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Research article

On the existence of almost periodic solutions of impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms

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Abstract: In this paper, by using the Mawhin's continuation theorem, some easily verifiable sufficient conditions are obtained to guarantee the existence of almost periodic solutions of impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms. Our result corrects the result obtained in [13]. An example and some remarks are given to illustrate the advantage of this paper.

Keywords: Mawhin's continuation theorem; almost periodic solution; impulsive differential equation; Lotka-Volterra system
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1. Introduction

Lotka-Volterra systems are very important mathematical models in the theory of mathematical biology, often deeply perturbed by activities of human exploitation, such as, crop-dusting, deforestation, hunting, harvesting. To accurately describe these systems, one need to use impulsive differential equations. Basic theory of impulsive differential equations can be found in monographs [1–3].

Over the years, much attention has been paid to the dynamical behaviors (such as, the permanence, extinction, global asymptotic behavior) of Lotka-Volterra systems with impulsive effects (see [4–6]). Thereinto, the existence of positive periodic solutions is important direction. Many important and interesting results can be found in [7–10] and references therein.

In paper [10], by applying the Mawhin's continuation theorem, the authors obtained the existence of periodic solutions of a non-autonomous Lotka-Volterra network-like predator-prey system with harvesting terms:

$$\begin{cases} N_i'(t) = N_i(t)[a_i(t) - b_i(t)N_i(t) - \sum_{r=1,r\neq i}^{n} c_{i,r}(t)N_r(t) - \sum_{j=1}^{m} d_{i,j}(t)N_{n+j}(t)] - h_i(t), \\ N_{n+j}'(t) = N_{n+j}(t)[\alpha_j(t) - \beta_j(t)N_{n+j}(t) - \sum_{l=1,l\neq j}^{m} \gamma_{l,j}(t)N_{n+l}(t) + \sum_{i=1}^{n} \delta_{i,j}(t)N_i(t)] - e_j(t). \end{cases} \tag{1.1}$$

The properties of almost periodic solutions of different classes of biological models have been extensively investigated during the years [11–15] since these states are more general that the pure periodicity and consistent with real world. In addition, as the generalization of periodicity, the interest in the investigation of almost periodic solutions of models is sometimes caused by the absence of periodic solutions [16].

In fact, the Mawhin's continuation theorem is one of the powerful and effective methods on the existence of periodic solutions to periodic systems (regardless of the systems with impulse or not). Concerning the almost periodic solutions, to our best knowledge, it is usually employed to prove the existence of almost periodic solutions for differential equations without impulse, such as [17, 18]. There are rarely articles applied this method to prove the existence of almost periodic solutions for impulsive differential equations, except [19, 20]. Li and Ye [19] considered the existence of almost periodic solutions of impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms:

$$\begin{cases} N_i'(t) = N_i(t)[a_i(t) - b_i(t)N_i(t) - \sum_{r=1,r\neq i}^{n} c_{i,r}(t)N_r(t - \tau_{i,r}(t)) \\ \qquad - \sum_{j=1}^{m} d_{i,j}(t)N_{n+j}(t - \sigma_{i,j}(t))] - h_i(t), t \neq t_k, i = 1, 2, ...n, \\ N_{n+j}'(t) = N_{n+j}(t)[\alpha_j(t) - \beta_j(t)N_{n+j}(t) - \sum_{l=1,l\neq j}^{m} \gamma_{l,j}(t)N_{n+l}(t - \theta_{l,j}(t)) \\ \qquad + \sum_{i=1}^{n} \delta_{i,j}(t)N_i(t - \nu_{i,j}(t))] - e_j(t), \quad t \neq t_k, j = 1, 2, ...m, \\ N_h(t_k^+) = (1 + \rho_{hk})N_h(t_k), \qquad h = 1, 2, ...n + m, k \in \mathbb{Z}^+. \end{cases} \tag{1.2}$$

where, $i = 1, 2, ...n$, $j = 1, 2, ...m$, $a_i(\cdot)$, $b_i(\cdot)$, and $h_i(\cdot)$ are the ith prey species birth rate, death rate and harvesting rate, respectively; $\alpha_j(\cdot)$, $\beta_j(\cdot)$ and $e_j(\cdot)$ stand for the jth predator species birth rate, death rate and harvesting rate, respectively; $c_{i,r}(\cdot)(i \neq r)$ represent the competition rate between the ith prey species and the rth prey species; $d_{i,j}(\cdot)$ represent the jth predator species predation rate on the ith prey species; $\gamma_{l,j}(\cdot)(l \neq j)$ stand for the competition rate between the lth predator species and the jth predator species; $\delta_{i,j}(\cdot)$ stand for the transformation rate between the ith prey species and the jth predator species; $\tau_{i,r}(\cdot), \sigma_{i,j}(\cdot), \theta_{l,j}(\cdot), \nu_{i,j}(\cdot)$ are the time delays; ρ_{hk} are the impulsive oscillations.

In order to obtain the existence of almost periodic solutions of Eq (1.2), Li and Ye [19] transformed Eq (1.2) into Eq (3.1) in [19].

By employing Lemmas 2.3 and 2.4 in [19] and the Mawhin's continuation theorem in coincidence degree theory, the existence of almost periodic solutions of Eq (3.1) in [19] was obtained. Based on this, the existence of almost periodic solutions of Eq (1.2) was obtained (Theorem 3.1 in [19]). In the proving process, Li and Ye acquiesced that the almost periodic solutions of Eq (3.1) in [19] belonged to C^1, which is also an important and necessary condition in order to use Lemmas 2.3 and 2.4. However, obviously, the solutions of Eq (3.1) in [19] don't belong to C^1.

Motivated by these, we still use the Mawhin's continuation theorem to investigate the existence of almost periodic solutions of Eq (1.2). The main contribution of this paper is: (1) The result of this paper corrects and generalizes the previous results in [10, 19]; (2) The method used in this paper can be applied to study the existence of almost periodic solutions of impulsive differential equations with linear impulsive perturbations.

