Mathematics

AIMS Mathematics, 6(5): 5193–5204.
DOI: 10.3934/math.2021308
Received: 04 December 2020
Accepted: 07 March 2021
Published: 11 March 2021

http://www.aimspress.com/journal/Math

Research article

An efficient modification to diagonal systematic sampling for finite populations

Muhammad Azeem[1], Muhammad Asif[1,*], Muhammad Ilyas[1], Muhammad Rafiq[1], Shabir Ahmad[2]

[1] Department of Statistics, University of Malakand, Chakdara, Lower Dir, Pakistan
[2] Department of Commerce and Management, University of Malakand, Chakdara, Lower Dir, Pakistan

* **Correspondence:** Email: m.asif@uom.edu.pk.

Abstract: This paper presents a new modified diagonal systematic sampling method for the linear trend situation. For numerical illustration, data from the literature have been used to compare the proposed method's efficiency with some existing sampling schemes. The findings show that the proposed new systematic sampling method is more efficient than the existing sampling schemes. Moreover, the improvement in efficiency has also been shown in the case of a perfect linear trend. Mathematical conditions under which the new method is more efficient than the existing sampling schemes have been derived.

Keywords: diagonal systematic sampling; efficiency; linear systematic sampling; linear trend; modified systematic sampling
Mathematics Subject Classification: 62D05

1. Introduction

Madow and Madow [1] originally developed the linear systematic sampling scheme, probability sampling. In this scheme, a sample of size n units is selected from a finite population of N units so that the first unit is drawn from the first k $(=N/n)$ units. After that, every kth unit of the population is selected in the sample. One of the limitations of LSS is requiring the population size to be a multiple of the sample size. Therefore, Lahiri, in 1951, suggested a circular-systematic sampling scheme [2]. Chang and Huang [3] developed a modified version of the systematic sampling called the remainder systematic sampling for situations where the population size is not a constant multiple sample size. Subramani [4] (2000) proposed a diagonal-systematic sampling where the units are drawn diagonally. Likewise, Sampath and Varalakshmi [5] proposed a diagonal-circular-systematic sampling scheme. Subramani [6] developed a generalized diagonal systematic sampling scheme. Subramani [7] proposed a modified version of the linear-systematic sampling for an odd sample size. The classical systematic sampling scheme was further improved by a generalized modified systematic sampling method suggested by Subramani and Gupta [8]. Their approach indicated practically advantageous as it does not require the population size to be a constant multiple of the required sample size. However, the sample mean was a biased estimator of the population mean. In the past, various other researchers also attempted to improve systematic sampling while estimating the population mean; for instance, such as studies [6,9–15].

This paper proposes a new systematic sampling method, combining both diagonal and linear systematic sampling procedures. It is shown that the proposed new systematic sampling scheme is more efficient as compared to the existing sampling schemes in the case of a linear trend based on the mean squared error (MSE) criteria. Following established literature, all efficiency comparisons are based on the estimation of the population mean. Our research findings show that the sample mean based on the proposed sampling scheme is more efficient than some commonly used systematic sampling schemes.

The rest of the paper is planned as follows. In the subsequent section, the proposed method is described. In section 3 theoretically proved that the proposed sampling scheme is more efficient as compared to other methods. Empirical results are provided in section 4. Finally, concluding remarks are given in section 5.

2. Proposed sampling method

Let the population consist of N units with labels *1, 2, 3, …, N,* and draw a sample of size n such that $N = nk = k \cdot k + (n-k)k$. The steps involved in the proposed method are as follows:

1) Divide the population of size N into two sets: Set-1 and Set-2, in such a way that Set-1 receives the first $k \times k = k^2$ units y_i (i=1, 2, …, k^2) thus forming a $k \times k$ square matrix, and Set-2 receives the remaining $(n-k)k$ units y_i ($i = k \cdot k + 1,\ k \cdot k + 2,\ k \cdot k + 3, ..., nk$).

2) In Set-1, arrange the units in a $k \times k$ square matrix. In Set-2, write the $(n-k)k$ units in a matrix of order $(n-k) \times k$, as shown in Table 1.

