Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 6(7): 7367–7385.
DOI:10.3934/math.2021432
Received: 23 December 2020
Accepted: 25 April 2021
Published: 30 April 2021

Research article

Intersectional soft gamma ideals of ordered gamma semigroups

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Abstract: In contemporary mathematics, parameterization tool like soft set theory precisely tackle complex problems of economics and engineering. In this paper, we demonstrate a novel approach of soft set theory i.e., intersectional soft (int-soft) sets of an ordered Γ-semigroup S and develop int-soft left (resp. right) Γ-ideals of S. Various classes like Γ-regular, left Γ-simple, right Γ-simple and some semilattices of an ordered Γ-semigroup S are characterize through int-soft left (resp. right) Γ-ideals of S. Particularly, a Γ-regular ordered Γ-semigroup S is a left Γ-simple if and only if every int-soft left Γ-ideal f_A of S is a constant function. Also, S is a semilattice of left (resp. right) Γ-simple Γ-semigroup if and only if for every int-soft left (resp. right) Γ-ideal f_A of S, $f_A(a) = f_A(a\alpha a)$ and $f_A(a\alpha b) = f_A(b\alpha a)$ for all $a, b \in S$ and $\alpha \in \Gamma$ hold.

Keywords: soft sets; int-soft left (right) Γ-ideals; Γ-regular ordered Γ-semigroups; left (right) Γ-simple ordered Γ-semigroups
Mathematics Subject Classification: 08Axx, 08A72, 16Wxx

1. Introduction

Unlike soft sets theory, most of the uncertainties theories such as fuzzy sets theory, probability theory and theory of rough sets are left behind due to the lack of parameterization. Despite the fact such theories can tackle various uncertainties problems but these theories have their own inherent limitations: incompatibility with the parameterization tools is one of the major problem associated with these theories. In order to overcome these implied challenges, Molodtsove [1] initiated the pioneering concept of soft set theory. This contemporary approach is free from the difficulties pointed out in the other theories of uncertainties particularly theories involving membership functions. Due to its dynamical nature, the soft sets successfully made its place and now extensively used in several applied fields like control engineering, information sciences, computer sciences, economics and decision making problems [2–11].

It is also worth-mentioning that due to the use of soft sets in algebraic framework changed the researchers approach toward algebraic structures and now these algebraic structures are extensively used in the aforementioned fields. For instance, Maji et al. [12] presented various operations of soft sets in algebraic framework which were further extended by Ali et al. [13, 14].

Note that ordered semigroups playing a key role in mathematics particularly in ordered theory [15]. Ordered semigroups are comprehensively studied by Kehyopulu [16–18]. Ordered gamma semigroups are the generalizations of ordered semigroups. Sen and Saha [19] were the first who initiated the concept of a gamma semigroup, and established a relation between regular gamma semigroup and gamma group. Beside this, several classical notions of semigroups have been extended to Γ-semigroup in [19–21]. Kwon and Lee [22] introduced the concept of Γ-ideals and weakly prime Γ-ideals in ordered Γ-semigroups. The notion of bi-Γ-ideal in Γ-semigroups was introduced by Chinram and Jirojkul in [23]. Dutta and Adhikari introduced the notion of prime Γ-ideal in Γ-semigroup in [24]. On the other hand the concepts of prime bi-Γ-ideal, strongly prime bi-Γ-ideal, semiprime bi-Γ-ideal, strongly irreducible and irreducible bi-Γ-ideals of Γ-semigroup are studied in [25]. Prince William et al. [26] provided the characterization of gamma semigroups in terms of bi-Γ-ideals (also refer to [27]). Jun et al. [28] investigated fuzzy ideals in gamma nearrings (also see [29]). In 2009, Iampan [30] gave the concept of (0-)minimal and maximal ordered bi-ideals in ordered Γ-semigroups, and give some characterizations theorems.

Jun and Song [31] applied the soft sets to one of abstract algebraic structures, the so-called semigroup. They took a semigroup as the parameter set for combining soft sets with semigroups. Their work is the continuation of [32]. Where they further discussed the properties and characterizations of int-soft left (right) ideals. More precisely, they introduce the notion of int-soft (generalized) bi-ideals and provide relations between int-soft generalized bi-ideals and int-soft semigroups. Khan and Sarwer [33] extended the concept of uni-soft ideals given in [34], by introducing some new ideals namely; uni-soft bi-ideals and uni-soft interior ideals of AG-groupoids and also discuss some related results. Jun et al. [35] introduced the notions of union-soft semigroups, union-soft l-ideals, and union-soft γ-ideals and determined various properties. They consider characterizations of a union-soft semigroup, a union-soft l-ideal,and a union-soft γ-ideal. Moreover, the concepts of union-soft products and union-soft semiprime soft sets are determined and their properties related to union-soft l-ideals and union-soft γ-ideals are investigated. Khan et al. [36] characterized weakly regular, intra-regular and semisimple ordered semigroups by the properties of their uni-soft ideals. Hamouda [37], developed the notions of soft left and soft right ideals, soft quasi-ideal and soft bi-ideal in ordered semigroups.

Recently, the int-soft's idea gain a centeral attention around the globe. Various applications of the said idea can be seen in a variety of research. For instance, Muhiuddin and Mahboob [38] developed a new ideal theory termed as Int-soft ideals in ordered semigroups. More precisely, the authors introduced int-soft left (right) ideals, int-soft interior ideals and int-soft bi-ideals in ordered semigroups and constructed various characterization theorems based on these newly developed idea. Ghosh et al. [39] investigated various properties of rings based on soft radical of a soft int-ideal, soft prime int-ideal, soft semiprime int-ideal. They broadly discussed that the direct and inverse images of soft prime (soft semiprime) int-ideals under homomorphism remains invariant. Further, Khan et al. [40] commence the notion of soft near-semirings with a varieties of essential results. They also explored several characterization theorems by using soft near-semiring homomorphism and soft near-semiring anti-homomorphism. Sezer et al. [41] gave the define of soft intersection semigroups, soft intersection left (right, two-sided) ideals and bi-ideals of semigroups. They extended their study by characterizing regular, intra-regular, completely regular, weakly regular and quasi-regular semigroups in terms of these newly developed ideals also refer to [42–45].

From the above discussion it is cleared that soft set theory is a remarkable mathematical tool dealing with uncertainity. The researchers working in this particular area of research are more interested to know how to link abstract algebra with these newely developed sets i.e., soft sets? Infact, several researchers worked on this, for instance, the theory of soft sets has been applied to rings, fields and modules [46, 47], groups [48], semigroups [49], ordered semigroups [32, 50] and hypervector space [51]. In this paper, we introduce a new notion worth applying to abstract algebraic structure. So we can provide the possibility of a new direction of soft sets based on abstract algebraic structure (ordered Γ-semigroup) in dealing with uncertainity. Infact, we have developed a new type of ideal theory in ordered Γ-semigroups based on soft sets. This new type of ideal theory will constitute a platform for other researchers to apply this conception in other algebraic structures as well. Particularly, we introduce some new types of soft Γ-ideals i.e., intersectional soft (int-soft) Γ-ideal of an ordered Γ-semigroup S and initiate int-soft left (resp. right) Γ-ideals of S. Several classes like Γ-regular, left Γ-simple, right Γ-simple and some semilattices of an ordered Γ-semigroup S are characterized by the properties of int-soft left (resp. right) Γ-ideals of S. Further, a Γ-regular ordered Γ-semigroup S is a left Γ-simple if and only if every int-soft left Γ-ideal f_A of S is a constant function. Moreover, S is a semilattice of left (resp. right) Γ-simple Γ-semigroup if and only if for every int-soft left (resp. right) Γ-ideal f_A of S, $f_A(a) = f_A(a\alpha a)$ and $f_A(a\alpha b) = f_A(b\alpha a)$ hold for all $a, b \in S$ and $\alpha \in \Gamma$.

2. Preliminaries

Definition 2.1. Suppose $S = \{x, y, z, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ are two non-empty sets and a function $S \times \Gamma \times S \longrightarrow S$ such that $(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$, then S is called *a Γ-semigroup* [19]. By an *ordered Γ-semigroup* $(S, \Gamma, \leq)$, we mean a Γ-semigroup S satisfying the following conditions:

(i) $(S, \leq)$ is a poset.

(ii) If $a, b, x \in S$ and $\alpha, \beta \in \Gamma$ then $a \leq b \Longrightarrow a\alpha x \leq b\alpha x$ and $x\beta a \leq x\beta b$.

