{M}_1^{-1}\Upsilon_1,\\ B &= \Phi_3^{-1}\mathcal{M}_2\mathcal{M}_1^{-1}\Phi_1 - \Phi_3^{-1}\Phi_2,\ C = \Phi_3^{-1}\mathcal{M}_2\mathcal{M}_1^{-1}U_1 - \Phi_3^{-1}U_2,\\ D &= \Phi_3^{-1}\mathcal{M}_2\mathcal{M}_1^{-1}\Upsilon_1 - \Phi_3^{-1}\Upsilon_2,\ \bar{H}_i = \mathcal{H}_i^{(1)} + \mathcal{H}_i^{(2)}(\Phi_3^{-1}\mathcal{M}_2\mathcal{M}_1^{-1}\Phi_1 - \Phi_3^{-1}\Phi_2),\\ E_i &= \mathcal{H}_i^{(2)}(\Phi_3^{-1}\mathcal{M}_2\mathcal{M}_1^{-1}U_1 - \Phi_3^{-1}U_2),\ F_i = \mathcal{H}_i^{(2)}(\Phi_3^{-1}\mathcal{M}_2\mathcal{M}_1^{-1}\Upsilon_1 - \Phi_3^{-1}\Upsilon_2),\\ \eta_{i,s} &= \Omega_{i,s}F_i\varpi_s + \vartheta_{i,s}. \end{aligned}$$

The noise sequences ϖ_s and $\eta_{i,s}$ obey

$$\mathbb{E}\left\{\begin{bmatrix} \varpi_s \\ \eta_{i,s} \end{bmatrix}\begin{bmatrix} \varpi_h^T & \eta_{j,h}^T \end{bmatrix}\right\} = \begin{bmatrix} Q_{\varpi,s} & S_{j,s} \\ S_{i,s}^T & Q_{\eta_{ij},s} \end{bmatrix}\delta_{sh},$$

especially when $i = j$, we define $Q_{\eta_{ii},s} = Q_{\eta_i,s}$, where

$$\begin{aligned} S_{j,s} &= \mathbb{E}\left\{\varpi_s\eta_{j,s}^T\right\} = Q_{\varpi,s}F_j^T\bar{\Omega}_{j,s}^T,\\ Q_{\eta_i,s} &= \mathbb{E}\left\{\eta_{i,s}\eta_{i,s}^T\right\} = \tilde{\Omega}_{i,s} \odot (F_iQ_{\varpi,s}F_i^T) + Q_{\vartheta_i,s},\\ Q_{\eta_{ij},s} &= \mathbb{E}\left\{\eta_{i,s}\eta_{j,s}^T\right\} = \vec{\Omega}_{ij,s} \odot (F_iQ_{\varpi,s}F_j^T), (i\neq j) \end{aligned}$$

with

$$\begin{aligned} \bar{\Omega}_{i,s} &= \mathbb{E}\{\Omega_{i,s}\} = \mathrm{diag}\{\mu_{i1,s}, \mu_{i2,s}, \cdots, \mu_{im_i,s}\},\\ \tilde{\Omega}_{i,s} &= \begin{bmatrix} \sigma_{i1,s}^2 + \mu_{i1,s}^2 & \mu_{i1,s}\mu_{i2,s} & \cdots & \mu_{i1,s}\mu_{im_i,s} \\ * & \sigma_{i2,s}^2 + \mu_{i2,s}^2 & \cdots & \mu_{i2,s}\mu_{im_i,s} \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & \sigma_{im_i,s}^2 + \mu_{im_i,s}^2 \end{bmatrix}, \end{aligned}$$

$$\vec{\Omega}_{ij,s}=\begin{bmatrix}\mu_{i1,s}\mu_{j1,s} & \mu_{i1,s}\mu_{j2,s} & \cdots & \mu_{i1,s}\mu_{jm_j,s}\\ * & \mu_{i2,s}\mu_{j2,s} & \cdots & \mu_{i2,s}\mu_{jm_j,s}\\ \vdots & \vdots & \ddots & \vdots\\ * & * & \cdots & \mu_{im_i,s}\mu_{jm_j,s}\end{bmatrix}.$$

Remark 1. *For the filtering problem of singular systems, the usual way is to convert the singular systems into nonsingular systems under the different assumptions. In [9], the full-order nonsingular systems have been introduced under the assumption that the systems are completely observable. Furthermore, a singular value decomposition method has been presented in [13], where the RONS have been obtained under the causal assumption. In addition, it should be noted that a fast-slow subsystem decomposition approach has been employed in [45] when the system is not causal. Accordingly, similar to [14] without the causal assumption and observability assumption, the singular systems (2.1) and (2.2) in this paper are converted into RONSs (3.1), (3.2) and (3.3). Moreover, it should be noted that it is not easy to estimate the system state due to the existence of the stochastic nonlinear effects.*

To proceed, we introduce the following assumption.

Assumption 7. *Matrices Γ_i, $\Pi_j(i,j=1,2,\cdots,m)$, C and R satisfy* $\det(\Psi-I_m)\neq 0$, *where*

$$\Psi=\left[\mathrm{tr}(\Gamma_i\Theta_j)\right]_{m\times m},\Theta_j=R\begin{bmatrix}0 & 0\\ 0 & C\Pi_jC^T\end{bmatrix}R^T.$$

Lemma 2. *For the original systems (2.1) and (2.2) under the conditions of Assumptions 1-6, the second-order moment matrix $q_{X_{s+1}}=\mathbb{E}\left\{X_{s+1}X^T_{s+1}\right\}$ of state X_{s+1} can be calculated by:*

$$q_{X_{s+1}}=R\begin{bmatrix}\mathbb{E}\left\{X_{1,s+1}X^T_{1,s+1}\right\} & \mathbb{E}\left\{X_{1,s+1}X^T_{2,s+1}\right\}\\ \mathbb{E}\left\{X_{2,s+1}X^T_{1,s+1}\right\} & \mathbb{E}\left\{X_{2,s+1}X^T_{2,s+1}\right\}\end{bmatrix}R^T, \tag{3.4}$$

where

$$q_{X_{1,s+1}}=\mathbb{E}\left\{X_{1,s+1}X^T_{1,s+1}\right\}=\bar{\Phi}q_{X_{1,s}}\bar{\Phi}^T+\bar{U}\sum_{l=1}^{m}\Pi_l\mathrm{tr}(q_{X_s}\Gamma_l)\bar{U}^T+\bar{\Gamma}Q_{\varpi,s}\bar{\Gamma}^T, \tag{3.5}$$

$$q_{X_{12,s+1}}=\mathbb{E}\left\{X_{1,s+1}X^T_{2,s+1}\right\}=q_{X_{1,s+1}}B^T, \tag{3.6}$$

$$q_{X_{2,s+1}}=\mathbb{E}\left\{X_{2,s+1}X^T_{2,s+1}\right\}=Bq_{X_{1,s+1}}B^T+C\sum_{l=1}^{m}\Pi_l\mathrm{tr}(q_{X_{s+1}}\Gamma_l)C^T+DQ_{\varpi,s+1}D^T. \tag{3.7}$$

The initial value is $q_{X_0}=\pi_0\pi_0^T+P_0$.

Proof. Substituting $X_{s+1}=R\begin{bmatrix}X^T_{1,s+1} & X^T_{2,s+1}\end{bmatrix}^T$ into the $q_{X_{s+1}}=\mathbb{E}\left\{X_{s+1}X^T_{s+1}\right\}$, we can obtain (3.4). Then, substituting (3.1) into the $q_{X_{1,s+1}}=\mathbb{E}\left\{X_{1,s+1}X^T_{1,s+1}\right\}$ and using Assumptions 4-6 and $X_{1,s}\perp\varpi_s$, we can obtain

$$\begin{aligned}q_{X_{1,s+1}}=&\bar{\Phi}\mathbb{E}\left\{X_{1,s}X^T_{1,s}\right\}\bar{\Phi}^T+\bar{U}\mathbb{E}\left\{f(X_s,\xi_s)f^T(X_s,\xi_s)\right\}\bar{U}^T+\bar{\Gamma}\mathbb{E}\left\{\varpi_s\varpi_s^T\right\}\bar{\Gamma}^T\\&+\left\{\bar{\Phi}\mathbb{E}\left\{X_{1,s}f^T(X_s,\xi_s)\right\}\bar{U}^T+\bar{\Phi}\mathbb{E}\left\{X_{1,s}\varpi_s^T\right\}\bar{\Gamma}^T+\bar{U}\mathbb{E}\left\{f(X_s,\xi_s)\varpi_s^T\right\}\bar{\Gamma}^T\right\}+\left\{*\right\}^T\end{aligned}$$

$$= \bar{\Phi} q_{X_{1,s}} \bar{\Phi}^T + \bar{U} \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{X_s} \Gamma_l) \bar{U}^T + \bar{\Gamma} Q_{\varpi,s} \bar{\Gamma}^T. \tag{3.8}$$

In the equation (3.8), the term $\{*\}$ represents the same item as the front neighboring term. Similarly, substituting (3.1) and (3.2) into $q_{X_{12,s+1}} = \mathbb{E}\left\{X_{1,s+1} X_{2,s+1}^T\right\}$ and substituting (3.2) into $q_{X_{2,s+1}} = \mathbb{E}\left\{X_{2,s+1} X_{2,s+1}^T\right\}$, we can obtain (3.6) and (3.7). □

Remark 2. *From Lemma 2, we can see that the second-order moment matrix q_{X_s} can not computed recursively due to the fact that $q_{X_{2,s}}$ contains $\sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{X_s} \Gamma_l)$. But in the subsequent calculations, it only needs to iterate $\sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{X_s} \Gamma_l)$, therefore, our objective is to solve it from equation (3.4). The calculation process is shown in the Appendix.*

Lemma 3. *For the ith RONSs (3.1), (3.2) and (3.3) under the conditions of Assumptions 4-6, the innovation $e_{i,s} = \mathcal{Y}_{i,s} - \hat{\mathcal{Y}}_{i,s|s-1}$ is calculated by*

$$e_{i,s} = (\Omega_{i,s} - \bar{\Omega}_{i,s}) \bar{H}_i X_{1,s} + \Omega_{i,s} E_i f(X_s, \xi_s) + \eta_{i,s} + \bar{\Omega}_{i,s} \bar{H}_i \tilde{X}_{i,s|s-1}^{(1)}. \tag{3.9}$$

The covariance $Q_{e_{i,s}} = \mathbb{E}\left\{e_{i,s} e_{i,s}^T\right\}$ of the innovation $e_{i,s}$ is calculated by

$$Q_{e_{i,s}} = \breve{\Omega}_{i,s} \odot (\bar{H}_i q_{X_{1,s}} \bar{H}_i^T) + \tilde{\Omega}_{i,s} \odot (E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{X_s} \Gamma_l) E_i^T) + Q_{\eta_{i,s}} + \bar{\Omega}_{i,s} \bar{H}_i P_{i,s|s-1}^{(1)} \bar{H}_i^T \bar{\Omega}_{i,s}^T, \tag{3.10}$$

where $\breve{\Omega}_{i,s} = \mathrm{diag}\left\{\sigma_{i1,s}^2, \cdots, \sigma_{im_i,s}^2\right\}$. The cross-covariance $Q_{e_{ij,s}} = \mathbb{E}\left\{e_{i,s} e_{j,s}^T\right\} (i \neq j)$ of the innovation $e_{i,s}$ and $e_{j,s}$ is computed by

$$Q_{e_{ij,s}} = \vec{\Omega}_{ij,s} \odot (E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{X_s} \Gamma_l) E_j^T) + Q_{\eta_{ij,s}} + \bar{\Omega}_{i,s} \bar{H}_i P_{ij,s|s-1}^{(1)} \bar{H}_j^T \bar{\Omega}_{j,s}^T. \tag{3.11}$$