3) Select two random numbers r_1 and r_2 where $1 \le r_1 \le k$ and $1 \le r_2 \le k$. In Set-1, the units are drawn so that the selected k units are the entries in the diagonal or broken diagonal of the matrix. In Set-2, the selected $n-k$ units are the elements of the r_2 column of the matrix. Finally, the selected units from both sets are combined to get the sample of size n.

Table 1. Arrangement of the units of the population in Set-1 and Set-2.

Set-1					**Set-2**				
S.No.	1	2	…	k	S.No.	1	2	…	k
1	y_1	y_2	...	y_k	k+1	y_{kk+1}	y_{kk+2}	...	$y_{kk+k=(k+1)k}$
2	y_{k+1}	y_{k+2}	…	y_{2k}	k+2	$y_{(k+1)k+1}$	$y_{(k+1)k+2}$	...	$y_{(k+2)k}$
3	y_{2k+1}	y_{2k+2}	…	y_{3k}	k+3	$y_{(k+2)k+1}$	$y_{(k+2)k+2}$	...	$y_{(k+3)k}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
k	$y_{(k-1)k+1}$	$y_{(k-1)k+2}$	…	y_{kk}	n	$y_{(n-1)k+1}$	$y_{(n-1)k+2}$	...	y_{nk}

The proposed sampling scheme has $k \times k = k^2$ possible samples, each of size n. For the proposed method, the first and second-order inclusion probabilities are given by:

$$\pi_i = \frac{1}{k} \tag{1}$$

and

$$\pi_{ij} = \begin{cases} \dfrac{1}{k}, & \text{if } i\text{th and } j\text{th units are from the same diagonal or broken diagonal of Set-1,} \\ \dfrac{1}{k}, & \text{if } i\text{th and } j\text{th units are from the same column of Set-2,} \\ \dfrac{1}{k^2}, & \text{if } i\text{th and } j\text{th units are from Set-1 and Set-2 respectively,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Generally, the selected sampling units are:

$$S_{r_1 r_2} = \begin{cases} y_{r_1}, y_{(k+1)+r_1}, \ldots, y_{(k-1)(k+1)+r_1}, y_{kk+r_2}, y_{(k+1)k+r_2}, \ldots, y_{(n-1)k+r_2}. & for \quad r_1 = 1 \\ y_{r_1}, y_{(k+1)+r_1}, \ldots y_{t(k+1)+r_1}, y_{(t+1)k+1}, y_{(t+2)k+2}, \ldots, y_{(k-1)k+k-t-1}, y_{kk+r_2}, y_{(k+1)k+r_2}, \ldots, y_{(n-1)k+r_2}. & for \quad r_1 > 1 \end{cases}$$

where $r_2 = 1, 2, \ldots, k$.

The sample mean is given by:

$$\bar{y}_{mdsy} = w_1 \bar{y}_1 + w_2 \bar{y}_2 \tag{3}$$

Where

$$\bar{y}_1 = \begin{cases} \frac{1}{k}\sum_{l=0}^{k-1} y_{l(k+1)+r_1}, & \text{if } r_1 = 1, \\ \frac{1}{k}\left(\sum_{i=0}^{t} y_{i(k+1)+r_1} + \sum_{i=1}^{k-t-1} y_{(t+i)k+i}\right), & \text{if } r_1 > 1. \end{cases} \tag{4}$$

Where $t=k-r_1$, and

$$\bar{y}_2 = \frac{1}{n-k}\sum_{l=k}^{n-1} y_{lk+r_2}, \qquad w_1 = \frac{k}{n},\ \ w_2 = \frac{n-k}{n},\ \ w_1 + w_2 = 1. \tag{5}$$

Theorem 1: Under the suggested modified diagonal systematic sampling design, the sample mean can be written in the form of Horvitz and Thompson [16] estimator $\bar{y}_{HT}$ is unbiased with variance:

$$Var\left(\bar{y}_{mdsy}\right) = \frac{1}{N^2}\left[k^4\left\{\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{1i} - \bar{Y}_1\right)^2\right\} + (n-k)^2 k^2\left\{\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{2i} - \bar{Y}_2\right)^2\right\}\right],$$

Where $\bar{y}_1$ and $\bar{y}_2$ are sample means of set-1 and set-2. Whereas, $\bar{Y}_1$ and $\bar{Y}_2$ are the means of all units of the set-1 and set-2, respectively. Further, k is the number of all possible samples.