For $A \subseteq S$, we denote $(A] := \{t \in S | t \leq h$ for some $h \in A\}$. If $A = \{a\}$, then we write $(a]$ instead of $(\{a\}]$. For $A, B \subseteq S$, we denote

$$A\Gamma B := \{a\alpha b | a \in A, b \in B, \alpha \in \Gamma\} \tag{2.1}$$

Definition 2.2. A non-empty subset A of an ordered Γ-semigroup S is called a *left* (resp. *right*) Γ-*ideal* of S if it satisfies

(i) $(\forall a, b \in S)(\forall b \in A)(a \leq b \Longrightarrow a \in A)$,

(ii) $S\Gamma A \subseteq A$ (resp. $A\Gamma S \subseteq A$).

If A is both left Γ-ideal and a right Γ-ideal of S then A is called Γ-ideal of S.

Definition 2.3. A non-empty subset A of an ordered Γ-semigroup S is called Γ-*subsemigroup* of S if it satisfies $A\Gamma A \subseteq A$.

Definition 2.4. A Γ-*subsemigroup* B of S is called bi-Γ-ideal of S, if it satisfies:

(i) $(\forall a, b \in S)(\forall b \in B)(a \leq b \Longrightarrow a \in B)$,
(ii) $B\Gamma S\Gamma B \subseteq B$.

Definition 2.5. A Γ-*subsemigroup* A of S is called $(1,2)$-Γ-ideal of S, if it satisfies:

(i) $(\forall a, b \in S)(\forall b \in A)(a \leq b \Longrightarrow a \in A)$,
(ii) $A\Gamma S\Gamma A\Gamma A \subseteq A$.

Soft Sets (Basic operations) In the last two decades, the uses of soft set theory is achieving another milestone in contemporary mathematics where several mathematical problems involving uncertainties in various field like decision making, automata theory, coding theory, economics and much others which can not be handle through ordinary mathematical tools (like fuzzy set theory, theory of probability etc) due to the lake of parameterization. The latest research in this direction and the new investigations of soft set theory is much productive due to the diverse applications of soft sets in the aforementioned fields [2–11]. It is important to note that Sezgin and Atagun [52] introduced some new operations on soft set theory and defined soft sets in the following way:

Suppose U be universal set, E be the set of parameters, $P(U)$ be the power set of U and A be a subset of E. Then a soft set f_A over U is an approximate function defined by:

Definition 2.6. Suppose U be universal set, E be the set of parameters, $P(U)$ be the power set of U and A be a subset of E. Then a soft set f_A over U is an approximate function defined by:

$$f_A : E \longrightarrow P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \notin A. \tag{2.2}$$

Symbolically a soft set over U is the set of ordered pairs

$$f_A = \{(x, f_A(x)) : x \in E,\ f_A(x) \in P(U)\}. \tag{2.3}$$

A soft set is a parameterized family of subsets of U, where $S(U)$ denotes the set of all soft sets.

Example 1. *Suppose Mr. Lee want to buy various business corners in newly developed supermarket having hundred business corners $\{c_1, c_2, ..., c_{100}\} = U$. For the said purpose, Mr. Lee has three different parameters in mind that are "beautiful (e_1)", "cheap (e_2)" and "good location (e_3)". These parameters are represented by the set $E = \{e_1, e_2, e_3\}$. Now for few corners he only consider $\{e_1, e_3\} = A$. Therefore, an approximate function $f_A : E \longrightarrow P(U)$ will image $f_A(e_2) = \emptyset$ as $e_2 \notin A$ and ultimately he will have only those choices from $P(U)$ which depend on e_1, e_3. Similarly, for any other subset of parameters, Mr. Lee can select a better corner for his business.*

Definition 2.7. Suppose $f_A, f_B \in S(U)$. Then f_A is said to be subset of f_B denoted by $f_A \widetilde{\subseteq} f_B$ if $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Also, two soft sets f_A, f_B are said to be equal denoted by $f_A \widetilde{=} f_B$, if $f_A \widetilde{\subseteq} f_B$ and $f_A \widetilde{\supseteq} f_B$ holds.

Definition 2.8. Let $f_A, f_B \in S(U)$,then the union of f_A and f_B, denoted by $f_A \widetilde{\cup} f_B$ is defined by $f_A \widetilde{\cup} f_B = f_{A\cup B}$, where $f_{A\cup B}(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.

Definition 2.9. If f_A, $f_B \in S(U)$, then the intersection of f_A and f_B, denoted by $f_A \widetilde{\cap} f_B$ is defined by $f_A \widetilde{\cap} f_B = f_{A\cap B}$, where $f_{A\cap B}(x) = f_A(x) \cap f_B(x)$ for all $x \in E$.

For any soft set f_A over U and $\gamma \subseteq U$, the γ-inclusive set is denoted by $i_A(f_A, \gamma)$ and is defined as

$$i_A(f_A, \gamma) = \{x \in A \mid f_A(x) \supseteq \gamma\}. \tag{2.4}$$

3. Int-soft left (resp. right) Γ-ideals

In this section, we introduce new types of Γ-ideals known as Int-soft left (resp. right) Γ-ideals of an ordered Γ-semigroup S. Ordinary left (resp. right) Γ-ideals are linked with these new types of Int-soft left (resp. right) Γ-ideals in ordered Γ-semigroup S. Several characterization theorems of an ordered Γ-semigroup are developed in terms of Int-soft left (resp. right) Γ-ideals.

Definition 3.1. Suppose $(S, \Gamma, \leq)$ is an ordered Γ-semigroup and f_A is a soft set over U, then f_A is called int-soft left (resp. right) Γ-ideal of S if:

(i) $(\forall\, x, y \in S)\,(x \leq y \longrightarrow f_A(x) \supseteq f_A(y))$.
(ii) $(\forall\, x, y \in S, \alpha \in \Gamma)\,(f_A(x\alpha y) \supseteq f_A(y)$ (resp. $f_A(x\alpha y) \supseteq f_A(x)))$.

An int-soft left and int-soft right Γ-ideal of S is called int-soft two sided Γ-ideal of S.

Theorem 3.2. *If $(S, \Gamma, \leq)$ is an ordered Γ-semigroup and f_A is a soft set of S, then the following conditions are equivalent:*

(1) f_A is an int-soft left (resp. right) Γ-ideal of S.
(2) For every $\gamma \subseteq U$, $i_A(f_A, \gamma)$ is a left (resp. right) Γ-ideal of S.

Proof. (1) $\Longrightarrow$ (2): Suppose f_A is an int-soft left Γ-ideal of S, we need to show that $i_A(f_A, \gamma)$ is a left Γ-ideal of S. For this let $a, b \in S$ such that $a \leq b$ and $b \in i_A(f_A, \gamma)$, then $f_A(b) \supseteq \gamma$. Since f_A is an int-soft left Γ-ideal of S, therefore, $f_A(a) \supseteq f_A(b)$. Thus

$$f_A(a) \supseteq f_A(b) \supseteq \gamma,$$

which implies that $f_A(a) \supseteq \gamma$, hence $a \in i_A(f_A, \gamma)$. Now let $x \in S$, $a \in i_A(f_A, \gamma)$ and $\alpha \in \Gamma$. Since f_A is an int-soft left Γ-ideal of S, therefore, $f_A(x\alpha a) \supseteq f_A(a)$, as $a \in i_A(f_A, \gamma) \Longrightarrow f_A(a) \supseteq \gamma$, then we have

$$f_A(x\alpha a) \supseteq f_A(a) \supseteq \gamma,$$

this implies that $f_A(x\alpha a) \supseteq \gamma$, thus $x\alpha a \in i_A(f_A, \gamma)$. Therefore, $i_A(f_A, \gamma)$ is left Γ-ideal of S.

(2) $\Longrightarrow$ (1): Assume that $i_A(f_A, \gamma)$ is left Γ-ideal of S, we need to show that f_A is an int-soft left Γ-ideal of S. For the said purpose let $x, y \in S$ with $x \leq y$. On contrary bases suppose that $f_A(x) \subset f_A(y)$, hence there exists some $\gamma_1 \subseteq U$ such that

$$f_A(x) \subset \gamma_1 \subseteq f_A(y),$$

this implies that $y \in i_A(f_A, \gamma_1)$ but $x \notin i_A(f_A, \gamma_1)$ which is contradiction to the fact that $i_A(f_A, \gamma)$ is left Γ-ideal of S. Hence $f_A(x) \supseteq f_A(y)$ hold for all $x, y \in S$ with $x \leq y$. Next, let $x, y \in S$, $\alpha \in \Gamma$ such that $f_A(x\alpha y) \subset f_A(y)$, again there exists some $\gamma_2 \subseteq U$ such that

$$f_A(x\alpha y) \subset \gamma_2 \subseteq f_A(y),$$

then $y \in i_A(f_A, \gamma_2)$ but $x\alpha y \notin i_A(f_A, \gamma_2)$ which is again contradiction to the fact that $i_A(f_A, \gamma)$ is left Γ-ideal of S. Thus $f_A(x\alpha y) \supseteq f_A(y)$ hold for $\forall\ x, y \in S, \alpha \in \Gamma$. Consequently, f_A is an int-soft left Γ-ideal of S.