Proof. According to the PT, we can easily obtain $\hat{\eta}_{i,s|s-1} = 0$. Then, we have

$$\begin{aligned} e_{i,s} &= \mathcal{Y}_{i,s} - \hat{\mathcal{Y}}_{i,s|s-1} \\ &= \Omega_{i,s} \bar{H}_i X_{1,s} + \Omega_{i,s} E_i f(X_s, \xi_s) + \eta_{i,s} - \bar{\Omega}_{i,s} \bar{H}_i \hat{X}_{i,s|s-1}^{(1)} \\ &= (\Omega_{i,s} - \bar{\Omega}_{i,s}) \bar{H}_i X_{1,s} + \Omega_{i,s} E_i f(X_s, \xi_s) + \eta_{i,s} + \bar{\Omega}_{i,s} \bar{H}_i \tilde{X}_{i,s|s-1}^{(1)}. \end{aligned} \tag{3.12}$$

Substituting (3.12) into $Q_{e_{i,s}} = \mathbb{E}\left\{e_{i,s} e_{i,s}^T\right\}$ yields

$$\begin{aligned} Q_{e_{i,s}} = &\mathbb{E}\left\{(\Omega_{i,s} - \bar{\Omega}_{i,s}) \bar{H}_i X_{1,s} X_{1,s}^T \bar{H}_i^T (\Omega_{i,s} - \bar{\Omega}_{i,s})^T\right\} + \mathbb{E}\left\{\Omega_{i,s} E_i f(X_s, \xi_s) f^T(X_s, \xi_s) E_i^T \Omega_{i,s}^T\right\} \\ &+ \mathbb{E}\left\{\eta_{i,s} \eta_{i,s}^T\right\} + \mathbb{E}\left\{\bar{\Omega}_{i,s} \bar{H}_i \tilde{X}_{i,s|s-1}^{(1)} \tilde{X}_{i,s|s-1}^{(1)T} \bar{H}_i^T \bar{\Omega}_{i,s}^T\right\} + \Big\{\mathbb{E}\left\{(\Omega_{i,s} - \bar{\Omega}_{i,s}) \bar{H}_i X_{1,s} f^T(X_s, \xi_s) E_i^T \Omega_{i,s}^T\right\} \\ &+ \mathbb{E}\left\{(\Omega_{i,s} - \bar{\Omega}_{i,s}) \bar{H}_i X_{1,s} \eta_{i,s}^T\right\} + \mathbb{E}\left\{(\Omega_{i,s} - \bar{\Omega}_{i,s}) \bar{H}_i X_{1,s} \tilde{X}_{i,s|s-1}^{(1)T} \bar{H}_i^T \bar{\Omega}_{i,s}^T\right\} + \mathbb{E}\left\{\Omega_{i,s} E_i f(X_s, \xi_s) \eta_{i,s}^T\right\} \\ &+ \mathbb{E}\left\{\Omega_{i,s} E_i f(X_s, \xi_s) \tilde{X}_{i,s|s-1}^{(1)T} \bar{H}_i^T \bar{\Omega}_{i,s}^T\right\} + \mathbb{E}\left\{\eta_{i,s} \tilde{X}_{i,s|s-1}^{(1)T} \bar{H}_i^T \bar{\Omega}_{i,s}^T\right\}\Big\} + \left\{*\right\}^T. \end{aligned} \tag{3.13}$$

By utilizing $\mathbb{E}\left\{\Omega_{i,s} - \bar{\Omega}_{i,s}\right\} = 0$, Assumptions 4-6 and $\mathbb{E}\left\{\tilde{X}_{i,s|s-1} \eta_{i,s}^T\right\} = 0$, we can obtain (3.10). Similarly, we also obtain (3.11). □

Remark 3. *Based on the above discussion, we see that the Hadamard product has been mainly used in Lemma 3. Note that the covariance $Q_{e_{i,s}}$ and the cross-covariance $Q_{e_{ij},s}$ of innovation can not be derived directly since the innovation $e_{i,s}$ contains a diagonal-matrix with random variable that is different from the single random variable. Before the derivation of the LFs, we give the Lemma 3 firstly by the following Lemma 1, where the Hadamard product makes the expression more concise in taking the expectations on the products of some random matrices.*

Lemma 4. *For the convenience of subsequent computations, we get*

$$\mathbb{E}\left\{\mathcal{X}_{1,s}e_{i,s}^T\right\} = P_{i,s|s-1}^{(1)}\bar{H}_i^T\bar{\Omega}_{i,s}^T, \tag{3.14}$$

$$\mathbb{E}\left\{\varpi_s e_{i,s}^T\right\} = S_{i,s}, \tag{3.15}$$

$$\mathbb{E}\left\{f(\mathcal{X}_s,\xi_s)e_{i,s}^T\right\} = \sum_{l=1}^{m}\Pi_l\mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)E_i^T\bar{\Omega}_{i,s}^T. \tag{3.16}$$

Proof. Substituting (3.9) into $\mathbb{E}\left\{\mathcal{X}_{1,s}e_{i,s}^T\right\}$ and using $\mathbb{E}\left\{\Omega_{i,s}-\bar{\Omega}_{i,s}\right\}=0$, $\mathcal{X}_{1,s}\perp\eta_{i,s}$ and $\hat{\mathcal{X}}_{i,s|s-1}^{(1)}\perp\tilde{\mathcal{X}}_{i,s|s-1}^{(1)}$, we can obtain

$$\begin{aligned}\mathbb{E}\left\{\mathcal{X}_{1,s}e_{i,s}^T\right\} &= \mathbb{E}\left\{\mathcal{X}_{1,s}\mathcal{X}_{1,s}^T\bar{H}_i^T(\Omega_{i,s}-\bar{\Omega}_{i,s})^T\right\}+\mathbb{E}\left\{\mathcal{X}_{1,s}f^T(\mathcal{X}_s,\xi_s)E_i^T\Omega_{i,s}^T\right\}\\ &\quad+\mathbb{E}\left\{\mathcal{X}_{1,s}\eta_{i,s}^T\right\}+\mathbb{E}\left\{\mathcal{X}_{1,s}\tilde{\mathcal{X}}_{i,s|s-1}^{(1)T}\bar{H}_i^T\bar{\Omega}_{i,s}^T\right\}\\ &= \mathbb{E}\left\{\tilde{\mathcal{X}}_{i,s|s-1}^{(1)}\tilde{\mathcal{X}}_{i,s|s-1}^{(1)T}\right\}\bar{H}_i^T\bar{\Omega}_{i,s}^T\\ &= P_{i,s|s-1}^{(1)}\bar{H}_i^T\bar{\Omega}_{i,s}^T.\end{aligned}$$

Substituting (3.9) into $\mathbb{E}\left\{\varpi_s e_{i,s}^T\right\}$ and using $\varpi_s\perp\tilde{\mathcal{X}}_{i,s|s-1}^{(1)}$, we can easily get (3.15). Similarly, we can prove (3.16). □

Lemma 5. *For the RONSs (3.1), (3.2) and (3.3) under the conditions of Assumptions 4-6, the covariance matrices $P_{i\mathcal{X}\varpi,s|s}^{(1)}$, $P_{ij\mathcal{X}\varpi,s|s}^{(1)}$, $(i\neq j)$, $P_{i\varpi,s|s}$ and $P_{ij\varpi,s|s}$, $(i\neq j)$ between the noise and the state are, respectively, computed by*

$$P_{i\mathcal{X}\varpi,s|s}^{(1)} = \mathbb{E}\left\{\tilde{\mathcal{X}}_{i,s|s}^{(1)}\tilde{\varpi}_{i,s|s}^T\right\} = -K_{i,s|s}^{(1)}Q_{e_i,s}K_{i\varpi,s|s}^T, \tag{3.17}$$

$$P_{ij\mathcal{X}\varpi,s|s}^{(1)} = \mathbb{E}\left\{\tilde{\mathcal{X}}_{i,s|s}^{(1)}\tilde{\varpi}_{j,s|s}^T\right\} = K_{i,s|s}^{(1)}Q_{e_{ij},s}K_{j\varpi,s|s}^T - K_{j,s|s}^{(1)}Q_{e_j,s}K_{j\varpi,s|s}^T - K_{i,s|s}^{(1)}Q_{e_i,s}K_{i\varpi,s|s}^T, \tag{3.18}$$

$$P_{i\varpi,s|s} = \mathbb{E}\left\{\tilde{\varpi}_{i,s|s}\tilde{\varpi}_{i,s|s}^T\right\} = Q_{\varpi,s} - K_{i\varpi,s|s}Q_{e_i,s}K_{i\varpi,s|s}^T, \tag{3.19}$$

$$\begin{aligned}P_{ij\varpi,s|s} &= \mathbb{E}\left\{\tilde{\varpi}_{i,s|s}\tilde{\varpi}_{j,s|s}^T\right\} = Q_{\varpi,s} - K_{i\varpi,s|s}Q_{e_i,s}K_{i\varpi,s|s}^T - K_{j\varpi,s|s}Q_{e_j,s}K_{j\varpi,s|s}^T\\ &\quad+K_{i\varpi,s|s}Q_{e_{ij},s}K_{j\varpi,s|s}^T,\end{aligned} \tag{3.20}$$

where the noise estimator $\hat{\varpi}_{i,s|s}$ is calculated by

$$\hat{\varpi}_{i,s|s} = K_{i\varpi,s|s}e_{i,s}, \tag{3.21}$$

the gain matrix is expressed as follows:

$$K_{i\varpi,s|s} = S_{i,s}Q_{e_i,s}^{-1}. \tag{3.22}$$

Proof. Let $P_{\mathcal{X}\mathcal{Y},\circ|\bullet}$ be the covariance among errors $\tilde{\mathcal{X}}_{\circ|\bullet}$ and $\tilde{\mathcal{Y}}_{\circ|\bullet}$. Based on the PT, we obtain $\hat{\mathcal{X}}_{\circ|\bullet}\perp\tilde{\mathcal{Y}}_{\circ|\bullet}$ and $\hat{\mathcal{Y}}_{\circ|\bullet}\perp\tilde{\mathcal{X}}_{\circ|\bullet}$. We have $P_{\mathcal{X}\mathcal{Y},\circ|\bullet} = \mathbb{E}\left\{\tilde{\mathcal{X}}_{\circ|\bullet}\tilde{\mathcal{Y}}^T_{\circ|\bullet}\right\} = \mathbb{E}\left\{\mathcal{X}_{\circ}\tilde{\mathcal{Y}}^T_{\circ|\bullet}\right\} = \mathbb{E}\left\{\tilde{\mathcal{X}}_{\circ|\bullet}\mathcal{Y}^T_{\circ}\right\}$. So, by using $\mathcal{X}_{1,s}\perp\varpi_s$, we have

$$P^{(1)}_{i\mathcal{X}\varpi,s|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s}\tilde{\varpi}^T_{i,s|s}\right\} = \mathbb{E}\left\{\mathcal{X}_{1,s}\tilde{\varpi}^T_{i,s|s}\right\} = -\mathbb{E}\left\{\mathcal{X}_{1,s}\hat{\varpi}^T_{i,s|s}\right\}. \tag{3.23}$$

Then, we obtain (3.21) easily, where $K_{i\varpi,s|s} = \mathbb{E}\left\{\varpi_s e^T_{i,s}\right\} Q^{-1}_{e_i,s}$. Substituting (3.21) into (3.23), and using $K^{(1)}_{i,s|s} = \mathbb{E}\left\{\mathcal{X}_{1,s}e^T_{i,s}\right\} Q^{-1}_{e_i,s}$, we obtain (3.17). Similarly, we can prove (3.18), (3.19) and (3.20). □

