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Research article

Soft version of compact and Lindelöf spaces using soft somewhere dense sets

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Abstract: Herein, we applied soft somewhere dense sets to initiate six sorts of soft spaces called almost (nearly, mildly) soft SD-compact and almost (nearly, mildly) soft SD-Lindelöf spaces. We study the master properties of these spaces and illustrate the relations between them with the help of examples. In addition, we clarify that the six soft spaces are equivalent under a soft SD-partition. Moreover, the relationships between the initiated spaces and enriched soft topological spaces and other well-known spaces such as soft S-connected are indicated.

Keywords: soft somewhere dense set; soft cs-dense set; almost soft SD-compact; almost soft SD-Lindelöf; nearly soft SD-compact; nearly soft SD-Lindelöf; mildly soft SD-compact; mildly soft SD-Lindelöf
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1. Introduction and preliminaries

Many researchers developed the theory of soft sets after it was established by Molodtsov [23] in 1999 as a new mathematical method for dealing with problems involving uncertainties. A first attempt was made by Maji et al. [21], in 2003, to formulate soft operators. He defined the null and absolute soft sets, a complement of soft set, and soft intersection and union between two soft sets. Ali et al. [2] made a major contribution, in 2009, through the soft set theory. Some soft operators were redefined such as the complement of a soft set and soft intersection between two soft sets, and new soft operators were initiated between two soft sets such as restricted union, restricted and extended intersection and restricted difference. In this sense, Al-shami and El-Shafei [9] relaxed constraints on a parameters set to implement generalized soft operators.

Shabir and Naz [27], in 2011, initiated soft topology and presented some properties of soft separation axioms. The work in [27] was continued by Min [22], he corrected a relationship between soft T_2 and soft T_3-spaces and analyzed the properties of soft regular spaces. In 2012, the relation between soft sets and fuzzy sets was pointed out by Zorlutuna et al. [29]. Also they introduced the soft point in order to analyze some properties of soft interior points and soft neighborhood systems. Aygünoğlu and Aygün [14] defined the notion of soft compact spaces and introduced the definition of enriched soft topological spaces which will play a remarkable role in this article. In [19], Hida provided a stronger description for soft compact spaces which defined in [14]. Al-shami et al. [10] investigated new types of covering properties called almost soft compact and approximately soft Lindelöf spaces.

Kharal and Ahmad [20] defined soft mappings and established main properties. Then, Zorlutuna and Çakir [30] explored the concept of soft continuous mappings. Asaad [13] presented the concept of soft extremally disconnected spaces and revealed main properties. In [28], the authors conducted a comparative study on soft separation axioms. Alcantud [1] discussed the countability axioms in the soft topologies. Recently, Al-shami applied the concepts of compactness and soft separation axioms to economic application [6] and information system [7].

At one and the same year, new form of a soft point was introduced in [15] and [24]. This form helps to simulate the followed manner in classical topology to soft topology and makes it easier to prove many soft topological properties. A comparative study on soft points was conducted in [26]. Al-shami [5] restudied main properties of soft separation axioms; especially, those introduced using the different types of soft points.

In 2017, Al-shami [3] initiated a new class of generalized open sets called somewhere dense sets, and in 2019, he and Noiri [11] applied to introduce new types of soft mappings. Al-shami [4] studied this class in soft topology in 2018. Then, he with co-authors [8, 17] exploited to study some concepts in soft setting. This study goes in this path of study by introducing new types of soft compact and Lindelöf spaces, namely almost (nearly, mildly) soft SD-compact and almost (nearly, mildly) soft SD-Lindelöf spaces. To clarify some features of these spaces and the relationships between them, we provided some illustrative examples. In addition, we define soft SD-partition and soft SD-hyperconnected spaces to study some properties that link them with the soft spaces that have been introduced, and we deduce some conclusions that associate these spaces with enriched soft topological spaces. Then we investigate when the six initiated soft spaces have a soft hereditary property. Finally, we elucidate that the soft SD-irresolute mappings keep the six initiated soft spaces.

Now, with a fixed parameters set Ω we recall some notions and conclusions that are mentioned in the various previous studies.

Definition 1.1. *([3, 11]) Let (W, σ) be a topological space and $M \subseteq W$. Then*

(i) *If $int(cl(M)) \neq \emptyset$, then M is called somewhere dense.*

(ii) *The complement of somewhere dense subset is cs-dense.*

(iii) *The union of all somewhere dense sets contained in M is called the S-interior and it is denoted by $S\,int(M)$.*

(iv) *The intersection of all cs-dense sets containing M is called the S-closure of M and it is denoted by $S\,cl(M)$.*

Definition 1.2. *[12] A topological space (W, σ) is called:*

(i) *Nearly SD-compact (resp. nearly SD-Lindelöf) if every somewhere dense cover of W has a finite (resp. countable) subcover in which its S-closure covers W.*

(ii) *Almost SD-compact(resp. almost SD-Lindelöf) if every somewhere dense cover of W has a finite (resp. countable) subcover which the S-closures of whose members cover W.*

Definition 1.3. *[23] If $M : \Omega \to 2^W$, then the pair (M, Ω) is called a soft set over W and it can be written as follows: $(M, \Omega) = \{(p, M(p)) : p \in \Omega$ and $M(p) \in 2^W\}$.*

Definition 1.4. *[21] Let (M, Ω) be a soft set:*

(i) *If $M(p) = W$ for each $p \in \Omega$, then (M, Ω) is called an absolute soft set and it is denoted by $\widetilde{W}$.*

(ii) *If $M(p) = \emptyset$ for each $p \in \Omega$, then (M, Ω) is called a null soft set and it is denoted by $\widetilde{\emptyset}$.*

Definition 1.5. *[18]*

(i) *(H, Ω) is a soft subset of (M, Ω), denoted by $(H, \Omega)\widetilde{\subseteq}(M, \Omega)$ if $H(p) \subseteq M(p)$ for each $p \in \Omega$.*

(ii) *(H, Ω) is a soft superset of (M, Ω) if $M(p) \subseteq H(p)$ for each $p \in \Omega$.*