The case for the right Γ-ideal can be proved in a similar way. □

Example 2. *Consider the ordered semigroup* $S = \{a_1, a_2, a_3, a_4, a_5, a_6\}$ *and let* $\Gamma = \{\alpha\}$ *be the set of binary operation such that*

α	a_1	a_2	a_3	a_4	a_5	a_6
a_1	a_1	a_1	a_1	a_4	a_1	a_1
a_2	a_1	a_2	a_2	a_4	a_2	a_2
a_3	a_1	a_2	a_3	a_4	a_5	a_5
a_4	a_1	a_1	a_4	a_4	a_4	a_4
a_5	a_1	a_2	a_3	a_4	a_5	a_5
a_6	a_1	a_2	a_3	a_4	a_5	a_6

$$\leq := \{(a_1, a_1), (a_2, a_2), (a_3, a_3), (a_4, a_4), (a_5, a_5), (a_6, a_6)(a_6, a_5)\}.$$

The covering relation $\leq := \{(a_6, a_5)\}$ is represented by Figure 1.

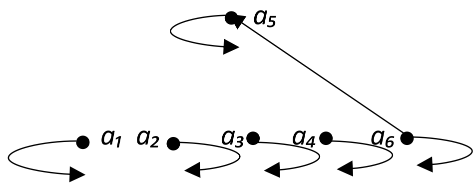

Figure 1. Converting relation.

Then, right Γ-ideals and left Γ-ideals of S are follows:

Right Γ-ideals $\longrightarrow$	$\{a_1, a_4\}$,	$\{a_1, a_2, a_4\}$,	S
Left Γ-ideals $\longrightarrow$	$\{a_1\}, \{a_4\}$,	$\{a_1, a_2\}, \{a_1, a_4\}$,	$\{a_1, a_2, a_4\}$,
	$\{a_1, a_2, a_3, a_4\}$,	$\{a_1, a_2, , a_4, a_5, a_6\}$,	S

Let $x \in S$, define an int-soft set f_A on $S = Z$ as follows:

S	a_1	a_2	a_4	a_3, a_5, a_6
$f_A(x)$	$\{0, \pm 1, \pm 2, ..., \pm 10\}$	$\{0, 2, 4, ..., 10\}$	$\{0, \pm 2, \pm 4, ..., \pm 10\}$	$\{0, 2, 4, 8\}$

Then for $\gamma \in Z$, γ-inclusive set is given by:

$\gamma \subseteq Z$	$i_A(f_A;\gamma)$
If $\gamma = \{2,4\}$	S
If $\gamma = \{2,4,10\}$	$\{a_1,a_2,a_4\}$
If $\gamma = \{2,4,-10\}$	$\{a_1,a_4\}$
If $\gamma = \{x \in Z \mid x > 10 \text{ or } x < -10\}$	$\emptyset$

Since, $i_A(f_A;\gamma)$ is a right Γ-ideal, hence using Theorem 12, f_A is an int-soft right Γ-ideal of S.

Let A be a non-empty subset of an ordered Γ-semigroup S, then the characteristic soft set $\mathbb{C}_A$ is a soft mapping i.e., $\mathbb{C}_A : S \longrightarrow P(U)$ defined by:

$$\mathbb{C}_A : x \longmapsto \begin{cases} U \text{ if } x \in A, \\ \emptyset \text{ if } x \notin A. \end{cases} \tag{3.1}$$

Lemma 3.3. *If A is a non-empty subset of an ordered Γ-semigroup S, then the following conditions are equivalent:*

(1) $\mathbb{C}_A$ is an int-soft left (resp. right) Γ-ideal of S.

(2) A is left (resp. right) Γ-ideal of S.

Proof. (1) $\Longrightarrow$ (2): Assume that $\mathbb{C}_A$ is an int-soft left Γ-ideal of S, we need to show that A is a left Γ-ideal of S. For this let $a, b \in S$ such that $a \leq b$ and $b \in A$, then $\mathbb{C}_A(b) = U$. Since $\mathbb{C}_A$ is an int-soft left Γ-ideal of S, therefore, $\mathbb{C}_A(a) \supseteq \mathbb{C}_A(b)$. Thus

$$\mathbb{C}_A(a) \supseteq \mathbb{C}_A(b) = U$$

which implies that $\mathbb{C}_A(a) \supseteq U$, But $\mathbb{C}_A(a) \subseteq U$ always hold. Thus $\mathbb{C}_A(a) = U$ it implies that $a \in A$. Now let $a, b \in S$, such that $b \in A$ and $\alpha \in \Gamma$. Since $\mathbb{C}_A$ is an int-soft left Γ-ideal of S, therefore, $\mathbb{C}_A(a\alpha b) \supseteq \mathbb{C}_A(b)$, as $b \in A \Longrightarrow \mathbb{C}_A(b) = U$, then we have

$$\mathbb{C}_A(a\alpha b) \supseteq \mathbb{C}_A(b) = U$$

this implies that $\mathbb{C}_A(a\alpha b) \supseteq U$, but $\mathbb{C}_A(a\alpha b) \subseteq U$ always hold. Thus $\mathbb{C}_A(a\alpha b) = U$ which implies that $a\alpha b \in A$. Therefore, A is left Γ-ideal of S.

(2) $\Longrightarrow$ (1): Consider A to be a left Γ-ideal of S. To show that $\mathbb{C}_A$ is an int-soft left Γ-ideal of S let $a, b \in S$ such that $a \leq b$, then we have the following cases:

Case I: Suppose both $a, b \in A$, then we have $\mathbb{C}_A(a) = U = \mathbb{C}_A(b)$, hence the inequality $\mathbb{C}_A(a) \supseteq \mathbb{C}_A(b)$ hold in this case.

Case II: If both $a, b \notin A$, then we have $\mathbb{C}_A(a) = \emptyset = \mathbb{C}_A(b)$, hence again the inequality $\mathbb{C}_A(a) \supseteq \mathbb{C}_A(b)$ hold in this case as well.

Case III: If $a \in A$ but $b \notin A$, then $\mathbb{C}_A(a) = U$ and $\mathbb{C}_A(b) = \emptyset$, so $\mathbb{C}_A(a) = U \supseteq \mathbb{C}_A(b)$. Thus $\mathbb{C}_A(a) \supseteq \mathbb{C}_A(b)$.

Case IV: If $b \in A$, then since A is left Γ-ideal of S and $a \leq b$, therefore $a \in A$. Hence it leads to Case I. Thus in all case $\mathbb{C}_A(a) \supseteq \mathbb{C}_A(b)$ holds for all $a, b \in S$ such that $a \leq b$.

Now let $a, b \in S$ and $\alpha \in \Gamma$. Again we have the following four cases;

Case I: Suppose both $a, b \in A$, then we have $\mathbb{C}_A(a) = U = \mathbb{C}_A(b)$, since A is left Γ-ideal of S and $b \in A$, then $a\alpha b \in A$ which implies $\mathbb{C}_A(a\alpha b) = U$. Hence the inequality $\mathbb{C}_A(a\alpha b) \supseteq \mathbb{C}_A(b)$ hold in this case.

Case II: If both $a, b \notin A$, then we have $\mathbb{C}_A(b) = \emptyset$, now if $a\alpha b \in A$, then $\mathbb{C}_A(a\alpha b) = U$ and it yields the inequality $\mathbb{C}_A(a\alpha b) = U \supseteq \mathbb{C}_A(b) = \emptyset$. If $a\alpha b \notin A$, then $\mathbb{C}_A(a\alpha b) = \emptyset = \mathbb{C}_A(b)$. Again in both cases $\mathbb{C}_A(a\alpha b) \supseteq \mathbb{C}_A(b)$ hold.