The remaining parts of this paper are organized as follows: The next section presents some preliminaries; In section 3, by employing the Mawhin's continuation theorem of coincidence degree theory, a criterion is established for the existence of almost periodic solutions of system (1.2). Finally, an example and its corresponding numerical simulation are presented to explain our theoretical result.

2. Preliminaries

Some preliminaries will be presented in this section in order to prove our main result in Section 3.

Definition 2.1. *([21]) $\varphi(\cdot) \in C(\mathbb{R}, \mathbb{R}^n)$ is said to be almost periodic in sense of Bohr if $\forall \varepsilon > 0$, there exists a relatively dense set $T(\varphi, \varepsilon)$ such that if $\tau \in T(\varphi, \varepsilon)$, then $|\varphi(t) - \varphi(t + \tau)| < \varepsilon$ for all $t \in \mathbb{R}$. Denote by $ap(\mathbb{R}, \mathbb{R}^n)$ all such functions.*

Definition 2.2. *([3]) Let $\phi(\cdot) = (\phi_1(\cdot), \phi_2(\cdot), ...\phi_n(\cdot))$ be a piecewise continuous function with first kind discontinuities at the points of a fixed sequence $\{t_k\}$, we call ϕ almost periodic if:*

1) $\{t_k\}$ is equipotentially almost periodic, that is, $\forall \varepsilon > 0$ there exists a relatively dense set of ε-almost periodic common for any sequences $\{t_k^j\}, t_k^j = t_{k+j} - t_k$;

2) $\forall \varepsilon > 0$, $\exists \delta > 0$ such that if the points t', t'' belong to the same interval of continuity and $|t' - t''| < \delta$, then $|\phi_i(t') - \phi_i(t'')| < \varepsilon$, i=1,2,...n;

3) $\forall \varepsilon > 0$, there exists a relatively dense set $T(\phi_i, \varepsilon)$ such that if $\tau \in T(\phi_i, \varepsilon)$, then $|\phi_i(t) - \phi_i(t+\tau)| < \varepsilon$ for all $t \in \mathbb{R}$ which satisfy the condition $|t - t_k| > \varepsilon, i = 1, 2, ...n, k = 0, \pm 1, \pm 2, ...$

Denote by $AP(\mathbb{R}, \mathbb{R}^n)$ all such functions.

Now we introduce some basic notations. Let g be continuous or piecewise continuous function, we denote $g^M = \sup_{t\in\mathbb{R}} |g(t)|$, $g^L = \inf_{t\in\mathbb{R}} |g(t)|$. Suppose $f \in ap(\mathbb{R}, \mathbb{R}^n)$ or $f \in AP(\mathbb{R}, \mathbb{R}^n)$, $a(\lambda, f)$, $\Lambda_f = \{\lambda, a(\lambda, f) \neq 0\}$ and $m(f)$ denote the Fourier coefficient, Fourier exponent set and the mean value of f, respectively. $mod(f)$ means the module of f.

Further theory of almost periodic functions can be found in the literatures [22–28]. Besides,The following lemmas are important for our result.

Lemma 2.3. *([20] Favard's theorem of AP function) Suppose $f \in AP$, there exist $\alpha_1 > \alpha > 0$ such that $\forall \lambda \in \Lambda_f$, $\alpha_1 > |\lambda| > \alpha$, $\Sigma_{i=1}^{\infty} |a(\lambda_i, f)| < +\infty$, then the primitive function of f is almost periodic function in sense of Bohr.*

Lemma 2.4. *([22]) Suppose $f, g \in ap$, the following two conditions are equivalent:*

1) $mod(f) \subset mod(g)$;

2) For any $\varepsilon > 0$, there exist $\delta > 0$ such that $T(g, \delta) \subset T(f, \varepsilon)$.

Lemma 2.5. *([23]) The necessary and sufficient condition that a family $\mathcal{F}$ of functions from ap be relatively compact is that the following properties hold true:*

1) $\mathcal{F}$ is equi-continuous;

2) $\mathcal{F}$ is equi-almost periodic;

3) For any $t \in \mathbb{R}$, the set of values of functions from $\mathcal{F}$ be relatively compact.

Let X and Z be real Banach spaces, $L : domL \subset X \to Z$ be a linear mapping, $N : X \to Z$ be a continuous mapping. L is called a Fredholm mapping of index zero if $dimKerL = codimImL < \infty$ and ImL is close in Z. If L is a Fredholm mapping of index zero, there are continuous projects $P : X \to X, Q : Z \to Z$ such that $ImP = KerL$, $ImL = KerQ = Im(I - Q)$. It follows that $L|domL \cap KerP : (I - P)X \to ImL$ is invertible. We denote the inverse of that map by K_p. If Ω is a open subset of X, the mapping N on $\Omega \times [0, 1]$ will be called $L-$compact on Ω if $QN(\bar{\Omega} \times [0, 1])$ is bounded and $K_p(I - Q)N : \bar{\Omega} \times [0, 1] \to X$ is compact. Since ImQ is isomorphic to $KerL$, there exist isomorphisms $J : ImQ \to KerL$. The following result is proved in [29].

Lemma 2.6. *(Mawhins continuation theorem) Let $\Omega \subset X$ be an open bounded set, L be a Fredholm mapping of index zero and N be $L-$compact on $\bar{\Omega} \times [0, 1]$. Assume*

1) For each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda N(x, \lambda)$ is such that $x \notin \partial\Omega$;

2) For each $x \in KerL \cap \partial\Omega, QN(x, 0) \neq 0$;

3) $deg(JQN(x, 0), KerL \cap \Omega, 0) \neq 0$.

Then $Lx = N(x, 1)$ has at least one solution in $domL \cap \bar{\Omega}$.