Proof. By definition

$$\bar{y}_{mdsy} = \frac{k^2}{N}\bar{y}_1 + \frac{(n-k)k}{N}\bar{y}_2 = \frac{1}{N}\left(k\sum_{i\in s_1} y_{1i} + k\sum_{i\in s_2} y_{2i}\right) \tag{6}$$

Where s_1 and s_2 denote the samples drawn from Set-1 and Set-2, respectively.

$$\bar{y}_{mdsy} = \frac{1}{N}\left(\sum_{i\in s_1}\frac{y_{1i}}{1/k} + \sum_{i\in s_2}\frac{y_{2i}}{1/k}\right) = \frac{1}{N}\sum_{i\in s}\frac{y_i}{\pi_i} = \bar{y}_{HT} \tag{7}$$

Where, s, is the sample obtained from the population (Set-I and Set-II). Taking expectation of both sides of (6) yields:

$$E\left(\bar{y}_{mdsy}\right) = \frac{k^2}{N}E\left(\bar{y}_1\right) + \frac{(n-k)k}{N}E\left(\bar{y}_2\right) \tag{8}$$

Now, $$E\left(\bar{y}_1\right) = E\left(\frac{1}{k}\sum_{i=1}^{k} y_{1i}\right) = \frac{1}{k}\sum_{i=1}^{k} E\left(y_{1i}\right)$$

Since each observation of the set-1 is equally likely to be in the sample selected. Therefore, assuming discrete uniform probability distribution, the probability of y_{1i} is equal to $1/k^2$ for all $i \in S_1$; hence,

$$E(\bar{y}_1) = E(\frac{1}{k}\sum_{i=1}^{k} y_{1i}) = E(y_{1i}) = \bar{Y}_1 \tag{9}$$

Similarly,

$$E\left(\bar{y}_2\right)=E\left(\frac{1}{(n-k)}\sum_{i=1}^{(n-k)}y_{2i}\right)=E\left(y_{2i}\right)=\bar{Y}_2 \tag{10}$$

Substituting (9) and (10) in (8) and simplification yields $E\left(\bar{y}_{mdsy}\right)=\bar{Y}$. Taking variance of both sides of (3) yields:

$$Var\left(\bar{y}_{mdsy}\right)=\frac{k^4}{N^2}Var\left(\bar{y}_1\right)+\frac{(n-k)^2k^2}{N^2}Var\left(\bar{y}_2\right) \tag{11}$$

Where

$$Var\left(\bar{y}_1\right)=\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{1i}-\bar{Y}_1\right)^2 \tag{12}$$

as there are *k* possible samples, each having identical probability equal to *1/k*.

$$Var\left(\bar{y}_2\right)=\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{2i}-\bar{Y}_2\right)^2 \tag{13}$$

Substituting (12) and (13) in (11), the variance of $\bar{y}_{mdsy}$ is obtained as:

$$Var\left(\bar{y}_{mdsy}\right)=\frac{1}{N^2}\left[k^4\left\{\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{1i}-\bar{Y}_1\right)^2\right\}+(n-k)^2k^2\left\{\frac{1}{k}\sum_{i=1}^{k}\left(\bar{y}_{2i}-\bar{Y}_2\right)^2\right\}\right] \tag{14}$$

Remark 1: Using the Sen-Yates-Grundy approach [17,18], the variance of $\bar{y}_{mdsy}$ can be written as:

$$Var\left(\bar{y}_{mdsy}\right)=\frac{1}{N^2}\left\{\frac{1}{2}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\left(\pi_i\pi_j-\pi_{ij}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\right\}=Var_{SYG}\left(\bar{y}_{HT}\right) \tag{15}$$

Remark 2: The Sen-Yates-Grundy estimator for (15) is given by:

$$var\left(\bar{y}_{mdsy}\right)=\frac{1}{N^2}\left\{\frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{\pi_i\pi_j-\pi_{ij}}{\pi_{ij}}\right)\left(\frac{y_i}{\pi_i}-\frac{y_j}{\pi_j}\right)^2\right\}=var_{SYG}\left(\bar{y}_{HT}\right) \tag{16}$$

The values of π_i and π_{ij} can be used from (1) and (2) in expressions (15) and (16) to obtain the sampling variance of the mean and its estimator under the modified diagonal systematic sampling scheme.

