AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(10): 18334–18359.
DOI: 10.3934/math.20221010
Received: 02 July 2022
Revised: 12 July 2022
Accepted: 25 July 2022
Published: 15 August 2022

Research article

Fractional view analysis of Kersten-Krasil'shchik coupled KdV-mKdV systems with non-singular kernel derivatives

M. Mossa Al-Sawalha[1], Rasool Shah[2], Adnan Khan[2], Osama Y. Ababneh[3] and Thongchai Botmart[4,*]

[1] Department of Mathematics, Faculty of Science, University of Ha'il, Ha'il 2440, Saudi Arabia
[2] Department of Mathematics, Abdul Wali Khan University, Mardan 23200, Pakistan
[3] Department of Mathematics, Faculty of Science, Zarqa University, Zarqa 13110, Jordan
[4] Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand

* **Correspondence:** Email: thongbo@kku.ac.th.

Abstract: The approximate solution of the Kersten-Krasil'shchik coupled Korteweg-de Vries-modified Korteweg-de Vries system is obtained in this study by employing a natural decomposition method in association with the newly established Atangana-Baleanu derivative and Caputo-Fabrizio derivative of fractional order. The Korteweg-de Vries equation is considered a classical super-extension in this system. This nonlinear model scheme is commonly used to describe waves in traffic flow, electromagnetism, electrodynamics, elastic media, multi-component plasmas, shallow water waves and other phenomena. The acquired results are compared to exact solutions to demonstrate the suggested method's effectiveness and reliability. Graphs and tables are used to display the numerical results. The results show that the natural decomposition technique is a very user-friendly and reliable method for dealing with fractional order nonlinear problems.

Keywords: Natural transform; Adomian decomposition method; Caputo-Fabrizio derivative; Atangana-Baleanu-Caputo operator; Korteweg-de Vries nonlinear system
Mathematics Subject Classification: 34A34, 35A20, 35A22, 44A10, 33B15

1. Introduction

Fractional calculus is as old as ordinary calculus, i. e. at three centuries; however, it is not widely used in research and engineering. In a letter to Leibniz dated September 30, 1695, L'Hopital introduced the concept of fractional-order derivatives. In Lacroix's writings, P. S. Laplace defined a fractional derivative of arbitrary order in 1812. Several researchers have developed excellent literature on fractional differentiation and integration operators for the purposes of extending scientific and technical areas, including Caputo [1], Oldham and Spanier [2], Carpinteri and Mainardi [3], Samko et al. [4], Ahmed et al. [5], Podlubny [6], Atangana and Alabaraoye [7], Kumar et al. [8], Yin et al. [9] and Arife et al. [10]. The beauty of fractional derivatives is that they are not local point properties. The genetic and nonlocal dispersed effects are taken into account in fractional calculus. This property makes it more accurate than the integer-order derivative description.

Due to their proven applications in science and engineering, fractional differential equations have grown in prominence and popularity. These equations, for example, are increasingly being utilized to simulate problems in signal processing, biology, fluid mechanics, acoustics, diffusion, electromagnetism and a wide range of other physical phenomena. The nonlocal quality of fractional differential equations is the most essential advantage of utilizing them in these and other applications. The integer order differential operator is well-known to be a local operator, whereas the fractional order differential operator is not. This indicates that a system's next state is determined not just by its current state, but also by all of its previous states. The theory of fractional differential equations better and more systematically describes natural occurrences [11–19].

Fractional coupled systems are commonly used to investigate the complex behavior of plasma that contains several components such as atoms, free electrons and ions. Many scholars have tried to assess this behavior. Recently, Paul Kersten and Joseph Krasil'shchik investigated the Korteweg-de Vries (KdV) equation and modified KdV (mKdV) equation, proposing absolute complexity between coupled KdV-mKdV nonlinear systems for the investigation of nonlinear system behavior [20–23]. Many scholars have proposed numerous variants of this Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system [24–28]. Among these variants, the nonlinear fractional Kersten-Krasil'shchik linked KdV-mKdV system provides a mathematical model for understanding the behavior of multi-component plasma for waves travelling along the positive *zeta* axis:

$$\begin{aligned} &D_{\chi}^{\alpha}\mathbb{F} + \mathbb{F}_{3\vartheta} - 6\mathbb{F}\mathbb{F}_{\vartheta} + 3\mathbb{G}\mathbb{G}_{3\vartheta} + 3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta} - 3\mathbb{F}_{\vartheta}\mathbb{G}^2 + 6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta} = 0,\ \ \chi > 0,\ \ \vartheta \in R,\ \ 0 < \alpha \le 1, \\ &D_{\chi}^{\alpha}\mathbb{G} + \mathbb{G}_{3\vartheta} - 3\mathbb{G}^2\mathbb{G}_{\vartheta} - 3\mathbb{F}\mathbb{G}_{\vartheta} + 3\mathbb{F}_{\vartheta}\mathbb{G} = 0,\ \ \chi > 0,\ \ \vartheta \in R,\ \ 0 < \alpha \le 1, \end{aligned} \tag{1.1}$$

where ϑ is a spatial coordinate and χ is a time coordinate. The fractional operator's order is represented by the factor α. The Caputo form is used to study this operator. When $\alpha = 1$, the fractional coupled system becomes a classical system, as follows:

$$\begin{aligned} &\mathbb{F}_{\chi} + \mathbb{F}_{3\vartheta} - 6\mathbb{F}\mathbb{F}_{\vartheta} + 3\mathbb{G}\mathbb{G}_{3\vartheta} + 3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta} - 3\mathbb{F}_{\vartheta}\mathbb{G}^2 + 6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta} = 0,\ \ \chi > 0,\ \ \vartheta \in R, \\ &\mathbb{G}_{\chi} + \mathbb{G}_{3\vartheta} - 3\mathbb{G}^2\mathbb{G}_{\vartheta} - 3\mathbb{F}\mathbb{G}_{\vartheta} + 3\mathbb{F}_{\vartheta}\mathbb{G} = 0,\ \ \chi > 0,\ \ \vartheta \in R. \end{aligned} \tag{1.2}$$

If $\mathbb{G} = 0$, the Kersten-Krasil'shchik linked KdV-mKdV system is converted to the well-known KdV system as follows:

$$\mathbb{F}_{\chi} + \mathbb{F}_{3\vartheta} - 6\mathbb{F}\mathbb{F}_{\vartheta} = 0,\ \ \chi > 0,\ \ \vartheta \in R. \tag{1.3}$$

When $\mathbb{F} = 0$, the Kersten-Krasil'shchik coupled KdV-mKdV system becomes the well-known mKdV system as follows:

$$\mathbb{G}_{\chi} + \mathbb{G}_{3\vartheta} - 3\mathbb{G}^2\mathbb{G}_{\vartheta} = 0,\ \ \chi > 0,\ \ \vartheta \in R. \tag{1.4}$$

As a result, the Kersten-Krasil'shchik linked KdV-mKdV system can be considered of as a combination of the KdV system and the mKdV system, which are represented by (1.2) to (1.4). We also investigate the following fractional nonlinear two component homogeneous time fractional coupled third order KdV system in this work as follows:

$$\begin{aligned} &D_{\chi}^{\alpha}\mathbb{F} - \mathbb{F}_{3\vartheta} - \mathbb{F}\mathbb{F}_{\vartheta} - \mathbb{G}\mathbb{G}_{\vartheta} = 0,\ \chi > 0,\ \vartheta \in R,\ 0 < \alpha \leq 1, \\ &D_{\chi}^{\alpha}\mathbb{G} + 2\mathbb{G}_{3\vartheta} - \mathbb{F}\mathbb{G}_{\vartheta} = 0,\ \chi > 0,\ \vartheta \in R,\ 0 < \alpha \leq 1, \end{aligned} \tag{1.5}$$

where χ is the temporal coordinate, ϑ is the spatial coordinate and α is the fractional operator's order factor. The Caputo form is used to study this operator. When $\alpha = 1$, the fractional coupled system becomes a classical system, as follows:

$$\begin{aligned} &\mathbb{F}_{\chi} - \mathbb{F}_{3\vartheta} - \mathbb{F}\mathbb{F}_{\vartheta} - \mathbb{G}\mathbb{G}_{\vartheta} = 0,\ \ \chi > 0,\ \ \vartheta \in R, \\ &\mathbb{G}_{\chi} + 2\mathbb{G}_{3\vartheta} - \mathbb{F}\mathbb{G}_{\vartheta} = 0,\ \ \chi > 0,\ \ \vartheta \in R. \end{aligned} \tag{1.6}$$

To solve the differential equations, we employ the natural decomposition method (NDM), which combines the natural transform (NT) and Adomian decomposition method and offers the approximate solution in series form. The proposed method has been implemented with the aid of two different fractional derivatives to solve two nonlinear systems. Many researchers have employed the NDM to obtain approximate analytical solutions, and it has provided reliable and closely convergent results. The calculations were done in Maple. The convergence of the proposed technique was also achieved by extending the concept discussed in [29, 30].

The organization of the present paper is as follows: Section 2 gives some basic definitions and the properties of the natural transform that is used in our present work. Section 3 handles the methodology of the proposed technique. In Section 4, we presented the convergence analysis of the suggested technique. Section 5 gives the implementation of the suggested approach to approximate the solution of the above systems. Finally, we discuss the obtained results and conclusion.

2. Preleminaries

In this section, we recall some basic definitions and results from fractional calculus.

Definition 2.1. *The Riemann-Liouville integral of a function $j \in C_{\mu}$, $\mu \geq -1$, having fractional-order is defined as [31]*

$$\begin{aligned} &I^{\alpha} j(\rho) = \frac{1}{\Gamma(\alpha)} \int_0^{\rho} (\rho - \mu)^{\alpha-1} j(\mu) d\mu,\ \ \alpha > 0,\ \ \rho > 0, \\ &\textit{and}\ \ I^0 j(\rho) = j(\rho). \end{aligned} \tag{2.1}$$

Definition 2.2. *The derivative of $j(\rho)$ with fractional-order in the Caputo sense is given as [31]*

$$D_{\rho}^{\alpha} j(\rho) = I^{m-\alpha} D^m j(\rho) = \frac{1}{\Gamma(m-\alpha)} \int_0^{\rho} (\rho - \mu)^{m-\alpha-1} D(j(\mu)) d\mu, \tag{2.2}$$

for $m - 1 < \alpha \leq m,\ \ m \in N,\ \ \rho > 0,\ \ j \in C_{\mu}^{m},\ \mu \geq -1.$

Definition 2.3. *The derivative of $j(\rho)$ with fractional-order in the Caputo-Fabrizio (CF) manner is given as [31]*

$$D_{\rho}^{\alpha} j(\rho) = \frac{F(\alpha)}{1-\alpha} \int_0^{\rho} \exp\left(\frac{-\alpha(\rho - \mu)}{1-\alpha}\right) D(j(\mu)) d\mu, \tag{2.3}$$

where $0 < \alpha < 1$ *and* $F(\alpha)$ *is a normalization function with* $F(0) = F(1) = 1$.

Definition 2.4. *The derivative of $j(\rho)$ with fractional-order in the Atangana-Baleanu-Caputo operator (ABC) manner is given as [31]*

$$D_{\rho}^{\alpha} j(\rho) = \frac{B(\alpha)}{1-\alpha} \int_0^{\rho} E_{\alpha}\left(\frac{-\alpha(\rho-\mu)^{\alpha}}{1-\alpha}\right) D(j(\mu)) d\mu, \tag{2.4}$$

where $0 < \alpha < 1$, $B(\alpha)$ is normalization function and $E_{\alpha}(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(\alpha^m+1)}$ is the Mittag-Leffler function.

Definition 2.5. *For the function $\mathbb{F}(\chi)$, the natural transformation is given as*

$$\mathbf{N}(\mathbb{F}(\chi)) = \mathcal{U}(\zeta,\tau) = \int_{-\infty}^{\infty} e^{-\zeta\chi}\mathbb{F}(\tau\chi)d\chi, \quad \zeta,\tau \in (-\infty,\infty), \tag{2.5}$$

and for $\chi \in (0,\infty)$, the natural transformation of $\mathbb{F}(\chi)$ is given as

$$\mathbf{N}(\mathbb{F}(\chi)H(\chi)) = \mathbf{N}^{+}\mathbb{F}(\chi) = \mathcal{U}^{+}(\zeta,\tau) = \int_{0}^{\infty} e^{-\zeta\chi}\mathbb{F}(\tau\chi)d\chi, \quad \zeta,\tau \in (0,\infty), \tag{2.6}$$

where $H(\chi)$ is the Heaviside function.

Definition 2.6. *For the function $\mathbb{F}(\zeta,\tau)$, the inverse natural transformation is given as*

$$\mathbf{N}^{-1}[\mathcal{U}(\zeta,\tau)] = \mathbb{F}(\chi), \quad \forall\chi \geq 0. \tag{2.7}$$

Lemma 2.1. *Let the $\mathbb{F}_1(\chi)$ and $\mathbb{F}_2(\chi)$ natural transformations be $\mathcal{U}_1(\zeta,\tau)$ and $\mathcal{U}_2(\zeta,\tau)$; so,*

$$\mathbf{N}[c_1\mathbb{F}_1(\chi) + c_2\mathbb{F}_2(\chi)] = c_1\mathbf{N}[\mathbb{F}_1(\chi)] + c_2\mathbf{N}[\mathbb{F}_2(\chi)] = c_1\mathcal{U}_1(\zeta,\tau) + c_2\mathcal{U}_2(\zeta,\tau), \tag{2.8}$$

where c_1 and c_2 constants.

