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Research article

Fundamental solutions for the conformable time fractional Phi-4 and space-time fractional simplified MCH equations

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Abstract: We construct new solitary structures for time fractional Phi-4 and space-time fractional simplified modified Camassa-Holm (MCH) equations, utilizing the unified solver technique. The time (space-time) fractional derivatives are defined via sense of the new conformable fractional derivative. The unified solver technique extract vital solutions in explicit way. The obtained solutions may be beneficial for explaining many complex phenomena arising in fluid mechanics, nuclear, plasma and particle physics. The unified solver method is a vital tool for handling further models arising in applied science and new physics. For detailed physical dynamical representation of our results, 3D and 2D profiles to some of the gained solutions are also illustrated using Matlab software.

Keywords: fractional Phi-4 equation; fractional MCH equation; unified solver method; solitons; conformable derivative; physical applications
Mathematics Subject Classification: 26A33, 34A45, 35C05, 35Q80, 35R11

1. Introduction

Fractional calculus plays a vital rule in many modern research problems in the field of nonlinear optics, plasma physics, solid state physics, biology, signal processing, economy, super-fluids and chemical engineering [1–8]. Nonlinear fractional partial differential equations (NFPDEs) are generalizations of classical differential equations of integer order. In particular, many complex physical phenomena in natural sciences can be analysed utilizing fractional differential equations

(NFPDEs). As a result, studying the existence of exact solutions for NFPDEs is of interest to many researchers. Accordingly, many different approaches proposed and developed to gain analytical and numerical solutions of NFPDEs such as, sub-equation approach [9], first integral approach [10], modified Kudryashov approach [11], exponential function approach [12], $(\frac{G'}{G})-$ expansion approach [13], variational iteration approach [14], fractional sub-equation approach [15] and tanh-sech approach [16].

There are various definitions of FPDEs, like, Grunwald-Letnikov, Caputo's fractional derivatives and Riemann-Liouville [17]. Khalil et al. [18] introduced a new unpretentious conformable fractional derivative. This definition is easier to take into account several traditional features that cannot be satisfied by the known fractional derivatives, such as, chain rule [19]. Recently, this new definition has gained significant attention because of its simplicity, thus various works has been done on it by many scientists. We recall the definition conformable derivative and some basic notions [18]:

Given a function $q : (0, \infty) \to \mathbb{R}$, hence the conformable fraction derivative of q of order α is

$$T_\alpha(q)(t) = \lim_{\epsilon\to 0} \frac{q(t + \epsilon t^{1-\alpha}) - q(t)}{\epsilon}, \quad t > 0, \;\; 0 < \alpha \leq 1.$$

The conformable fractional derivative satisfies:

(i) $T_\alpha(a\,u + b\,q) = aT_\alpha(u) + bT_\alpha(q), \;\; a, b \in \mathbb{R}$,
(ii) $T_\alpha(t^n) = nt^{n-\alpha}, \;\; n \in \mathbb{R}$,
(iii) $T_\alpha(u\,q) = u\,T_\alpha(q) + q\,T_\alpha(u)$,
(vi) $T_\alpha(\frac{u}{q}) = \frac{q\,T_\alpha(u) - u\,T_\alpha(q)}{q^2}$,
(v) if q is differentiable, thus $T_\alpha(q)(t) = t^{1-\alpha}\frac{dq}{dt}$.

Theorem 1. *[18] Let $u, q : (0, \infty) \to \mathbb{R}$ be differentiable and also α-differentiable, then the following rule holds:*

$$T_\alpha(u \circ q)(t) = t^{1-\alpha} q'(t) u'(q(t)). \tag{1.1}$$

Recently, the conformable fractional calculus topic is widely used in defining many various fractal problems in fractal-media [11].

We concerned with the following time fractional Phi-4 equation [20, 21]:

$$D_t^\alpha u - \; D_{xx}^\alpha u + \varepsilon^2\, u + \sigma u^3 = 0, \;\; \varepsilon, \; \sigma \in \mathbb{R}\,. \tag{1.2}$$

The Klein-Gordon (KG) model admits many vital applications in applied science [22]. Eq (1.2) is a special case of KG model that depicts complex phenomenon in particle physics in which kink and anti-kinksolitary waves interact [23]. Eq (1.2) plays a crucial role in particle and nuclear physics [23]. Rezazadeh et al. [20] introduced new solutions for Eq (1.2) utilizing extended direct algebraic method. Tariq and Akram [24] implemented tanh method to establish various solutions to Eq (1.2). Korpinar [21] applied different types of mapping methods to extract different solutions to Eq (1.2) by process of conformable fractional derivative operator.