Definition 1.6. *[2] The relative complement of a soft set (M, Ω), denoted by $(M, \Omega)^c$, is defined by $(M, \Omega)^c = (M^c, \Omega)$, where a mapping $M^c : \Omega \to 2^W$ is given by $M^c(p) = W - M(p)$, for each $p \in \Omega$.*

Definition 1.7. *[2, 21] Let (M, Ω) and (F, Ω) be two soft sets. Then:*

(i) *$(M, \Omega)\widetilde{\bigcup}(F, \Omega) = (H, \Omega)$, where $H(p) = M(p) \bigcup F(p)$, for each $p \in \Omega$.*

(ii) *$(M, \Omega)\widetilde{\bigcap}(F, \Omega) = (H, \Omega)$, where $H(p) = M(p) \bigcap F(p)$, for each $p \in \Omega$.*

Definition 1.8. *([16, 27]) Let (M, Ω) be a soft set. Then:*

(i) *$t \in (M, \Omega)$ if $t \in M(p)$, for each $p \in \Omega$.*

(ii) *$t \Subset (M, \Omega)$ if $t \in M(p)$, for some $p \in \Omega$.*

Definition 1.9. *[27] A family σ of soft sets of W with a parameters set Ω is called a soft topology on W if :*

(i) *The members of σ include absolute soft set $\widetilde{W}$ and the null soft set $\widetilde{\emptyset}$.*

(ii) *The member of σ include the soft union of an arbitrary number of soft sets in σ.*

(iii) *The member of σ include the soft intersection of a finite number of soft sets in σ.*

Then (W, σ, Ω) is said to be a soft topological space (briefly, $soft_{TS}$). Each member in σ is called soft open and its relative complement is called soft closed.

Proposition 1.10. *[27] Let (W, σ, Ω) be a $soft_{TS}$. Then $\sigma_p = \{M(p) : (M, \Omega) \in \sigma\}$ defines a topology on W, for each $p \in \Omega$.*

Definition 1.11. *[27] Let (M, Ω) be a soft subset of a $soft_{TS}$ (W, σ, Ω). Then $(cl(M), \Omega)$ is defined by $cl(M)(p) = cl(M(p))$, where $cl(M(p))$ is the closure of $M(p)$ in (W, σ_p) for each $p \in \Omega$.*

Proposition 1.12. *[27] Let (M, Ω) be a soft subset of a soft$_{TS}$ (W, σ, Ω). Then:*

(i) $(cl(M), \Omega) \widetilde{\subseteq} cl(M, \Omega)$.

(ii) $(cl(M), \Omega) = cl(M, \Omega)$ *iff* $(cl(M), \Omega)^c$ *is soft closed.*

Definition 1.13. *A soft set (M, Ω) is called:*

(i) *Soft point [15, 24] if there are $p \in \Omega$ and $t \in W$ with $M(p) = \{t\}$ and $M(q) = \emptyset$, for each $q \in \Omega - \{p\}$. A soft point is briefly denoted by P_p^t.*

(ii) *Pseudo constant [25] if $M(p) = W$ or $\emptyset$, for each $p \in \Omega$. A family of all pseudo constant soft sets is briefly denoted by $CS(W, \Omega)$.*

Definition 1.14. *[14] If* **(i)** *of Definition 1.9 is replaced by the following condition: $(M, \Omega) \in \sigma$, for all $(M, \Omega) \in CS(W, \Omega)$, then a soft topology σ on W is said to be enriched. In this case, the triple (W, σ, Ω) is called an enriched soft$_{TS}$ over W.*

Definition 1.15. *[15] A soft set (M, Ω) is called finite (resp. countable) if $M(p)$ is finite (resp. countable) for each $p \in \Omega$. A soft set is called infinite (resp. uncountable) if it is not finite (resp. countable).*

Definition 1.16. *[10] A soft$_{TS}$ (W, σ, Ω) is said to be almost soft compact (resp. almost soft Lindelöf) if every soft open cover $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ of $\widetilde{W}$ has a finite (resp. countable) subcover with $\widetilde{W} = \bigcup_{i \in \Lambda} cl(H_i, \Omega)$, where Λ is a finite (resp. countable) set.*

Definition 1.17. *[4] A soft subset (M, Ω) of (W, σ, Ω) is said to be soft somewhere dense if there is a non null soft open set (H, Ω) such that $(H, \Omega) \widetilde{\subseteq} cl(M, \Omega)$. The complement of a soft somewhere dense set is said to be soft cs-dense.*

Definition 1.18. *[4] Let (M, Ω) be a soft subset of soft$_{TS}$ (W, σ, Ω). Then*

(i) *The union of all soft somewhere dense sets contained in (M, Ω) is called the soft S-interior and it is denoted by $S\,int(M, \Omega)$.*

(ii) *The intersection of all soft cs-dense sets containing (M, Ω) is called the soft S-closure of (M, Ω) and it is denoted by $S\,cl(M, \Omega)$.*

Proposition 1.19. *[17] The S-closure operator of any soft set (M, Ω) defined by the following rule:*

$$S\,cl(M, \Omega) = \begin{cases} \widetilde{X} & : \ (M, \Omega) \text{ is only soft somewhere dense} \\ (M, \Omega) & : \ \text{otherwise} \end{cases}$$

Theorem 1.20. *[17] A subset (M, Ω) of an enriched soft$_{TS}$ (W, σ, Ω) is:*

(i) *Soft somewhere dense iff there is a somewhere dense subset G of (W, σ_c) with $M(c) = G$.*

(ii) *Soft cs-dense iff there is a cs-dense subset G of (W, σ_c) with $M(c) = G$.*

Definition 1.21. *[17] A soft$_{TS}$ (W, σ, Ω) is soft S-connected iff the only soft somewhere dense and soft cs-dense subsets of (W, σ, Ω) are $\widetilde{\emptyset}$ and $\widetilde{W}$. In this work, we will use SD-connected instead of S-connected.*

Definition 1.22. *[4] A soft map $\Psi_\Theta : (W, \sigma_W, \Omega) \to (Z, \sigma_Z, \Omega)$ is said to be soft SD-continuous (resp. soft SD-irresolute) if $\Psi_\Theta^{-1}(M, \Omega)$ is the null soft set or a soft somewhere dense set where (M, Ω) is soft open (resp. soft somewhere dense) set.*