Lemma 2.2. *Let the $\mathbb{F}_1(\chi)$ and $\mathbb{F}_2(\chi)$ inverse natural transformations be $\mathbb{F}_1(\zeta,\tau)$ and $\mathbb{F}_2(\zeta,\tau)$; so,*

$$\{N\}^{-1}[c_1\mathcal{U}_1(\zeta,\tau) + c_2\mathcal{U}_2(\zeta,\tau)] = c_1\mathbf{N}^{-1}[\mathcal{U}_1(\zeta,\tau)] + c_2\mathbf{N}^{-1}[\mathcal{U}_2(\zeta,\tau)] = c_1\mathbb{F}_1(\chi) + c_2\mathbb{F}_2(\chi), \tag{2.9}$$

where c_1 and c_2 constants.

Definition 2.7. *The NT of $D_{\chi}^{\alpha}\mathbb{F}(\chi)$ in the Caputo sense is stated as [31]*

$$\mathbf{N}[D_{\chi}^{\alpha}\mathbb{F}(\chi)] = \left(\frac{\zeta}{\tau}\right)^{\alpha}\left(\mathbf{N}[\mathbb{F}(\chi)] - \left(\frac{1}{\zeta}\right)\mathbb{F}(0)\right). \tag{2.10}$$

Definition 2.8. *The NT of $D_{\chi}^{\alpha}\mathbb{F}(\chi)$ in the CF sense is defined as [31]*

$$\mathbf{N}[D_{\chi}^{\alpha}\mathbb{F}(\chi)] = \frac{1}{1-\alpha+\alpha(\frac{\tau}{\zeta})}\left(\mathbf{N}[\mathbb{F}(\chi)] - \left(\frac{1}{\zeta}\right)\mathbb{F}(0)\right). \tag{2.11}$$

Definition 2.9. *The NT of $D_{\chi}^{\alpha}\mathbb{F}(\chi)$ in the ABC manner is given as [31]*

$$\mathbf{N}[D_{\chi}^{\alpha}\mathbb{F}(\chi)] = \frac{M[\alpha]}{1-\alpha+\alpha(\frac{\tau}{\zeta})^{\alpha}}\left(\mathbf{N}[\mathbb{F}(\chi)] - \left(\frac{1}{\zeta}\right)\mathbb{F}(0)\right), \tag{2.12}$$

with $M[\alpha]$ denoting a normalization function.

3. Methodology

This section is concerned with the general procedure for numerical treatment of the below equation.

$$D_{\chi}^{\alpha}\mathbb{F}(\vartheta,\chi) = \mathcal{L}(\mathbb{F}(\vartheta,\chi)) + N(\mathbb{F}(\vartheta,\chi)) + h(\vartheta,\chi) = M(\vartheta,\chi), \tag{3.1}$$

where the initial condition

$$\mathbb{F}(\vartheta,0) = \phi(\vartheta) \tag{3.2}$$

has $\mathcal{L}$ and N linear and nonlinear terms, and where $h(\vartheta,\chi)$ represents the source term.

3.1. Case I ($NTDM_{CF}$)

By means of the NT and CF fractional derivative, Eq (3.1) can be stated as

$$\frac{1}{p(\alpha,\tau,\zeta)}\left(\mathbf{N}[\mathbb{F}(\vartheta,\chi)] - \frac{\phi(\vartheta)}{\zeta}\right) = \mathbf{N}[M(\vartheta,\chi)], \tag{3.3}$$

with

$$p(\alpha,\tau,\zeta) = 1-\alpha+\alpha(\frac{\tau}{\zeta}). \tag{3.4}$$

By taking the natural inverse transform, we get

$$\mathbb{F}(\vartheta,\chi) = \mathbf{N}^{-1}\left(\frac{\phi(\vartheta)}{\zeta} + p(\alpha,\tau,\zeta)\mathbf{N}[M(\vartheta,\chi)]\right). \tag{3.5}$$

Assume that for $\mathbb{F}(\vartheta,\chi)$, the series form solution is determined as

$$\mathbb{F}(\vartheta,\chi) = \sum_{i=0}^{\infty}\mathbb{F}_i(\vartheta,\chi), \tag{3.6}$$

and the $N(\mathbb{F}(\vartheta,\chi))$ decomposition is given as

$$N(\mathbb{F}(\vartheta,\chi)) = \sum_{i=0}^{\infty}A_i(\mathbb{F}_0,...,\mathbb{F}_i), \tag{3.7}$$

with A_i representing the Adomian polynomials, which is illustrated as

$$A_n = \frac{1}{n!}\frac{d^n}{d\varepsilon^n}N(\varepsilon,\Sigma_{k=0}^{n}\varepsilon^k\mathbb{F}_k)|_{\varepsilon=0}.$$

By putting Eqs (3.6) and (3.7) into Eq (3.5), we have

$$\sum_{i=0}^{\infty}\mathbb{F}_i(\vartheta,\chi) = \mathbf{N}^{-1}\left(\frac{\phi(\vartheta)}{\zeta} + p(\alpha,\tau,\zeta)\mathbf{N}[h(\vartheta,\chi)]\right) + \mathbf{N}^{-1}\left(p(\alpha,\tau,\zeta)\mathbf{N}\left[\sum_{i=0}^{\infty}\mathcal{L}(\mathbb{F}_i(\vartheta,\chi)) + A_i\right]\right). \tag{3.8}$$

From Eq (3.8), we obtain

$$
\begin{aligned}
\mathbb{F}_0^{CF}(\vartheta,\chi) =& \mathbf{N}^{-1}\left(\frac{\phi(\vartheta)}{\zeta} + p(\alpha,\tau,\zeta)\mathbf{N}[h(\vartheta,\chi)]\right),\\
\mathbb{F}_1^{CF}(\vartheta,\chi) =& \mathbf{N}^{-1}\left(p(\alpha,\tau,\zeta)\mathbf{N}\left[\mathcal{L}(\mathbb{F}_0(\vartheta,\chi)) + A_0\right]\right),\\
&\vdots\\
\mathbb{F}_{l+1}^{CF}(\vartheta,\chi) =& \mathbf{N}^{-1}\left(p(\alpha,\tau,\zeta)\mathbf{N}\left[\mathcal{L}(\mathbb{F}_l(\vartheta,\chi)) + A_l\right]\right),\quad l = 1,2,3,\cdots.
\end{aligned}
\tag{3.9}
$$

In this way, the solution of Eq (3.1) is obtained by putting Eq (3.9) into Eq (3.6) to solve for the $NTDM_{CF}$,

$$
\mathbb{F}^{CF}(\vartheta,\chi) = \mathbb{F}_0^{CF}(\vartheta,\chi) + \mathbb{F}_1^{CF}(\vartheta,\chi) + \mathbb{F}_2^{CF}(\vartheta,\chi) + \cdots. \tag{3.10}
$$

3.2. *Case II* ($NTDM_{ABC}$)

By means of the NT and ABC fractional derivative, Eq (3.1) can be stated as

$$
\frac{1}{q(\alpha,\tau,\zeta)}\left(\mathbf{N}[\mathbb{F}(\vartheta,\chi)] - \frac{\phi(\vartheta)}{\zeta}\right) = \mathbf{N}[M(\vartheta,\chi)], \tag{3.11}
$$

with

$$
q(\alpha,\tau,\zeta) = \frac{1-\alpha+\alpha(\frac{\tau}{\zeta})^{\alpha}}{B(\alpha)}. \tag{3.12}
$$

By taking the natural inverse transform, we get

$$
\mathbb{F}(\vartheta,\chi) = \mathbf{N}^{-1}\left(\frac{\phi(\vartheta)}{\zeta} + q(\alpha,\tau,\zeta)\mathbf{N}[M(\vartheta,\chi)]\right). \tag{3.13}
$$

In terms of the Adomain decomposition, we have

$$
\sum_{i=0}^{\infty}\mathbb{F}_i(\vartheta,\chi) = \mathbf{N}^{-1}\left(\frac{\phi(\vartheta)}{\zeta} + q(\alpha,\tau,\zeta)\mathbf{N}[h(\vartheta,\chi)]\right) + \mathbf{N}^{-1}\left(q(\alpha,\tau,\zeta)\mathbf{N}\left[\sum_{i=0}^{\infty}\mathcal{L}(\mathbb{F}_i(\vartheta,\chi)) + A_i\right]\right). \tag{3.14}
$$

From Eq (3.8), we have the following:

$$
\begin{aligned}
\mathbb{F}_0^{ABC}(\vartheta,\chi) =& \mathbf{N}^{-1}\left(\frac{\phi(\vartheta)}{\zeta} + q(\alpha,\tau,\zeta)\mathbf{N}[h(\vartheta,\chi)]\right),\\
\mathbb{F}_1^{ABC}(\vartheta,\chi) =& \mathbf{N}^{-1}\left(q(\alpha,\tau,\zeta)\mathbf{N}\left[\mathcal{L}(\mathbb{F}_0(\vartheta,\chi)) + A_0\right]\right),\\
&\vdots\\
\mathbb{F}_{l+1}^{ABC}(\vartheta,\chi) =& \mathbf{N}^{-1}\left(q(\alpha,\tau,\zeta)\mathbf{N}\left[\mathcal{L}(\mathbb{F}_l(\vartheta,\chi)) + A_l\right]\right),\quad l = 1,2,3,\cdots.
\end{aligned}
\tag{3.15}
$$

In this way, the solution of Eq (3.1) is obtained to solve for the $NTDM_{ABC}$:

$$
\mathbb{F}^{ABC}(\vartheta,\chi) = \mathbb{F}_0^{ABC}(\vartheta,\chi) + \mathbb{F}_1^{ABC}(\vartheta,\chi) + \mathbb{F}_2^{ABC}(\vartheta,\chi) + \cdots. \tag{3.16}
$$

4. Convergence analysis

This section is concerned with the $NTDM_{ABC}$ and $NTDM_{CF}$ convergence and uniqueness.

Theorem 4.1. *Let* $|\mathcal{L}(\mathbb{F}) - \mathcal{L}(\mathbb{F}^*)| < \gamma_1|\mathbb{F} - \mathbb{F}^*|$ *and* $|N(\mathbb{F}) - N(\mathbb{F}^*)| < \gamma_2|\mathbb{F} - \mathbb{F}^*|$, *where* $\mathbb{F} := \mathbb{F}(\mu, \chi)$ *and* $\mathbb{F}^* := \mathbb{F}^*(\mu, \chi)$ *are two variable functions values, and* γ_1, γ_2 *are Lipschitz constants.*

The operators $\mathcal{L}$ *and* N *are given in Eq (3.1). Thus, the solution for the* $NTDM_{CF}$ *is unique for Eq (3.1) when* $0 < (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\chi) < 1$ *for all* χ.

Proof. Let $K = (C[J], \|.\|)$, where norm $\|\phi(\chi)\| = max_{\chi\in J}|\phi(\chi)|$ is the Banach space and $\forall$ continuous on the $J = [0, T]$ interval. Let $I : K \to K$ be a nonlinear mapping with

$$\mathbb{F}^C_{l+1} = \mathbb{F}^C_0 + \mathbf{N}^{-1}[p(\alpha, \tau, \zeta)\mathbf{N}[\mathcal{L}(\mathbb{F}_l(\mu, \chi)) + N(\mathbb{F}_l(\mu, \chi))]], \quad l \geq 0.$$

$$\begin{aligned}
\|I(\mathbb{F}) - I(\mathbb{F}^*)\| &\leq max_{\chi\in J}|\mathbf{N}^{-1}\Big[p(\alpha, \tau, \zeta)\mathbf{N}[\mathcal{L}(\mathbb{F}) - \mathcal{L}(\mathbb{F}^*)] + p(\alpha, \tau, \zeta)\mathbf{N}[N(\mathbb{F}) - N(\mathbb{F}^*)]|\Big] \\
&\leq max_{\chi\in J}\Big[\gamma_1\mathbf{N}^{-1}[p(\alpha, \tau, \zeta)\mathbf{N}[|\mathbb{F} - \mathbb{F}^*|]] + \gamma_2\mathbf{N}^{-1}[p(\alpha, \tau, \zeta)\mathbf{N}[|\mathbb{F} - \mathbb{F}^*|]]\Big] \\
&\leq max_{\chi\in J}(\gamma_1 + \gamma_2)\Big[\mathbf{N}^{-1}[p(\alpha, \tau, \zeta)\mathbf{N}|\mathbb{F} - \mathbb{F}^*|]\Big] \\
&\leq (\gamma_1 + \gamma_2)\Big[\mathbf{N}^{-1}[p(\alpha, \tau, \zeta)\mathbf{N}\|\mathbb{F} - \mathbb{F}^*\|]\Big] \\
&= (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\chi)\|\mathbb{F} - \mathbb{F}^*\|.
\end{aligned} \tag{4.1}$$

So, I is a contraction as $0 < (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\chi) < 1$. Thus, by means of the Banach fixed point theorem, the solution of Eq (3.1) is unique. □

Theorem 4.2. *According to the above theorem, the solution of Eq (3.1) is unique for the* $NTDM_{ABC}$ *when* $0 < (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\frac{\chi^\alpha}{\Gamma(\alpha+1)}) < 1$ *for all* χ.