Case III: If $a \in A$ but $b \notin A$, then $\mathbb{C}_A(b) = \emptyset$, so if $a\alpha b \in A$, then $\mathbb{C}_A(a\alpha b) = U$ and it yields the inequality $\mathbb{C}_A(a\alpha b) = U \supseteq \mathbb{C}_A(b) = \emptyset$. If $a\alpha b \notin A$, then $\mathbb{C}_A(a\alpha b) = \emptyset = \mathbb{C}_A(b)$. Hence $\mathbb{C}_A(a\alpha b) \supseteq \mathbb{C}_A(b)$ hold.

Case IV: If $b \in A$, $a \notin A$, then since A is left Γ-ideal of S, $b \in A$ and $a \in S$, then $a\alpha b \in A$ (must be). Hence, $\mathbb{C}_A(a\alpha b) = U = \mathbb{C}_A(b)$. Thus in all cases $\mathbb{C}_A(a\alpha b) \supseteq \mathbb{C}_A(b)$ holds for $a, b \in S$ and $\alpha \in \Gamma$. Consequently, $\mathbb{C}_A$ is an int-soft left Γ-ideal of S.

The case for right Γ-ideal can be proved in a similar way. □

Definition 3.4. A subset P of an ordered Γ-semigroups S is called Γ-semiprime, if for $a \in S$ and $\alpha \in \Gamma$, $a\alpha a \in P$ implies $a \in P$ or equivalently, $A \subseteq S, A\Gamma A \subseteq P \Longrightarrow A \subseteq P$.

4. Int-soft Γ-regular ordered Γ-semigroups

In this section, we characterize Γ-regular ordered Γ-semigroups by the properties of int-soft left (resp. right) Γ-ideal of S.

Definition 4.1. An ordered Γ-semigroup S is *regular* if for every $a \in S$ and $\alpha, \beta \in \Gamma$ there exists $x \in S$ such that $a \leq a\alpha x\beta a$ or equivalently, (i) $a \in (a\Gamma S\Gamma a]$ for all $a \in S$ and (ii) $A \subseteq (A\Gamma S\Gamma A]$ for all $A \subseteq S$.

An ordered Γ-semigroup S is left (resp. right) Γ-simple, if every *left* (resp. *right*) Γ*-ideal* A of S, we have $A = S$. S is Γ-simple, if it is both left Γ-simple and right Γ-simple.

Theorem 4.2. *If S is an Γ-regular ordered Γ-semigroup, then the following conditions are equivalent:*

(1) S is left Γ-simple.

(2) Every int-soft left Γ-ideal f_A of S is a constant mapping.

Proof. (1) $\Longrightarrow$ (2): Let S be a left Γ-simple ordered Γ-semigroup, f_A is an int-soft left Γ-ideal of S and $a \in S$. Consider the set

$$E_S = \{e \in S \mid e\alpha e \geq e \text{ for } \alpha \in \Gamma\}.$$

Then E_S is non-empty set because S is an Γ-regular ordered Γ-semigroup and $a \in S$, hence there exists $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$ it can also be written as $a\alpha x\beta a \geq a$ which implies that $(a\alpha x\beta a)\alpha x \geq a\alpha x$, thus $(a\alpha x)\beta(a\alpha x) \geq a\alpha x$ implies that $a\alpha x \in E_S$. Hence $E_S \neq \emptyset$. Let $t, e \in E_S$, since S is left Γ-simple and $t \in S$ so we have $(S\alpha t] = S$, also as $e \in E_S$ it implies that $e \in S$, thus $e \in (S\alpha t]$. So there exists $z \in S$ such that $e \leq z\alpha t$, therefore, $e\beta e \leq (z\alpha t)\beta(z\alpha t) = (z\alpha t\beta z)\alpha t$. Now since f_A is an int-soft left Γ-ideal of S and $e\beta e \leq (z\alpha t\beta z)\alpha t$, then we have

$$f_A(e\beta e) \supseteq f_A((z\alpha t\beta z)\alpha t) \supseteq f_A(t),$$

now since $e \in E_S$, so $e\beta e \geq e$ for some $\beta \in \Gamma$. Thus

$$f_A(e) \supseteq f_A(e\beta e) \supseteq f_A(t),$$

Also, as S is left Γ-simple and $e \in S$, therefore, we have $(S\alpha e] = S$, also as $t \in E_S$ it implies that $t \in S$, thus $t \in (S\alpha t]$. So there exists $z \in S$ such that $t \leq z\alpha e$, therefore, $t\beta t \leq (z\alpha e)\beta(z\alpha e) = (z\alpha e\beta z)\alpha e$. As f_A is an int-soft left Γ-ideal of S and $t\beta t \leq (z\alpha e\beta z)\alpha e$, hence

$$f_A(t\beta t) \supseteq f_A((z\alpha e\beta z)\alpha e) \supseteq f_A(e),$$

now as $t \in E_S$, so $t\beta t \geq t$ for some $\beta \in \Gamma$. Thus

$$f_A(t) \supseteq f_A(t\beta t) \supseteq f_A(e).$$

Hence, $f_A(t) = f_A(e)$ for all $t, e \in E_S$. Therefore, f_A is a constant mapping on E_S. Now let $a \in S$, since S is an Γ-regular so there exists $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$, therefore we have

$$x\beta a \leq x\beta(a\alpha x\beta a) = (x\beta a)\alpha(x\beta a)$$

it implies that $(x\beta a)\alpha(x\beta a) \geq x\beta a$ for some $\alpha \in \Gamma$. Hence $x\beta a \in E_S$, so $f_A(x\beta a) = f_A(t)$, as f_A is an int-soft left Γ-ideal of S. Therefore, $f_A(x\beta a) \supseteq f_A(a)$ which shows that $f_A(t) \supseteq f_A(a)$. On the other hand, since S is an Γ-simple and $t \in S$, so $S = (S\alpha t]$ for $\alpha \in \Gamma$. As $a \in S$ implies that $a \in (S\alpha t]$, hence there exists some $s \in S$ such that $a \leq s\alpha t$. Now since f_A is an int-soft left Γ-ideal of S. Therefore, $f_A(a) \supseteq f_A(s\alpha t)$ and $f_A(s\alpha t) \supseteq f_A(t)$ it leads to $f_A(a) \supseteq f_A(t)$. Thus $f_A(a) = f_A(t)$ for $a, t \in S$. Ultimately, an int-soft left Γ-ideal f_A of S is a constant mapping.

(2) $\Longrightarrow$ (1): Let $a \in S$, then $(S\alpha a]$ is left Γ-ideal of S for some $\alpha \in \Gamma$. Also,

$$\begin{aligned} S\Gamma(S\alpha a] &= (S]\Gamma(S\alpha a] \\ &\subseteq (S\Gamma S\alpha a] \subseteq (S\alpha a]. \end{aligned}$$

If $x \in (S\alpha a]$ and $x \leq y$ for some $y \in S$, then $y \in ((S\alpha a]] = (S\alpha a]$. Since $(S\alpha a]$ is left Γ-ideal of S so by Lemma 14, $\mathbb{C}_{(S\alpha a]}$ is an int-soft left Γ-ideal of S. But by given hypothesis every int-soft left Γ-ideal of S is constant mapping, therefore $\mathbb{C}_{(S\alpha a]}$ is a constant mapping. Hence $\mathbb{C}_{(S\alpha a]}(x) = 1$ if $x \in (S\alpha a]$ and $\mathbb{C}_{(S\alpha a]}(x) = 0$ if $x \notin (S\alpha a]$. Assume that $(S\alpha a] \subset S$ and $t \in S$ such that $t \notin (S\alpha a]$. It implies that $\mathbb{C}_{(S\alpha a]}(t) = 0$. On the other hand, $a\alpha a \in (S\alpha a]$ so $\mathbb{C}_{(S\alpha a]}(a\alpha a) = 1$. It shows that $\mathbb{C}_{(S\alpha a]}$ is not a constant mapping which is a contradiction. Thus $(S\alpha a] = S$. Therefore, S is left Γ-simple. □

Theorem 4.3. *If S is an Γ-regular ordered Γ-semigroup, then the following conditions are equivalent:*

(1) S is right Γ-simple.
(2) Every int-soft right ideal f_A of S is a constant mapping.

Proof. The proof follows from Theorem 17. □

Combining Theorem 17 and Theorem 18, we have the following corollary.

Corollary 1. *An Γ-regular ordered Γ-semigroup S is Γ-simple if and only if every int-soft Γ-ideal of S is a constant map.*

Definition 4.4. An ordered Γ-semigroup $(S, \Gamma, \leq)$ is left (resp. right) Γ-regular if for every $a \in S$ and $\alpha, \beta \in \Gamma$ there exists $x \in S$ such that $a \leq x\alpha a\beta a$ (resp. $a \leq a\alpha a\beta x$) or equivalently, $a \in (S\alpha a\beta a]$ (resp. $a \in (a\alpha a\beta S])$ for all $a \in S$, and $A \subseteq (S\Gamma A\Gamma A]$ (resp. $A \subseteq (A\Gamma A\Gamma S])$ for all $A \subseteq S$.