Theorem 1.23. *[4] Let $\Psi_\Theta : (W, \sigma_W, \Omega) \to (Z, \sigma_Z, \Omega)$ be a soft map. Then the following properties are equivalent:*

(i) *Ψ_Θ is soft SD-continuous.*

(ii) *The inverse image of every soft closed subset of (Z, σ_Z, Ω) is $\widetilde{W}$ or soft cs-dense.*

(iii) *$S\,cl(\Psi_\Theta^{-1}(M, \Omega)) \widetilde{\subseteq} \Psi_\Theta^{-1}(cl(M, \Omega))$ for each $(M, \Omega) \widetilde{\subseteq} \widetilde{Z}$.*

(iv) *$\Psi_\Theta(S\,cl(H, \Omega)) \widetilde{\subseteq} cl(\Psi_\Theta(H, \Omega))$ for each $(H, \Omega) \widetilde{\subseteq} \widetilde{W}$.*

(v) *$\Psi_\Theta^{-1}(int(M, \Omega)) \widetilde{\subseteq} S\,int(\Psi_\Theta^{-1}(M, \Omega))$ for each $(M, \Omega) \widetilde{\subseteq} \widetilde{Z}$.*

2. Almost soft SD-compact and almost soft SD-Lindelöf spaces

In this section, we will use soft somewhere dense sets to define two new spaces, namely almost soft SD-compact and almost soft SD-Lindelöf spaces, also, we study some of their basic properties.

Definition 2.1. *A family of soft somewhere dense subsets of (W, σ, Ω) is called a soft somewhere dense cover (briefly, SD-cover) of $\widetilde{W}$ if $\widetilde{W}$ is a soft subset of this family.*

Definition 2.2. *A soft$_{TS}$ (W, σ, Ω) is said to be almost soft SD-compact (resp. almost soft SD-Lindelöf) if for every soft SD-cover $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ of (W, σ, Ω), there is a finite (resp. countable) set $\Lambda \subseteq I$ with $\widetilde{W} = \widetilde{\bigcup}_{i \in \Lambda} S\,cl(H_i, \Omega)$.*

The following example presents a soft topological space which is almost soft SD-compact but not soft compact.

Example 2.3. *Let the set of real numbers $\mathbb{R}$ be the universal set and $\Omega = \{p, q\}$ be a set of parameters. Then $\sigma = \{\widetilde{\emptyset}, \widetilde{\mathbb{R}}, (E, \Omega)\}$ is a soft topology on $\mathbb{R}$, where $(E, \Omega) = \{(p, \{1\}), (q, \emptyset)\}$. Now, every soft superset of (E, Ω) is soft somewhere dense, but not soft cs-dense. Also, any soft SD-cover of $(\mathbb{R}, \sigma, \Omega)$ is superset of (E, Ω). From Proposition 1.19, we find that soft S-closure of any member of the soft SD-cover is the absolute soft set $\mathbb{R}$. Thus, $(\mathbb{R}, \sigma, \Omega)$ is almost soft SD-compact.*

Proposition 2.4. *If a soft$_{TS}$ (W, σ, Ω) is almost soft SD-compact, then (W, σ, Ω) is almost soft SD-Lindelöf.*

Proof. It follows from Definition 2.2. □

Proposition 2.5. *If a soft$_{TS}$ (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf), then (W, σ, Ω) is almost soft compact (resp. almost soft Lindelöf).*

Proof. Assume that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ which is a soft open cover of (W, σ, Ω). Then it is a soft SD-cover of almost soft SD-compact space (W, σ, Ω), so there is a finite subcover with $\widetilde{W} = \widetilde{\bigcup}_{i=1}^{n} S\,cl(H_i, \Omega) \widetilde{\subseteq} \widetilde{\bigcup}_{i=1}^{n} cl(H_i, \Omega)$. Hence, (W, σ, Ω) is almost soft compact.

Similarly, the proof is given in the case of almost soft SD-Lindelöf. □

The following example demonstrates that the converses of Proposition 2.4 and 2.5 are not valid.

Example 2.6. *Let $\Omega = \{p, q\}$ be a set of parameters. Define a soft topology on the natural numbers $\mathbb{N}$ by $\sigma = \{\widetilde{\emptyset}, \widetilde{W}, (E, \Omega), (H, \Omega)\}$, where*

$$(E, \Omega) = \{(p, \{1\}), (q, \{2\})\} \text{ and } (H, \Omega) = \{(p, \mathbb{N} - \{1\}), (q, \mathbb{N} - \{2\})\}.$$

All soft points in $\widetilde{\mathbb{N}}$ construct a soft SD-cover of $(\mathbb{N}, \sigma, \Omega)$. Since every soft subset of $(\mathbb{N}, \sigma, \Omega)$ (except the null and absolute soft sets) is both soft somewhere dense and soft cs-dense. Therefore, $(\mathbb{N}, \sigma, \Omega)$ is not almost soft SD-compact.

Proposition 2.7. *If a soft$_{TS}$ (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf), then a finite (resp. countable) union of subsets of (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf).*

Proof. It is obvious. □

Proposition 2.8. *If a soft$_{TS}$ (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf), then every soft cs-dense subset of (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf).*

Proof. Assume that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ which is a soft SD-cover of a soft cs-dense subset (M, Ω) of (W, σ, Ω). Now, $\{(H_i, \Omega) : i \in I\} \widetilde{\bigcup} (M^c, \Omega)$ is a soft SD-cover of an almost soft SD-compact space (W, σ, Ω). So $\widetilde{W} = \widetilde{\bigcup}_{i=1}^{n} Scl(H_i, \Omega)\ \widetilde{\bigcup} (M^c, \Omega)$. This means that $(M, \Omega) \widetilde{\subseteq} \widetilde{\bigcup}_{i=1}^{n} Scl(H_i, \Omega)$ and hence (M, Ω) is almost soft SD-compact.
The case between parentheses can be achieved similarly. □

Corollary 2.9. *The soft intersection of a soft cs-dense and almost soft SD-compact (resp. almost soft SD-Lindelöf) sets is almost soft SD-compact (resp. almost soft SD-Lindelöf).*

In Example 2.3, a soft subset $(H, \Omega) = \{(p, \{1\}), (q, \{2\})\}$ of $(\mathbb{R}, \sigma, \Omega)$ is almost soft SD-compact; however, it is not soft cs-dense. This demonstrates that the converse of Proposition 2.8 is not valid.