Proof. Now, from the theorem above, let $K = (C[J], \|.\|)$ be the Banach space that is $\forall$ continuous on the $J = [0, T]$ interval. Let $I : K \to K$ be the nonlinear mapping with

$$\mathbb{F}^C_{l+1} = \mathbb{F}^C_0 + \mathbf{N}^{-1}[p(\alpha, \tau, \zeta)\mathbf{N}[\mathcal{L}(\mathbb{F}_l(\mu, \chi)) + N(\mathbb{F}_l(\mu, \chi))]], \quad l \geq 0.$$

$$\begin{aligned}
\|I(\mathbb{F}) - I(\mathbb{F}^*)\| &\leq max_{\chi\in J}|\mathbf{N}^{-1}\Big[q(\alpha, \tau, \zeta)\mathbf{N}[\mathcal{L}(\mathbb{F}) - \mathcal{L}(\mathbb{F}^*)] + q(\alpha, \tau, \zeta)\mathbf{N}[N(\mathbb{F}) - N(\mathbb{F}^*)]|\Big] \\
&\leq max_{\chi\in J}\Big[\gamma_1\mathbf{N}^{-1}[q(\alpha, \tau, \zeta)\mathbf{N}[|\mathbb{F} - \mathbb{F}^*|]] + \gamma_2\mathbf{N}^{-1}[q(\alpha, \tau, \zeta)\mathbf{N}[|\mathbb{F} - \mathbb{F}^*|]]\Big] \\
&\leq max_{\chi\in J}(\gamma_1 + \gamma_2)\Big[\mathbf{N}^{-1}[q(\alpha, \tau, \zeta)\mathbf{N}|\mathbb{F} - \mathbb{F}^*|]\Big] \\
&\leq (\gamma_1 + \gamma_2)\Big[\mathbf{N}^{-1}[q(\alpha, \tau, \zeta)\mathbf{N}\|\mathbb{F} - \mathbb{F}^*\|]\Big] \\
&= (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\frac{\chi^\alpha}{\Gamma(\alpha + 1)})\|\mathbb{F} - \mathbb{F}^*\|.
\end{aligned} \tag{4.2}$$

So, I is a contraction as $0 < (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\frac{\chi^\alpha}{\Gamma(\alpha+1)}) < 1$. Thus, by means of the Banach fixed point theorem, the solution of Eq (3.1) is unique. □

Theorem 4.3. *Let* $\mathcal{L}$ *and* N *be Lipschitz functions as given in the above theorems; then, the solution for the* $NTDM_{CF}$ *is convergent for Eq (3.1).*

Proof. Let us consider K to be the Banach space as stated above and let $\mathbb{F}_m = \sum_{r=0}^{m} \mathbb{F}_r(\mu, \chi)$. To prove that $\mathbb{F}_m$ is a Cauchy sequence in $\mathbf{H}$, let

$$\begin{aligned}
\|\mathbb{F}_m - \mathbb{F}_n\| &= max_{\chi\in J}|\sum_{r=n+1}^{m} \mathbb{F}_r|, \quad n = 1, 2, 3, \cdots \\
&\leq max_{\chi\in J}\left|\mathbf{N}^{-1}\left[p(\alpha,\tau,\zeta)\mathbf{N}\left[\sum_{r=n+1}^{m}(\mathcal{L}(\mathbb{F}_{r-1}) + N(\mathbb{F}_{r-1}))\right]\right]\right| \\
&= max_{\chi\in J}\left|\mathbf{N}^{-1}\left[p(\alpha,\tau,\zeta)\mathbf{N}\left[\sum_{r=n+1}^{m-1}(\mathcal{L}(\mathbb{F}_{r}) + N(\mathbb{F}_{r}))\right]\right]\right| \\
&\leq max_{\chi\in J}|\mathbf{N}^{-1}[p(\alpha,\tau,\zeta)\mathbf{N}[(\mathcal{L}(\mathbb{F}_{m-1}) - \mathcal{L}(\mathbb{F}_{n-1}) + N(\mathbb{F}_{m-1}) - N(\mathbb{F}_{n-1}))]]| \\
&\leq \gamma_1 max_{\chi\in J}|\mathbf{N}^{-1}[p(\alpha,\tau,\zeta)\mathbf{N}[(\mathcal{L}(\mathbb{F}_{m-1}) - \mathcal{L}(\mathbb{F}_{n-1}))]]| \\
&\quad + \gamma_2 max_{\chi\in J}|\mathbf{N}^{-1}[p(\alpha,\tau,\zeta)\mathbf{N}[(N(\mathbb{F}_{m-1}) - N(\mathbb{F}_{n-1}))]]| \\
&= (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\chi)\|\mathbb{F}_{m-1} - \mathbb{F}_{n-1}\|.
\end{aligned} \tag{4.3}$$

Let $m = n + 1$; then,

$$\|\mathbb{F}_{n+1} - \mathbb{F}_n\| \leq \gamma\|\mathbb{F}_n - \mathbb{F}_{n-1}\| \leq \gamma^2\|\mathbb{F}_{n-1}\mathbb{F}_{n-2}\| \leq \cdots \leq \gamma^n\|\mathbb{F}_1 - \mathbb{F}_0\|, \tag{4.4}$$

where $\gamma = (\gamma_1 + \gamma_2)(1 - \alpha + \alpha\chi)$. Thus, we have that

$$\begin{aligned}
&\|\mathbb{F}_m - \mathbb{F}_n\| \leq \|\mathbb{F}_{n+1} - \mathbb{F}_n\| + \|\mathbb{F}_{n+2}\mathbb{F}_{n+1}\| + \cdots + \|\mathbb{F}_m - \mathbb{F}_{m-1}\|, \\
&(\gamma^n + \gamma^{n+1} + \cdots + \gamma^{m-1})\|\mathbb{F}_1 - \mathbb{F}_0\| \leq \gamma^n\left(\frac{1-\gamma^{m-n}}{1-\gamma}\right)\|\mathbb{F}_1\|.
\end{aligned} \tag{4.5}$$

As $0 < \gamma < 1$, we have that $1 - \gamma^{m-n} < 1$. Thus,

$$\|\mathbb{F}_m - \mathbb{F}_n\| \leq \frac{\gamma^n}{1-\gamma} max_{\chi\in J}\|\mathbb{F}_1\|. \tag{4.6}$$

Since $\|\mathbb{F}_1\| < \infty$, $\|\mathbb{F}_m - \mathbb{F}_n\| \to 0$ when $n \to \infty$. In this way, $\mathbb{F}_m$ is a Cauchy sequence in K and is convergent. □

Theorem 4.4. *Let $\mathcal{L}$ and N be Lipschitz functions as given in the above theorems; then, the solution for the $NTDM_{ABC}$ is convergent for Eq (3.1).*

Proof. Suppose $\mathbb{F}_m = \sum_{r=0}^{m} \mathbb{F}_r(\mu, \chi)$. To prove that $\mathbb{F}_m$ is a Cauchy sequence in K, let

$$\begin{aligned}
\|\mathbb{F}_m - \mathbb{F}_n\| &= max_{\chi\in J}|\sum_{r=n+1}^{m} \mathbb{F}_r|, \quad n = 1, 2, 3, \cdots \\
&\leq max_{\chi\in J}\left|\mathbf{N}^{-1}\left[q(\alpha,\tau,\zeta)\mathbf{N}\left[\sum_{r=n+1}^{m}(\mathcal{L}(\mathbb{F}_{r-1}) + N(\mathbb{F}_{r-1}))\right]\right]\right| \\
&= max_{\chi\in J}\left|\mathbf{N}^{-1}\left[q(\alpha,\tau,\zeta)\mathbf{N}\left[\sum_{r=n+1}^{m-1}(\mathcal{L}(\mathbb{F}_{r}) + N(u_{r}))\right]\right]\right| \\
&\leq max_{\chi\in J}|\mathbf{N}^{-1}[q(\alpha,\tau,\zeta)\mathbf{N}[(\mathcal{L}(\mathbb{F}_{m-1}) - \mathcal{L}(\mathbb{F}_{n-1}) + N(\mathbb{F}_{m-1}) - N(\mathbb{F}_{n-1}))]]|
\end{aligned}$$

$$
\begin{aligned}
&\leq \gamma_1 max_{\chi\in J}|\mathbf{N}^{-1}[q(\alpha,\tau,\zeta)\mathbf{N}[(\mathcal{L}(\mathbb{F}_{m-1})-\mathcal{L}(\mathbb{F}_{n-1}))]]| \\
&\quad + \gamma_2 max_{\chi\in J}|\mathbf{N}^{-1}[p(\alpha,\tau,\zeta)\mathbf{N}[(N(\mathbb{F}_{m-1})-N(\mathbb{F}_{n-1}))]]| \\
&= (\gamma_1+\gamma_2)(1-\alpha+\alpha\frac{\chi^{\alpha}}{\Gamma(\alpha+1)})||\mathbb{F}_{m-1}-\mathbb{F}_{n-1}||.
\end{aligned} \tag{4.7}
$$

Suppose $m = n + 1$; thus,

$$
||\mathbb{F}_{n+1}-\mathbb{F}_n|| \leq \gamma||\mathbb{F}_n-\mathbb{F}_{n-1}|| \leq \gamma^2||\mathbb{F}_{n-1}-\mathbb{F}_{n-2}|| \leq \cdots \leq \gamma^n||\mathbb{F}_1-\mathbb{F}_0||, \tag{4.8}
$$

where $\gamma = (\gamma_1+\gamma_2)(1-\alpha+\alpha\frac{\chi^{\alpha}}{\Gamma(\alpha+1)})$. Thus, we have that

$$
\begin{aligned}
||\mathbb{F}_m-\mathbb{F}_n|| &\leq ||\mathbb{F}_{n+1}-\mathbb{F}_n|| + ||\mathbb{F}_{n+2}-\mathbb{F}_{n+1}|| + \cdots + ||\mathbb{F}_m-\mathbb{F}_{m-1}||, \\
&\leq (\gamma^n+\gamma^{n+1}+\cdots+\gamma^{m-1})||\mathbb{F}_1-\mathbb{F}_0|| \\
&\leq \gamma^n\left(\frac{1-\gamma^{m-n}}{1-\gamma}\right)||\mathbb{F}_1||.
\end{aligned} \tag{4.9}
$$

As $0 < \gamma < 1$, we have that $1-\gamma^{m-n} < 1$. Thus,

$$
||\mathbb{F}_m-\mathbb{F}_n|| \leq \frac{\gamma^n}{1-\gamma} max_{\chi\in J}||\mathbb{F}_1||. \tag{4.10}
$$

Since $||\mathbb{F}_1|| < \infty$, $||\mathbb{F}_m-\mathbb{F}_n|| \to 0$ when $n \to \infty$. In this way, $\mathbb{F}_m$ is a Cauchy sequence in K and convergent. □

5. Applications

Example 5.1. Let us consider the fractional Kersten-Krasil'shchik coupled KdV-mKdV nonlinear system as

$$
\begin{aligned}
&D^{\alpha}_{\chi}\mathbb{F}+\mathbb{F}_{3\vartheta}-6\mathbb{F}\mathbb{F}_{\vartheta}+3\mathbb{G}\mathbb{G}_{3\vartheta}+3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta}-3\mathbb{F}_{\vartheta}\mathbb{G}^2+6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta}=0,\quad \chi>0,\quad \vartheta\in R,\quad 0<\alpha\leq 1, \\
&D^{\alpha}_{\chi}\mathbb{G}+\mathbb{G}_{3\vartheta}-3\mathbb{G}^2\mathbb{G}_{\vartheta}-3\mathbb{F}\mathbb{G}_{\vartheta}+3\mathbb{F}_{\vartheta}\mathbb{G}=0,
\end{aligned} \tag{5.1}
$$

where the initial conditions are as follows:

$$
\begin{aligned}
&\mathbb{F}(\vartheta,0)=c-2c\,\mathrm{sech}^2(\sqrt{c}\vartheta),\quad c>0, \\
&\mathbb{G}(\vartheta,0)=2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta).
\end{aligned} \tag{5.2}
$$

By taking the NT, we get

$$
\begin{aligned}
&\mathbf{N}[D^{\alpha}_{\chi}\mathbb{F}(\vartheta,\chi)] = -\mathbf{N}\left[\mathbb{F}_{3\vartheta}-6\mathbb{F}\mathbb{F}_{\vartheta}+3\mathbb{G}\mathbb{G}_{3\vartheta}+3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta}-3\mathbb{F}_{\vartheta}\mathbb{G}^2+6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta}\right], \\
&\mathbf{N}[D^{\alpha}_{\chi}\mathbb{G}(\vartheta,\chi)] = -\mathbf{N}\left[\mathbb{G}_{3\vartheta}-3\mathbb{G}^2\mathbb{G}_{\vartheta}-3\mathbb{F}\mathbb{G}_{\vartheta}+3\mathbb{F}_{\vartheta}\mathbb{G}\right].
\end{aligned} \tag{5.3}
$$

Thus, we have that

$$
\begin{aligned}
&\frac{1}{\zeta^{\alpha}}\mathbf{N}[\mathbb{F}(\vartheta,\chi)]-\zeta^{2-\alpha}\mathbb{F}(\vartheta,0) = -\mathbf{N}\left[\mathbb{F}_{3\vartheta}-6\mathbb{F}\mathbb{F}_{\vartheta}+3\mathbb{G}\mathbb{G}_{3\vartheta}+3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta}-3\mathbb{F}_{\vartheta}\mathbb{G}^2+6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta}\right], \\
&\frac{1}{\zeta^{\alpha}}\mathbf{N}[\mathbb{G}(\vartheta,\chi)]-\zeta^{2-\alpha}\mathbb{F}(\vartheta,0) = -\mathbf{N}\left[\mathbb{G}_{3\vartheta}-3\mathbb{G}^2\mathbb{G}_{\vartheta}-3\mathbb{F}\mathbb{G}_{\vartheta}+3\mathbb{F}_{\vartheta}\mathbb{G}\right].
\end{aligned} \tag{5.4}
$$

By simplification, we get

$$\begin{aligned}
&\mathbf{N}[\mathbb{F}(\vartheta,\chi)]\\
=&\zeta^2\Big[c-2c\,\mathrm{sech}^2(\sqrt{c}\vartheta)\Big]-\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^2}\mathbf{N}\Big[\mathbb{F}_{3\vartheta}-6\mathbb{F}\mathbb{F}_{\vartheta}+3\mathbb{G}\mathbb{G}_{3\vartheta}+3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta}-3\mathbb{F}_{\vartheta}\mathbb{G}^2+6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta}\Big],\\
&\mathbf{N}[\mathbb{G}(\vartheta,\chi)]=\zeta^2\Big[2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta)\Big]-\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^2}\mathbf{N}\Big[\mathbb{G}_{3\vartheta}-3\mathbb{G}^2\mathbb{G}_{\vartheta}-3\mathbb{F}\mathbb{G}_{\vartheta}+3\mathbb{F}_{\vartheta}\mathbb{G}\Big].
\end{aligned} \tag{5.5}$$