Theorem 1. *For the RONSs (3.1), (3.2) and (3.3) under the conditions of Assumptions 4-6, the LFs and one-step predictors for the states $\mathcal{X}_{1,s}$ and $\mathcal{X}_{2,s}$ are given by*

$$\begin{aligned}
\hat{\mathcal{X}}^{(1)}_{i,s|s} &= \hat{\mathcal{X}}^{(1)}_{i,s|s-1} + K^{(1)}_{i,s|s}e_{i,s}, &(3.24)\\
\hat{\mathcal{X}}^{(1)}_{i,s+1|s} &= \bar{\Phi}\hat{\mathcal{X}}^{(1)}_{i,s|s} + \bar{U}K_{if,s|s}e_{i,s} + \bar{\Gamma}\hat{\varpi}_{i,s|s}, &(3.25)\\
\hat{\mathcal{X}}^{(2)}_{i,s|s} &= \hat{\mathcal{X}}^{(2)}_{i,s|s-1} + K^{(2)}_{i,s|s}e_{i,s}, &(3.26)\\
\hat{\mathcal{X}}^{(2)}_{i,s+1|s} &= B\hat{\mathcal{X}}^{(1)}_{i,s+1|s}, &(3.27)
\end{aligned}$$

where the gain matrices $K^{(1)}_{i,s|s}$, $K^{(2)}_{i,s|s}$ and $K_{if,s|s}$ are, respectively, expressed by

$$\begin{aligned}
K^{(1)}_{i,s|s} &= P^{(1)}_{i,s|s-1}\bar{H}^T_i\bar{\Omega}^T_{i,s}Q^{-1}_{e_i,s}, &(3.28)\\
K_{if,s|s} &= \sum_{l=1}^{m}\Pi_l \mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)E^T_i\bar{\Omega}^T_{i,s}Q^{-1}_{e_i,s}, &(3.29)\\
K^{(2)}_{i,s|s} &= \Big[BP^{(1)}_{i,s|s-1}\bar{H}^T_i\bar{\Omega}^T_{i,s} + C\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)E^T_i\bar{\Omega}^T_{i,s} + DS_{i,s}\Big]Q^{-1}_{e_i,s}. &(3.30)
\end{aligned}$$

The filtering error covariance (FEC) $P^{(1)}_{i,s|s}$ and the prediction error covariance (PEC) $P^{(1)}_{i,s+1|s}$ for state $\mathcal{X}_{1,s}$ are computed as

$$\begin{aligned}
P^{(1)}_{i,s|s} &= P^{(1)}_{i,s|s-1} - K^{(1)}_{i,s|s}Q_{e_i,s}K^{(1)T}_{i,s|s}, &(3.31)\\
P^{(1)}_{i,s+1|s} &= \bar{\Phi}P^{(1)}_{i,s|s}\bar{\Phi}^T + \bar{U}\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)\bar{U}^T + \bar{\Gamma}P_{i\varpi,s|s}\bar{\Gamma}^T + \bar{U}K_{if,s|s}Q_{e_i,s}K^T_{if,s|s}\bar{U}^T\\
&\quad+\Big\{-\bar{\Phi}K^{(1)}_{i,s|s}\bar{\Omega}_{i,s}E_i\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)\bar{U}^T + \bar{\Phi}P^{(1)}_{i\mathcal{X}\varpi,s|s}\bar{\Gamma}^T - \bar{U}\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)E^T_i\bar{\Omega}^T_{i,s}K^T_{i\varpi,s|s}\bar{\Gamma}^T\\
&\quad-\bar{U}\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)E^T_i\bar{\Omega}^T_{i,s}K^T_{if,s|s}\bar{U}^T\Big\} + \Big\{*\Big\}^T. &(3.32)
\end{aligned}$$

The filtering error cross-covariance (FECC) $P^{(1)}_{ij,s|s}$ and the prediction error cross-covariance (PECC) $P^{(1)}_{ij,s+1|s}$ among the ith and the jth sensor ($i \neq j$) for state $\mathcal{X}_{1,s}$ are computed as

$$P^{(1)}_{ij,s|s} = P^{(1)}_{ij,s|s-1} - P^{(1)}_{ij,s|s-1}\bar{H}^T_j\bar{\Omega}^T_{j,s}K^{(1)T}_{j,s|s} - K^{(1)}_{i,s|s}\bar{\Omega}_{i,s}\bar{H}_iP^{(1)}_{ij,s|s-1} + K^{(1)}_{i,s|s}Q_{e_{ij},s}K^{(1)T}_{j,s|s}, \tag{3.33}$$

$$
\begin{aligned}
P^{(1)}_{ij,s+1|s} &= \bar{\Phi} P^{(1)}_{ij,s|s} \bar{\Phi}^T + \bar{U} \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) \bar{U}^T + \bar{\Gamma} P_{ij\varpi,s|s} \bar{\Gamma}^T + \bar{U} K_{if,s|s} Q_{e_{ij},s} K^T_{jf,s|s} \bar{U}^T \\
&\quad - \bar{\Phi} K^{(1)}_{i,s|s} \bar{\Omega}_{i,s} E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) \bar{U}^T + \bar{\Phi} P^{(1)}_{ij\mathcal{X}\varpi,s|s} \bar{\Gamma}^T - \bar{\Phi} P^{(1)}_{ij,s|s-1} \bar{H}^T_j \bar{\Omega}^T_{j,s} K^T_{jf,s|s} \bar{U}^T \\
&\quad + \bar{\Phi} K^{(1)}_{i,s|s} Q_{e_{ij},s} K^T_{jf,s|s} \bar{U}^T - \bar{U} \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) E^T_j \bar{\Omega}^T_{j,s} K^T_{j\varpi,s|s} \bar{\Gamma}^T \\
&\quad - \bar{U} \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) E^T_j \bar{\Omega}^T_{j,s} K^T_{jf,s|s} \bar{U}^T - \bar{\Gamma} K_{j\varpi,s|s} Q_{e_j,s} K^T_{jf,s|s} \bar{U}^T + \bar{\Gamma} K_{i\varpi,s|s} Q_{e_{ij},s} K^T_{jf,s|s} \bar{U}^T \\
&\quad - \bar{U} \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) E^T_j \bar{\Omega}^T_{j,s} K^{(1)T}_{j,s|s} \bar{\Phi}^T + \bar{\Gamma} P^{(1)}_{ij\varpi\mathcal{X},s|s} \bar{\Phi}^T - \bar{U} K_{if,s|s} \bar{\Omega}_{i,s|s} \bar{H}_i P^{(1)}_{ij,s|s-1} \bar{\Phi}^T \\
&\quad + \bar{U} K_{if,s|s} Q_{e_{ij},s} K^{(1)T}_{j,s|s} \bar{\Phi}^T - \bar{\Gamma} K_{i\varpi,s|s} \bar{\Omega}_{i,s} E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) \bar{U}^T - \bar{U} K_{if,s|s} \bar{\Omega}_{i,s} E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) \bar{U}^T \\
&\quad - \bar{U} K_{if,s|s} Q_{e_i,s} K^T_{i\varpi,s|s} \bar{\Gamma}^T + \bar{U} K_{if,s|s} Q_{e_{ij},s} K^T_{j\varpi,s|s} \bar{\Gamma}^T.
\end{aligned} \tag{3.34}
$$

The FEC $P^{(2)}_{i,s|s}$ and the FECC $P^{(2)}_{ij,s|s}$ among the ith and the jth sensor ($i \neq j$) for state $\mathcal{X}_{2,s}$ are computed as

$$
\begin{aligned}
P^{(2)}_{i,s|s} &= B P^{(1)}_{i,s|s-1} B^T + C \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) C^T + D Q_{\varpi,s} D^T + K^{(2)}_{i,s|s} Q_{e_i,s} K^{(2)T}_{i,s|s} + \Big\{ - B K^{(1)}_{i,s|s} Q_{e_i,s} K^{(2)T}_{i,s|s} \\
&\quad - C \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) E^T_i \bar{\Omega}^T_{i,s} K^{(2)T}_{i,s|s} - D S_{i,s} K^{(2)T}_{i,s|s} \Big\} + \Big\{ * \Big\}^T,
\end{aligned} \tag{3.35}
$$

$$
\begin{aligned}
P^{(2)}_{ij,s|s} &= B P^{(1)}_{ij,s|s-1} B^T + C \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) C^T + D Q_{\varpi,s} D^T + K^{(2)}_{i,s|s} Q_{e_{ij},s} K^{(2)T}_{j,s|s} - B P^{(1)}_{ij,s|s-1} \bar{H}^T_j \bar{\Omega}^T_{j,s} K^{(2)T}_{j,s|s} \\
&\quad - C \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) E^T_j \bar{\Omega}^T_{j,s} K^{(2)T}_{j,s|s} - D S_{j,s} K^{(2)T}_{j,s|s} - K^{(2)}_{i,s|s} \bar{\Omega}_{i,s} \bar{H}_i P^{(1)}_{ij,s|s-1} B^T \\
&\quad - K^{(2)}_{i,s|s} \bar{\Omega}_{i,s} E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) C^T - K^{(2)}_{i,s|s} S^T_{i,s} D^T.
\end{aligned} \tag{3.36}
$$

The correlation matrices $P^{(1,2)}_{i,s|s}$ and $P^{(1,2)}_{ij,s|s}$ ($i \neq j$) between the $\tilde{\mathcal{X}}^{(1)}_{i,s|s}$ and the $\tilde{\mathcal{X}}^{(2)}_{j,s|s}$ are computed as

$$
P^{(1,2)}_{i,s|s} = P^{(1)}_{i,s|s-1} B^T - K^{(1)}_{i,s|s} Q_{e_i,s} K^{(1)T}_{i,s|s} B^T + P^{(1)}_{i\mathcal{X}\varpi,s|s} D^T - K^{(1)}_{i,s|s} \bar{\Omega}_{i,s} E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) C^T, \tag{3.37}
$$

$$
\begin{aligned}
P^{(1,2)}_{ij,s|s} &= P^{(1)}_{ij,s|s-1} B^T - K^{(1)}_{i,s|s} \bar{\Omega}_{i,s} \bar{H}_i P^{(1)}_{ij,s|s-1} B^T + P^{(1)}_{i\mathcal{X}\varpi,s|s} D^T - K^{(1)}_{i,s|s} \bar{\Omega}_{i,s} E_i \sum_{l=1}^{m} \Pi_l \mathrm{tr}(q_{\mathcal{X}_s} \Gamma_l) C^T \\
&\quad - P^{(1)}_{ij,s|s-1} \bar{H}^T_j \bar{\Omega}^T_{j,s} K^{(2)T}_{j,s|s} + K^{(1)}_{i,s|s} Q_{e_{ij},s} K^{(2)T}_{j,s|s}.
\end{aligned} \tag{3.38}
$$

The initial values are $\hat{\mathcal{X}}^{(1)}_{i,0|-1} = [I_{n_1}\ 0] R^{-1} \pi_0$ and $P^{(1)}_{i,0|-1} = P^{(1)}_{ij,0|-1} = [I_{n_1}\ 0] R^{-1} P_0 (R^T)^{-1} [I_{n_1}\ 0]^T$.

Proof. According to the PT, we obtain (3.24), (3.25), (3.26) and (3.27), where $K^{(1)}_{i,s|s}$, $K_{if,s|s}$, $K^{(2)}_{i,s|s}$ are gain matrices. Substituting (3.14) and (3.16) into the gain matrices $K^{(1)}_{i,s|s} = \mathbb{E}\left\{\mathcal{X}_{1,s}e^T_{i,s}\right\} Q^{-1}_{e_i,s}$ and $K_{if,s|s} = \mathbb{E}\left\{f(\mathcal{X}_s, \xi_s)e^T_{i,s}\right\} Q^{-1}_{e_i,s}$, respectively, we obtain (3.28) and (3.29) immediately. Substituting (3.2) into $K^{(2)}_{i,s|s} = \mathbb{E}\left\{\mathcal{X}_{2,s}e^T_{i,s}\right\} Q^{-1}_{e_i,s}$, we can obtain

$$K^{(2)}_{i,s|s} = \left[B\mathbb{E}\left\{\mathcal{X}_{1,s}e^T_{i,s}\right\} + C\mathbb{E}\left\{f(\mathcal{X}_s, \xi_s)e^T_{i,s}\right\} + D\mathbb{E}\left\{\varpi_s e^T_{i,s}\right\}\right]Q^{-1}_{e_i,s}, \tag{3.39}$$

then, substituting (3.14), (3.15) and (3.16) into (3.39), we can easily obtain (3.30).