Definition 2.10. *Let $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ be a family of soft sets. If $\widetilde{\bigcap}_{i \in \Lambda} Sint(E_i, \Omega) \neq \widetilde{\emptyset}$ for any finite (resp. countable) set Λ, then $\mathfrak{E}$ is said to have the first type of finite (resp. countable) SD-intersection property.*

Theorem 2.11. *A soft$_{TS}$ (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf) iff $\widetilde{\bigcap}_{i \in I}(E_i, \Omega) \neq \widetilde{\emptyset}$ for every family $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ of soft cs-dense sets has the first type of finite (resp. countable) SD-intersection property.*

Proof. By contrary, suppose that $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ is a soft cs-dense subsets of (W, σ, Ω) with $\widetilde{\bigcap}_{i \in I}(E_i, \Omega) = \widetilde{\emptyset}$. Since (W, σ, Ω) is almost soft SD-compact and $\widetilde{W} = \widetilde{\bigcup}_{i \in I}(E_i^c, \Omega)$, then $\widetilde{W} = \widetilde{\bigcup}_{i=1}^{n} Scl(E_i^c, \Omega)$. Therefore, $\widetilde{\emptyset} = (\widetilde{\bigcup}_{i=1}^{n} Scl(E_i^c, \Omega))^c = \widetilde{\bigcap}_{i=1}^{n} Sint(E_i, \Omega)$, which is a contradiction.
Conversely, let $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ be a soft SD-cover of (W, σ, Ω). Then $\widetilde{\emptyset} = \widetilde{\bigcap}_{i \in I}(H_i^c, \Omega)$ and so by the first type of finite SD-intersection property, we have $\widetilde{\emptyset} = \widetilde{\bigcap}_{i=1}^{n} Sint(H_i^c, \Omega)$. Therefore, $\widetilde{W} = \widetilde{\bigcup}_{i=1}^{n} Scl(H_i, \Omega)$ and hence (W, σ, Ω) is almost soft SD-compact.
The case between parentheses can be achieved similarly. □

Theorem 2.12. *Let $\Psi_\Theta : (W, \sigma_W, \Omega) \to (Z, \sigma_Z, \Omega)$ be a soft SD-continuous map. Then the image of an almost soft SD- compact (resp. almost soft SD-Lindelöf) set is almost soft compact (resp. almost soft Lindelöf).*

Proof. Assume that (E, Ω) is an almost soft SD-Lindelöf subset of (W, σ, Ω) and $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ is a soft open cover of $\Psi_\Theta(E, \Omega)$. Now, for each $i \in I$, $\Psi_\Theta^{-1}(H_i, \Omega)$ is a null soft set or soft somewhere dense with $(E, \Omega)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in I}\Psi_\Theta^{-1}(H_i, \Omega)$. So there is a countable set Λ with $(E, \Omega)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in\Lambda} S\,cl(\Psi_\Theta^{-1}(H_i, \Omega))$; therefore, $\Psi_\Theta(E, \Omega)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in\Lambda}\Psi_\Theta(S\,cl(\Psi_\Theta^{-1}(H_i, \Omega)))$. By Theorem 1.23, $\Psi_\Theta(S\,cl(\Psi_\Theta^{-1}(H_i, \Omega)))$ $\widetilde{\subseteq} cl(\Psi_\Theta(\Psi_\Theta^{-1}(H_i, \Omega)))\widetilde{\subseteq} cl(H_i, \Omega)$. Thus, $\Psi_\Theta(E, \Omega)\widetilde{\subseteq}\ \widetilde{\bigcup}_{i\in\Lambda} cl(H_i, \Omega)$. Hence, $\Psi_\Theta(E, \Omega)$ is almost soft Lindelöf.
The case of almost soft SD-compact can be achieved similarly . □

Corollary 2.13. *The soft SD-irresolute image of an almost soft SD-compact (resp. almost soft SD-Lindelöf) set is almost soft SD-compact (resp. almost soft SD-Lindelöf).*

Definition 2.14. *Let (W, σ, Ω) be a soft$_{TS}$. For each soft subset (M, Ω) of (W, σ, Ω), define $(S\,cl(M), \Omega)$ as $S\,cl(M)(p) = S\,cl(M(p))$, where $S\,cl(M(p))$ is the S-closure of $M(p)$ in (W, σ_p) for each $p \in \Omega$.*

Proposition 2.15. *Let (W, σ, Ω) be a soft$_{TS}$. If (M, Ω) is a soft subset of (W, σ, Ω). Then*

(i) $(S\,cl(M), \Omega)\widetilde{\subseteq} S\,cl(M, \Omega)$.

(ii) $(S\,cl(M), \Omega) = S\,cl(M, \Omega)$ *iff* $(S\,cl(M), \Omega)$ *is soft cs-dense.*

Theorem 2.16. *If an enriched soft$_{TS}$ (W, σ, Ω) is an almost soft SD-compact (resp. almost soft SD-Lindelöf) space, then (W, σ_p) is almost SD-compact (resp. almost SD-Lindelöf), for each $p \in \Omega$.*

Proof. Assume that $\{H_i(p) : i \in I\}$ is a somewhere dense cover for (W, σ_p). We construct a soft SD-cover $\{(E_i, \Omega) : i \in I\}$ for (W, σ, Ω) with $E_i(p) = H_i(p)$ and for $p' \neq p$, $E_i(p') = W$. Since (W, σ, Ω) is almost soft SD-compact, then $\widetilde{W} = \bigcup_{i=1}^{n} S\,cl(E_i, \Omega) = \bigcup_{i=1}^{n}(S\,cl(E_i), \Omega)$. Therefore, $W = \bigcup_{i=1}^{n} S\,cl(E_i(p)) = \bigcup_{i=1}^{n} S\,cl(H_i(p))$ and so (W, σ_p) is an almost SD-compact space.
The case between parentheses can be achieved similarly. □

Proposition 2.17. *Let Ω be a finite (resp. countable) parameter set. For each $p \in \Omega$, if (W, σ_p) is almost SD-compact (resp. almost SD-Lindelöf), then (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf).*