3. Main results

In this section, by means of the Mawhin's continuation theorem of coincidence degree theory, we investigate the existence of almost periodic solutions of Eq (1.2). To do so, we firstly consider the following equation:

$$\begin{cases} y_i'(t) = y_i(t)[a_i(t) - \bar{b}_i(t)y_i(t) - \sum_{r=1,r\neq i}^{n} \bar{c}_{i,r}(t)y_r(t - \tau_{i,r}(t)) \\ \qquad\qquad - \sum_{j=1}^{m} \bar{d}_{i,j}(t)y_{n+j}(t - \sigma_{i,j}(t))] - \bar{h}_i(t), i = 1, 2, ...n, \\ y_{n+j}'(t) = y_{n+j}(t)[\alpha_j(t) - \bar{\beta}_j(t)y_{n+j}(t) - \sum_{l=1,l\neq j}^{m} \bar{\gamma}_{l,j}(t)y_{n+l}(t - \theta_{l,j}(t)) \\ \qquad\qquad + \sum_{i=1}^{n} \bar{\delta}_{i,j}(t)y_i(t - \nu_{i,j}(t))] - \bar{e}_j(t), \quad j = 1, 2, ...m, \end{cases} \tag{3.1}$$

where

$$\bar{b}_i(t) = b_i(t) \prod_{0<t_k<t}(1 + \rho_{ik}), \quad \bar{h}_i(t) = h_i(t) \prod_{0<t_k<t}(1 + \rho_{ik})^{-1}, i, r = 1, 2, ..n,$$
$$\bar{c}_{i,r}(t) = c_{i,r}(t) \prod_{0<t_k<t-\tau_{i,r}(t)}(1 + \rho_{rk}), \quad \bar{d}_{i,j}(t) = d_{i,j}(t) \prod_{0<t_k<t-\sigma_{i,j}(t)}(1 + \rho_{n+j,k}), r \neq i;$$
$$\bar{\beta}_j(t) = \beta_j(t) \prod_{0<t_k<t}(1 + \rho_{n+j,k}), \quad \bar{e}_j(t) = e_j(t) \prod_{0<t_k<t}(1 + \rho_{n+j,k})^{-1}, l, j = 1, 2, ...m,$$
$$\bar{\gamma}_{l,j}(t) = \gamma_{l,j}(t) \prod_{0<t_k<t-\theta_{l,j}(t)}(1 + \rho_{n+l,k}), \quad \bar{\delta}_{i,j}(t) = \delta_{i,j}(t) \prod_{0<t_k<t-\nu_{i,j}(t)}(1 + \rho_{ik}), l \neq j.$$

Remark 3.1. In paper [19], the authors define $\bar{c}_{i,r}(t), \bar{d}_{i,j}(t), \bar{\gamma}_{l,j}(t)$ and $\bar{\delta}_{i,j}(t)$ in other forms. We think those should be the forms above. Throughout this paper, we suppose the following hypotheses hold:

$(H1)$ $a_i(\cdot)$, $b_i(\cdot)$, $c_{i,r}(\cdot)$, $\tau_{i,r}(\cdot)$, $d_{i,j}(\cdot), \sigma_{i,j}(\cdot), h_i(\cdot)$, $\alpha_j(\cdot), \beta_j(\cdot), \gamma_{l,j}(\cdot), \theta_{l,j}(\cdot)\delta_{i,j}(\cdot), \nu_{i,j}(\cdot)$ and $e_j(\cdot)$ are all positive almost periodic functions in sense of Bohr, $i, r = 1, 2, ...n, l, j, = 1, 2, ...m$. $\{\rho_{hk}\}$ are almost periodic sequences, $h = 1, 2, ..., n + m$, $\{t_k\}$is an equipotentially almost periodic sequence.

$(H2)$ $\prod_{0<t_k<t}(1+\rho_{ik})$, $\prod_{0<t_k<t}(1+\rho_{n+j,k})$, $\prod_{0<t_k<t-\tau_{i,r}(t)}(1+\rho_{rk})$, $\prod_{0<t_k<t-\sigma_{i,j}(t)}(1+\rho_{n+j,k})$, $\prod_{0<t_k<t-\theta_{l,j}(t)}(1+\rho_{n+l,k})$, and $\prod_{0<t_k<t-\nu_{i,j}(t)}(1 + \rho_{ik})$ are all positive almost periodic functions, $\inf_{t\in\mathbb{R}} \prod_{0<t_k<t} |(1 + \rho_{i,k})| > 0, \inf_{t\in\mathbb{R}} \prod_{0<t_k<t} |(1 + \rho_{n+j,k})| > 0$, $i, r = 1, 2, ..n, l, j = 1, 2, ...m$.

$(H3)$ $m(\bar{b}_i) > 0, m(\bar{\beta}_j) > 0$, $m(a_i)^2 - 4m(\bar{b}_i)m(\bar{h}_i) > 0, m(\alpha_j)^2 - 4m(\bar{\beta}_j)m(\bar{e}_j) > 0$, $i = 1, 2, ..n, j = 1, 2, ...m$.

The solutions of Eqs (1.2) and (3.1) satisfy the following relations:

Lemma 3.2. *Suppose (H2) is satisfied, the following results hold:*
1) If $N(t) = (N_1(t), ..., N_{n+m}(t))$ is a positive AP solution of Eq (1.2), then

$$y(t) = (y_1(t), ..., y_{n+m}(t)) = (\prod_{0<t_k<t}(1+\rho_{1k})^{-1}N_1(t), ..., \prod_{0<t_k<t}(1+\rho_{n+m,k})^{-1}N_{n+m}(t))$$

is a positive ap solution of Eq (3.1);
2) If $y(t) = (y_1(t), ..., y_{n+m}(t))$ is a positive ap solution of Eq (3.1), then

$$N(t) = (N_1(t), ..., N_{n+m}(t)) = (\prod_{0<t_k<t}(1+\rho_{1k})y_1(t), ..., \prod_{0<t_k<t}(1+\rho_{n+m,k})y_{n+m}(t))$$

is a positive AP solution of Eq (1.2).