We also consider space-time fractional MCH equation [25] given as follows

$$D_t^\alpha \psi + \lambda\, D_t^\alpha \psi - D_{xxt}^{3\alpha} \psi + \mu D_x^\alpha \psi^3 = 0, \;\; \lambda, \; (\mu > 0) \in \mathbb{R}\,. \tag{1.3}$$

Rezazadeh et al., [25] applied new extended direct algebraic technique to establish new solutions of Eq (1.3). Shakeel et al. [26] introduced some solutions via modified Riemann-Liouville derivative by Jumarie.

Consider NFPDEs as follows

$$\Lambda(\Xi, D_t^\alpha \Xi, D_x^\alpha \Xi, D_t^\alpha D_x^\alpha \Xi, D_x^\alpha D_x^\alpha \Xi, ...) = 0, \quad 0 < \alpha \leq 1. \tag{1.4}$$

Using the wave transformation:

$$\Xi(x,t) = \Xi(\xi), \qquad \xi = \frac{x^\alpha}{\alpha} - \gamma \frac{t^\alpha}{\alpha}, \tag{1.5}$$

converts Eq (1.4) to the following ODE:

$$\Gamma(\Xi, \Xi', \Xi'', \Xi''', ...) = 0. \tag{1.6}$$

It can be seen that in applied science and new physics, various models in form of Eq (1.4) are converted to the ODE:

$$L\Xi'' + M\Xi^3 + N\Xi = 0, \tag{1.7}$$

see for example [27–34]. In [35], we introduced a unified solver method to solve NFPDE (1.4). Specifically, this solver presents the complete wave structure of different types of NFPDEs. This solver will be utilized as a box solver for considering various models in many natural and physical sciences. In contrast with other approaches, the presented unified solver admits many advantages, like, it averts complex and tedious computations, and supplies us exact solutions in an explicit way. The proposed unified solver is direct, sturdy, efficacious and adequate. Indeed, the proposed solver will be very significant for mathematicians, physicists and engineers.

Our study is motivated to grasp new solutions for time fractional Phi-4 and space-time fractional simplified modified Camassa-Holm (MCH) equations by utilizing the unified solver technique. This method presents some families of solutions with free physical parameters. These solutions may be applicable to understand various interesting phenomena in natural sciences, such as nuclear physics, plasma physics, particle physics, modeling of deep water and superfluid. Our study will show that the proposed unified solver can be applied for many other nonlinear fractional partial differential equations arising in natural sciences.

The rest of this article is organized as follows: Section 2 introduces the unified solver method. In Section 3, this solver is implemented to establish new solutions for time fractional Phi-4 and space-time fractional simplified MCH models. The physical interpretation for some obtained solutions is reported to illustrate the dynamical of waves structures in Section 4. The summary of the work is reported in Section 5.

2. Description of the method

We introduce the unified solver for equation

$$L\Xi'' + M\Xi^3 + N\Xi = 0. \tag{2.1}$$

As in [36, 37], balancing Ξ'' and Ξ^3, yields the solution of Eq (2.1) in the form :

$$\Xi = A_0 + A_1 sn(\zeta, m) + B_1 cn(\zeta, m). \tag{2.2}$$

From Eq (2.2) we get:

$$\Xi' = A_1 cn(\zeta, m)\, dn(\zeta, m) - B_1 sn(\zeta, m)\, dn(\zeta, m), \tag{2.3}$$

$$\begin{aligned}\Xi'' = & - m^2 sn(\zeta, m, m)\, A_1 + 2\, A_1 sn(\zeta, m)^3 m^2 + 2\, m^2 sn(\zeta, m)^2 cn(\zeta, m)\, B_1 \\ & - A_1 sn(\zeta, m) - B_1 cn(\zeta, m).\end{aligned} \tag{2.4}$$

Superseding Eq (2.2) & Eq (2.4) into Eq (2.1) and equating the coefficients of sn^3, sn^2cn, sn^2, $sncn$, sn, cn, sn^0 with zero, yields some algebraic equations. Solving these equations for A_0, A_1 and B_1, gives two main cases:
(1) For $m = 0$ (or $L = 0$) a constant solution will appears and given in the form:

$$\Xi_0(\zeta) = \pm\sqrt{\frac{-N}{M}} \tag{2.5}$$

(2) For $0 < m < 1$ we get four families of solutions :

Family I.

$$A_0 = 0, A_1 = \pm\sqrt{\frac{2(L-N)}{M}}, B_1 = 0, m = \sqrt{\frac{N}{L} - 1}.$$

The first family of solutions takes the form:

$$\Xi_1(\zeta) = \pm\sqrt{\frac{2(L-N)}{M}}\mathrm{sn}(\zeta, \sqrt{\frac{N}{L} - 1}), \tag{2.6}$$

where $1 < \frac{N}{L} < 2$.