An ordered Γ-semigroup S is called completely Γ-regular, if it is both Γ-regular, left Γ-regular and right Γ-regular.

Lemma 4.5. *An ordered* Γ*-semigroup* S *is completely* Γ*-regular if and only if* $A \subseteq (A\Gamma A\Gamma S\Gamma A\Gamma A]$ *for every* $A \subseteq S$ *or, equivalently, if and only if* $a \in (a\alpha a\beta S\gamma a\delta a]$ *for every* $a \in S$ *where* $\alpha, \beta, \gamma, \delta \in \Gamma$.

Proof. Let $A \subseteq S$, then $A \subseteq (A\Gamma S\Gamma A]$. Since S is completely Γ-regular, therefore it is Γ-regular, left Γ-regular and right Γ-regular i.e., $A \subseteq (A\Gamma A\Gamma S]$ and $A \subseteq (S\Gamma A\Gamma A]$. Hence we have

$$\begin{aligned} A &\subseteq ((A\Gamma A\Gamma S]\Gamma S\Gamma(S\Gamma A\Gamma A]] \\ &= ((A\Gamma A\Gamma S)\Gamma S\Gamma(S\Gamma A\Gamma A)] \\ &\subseteq (A\Gamma A\Gamma S\Gamma A\Gamma A]. \end{aligned}$$

Conversely, let $A \subseteq S$ such that $A \subseteq (A\Gamma A\Gamma S\Gamma A\Gamma A]$, then

$$A \subseteq (A\Gamma A\Gamma S\Gamma A\Gamma A] \subseteq (A\Gamma S\Gamma A],$$

$$A \subseteq (A\Gamma A\Gamma S\Gamma A\Gamma A] \subseteq (A\Gamma A\Gamma S]$$

and

$$A \subseteq (A\Gamma A\Gamma S\Gamma A\Gamma A] \subseteq (S\Gamma A\Gamma A].$$

Therefore, S is Γ-regular, left Γ-regular and right Γ-regular implies that S is completely Γ-regular. □

Theorem 4.6. *An ordered* Γ*-semigroup* S *is left* Γ*-regular if and only if for each int-soft left* Γ*-ideal* f_A *of* S*, we have* $f_A(a) = f_A(a\alpha a)$ *for all* $a \in S$ *and* $\alpha \in \Gamma$.

Proof. Assume that f_A is an int-soft left Γ-ideal and let $a \in S$. Since S is left Γ-regular, therefore there exists $x \in S$ such that $a \leq x\beta a\alpha a$ for some $\beta, \alpha \in \Gamma$. Also, as f_A is an int-soft left Γ-ideal. So we have

$$\begin{aligned} f_A(a) &\supseteq f_A(x\beta a\alpha a) \\ &= f_A(x\beta(a\alpha a)) \\ &\supseteq f_A(a\alpha a) \supseteq f_A(a), \end{aligned}$$

it shows that $f_A(a) = f_A(a\alpha a)$ for all $a \in S$ and $\alpha \in \Gamma$.

Conversely, let $a \in S$, we consider left Γ-ideal $L(a\alpha a) = (a\alpha a \cup S\beta a\alpha a]$ of S generated by $a\alpha a$. Then by Lemma 14, $\mathbb{C}_{L(a\alpha a)}$ is an int-soft left Γ-ideal of S. By hypothesis, $\mathbb{C}_{L(a\alpha a)}(a) = \mathbb{C}_{L(a\alpha a)}(a\alpha a)$. Now as $a\alpha a \in L(a\alpha a)$ so $\mathbb{C}_{L(a\alpha a)}(a\alpha a) = 1$ which implies that $\mathbb{C}_{L(a\alpha a)}(a) = 1$. Hence $a \in L(a\alpha a) = (a\alpha a \cup S\beta a\alpha a]$, therefore $a \leq y$ for some $y \in a\alpha a \cup S\beta a\alpha a$. Now if $y = a\alpha a$, then

$$a \leq y = a\alpha a \leq a\alpha y = a\alpha a\alpha a \in S\alpha a\alpha a$$

and $a \in (S\alpha(a\alpha a)]$. If $y = x\beta(a\alpha a)$ for some $x \in S$ and $\beta \in \Gamma$. It implies that

$$a \leq y = x\beta(a\alpha a) \in S\beta(a\alpha a),$$

which implies that $a \in (S\beta(a\alpha a)]$ for $\beta \in \Gamma$. Thus S is left Γ-regular ordered Γ-semigroup. □

Theorem 4.7. *An ordered* Γ*-semigroup* S *is right* Γ*-regular if and only if for each int-soft right* Γ*-ideal* f_A *of* S*, we have* $f_A(a) = f_A(a\alpha a)$ *for all* $a \in S$ *and* $\alpha \in \Gamma$.

Proof. Proof follows from Theorem 22. □

Definition 4.8. Suppose $(S, \Gamma, \leq)$ is an ordered Γ-semigroup and f_A is a soft set over U, then f_A is called int-soft bi-Γ-ideal of S if:

(i) $(\forall\, x, y \in S)\,(x \leq y \longrightarrow f_A\,(x) \supseteq f_A\,(y))$.
(ii) $(\forall\, x, y \in S, \alpha \in \Gamma)\,(f_A\,(x\alpha y) \supseteq f_A\,(x) \cap f_A\,(y))$.
(ii) $(\forall\, x, y, z \in S, \alpha, \beta \in \Gamma)\,(f_A\,(x\alpha y\beta z) \supseteq f_A\,(x) \cap f_A\,(z))$.

Definition 4.9. Suppose $(S, \Gamma, \leq)$ is an ordered Γ-semigroup and f_A is a soft set over U, then f_A is called int-soft $(1, 2)$-Γ-ideal of S if:

(i) $(\forall\, x, y \in S)\,(x \leq y \longrightarrow f_A\,(x) \supseteq f_A\,(y))$.
$(\forall\, x, y \in S, \alpha \in \Gamma)\,(x \leq y \longrightarrow f_A\,(x\alpha y) \supseteq f_A\,(x) \cap f_A\,(y))$.
(ii) $(\forall\, x, y, z, a \in S, \alpha, \beta, \gamma \in \Gamma)\,(f_A\,(x\alpha a\beta(y\gamma z)) \supseteq f_A\,(x) \cap f_A\,(y) \cap f_A\,(z))$.

Theorem 4.10. *If* S *is an ordered* Γ*-semigroup, then the following conditions are equivalent:*

(1) S is completely Γ-regular.
(2) For every int-soft bi-Γ-ideal f_A of S, we have, $f_A(a) = f_A(a\alpha a)$ for all $a \in S$ and $\alpha \in \Gamma$.
(3) For every int-soft left Γ-ideal f_B and int-soft right Γ-ideal f_C of S, we have $f_B(a) = f_B(a\alpha a)$, $f_C(a) = f_C(a\alpha a)$ for all $a \in S$ and $\alpha \in \Gamma$.

Proof. Proof follows from Lemma 21, Theorem 22 and Theorem 23. □

An ordered Γ-semigroup S is called left (resp. right) Γ-duo if every left (resp. right) Γ-ideal of S is a two-sided Γ-ideal of S, and Γ-duo if every its Γ-ideal is both left and right Γ-duo.

Definition 4.11. An ordered Γ-semigroup S is called int-soft left (resp. right) Γ-duo if every int-soft left (resp. right) Γ-ideal of S is an int-soft two-sided Γ-ideal of S. An ordered Γ-semigroup S is called int-soft Γ-duo if it is both int-soft left and int-soft right Γ-duo.