Proof. If $N(t) = (N_1(t), ..., N_{n+m}(t))$ is a positive AP solution of Eq (1.2), for any $i = 1, 2, ..., n+m$,

$$y_i(t_k^+) = \prod_{0<t_l<t_k^+}(1+\rho_{il})^{-1}N_i(t_k^+) = \prod_{0<t_l<t_k^+}(1+\rho_{il})^{-1}(1+\rho_{ik})N_i(t_k) = y_i(t_k).$$

Since $\prod_{0<t_k<t}(1+\rho_{ik}) \in AP$, and $\inf_{t\in\mathbb{R}} \prod_{0<t_k<t} |(1+\rho_{ik})| > 0$, from [20], we know $\prod_{0<t_k<t}(1+\rho_{ik})^{-1} \in AP$, $i = 1, ...n+m$. Then, $y(t) = (y_1(t), ..., y_{n+m}(t)) \in$ ap; If $y_i(\cdot) \in ap$, it follow from $(H2)$ that $N_i(t) = \prod_{0<t_k<t}(1+\rho_{ik})y_i(t) \in AP$, $i = 1, ...n+m$. Similar as [19], the rest proof of Lemma 3.2 can be obtained easily, we omit it here. □

By making the transformations $y_i(t) = e^{x_i(t)}$, $y_{n+j}(t) = e^{x_{n+j}(t)}, i = 1, 2, ...n, j = 1, ..., m$, system (3.1) is changed into:

$$\begin{cases} x_i'(t) = a_i(t) - \bar{b}_i(t)e^{x_i(t)} - \sum_{r=1,r\neq i}^{n} \bar{c}_{i,r}(t)e^{x_r(t-\tau_{i,r}(t))} \\ \qquad\qquad - \sum_{j=1}^{m} \bar{d}_{i,j}(t)e^{x_{n+j}(t-\sigma_{i,j}(t))} - \bar{h}_i(t)e^{-x_i(t)}, i = 1, 2, ...n, \\ x_{n+j}'(t) = \alpha_j(t) - \bar{\beta}_j(t)e^{x_{n+j}(t)} - \sum_{l=1,l\neq j}^{m} \bar{\gamma}_{l,j}(t)e^{x_{n+l}(t-\theta_{l,j}(t))} \\ \qquad\qquad + \sum_{i=1}^{n} \bar{\delta}_{i,j}(t)e^{x_i(t-\nu_{i,j}(t))} - \bar{e}_j(t)e^{-x_{n+j}(t)}, j = 1, 2, ...m. \end{cases} \tag{3.2}$$

Obviously, the existence of ap solutions of Eq (3.2) can lead to the existence of strictly positive ap solutions of Eq (3.1). Then, it follow from Lemma 3.2 that there exist strictly positive AP solutions of Eq (1.2). Due to this, we concentrate on solving the existence of ap solutions of Eq (3.2). we take

$$X_1 = \{x = (x_1, ..., x_{m+n}) \in \text{ap} : mod(x_i) \subset mod(F), \forall\lambda \in \Lambda_{x_i}, \alpha_1 > |\lambda| > \alpha, i = 1, ..., m+n\} \cup \{0\},$$
$$Z_1 = \{z = (z_1, ...z_{m+n}) \in \text{AP}, \text{z}_i(\cdot) \text{ are piecewise continuous with discontinuous points}\{t_k\},$$
$$mod(z_i) \subset mod(F), \forall\lambda \in \Lambda_{z_i}, \alpha_1 > |\lambda| > \alpha, \textstyle\sum_{j=1}^{\infty} |a(\lambda_j, z_i)| < +\infty, i = 1, ..., m+n\} \cup \{0\},$$
$$Z_2 = X_2 = \{x = (h_1, h_2, ..., h_{m+n}) \in \mathbb{R}^{m+n}\},$$

where, α and α_1 are given positive constants, F is given almost periodic function in sense of Bohr. Define $X = X_1 \oplus X_2, Z = Z_1 \oplus Z_2$ with the norm $||\phi|| = \max_{1\le i\le n+m} \sup_{t\in\mathbb{R}} |\phi_i(t)|, \phi \in X$ or Z.

By using Lemma 2.3, similar to the proof of Lemma 3.3 in [20], we can obtain

Lemma 3.3. *X and Z are Banach spaces equipped with the norm $\|\cdot\|$.*

Lemma 3.4. *Let*

$$L: X \to Z, L(x_1, x_2, ..., x_{m+n}) = (\frac{dx_1}{dt}, \frac{dx_2}{dt}, ..., \frac{dx_{m+n}}{dt}),$$

then L is a Fredholm mapping of index zero.

Remark 3.5. (1) In Lemma 3.4 in [19], the authors took $\mathbb{X} = \mathbb{Z} = V_1 \bigoplus V_2$, $L : \mathbb{X} \to \mathbb{Z}, Lz = z' = (z_1', z_2', ..., z_{n+m}')$. Obviously, z' should belong to a piecewise continuous function space, hence, $\mathbb{Z}$ appeared in Lemma 3.3 in [19] was not suitable.

(2) If $f \in ap$, $\forall \lambda \in \Lambda_f, |\lambda| > \alpha > 0$, then f has ap primitive function. It doesn't hold for AP function. Lemma 2.3 implies that if $f \in AP$, $\forall \lambda \in \Lambda_f, \alpha_1 > |\lambda| > \alpha > 0$, $\sum_{i=1}^{\infty} |a(\lambda_i, z)| < +\infty$, then f has ap primitive function. That is the reason why we take Z_1 like that.