Family II.

$$A_0 = 0, A_1 = 0, B_1 = \pm\sqrt{\frac{L-N}{M}}, m = \sqrt{\frac{1}{2}(1 - \frac{N}{L})}.$$

The second family of solutions takes the form:

$$\Xi_2(\zeta) = \pm\sqrt{\frac{L-N}{M}}\mathrm{cn}(\zeta, \sqrt{\frac{1}{2}(1 - \frac{N}{L})}), \tag{2.7}$$

where $0 < \frac{N}{L} < 2$.

Family III.

$$A_0 = 0, A_1 = \pm\sqrt{\frac{N-L}{M}}, B_1 = -\sqrt{\frac{L-N}{M}}, m = \sqrt{2(1 - \frac{N}{L})}.$$

The third family of solutions is

$$\Xi_3(\zeta) = \pm\sqrt{\frac{N-L}{M}}\mathrm{sn}(\zeta, \sqrt{2(1 - \frac{N}{L})} - \sqrt{\frac{L-N}{M}}\mathrm{cn}(\zeta, \sqrt{2(1 - \frac{N}{L})}), \tag{2.8}$$

where $\frac{1}{2} < \frac{N}{L} < 1$.

Family IV.

$$A_0 = 0, A_1 = \pm\sqrt{\frac{N-L}{M}}, B_1 = \sqrt{\frac{Ł-N}{M}}, m = \sqrt{2(1-\frac{N}{L})}.$$

The fourth family of solutions is

$$\Xi_4(\zeta) = \pm\sqrt{\frac{N-L}{M}}\text{sn}(\zeta, \sqrt{2(1-\frac{N}{L})}) + \sqrt{\frac{L-N}{M}}\text{cn}(\zeta, \sqrt{2(1-\frac{N}{L})}), \tag{2.9}$$

where $\frac{1}{2} < \frac{N}{L} < 1$.

Hyperbolic functions solutions

From the previous families of elliptic functions solutions we find that hyperbolic functions solutions can appear for certain values of $\frac{N}{L}$ corresponding to $m = 1$ as follows:

- For $N = 2L$ Eq (2.6) transformed to:

$$\Xi_1(\zeta) = \pm\sqrt{\frac{-2L}{M}}\tanh(\zeta). \tag{2.10}$$

- For $N = -L$, Eq (2.7) turns into

$$\Xi_2(\zeta) = \pm\sqrt{\frac{2L}{M}}\text{sech}(\zeta). \tag{2.11}$$

- For $N = L/2$, Eq (2.8) and Eq. (2.9) turn into the forms

$$\Xi_3(\zeta) = \pm\sqrt{\frac{-L}{2M}}\tanh(\zeta) - \sqrt{\frac{L}{2M}}\text{sech}(\zeta), \tag{2.12}$$

$$\Xi_4(\zeta) = \pm\sqrt{\frac{-L}{2M}}\tanh(\zeta) + \sqrt{\frac{L}{2M}}\text{sech}(\zeta), \tag{2.13}$$

respectively.

3. Application

We implemented the unified solver to extract solutions for time fractional Phi-4 and space-time fractional MCH models. It can be seen from the construction of solutions the importance and convenience of the solver.

3.1. The time fractional Phi-4 equation

Utilizing the wave transformation [21]:

$$u(x,t) = U(\zeta),\ \zeta = x^\alpha - \delta\frac{t^\alpha}{\alpha}. \tag{3.1}$$

Superseding Eq (3.3) into Eq (1.3), gives

$$U'' + MU^3 + NU = 0, \tag{3.2}$$

where $L = \delta^2 - 1, M = \sigma$ and $N = \varepsilon^2$. In light of the unified solver method, the solutions of Eq (1.2) given as:

(1) For $m = 0$ in that case $\delta = \pm 1$ and $\zeta = x^\alpha \mp \frac{t^\alpha}{\alpha}$. The solution of Eqs (3.2) and (1.6) respectively are given by:

$$u(x,t) = \pm\sqrt{\frac{-\varepsilon^2}{\sigma}}. \tag{3.3}$$

(2) For $0 < m < 1$ we get four families of solutions :

Family I.