Theorem 4.12. *An* Γ*-regular ordered* Γ*-semigroup is left (right)* Γ*-duo if and only if it is int-soft left (right)* Γ*-duo.*

Proof. Let S be a left Γ-duo and f_A is an int-soft left Γ-ideal of S. Assume $a, b \in S$, then the set $(S\alpha a]$ is a left Γ-ideal of S. Infact,

$$S\Gamma(S\alpha a] = (S]\Gamma(S\alpha a] \subseteq (S\Gamma S\alpha a] \subseteq (S\alpha a]$$

and if $x \in (S\alpha a]$, then there exists some $y \in S$ such that $y \leq x$, thus $y \in ((S\alpha a]] = (S\alpha a]$. Since S is left Γ-duo, then $(S\alpha a]$ is two sided Γ-ideal of S. Also, as S is Γ-regular ordered Γ-semigroup so there exists $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$ it implies that $a\gamma b \leq (a\alpha x\beta a)\,\gamma b$ for some $b \in S$ and $\gamma \in \Gamma$. Therefore,

$$\begin{aligned} a\gamma b &\leq (a\alpha x\beta a)\,\gamma b \in (a\alpha S\beta a)\,\gamma b \\ &\subseteq (S\alpha a)\gamma S \subseteq (S\alpha a]\Gamma S \subseteq (S\alpha a], \end{aligned}$$

it implies that $a\gamma b \in ((S\alpha a]] = (S\alpha a]$ and so $a\gamma b \leq x\alpha a$ for some $x \in S$ and $\alpha \in \Gamma$. Since f_A is an int-soft left Γ-ideal of S, so we have

$$f_A(a\gamma b) \supseteq f_A(x\alpha a) \supseteq f_A(a).$$

Also, let $x, y \in S$ such that $x \leq y$, then $f_A(x) \supseteq f_A(y)$ (f_A being an int-soft left Γ-ideal of S). Thus f_A is an int-soft right Γ-ideal of S and ultimately S is an int-soft left Γ-duo.

Conversely, suppose S is an int-soft left Γ-duo and A is left Γ-ideal of S, then by Lemma 14, $\mathbb{C}_A$ is an int-soft left Γ-ideal of S. By hypothesis, $\mathbb{C}_A$ is an int-soft right Γ-ideal of S. Thus by Lemma 14, A is a right Γ-ideal of S. Hence S is a left Γ-duo.

The case for right Γ-duo can be proved in a similar way. □

Proposition 1. *In a Γ-regular ordered Γ-semigroup every bi-Γ-ideal is a right (left) Γ-ideal if and only if every its int-soft bi-Γ-ideal is an int-soft right (left) Γ-ideal.*

Proof. Suppose S is an Γ-regular ordered Γ-semigroup, $a, b \in S$ and f_A is an int-soft bi-Γ-ideal. Then $(a\alpha S\beta a]$ is a bi-Γ-ideal of S. In fact, $(a\alpha S\beta a]\Gamma(a\alpha S\beta a] \subseteq (a\alpha S\beta a]$, $(a\alpha S\beta a]\Gamma(S]\Gamma(a\alpha S\beta a] \subseteq (a\alpha S\beta a]$ and if $x \in (a\alpha S\beta a]$ and $y \in S$ such that $x \leq y$, then $y \in ((a\alpha S\beta a]] = (a\alpha S\beta a]$. Since $(a\alpha S\beta a]$ is a bi-Γ-ideal of S, hence by hypothesis, $(a\alpha S\beta a]$ is a right Γ-ideal of S. Also as S is an Γ-regular ordered Γ-semigroup and $a \in S$, therefore there exists $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a \leq a\alpha x\beta a$, then $a\gamma b \leq (a\alpha x\beta a)\gamma b$ for $\gamma \in \Gamma$. Thus

$$\begin{aligned} a\gamma b &\leq (a\alpha x\beta a)\gamma b \in (a\alpha S\beta a)\gamma S \\ &\subseteq (a\alpha S\beta a]\Gamma S \subseteq (a\alpha S\beta a], \end{aligned}$$

it implies that $a\gamma b \leq a\alpha z\beta a$ for some $z \in S$ and $\alpha, \beta \in \Gamma$. Since f_A is an int-soft bi-Γ-ideal. Therefore,

$$\begin{aligned} f_A(a\gamma b) &\supseteq f_A(a\alpha z\beta a) \\ &\supseteq f_A(a) \cap f_A(a) \\ &= f_A(a), \end{aligned}$$

also since f_A is an int-soft bi-Γ-ideal, so for $x, y \in S$ with $x \leq y$, $f_A(x) \supseteq f_A(y)$ hold. Consequently, f_A is an int-soft right Γ-ideal of S.

Conversely, if B is a bi-Γ-ideal of S, then by Lemma 14, $\mathbb{C}_B$ is an int-soft bi-Γ-ideal. Using hypothesis, $\mathbb{C}_B$ is an int-soft right Γ-ideal and again by Lemma 14, B is a right Γ-ideal of S.

The case for left Γ-ideal of S can be proved in a similar way. □

Proposition 2. *Every int-soft bi-Γ-ideal of an ordered Γ-semigroup S is an int-soft $(1,2)$-Γ-ideal of S.*

Proof. Assume that f_A is an int-soft bi-Γ-ideal of an ordered Γ-semigroup S, let $x, y, z, a \in S$ and $\alpha, \beta, \gamma, \xi \in \Gamma$, then we have

$$\begin{aligned} f_A(x\alpha a\beta(y\xi z)) &= f_A((x\alpha a\beta y)\xi z) \\ &\supseteq f_A(x\alpha a\beta y) \cap f_A(z) \\ &\supseteq [f_A(x) \cap f_A(y)] \cap f_A(z) \\ &= f_A(x) \cap f_A(y) \cap f_A(z). \end{aligned}$$

Also, since f_A is an int-soft bi-Γ-ideal, so for $x, y \in S$ with $x \leq y$, $f_A(x) \supseteq f_A(y)$ hold. Hence, f_A is an int-soft $(1,2)$-Γ-ideal of S. □

Corollary 2. *Every int-soft* Γ*-ideal of an ordered* Γ*-semigroup* S *is an int-soft* $(1,2)$-Γ*-ideal of* S.

Proof. Suppose that f_A is an int-soft Γ-ideal of an ordered Γ-semigroup S, let $x, y, z, a \in S$ and $\alpha, \beta, \gamma, \xi \in \Gamma$, then we have

$$f_A(x\alpha a\beta(y\xi z)) = f_A((x\alpha a\beta y)\xi z) \supseteq f_A(z) \because f_A \text{ is an int-soft left } \Gamma\text{-ideal}$$

also,

$$f_A(x\alpha a\beta(y\xi z)) = f_A(x\alpha(a\beta y\xi z)) \supseteq f_A(x) \because f_A \text{ is an int-soft right } \Gamma\text{-ideal}$$

and

$$\begin{aligned} f_A(x\alpha a\beta(y\xi z)) &= f_A((x\alpha a\beta y)\xi z) \supseteq f_A(x\alpha a\beta y) \because f_A, \text{ int-soft right } \Gamma\text{-ideal} \\ &= f_A((x\alpha a\beta)y) \supseteq f_A(y) \because f_A \text{ is an int-soft left } \Gamma\text{-ideal.} \end{aligned}$$

Consequently, $f_A(x\alpha a\beta(y\xi z)) \supseteq f_A(x) \cap f_A(y) \cap f_A(z)$. Also, since f_A is an int-soft Γ-ideal, so for $x, y \in S$ with $x \leq y$, $f_A(x) \supseteq f_A(y)$ hold. Hence, f_A is an int-soft $(1,2)$-Γ-ideal of S. □

The converse of the Proposition 30 is not true in general. However, if S is an Γ-regular ordered Γ-semigroup, then we have the following result.

Proposition 3. *An int-soft* $(1,2)$-Γ*-ideal of* Γ*-regular ordered* Γ*-semigroup* S *is an int-soft bi-*Γ*-ideal of* S.

Proof. Suppose S is an Γ-regular ordered Γ-semigroup and f_A is an int-soft $(1,2)$-Γ-ideal of S. Let $x, y, a \in S$ and $\alpha, \beta \in \Gamma$. Since S is an Γ-regular and $(x\alpha S\beta x]$ is a bi-Γ-ideal of S, therefore by Proposition 29, it is a right Γ-ideal of S. Thus

$$x\alpha a \leq (x\alpha S\beta x)\alpha a \in (x\alpha S\beta x)\Gamma S \subseteq (x\alpha S\beta x],$$

therefore, $x\alpha a \leq x\alpha y\beta x$ for some $y \in S$ and $\alpha, \beta \in \Gamma$. Thus $x\alpha a\gamma y \leq (x\alpha y\beta x)\gamma y$ where $\gamma \in \Gamma$. Hence

$$\begin{aligned} f_A(x\alpha a\gamma y) &\supseteq f_A((x\alpha y\beta x)\gamma y) \\ &\supseteq f_A(x\alpha y\beta x) \cap f_A(y) \because f_A \text{ is an int-soft } (1,2)\text{-}\Gamma\text{-ideal} \\ &\supseteq f_A(x) \cap f_A(x) \cap f_A(y) \\ &= f_A(x) \cap f_A(y). \end{aligned}$$

As f_A is an int-soft $(1,2)$-Γ-ideal, so for $x, y \in S$ with $x \leq y$, $f_A(x) \supseteq f_A(y)$ hold. Hence, f_A is an int-soft bi-Γ-ideal of S. □

5. Semilattices of left Γ-simple ordered Γ-semigroups

In this section, we introduce semilattices of left Γ-simple ordered Γ-semigroups. Various characterization theorems using semilattice of left Γ-simple Γ-semigroups are determined.