From (3.25), the prediction error can be obtained for the state $\mathcal{X}_{1,s}$ as follows:

$$\tilde{\mathcal{X}}^{(1)}_{i,s+1|s} = \bar{\Phi}\tilde{\mathcal{X}}^{(1)}_{i,s|s} + \bar{U}f(\mathcal{X}_s, \xi_s) + \bar{\Gamma}\tilde{\varpi}_{i,s|s} - \bar{U}K_{if,s|s}e_{i,s}. \tag{3.40}$$

Substituting (3.40) into the PEC $P^{(1)}_{i,s+1|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s+1|s}\tilde{\mathcal{X}}^{(1)T}_{i,s+1|s}\right\}$ for state $\mathcal{X}_{1,s}$, we get

$$\begin{aligned}
P^{(1)}_{i,s+1|s} = {} & \bar{\Phi}P^{(1)}_{i,s|s}\bar{\Phi}^T + \bar{U}\mathbb{E}\left\{f(\mathcal{X}_s, \xi_s)f^T(\mathcal{X}_s, \xi_s)\right\}\bar{U}^T + \bar{\Gamma}P_{i\varpi,s|s}\bar{\Gamma}^T + \bar{U}K_{if,s|s}Q_{e_i,s}K^T_{if,s|s}\bar{U}^T \\
& + \Big\{\bar{\Phi}\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s}f^T(\mathcal{X}_s, \xi_s)\right\}\bar{U}^T + \bar{\Phi}P^{(1)}_{i\mathcal{X}\varpi,s|s}\bar{\Gamma}^T - \bar{\Phi}\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s}e^T_{i,s}\right\}K^T_{if,s|s}\bar{U}^T \\
& + \bar{U}\mathbb{E}\left\{f(\mathcal{X}_s, \xi_s)\tilde{\varpi}^T_{i,s|s}\right\}\bar{\Gamma}^T - \bar{U}\mathbb{E}\left\{f(\mathcal{X}_s, \xi_s)e^T_{i,s}\right\}K^T_{if,s|s}\bar{U}^T - \bar{\Gamma}\mathbb{E}\left\{\tilde{\varpi}_{i,s|s}e^T_{i,s}\right\}K^T_{if,s|s}\bar{U}^T\Big\} \\
& + \Big\{*\Big\}^T.
\end{aligned} \tag{3.41}$$

From (3.24), we have the local filtering error equation for state $\mathcal{X}_{1,s}$ as follows:

$$\tilde{\mathcal{X}}^{(1)}_{i,s|s} = \mathcal{X}_{1,s} - \hat{\mathcal{X}}^{(1)}_{i,s|s} = \tilde{\mathcal{X}}^{(1)}_{i,s|s-1} - K^{(1)}_{i,s|s}e_{i,s}. \tag{3.42}$$

Therefore, by using (3.16) and (3.42), we obtain

$$\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s}f^T(\mathcal{X}_s, \xi_s)\right\} = -K^{(1)}_{i,s|s}\bar{\Omega}_{i,s}E_i\sum_{l=1}^{m}\Pi_l\mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l). \tag{3.43}$$

Moreover, by using (3.21), we have

$$\mathbb{E}\left\{f(\mathcal{X}_s, \xi_s)\tilde{\varpi}^T_{i,s|s}\right\} = -\sum_{l=1}^{m}\Pi_l\mathrm{tr}(q_{\mathcal{X}_s}\Gamma_l)E^T_i\bar{\Omega}^T_{i,s}K^T_{i\varpi,s|s}. \tag{3.44}$$

Then, by using (3.14), $\hat{\mathcal{X}}^{(1)}_{i,s|s-1}\perp e_{i,s}$ and $K^{(1)}_{i,s|s} = \mathbb{E}\left\{\mathcal{X}_{1,s}e^T_{i,s}\right\} Q^{-1}_{e_i,s}$, we obtain

$$\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s}e^T_{i,s}\right\} = 0. \tag{3.45}$$

Similarly, by using (3.15) and (3.22), we arrive at

$$\mathbb{E}\left\{\tilde{\varpi}_{i,s|s}e^T_{i,s}\right\} = 0. \tag{3.46}$$

Subsequently, substituting (3.16), (3.43), (3.44), (3.45) and (3.46) into (3.41), we get (3.32).

Substituting (3.42) into the FEC $P^{(1)}_{i,s|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s}\tilde{\mathcal{X}}^{(1)T}_{i,s|s}\right\}$, we have

$$P^{(1)}_{i,s|s} = P^{(1)}_{i,s|s-1} - \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s-1}e^T_{i,s}\right\}K^{(1)T}_{i,s|s} - K^{(1)}_{i,s|s}\mathbb{E}\left\{e_{i,s}\tilde{\mathcal{X}}^{(1)T}_{i,s|s-1}\right\} + K^{(1)}_{i,s|s}Q_{e_i,s}K^{(1)T}_{i,s|s}. \tag{3.47}$$

Next, by using $\hat{\mathcal{X}}^{(1)}_{i,s|s-1} \perp e_{i,s}$ and $K^{(1)}_{i,s|s} = \mathbb{E}\left\{\mathcal{X}_{1,s} e^T_{i,s}\right\} Q^{-1}_{e_i,s}$, we obtain (3.31) from (3.47). Similarly, we can prove (3.33) and (3.34).

From (3.2) and (3.26), we get the local filtering error of state $\mathcal{X}_{2,s}$ as follows:

$$\tilde{\mathcal{X}}^{(2)}_{i,s|s} = B\tilde{\mathcal{X}}^{(1)}_{i,s|s-1} + Cf(\mathcal{X}_s, \xi_s) + D\varpi_s - K^{(2)}_{i,s|s} e_{i,s}. \tag{3.48}$$

Then, substituting (3.48) into the FEC $P^{(2)}_{i,s|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(2)}_{i,s|s} \tilde{\mathcal{X}}^{(2)T}_{i,s|s}\right\}$, and utilizing $\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s-1} e^T_{j,s}\right\} = P^{(1)}_{ij,s|s-1} \bar{H}^T_j \bar{\Omega}^T_{j,s}$, we get (3.35) easily. Similarly, we can prove (3.36).

Substituting (3.48) into $P^{(1,2)}_{i,s|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \tilde{\mathcal{X}}^{(2)T}_{i,s|s}\right\}$, we can obtain

$$P^{(1,2)}_{i,s|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \tilde{\mathcal{X}}^{(1)T}_{i,s|s-1}\right\} B^T + \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} f^T(\mathcal{X}_s, \xi_s)\right\} C^T + \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \varpi^T_s\right\} D^T - \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} e^T_{i,s}\right\} K^{(2)T}_{i,s|s}. \tag{3.49}$$

Moreover, by using (3.42), $\hat{\mathcal{X}}^{(1)}_{i,s|s-1} \perp e_{i,s}$ and $\tilde{\mathcal{X}}^{(1)}_{i,s|s} \perp \hat{\varpi}_{i,s|s}$, we can easily get

$$\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \tilde{\mathcal{X}}^{(1)T}_{i,s|s-1}\right\} = P^{(1)}_{i,s|s-1} - K^{(1)}_{i,s|s} Q_{e_i,s} K^{(1)T}_{i,s|s}, \tag{3.50}$$

$$\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \varpi^T_s\right\} = P^{(1)}_{i\mathcal{X}\varpi,s|s}. \tag{3.51}$$

Then, substituting (3.43), (3.45), (3.50) and (3.51) into (3.49), we get (3.37).

Substituting (3.48) into $P^{(1,2)}_{ij,s|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \tilde{\mathcal{X}}^{(2)T}_{j,s|s}\right\}$, we can get

$$P^{(1,2)}_{ij,s|s} = \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \tilde{\mathcal{X}}^{(1)T}_{j,s|s-1}\right\} B^T + \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} f^T(\mathcal{X}_s, \xi_s)\right\} C^T + \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \varpi^T_s\right\} D^T - \mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} e^T_{j,s}\right\} K^{(2)T}_{j,s|s}. \tag{3.52}$$

Then, we have

$$\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} \tilde{\mathcal{X}}^{(1)T}_{j,s|s-1}\right\} = P^{(1)}_{ij,s|s-1} - K^{(1)}_{i,s|s} \bar{\Omega}_{i,s} \bar{H}_i P^{(1)}_{ij,s|s-1}, \tag{3.53}$$

$$\mathbb{E}\left\{\tilde{\mathcal{X}}^{(1)}_{i,s|s} e^T_{j,s}\right\} = P^{(1)}_{ij,s|s-1} \bar{H}^T_j \bar{\Omega}^T_{j,s} - K^{(1)}_{i,s|s} Q_{e_{ij},s}. \tag{3.54}$$

Therefore, substituting (3.43), (3.51), (3.53) and (3.54) into (3.52), we can prove (3.38). □

Remark 4. *From Theorem 1, we have derived the optimal LFs on the basis of the IAA, which is a common method to obtain the optimal linear estimate in the sense of LMV. The existing results in [37], [43] and [46] with stochastic nonlinearity and fading measurements have been generalized in this paper. We can see that the proposed LFs are dependent on the noise filters, which is different from the suboptimal Kalman-type recursive filter in [37]. Moreover, it should be noted that the filters in [43] and [46] also have been presented by utilizing the IAA. However, the above literatures only take normal systems into consideration. Hence, in this paper, we have considered the fusion filtering problem for MSSSs with stochastic nonlinearity and fading measurements by employing the IAA.*

In the fusion center, based on the LFs and covariance matrices proposed above, we have the following reduced-order DFFs for two RONSs (3.1), (3.2) and (3.3) by using the MWFA.

Theorem 2. *For RONSs (3.1), (3.2) and (3.3), we have the reduced-order DFFs as follows:*

$$\hat{\mathcal{X}}^{(k)}_{0,s|s} = \Lambda_{k,s} \hat{\aleph}_{k,s|s}, k = 1, 2$$

where $\Lambda_{k,s} = (\epsilon_{n_k}^T \Sigma_{k,s|s}^{-1} \epsilon_{n_k})^{-1} \epsilon_{n_k}^T \Sigma_{k,s|s}^{-1}$, $\hat{\aleph}_{k,s|s} = \begin{bmatrix} \hat{X}_{1,s|s}^{(k)T} & \hat{X}_{2,s|s}^{(k)T} & \cdots & \hat{X}_{N,s|s}^{(k)T} \end{bmatrix}^T$ $\epsilon_{n_k} = \begin{bmatrix} I_{n_k} & \cdots & I_{n_k} \end{bmatrix}^T$, $\Sigma_{k,s|s} = \begin{bmatrix} P_{ij,s|s}^{(k)} \end{bmatrix}_{Nn_k \times Nn_k}$, *whose* $n_k \times n_k$ *sub-block in the* (i, j) *place is* $P_{ij,s|s}^{(k)}$. *The covariance of* $\hat{X}_{0,s|s}^{(k)}$ *are computed by*

$$P_{0,s|s}^{(k)} = (\epsilon_{n_k}^T \Sigma_{k,s|s}^{-1} \epsilon_{n_k})^{-1},$$

and we have $P_{0,s|s}^{(k)} \leq P_{i,s|s}^{(k)} (i = 1, 2, \cdots, N)$.