The first family of solutions is

$$u_1(x,t) = \pm\sqrt{\frac{2(\delta^2 - \varepsilon^2 - 1)}{\sigma}}\mathrm{sn}(x^\alpha - \frac{\delta}{\alpha}t^\alpha, \sqrt{\frac{\varepsilon^2}{\delta^2 - 1} - 1}), \tag{3.4}$$

where $\varepsilon^2 > \delta^2 > \frac{\varepsilon^2}{2}$.

Family II.

The second family of solutions is

$$u_2(x,t) = \pm\sqrt{\frac{\delta^2 - \varepsilon^2 - 1}{\sigma}}\mathrm{cn}(x^\alpha - \frac{\delta}{\alpha}t^\alpha, \sqrt{\frac{1}{2}(1 - \frac{\varepsilon^2}{\delta^2 - 1})}), \tag{3.5}$$

where $\delta^2 \geq 1 + \varepsilon^2$ or $\delta^2 \leq 1 - \varepsilon^2$.

Family III.

The third family of solutions is

$$\begin{aligned} u_3(x,t) = \ & \pm \ \sqrt{\frac{\varepsilon^2 - \delta^2 + 1}{\sigma}}\mathrm{sn}(x^\alpha - \frac{\delta}{\alpha}t^\alpha, \sqrt{2(1 - \frac{\varepsilon^2}{\delta^2 - 1})}) \\ & - \sqrt{\frac{\delta^2 - \varepsilon^2 - 1}{\sigma}}\mathrm{cn}(x^\alpha - \frac{\delta}{\alpha}t^\alpha, \sqrt{2(1 - \frac{\varepsilon^2}{\delta^2 - 1})}), \end{aligned} \tag{3.6}$$

where $\varepsilon^2 < \delta^2 < 2\varepsilon^2$.

Family IV.

The fourth family of solutions is

$$\begin{aligned} u_4(x,t) = \ & \pm \ \sqrt{\frac{\varepsilon^2 - \delta^2 + 1}{\beta}}\mathrm{sn}(x^\alpha - \frac{\delta}{\alpha}t^\alpha, \sqrt{2(1 - \frac{\varepsilon^2}{\delta^2 - 1})}) \\ & + \sqrt{\frac{\delta^2 - \varepsilon^2 - 1}{\sigma}}\mathrm{cn}(x^\alpha - \frac{\delta}{\alpha}t^\alpha, \sqrt{2(1 - \frac{\varepsilon^2}{\delta^2 - 1})}), \end{aligned} \tag{3.7}$$

where $\varepsilon^2 < \delta^2 < 2\varepsilon^2$.

Hyperbolic functions solutions

These solutions are given as follows:

- For $\delta = \pm\sqrt{\frac{\varepsilon^2+2}{2}}$ Eq (3.4) transformed to:

$$u_1(x,t) = \pm\sqrt{\frac{-\varepsilon^2}{\sigma}}\ \tanh(x^\alpha \mp \frac{\sqrt{\frac{\varepsilon^2+2}{2}}}{\alpha}t^\alpha). \tag{3.8}$$

-For $\delta = \pm\sqrt{1-\varepsilon^2}$ Eq (3.5) turns into

$$u_2(x,t) = \pm\sqrt{\frac{-2\varepsilon^2}{\sigma}}\,\mathrm{sech}(x^\alpha \mp \frac{\sqrt{1-\varepsilon^2}}{\alpha}t^\alpha). \tag{3.9}$$

- For $\delta^2 = 2\varepsilon^2 + 1$ Eq (3.6) and Eq (3.7) turn into the forms

$$u_3(x,t) = \pm\sqrt{\frac{-\varepsilon^2}{\beta}}\,\tanh(x^\alpha \mp \frac{\sqrt{2\varepsilon^2+1}}{\alpha}t^\alpha) - \sqrt{\frac{\varepsilon^2}{\sigma}}\,\mathrm{sech}(x^\alpha \mp \frac{\sqrt{2\varepsilon^2+1}}{\alpha}t^\alpha), \tag{3.10}$$

$$u_4(x,t) = \pm\sqrt{\frac{-\varepsilon^2}{\beta}}\,\tanh(x^\alpha \mp \frac{\sqrt{2\varepsilon^2+1}}{\alpha}t^\alpha) + \sqrt{\frac{\varepsilon^2}{\sigma}}\,\mathrm{sech}(x^\alpha \mp \frac{\sqrt{2\varepsilon^2+1}}{\alpha}t^\alpha). \tag{3.11}$$