Definition 5.1. A Γ*-subsemigroup* F of S is called Γ-filter of S, if it satisfies:

(i) $(\forall\, x, y \in S, \alpha \in \Gamma)\,(x\alpha y \in F \implies x \in F \text{ and } y \in F)$

(ii) $(\forall\, x, z \in S)\,(\forall x \in F)(x \leq z \Longrightarrow z \in F)$,

Note that for any $x \in S$, we denote by $N(x)$ the filter of S generated by x. $\mathbb{N}$ denotes the equivalence relation on S which is denoted by

$$\mathbb{N} = \{(a, b) \in S \times S \mid N(x) = N(y)\}. \tag{5.1}$$

Definition 5.2. An equivalence relation ξ on ordered Γ-semigroup S is called Γ-congruence if $(a, b) \in \xi$ implies $(a\alpha c, b\alpha c) \in \xi$ and $(c\alpha a, c\alpha b) \in \xi$ for every $c \in S$ and $\alpha \in \Gamma$. A Γ-congruence ξ on S is called semilattice Γ-congruence if $(a\alpha a, a) \in \xi$ and $(a\alpha b, b\alpha a) \in \xi$ for each $a, b \in S$ and $\alpha \in \Gamma$. If ξ is a semilattice Γ-congruence on S then the ξ-class $(x)_\xi$ of S containing x is a Γ-subsemigroup of S for every $x \in S$.

Lemma 5.3. *Let S be an ordered Γ-semigroup. Then $(x)_{\mathbb{N}}$ is a left Γ-simple Γ-subsemigroup of S, for every $x \in S$ if and only every left Γ-ideal of S is a right Γ-ideal of S and it is Γ-semiprime.*

An ordered Γ-semigroup S is called a semilattice of left Γ-simple Γ-semigroups if there exists a semilattice Γ-congruence ξ on S such that the ξ-class $(x)_\xi$ of S containing x is a left Γ-simple Γ-subsemigroup of S for every $x \in S$ or, equivalently, if there exists a semilattice Y and a family $\{S_\alpha\}_{\alpha\in Y}$ of left Γ-simple Γ-subsemigroups of S such that

(1) $S_\alpha \cap S_\beta = \emptyset$ for all $\alpha, \beta \in Y$ such that $\alpha \neq \beta$,

(2) $S = \bigcup\limits_{\alpha\in Y} S_\alpha$,

(3) $S_\alpha \Gamma S_\beta \subseteq S_{\alpha\beta}$ for all $\alpha, \beta \in Y$.

Note that in ordered Γ-semigroup the semilattice Γ-congruences are defined exactly same as in the case of Γ-semigroups without order so the two definitions are equivalent.

Theorem 5.4. *An ordered Γ-semigroup $(S, \Gamma, \leq)$ is a semilattice of left Γ-simple Γ-semigroups if and only if for all left Γ-ideals A, B of S we have*

$$(A\Gamma A] = A \text{ and } (A\Gamma B] = (B\Gamma A].$$

Proof. Assume that S is a semilattice of left Γ-simple Γ-semigroups and A, B are left Γ-ideals of S, then there exists a semilattice Y and a family $\{S_\alpha\}_{\alpha\in Y}$ of left Γ-simple Γ-subsemigroups of S such that for all $\alpha, \beta \in Y$ the following conditions are satisfied:

(1) $S_\alpha \cap S_\beta = \emptyset$ where $\alpha \neq \beta$,

(2) $S = \bigcup\limits_{\alpha\in Y} S_\alpha$,

(3) $S_\alpha \Gamma S_\beta \subseteq S_{\alpha\beta}$.

Now let $a \in A$, then $a \in A \subseteq S = \bigcup\limits_{\alpha\in Y} S_\alpha$, therefore there exists $\alpha \in Y$ such that $a \in S_\alpha$. As S_α is left Γ-simple, so we have $(S_\alpha \beta b] = \{x \in S \mid \exists y \in S_\alpha : x \leq y\gamma b \text{ for some } \beta, \gamma \in \Gamma\}$ for all $b \in S_\alpha$. Now as $a \in S_\alpha$, so $S_\alpha = (S_\alpha \beta a]$ which implies that $a \leq x\gamma a$ for some $\gamma \in \Gamma$ and $x \in S_\alpha$. Since $x \in S_\alpha = (S_\alpha \beta a]$, hence $x \leq y\delta a$ for some $y \in S_\alpha$ and $\delta \in \Gamma$. Thus $a \leq x\gamma a \leq (y\delta a)\gamma a \in (S\Gamma A)\Gamma A \subseteq A\Gamma A$ (A being left Γ-ideals of S). Implies that $a \in (A\Gamma A]$. Hence $A \subseteq (A\Gamma A]$. Also, as A is Γ-subsemigroup of S, so $A\Gamma A \subseteq A$. Thus $(A\Gamma A] \subseteq (A] = A$. Now let $x \in (A\Gamma B]$, then there exist some $a \in A, b \in B$ and $\alpha \in \Gamma$ such that $x \leq a\gamma b$. Since $a, b \in S = \bigcup\limits_{\alpha\in Y} S_\alpha$, then there exist $\alpha, \beta \in Y$ such that $a \in S_\alpha$ and $b \in S_\beta$. Thus $a\gamma b \in S_\alpha \Gamma S_\beta \subseteq S_{\alpha\beta}$ and $b\gamma a \in S_\beta \Gamma S_\alpha \subseteq S_{\beta\alpha} = S_{\alpha\beta}$ (since $\alpha, \beta \in Y, Y$ is semilattice). Since

$S_{\alpha\beta}$ is left Γ-simple, implies that $S_{\alpha\beta} = (S_{\alpha\beta}\delta c]$ for some $c \in S_{\alpha\beta}, \delta \in \Gamma$. Hence $a\gamma b \in (S_{\alpha\beta}\delta b\gamma a]$ where $\delta, \gamma \in \Gamma$. Therefore, $a\gamma b \leq y\delta b\gamma a$ for some $y \in S_{\alpha\beta}$ and $\delta, \gamma \in \Gamma$. As B is left Γ-ideal of S, so $y\delta b\gamma a \in (S\Gamma B)\Gamma A \subseteq B\Gamma A$, then $x \leq a\gamma b \leq y\delta b\gamma a \in (B\Gamma A]$ implies that $x \in (B\Gamma A]$. Hence $(A\Gamma B] \subseteq (B\Gamma A]$, in a similar way we can show that $(B\Gamma A] \subseteq (A\Gamma B]$. Therefore, $(A\Gamma B] = (B\Gamma A]$.

Conversely, since $\mathbb{N}$ is a semilattice Γ-congruence on S, which is equivalent to the fact that $(x)_{\mathbb{N}}, \forall x \in S$, is a left Γ-simple Γ-subsemigrup of S. By Lemma 35, it is enough to prove that every left Γ-ideal is right Γ-ideal and Γ-semiprime. Suppose L be a left Γ-ideal of S. Then $L\Gamma S \subseteq (L\Gamma S] = (S\Gamma L] \subseteq (L] = L$. If $x \in L$, then $y \leq x$ for some $y \in S$. Now as L is left Γ-ideal and $x \in L$ it implies that $y \in L$. Therefore, L is right Γ-ideal of S. Now let $x \in S$ such that $x\alpha x \in L$ where $\alpha \in \Gamma$. Consider the bi-Γ-ideal $B(x)$ of S generated by x. Thus for $\alpha, \beta, \gamma, \delta \in \Gamma$, we have

$$\begin{aligned} B((x)\alpha(x)) &= (x \cup x\alpha x \cup x\alpha S\beta x]\Gamma(x \cup x\alpha x \cup x\alpha S\beta x] \\ &\subseteq ((x \cup x\alpha x \cup x\alpha S\beta x)\Gamma(x \cup x\alpha x \cup x\alpha S\beta x)] \\ &= (x\alpha x \cup x\alpha x\alpha x \cup x\alpha S\beta x\gamma x \cup x\alpha x\alpha x\alpha x \cup x\alpha S\beta x\gamma x\delta x \cup x\gamma x\beta S\alpha x \\ &\quad \cup x\gamma x\delta x\alpha S\beta x \cup x\alpha S\beta x\gamma x\delta S\rho x]. \end{aligned}$$