Let

$$P: X \to X, P(x_1, ..., x_n, ..., x_{m+n}) = (m(x_1), ..., m(x_n), ..., m(x_{m+n})),$$

$$Q: Z \to Z, Q(z_1, ..., z_n, ..., z_{m+n}) = (m(z_1), ..., m(z_n), ..., m(z_{m+n})),$$

$$N: X \times [0,1] \to Z, N(x_1, ..., x_n, ..., x_{m+n}, \lambda) = (N(x_1, \lambda), ..., N(x_n, \lambda), ...N(x_{m+n}, \lambda)),$$

where

$$\begin{aligned} N(x_i(t), \lambda) &= a_i(t) - \bar{b}_i(t)e^{x_i(t)} - \lambda \sum_{r=1, r\neq i}^{n} \bar{c}_{i,r}(t)e^{x_r(t-\tau_{i,r}(t))} \\ &\quad -\lambda \sum_{j=1}^{m} \bar{d}_{i,j}(t)e^{x_{n+j}(t-\sigma_{i,j}(t))} - \bar{h}_i(t)e^{-x_i(t)}, i = 1, 2, ...n, \\ N(x_{n+j}(t), \lambda) &= \alpha_j(t) - \bar{\beta}_j(t)e^{x_{n+j}(t)} - \lambda \sum_{l=1, l\neq j}^{m} \bar{\gamma}_{l,j}(t)e^{x_{n+l}(t-\theta_{l,j}(t))} \\ &\quad +\lambda \sum_{i=1}^{n} \bar{\delta}_{i,j}(t)e^{x_i(t-\nu_{i,j}(t))} - \bar{e}_j(t)e^{-x_{n+j}(t)}, j = 1, ..., m, \end{aligned}$$

then we have:

Lemma 3.6. *N is L-compact on $\bar{\Omega}$, (Ω is an open, bounded subset of X).*

Proof. Firstly, it is easy to prove that P and Q are continuous projectors such that

$$ImP = KerL, ImL = Im(I - Q) = KerQ,$$

where I is identity mapping. Hence, $L|domL \cap KerP : (I - P)X \to ImL$ is invertible. We denote the inverse of that map by K_p. $K_p : ImL \to KerP \cap DomL$ has the form:

$$K_p z = K_p(z_1, z_2, ..., z_{n+m}) = (\int_0^t z_1(s)ds - m(\int_0^t z_1(s)ds), ..., \int_0^t z_{m+n}(s)ds - m(\int_0^t z_{m+n}(s)ds))$$

then,

$$QN(x,\lambda) = (QN(x_1,\lambda), ..., QN(x_{m+n},\lambda)),$$

$$K_p(I-Q)N(x,\lambda) = \big(f(x_1(t)) - Qf(x_1(t)), ..., f(x_{m+n}(t)) - Qf(x_{m+n}(t))\big),$$

where

$$f(x_i(t)) = \int_0^t (N(x_i(s),\lambda) - QN(x_i(s),\lambda))ds, i = 1,2,...,n+m.$$

Obviously, QN and $(I-Q)N$ are continuous, so is K_p. In fact, for any $z = (z_1, z_2, ..., z_{m+n}) \in Z_1 = ImL$, according to Lemma 2.3, we know $\int_0^t z_i(s)ds \in ap$. Besides, we have:

$$\Lambda_{\int_0^t z_i(s)ds} \backslash \{0\} = \Lambda_{z_i} = \Lambda_{\int_0^t z_i(s)ds - m(\int_0^t z_i(s)ds)}, i = 1,2,..n+m.$$

Since $mod(z_i) \subset mod(F)$, then $mod(\int_0^t z_i(s)ds) \subset mod(F)$. It follows from Lemma 2.4 that for any $\varepsilon > 0$, there exists $\delta > 0$ such that $T(F,\delta) \subset T(\int_0^t z_i(s)ds, \varepsilon)$. Let l be the inclusion interval of $T(F,\delta)$. For any $t \notin [0,l]$, there exists $\xi \in T(F,\delta) \subset T(\int_0^t z_i(s)ds, \varepsilon)$ such that $t + \xi \in [0,l]$, hence, for any $i = 1,2,...n+m$,

$$\begin{aligned}
\sup_{t\in\mathbb{R}} \Big|\int_0^t z_i(s)ds\Big| &\leq \sup_{t\in[0,l]} \Big|\int_0^t z_i(s)ds\Big| + \sup_{t\notin[0,l]} \Big|\int_0^t z_i(s)ds - \int_0^{t+\xi} z_i(s)ds\Big| + \sup_{t\notin[0,l]} \Big|\int_0^{t+\xi} z_i(s)ds\Big| \\
&\leq 2\sup_{t\in[0,l]} \int_0^t |z_i(s)|ds + \sup_{t\notin[0,l]} \Big|\int_0^t z_i(s)ds - \int_0^{t+\xi} z_i(s)ds\Big| \\
&\leq 2\int_0^l |z_i(s)|ds + \varepsilon.
\end{aligned}$$

We can conclude that K_p is continuous, and consequently, $K_p(I-Q)N$ is also continuous. In addition, we also have $K_p(I-Q)N(x,\lambda)$ is uniformly bounded in $\bar{\Omega} \times [0,1]$, $QN(\bar{\Omega} \times [0,1])$ is bounded and $K_p(I-Q)N(x,\lambda)$ is equicontinuous in $\bar{\Omega} \times [0,1]$. For any $x = (x_1, ..., x_{n+m}) \in \Omega$, $\lambda \in [0,1]$, since

$$(I-Q)N(x,\lambda) \in Z_1 = ImL, \Lambda_{K_p(I-Q)N(x_i,\lambda)} = \Lambda_{(I-Q)N(x_i,\lambda)},$$

then, $mod(K_p(I-Q)N(x_i,\lambda)) = mod((I-Q)N(x_i,\lambda)) \subset mod(F)$. For any $\varepsilon > 0$, $\exists\delta > 0$ such that $T(F,\delta) \subset T(K_p(I-Q)N(x_i,\lambda),\varepsilon), i = 1,...,n+m$, hence, $K_p(I-Q)N$ is equi-almost periodic in $\bar{\Omega} \times [0,1]$. According to Lemma 2.5, we can immediately conclude that $K_p(I-Q)N\bar{\Omega}$ is compact, thus N is L-compact on $\bar{\Omega}$. □