3.2. The space-time fractional simplified MCH equation

Using the wave transformation [25]:

$$\psi(x,t) = \Psi(\zeta),\ \zeta = \frac{x^\alpha}{\alpha} - \gamma\frac{t^\alpha}{\alpha}. \tag{3.12}$$

Superseding Eq (3.12) into Eq (1.3), gives

$$L\Psi'' + M\Psi^3 + N\Psi = 0, \tag{3.13}$$

$L = \gamma, M = \mu$ and $N = \lambda - \gamma$. In light of the unified solver, the solutions of Eq (1.3) gives as:
(1) For $m = 0$ (or $\gamma = 0$) a constant solution will appears and given in the form:

$$\psi_0(x,t) = \pm\sqrt{\frac{-\lambda}{\mu}}. \tag{3.14}$$

(2) For $0 < m < 1$ we get four families of solutions :
Family I.

$$\psi_1(x,t) = \pm\sqrt{\frac{2\gamma-\lambda}{\mu}}\mathrm{sn}(\frac{x^\alpha}{\alpha} - \gamma\frac{t^\alpha}{\alpha}, \sqrt{\frac{\lambda}{\gamma} - 2}), \tag{3.15}$$

where $\frac{\lambda}{3} < \gamma < \frac{\lambda}{2}$.
Family II.
The second family of solutions is

$$\psi_2(x,t) = \pm\sqrt{\frac{2\gamma-\lambda}{\mu}}\mathrm{cn}(\frac{x^\alpha}{\alpha} - \gamma\frac{t^\alpha}{\alpha}, \sqrt{\frac{1}{2}(2 - \frac{\lambda}{\gamma})}), \tag{3.16}$$

where $\frac{\lambda}{3} < \gamma < \lambda$.

Family III.
The third family of solutions is

$$\psi_3(x,t) = \pm\sqrt{\frac{\lambda-2\gamma}{\mu}}\mathrm{sn}(\frac{x^\alpha}{\alpha}-\gamma\frac{t^\alpha}{\alpha},\sqrt{2(2-\frac{\lambda}{\gamma})}) - \sqrt{\frac{2\gamma-\lambda}{\mu}}\mathrm{cn}(\frac{x^\alpha}{\alpha}-\gamma\frac{t^\alpha}{\alpha},\sqrt{2(2-\frac{\lambda}{\gamma})}), \quad (3.17)$$

where $\frac{\lambda}{2} < \gamma < \frac{2\lambda}{3}$.
Family IV.
The fourth family of solutions is

$$\psi_4(x,t) = \pm\sqrt{\frac{\lambda-2\gamma}{\mu}}\mathrm{sn}(\frac{x^\alpha}{\alpha}-\gamma\frac{t^\alpha}{\alpha},\sqrt{2(2-\frac{\lambda}{\gamma})}) + \sqrt{\frac{2\gamma-\lambda}{\mu}}\mathrm{cn}(\frac{x^\alpha}{\alpha}-\gamma\frac{t^\alpha}{\alpha},\sqrt{2(2-\frac{\lambda}{\gamma})}), \quad (3.18)$$

where $\frac{\lambda}{3} < \gamma < \frac{\lambda}{2}$.
Hyperbolic functions solutions
For certain values of γ the hyperbolic functions solutions are given as:
- For $\gamma = \lambda/3$ Eq (3.15) transformed to:

$$\psi_{1,2}(x,t) = \pm\sqrt{\frac{-2\lambda}{3\mu}}\,tanh(\frac{x^\alpha}{\alpha}-\frac{\lambda}{3\alpha}t^\alpha). \quad (3.19)$$

- For $\lambda = 0$, Eq (3.16) turns into

$$\psi_{3,4}(x,t) = \pm\sqrt{\frac{2\gamma}{\mu}}\,sech(\frac{x^\alpha}{\alpha}-\gamma\frac{t^\alpha}{\alpha}). \quad (3.20)$$