Proof. Suppose that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ is a soft SD-cover of (W, σ, Ω). Then for each $p \in \Omega$, $W = \bigcup_{i\in I} H_i(p)$. Since for each $p \in \Omega$, (W, σ_p) is almost SD-compact then $W = \bigcup_{i=1}^{n_1} S\,cl(H_i(p_1))$, $W = \bigcup_{i=n_1+1}^{n_2} S\,cl(H_i(p_2))$,..., $W = \bigcup_{i=n_{m-1}+1}^{n_m} S\,cl(H_i(p_m))$. Therefore, $\widetilde{W} = \widetilde{\bigcup}_{i=1}^{n_m} S\,cl(H_i, \Omega)$ and hence (W, σ, Ω) is almost soft SD-compact.
A similar technique to prove the case between parentheses. □

Proposition 2.18. *If an enriched soft$_{TS}$ (W, σ, Ω) is an almost soft SD-compact (resp. almost soft SD-Lindelöf) space, then Ω is finite (resp. countable).*

Proof. It is obvious. □

3. Nearly soft SD-compact and nearly soft SD-Lindelöf spaces

In this section, we present generalizations for almost soft SD-compact and almost soft SD-Lindelöf spaces called nearly soft SD-compact and nearly soft SD-Lindelöf spaces. Also, some properties of those generalizations are studied.

Definition 3.1. *A soft$_{TS}$ (W, σ, Ω) is said to be nearly soft SD-compact (resp. nearly soft SD-Lindelöf) if for every soft SD-cover $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ of (W, σ, Ω), there is a finite (resp. countable) set $\Lambda \subseteq I$ with $\widetilde{W} = S\,cl(\widetilde{\bigcup}_{i\in\Lambda}(H_i, \Omega))$.*

Proposition 3.2. *If a soft$_{TS}$ (W, σ, Ω) is nearly soft SD-compact, then (W, σ, Ω) is nearly soft SD-Lindelöf.*

Proof. It follows from Definition 3.1. □

Definition 3.3. *A soft set (M, Ω) of a soft$_{TS}$ (W, σ, Ω) is called soft SD-dense set if $S\,cl(M, \Omega) = \widetilde{W}$.*

The following example shows that the converse of Proposition 3.2 is not true.

Example 3.4. *Let $\Omega = \{p, q\}$ be a set of parameters. Consider $(\mathbb{R}, \sigma, \Omega)$ as a soft$_{TS}$ where $\sigma = \{\widetilde{\emptyset}, (H, \Omega)\widetilde{\subseteq}\widetilde{\mathbb{R}}$ with for each $p \in \Omega$, $H(p) = n$ or their soft union, where $n \in \mathbb{N}\}$. Then (H, Ω) is soft somewhere dense iff each component contains a natural number. Set $\mathfrak{H} = \sigma$, then $\mathfrak{H}$ is a soft SD-cover of $\widetilde{\mathbb{R}}$. Now, any soft SD-cover of $\widetilde{\mathbb{R}}$ which has a countable soft somewhere dense subset of $\mathfrak{H}$ contains a soft open set $\{(p_1, \mathbb{N}), (p_2, \mathbb{N})\}$. This soft somewhere dense set is soft SD-dense; hence, $(\mathbb{R}, \sigma, \Omega)$ is nearly soft SD-Lindelöf. However, $(\mathbb{R}, \sigma, \Omega)$ is not nearly soft SD-compact since $\mathfrak{H}$ has no a finite subcover which its soft SD-closure covers $\widetilde{\mathbb{R}}$.*

Proposition 3.5. *If a soft$_{TS}$ (W, σ, Ω) is nearly soft SD-compact (resp. nearly soft SD-Lindelöf), then a finite (resp. countable) union of soft subsets of (W, σ, Ω) is nearly soft SD-compact (resp. nearly soft SD-Lindelöf).*

Proof. Assume that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ which is a soft SD-cover of $\widetilde{\bigcup}_{k\in\Lambda}(E_k, \Omega)$ where $\{(E_k, \Omega) : k \in \Lambda\}$ is a family of nearly soft SD-Lindelöf subsets of (W, σ, Ω). So there is countable sets I_k^* with $(E_1, \Omega)\widetilde{\subseteq}S\,cl(\widetilde{\bigcup}_{i\in I_1^*}(H_i, \Omega))$, ..., $(E_n, \Omega)\widetilde{\subseteq}S\,cl(\widetilde{\bigcup}_{i\in I_n^*}(H_i, \Omega))$, ... Therefore, $\widetilde{\bigcup}_{k\in\Lambda}(E_k, \Omega)\widetilde{\subseteq}S\,cl(\widetilde{\bigcup}_{i\in I_1^*}(H_i, \Omega))\widetilde{\bigcup}\ldots\widetilde{\bigcup}S\,cl(\widetilde{\bigcup}_{i\in I_n^*}(H_i, \Omega))\widetilde{\bigcup}\ldots\widetilde{\subseteq}S\,cl\ (\widetilde{\bigcup}_{i\in \bigcup_{k\in\Lambda} I_k^*}(H_i, \Omega))$ where $\bigcup_{k\in\Lambda} I_k^*$ is countable.
The case of a nearly soft SD-compact space can be achieved similarly. □

Proposition 3.6. *If a soft$_{TS}$ (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf), then (W, σ, Ω) is nearly soft SD-compact (resp. nearly soft SD-Lindelöf).*

Proof. It follows from $\widetilde{\bigcup}_{i\in I}S\,cl(H_i, \Omega)\widetilde{\subseteq}S\,cl(\widetilde{\bigcup}_{i\in I}(H_i, \Omega))$, where (H_i, Ω) is a soft subset of (W, σ, Ω). □

A soft$_{TS}$ (W, σ, Ω) is called soft SD-hyperconnected if it does not contain two disjoint soft somewhere dense sets.

Corollary 3.7. *If a soft$_{TS}$ (W, σ, Ω) is soft SD-hyperconnected, then (W, σ, Ω) is nearly soft SD-Lindelöf.*

The following example demonstrates the converse of Proposition 3.6 is not true.