Combining Lemmas 3.3–3.6, for Eq (3.2), we have the result:

Lemma 3.7. *If (H1)–(H3) are all satisfied, then Eq (3.2) has at least one almost periodic solution (in sense of Bohr).*

Proof. Define the isomorphism $J : ImQ \to KerL$ be an identity mapping. We search for an appropriate bounded open subset Ω for the application of Lemma 2.6. Corresponding to operator equation $Lx = \lambda N(x,\lambda), \lambda \in (0,1)$ we have:

$$\begin{aligned}
x_i'(t) &= \lambda(a_i(t)-\bar{b}_i(t)e^{x_i(t)}-\lambda\sum_{r=1,r\neq i}^{n}\bar{c}_{i,r}(t)e^{x_r(t-\tau_{i,r}(t))}\\
&\quad -\lambda\sum_{j=1}^{m}\bar{d}_{i,j}(t)e^{x_{n+j}(t-\sigma_{i,j}(t))}-\bar{h}_i(t)e^{-x_i(t)}),\, i=1,2,...n, \qquad (3.3)\\
x_{n+j}'(t) &= \lambda(\alpha_j(t)-\bar{\beta}_j(t)e^{x_{n+j}(t)}-\lambda\sum_{l=1,l\neq j}^{m}\bar{\gamma}_{l,j}(t)e^{x_{n+l}(t-\theta_{l,j}(t))}\\
&\quad +\lambda\sum_{i=1}^{n}\bar{\delta}_{i,j}(t)e^{x_i(t-\nu_{i,j}(t))}-\bar{e}_j(t)e^{-x_{n+j}(t)}),\, j=1,2,...m. \qquad (3.4)
\end{aligned}$$

If $x=(x_1,...,x_{n+m})\in X$ is an almost periodic solution of system (3.2), then

$$m(e^{x_k(t)}x_k'(t))=\lim_{T\to\infty}\frac{1}{T}\int_0^T e^{x_k(s)}x_k'(s)ds=\lim_{T\to\infty}\frac{e^{z_k(T)}-e^{z_k(0)}}{T}=0, k=1,..,n+m.$$

Multiplying $e^{x_i(t)}, i=1,...,n$, and $e^{x_{n+j}(t)}, j=1,2,...m$, on both sides of the Eqs (3.3) and (3.4), respectively, then, taking the limit mean, we can obtain:

$$\begin{aligned}
&\lambda m(\sum_{r=1,r\neq i}^{n}\bar{c}_{i,r}(t)e^{x_r(t-\tau_{i,r}(t))}e^{x_i(t)}+\sum_{j=1}^{m}\bar{d}_{i,j}(t)e^{x_{n+j}(t-\sigma_{i,j}(t))}e^{x_i(t)})\\
=&m(a_i(t)e^{x_i(t)}-\bar{b}_i(t)e^{2x_i(t)}-\bar{h}_i(t)), \qquad (3.5)\\
&\lambda m(\sum_{l=1,l\neq j}^{m}\bar{\gamma}_{l,j}(t)e^{x_{n+l}(t-\theta_{l,j}(t))}e^{x_{n+j}(t)}-\sum_{i=1}^{n}\bar{\delta}_{i,j}(t)e^{x_i(t-\nu_{i,j}(t))}e^{x_{n+j}(t)})\\
=&m(\alpha_j(t)e^{x_{n+j}(t)}-\bar{\beta}_j(t)e^{2x_{n+j}(t)}-\bar{e}_j(t)). \qquad (3.6)
\end{aligned}$$

From Eq (3.5), we can obtain:

$$m(a_i(t)e^{x_i(t)}-\bar{b}_i(t)e^{2x_i(t)}-\bar{h}_i(t))\geq 0,$$

hence,

$$0\geq m(\bar{b}_i(t)e^{2x_i(t)}-a_i(t)e^{x_i(t)}+\bar{h}_i(t))\geq m(\bar{b}_i^Le^{2x_i(t)}-a_i^Me^{x_i(t)}+\bar{h}_i^L).$$

We assert that there exist $\xi_i, i=1,...,n$ such that

$$\frac{a_i^M-\sqrt{(a_i^M)^2-4\bar{b}_i^L\bar{h}_i^L}}{4\bar{b}_i^L}\leq e^{x_i(\xi_i)}\leq\frac{3a_i^M+\sqrt{(a_i^M)^2-4\bar{b}_i^L\bar{h}_i^L}}{4\bar{b}_i^L}, i=1,...n,$$

then, there exist at least one $\xi_i, i=1,...,n$ such that

$$|x_i(\xi_i)|<|\ln\frac{a_i^M-\sqrt{(a_i^M)^2-4\bar{b}_i^L\bar{h}_i^L}}{4\bar{b}_i^L}|+|\ln\frac{3a_i^M+\sqrt{(a_i^M)^2-4\bar{b}_i^L\bar{h}_i^L}}{4\bar{b}_i^L}|, i=1,...n. \qquad (3.7)$$