Proof. From the optimal fusion estimation algorithm [26], the proof is complete. □

Theorem 3. *For the original systems (2.1) and (2.2), the DFF has the following form:*

$$\hat{X}_{0,s|s} = R \begin{bmatrix} \hat{X}_{0,s|s}^{(1)T} & \hat{X}_{0,s|s}^{(2)T} \end{bmatrix}^T.$$

The covariance of $\hat{X}_{0,s|s}$ *is calculated by*

$$P_{0,s|s} = R \begin{bmatrix} P_{0,s|s}^{(1)} & P_{0,s|s}^{(1,2)} \\ P_{0,s|s}^{(2,1)} & P_{0,s|s}^{(2)} \end{bmatrix} R^T,$$

where the FEC $P_{0,s|s}^{(1,2)}$ *between* $\tilde{X}_{0,s|s}^{(1)}$ *and* $\tilde{X}_{0,s|s}^{(2)}$ *is calculated by*

$$P_{0,s|s}^{(1,2)} = (\epsilon_{n_1}^T \Sigma_{1,s|s}^{-1} \epsilon_{n_1})^{-1} \epsilon_{n_1}^T \Sigma_{1,s|s}^{-1} \Sigma_{12,s|s} \Sigma_{2,s|s}^{-1} \epsilon_{n_2} (\epsilon_{n_2}^T \Sigma_{2,s|s}^{-1} \epsilon_{n_2})^{-1},$$

where $P_{0,s|s}^{(1,2)} = P_{0,s|s}^{(2,1)T}$, $\Sigma_{12,s|s} = \begin{bmatrix} P_{ij,s|s}^{(1,2)} \end{bmatrix}_{Nn_1 \times Nn_2}$.

Proof. From the transformation $X_s = R \begin{bmatrix} X_{1,s}^T & X_{2,s}^T \end{bmatrix}^T$, we can prove it easily. □

Remark 5. *So far, we have encountered some obstacles in deriving the distributed matrix-weighted fusion filter. For example, (1) how to handle the diagonal-matrix with random variable exists in the measurement equations; (2) how to derive the FECC between any two LFs by using the IAA. In order to overcome these difficulties, the Hadamard product has been introduced to make the calculation more convenient in taking the expectations on the products of some random matrices. In addition, the FECCs are obtained by solving the cross-covariance matrices between state and noises, innovation and state.*

The distributed fusion filtering algorithm for the singular systems can be given by the following steps:

Step 1. Set the initial values $\hat{X}_{i,0|-1}^{(1)} = [I_{n_1}\ 0] R^{-1} \pi_0$ and $P_{i,0|-1}^{(1)} = P_{ij,0|-1}^{(1)} = [I_{n_1}\ 0] R^{-1} P_0 (R^T)^{-1} [I_{n_1}\ 0]^T$, where $i, j = 1, 2, \cdots, N; i \neq j$, $q_{X_0} = \pi_0 \pi_0^T + P_0$.

Step 2. Compute the second-order moment matrix q_{X_s} by Lemma 2, and the innovation $e_{i,s}$ by (3.9).

Step 3. Compute the innovation covariance matrices $Q_{e_{i,s}}$ and $Q_{e_{ij,s}}$ by Lemma 3, the gain matrices $K_{i,s|s}^{(1)}$ by (3.28), $K_{if,s|s}$ by (3.29), $K_{i,s|s}^{(2)}$ by (3.30), the covariance matrices $P_{iX\varpi,s|s}^{(1)}$, $P_{ijX\varpi,s|s}^{(1)}$, $P_{i\varpi,s|s}$ and $P_{ij\varpi,s|s}$ by Lemma 5.

Step 4. Compute the FECs $P_{i,s|s}^{(1)}$ by (3.31), $P_{i,s|s}^{(2)}$ by (3.35), the FECCs $P_{ij,s|s}^{(1)}$ by (3.33), $P_{ij,s|s}^{(2)}$ by (3.36), the PEC $P_{i,s+1|s}^{(1)}$ by (3.32), the PECC $P_{ij,s+1|s}^{(1)}$ by (3.34), the correlation matrices $P_{i,s|s}^{(1,2)}$ by (3.37), $P_{ij,s|s}^{(1,2)}$ by (3.38).

Step 5. Compute the LFs $\hat{X}^{(1)}_{i,s|s}$ by (3.24), $\hat{X}^{(2)}_{i,s|s}$ by (3.26), the local predictors $\hat{X}^{(1)}_{i,s+1|s}$ by (3.25), $\hat{X}^{(2)}_{i,s+1|s}$ by (3.27).
Step 6. Compute the reduced-order fusion filters $\hat{X}^{(k)}_{0,s|s}$ $(k = 1, 2)$ by Theorem 2.
Step 7. Compute the DFF $\hat{X}_{0,s|s}$ for original singular systems by Theorem 3.
Step 8. Let $s = s + 1$, return step 2.

4. An illustrative example

In this section, we provide a simulation example to illustrate the validity of the proposed distributed fusion filtering algorithm.

As we all know, the nonlinear RLC circuits can be molded by the nonlinear singular systems as in [47]. we consider the class of singular systems (2.1) and (2.2) with three sensors, where the related parameters are

$$\mathcal{M} = \begin{bmatrix} -1.8 & 0 & 0 & 0 \\ 1.2 & 0 & 2 & 0 \\ -2.38 & 0 & -1 & 0 \\ -1.44 & 0 & 1.4 & 0 \end{bmatrix}, \Phi = \begin{bmatrix} 0.6 & 0 & -0.5 & 0 \\ -2.8 & 0 & 1 & 0 \\ 5.22 & -2 & -1.45 & 0 \\ -3.36 & -1 & 0.2 & 2 \end{bmatrix}, \Upsilon = \begin{bmatrix} -4 & -1 \\ 0.8 & 3.2 \\ 0.08 & -2.38 \\ 0.96 & -4.66 \end{bmatrix},$$

$$\mathcal{H}_1 = \begin{bmatrix} 1 & 1 & 0.5 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \mathcal{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \mathcal{H}_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0.1 & 1 & 1 \end{bmatrix}.$$

We choose the nonsingular matrices

$$U = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 0 \\ 0.5 & 0.9 & 1 & 0 \\ 0 & -1.2 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then, we have the standard form as follows:

$$\begin{bmatrix} \mathcal{M}_1 & 0 \\ \mathcal{M}_2 & 0 \end{bmatrix} \begin{bmatrix} X_{1,s+1} \\ X_{2,s+1} \end{bmatrix} = \begin{bmatrix} \Phi_1 & 0 \\ \Phi_2 & \Phi_3 \end{bmatrix} \begin{bmatrix} X_{1,s} \\ X_{2,s} \end{bmatrix} + \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} f(X_s, \xi_s) + \begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \end{bmatrix} \varpi_s,$$

$$\mathcal{Y}_{i,s} = \Omega_{i,s} \begin{bmatrix} \mathcal{H}^{(1)}_i & \mathcal{H}^{(2)}_i \end{bmatrix} \begin{bmatrix} X_{1,s} \\ X_{2,s} \end{bmatrix} + \vartheta_{i,s}, i = 1, 2, 3$$

where

$$\mathcal{M}_1 = \begin{bmatrix} 3.6 & 0 \\ -1.2 & 1 \end{bmatrix}, \mathcal{M}_2 = \begin{bmatrix} -2.2 & 0.8 \\ 0 & -1 \end{bmatrix}, \Phi_1 = \begin{bmatrix} -1.2 & 1 \\ -0.8 & 0 \end{bmatrix}, \Phi_2 = \begin{bmatrix} 3 & -0.8 \\ 0 & -1 \end{bmatrix}, \Phi_3 = \begin{bmatrix} -2 & 0 \\ -1 & 2 \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} 0.8 & 2 \\ 0 & 0.6 \end{bmatrix}, \Upsilon_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & -0.82 \end{bmatrix}, \mathcal{H}^{(1)}_1 = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \mathcal{H}^{(2)}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathcal{H}^{(1)}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathcal{H}^{(2)}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$\mathcal{H}_3^{(1)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \mathcal{H}_3^{(2)} = \begin{bmatrix} 1 & 1 \\ 0.1 & 1 \end{bmatrix}, U_1 = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 0 \end{bmatrix}, U_2 = \begin{bmatrix} 0.5 & 0.9 & 1 & 0 \\ 0 & -1.2 & 0 & 1 \end{bmatrix}.$$

We set $Q_{\varpi,s} = I_2$, $Q_{\vartheta_1,s} = 5I_2$, $Q_{\vartheta_2,s} = 10I_2$, $Q_{\vartheta_3,s} = 15I_2$. The function $f(\mathcal{X}_s, \xi_s)$ is given as follows:

$$\begin{aligned} f(\mathcal{X}_s, \xi_s) = & \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.5 \end{bmatrix} [0.2\text{sign}(\bar{\mathcal{X}}_{1,s})\bar{\mathcal{X}}_{1,s}\xi_{1,s} + 0.3\text{sign}(\bar{\mathcal{X}}_{2,s})\bar{\mathcal{X}}_{2,s}\xi_{2,s} + 0.4\text{sign}(\bar{\mathcal{X}}_{3,s})\bar{\mathcal{X}}_{3,s}\xi_{3,s} \\ & +0.5\text{sign}(\bar{\mathcal{X}}_{4,s})\bar{\mathcal{X}}_{4,s}\xi_{4,s}], \end{aligned}$$

where $\bar{\mathcal{X}}_{i,s}$ and $\xi_{i,s}$ ($i = 1, 2, 3, 4$) represent the ith component of $\mathcal{X}_s$ and ξ_s, respectively. $\xi_{i,s}$ are uncorrelated Gaussian white noises with zero-mean and unity variances. We can easily get

$$\Pi_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.5 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.5 \end{bmatrix}^T, \Gamma_1 = \text{diag}\{0.04, 0.09, 0.16, 0.25\}.$$

Let $\Omega_{1,s} = \text{diag}\{\alpha_{11,s}, \alpha_{12,s}\}$, $\Omega_{2,s} = \text{diag}\{\alpha_{21,s}, \alpha_{22,s}\}$, $\Omega_{3,s} = \text{diag}\{\alpha_{31,s}, \alpha_{32,s}\}$, where the probability density functions $p_{ik,s}(l)$ ($i = 1, 2, 3; k = 1, 2$) in the interval $[0, 1]$ for $\alpha_{ik,s}$ ($i = 1, 2, 3$) satisfy

$$p_{1k,s}(l) = \begin{cases} 0.05, & l = 0 \\ 0.1, & l = 0.5 \\ 0.85, & l = 1 \end{cases}, \quad p_{2k,s}(l) = \begin{cases} 0.05, & l = 0 \\ 0.1, & l = 0.2 \\ 0.35, & l = 0.6 \\ 0.5, & l = 1 \end{cases}, \quad p_{3k,s}(l) = \begin{cases} 0.05, & l = 0 \\ 0.1, & l = 0.2 \\ 0.2, & l = 0.5 \\ 0.3, & l = 0.8 \\ 0.35, & l = 1 \end{cases},$$

with the mathematical expectations and variances $\mu_1 = 0.9$, $\mu_2 = 0.73$, $\mu_3 = 0.71$, $\sigma_1^2 = 0.065$, $\sigma_2^2 = 0.0971$ and $\sigma_3^2 = 0.0919$. In the simulation, we chose the collection of 80 data points, the initial value is a zero-mean Gaussian variable with $P_0 = 0.1I_4$. For every RONS, applying Theorem 1, we have the LFs $\hat{\mathcal{X}}_{i,s|s}^{(1)}$ and $\hat{\mathcal{X}}_{i,s|s}^{(2)}$, where $i = 1, 2, 3$. Then applying Theorem 2, we obtain the DFFs weighted by matrices as follows: $\hat{\mathcal{X}}_{0,s|s}^{(1)}$ and $\hat{\mathcal{X}}_{0,s|s}^{(2)}$. Subsequently, by applying Theorem 3, we can obtain DFF $\hat{\mathcal{X}}_{0,s|s}$ of the original systems. The simulation results are given in Figures 1–9.