Now since $x\alpha x \in L, x\alpha x\alpha x \in S\Gamma L \subseteq L, x\alpha S\beta x\gamma x \in S\Gamma L \subseteq L, x\alpha x\alpha x\alpha x \in S\Gamma L \subseteq L$. Therefore, $B((x)\alpha(x)) \subseteq (L \cup L\Gamma S] = (L] = L$, so $(B((x)\alpha(x))] \subseteq (L] = L$ and $x \in L$, Thus L is Γ-semiprime. □

Theorem 5.5. *An ordered* Γ*-semigroup* $(S, \Gamma, \leq)$ *is a semilattice of left (right)* Γ*-simple* Γ*-semigroups if and only if for every int-soft left (right)* Γ*-ideal* f_A *of* S *and all* $a, b \in S$*, we have*

$$(i)\quad f_A(a) = f_A(a\alpha a) \ \text{and (ii)}\quad f_A(a\alpha b) = f_A(b\alpha a) \ \text{where}\ \alpha \in \Gamma.$$

Proof. Assume that S is a semilattice of left Γ-simple Γ-semigroups, then there exists a semilattice Y and a family $\{S_\alpha\}_{\alpha\in Y}$ of left Γ-simple Γ-subsemigroups of S such that for all $\alpha, \beta \in Y$ the following conditions are satisfied:

(1) $S_\alpha \cap S_\beta = \emptyset$ where $\alpha \neq \beta$,

(2) $S = \bigcup_{\alpha\in Y} S_\alpha$,

(3) $S_\alpha \Gamma S_\beta \subseteq S_{\alpha\beta}$.

Suppose f_A is an int-soft left Γ-ideal of S and $a \in S$, then there exists $\alpha \in Y$ such that $a \in S_\alpha$. Since S_α is left Γ-simple,so we have $S_\alpha = (S_\alpha \alpha a]$. Therefore, $a \leq x\alpha a$ for some $x \in S_\alpha$ and $\alpha \in \Gamma$. Now as $x \in S_\alpha$, then $x \in S_\alpha = (S_\alpha \alpha a]$, it implies that $x \leq y\alpha a$ for some $y \in S_\alpha$. Thus $a \leq x\alpha a \leq (y\alpha a)\alpha a = y\alpha(a\alpha a)$ which implies that for $y \in S$, $a \in (S\alpha(a\alpha a)]$. Therefore, by Theorem 22, $f_A(a) = f_A(a\alpha a)$. Also, if $a, b \in S$, then by (i),

$$\begin{aligned} f_A(a\alpha b) &= f_A((a\alpha b)\alpha(a\alpha b)) \\ &= f_A(a\alpha(b\alpha a)\alpha b) \\ &\supseteq f_A(b\alpha a). \end{aligned}$$

Similarly, $f_A(b\alpha a) \supseteq f_A(a\alpha b)$. Hence, $f_A(a\alpha b) = f_A(b\alpha a)$ hold for all $a, b \in S$ and $\alpha \in \Gamma$.

Conversely, suppose that f_A is an int-soft left Γ-ideal of S such that $f_A(a) = f_A(a\alpha a)$ and $f_A(a\alpha b) = f_A(b\alpha a)$ hold for all $a, b \in S$ and $\alpha \in \Gamma$. Then using Theorem 22 and (i), S is left Γ-regular. Assume A to be a left Γ-ideal of S and let $a \in A$. Then $a \in S$, since S is left Γ-regular, so there exists $x \in S$

such that $a \leq x\alpha(a\beta a)$ for $\alpha, \beta \in S$. It implies that $a \leq x\alpha(a\beta a) = (x\alpha a)\beta a \in (S\Gamma A)\Gamma A \subseteq A\Gamma A$. Thus $a \in (A\Gamma A]$ and $A \subseteq (A]$, also, as A is left Γ-ideal of S. Therefore, $A\Gamma A \subseteq S\Gamma A \subseteq A = (A]$. Hence, $(A\Gamma A] \subseteq (A]$. Now, let A and B be left Γ-ideals of S and let $x \in (B\Gamma A]$ then $x \leq b\alpha a$ for some $a \in A$ and $b \in B$ and $\alpha \in \Gamma$. We consider the left Γ-ideal $L(a\alpha b)$ generated by $a\alpha b$. That is, the set $L(a\alpha b) = (a\alpha b \cup S\beta a\alpha b]$. Then by Lemma 14, the characteristic function $\mathbb{C}_{L(a\alpha b)}$ of $L(a\alpha b)$ is an int-soft left Γ-ideal of S. By hypothesis, we have $\mathbb{C}_{L(a\alpha b)}(a\alpha b) = \mathbb{C}_{L(a\alpha b)}(b\alpha a)$. Since $a\alpha b \in L(a\alpha b)$, we have$\mathbb{C}_{L(a\alpha b)}(a\alpha b) = 1$ and $\mathbb{C}_{L(a\alpha b)}(b\alpha a) = 1$ and hence $b\alpha a \in L(a\alpha b) = (a\alpha b \cup S\beta a\alpha b]$. Then $b\alpha a \leq a\alpha b$ or $b\alpha a \leq y\beta a\alpha b$ for some $y \in S$ and $\alpha, \beta \in \Gamma$. If $b\alpha a \leq a\alpha b$, then $x \leq a\alpha b \in A\Gamma B$ and $x \in (A\Gamma B]$. If $b\alpha a \leq y\beta a\alpha b$, then $x \leq y\beta a\alpha b \in (S\Gamma A)\Gamma B \subseteq A\Gamma B$ and $x \in (A\Gamma B]$. Thus $(B\Gamma A] \subseteq (A\Gamma B]$. Similarly, we can prove that $(A\Gamma B] \subseteq (B\Gamma A]$. Therefore, $(A\Gamma B] = (B\Gamma A]$ and by Theorem 36, it follows that S is a semilattice of left Γ-simple semigroups. □

Theorem 5.6. *Let $(S, \Gamma, \leq)$ be an ordered Γ-semigroup and f_A an int-soft left (resp. right) Γ-ideal of S, $a \in S$ such that $a \leq a\alpha a$. Then $f_A(a) = f_A(a\alpha a)$.*

Proof. Suppose S be an ordered Γ-semigroup and f_A is an int-soft left Γ-ideal of S, $a \in S$ such that $a \leq a\alpha a$. Then

$$\begin{aligned} f_A(a) &\supseteq f_A(a\alpha a) \\ &\supseteq f_A(a) \quad \because f_A \text{ being an int-soft left } \Gamma\text{-ideal.} \end{aligned}$$

Consequently, $f_A(a) = f_A(a\alpha a)$. □

6. Conclusions

In modern era, most of the uncertainty theories such as fuzzy sets theory, probability theory and theory of rough sets can not tackle various problems of engineering and sciences due to the lack of parameterization. Soft set theory is one of the most reliable mathematical tool to handle such uncertainty problems of engineering and sciences. Due to the parameterization nature, soft sets have numerous applications in applied fields like decision making problems, control engineering, structural engineering, automata theory and economics. In this study, we have initiated a new type of soft set theory in ordered gamma semigroups S i.e., intersectional soft (int-soft) sets theory of S. Particularly, we have introduced int-soft left (resp. right) Γ-semigroup of S. The main contribution of this research work is:

↦ Several classes of ordered gamma semigroups like Γ-regular, left Γ-simple, right Γ-simple are characterized through int-soft left (resp. right) Γ-ideals.

↦ Semilattices of ordered Γ-semigroups are characterized through these newly developed Γ-ideals.

↦ It is shown that a Γ-regular ordered Γ-semigroup is left Γ-simple if and only if every int-soft left Γ-ideals of S is a constant function.

Beside this, these newly developed int-soft Γ-ideals theory can be further used to investigate other ideals like int-soft bi- (resp. generalized bi-, interior, quasi) Γ-ideals of ordered Γ-semigroup and other algebraic structures as well. Further, the proposed methods can also be extended to Pythagorean fuzzy uncertain environments. Such as: Pythagorean fuzzy interactive Hamacher power aggregation operators for assessment of express service quality with entropy weight, Soft Comput.

Acknowledgments

This Research is part of NRPU project 6831 by HEC Pakistan. The first author would like to thank the support of HEC through NRPU project 6831, and the Fundamental Research Funds for the central Universities (No. 20lgpy137).

Conflict of interest

The authors declare no conflict of interest.

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