By taking the inverse NT, we have that

$$\begin{aligned}
&\mathbb{F}(\vartheta,\chi)\\
=&\Big[c-2c\,\mathrm{sech}^2(\sqrt{c}\vartheta)\Big]-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^2}\mathbf{N}\Big\{\mathbb{F}_{3\vartheta}-6\mathbb{F}\mathbb{F}_{\vartheta}+3\mathbb{G}\mathbb{G}_{3\vartheta}+3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta}-3\mathbb{F}_{\vartheta}\mathbb{G}^2+6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta}\Big\}\Big],\\
&\mathbb{G}(\vartheta,\chi)=\Big[2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta)\Big]-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^2}\mathbf{N}\Big\{\mathbb{G}_{3\vartheta}-3\mathbb{G}^2\mathbb{G}_{\vartheta}-3\mathbb{F}\mathbb{G}_{\vartheta}+3\mathbb{F}_{\vartheta}\mathbb{G}\Big\}\Big].
\end{aligned} \tag{5.6}$$

Solution for the NDM_{CF}:

Assume that for the unknown functions $\mathbb{F}(\vartheta,\chi)$ and $\mathbb{G}(\vartheta,\chi)$, the series form solutions are given as

$$\mathbb{F}(\vartheta,\chi)=\sum_{l=0}^{\infty}\mathbb{F}_l(\vartheta,\chi)\ \textit{and}\ \mathbb{G}(\vartheta,\chi)=\sum_{l=0}^{\infty}\mathbb{F}_l(\vartheta,\chi). \tag{5.7}$$

The nonlinear components in terms of the Adomian polynomials are given as $-6\mathbb{F}\mathbb{F}_{\vartheta}+3\mathbb{G}\mathbb{G}_{3\vartheta}=\sum_{m=0}^{\infty}\mathcal{A}_m$, $3\mathbb{G}_{\vartheta}\mathbb{G}_{2\vartheta}-3\mathbb{F}_{\vartheta}\mathbb{G}^2=\sum_{m=0}^{\infty}\mathcal{B}_m$, $6\mathbb{F}\mathbb{G}\mathbb{G}_{\vartheta}=\sum_{m=0}^{\infty}C_m$ and $-3\mathbb{G}^2\mathbb{G}_{\vartheta}-3\mathbb{F}\mathbb{G}_{\vartheta}+3\mathbb{F}_{\vartheta}\mathbb{G}=\sum_{m=0}^{\infty}\mathcal{D}_m$. With the help of these terms, Eq (5.6) can be stated as

$$\begin{aligned}
&\sum_{l=0}^{\infty}\mathbb{F}_{l+1}(\vartheta,\chi)=c-2c\,\mathrm{sech}^2(\sqrt{c}\vartheta)-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^2}\mathbf{N}\Big\{\mathbb{F}_{3\vartheta}+\sum_{l=0}^{\infty}\mathcal{A}_l+\sum_{l=0}^{\infty}\mathcal{B}_l+\sum_{l=0}^{\infty}C_l\Big\}\Big],\\
&\sum_{l=0}^{\infty}\mathbb{G}_{l+1}(\vartheta,\chi)=2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta)-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^2}\mathbf{N}\Big\{\mathbb{F}_{3\vartheta}+\sum_{l=0}^{\infty}\mathcal{D}_l\Big\}\Big].
\end{aligned} \tag{5.8}$$

By comparing both sides of Eq (5.8), we have that

$$\begin{aligned}
\mathbb{F}_0(\vartheta,\chi)&=c-2c\,\mathrm{sech}^2(\sqrt{c}\vartheta),\\
\mathbb{G}_0(\vartheta,\chi)&=2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta),
\end{aligned}$$

$$\begin{aligned}
\mathbb{F}_1(\vartheta,\chi)&=8c^{\frac{5}{2}}\sinh(\sqrt{c}\vartheta)\,\mathrm{sech}^3(\sqrt{c}\vartheta)\,(\alpha(\chi-1)+1)\,,\\
\mathbb{G}_1(\vartheta,\chi)&=-4c^2\sinh(\sqrt{c}\vartheta)\,\mathrm{sech}^2(\sqrt{c}\vartheta)\,(\alpha(\chi-1)+1)\,,
\end{aligned} \tag{5.9}$$

$$\begin{aligned}
\mathbb{F}_2(\vartheta,\chi)&=-16c^4[2\cosh^2(\sqrt{c}\vartheta)-3]\,\mathrm{sech}^4(\sqrt{c}\vartheta)\Big((1-\alpha)^2+2\alpha(1-\alpha)\chi+\frac{\alpha^2\chi^2}{2}\Big),\\
\mathbb{G}_2(\vartheta,\chi)&=8c^{\frac{7}{2}}[\cosh^2(\sqrt{c}\vartheta)-2]\,\mathrm{sech}^3(\sqrt{c}\vartheta)\Big((1-\alpha)^2+2\alpha(1-\alpha)\chi+\frac{\alpha^2\chi^2}{2}\Big).
\end{aligned} \tag{5.10}$$

In this way, given ($l \geq 3$), the remaining terms for $\mathbb{F}_l$ and $\mathbb{G}_l$ are easy to get. Thus, the series form solutions are given as

$$
\begin{aligned}
\mathbb{F}(\vartheta,\chi) &= \sum_{l=0}^{\infty} \mathbb{F}_l(\vartheta,\chi) = \mathbb{F}_0(\vartheta,\chi) + \mathbb{F}_1(\vartheta,\chi) + \mathbb{F}_2(\vartheta,\chi) + \cdots, \\
\mathbb{F}(\vartheta,\chi) &= c - 2c\,\mathrm{sech}^2(\sqrt{c}\vartheta) + 8c^{\frac{5}{2}} \sinh(\sqrt{c}\vartheta)\,\mathrm{sech}^3(\sqrt{c}\vartheta)\,(\alpha(\chi-1)+1) \\
&\quad - 16c^4[2\cosh^2(\sqrt{c}\vartheta) - 3]\,\mathrm{sech}^4(\sqrt{c}\vartheta)\Big((1-\alpha)^2 + 2\alpha(1-\alpha)\chi + \frac{\alpha^2\chi^2}{2}\Big) + \cdots, \\
\mathbb{G}(\vartheta,\chi) &= \sum_{l=0}^{\infty} \mathbb{G}_l(\vartheta,\chi) = \mathbb{G}_0(\vartheta,\chi) + \mathbb{G}_1(\vartheta,\chi) + \mathbb{G}_2(\vartheta,\chi) + \cdots, \\
\mathbb{G}(\vartheta,\chi) &= 2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta) - 4c^2 \sinh(\sqrt{c}\vartheta)\,\mathrm{sech}^2(\sqrt{c}\vartheta)\,(\alpha(\chi-1)+1) \\
&\quad + 8c^{\frac{7}{2}}[\cosh^2(\sqrt{c}\vartheta) - 2]\,\mathrm{sech}^3(\sqrt{c}\vartheta)\Big((1-\alpha)^2 + 2\alpha(1-\alpha)\chi + \frac{\alpha^2\chi^2}{2}\Big) + \cdots.
\end{aligned}
\tag{5.11}
$$

Solution for the NDM_{ABC}**:**

Assume that for the unknown functions $\mathbb{F}(\vartheta,\chi)$ and $\mathbb{G}(\vartheta,\chi)$, the series form solutions respectively are given as

$$
\begin{aligned}
\mathbb{F}(\vartheta,\chi) &= \sum_{l=0}^{\infty} \mathbb{F}_l(\vartheta,\chi), \\
\mathbb{G}(\vartheta,\chi) &= \sum_{l=0}^{\infty} \mathbb{F}_l(\vartheta,\chi).
\end{aligned}
\tag{5.12}
$$

The nonlinear components in terms of the Adomian polynomials are given as $-6\mathbb{F}\mathbb{F}_\vartheta + 3\mathbb{G}\mathbb{G}_{3\vartheta} = \sum_{m=0}^{\infty} \mathcal{A}_m$, $3\mathbb{G}_\vartheta\mathbb{G}_{2\vartheta} - 3\mathbb{F}_\vartheta\mathbb{G}^2 = \sum_{m=0}^{\infty} \mathcal{B}_m$, $6\mathbb{F}\mathbb{G}\mathbb{G}_\vartheta = \sum_{m=0}^{\infty} C_m$ and $-3\mathbb{G}^2\mathbb{G}_\vartheta - 3\mathbb{F}\mathbb{G}_\vartheta + 3\mathbb{F}_\vartheta\mathbb{G} = \sum_{m=0}^{\infty} \mathcal{D}_m$. With the help of these terms, Eq (5.6) can be stated as

$$
\begin{aligned}
\sum_{l=0}^{\infty} \mathbb{F}_{l+1}(\vartheta,\chi) &= c - 2c\,\mathrm{sech}^2(\sqrt{c}\vartheta) - \mathbf{N}^{-1}\left[\frac{\tau^\alpha(\zeta^\alpha + \alpha(\tau^\alpha - \zeta^\alpha))}{\zeta^{2\alpha}}\mathbf{N}\left\{\mathbb{F}_{3\vartheta} + \sum_{l=0}^{\infty}\mathcal{A}_l + \sum_{l=0}^{\infty}\mathcal{B}_l + \sum_{l=0}^{\infty} C_l\right\}\right], \\
\sum_{l=0}^{\infty} \mathbb{G}_{l+1}(\vartheta,\chi) &= 2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta) - \mathbf{N}^{-1}\left[\frac{\tau^\alpha(\zeta^\alpha + \alpha(\tau^\alpha - \zeta^\alpha))}{\zeta^{2\alpha}}\mathbf{N}\left\{\mathbb{F}_{3\vartheta} + \sum_{l=0}^{\infty}\mathcal{D}_l\right\}\right].
\end{aligned}
\tag{5.13}
$$

By comparing both sides of Eq (5.13), we have that

$$
\begin{aligned}
\mathbb{F}_0(\vartheta,\chi) &= c - 2c\,\mathrm{sech}^2(\sqrt{c}\vartheta), \\
\mathbb{G}_0(\vartheta,\chi) &= 2\sqrt{c}\,\mathrm{sech}(\sqrt{c}\vartheta),
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{F}_1(\vartheta,\chi) &= 8c^{\frac{5}{2}} \sinh(\sqrt{c}\vartheta)\,\mathrm{sech}^3(\sqrt{c}\vartheta)\left(1 - \alpha + \frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right), \\
\mathbb{G}_1(\vartheta,\chi) &= -4c^2 \sinh(\sqrt{c}\vartheta)\,\mathrm{sech}^2(\sqrt{c}\vartheta)\left(1 - \alpha + \frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right),
\end{aligned}
\tag{5.14}
$$

$$\begin{aligned}
\mathbb{F}_2(\vartheta,\chi) &= -16c^4[2\cosh^2(\sqrt{c}\vartheta)-3]\operatorname{sech}^4(\sqrt{c}\vartheta)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right],\\
\mathbb{G}_2(\vartheta,\chi) &= 8c^{\frac{7}{2}}[\cosh^2(\sqrt{c}\vartheta)-2]\operatorname{sech}^3(\sqrt{c}\vartheta)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right].
\end{aligned}\tag{5.15}$$

In this way, given ($l \geq 3$), the remaining terms for $\mathbb{F}_l$ and $\mathbb{G}_l$ are easy to get. Thus, the series form solutions are given as

$$\begin{aligned}
\mathbb{F}(\vartheta,\chi) &= \sum_{l=0}^{\infty}\mathbb{F}_l(\vartheta,\chi) = \mathbb{F}_0(\vartheta,\chi)+\mathbb{F}_1(\vartheta,\chi)+\mathbb{F}_2(\vartheta,\chi)+\cdots,\\
\mathbb{F}(\vartheta,\chi) &= c-2c\operatorname{sech}^2(\sqrt{c}\vartheta)+8c^{\frac{5}{2}}\sinh(\sqrt{c}\vartheta)\operatorname{sech}^3(\sqrt{c}\vartheta)\left(1-\alpha+\frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right)\\
&\quad -16c^4[2\cosh^2(\sqrt{c}\vartheta)-3]\operatorname{sech}^4(\sqrt{c}\vartheta)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right]+\cdots,\\
\mathbb{G}(\vartheta,\chi) &= \sum_{l=0}^{\infty}\mathbb{G}_l(\vartheta,\chi) = \mathbb{G}_0(\vartheta,\chi)+\mathbb{G}_1(\vartheta,\chi)+\mathbb{G}_2(\vartheta,\chi)+\cdots,\\
\mathbb{G}(\vartheta,\chi) &= 2\sqrt{c}\operatorname{sech}(\sqrt{c}\vartheta)-4c^2\sinh(\sqrt{c}\vartheta)\operatorname{sech}^2(\sqrt{c}\vartheta)\left(1-\alpha+\frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right)\\
&\quad +8c^{\frac{7}{2}}[\cosh^2(\sqrt{c}\vartheta)-2]\operatorname{sech}^3(\sqrt{c}\vartheta)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right]+\cdots.
\end{aligned}\tag{5.16}$$

When $\alpha = 1$, we get the exact solution as

$$\begin{aligned}
\mathbb{F}(\vartheta,\chi) &= c-2c\operatorname{sech}^2(\sqrt{c}(\vartheta+2c\chi)),\\
\mathbb{G}(\vartheta,\chi) &= 2\sqrt{c}\operatorname{sech}(\sqrt{c}(\vartheta+2c\chi)).
\end{aligned}\tag{5.17}$$