The proposed DFF is shown in Figures 1–4. As we can see, it has the effective estimation performance, where the true value is represented by solid curves, and the dashed curves denote the DFF. To show the performance of DFF algorithm, in Figures 5–8, the precision is compared by plotting the mean square errors (MSEs), where the MSEs of 1000 times (i.e., $\frac{1}{1000}\sum_{l=1}^{1000}(\mathcal{X}_s^l - \hat{\mathcal{X}}_{i,s|s}^l)^2$ with $i = 0, 1, 2, 3$) are for LFs when $i = 1, 2, 3$ and DFF when $i = 0$. From Figures 5–8, we can easily obtain that the DFF has better precision than the LFs. Thereby, it is easy to see that the proposed fusion algorithm in this paper has better estimation performance from the simulation results.

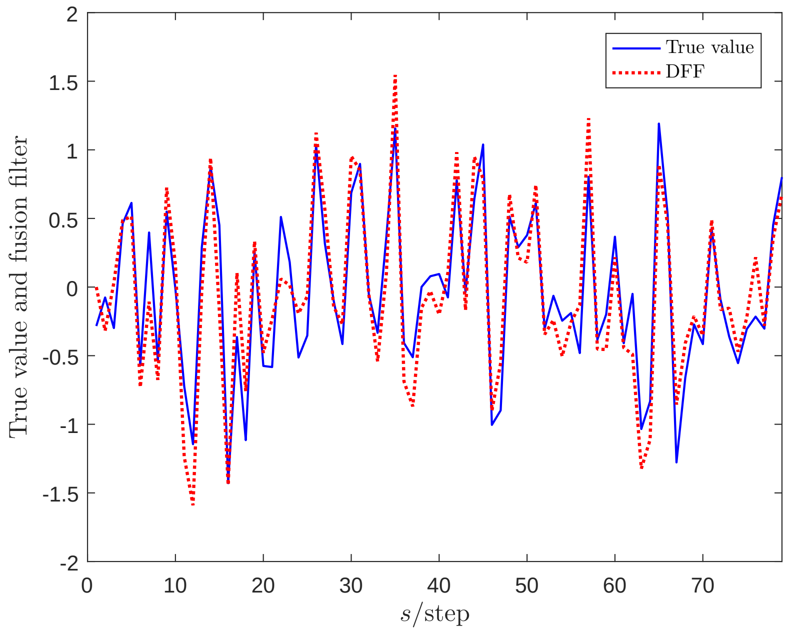

Figure 1. The first component of the true state X_s and its fusion filter $\hat{X}_{0,s|s}$.

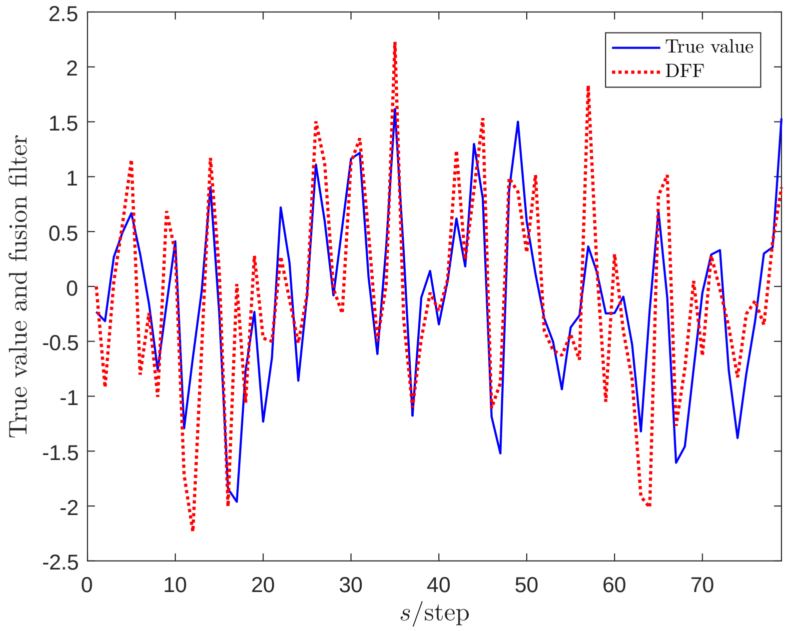

Figure 2. The second component of the true state X_s and its fusion filter $\hat{X}_{0,s|s}$.

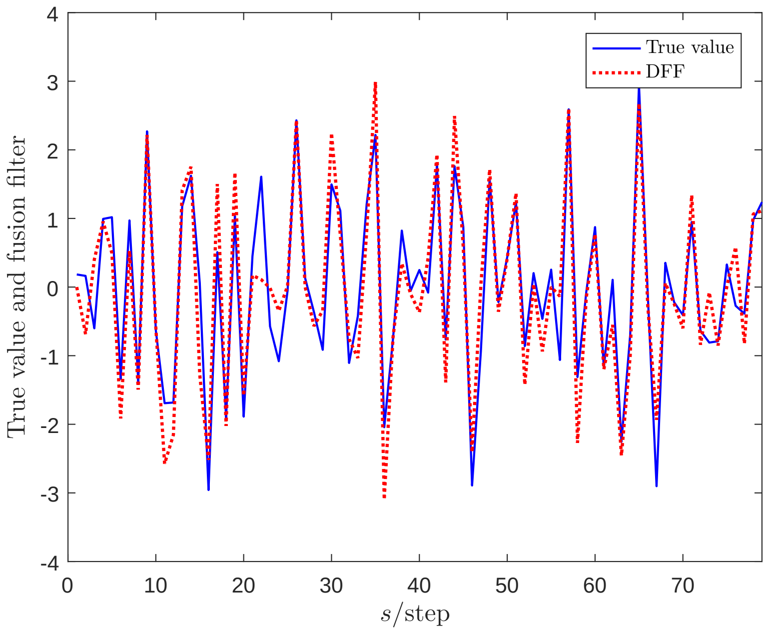

Figure 3. The third component of the true state X_s and its fusion filter $\hat{X}_{0,s|s}$.

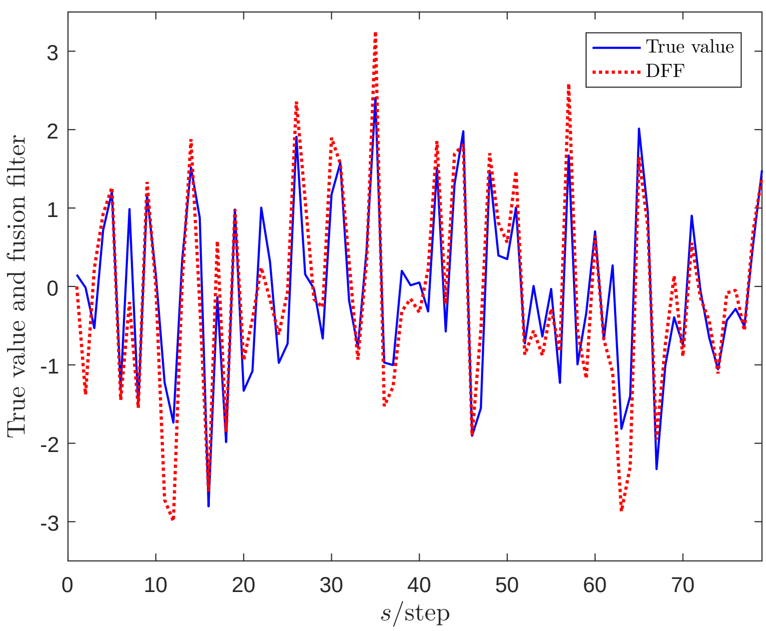

Figure 4. The fourth component of the true state X_s and its fusion filter $\hat{X}_{0,s|s}$.

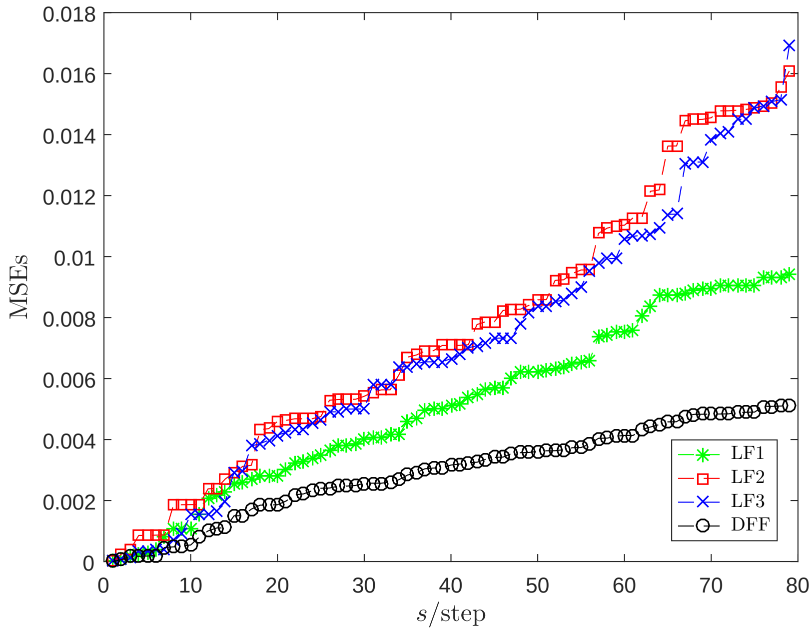

Figure 5. MSEs of the first component of state X_s for LFs and DFF.

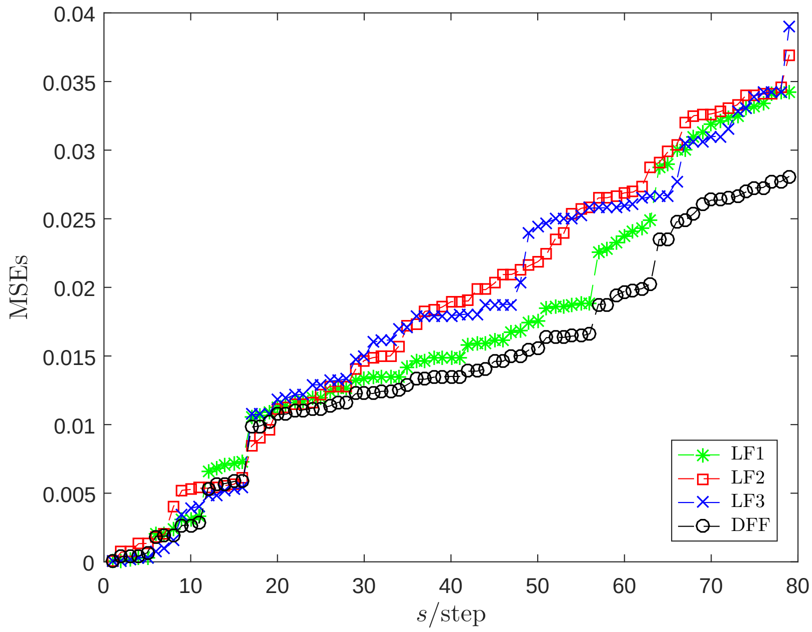

Figure 6. MSEs of the second component of state X_s for LFs and DFF.