Since $x_i'\in Z_1$, $\int_a^t x_i'(s)ds\in ap, mod(\int_a^t x_i'(s)ds)\subset mod(F), \forall a\in\mathbb{R}, i=1,...,n$, it follows from Lemma 2.4 that for $\varepsilon=1$, there exist $\delta>0$ such that $T(F,\delta)\subset T(\int_a^t x_i'(s)ds,1)$. Let l be the inclusion interval of $T(F,\delta)$. Same argument as Lemma 3.6 can drive that

$$
\begin{aligned}
|x_i(t)| &= |x_i(\xi_i) + \int_{\xi_i}^{t} x_i'(s)ds| \\
&\le |x_i(\xi_i)| + |\int_{\xi_i}^{t} x_i'(s)ds| \\
&\le |x_i(\xi_i)| + 1 + 2\int_{\xi_i}^{\xi_i+l} |x_i'(s)|ds, \qquad (3.8)
\end{aligned}
$$

$$
\begin{aligned}
\int_{\xi_i}^{\xi_i+l} |x_i'(s)|ds &= \int_{\xi_i}^{\xi_i+l} |a_i(s) - \bar{b}_i(s)e^{x_i(s)} - \lambda \sum_{r=1,r\neq i}^{n} \bar{c}_{i,r}(t)e^{x_r(s-\tau_{i,r}(s))}ds \\
&- \lambda \sum_{j=1}^{m} \bar{d}_{i,j}(t)e^{x_{n+j}(s-\sigma_{i,j}(s))} - \bar{h}_i(s)e^{-x_i(s)}| \\
&\le \int_{\xi_i}^{\xi_i+l} |a_i(s)|ds. \qquad (3.9)
\end{aligned}
$$

Combining (3.7)–(3.9), we know that for any $t \in \mathbb{R}$, $i = 1, ..., n$

$$
|x_i(t|| \le 3la_i^M + 3 + |\ln \frac{a_i^M - \sqrt{(a_i^M)^2 - 4\bar{b}_i^L\bar{h}_i^L}}{4\bar{b}_i^L}| + |\ln \frac{3a_i^M + \sqrt{(a_i^M)^2 - 4\bar{b}_i^L\bar{h}_i^L}}{4\bar{b}_i^L}| \triangleq x_i^M.
$$

Similar argument as above, from Eq (3.6), we know that:

$$
m(\alpha_j(t)e^{x_{n+j}(t)} - \bar{\beta}_j(t)e^{2x_{n+j}(t)} - \bar{e}_j(t)) \ge m(-\sum_{i=1}^{n} \bar{\delta}_{i,j}(t)e^{x_i(t-\nu_{i,j}(t))}e^{x_{n+j}(t)}), \qquad (3.10)
$$

hence,

$$
m(\alpha_j^M e^{x_{n+j}(t)} - \bar{\beta}_j^L e^{2x_{n+j}(t)} - \bar{e}_j^L) \ge m(-\sum_{i=1}^{n} \bar{\delta}_{i,j}^M e^{x_i^M} e^{x_{n+j}(t)}),
$$

then,

$$
m(\bar{\beta}_j^L e^{2x_{n+j}(t)} - (\alpha_j^M + \sum_{i=1}^{n} \bar{\delta}_{i,j}^M e^{x_i^M})e^{x_{n+j}(t)} + \bar{e}_j^L) \le 0.
$$

We assert that there exist ξ_{n+j}, $j = 1, ...m$, such that

$$
\begin{aligned}
\frac{(\alpha_j^M + \sum_{i=1}^{n} \bar{\delta}_{i,j}^M e^{x_i^M}) - \sqrt{(\alpha_j^M + \sum_{i=1}^{n} \bar{\delta}_{i,j}^M e^{x_i^M})^2 - 4\bar{\beta}_j^L\bar{e}_j^L}}{4\bar{\beta}_j^L} &\le e^{x_{n+j}(\xi_{n+j})}, \\
e^{x_{n+j}(\xi_{n+j})} &\le \frac{3(\alpha_j^M + \sum_{i=1}^{n} \bar{\delta}_{i,j}^M e^{x_i^M}) + \sqrt{(\alpha_j^M + \sum_{i=1}^{n} \bar{\delta}_{i,j}^M e^{x_i^M})^2 - 4\bar{\beta}_j^L\bar{e}_j^L}}{4\bar{\beta}_j^L},
\end{aligned}
$$

then

$$
\begin{aligned}
|x_{n+j}(\xi_{n+j})| \leq \quad & |\ln \frac{(\alpha_j^M+\sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M})-\sqrt{(\alpha_j^M+\sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M})^2-4\bar{\beta}_j^L\bar{e}_j^L}}{4\bar{\beta}_j^L}| \\
& +|\ln \frac{3(\alpha_j^M+\sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M})+\sqrt{(\alpha_j^M+\sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M})^2-4\bar{\beta}_j^L\bar{e}_j^L}}{4\bar{\beta}_j^L}|.
\end{aligned}
$$

Similarly, for any $t \in \mathbb{R}$, $j = 1, ..., m$, we have

$$
\begin{aligned}
|x_{n+j}(t)| &\leq |x_{n+j}(\xi_{n+j})| + 1 + 2\int_{\xi_{n+j}}^{\xi_{n+j}+l} |x'_{n+j}(s)|ds \\
&\leq |x_{n+j}(\xi_{n+j})| + 1 + 2\int_{\xi_{n+j}}^{\xi_{n+j}+l} |\alpha_j(s) + \sum_{i=1}^n \bar{\delta}_{i,j}(s)e^{x_i^M}|ds \\
&\leq 3 + 3l(\alpha_j^M + \sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M}) \\
&\quad +|\ln \frac{(\alpha_j^M + \sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M}) - \sqrt{(\alpha_j^M + \sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M})^2 - 4\bar{\beta}_j^L\bar{e}_j^L}}{4\bar{\beta}_j^L}| \\
&\quad +|\ln \frac{3(\alpha_j^M + \sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M}) + \sqrt{(\alpha_j^M + \sum_{i=1}^n \bar{\delta}_{i,j}^M e^{x_i^M})^2 - 4\bar{\beta}_j^L\bar{e}_j^L}}{4\bar{\beta}_j^L}| \triangleq x_{n+j}^M. \qquad (3.11)
\end{aligned}
$$