Example 5.2. Let us consider the time-fractional homogeneous two component coupled third order KdV system as

$$\begin{aligned}
&D_\chi^\alpha\mathbb{F}-\mathbb{F}_{3\vartheta}-\mathbb{F}\mathbb{F}_\vartheta-\mathbb{G}\mathbb{G}_\vartheta = 0,\ \ \chi>0,\ \ \vartheta\in R,\ \ 0<\alpha\leq 1,\\
&D_\chi^\alpha\mathbb{G}+2\mathbb{G}_{3\vartheta}-\mathbb{F}\mathbb{G}_\vartheta = 0,
\end{aligned}\tag{5.18}$$

where the initial conditions are as follows:

$$\begin{aligned}
\mathbb{F}(\vartheta,0) &= 3-6\tanh^2\left(\frac{\vartheta}{2}\right),\\
\mathbb{G}(\vartheta,0) &= -3c\sqrt{2}\tanh\left(\frac{\vartheta}{2}\right).
\end{aligned}\tag{5.19}$$

By taking the NT, we get

$$\begin{aligned}
\mathbf{N}[D_\chi^\alpha\mathbb{F}(\vartheta,\chi)] &= -\mathbf{N}\left[-\mathbb{F}_{3\vartheta}-\mathbb{F}\mathbb{F}_\vartheta-\mathbb{G}\mathbb{G}_\vartheta\right],\\
\mathbf{N}[D_\chi^\alpha\mathbb{G}(\vartheta,\chi)] &= -\mathbf{N}\left[2\mathbb{G}_{3\vartheta}-\mathbb{F}\mathbb{G}_\vartheta\right].
\end{aligned}\tag{5.20}$$

Thus, we have

$$\begin{aligned}&\frac{1}{\zeta^{\alpha}}\mathbf{N}[\mathbb{F}(\vartheta,\chi)]-\zeta^{2-\alpha}\mathbb{F}(\vartheta,0)=-\mathbf{N}\Big[-\mathbb{F}_{3\vartheta}-\mathbb{F}\mathbb{F}_{\vartheta}-\mathbb{G}\mathbb{G}_{\vartheta}\Big],\\&\frac{1}{\zeta^{\alpha}}\mathbf{N}[\mathbb{G}(\vartheta,\chi)]-\zeta^{2-\alpha}\mathbb{F}(\vartheta,0)=-\mathbf{N}\Big[2\mathbb{G}_{3\vartheta}-\mathbb{F}\mathbb{G}_{\vartheta}\Big].\end{aligned}\tag{5.21}$$

By simplification, we get

$$\begin{aligned}&\mathbf{N}[\mathbb{F}(\vartheta,\chi)]=\zeta^{2}\Big[3-6\tanh^{2}\Big(\frac{\vartheta}{2}\Big)\Big]-\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^{2}}\mathbf{N}\Big[-\mathbb{F}_{3\vartheta}-\mathbb{F}\mathbb{F}_{\vartheta}-\mathbb{G}\mathbb{G}_{\vartheta}\Big],\\&\mathbf{N}[\mathbb{G}(\vartheta,\chi)]=\zeta^{2}\Big[-3c\sqrt{2}\tanh\Big(\frac{\vartheta}{2}\Big)\Big]-\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^{2}}\mathbf{N}\Big[2\mathbb{G}_{3\vartheta}-\mathbb{F}\mathbb{G}_{\vartheta}\Big].\end{aligned}\tag{5.22}$$

By taking the inverse NT, we have that

$$\begin{aligned}&\mathbb{F}(\vartheta,\chi)=3-6\tanh^{2}\Big(\frac{\vartheta}{2}\Big)-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^{2}}\mathbf{N}\Big\{-\mathbb{F}_{3\vartheta}-\mathbb{F}\mathbb{F}_{\vartheta}-\mathbb{G}\mathbb{G}_{\vartheta}\Big\}\Big],\\&\mathbb{G}(\vartheta,\chi)=\Big[-3c\sqrt{2}\tanh\Big(\frac{\vartheta}{2}\Big)\Big]-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^{2}}\mathbf{N}\Big\{2\mathbb{G}_{3\vartheta}-\mathbb{F}\mathbb{G}_{\vartheta}\Big\}\Big].\end{aligned}\tag{5.23}$$

Solution for the $NTDM_{CF}$:

Assume that for the unknown functions $\mathbb{F}(\vartheta,\chi)$ and $\mathbb{G}(\vartheta,\chi)$, the series form solution are given as

$$\mathbb{F}(\vartheta,\chi)=\sum_{l=0}^{\infty}\mathbb{F}_{l}(\vartheta,\chi) and \mathbb{G}(\vartheta,\chi)=\sum_{l=0}^{\infty}\mathbb{F}_{l}(\vartheta,\chi).\tag{5.24}$$

The nonlinear components in terms of the Adomian polynomials are given as $-\mathbb{F}\mathbb{F}_{\vartheta}-\mathbb{G}\mathbb{G}_{\vartheta}=\sum_{m=0}^{\infty}\mathcal{A}_{m}$ and $\mathbb{F}\mathbb{G}_{\vartheta}=\sum_{m=0}^{\infty}\mathcal{B}_{m}$. With the help of these terms, Eq (5.23) can be stated as

$$\begin{aligned}&\sum_{l=0}^{\infty}\mathbb{F}_{l+1}(\vartheta,\chi)=3-6\tanh^{2}\Big(\frac{\vartheta}{2}\Big)-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^{2}}\mathbf{N}\Big\{-\mathbb{F}_{3\vartheta}+\sum_{l=0}^{\infty}\mathcal{A}_{l}\Big\}\Big],\\&\sum_{l=0}^{\infty}\mathbb{G}_{l+1}(\vartheta,\chi)=-3c\sqrt{2}\tanh\Big(\frac{\vartheta}{2}\Big)-\mathbf{N}^{-1}\Big[\frac{\alpha(\zeta-\alpha(\zeta-\alpha))}{\zeta^{2}}\mathbf{N}\Big\{2\mathbb{G}_{3\vartheta}-\sum_{l=0}^{\infty}\mathcal{B}_{l}\Big\}\Big].\end{aligned}\tag{5.25}$$

By comparing both sides of Eq (5.25), we have that

$$\mathbb{F}_{0}(\vartheta,\chi)=3-6\tanh^{2}\Big(\frac{\vartheta}{2}\Big),$$

$$\mathbb{G}_{0}(\vartheta,\chi)=-3c\sqrt{2}\tanh\Big(\frac{\vartheta}{2}\Big),$$

$$\begin{aligned}&\mathbb{F}_{1}(\vartheta,\chi)=6\,\mathrm{sech}^{2}\Big(\frac{\vartheta}{2}\Big)\tanh\Big(\frac{\vartheta}{2}\Big)(\alpha(\chi-1)+1),\\&\mathbb{G}_{1}(\vartheta,\chi)=3c\sqrt{2}\,\mathrm{sech}^{2}\Big(\frac{\vartheta}{2}\Big)\tanh\Big(\frac{\vartheta}{2}\Big)(\alpha(\chi-1)+1),\end{aligned}\tag{5.26}$$

$$\mathbb{F}_2(\vartheta,\chi) = 3[2 + 7\operatorname{sech}^2\left(\frac{\vartheta}{2}\right) - 15\operatorname{sech}^4\left(\frac{\vartheta}{2}\right)]\operatorname{sech}^2\left(\frac{\vartheta}{2}\right)\Big((1-\alpha)^2 + 2\alpha(1-\alpha)\chi + \frac{\alpha^2\chi^2}{2}\Big),$$
$$\mathbb{G}_2(\vartheta,\chi) = \frac{3c\sqrt{2}}{2}\left[2 + 21\operatorname{sech}^2\left(\frac{\vartheta}{2}\right) - 24\operatorname{sech}^4\left(\frac{\vartheta}{2}\right)\right]\operatorname{sech}^2\left(\frac{\vartheta}{2}\right)\Big((1-\alpha)^2 + 2\alpha(1-\alpha)\chi + \frac{\alpha^2\chi^2}{2}\Big). \tag{5.27}$$

In this way, given ($l \geq 3$), the remaining terms for $\mathbb{F}_l$ and $\mathbb{G}_l$ are easy to get. Thus, the series form solutions are given as

$$\mathbb{F}(\vartheta,\chi) = \sum_{l=0}^{\infty}\mathbb{F}_l(\vartheta,\chi) = \mathbb{F}_0(\vartheta,\chi) + \mathbb{F}_1(\vartheta,\chi) + \mathbb{F}_2(\vartheta,\chi) + \cdots,$$
$$\mathbb{F}(\vartheta,\chi) = 3 - 6\tanh^2\left(\frac{\vartheta}{2}\right) + 6\operatorname{sech}^2\left(\frac{\vartheta}{2}\right)\tanh\left(\frac{\vartheta}{2}\right)(\alpha(\chi-1)+1)$$
$$+ 3[2 + 7\operatorname{sech}^2\left(\frac{\vartheta}{2}\right) - 15\operatorname{sech}^4\left(\frac{\vartheta}{2}\right)]\operatorname{sech}^2\left(\frac{\vartheta}{2}\right)\Big((1-\alpha)^2 + 2\alpha(1-\alpha)\chi + \frac{\alpha^2\chi^2}{2}\Big) + \cdots,$$
$$\mathbb{G}(\vartheta,\chi) = \sum_{l=0}^{\infty}\mathbb{G}_l(\vartheta,\chi) = \mathbb{G}_0(\vartheta,\chi) + \mathbb{G}_1(\vartheta,\chi) + \mathbb{G}_2(\vartheta,\chi) + \cdots,$$
$$\mathbb{G}(\vartheta,\chi) = -3c\sqrt{2}\tanh\left(\frac{\vartheta}{2}\right) + 3c\sqrt{2}\operatorname{sech}^2\left(\frac{\vartheta}{2}\right)\tanh\left(\frac{\vartheta}{2}\right)(\alpha(\chi-1)+1)$$
$$+ \frac{3c\sqrt{2}}{2}\left[2 + 21\operatorname{sech}^2\left(\frac{\vartheta}{2}\right) - 24\operatorname{sech}^4\left(\frac{\vartheta}{2}\right)\right]\operatorname{sech}^2\left(\frac{\vartheta}{2}\right)\Big((1-\alpha)^2 + 2\alpha(1-\alpha)\chi + \frac{\alpha^2\chi^2}{2}\Big) + \cdots. \tag{5.28}$$

Solution for the $NTDM_{ABC}$:
Assume that for the unknown functions $\mathbb{F}(\vartheta,\chi)$ and $\mathbb{G}(\vartheta,\chi)$, the series form solutions are given as

$$\mathbb{F}(\vartheta,\chi) = \sum_{l=0}^{\infty}\mathbb{F}_l(\vartheta,\chi),$$
$$\mathbb{G}(\vartheta,\chi) = \sum_{l=0}^{\infty}\mathbb{F}_l(\vartheta,\chi). \tag{5.29}$$

The nonlinear components in terms of the Adomian polynomials are given as $-\mathbb{F}\mathbb{F}_\vartheta - \mathbb{G}\mathbb{G}_\vartheta = \sum_{m=0}^{\infty}\mathcal{A}_m$ and $\mathbb{F}\mathbb{G}_\vartheta = \sum_{m=0}^{\infty}\mathcal{B}_m$. Given these terms, Eq (5.23) can be stated as

$$\sum_{l=0}^{\infty}\mathbb{F}_{l+1}(\vartheta,\chi) = 3 - 6\tanh^2\left(\frac{\vartheta}{2}\right) + \mathbf{N}^{-1}\left[\frac{\tau^\alpha(\zeta^\alpha + \alpha(\tau^\alpha - \zeta^\alpha))}{\zeta^{2\alpha}}\mathbf{N}\left\{-\mathbb{F}_{3\vartheta} + \sum_{l=0}^{\infty}\mathcal{A}_l\right\}\right],$$
$$\sum_{l=0}^{\infty}\mathbb{G}_{l+1}(\vartheta,\chi) = -3c\sqrt{2}\tanh\left(\frac{\vartheta}{2}\right) + \mathbf{N}^{-1}\left[\frac{\tau^\alpha(\zeta^\alpha + \alpha(\tau^\alpha - \zeta^\alpha))}{\zeta^{2\alpha}}\mathbf{N}\left\{2\mathbb{G}_{3\vartheta} - \sum_{l=0}^{\infty}\mathcal{B}_l\right\}\right]. \tag{5.30}$$

By comparing both sides of Eq (5.30), we have that

$$\mathbb{F}_0(\vartheta,\chi) = 3 - 6\tanh^2\left(\frac{\vartheta}{2}\right),$$
$$\mathbb{G}_0(\vartheta,\chi) = -3c\sqrt{2}\tanh\left(\frac{\vartheta}{2}\right),$$

$$\begin{aligned}\mathbb{F}_1(\vartheta,\chi) &= 6\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\tanh\left(\frac{\vartheta}{2}\right)\left(1-\alpha+\frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right),\\ \mathbb{G}_1(\vartheta,\chi) &= 3c\sqrt{2}\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\tanh\left(\frac{\vartheta}{2}\right)\left(1-\alpha+\frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right),\end{aligned} \tag{5.31}$$

$$\begin{aligned}&\mathbb{F}_2(\vartheta,\chi) = 3[2+7\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)-15\,\mathrm{sech}^4\left(\frac{\vartheta}{2}\right)]\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right],\\ &\mathbb{G}_2(\vartheta,\chi)\\ &=\frac{3c\sqrt{2}}{2}\left[2+21\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)-24\,\mathrm{sech}^4\left(\frac{\vartheta}{2}\right)\right]\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right].\end{aligned} \tag{5.32}$$