- For $\gamma = 2\lambda/3$, Eq (3.17) and Eq (3.18) turn into the forms

$$\psi_{5,6}(x,t) = \pm\sqrt{\frac{-\lambda}{3\mu}}\,\tanh(\frac{x^\alpha}{\alpha}-\frac{2\lambda}{3\alpha}t^\alpha) - \sqrt{\frac{\lambda}{3\mu}}\,\mathrm{sech}(\frac{x^\alpha}{\alpha}-\frac{2\lambda}{3\alpha}t^\alpha), \quad (3.21)$$

$$\psi_{7,8}(x,t) = \pm\sqrt{\frac{-\lambda}{3\mu}}\,\tanh(\frac{x^\alpha}{\alpha}-\frac{2\lambda}{3\alpha}t^\alpha) + \sqrt{\frac{\lambda}{3\mu}}\,\mathrm{sech}(\frac{x^\alpha}{\alpha}-\frac{2\lambda}{3\alpha}t^\alpha). \quad (3.22)$$

Remark 1. *The proposed unified solver technique in the work can be applied for a large class of nonlinear fractional partial differential equations, see for example [38, 39]. In future works, we will try to modify this solver to solve delay partial differential equations [40, 41].*

4. Physical interpretation

We introduce some profile pictures for some solutions of time fractional Phi-4 and space-time fractional MCH. The new solitary structures for these equations are reported in the explicit way.

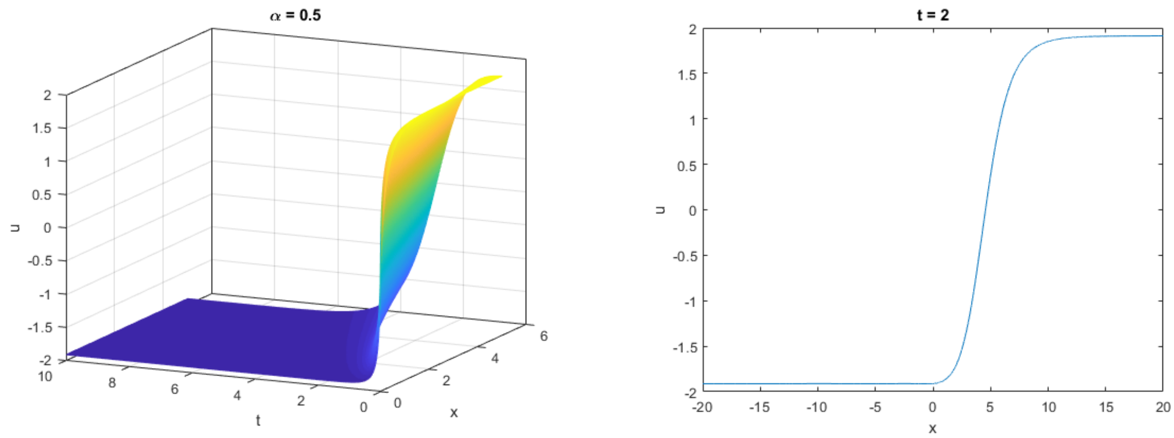

Figure 1. Profile of 3D & 2D of $u = u_1$ with $\alpha = 0.5$.

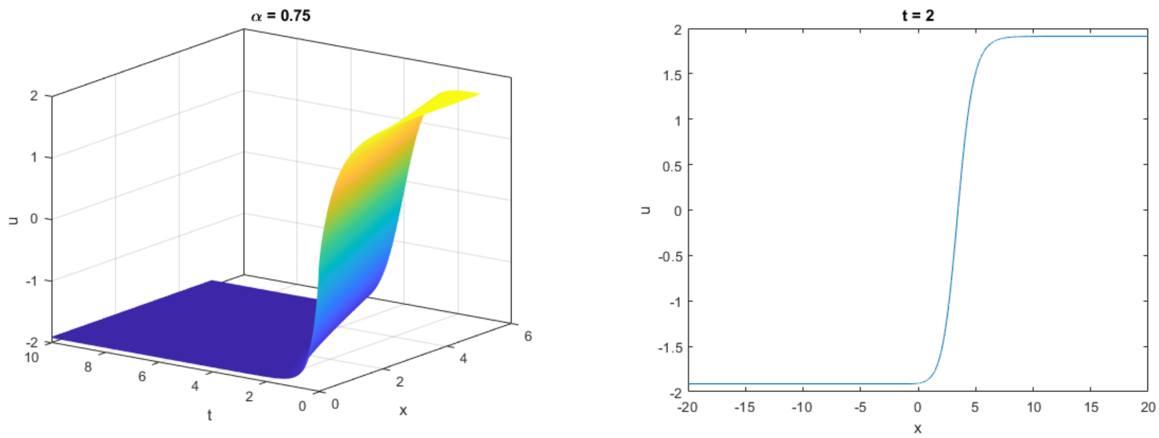

Figure 2. Profiles of 3D & 2D of $u = u_1$ with $\alpha = 0.75$.