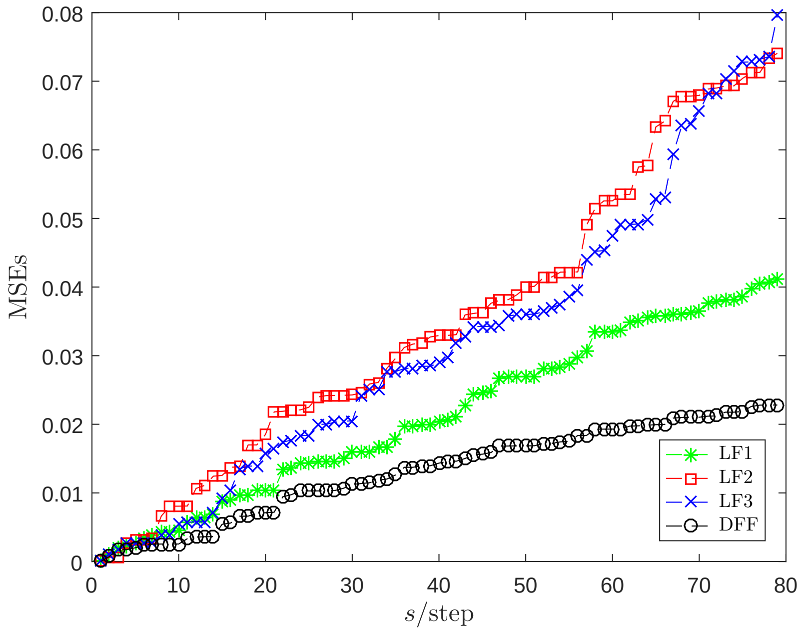

Figure 7. MSEs of the third component of state $\mathcal{X}_s$ for LFs and DFF.

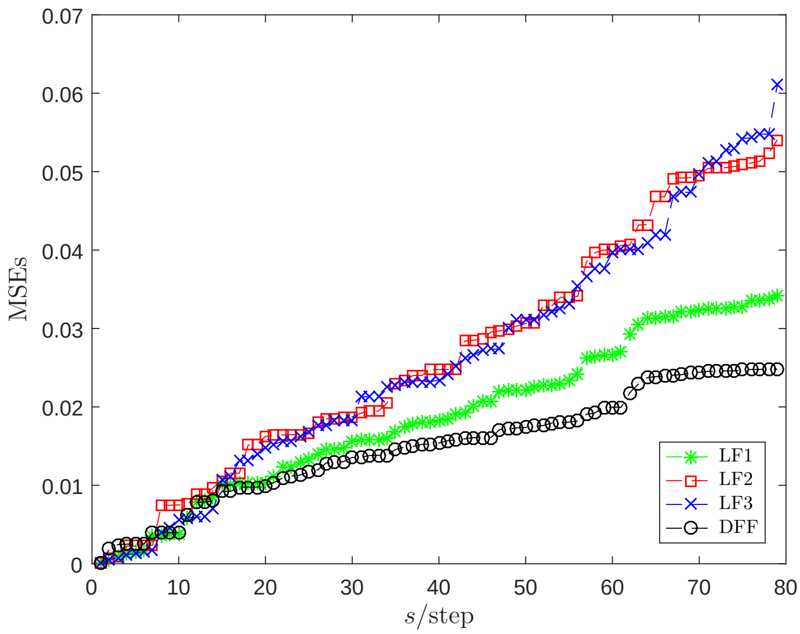

Figure 8. MSEs of the fourth component of state $\mathcal{X}_s$for LFs and DFF.

In addition, in order to illustrate the impacts from the fading measurements, Figure 9 shows the comparison of MSEs (i.e., $\frac{1}{1000}\sum_{l=1}^{1000}\sum_{k=1}^{4}(\mathcal{X}_s^{l,k}-\hat{\mathcal{X}}_{0,s|s}^{l,k})^2$) for DFF under different fading cases, where $\mathcal{X}_s^{l,k}$ and $\hat{\mathcal{X}}_{0,s|s}^{l,k}$ represent the kth component of $\mathcal{X}_s^l$ and $\hat{\mathcal{X}}_{0,s|s}^l$, respectively. Then, we consider the following different cases: Case 1: $\mu_1 = 0.2$, $\mu_2 = 0.4$, $\mu_3 = 0.5$; Case 2: $\mu_1 = 0.5$, $\mu_2 = 0.5$, $\mu_3 = 0.5$; Case 3: $\mu_1 = 0.5$, $\mu_2 = 0.5$, $\mu_3 = 0.97$. It is obviously observed that Case 3 is better than Case 2, while Case 1 is the worst. Owing to the dependence between the filtering error and fading probabilities, we can derive that the better estimations are obtained when the values μ_i $(i = 1, 2, 3)$ increase. Hence, we further illustrate the effectiveness of the proposed fusion filtering scheme.

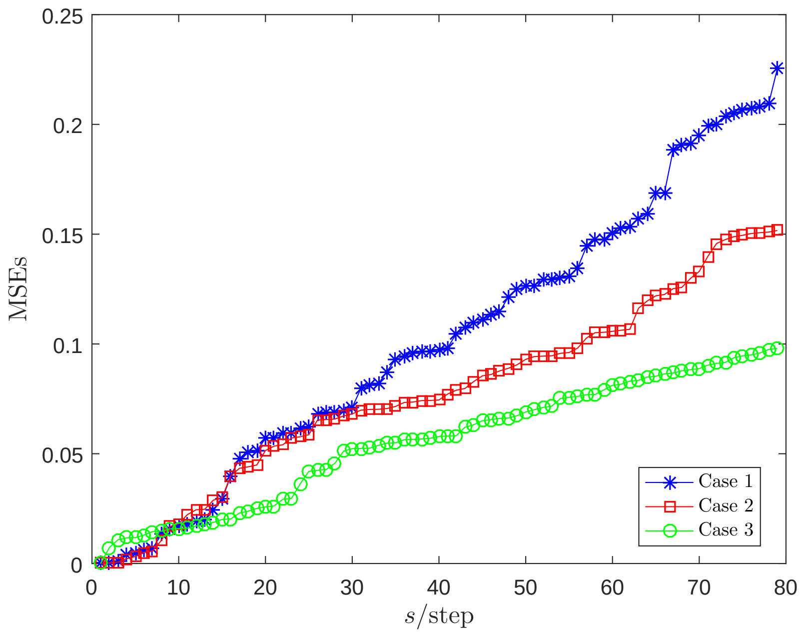

Figure 9. MSEs for DFF under different fading probabilities.

5. Conclusions

In this paper, we have investigated the fusion filtering problem for MSSSs in the presence of fading measurements and stochastic nonlinearity. The phenomenon of fading measurements has been described by a diagonal matrix with a set of random variables. By using the nonsingular transformations, the singular systems have been converted into two RONSs. For subsystems, the distributed optimal fusion filters have been presented on the basis of the matrices weighted fusion criterion. It is worth pointing out that the matrices weighted fusion filter is unbiased and optimal, and the computation burden of fusion center is reduced by using the parallel structure. In simulation, by comparing the DFF with LFs, we can easily obtain the DFF with better precision. In some special fields, the nonsingular matrix $\mathcal{M}$ is a non-square matrix. In future work, we will consider the research of non-square networked singular systems.

Appendix

From (3.7), we can see that $q\chi_{2,s+1}$ contains $\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q\chi_{s+1}\Gamma_l)$, then substituting $q\chi_{2,s+1}$ into (3.4), we get

$$\begin{aligned} q\chi_{s+1} &= R\begin{bmatrix} q\chi_{1,s+1} & q\chi_{12,s+1} \\ q\chi_{21,s+1} & Bq\chi_{1,s+1}B^T + DQ_{\varpi,s+1}D^T \end{bmatrix}R^T + R\begin{bmatrix} 0 & 0 \\ 0 & C\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q\chi_{s+1}\Gamma_l)C^T \end{bmatrix}R^T \\ &= \Delta_{s+1} + \mathrm{tr}(q\chi_{s+1}\Gamma_1)\Theta_1 + \mathrm{tr}(q\chi_{s+1}\Gamma_2)\Theta_2 + \cdots + \mathrm{tr}(q\chi_{s+1}\Gamma_m)\Theta_m, \end{aligned} \tag{5.1}$$

where

$$\begin{aligned} \Delta_{s+1} &= R\begin{bmatrix} q\chi_{1,s+1} & q\chi_{12,s+1} \\ q\chi_{21,s+1} & Bq\chi_{1,s+1}B^T + DQ_{\varpi,s+1}D^T \end{bmatrix}R^T, \\ \Theta_l &= R\begin{bmatrix} 0 & 0 \\ 0 & C\Pi_l C^T \end{bmatrix}R^T, l = 1, 2, \cdots, m. \end{aligned}$$

Then, multiplying the right side of Γ_l $(l = 1, 2, \cdots, m)$ to (5.1), we obtain m equations as follows:

$$\begin{cases} q\chi_{s+1}\Gamma_1 = \Delta_{s+1}\Gamma_1 + \mathrm{tr}(q\chi_{s+1}\Gamma_1)\Theta_1\Gamma_1 + \cdots + \mathrm{tr}(q\chi_{s+1}\Gamma_m)\Theta_m\Gamma_1 \\ \qquad\vdots \\ q\chi_{s+1}\Gamma_m = \Delta_{s+1}\Gamma_m + \mathrm{tr}(q\chi_{s+1}\Gamma_1)\Theta_1\Gamma_m + \cdots + \mathrm{tr}(q\chi_{s+1}\Gamma_m)\Theta_m\Gamma_m \end{cases}. \tag{5.2}$$

Calculating the trace of equations (5.2), we get

$$\begin{bmatrix} \mathrm{tr}(\Gamma_1\Theta_1) - 1 & \mathrm{tr}(\Gamma_1\Theta_2) & \cdots & \mathrm{tr}(\Gamma_1\Theta_m) \\ \mathrm{tr}(\Gamma_2\Theta_1) & \mathrm{tr}(\Gamma_2\Theta_2) - 1 & \cdots & \mathrm{tr}(\Gamma_2\Theta_m) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{tr}(\Gamma_m\Theta_1) & \mathrm{tr}(\Gamma_m\Theta_2) & \cdots & \mathrm{tr}(\Gamma_m\Theta_m) - 1 \end{bmatrix}\begin{bmatrix} \mathrm{tr}(q\chi_{s+1}\Gamma_1) \\ \mathrm{tr}(q\chi_{s+1}\Gamma_2) \\ \vdots \\ \mathrm{tr}(q\chi_{s+1}\Gamma_m) \end{bmatrix} = \begin{bmatrix} -\mathrm{tr}(\Delta_{s+1}\Gamma_1) \\ -\mathrm{tr}(\Delta_{s+1}\Gamma_2) \\ \vdots \\ -\mathrm{tr}(\Delta_{s+1}\Gamma_m) \end{bmatrix}. \tag{5.3}$$

According to Assumption 7, we obtain that the coefficient matrix of (5.3) is invertible, so we get

$$\begin{bmatrix} \mathrm{tr}(q\chi_{s+1}\Gamma_1) \\ \mathrm{tr}(q\chi_{s+1}\Gamma_2) \\ \vdots \\ \mathrm{tr}(q\chi_{s+1}\Gamma_m) \end{bmatrix} = (\Psi - I_m)^{-1}\begin{bmatrix} -\mathrm{tr}(\Delta_{s+1}\Gamma_1) \\ -\mathrm{tr}(\Delta_{s+1}\Gamma_2) \\ \vdots \\ -\mathrm{tr}(\Delta_{s+1}\Gamma_m) \end{bmatrix}, \tag{5.4}$$

then, multiplying the left side of $\begin{bmatrix} \Pi_1 & \Pi_2 & \cdots & \Pi_m \end{bmatrix}$ to (5.4), we obtain

$$\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q\chi_{s+1}\Gamma_l) = \begin{bmatrix} \Pi_1 & \Pi_2 & \cdots & \Pi_m \end{bmatrix}(\Psi - I_m)^{-1}\begin{bmatrix} -\mathrm{tr}(\Delta_{s+1}\Gamma_1) \\ -\mathrm{tr}(\Delta_{s+1}\Gamma_2) \\ \vdots \\ -\mathrm{tr}(\Delta_{s+1}\Gamma_m) \end{bmatrix}. \tag{5.5}$$

From the above discussion, we can see that the right-hand side of Eq (5.5) contains $\sum_{l=1}^{m}\Pi_l \mathrm{tr}(q\chi_s\Gamma_l)$, and the Eq (5.5) can be computed recursively.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 12171124 and 72001059, the Talent Training Project of Reform and Development Foundation for Local Universities from Central Government of China-Youth Talent Project, the Fundamental Research Foundation for Universities of Heilongjiang Province of China under Grant 2019-KYYWF-0215, the University Nursing Program for Young Scholars with Creative Talents in Heilongjiang Province of China under Grant UNPYSCT-2020186, and the Alexander von Humboldt Foundation of Germany.