In this way, given ($l \geq 3$), the remaining terms for $\mathbb{F}_l$ and $\mathbb{G}_l$ are easy to get. Thus, the series form solutions are given as

$$\begin{aligned}&\mathbb{F}(\vartheta,\chi) = \sum_{l=0}^{\infty}\mathbb{F}_l(\vartheta,\chi) = \mathbb{F}_0(\vartheta,\chi)+\mathbb{F}_1(\vartheta,\chi)+\mathbb{F}_2(\vartheta,\chi)+\cdots,\\ &\mathbb{F}(\vartheta,\chi) = 3-6\tanh^2\left(\frac{\vartheta}{2}\right)+6\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\tanh\left(\frac{\vartheta}{2}\right)\left(1-\alpha+\frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right)+3[2+7\\ &\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)-15\,\mathrm{sech}^4\left(\frac{\vartheta}{2}\right)]\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right]+\cdots,\\ &\mathbb{G}(\vartheta,\chi) = \sum_{l=0}^{\infty}\mathbb{G}_l(\vartheta,\chi) = \mathbb{G}_0(\vartheta,\chi)+\mathbb{G}_1(\vartheta,\chi)+\mathbb{G}_2(\vartheta,\chi)+\cdots,\\ &\mathbb{G}(\vartheta,\chi) = -3c\sqrt{2}\tanh\left(\frac{\vartheta}{2}\right)+3c\sqrt{2}\,\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\tanh\left(\frac{\vartheta}{2}\right)\left(1-\alpha+\frac{\alpha\chi^\alpha}{\Gamma(\alpha+1)}\right)+\frac{3c\sqrt{2}}{2}[2+21\\ &\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)-24\,\mathrm{sech}^4\left(\frac{\vartheta}{2}\right)\Big]\mathrm{sech}^2\left(\frac{\vartheta}{2}\right)\left[\frac{\alpha^2\chi^{2\alpha}}{\Gamma(2\alpha+1)}+2\alpha(1-\alpha)\frac{\chi^\alpha}{\Gamma(\alpha+1)}+(1-\alpha)^2\right]+\cdots.\end{aligned} \tag{5.33}$$

When $\alpha = 1$, we get the exact solution as

$$\begin{aligned}\mathbb{F}(\vartheta,\chi) &= 3-6\tanh^2\left(\frac{\vartheta+\chi}{2}\right),\\ \mathbb{G}(\vartheta,\chi) &= -3c\sqrt{2}\tanh\left(\frac{\vartheta+\chi}{2}\right).\end{aligned} \tag{5.34}$$

6. Results and discussion

In Figure 1, the actual and suggested methods solutions of $\mathbb{F}(\vartheta,\chi)$ are calculated at $\alpha = 1$. Figure 2 gives the graphical layouts of $\mathbb{F}(\vartheta,\chi)$ when $\alpha = 0.8$ and 0.6. In Figure 3, the 2D and 3D behaviors of $\mathbb{F}(\vartheta,\chi)$ for different fractional orders are respectively given. In Figure 4, the actual and suggested methods solutions of $\mathbb{G}(\vartheta,\chi)$ are calculated at $\alpha = 1$. Figure 5 gives the graphical layouts of $\mathbb{G}(\vartheta,\chi)$ when $\alpha = 0.8$ and 0.6. In Figure 6, the 2D and 3D behaviors of $\mathbb{G}(\vartheta,\chi)$ for different fractional orders are respectively given. The graphical layout of Example 5.1 confirms that our solutions are in a strong agreement with the exact solution. Similarly, in Figures 7, the actual and suggested methods solutions of $\mathbb{F}(\vartheta,\chi)$ are respectively presented for $\alpha = 1$. Figures 8 gives the graphical layouts of $\mathbb{F}(\vartheta,\chi)$ when $\alpha = 0.8$ and 0.6, while Figures 9 shows the 2D and 3D behaviors of $\mathbb{F}(\vartheta,\chi)$ for different fractional orders, respectively. Also, in Figures 10, the actual and suggested methods solutions of $\mathbb{G}(\vartheta,\chi)$ are respectively presented for $\alpha = 1$. Figures 11 gives the graphical layouts of $\mathbb{G}(\vartheta,\chi)$ when $\alpha = 0.8$ and 0.6, while Figures 12 shows the 2D and 3D behaviors of $\mathbb{G}(\vartheta,\chi)$ for different fractional orders, respectively. In the same way, the graphical layout of Example 5.2 confirms that our solutions are in a strong agreement with the exact solution. All the figures have been drawn at c=0.1 and $\chi = 0.01$ within the domain $-5 \leq \vartheta \geq 5$. Furthermore, Tables 1 and 2 shows the approximate solution of Example 5.1 at different values of ϑ and χ. Tables 3 and 4 show a numerical comparison of the reduced differential transform method and proposed method in terms of the absolute error for Example 5.1. Finally, Tables 5 and 6 show the approximate solution of Example 5.2 at different values of ϑ and χ. From the figures and tables, it can be observed that our methods solution and the exact solution are very close to each other and possess a higher degree of accuracy.

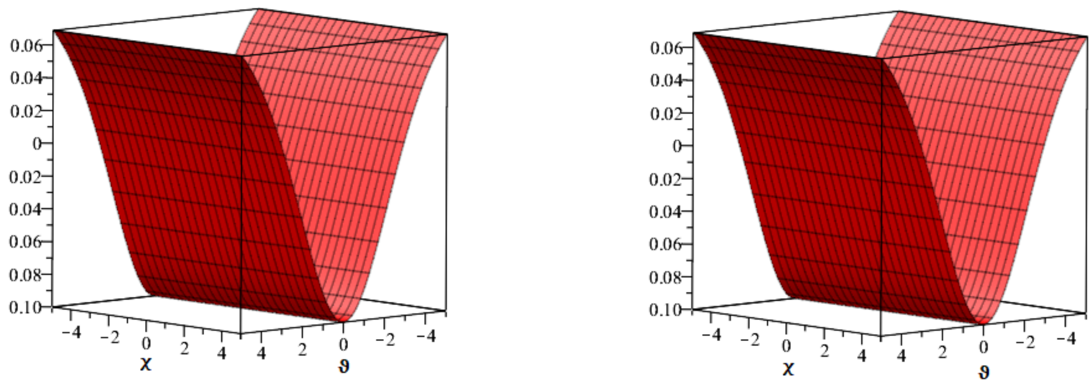


Figure 1. Exact and proposed method solution at $\alpha = 1$ for Example 5.1.

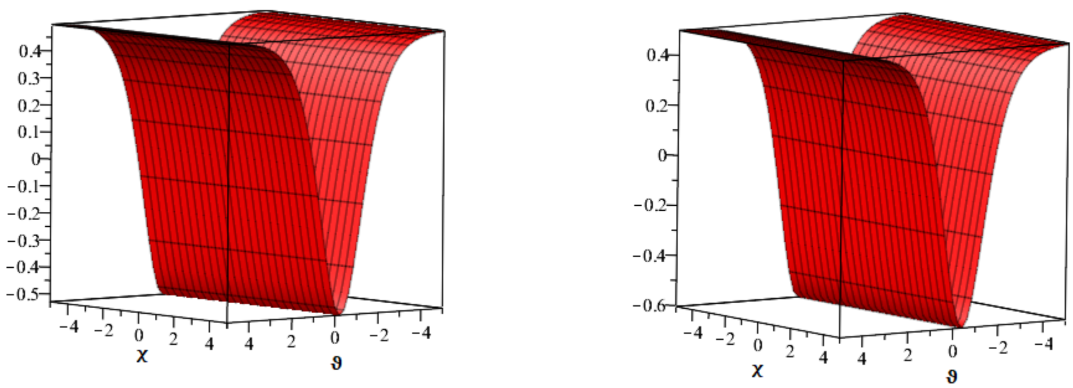


Figure 2. Proposed method solution at $\alpha = 0.8, 0.6$ for Example 5.1.

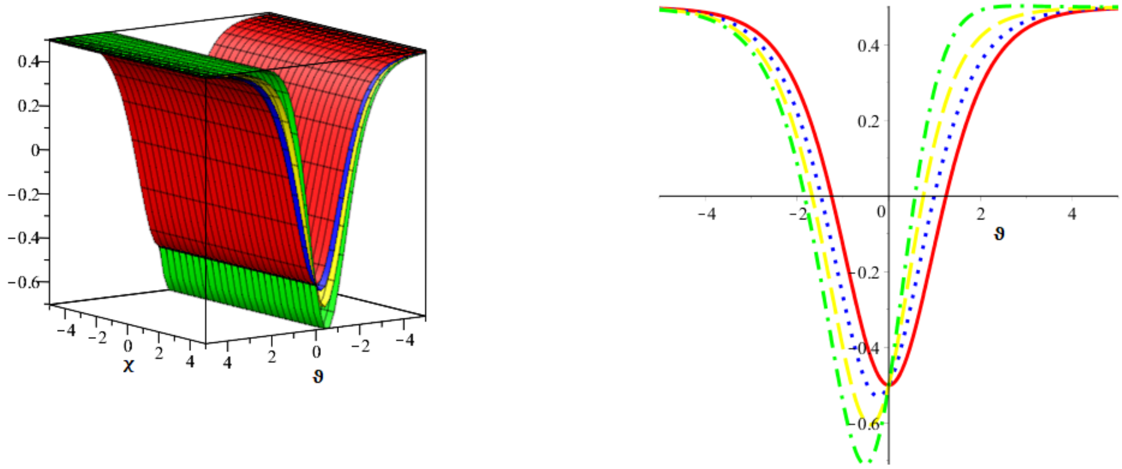


Figure 3. Proposed method solution at different values of α for Example 5.1.

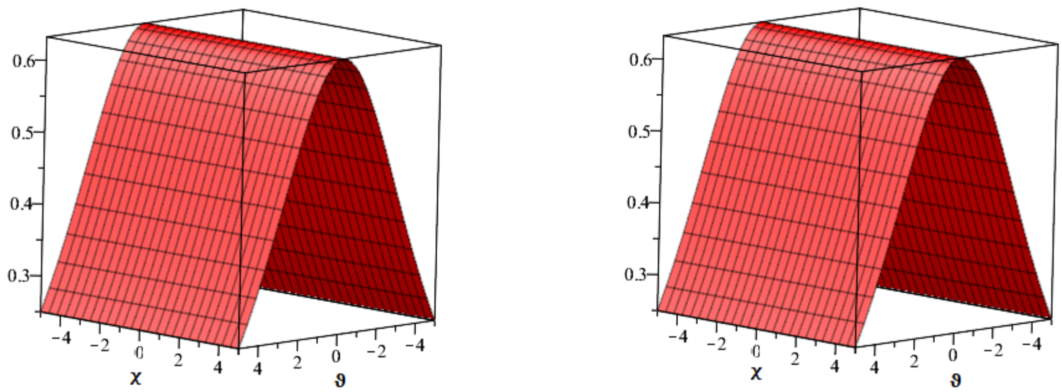


Figure 4. Exact and proposed method solution at $\alpha = 1$ for Example 5.1.

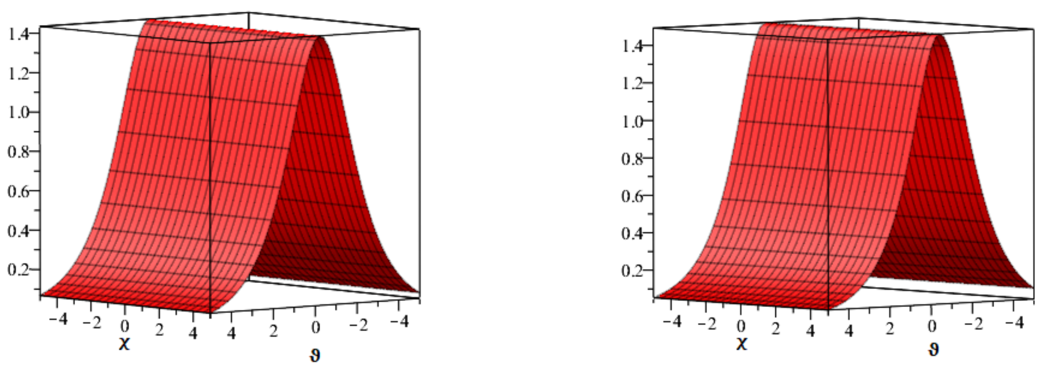


Figure 5. Proposed method solution at $\alpha = 0.8, 0.6$ for Example 5.1.

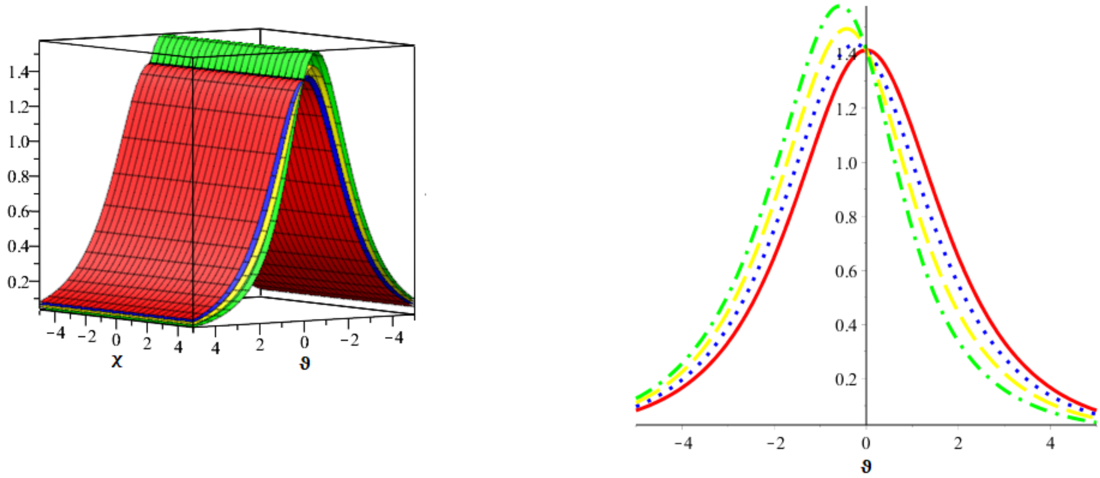


Figure 6. Proposed method solution at different values of α for Example 5.1.