Namely, we presented some new hyperbolic function solutions. The way of behaving of the gained solutions; solitons, localized, explosive, rough, periodic, dissipative, etc., is based on the values of physical parameters. For example, the behaviour of wave varies from compressive to rarefactive at critical points and stability regions changed to unstable regions at critical values of wave number see for example [42, 43]. These presented solutions realize very important fact for realization the qualitative interpretations for various phenomena in our nature.

The hyperbolic function solutions depict the ranges and altitudes of seismic sea waves as illustrated in Figure 3. The waves would be more grave to the nature if the amplitudes of the wave are high. To reduce the calamitous power of these massive natural disasters or to turn them into beneficial energy sources, we should consider the mathematical configuration of such natural troubles. Based on presented analysis in this work, we can applied the proposed unified solver for further NFPDEs arising in new physics and applied sciences.

Here, we illustrate some 2-D and 3-D profile pictures for some selected solutions of Eqs (1.2) and (1.3). Namely, these figures prescribe the dynamical behaviour of some selected solutions with certain values of physical parameters. Figures 1–6 show respectively, the evolution of kink-solitary wave structures of Eqs (3.8) & (3.19) with $\alpha = 0.5, \alpha = 0.75$ & $\alpha = 1$.

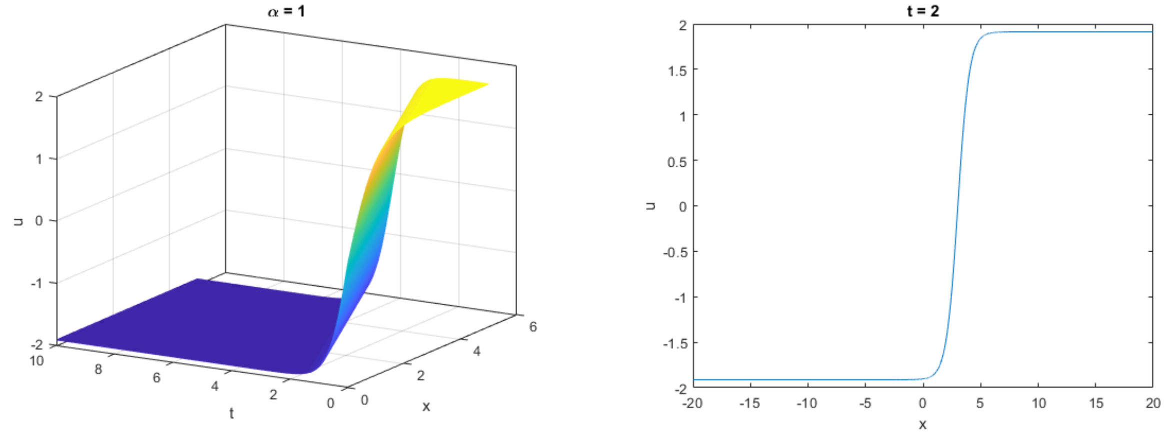

Figure 3. Profiles of 3D & 2D of $u = u_1$ with $\alpha = 1$.

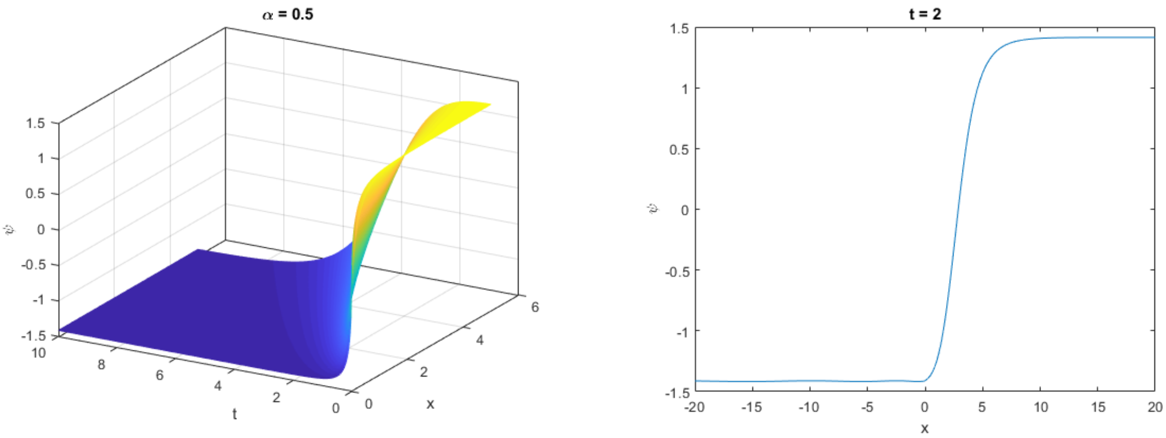

Figure 4. Profiles of 3D & 2D of $\psi = \psi_1$ with $\alpha = 0.5$.