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. L. Dai, *Singular control systems*, Springer-Verlag, 1989. doi: 10.1007/BFb0002475.
2. I. Dassios, Optimal solutions for non-consistent singular linear systems of fractional nabla difference equations, *Circuits Syst. Signal Process.,* **34** (2015), 1969–1797. doi: 10.1007/s00034-014-9930-2.
3. I. Dassios, G. Kalogeropoulos, On a non-homogeneous singular linear discrete time system with a singular matrix pencil, *Circuits Syst. Signal Process.,* **32** (2013), 1615–1635. doi: 10.1007/s00034-012-9541-8.
4. I. Dassios, On non homogeneous linear generalized linear discrete time systems, *Circuits Syst. Signal Process.,* **31** (2012), 1699–1712. doi: 10.1007/s00034-012-9400-7.
5. Z. Feng, Z. Jiang, W. Zheng, Reachable set synthesis of singular Markovian jump systems, *J. Frankl. Inst. Eng. Appl. Math.,* **357** (2020), 13785–13799. doi: 10.1016/j.jfranklin.2020.09.042.
6. Q. Wu, Q. Song, Z. Zhao, Y. Liu, F. E. Alsaadi, Stabilization of T-S fuzzy fractional rectangular descriptor time-delay system, *Int. J. Syst. Sci.,* **52** (2021), 2268–2282. doi: 10.1080/00207721.2021.1882613.
7. S. Sun, J. Ma, Optimal filtering and smoothing for discrete-time stochastic singular systems, *Signal Process.,* **87** (2007), 189–201. doi: 10.1016/j.sigpro.2006.05.007.
8. X. Yu, J. Li, Optimal filtering and a smoothing algorithm for a singular system with a complex stochastic uncertain parameter matrix, *IEEE Trans. Circuits Syst. II-Express Briefs,* **67** (2020), 780–784. doi: 10.1109/TCSII.2019.2922010.
9. S. Sun, J. Ma, Distributed reduced-order optimal fusion Kalman filters for stochastic singular systems, *ACTA Automatica Sinica,* **32** (2006), 286–290. doi: 10.16383/j.aas.2006.02.017.
10. S. Sun, Y. Shen, J. Ma, Optimal fusion reduced-order Kalman estimators for discrete-time stochastic singular systems, *Control Intell. Syst.,* **36** (2008), 1–9. doi: 10.2316/Journal.201.2008.1.201-1614.
11. J. Feng, M. Zeng, Descriptor recursive estimation for multiple sensors with different delay rates, *Int. J. Control,* **84** (2011), 584–596. doi: 10.1080/00207179.2011.563321.

12. Y. Dou, C. Ran, Y. Gao, Weighted measurement fusion Kalman estimator for multisensor descriptor system, *Int. J. Syst. Sci.,* **47** (2016), 2722–2732. doi: 10.1080/00207721.2015.1018368.

13. M. M. Fahmy, J. O'Reilly, Observers for descriptor systems, *Int. J. Control,* **49** (1989), 2013–2028. doi: 10.1080/00207178908559759.

14. V. L. Syrmos, F. L. Lewis, Robust eigenvalue assignment for generalized systems, *Automatica,* **28** (1992), 1223–1228. doi: 10.1016/0005-1098(92)90064-M.

15. J. Sun, C. Zhang, J. Gu, Decentralized optimal fusion filtering for multi-sensor multi-delay singular systems, *Circuits Syst. Signal Process.,* **31** (2012), 163–176. doi: 10.1007/s00034-011-9313-x.

16. C. Zhu, Y. Xia, L. Xie, L. Yan, Optimal linear estimation for systems with transmission delays and packet dropouts, *IET Signal Process.,* **7** (2013), 814–823. doi: 10.1049/iet-spr.2012.0348.

17. S. Sun, J. Ma, Linear estimation for networked control systems with random transmission delays and packet dropouts, *Inf. Sci.,* **269** (2014), 349–365. doi: 10.1016/j.ins.2013.12.055.

18. J. Linares-Pérez, R. Caballero-Águila, I. García-Garrido, Optimal linear filter design for systems with correlation in the measurement matrices and noises: recursive algorithm and applications, *Int. J. Syst. Sci.,* **45** (2014), 1548–1562. doi: 10.1080/00207721.2014.909093.

19. S. Wang, Z. Wang, H. Dong, F. E. Alsaadi, Recursive state estimation for linear systems with lossy measurements under time-correlated multiplicative noises, *J. Frankl. Inst. Eng. Appl. Math.,* **357** (2020), 1887–1908. doi: 10.1016/j.jfranklin.2019.11.031.

20. B. D. O. Anderson, J. B. Moore, *Optimal filtering*, New Jersey: Prentice-Hall, 1979. doi: 10.1007/978-94-011-4691-3.

21. Y. Li, H. Li, Optimal state estimation for finite-field networks with stochastic disturbances, *Neurocomputing,* **414** (2020), 238–244. doi: 10.1016/j.neucom.2020.07.065.

22. Y. Dou, C. Ran, WMF self-tuning Kalman estimators for multisensor singular system, *Int. J. Syst. Sci.,* **50** (2019), 1873–1888. doi: 10.1080/00207721.2019.1645234.

23. Q. Gan, C. J. Harris, Comparison of two measurement fusion methods for Kalman-filter-based multisensor data fusion, *IEEE Trans. Aerosp. Electron. Syst.,* **37** (2001), 273–279. doi: 10.1109/7.913685.

24. S. Sun, Multi-sensor optimal information fusion Kalman filter with application, *Aerosp. Sci. Technol.,* **8** (2003), 57–62. doi: 10.1016/j.ast.2003.08.003.

25. S. Sun, Multi-sensor information fusion white noise filter weighted by scalars based on Kalman predictor, *Automatica,* **40** (2004), 1447–1453. doi: 10.1016/j.automatica.2004.03.012.

26. S. Sun, Z. Deng, Multi-sensor optimal information fusion Kalman filter, *Automatica,* **40** (2004), 1017–1023. doi: 10.1016/j.automatica.2004.01.014.

27. J. Ma, S. Sun, Information fusion estimators for systems with multiple sensors of different packet dropout rates, *Inf. Fusion,* **12** (2011), 213–222. doi: 10.1016/j.inffus.2010.11.003.

28. H. Tan, B. Shen, K. Peng, H. Liu, Robust recursive filtering for uncertain stochastic systems with amplify-and-forward relays, *Int. J. Syst. Sci.,* **51** (2020), 1188–1199. doi: 10.1080/00207721.2020.1754960.

29. L. Liu, L. Ma, J. Zhang, Y. Bo, Distributed non-fragile set-membership filtering for nonlinear systems under fading channels and bias injection attacks, *Int. J. Syst. Sci.,* **52** (2021), 1192–1205. doi: 10.1080/00207721.2021.1872118.

30. Y. Liu, F. Han, N. Hou, H. Dong, H. Liu, Set-membership filtering for piecewise linear systems with censored measurements under Round-Robin protocol, *Int. J. Syst. Sci.,* **51** (2020), 1578–1588. doi: 10.1080/00207721.2020.1768453.

31. C. Ran, S. Sun, Y. Dou, WMF reduced-order robust estimators for multisensor descriptor systems, *IET Contr. Theory Appl.,* **12** (2018), 2232–2244. doi: 10.1049/iet-cta.2018.5498.

32. Y. Tang, W. Sun, D. Liu, X. Li, Finite-time simultaneous stabilization for stochastic port-controlled Hamiltonian systems over delayed and fading channels, *Complexity,* **2020** (2020), 1–12. doi: 10.1155/2020/6387025.

33. J. Hu, Z. Wang, D. Chen, F. E. Alsaadi, Estimation, filtering and fusion for networked systems with network-induced phenomena: New progress and prospects, *Inf. Fusion,* **31** (2016), 65–75. doi: 10.1016/j.inffus.2016.01.001.

34. S. Sun, H. Lin, J. Ma, X. Li, Multi-sensor distributed fusion estimation with applications in networked systems: a review paper, *Inf. Fusion,* **38** (2017), 122–134. doi: 10.1016/j.inffus.2017.03.006.

35. Z. Wang, F. Yang, D. Ho, X. Liu, Robust finite-horizon filtering for stochastic systems with missing measurements, *IEEE Signal Process. Lett.,* **12** (2005), 437–440. doi: 10.1109/LSP.2005.847890.

36. J. Hu, Z. Wang, S. Liu, H. Gao, A variance-constrained approach to recursive state estimation for time-varying complex networks with missing measurements, *Automatica,* **64** (2016), 155–162. doi: 10.1016/j.automatica.2015.11.008.

37. J. Hu, Z. Wang, H. Gao, Recursive filtering with random parameter matrices, multiple fading measurements and correlated noises, *Automatica,* **49** (2013), 3440–3448. doi: 10.1016/j.automatica.2013.08.021.

38. C. Pang, S. Sun, Fusion predictors for multisensor stochastic uncertain systems with missing measurements and unknown measurement disturbances, *IEEE Sens. J.,* **15** (2015), 4346–4354. doi: 10.1109/JSEN.2015.2416511.

39. X. Li, X. Fu, R. Rakkiyappan, Delay-dependent stability analysis for a class of dynamical systems with leakage delay and nonlinear perturbations, *Appl. Math. Comput.,* **226** (2014), 10–19. doi: 10.1016/j.amc.2013.10.004.

40. X. Lu, H. Li, An improved stability theorem for nonlinear systems on time scales with application to multi-agent systems, *IEEE Trans. Circuits Syst. II-Express Briefs,* **67** (2020), 3277–3281. doi: 10.1109/TCSII.2020.2983180.

41. X. Lu, H. Li, Prescribed finite-time H_∞ control for nonlinear descriptor systems, *IEEE Trans. Circuits Syst. II-Express Briefs,* **68** (2021), 2917–2921. doi: 10.1109/TCSII.2021.3060550.

42. S. Wang, H. Fang, X. Tian, Recursive estimation for nonlinear stochastic systems with multi-step transmission delays, multiple packet dropouts and correlated noises, *Signal Process.,* **115** (2015), 164–175. doi: 10.1016/j.sigpro.2015.03.022.

43. S. Wang, H. Fang, X. Liu, Distributed state estimation for stochastic non-linear systems with random delays and packet dropouts, *IET Contr. Theory Appl.*, **9** (2015), 2657–2665. doi: 10.1049/iet-cta.2015.0257.

44. R. A. Horn, C. R. Johnson, *Topics in matrix analysis*, New York: Cambridge University Press, 1991. doi: 10.1017/CBO9780511840371.

45. L. Dai, Observers for discrete singular systems, *IEEE Trans. Autom. Control,* **33** (1988), 187–191. doi: 10.1109/9.387.

46. S. Sun, T. Tian, H. Lin, State estimators for systems with random parameter matrices, stochastic nonlinearities, fading measurements and correlated noises, *Inf. Sci.,* **397** (2017), 118–136. doi: 10.1016/j.ins.2017.02.048.

47. P. C. Müller, Some aspects of the optimal control of non-linear descriptor systems, *J. Appl. Math. Mech.,* **65** (2001), 769–776. doi: 10.1016/S0021-8928(01)00082-X.

AIMS Press