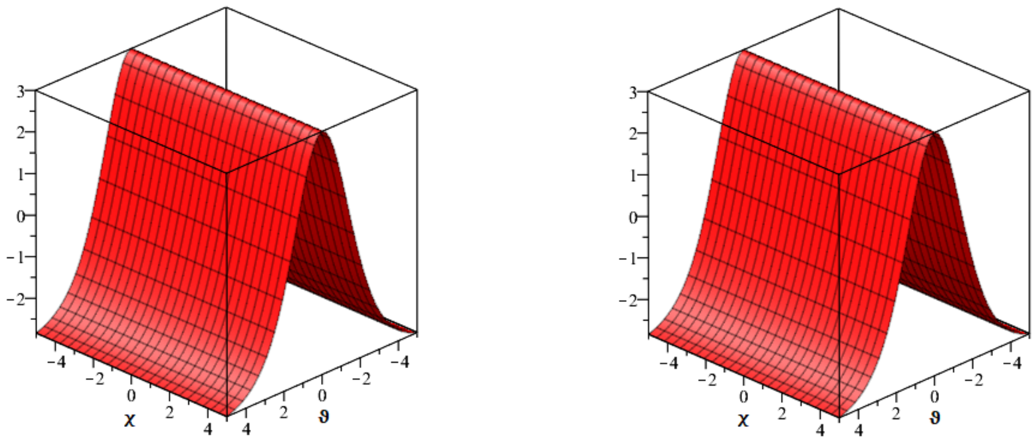


Figure 7. Exact and proposed method solution at $\alpha = 1$ for Example 5.2.

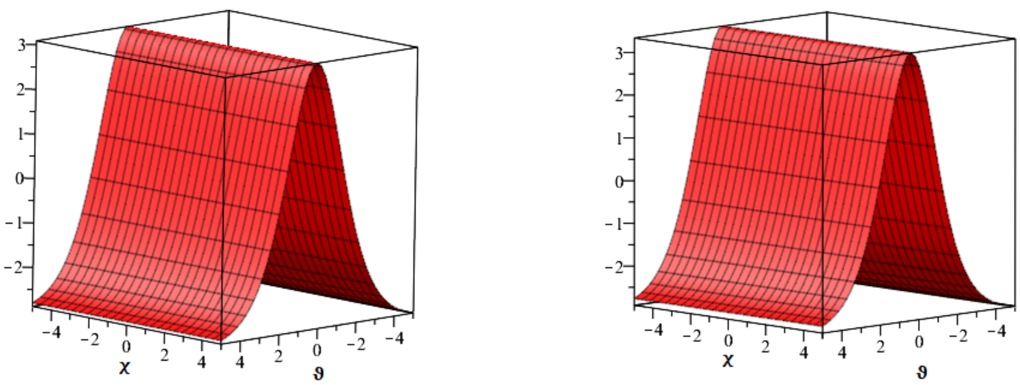


Figure 8. Proposed method solution at $\alpha = 0.8, 0.6$ for Example 5.2.

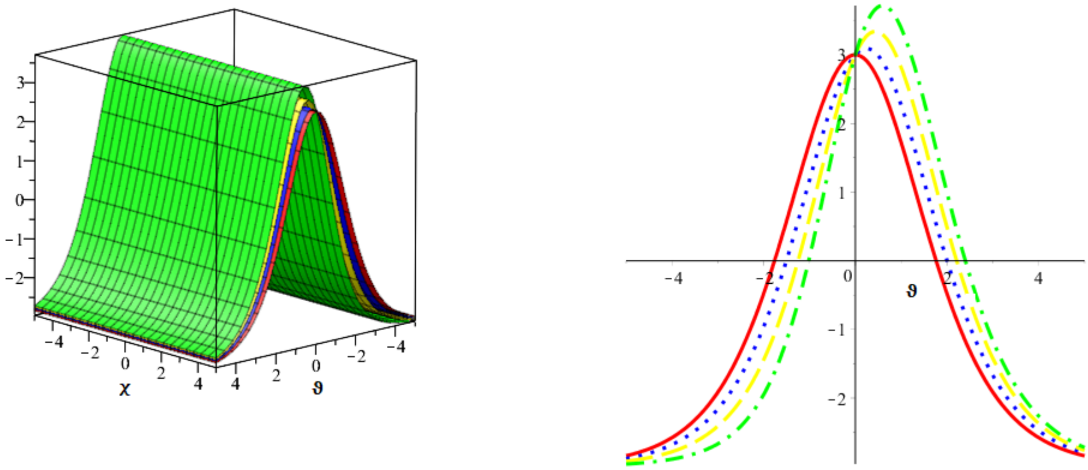


Figure 9. Proposed method solution at different values of α for Example 5.2.

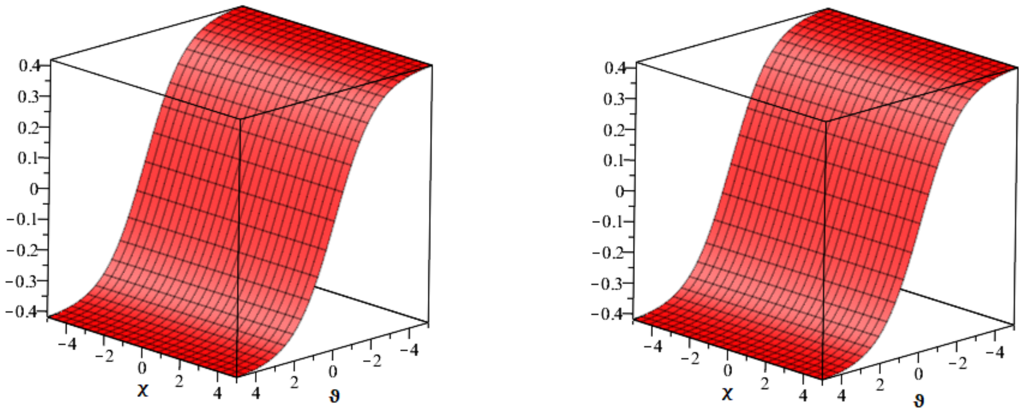


Figure 10. Exact and proposed method solution at $\alpha = 1$ for Example 5.2.

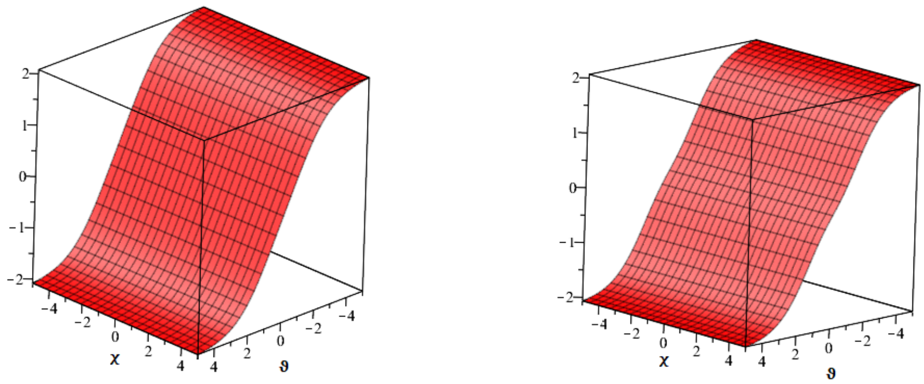
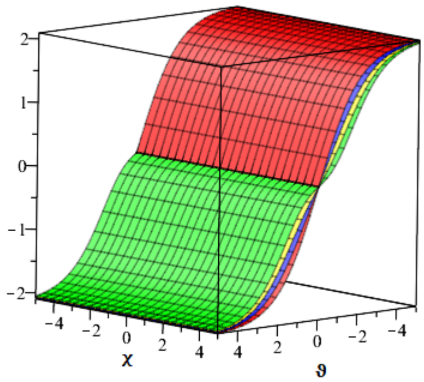


Figure 11. Proposed method solution at $\alpha = 0.8, 0.6$ for Example 5.2.



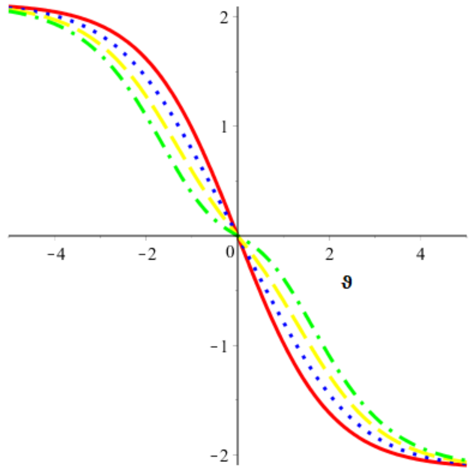


Figure 12. Proposed method solution at different values of α for Example 5.2.

Table 1. For Example 5.1, the suggested technique solution for $\mathbb{F}(\vartheta, \chi)$ given c=0.1 at various fractional orders.

(ϑ,χ)	$(NTDM_{ABC})$ at $\alpha = 0.5$	$(NTDM_{CF})$ at $\alpha = 0.75$	$(NTDM_{ABC})$ at $\alpha = 1$	$(NTDM_{CF})$ at $\alpha = 1$	Exact result
(0.2,0.01)	-0.099152	-0.099184	-0.099200	-0.099200	-0.099200
(0.4,0.02)	-0.096736	-0.096799	-0.096830	-0.096830	-0.096830
(0.6,0.03)	-0.092826	-0.092918	-0.092964	-0.092964	-0.092964
(0.2,0.01)	-0.099150	-0.099182	-0.099198	-0.099198	-0.099198
(0.4,0.02)	-0.096732	-0.096795	-0.096827	-0.096827	-0.096827
(0.6,0.03)	-0.092821	-0.092913	-0.092960	-0.092960	-0.092960
(0.2,0.01)	-0.099148	-0.099181	-0.099197	-0.099197	-0.099197
(0.4,0.02)	-0.096728	-0.096792	-0.096824	-0.096824	-0.096824
(0.6,0.03)	-0.092815	-0.092909	-0.092955	-0.092955	-0.092955
(0.2,0.01)	-0.099147	-0.099179	-0.099195	-0.099195	-0.099195
(0.4,0.02)	-0.096725	-0.096789	-0.096821	-0.096821	-0.096821
(0.6,0.03)	-0.092810	-0.092904	-0.092951	-0.092951	-0.092951
(0.2,0.01)	-0.099145	-0.099177	-0.099194	-0.099194	-0.099194
(0.4,0.02)	-0.096721	-0.096786	-0.096818	-0.096818	-0.096818
(0.6,0.03)	-0.092805	-0.092899	-0.092946	-0.092946	-0.092946

Table 2. For Example 5.1, the suggested technique solution for $\mathbb{G}(\vartheta, \chi)$ given c=0.1 at various fractional orders.

(ϑ,χ)	$(NTDM_{ABC})$ at α =0.5	$(NTDM_{CF})$ at α =0.75	$(NTDM_{ABC})$ at $\alpha = 1$	$(NTDM_{CF})$ at $\alpha = 1$	Exact result
(0.2,0.01)	0.631114	0.631164	0.631190	0.631190	0.631190
(0.4,0.02)	0.627273	0.627374	0.627424	0.627424	0.627424
(0.6,0.03)	0.621009	0.621158	0.621232	0.621232	0.621232
(0.2,0.01)	0.631111	0.631162	0.631187	0.631187	0.631187
(0.4,0.02)	0.627267	0.627368	0.627419	0.627419	0.627419
(0.6,0.03)	0.621001	0.621150	0.621224	0.621224	0.621224
(0.2,0.01)	0.631108	0.631159	0.631185	0.631185	0.631185
(0.4,0.02)	0.627262	0.627363	0.627414	0.627414	0.627414
(0.6,0.03)	0.620992	0.621142	0.621217	0.621217	0.621217
(0.2,0.01)	0.631105	0.631156	0.631182	0.631182	0.631182
(0.4,0.02)	0.627256	0.627358	0.627409	0.627409	0.627409
(0.6,0.03)	0.620984	0.621135	0.621210	0.621210	0.621210
(0.2,0.01)	0.631102	0.631154	0.631180	0.631180	0.631180
(0.4,0.02)	0.627250	0.627353	0.627404	0.627404	0.627404
(0.6,0.03)	0.620976	0.621127	0.621202	0.621202	0.621202

Table 3. For Example 5.1, the suggested technique absolute error comparison with a reduced differential transform method (RDTM) for $\mathbb{F}(\vartheta, \chi)$ at c=1.

χ	ϑ	$\|RDTM\|$	$\|NTDM_{CF}\|$	$\|NTDM_{CF}\|$
	-2	9.00000000E-09	8.5920000000E-09	8.5920000000E-09
	-1	2.22440000E-06	1.3071200000E-08	1.3071200000E-08
0.05	0	7.51000000E-07	2.0000000000E-08	2.0000000000E-08
	1	2.29510000E-06	1.2990800000E-07	1.2990800000E-07
	2	2.77000000E-08	7.9080000000E-08	7.9080000000E-08
	-2	7.14000000E-08	3.5763000000E-09	3.5763000000E-09
	-1	6.89230000E-05	5.2446300000E-08	5.2446300000E-08
0.1	0	4.73670000E-05	7.9998000000E-07	7.9998000000E-07
	1	7.35887000E-05	5.1802700000E-08	5.1802700000E-08
	2	1.11602000E-06	3.0257000000E-07	3.0257000000E-07
	-2	3.75120000E-06	8.3575000000E-08	8.3575000000E-08
	-1	4.99186000E-04	1.1836350000E-07	1.1836350000E-07
0.15	0	5.26076000E-04	1.7998900000E-07	1.7998900000E-07
	1	5.56398000E-04	1.1619250000E-06	1.1619250000E-06
	2	9.98978000E-06	6.5015000000E-08	6.5015000000E-08

Table 4. For Example 5.1, the suggested technique absolute error comparison for $\mathbb{G}(\vartheta,\chi)$ at c=1.