Example 3.8. *Let $\Omega = \{p, q\}$ be a set of parameters. Consider $(\mathbb{R}, \sigma, \Omega)$ as a soft$_{TS}$ with $\sigma = \{\widetilde{\emptyset}, \widetilde{\mathbb{R}}, (E_1, \Omega), (E_2, \Omega), (E_3, \Omega)\}$, where for each $r \in \Omega$, $E_1(r) = \{1\}, E_2(r) = \{2\}, E_3(r) = \{1, 2\}$. Then (E, Ω) is soft somewhere dense iff $1 \Subset (E, \Omega)$ or $2 \Subset (E, \Omega)$. Set $\mathfrak{E} = \{(E, \Omega)$ is finite such that there is only one parameter $r \in \Omega$ with $1 \in E(r)$ or $2 \in E(r)\}$, then $\mathfrak{E}$ is a soft SD-cover of $\widetilde{\mathbb{R}}$. Now, any soft SD-cover of $\widetilde{\mathbb{R}}$ contains four soft somewhere dense subsets of $\mathfrak{E}$ which contains a soft open set (E_3, Ω). Since a soft somewhere dense set (E_3, Ω) is soft SD-dense, then $(\mathbb{R}, \sigma, \Omega)$ is nearly soft SD-compact. However, $(\mathbb{R}, \sigma, \Omega)$ is not almost soft SD-Lindelöf since $\mathfrak{E}$ is a soft SD-cover does not have a countable subcover in which its soft SD-closure of whose members covers $\widetilde{\mathbb{R}}$.*

Definition 3.9. *Let $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ be a family of soft sets. If $S\,int(\widetilde{\bigcap}_{i\in\Lambda}(E_i, \Omega)) \neq \widetilde{\emptyset}$ for any finite (resp. countable) set Λ, then $\mathfrak{E}$ is said to have the second type of finite (resp. countable) SD-intersection property.*

Note that if a family $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ has the second type of finite (resp. countable) SD-intersection property, then it has the first type of finite (resp. countable) SD-intersection property.

Theorem 3.10. *A soft$_{TS}$ (W, σ, Ω) is nearly soft SD-compact (resp. nearly soft SD-Lindelöf) iff $\widetilde{\bigcap}_{i\in I}(E_i, \Omega) \neq \widetilde{\emptyset}$ for every family $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ of soft cs-dense sets has the second type of finite (resp. countable) SD-intersection property.*

Proof. We give the proof when (W, σ, Ω) is nearly soft SD-compact. The other case can be made similarly.
By contrary, suppose that $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ is a family of soft cs-dense subsets of (W, σ, Ω) with $\widetilde{\bigcap}_{i\in I}(E_i, \Omega) = \widetilde{\emptyset}$. Since (W, σ, Ω) is nearly soft SD-compact and $\widetilde{W} = \widetilde{\bigcup}_{i\in I}(E_i^c, \Omega)$, then $\widetilde{W} = S\,cl(\widetilde{\bigcup}_{i=1}^{n}(E_i^c, \Omega))$. Therefore, $\widetilde{\emptyset} = (S\,cl(\widetilde{\bigcup}_{i=1}^{n}(E_i^c, \Omega)))^c = S\,int(\widetilde{\bigcap}_{i=1}^{n}(E_i, \Omega))$, which is a contradiction.
Conversely, it follows from Theorem 2.11 and Proposition 3.6. □

Theorem 3.11. *If an enriched soft$_{TS}$ (W, σ, Ω) is nearly soft SD-compact (resp. nearly soft SD-Lindelöf), then (W, σ_p) is nearly SD-compact (resp. nearly SD-Lindelöf), for each $p \in \Omega$.*

Proof. Assume that $\{H_i(p) : i \in I\}$ is a somewhere dense cover for (W, σ_p). We construct a soft SD-cover $\{(E_i, \Omega) : i \in I\}$ for (W, σ, Ω) with $E_i(p) = H_i(p)$ and for each $p' \neq p$, $E_i(p') = W$. Since (W, σ, Ω) is nearly soft SD-Lindelöf, there is a countable set Λ with $\widetilde{W} = S\,cl(\widetilde{\bigcup\limits_{i\in\Lambda}}(E_i, \Omega))\widetilde{\subseteq}(S\,cl(\bigcup E_i), \Omega)$. Therefore, $W = S\,cl(\bigcup\limits_{i\in\Lambda} E_i(p_1)) = S\,cl(\bigcup\limits_{i\in\Lambda} H_i(p_1))$ and so (W, σ_p) is nearly SD-Lindelöf.
The case of a nearly soft SD-compact space can be achieved similarly. □

Proposition 3.12. *A soft$_{TS}$ (W, σ, Ω) is nearly soft SD-compact (resp. nearly soft SD-Lindelöf), if there is a finite (resp. countable) soft SD-dense subset of (W, σ, Ω), where Ω is finite (resp. countable).*

Proof. Assume that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ is a soft SD-cover of (W, σ, Ω) and (E, Ω) is a finite (countable) soft SD-dense subset of (W, σ, Ω). Now, for each $P_{p_s}^{q_s} \in (E, \Omega)$, there is $(H_{q_s}, \Omega) \in \mathfrak{H}$ containing $P_{p_s}^{q_s}$; hence, $\widetilde{W} = S\,cl(\widetilde{\bigcup}(H_{q_s}, \Omega))$. The collection $\{(H_s, \Omega)\}$ is finite (countable) because (E, Ω) and Ω are finite (countable). □

By using Theorem 2.12, we can prove the following theorem.

Theorem 3.13. *Let $\Psi_\Theta : (W, \sigma_W, \Omega) \rightarrow (Z, \sigma_Z, \Omega)$ be a soft SD-irresolute map. Then the image of a nearly soft SD-compact (resp. nearly soft SD-Lindelöf) set is nearly soft SD-compact (resp. nearly soft SD-Lindelöf).*

4. Mildly soft SD-compact and mildly soft SD-Lindelöf spaces

In this section, we introduce another generalizations of almost soft SD-compact and almost soft SD-Lindelöf spaces, we put some restrictions to become all of the soft spaces that introduced are equivalent.