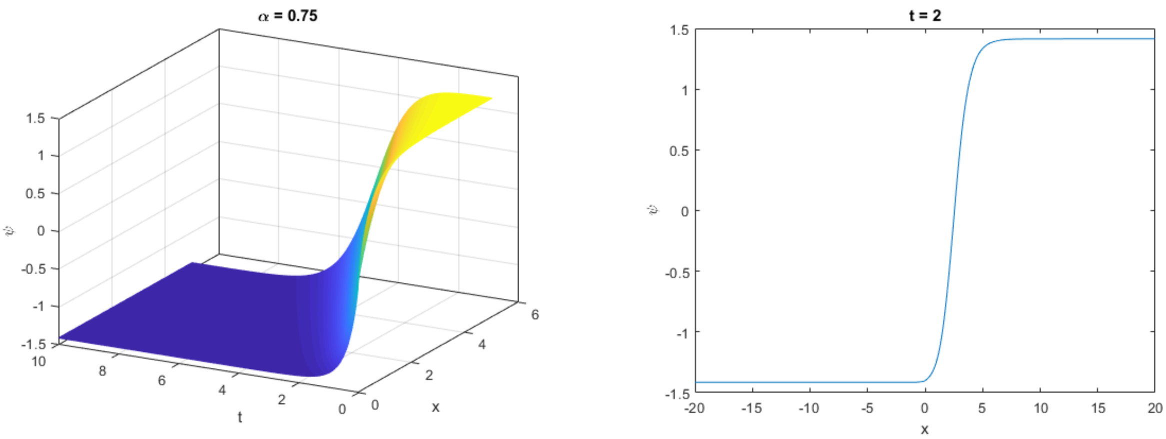

Figure 5. Profiles of 3D & 2D of $\psi = \psi_1$ with $\alpha = 0.75$.

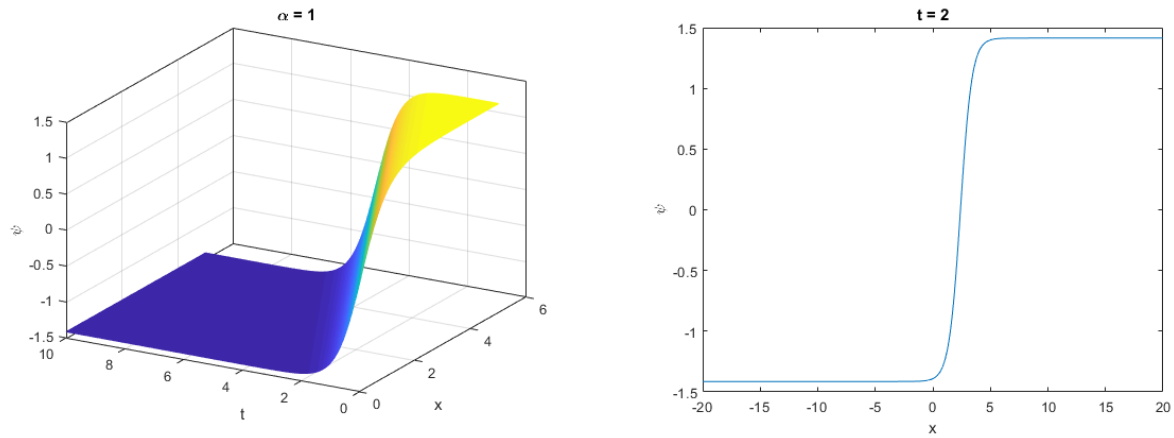

Figure 6. Profiles of 3D & 2D of $\psi = \psi_1$ with $\alpha = 1$.

5. Conclusions

In this work, we applied the unified solver technique to extract new solutions for time fractional Phi-4 and space-time fractional MCH models. The solutions are presented in explicit way. These solutions have vital applications in superfluid, particle and nuclear physics. The presented solver is very simple, direct, powerful, concise and efficient. It is quite capable and almost well suited for using this solver to construct solutions for other models of NFPDEs arising in applied sciences. Finally, we have presented some graphs in order to illustrate the dynamics behaviour of presented solutions.

Conflict of interest

The authors declare no conflict of interest.

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