χ	ϑ	$\|RDTM\|$	$\|NTDM_{CF}\|$	$\|NTDM_{CF}\|$
0.05	-2	1.1860000E-07	9.8320000000E-09	9.8320000000E-09
	-1	2.1200000E-07	2.4506000000E-08	2.4506000000E-08
	0	1.6800000E-07	3.1623000000E-08	3.1623000000E-08
	1	2.8200000E-07	2.4420000000E-09	2.4420000000E-09
	2	1.0710000E-07	9.7370000000E-08	9.7370000000E-08
0.1	-2	4.0069000E-06	3.9524000000E-08	3.9524000000E-08
	-1	5.3900000E-06	9.8199000000E-07	9.8199000000E-07
	0	1.0671000E-05	1.2648900000E-08	1.2648900000E-08
	1	9.8860000E-06	9.7508000000E-08	9.7508000000E-08
	2	3.2565000E-06	3.8752000000E-07	3.8752000000E-07
0.15	-2	3.2082600E-05	8.9365000000E-08	8.9365000000E-08
	-1	2.8970000E-05	2.2133100000E-07	2.2133100000E-07
	0	1.1917600E-04	2.8459400000E-07	2.8459400000E-07
	1	8.0549000E-05	2.1900500000E-07	2.1900500000E-07
	2	2.3525200E-05	8.6759000000E-06	8.6759000000E-06

Table 5. For Example 5.2, the suggested technique solution for $\mathbb{F}(\vartheta,\chi)$ given c=0.1 at various fractional orders.

(ϑ,χ)	$(NTDM_{ABC})$ at α =0.5	$(NTDM_{CF})$ at α =0.75	$(NTDM_{ABC})$ at $\alpha = 1$	$(NTDM_{CF})$ at $\alpha = 1$	Exact result
(0.2,0.01)	2.952239	2.946318	2.940397	2.940397	2.940397
(0.4,0.01)	2.789020	2.777639	2.766257	2.766257	2.766257
(0.6,0.01)	2.522812	2.506817	2.490821	2.490821	2.490821
(0.2,0.02)	2.952239	2.946318	2.940397	2.940397	2.940397
(0.4,0.02)	2.789020	2.777639	2.766257	2.766257	2.766257
(0.6,0.02)	2.522812	2.506817	2.490821	2.490821	2.490821
(0.2,0.03)	2.952239	2.946318	2.940397	2.940397	2.940397
(0.4,0.03)	2.789020	2.777639	2.766257	2.766257	2.766257
(0.6,0.03)	2.522812	2.506817	2.490821	2.490821	2.490821
(0.2,0.04)	2.952239	2.946318	2.940397	2.940397	2.940397
(0.4,0.04)	2.789020	2.777639	2.766257	2.766257	2.766257
(0.6,0.04)	2.522812	2.506817	2.490821	2.490821	2.490821
(0.2,0.05)	2.952239	2.946318	2.940397	2.940397	2.940397
(0.4,0.05)	2.789020	2.777639	2.766257	2.766257	2.766257
(0.6,0.05)	2.522812	2.506817	2.490821	2.490821	2.490821

Table 6. For Example 5.2, the suggested technique solution for $\mathbb{G}(\vartheta, \chi)$ given c=0.1 at various fractional orders.

(ϑ,χ)	$(NTDM_{ABC})$ at α =0.5	$(NTDM_{CF})$ at α =0.75	$(NTDM_{ABC})$ at $\alpha = 1$	$(NTDM_{CF})$ at $\alpha = 1$	Exact result
(0.2,0.01)	-0.000410	-0.000418	-0.000422	-0.000422	-0.000422
(0.4,0.01)	-0.000813	-0.000829	-0.000837	-0.000837	-0.000837
(0.6,0.01)	-0.001202	-0.001224	-0.001235	-0.001235	-0.001235
(0.2,0.02)	-0.000410	-0.000418	-0.000422	-0.000422	-0.000422
(0.4,0.02)	-0.000813	-0.000829	-0.000837	-0.000837	-0.000837
(0.6,0.02)	-0.001201	-0.001224	-0.001235	-0.001235	-0.001235
(0.2,0.03)	-0.000410	-0.000418	-0.000422	-0.000422	-0.000422
(0.4,0.03)	-0.000813	-0.000829	-0.000837	-0.000837	-0.000837
(0.6,0.03)	-0.001201	-0.001224	-0.001235	-0.001235	-0.001235
(0.2,0.04)	-0.000410	-0.000418	-0.000422	-0.000422	-0.000422
(0.4,0.04)	-0.000813	-0.000829	-0.000837	-0.000837	-0.000837
(0.6,0.04)	-0.001201	-0.001224	-0.001235	-0.001235	-0.001235
(0.2,0.05)	-0.000410	-0.000418	-0.000422	-0.000422	-0.000422
(0.4,0.05)	-0.000813	-0.000829	-0.000837	-0.000837	-0.000837
(0.6,0.05)	-0.001201	-0.001224	-0.001235	-0.001235	-0.001235

7. Conclusions

The main goal of this work as to develop a fractional-order Kersten-Krasil' shchik coupled KdV-mKdV nonlinear system approximate analytical solution. Using the NDM, we were able to achieve this goal. The numerical solutions can be achieved in two steps. The targeted problems were first simplified using the NT; then, the decomposition approach was employed to get the solutions. The method's fundamental benefit is that it gives the user an analytical approximation, and in many cases an exact solution, in a quickly converging sequence with elegantly computed terms. The suggested method has been applied to obtain the solution of the given two problems. The method's small computational size in comparison to the computational size required by other numerical methods, as well as its rapid convergence, demonstrate that it is reliable and a significant improvement on existing methods in terms of solving the generalized Kersten-Krasil' shchik coupled KdV-mKdV equation. The results we obtained have been illustrated with the help of plots and tables which confirm the validity of the proposed method. Furthermore, the proposed method is simple, straightforward, and requires minimal computational time; it may also be extended to solve other fractional-order partial differential equations.

Acknowledgements

This research received funding support from the NSRF via the Program Management Unit for Human Resources & Institutional Development, Research and Innovation.

Conflict of interest

The authors declare that they have no competing interests.

References

1. M. Caputo, Linear models of dissipation whose Q is almost frequency independent–II, *Geophys. J. Int.*, **13** (1967), 529–539. https://doi.org/10.1111/j.1365-246X.1967.tb02303.x

2. K. B. Oldham, J. Spanier, *The fractional calculus: Theory and applications of differentiation and integration to arbitrary order*, Academic Press, 1974.

3. R. Gorenflo, F. Mainardi, Fractional calculus, In: *Fractals and fractional calculus in continuum mechanics*, Vienna: Springer, 1997, 223–276. https://doi.org/10.1007/978-3-7091-2664-6_5

4. S. G. Samko, A. A. Kilbas, O. I. Marichev, *Fractional integrals and derivatives*, Gordon and Breach Science Publishers, 1993.

5. E. Ahmed, A. M. A. El-Sayed, H. A. A. El-Saka, Equilibrium points, stability and numerical solutions of fractional-order predator–prey and rabies models, *J. Math. Anal. Appl.*, **325** (2007), 542–553. https://doi.org/10.1016/j.jmaa.2006.01.087

6. I. Podlubny, *Fractional differential equations*, Mathematics in Science and Engineering, Vol. 198, Academic Press, 1999.

7. A. Atangana, E. Alabaraoye, Solving a system of fractional partial differential equations arising in the model of HIV infection of $CD4^{+}$ cells and attractor one-dimensional Keller-Segel equations, *Adv. Differ. Equ.*, **2013** (2013), 1–14. https://doi.org/10.1186/1687-1847-2013-94

8. S. Kumar, H. Kocak, A. Yıldırım, A fractional model of gas dynamics equations and its analytical approximate solution using Laplace transform, *Z. Naturforschung A*, **67** (2012), 389–396. https://doi.org/10.5560/zna.2012-0038

9. F. Yin, J. Q. Song, X. Q. Cao, A general iteration formula of VIM for fractional heat- and wave-like equations, *J. Appl. Math.*, **2013** (2013), 1–9. https://doi.org/10.1155/2013/428079

10. A. S. Arife, S. K. Vanani, F. Soleymani, The Laplace homotopy analysis method for solving a general fractional diffusion equation arising in nano-hydrodynamics, *J. Comput. Theor. Nanos.*, **10** (2013), 33–36.

11. H. Beyer, S. Kempfle, Definition of physically consistent damping laws with fractional derivatives, *ZAMM Z. Angew. Math. Mech.*, **75** (1995), 623–635. https://doi.org/10.1002/zamm.19950750820

12. M. K. Alaoui, R. Fayyaz, A. Khan, R. Shah, M. S. Abdo, Analytical investigation of Noyes-Field model for time-fractional Belousov-Zhabotinsky reaction, *Complexity*, **2021** (2021), 1–21. https://doi.org/10.1155/2021/3248376

13. K. Nonlaopon, A. M. Alsharif, A. M. Zidan, A. Khan, Y. S. Hamed, R. Shah, Numerical investigation of fractional-order Swift-Hohenberg equations via a Novel transform, *Symmetry*, **13** (2021), 1263. https://doi.org/10.3390/sym13071263

14. Hajira, H. Khan, A. Khan, P. Kumam, D. Baleanu, M. Arif, An approximate analytical solution of the Navier-Stokes equations within Caputo operator and Elzaki transform decomposition method, *Adv. Differ. Equ.*, **2020** (2020), 1–23. https://doi.org/10.1186/s13662-020-03058-1

15. K. S. Miller, B. Ross, *An introduction to the fractional calculus and fractional differential equations*, Wiley, 1993.
16. N. A. Shah, E. R. El-Zahar, A. Akgül, A. Khan, J. Kafle, Analysis of fractional-order regularized long-wave models via a Novel transform, *J. Funct. Space.*, **2022** (2022), 1–16. https://doi.org/10.1155/2022/2754507
17. A. A. Alderremy, S. Aly, R. Fayyaz, A. Khan, R. Shah, N. Wyal, The analysis of fractional-order nonlinear systems of third order KdV and Burgers equations via a Novel transform, *Complexity*, **2022** (2022), 1–24. https://doi.org/10.1155/2022/4935809
18. A. Khan, U. Nigar, H. Chaudhary, Secure communication and synchronization dynamics in chaotic Chua's system via adaptive sliding mode control technique, *Int. J. Appl. Comput. Math.*, **8** (2022), 1–20. https://doi.org/10.1007/s40819-022-01378-2
19. M. Türkyilmazoğlu, Nonlinear problems via a convergence accelerated decomposition method of Adomian, 2021.
20. M. Naeem, A. M. Zidan, K. Nonlaopon, M. I. Syam, Z. Al-Zhour, R. Shah, A new analysis of fractional-order equal-width equations via novel techniques, *Symmetry*, **13** (2021), 1–20. https://doi.org/10.3390/sym13050886
21. N. Iqbal, A. Akgül, R. Shah, A. Bariq, M. M. Al-Sawalha, A. Ali, On solutions of fractional-order gas dynamics equation by effective techniques, *J. Funct. Space.*, **2022** (2022), 1–14. https://doi.org/10.1155/2022/3341754
22. P. Sunthrayuth, A. M. Zidan, S. W. Yao, R. Shah, M. Inc, The comparative study for solving fractional-order Fornberg-Whitham equation via ρ-Laplace transform, *Symmetry*, **13** (2021), 1–15. https://doi.org/10.3390/sym13050784
23. M. Alesemi, N. Iqbal, T. Botmart, Novel analysis of the fractional-order system of non-linear partial differential equations with the exponential-decay kernel, *Mathematics*, **10** (2022), 1–17. https://doi.org/10.3390/math10040615
24. A. F. Qasim, M. O. Al-Amr, Approximate solution of the Kersten-Krasil'shchik coupled Kdv-mKdV system via reduced differential transform method, *Eurasian J. Sci. Eng.*, **4** (2018), 1–9.
25. A. K. Kalkanli, S. Y. Sakovich, I. Yurdusen, Integrability of Kersten-Krasil'shchik coupled KdV-mKdV equations: singularity analysis and Lax pair, *J. Math. Phys.*, **44** (2003), 1703–1708. https://doi.org/10.1063/1.1558903
26. Y. C. Hon, E. G. Fan, Solitary wave and doubly periodic wave solutions for the Kersten-Krasil'shchik coupled KdV-mKdV system, *Chaos Solitons Fract.*, **19** (2004), 1141–1146. https://doi.org/10.1016/S0960-0779(03)00302-3
27. A. Goswami, Sushila, J. Singh, D. Kumar, Numerical computation of fractional Kersten-Krasil'shchik coupled KdV-mKdV system occurring in multi-component plasmas, *AIMS Math.*, **5** (2020), 2346–2368. https://doi.org/10.3934/math.2020155
28. W. J. Rui, X. M. Qi, Bilinear approach to quasi-periodic wave solutions of the Kersten-Krasil'shchik coupled KdV-mKdV system, *Bound. Value Probl.*, **2016** (2016), 1–13. https://doi.org/10.1186/s13661-016-0634-3

29. M. K. Alaoui, K. Nonlaopon, A. M. Zidan, A. Khan, R. Shah, Analytical investigation of fractional-order Cahn-Hilliard and Gardner equations using two novel techniques, *Mathematics*, **10** (2022), 1–19. https://doi.org/10.3390/math10101643

30. T. Botmart, R. P. Agarwal, M. Naeem, A. Khan, R. Shah, On the solution of fractional modified Boussinesq and approximate long wave equations with non-singular kernel operators, *AIMS Math.*, **7** (2022), 12483–12513. https://doi.org/10.3934/math.2022693

31. M. X. Zhou, A. S. V. R. Kanth, K. Aruna, K. Raghavendar, H. Rezazadeh, M. Inc, et al., Numerical solutions of time fractional Zakharov-Kuznetsov equation via natural transform decomposition method with nonsingular kernel derivatives, *J. Funct. Space.*, **2021** (2021), 1–17. https://doi.org/10.1155/2021/9884027

AIMS Press