Taking $\Omega = \{x \in X, \|x\| \leq 2\max\{x_1^M, ..., x_{n+m}^M\}\}$, combining Lemma 2.6 in [9] and condition (H3), it is easy to prove that that Ω satisfies all the requirements in Lemma 2.6, hence system (3.2) has at least one almost periodic solution in Ω. The proof of the theorem is complete. □

From Lemma 3.7, we know that system (3.1) has at least one strictly positive almost periodic solution in Ω. It follows from Lemma 3.2 that:

Theorem 3.8. *If (H1)–(H3) are all satisfied, then Eq (1.2) has at least one strictly positive almost periodic solution.*

Remark 3.9. (1) In order to obtain the existence of almost periodic solutions of Eq (1.2), Li and Ye [19] transformed Eq (1.2) into Eq (3.1) in [19]. Unfortunately, Eq (3.1) in [19] was not correct. Besides, They acquiesced that the almost periodic solution of Eq (3.1) in [19] belonged to C^1, which was also an important condition to get the existence of almost periodic solutions of Eq (1.2). However, obviously, the solution of Eq (3.1) in [19] don't belong to C^1.

(2) Compared this paper with [10], Eq (1.2) is more general. Moreover, it is more realistic to study almost periodic solution for a model than periodic solution. The sufficient conditions for the existence of almost periodic solutions are more easier verification. Therefore, this paper corrects and generalizes the previous results.

(3) The aim of this paper is to apply coincidence degree to study the existence of almost periodic solution of system (1.2). we set new functional space Z_1, and use the Favard's theorem of AP function to realize our purpose. Our method used in this paper can be applied to study the existence of almost periodic solution of impulsive differential equations with linear impulsive perturbations.

4. Example

In this section, we present an example to demonstrate Theorem 3.8 obtained in previous section. Consider the following impulsive Lotka-Volterra predator-prey system (4.1) with harvesting terms:

$$\begin{cases} N_1'(t) = N_1(t)[(5+\sin t) - \frac{1+\cos \pi t}{2} N_1(t) - d_{1,1}(t)N_2(t) - d_{1,2}(t)N_3(t)] - \frac{1+\sin \pi t}{2}, t \neq t_k, \\ N_2'(t) = N_2(t)[(5+\sin 2t) - \frac{1+\cos 2\pi t}{2} N_2(t) - \gamma_{2,1}(t)N_3(t) + \delta_{1,1}(t)N_1(t)] - \frac{1+\sin 2\pi t}{2}, t \neq t_k, \\ N_3'(t) = N_3(t)[(5+\sin 3t) - \frac{1+\cos 3\pi t}{2} N_3(t) - \gamma_{1,2}(t)N_2(t) + \delta_{1,2}(t)N_1(t)] - \frac{1+\sin 4\pi t}{2}, t \neq t_k, \\ N_h(t_k^+) = (1+\rho_{hk})N_h(t_k), \qquad h = 1,2,3, \end{cases}$$

where, $d_{1,1}(t) = 0.1, d_{1,2}(t) = 0.04, \gamma_{1,2}(t) = 0.04, \gamma_{2,1}(t) = 0.15, \delta_{1,1}(t) = 0.6, \delta_{1,2}(t) = 0.8$, $t_k = \{1,2,3,...\}, \rho_{hk} = \{-\frac{1}{2}, 1, -\frac{1}{2}, 1, -\frac{1}{2}, 1, ...\}, h = 1,2,3.$

Obviously, condition $(H1)$ is satisfied. Moreover, $\prod_{0<t_k<t}(1+\rho_{ik})$ are positive piecewise continuous periodic functions, $\inf_{t\in\mathbb{R}} \prod_{0<t_k<t} |(1+\rho_{i,k})| = \frac{1}{2} > 0$, $i = 1,2,3$. Hence, condition $(H2)$ holds. Besides, we have

$$\begin{aligned} &m(a_1) = m(\alpha_1) = m(\alpha_2) = 5, \\ &m(\bar{b}_1) = m(\bar{\beta}_1) = m(\bar{\beta}_2) = m(\bar{e}_1) = m(\bar{e}_2) = \frac{3}{8}, \\ &m(\bar{h}_i) = \frac{3}{8} + \frac{3\pi}{2}. \end{aligned}$$

Hence, condition $(H3)$ holds. It follows from Theorem 3.8 that system (4.1) has at least one strictly positive almost periodic solution. By matlab, we can give the simulation of Eq (4.1), see Figure 1.

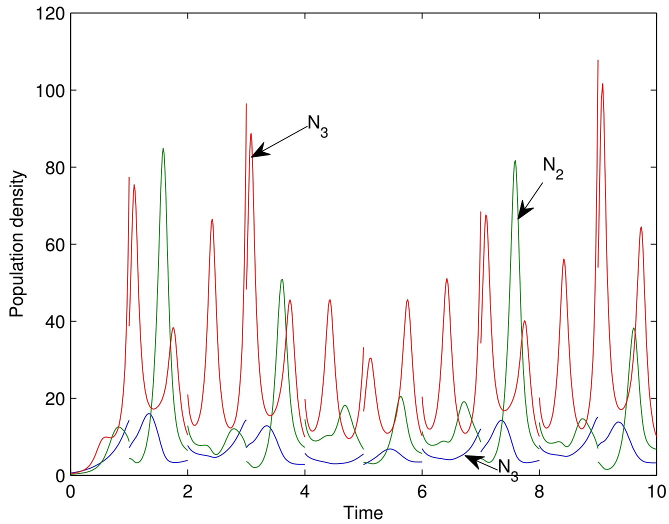

Figure 1. The existence of almost periodic solution for system (4.1).

5. Conclusions

Based on the Mawhins continuation theorem, the existence of almost periodic solutions of impulsive non-autonomous Lotka-Volterra predator prey system with harvesting terms was obtained. An example and some remarks are given to illustrate the advantage of this paper.

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Conflict of interest

The authors declared that they have no conflicts of interest to this work.

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