Definition 4.1. *A soft subset of a soft$_{TS}$ (W, σ, Ω) is called a soft SC-set if it is both soft somewhere dense and soft cs-dense.*

Definition 4.2. *A family of soft SC-subsets of a soft$_{TS}$ (W, σ, Ω) is called soft SC-cover of $\widetilde{W}$ if $\widetilde{W}$ is a soft subset of this family.*

Definition 4.3. *A soft$_{TS}$ (W, σ, Ω) is said to be mildly soft SD-compact (resp. mildly soft SD-Lindelöf) if for every soft SC-cover $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ of $(W\sigma, \Omega)$, there is a finite (resp. countable) set $\Lambda \subseteq I$ with $\widetilde{W} = \widetilde{\bigcup}_{i\in\Lambda}(H_i, \Omega)$.*

From Definition 4.3 we can get the following proposition.

Proposition 4.4. *If a soft$_{TS}$ (W, σ, Ω) is mildly soft SD-compact, then (W, σ, Ω) is mildly soft SD-Lindelöf.*

Example 3.4 illustrates that the converse of Proposition 4.4 is not true.

Proposition 4.5. *If a soft$_{TS}$ (W, σ, Ω) is mildly soft SD-compact (resp. mildly soft SD-Lindelöf), then a finite (resp. countable) union of subsets of (W, σ, Ω) is mildly soft SD-compact (resp. mildly soft SD-Lindelöf).*

Proof. It is obvious. □

Proposition 4.6. *If a soft$_{TS}$ (W, σ, Ω) is almost soft SD-compact (resp. almost soft SD-Lindelöf), then (W, σ, Ω) is mildly soft SD-compact (resp. mildly soft SD-Lindelöf).*

Proof. Assume that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ is a soft SC-cover of (W, σ, Ω). Since (W, σ, Ω) is almost soft SD-Lindelöf, there is a countable set Λ with $\widetilde{W} = \widetilde{\bigcup}_{i\in\Lambda} S\,cl(H_i, \Omega) = \widetilde{\bigcup}_{i\in\Lambda}(H_i, \Omega)$. Thus, (W, σ, Ω) is mildly soft SD-Lindelöf.
The other case can be achieved similarly. □

Proposition 4.7. *If a soft$_{TS}$ (W, σ, Ω) is soft SD-hyperconnected, then the following are equivalent:*

(i) (W, σ, Ω) *is almost soft SD-compact.*

(ii) (W, σ, Ω) *is almost soft SD-Lindelöf.*

(iii) (W, σ, Ω) *is nearly soft SD-compact.*

(iv) (W, σ, Ω) *is nearly soft SD-Lindelöf.*

(v) (W, σ, Ω) *is mildly soft* SD*-compact.*

(vi) (W, σ, Ω) *is mildly soft* SD*-Lindelöf.*

Proof. Since (W, σ, Ω) is soft SD-hyperconnected, then the soft S-closure of any non null soft open set is $\widetilde{W}$ and the only soft SC-sets are $\widetilde{W}$ and $\widetilde{\emptyset}$, so the proof becomes clear. □

Proposition 4.8. *If a soft*$_{TS}$ (W, σ, Ω) *is soft* SD*-connected, then* (W, σ, Ω) *is mildly soft* SD*-compact.*

Proof. Since the only soft SC-subsets of a soft SD-connected space (W, σ, Ω) are $\widetilde{W}$ and $\widetilde{\emptyset}$, then (W, σ, Ω) is mildly soft SD-compact. □

If we replace the natural numbers set $\mathbb{N}$ in Example 2.6 by a set $W = \{1, 2, 3\}$, then we obtain (W, σ, Ω) is mildly soft SD-compact. But it is soft SD-disconnected. This shows that the converse of the above proposition fails.

To illustrate that not every nearly soft SD-Lindelöf space is mildly soft SD-Lindelöf, we give the following example.

Example 4.9. *Define a soft*$_{TS}$ $(\mathbb{R}, \sigma, \Omega)$ *as in Example 3.8. It was shown that* $(\mathbb{R}, \sigma, \Omega)$ *is nearly soft* SD*-Lindelöf space. On the other hand,* $(\mathbb{R}, \sigma, \Omega)$ *is not a mildly soft* SD*-Lindelöf since the family* $\mathfrak{E}$ *is a soft* SC*-cover of* $\widetilde{\mathbb{R}}$ *has no a countable subcover.*

Theorem 4.10. *A soft*$_{TS}$ (W, σ, Ω) *is mildly soft* SD*-compact (resp. mildly soft* SD*-Lindelöf) iff* $\widetilde{\bigcap}_{i \in I}(E_i, \Omega) \neq \widetilde{\emptyset}$ *for every family* $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ *of soft* SC*-subsets of* (W, σ, Ω) *that have the finite (resp. countable) intersection property.*

Proof. By contrary, suppose that $\mathfrak{E} = \{(E_i, \Omega) : i \in I\}$ is a family of soft SC-subsets of $\widetilde{W}$ with $\widetilde{\bigcap}_{i \in I}(E_i, \Omega) = \widetilde{\emptyset}$. Since (W, σ, Ω) is mildly soft SD-compact and $\widetilde{W} = \widetilde{\bigcup}_{i \in I}(E_i^c, \Omega)$, then $\widetilde{W} = \widetilde{\bigcup}_{i=1}^{n}(E_i^c, \Omega)$. Therefore, $\widetilde{\bigcap}_{i=1}^{n}(E_i, \Omega) = \widetilde{\emptyset}$, which is a contradiction.

Conversely, let $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ be a soft SC-cover of $\widetilde{W}$. Suppose $\mathfrak{H}$ has no finite subcover of $\widetilde{W}$, so for each $n \in \mathbb{N}$, $\widetilde{W} - \widetilde{\bigcup}_{i=1}^{n}(H_i, \Omega) \neq \widetilde{\emptyset}$. Now, $\{(H_i^c, \Omega) : i \in I\}$ is a family of soft SC-subsets of $\widetilde{W}$ which has the finite intersection property, so $\widetilde{\bigcap}_{i \in I}(H_i^c, \Omega) \neq \widetilde{\emptyset}$. Therefore, $\widetilde{W} \neq \widetilde{\bigcup}_{i \in I}(H_i, \Omega)$, which is a contradiction. Hence, (W, σ, Ω) is mildly soft SD-compact.

The case between parentheses can be achieved similarly. □

Proposition 4.11. *Let* $\Psi_\Theta : (W, \sigma_W, \Omega) \to (Z, \sigma_Z, \Omega)$ *be a soft* SD*-irresolute map. Then the image of mildly soft* SD*-compact (resp. mildly soft* SD*-Lindelöf) set is mildly soft* SD*-compact (resp. mildly soft* SD*-Lindelöf).*

Proof. By using a similar technique of the proof of Theorem 2.12, the proposition holds. □

The proof of the following two propositions is obvious, so it will be omitted.

Proposition 4.12. *If a soft*$_{TS}$ (W, σ, Ω) *is mildly soft* SD*-compact (resp. mildly soft* SD*-Lindelöf), then every soft* SC*-subset* (M, Ω) *is mildly soft* SD*-compact (resp. mildly soft* SD*-Lindelöf).*

Proposition 4.13. *The soft intersection of soft* SC*-sets and mildly soft* SD*-compact (resp. mildly soft* SD*-Lindelöf) is mildly soft* SD*-compact (resp. mildly soft* SD*-Lindelöf).*

Definition 4.14. *A* $soft_{TS}$ (W, σ, Ω) *is said to be a soft* SD*-partition if a soft set is soft somewhere dense iff it is soft* cs*-dense.*

Theorem 4.15. *If a* $soft_{TS}$ (W, σ, Ω) *is soft* SD*-partition, then the following are equivalent:*

(i) (W, σ, Ω) *is almost soft* SD*-Lindelöf (resp. almost soft* SD*-compact).*

(ii) (W, σ, Ω) *is nearly soft* SD*-Lindelöf (resp. nearly soft* SD*-compact).*

(iii) (W, σ, Ω) *is mildly soft* SD*-Lindelöf (resp. mildly soft* SD*-compact).*

Proof. **(i)** → **(ii)** It follows from Proposition 3.6.
(ii) → **(iii)** Assume that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ is an SC-cover of a nearly soft SD-Lindelöf space (W, σ, Ω). Then there is a countable set $\Lambda \subseteq I$ with $\widetilde{W} = Scl(\widetilde{\bigcup}_{i\in\Lambda}(H_i, \Omega))$. Now, since (W, σ, Ω) is a soft SD-partition, then $Scl(\widetilde{\bigcup}_{i\in\Lambda}(H_i, \Omega)) = \widetilde{\bigcup}_{i\in\Lambda}(H_i, \Omega)$. Hence, (W, σ, Ω) is mildly soft SD-Lindelöf.
(iii) → **(i)** Assume that $\mathfrak{H} = \{(H_i, \Omega) : i \in I\}$ is a soft SD-cover of a soft SD-partition space (W, σ, Ω). Then $\mathfrak{H}$ is an SC-cover of a mildly soft SD-Lindelöf space (W, σ, Ω) , so there is a countable set $\Lambda \subseteq I$ with $\widetilde{W} = \widetilde{\bigcup}_{i\in\Lambda} Scl(H_i, \Omega)$.
The case between parentheses can be achieved similarly. □

Definition 4.16. *Let* (M, Ω) *be a soft subset of a* $soft_{TS}$ (W, σ, Ω)*. For each* $p \in \Omega$*, define* $(Sint(M), \Omega)$ *as* $Sint(M)(p) = Sint(M(p))$ *where* $Sint(M(p))$ *is the* SD*-interior of* $M(p)$ *in* (W, σ_p)*.*

Proposition 4.17. *Let* (W, σ, Ω) *be a* $soft_{TS}$*. If* (M, Ω) *is a soft subset of* $\widetilde{W}$*. Then*

(i) $Sint(M, \Omega) \widetilde{\subseteq} (Sint(M), \Omega)$.

(ii) $Sint(M, \Omega) = (Sint(M), \Omega)$ *iff* $(Sint(M), \Omega)$ *is soft somewhere dense.*

Proof. **(i)** For every $p \in \Omega$, $Sint(M(p))$ is the largest somewhere dense subset of (W, σ_p) contained in $M(p)$. Set $Sint(M, \Omega) = (E, \Omega)$. We infer that $E(p) \subseteq Sint(M(p)) = Sint(M)(p)$ since $E(p)$ is a somewhere dense subset of (W, σ_p) contained in $M(p)$. Hence, $Sint(M, \Omega) \widetilde{\subseteq} (Sint(M), \Omega)$.

(ii) If $(Sint(M), \Omega) = Sint(M, \Omega)$, then $(Sint(M), \Omega)$ is a soft somewhere dense set. Conversely, from **(i)**, we obtain $Sint(M, \Omega) \widetilde{\subseteq} (Sint(M), \Omega)$. Now, we need to show that $(Sint(M), \Omega) \widetilde{\subseteq} Sint(M, \Omega)$. Let $(Sint(M), \Omega)$ be a soft somewhere dense set. Since $(Sint(M), \Omega)$ contained in (M, Ω) and from Definition 4.16 we conclude that $(Sint(M), \Omega) \widetilde{\subseteq} Sint(M, \Omega)$. Hence, $(Sint(M), \Omega) = Sint(M, \Omega)$. □

The proof of the following proposition is obvious, so it will be omitted.

Proposition 4.18. *If an enriched* $soft_{TS}$ (W, σ, Ω) *is soft mildly* SD*-compact (resp. soft mildly* SD*-Lindelöf), then* Ω *is finite (resp. countable).*

5. Conclusions

We define some new types of soft spaces based on soft somewhere dense sets, namely, almost (nearly, mildly) soft SD-compactness and almost (nearly, mildly) soft SD-Lindelöfness. We use examples to illustrate the relationships between these concepts, and we analyze the image of these spaces under soft SD-continuous and soft SD-irresolute mappings.

The different types of soft compact and Lindelöf spaces introduced herein help us to classify soft structures into new different families which help us to model some real-life problems as those given in [7]. Also, we can apply soft somewhere dense sets and these families to initiate new types of approximations and accuracy measures in the content of rough sets models. Finally, the given concepts herein allow us to study many results induced from their interaction with some soft topological notions such as soft Menger and soft connected spaces.

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Conflict of interest

The authors declare that they have no competing interests.

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