3. Theoretical results

The term linear trend implies that the population units follow an arithmetic progression, either increasing or decreasing order. There are practical situations where a moderate or high degree of a linear trend may be observed. For example, educational institutes usually offer admissions on a merit basis to various academic programs and allocate roll numbers to students based on their previous grades. Usually, intelligent students occupied the top enrollment numbers. Therefore, in a test, if students' test scores are recorded in order of their enrollment numbers, a high degree of a linear trend in the data is expected as the enrollment numbered students are expected to perform better.

Likewise, consider the milk yield data, recorded daily, start from calving. It is expected that the milk yield will be decreasing over time, leading to a linear trend in the data set. Let the $N=nk=k{\cdot}k+(n-k)k$ units of the finite population follow a perfect linear trend. That is,

$$y_i = a + ib, \quad \text{for} \quad i = 1,\ 2,\ 3,\ \ldots,N \tag{17}$$

The variance of the mean based on a simple random sampling scheme in the case of linear trend is given by,

$$Var\left(\bar{y}_r\right) = \left(k-1\right)\left(N+1\right)\frac{b^2}{12} \tag{18}$$

The variance of the mean in linear-systematic sampling is given by,

$$Var\left(\bar{y}_{sy}\right) = \left(k-1\right)\left(k+1\right)\frac{b^2}{12} \tag{19}$$

The variance of the mean in diagonal-systematic sampling is given by,

$$Var\left(\bar{y}_{dsy}\right) = \left(k-n\right)\left[n\left(k-n\right)+2\right]\frac{b^2}{12n} \tag{20}$$

The variance of the remainder systematic sampling is given by,

$$Var\left(\bar{y}_{rsy}\right) = k\left[\left(n-r\right)^2 k\left(k^2-1\right) + r^2\left(k+1\right)^2\left(k+2\right)\right]\frac{b^2}{12N^2} \tag{21}$$

where $N=nk+r$. The variance of the sample means based on Subramani's modified systematic sampling scheme is given by,

$$Var\left(\bar{y}_{msy}\right) = \left(\frac{\left(n-1\right)^2+1}{n^2}\right)\left(k-1\right)\left(k+1\right)\frac{b^2}{12} \tag{22}$$

Finally, the variance of the mean in the proposed method is obtained as:

$$Var\left(\bar{y}_{mdsy}\right)=w_1^2 Var\left(\bar{y}_1\right)+w_2^2 Var\left(\bar{y}_2\right) \tag{23}$$

Since there are $k\times k=k^2$ units in Set-1, this implies putting $k=n$ in (20) leads to

$$Var\left(\bar{y}_1\right)=0 \tag{24}$$

Also, there are $(n\text{-}k)k$ units in Set-2 where linear systematic sampling is used. Since the RHS of (19) is independent of n, so,

$$Var\left(\bar{y}_2\right)=Var\left(\bar{y}_{sy}\right)=\left(k-1\right)\left(k+1\right)\frac{b^2}{12} \tag{25}$$

Using (24) and (25) in (23), the variance of $\bar{y}_{mdsy}$ in the case of linear trend is obtained as,

$$Var\left(\bar{y}_{mdsy}\right)=\left(\frac{n-k}{n}\right)^2\left(k-1\right)\left(k+1\right)\frac{b^2}{12} \tag{26}$$

3.1. Simple random and linear systematic sampling

If the units of the population follow a linear trend, the proposed modified diagonal systematic sampling scheme is more efficient than the simple random sampling scheme if,

$$Var\left(\bar{y}_{mdsy}\right)<Var\left(\bar{y}_r\right) \tag{27}$$

Using (18) and (26) in (27) and on simplification, the condition reduces to

$$\left(\frac{n-k}{n}\right)^2\left(k+1\right)<N+1 \tag{28}$$

Condition (28) always holds. Therefore, the proposed modified diagonal systematic sampling scheme is always more efficient than a simple random sampling scheme.
The proposed sampling scheme is more efficient than a linear systematic sampling scheme if

$$Var\left(\bar{y}_{mdsy}\right)<Var\left(\bar{y}_{sy}\right) \tag{29}$$

Using (19) and (26) in (29),

$$\left(\frac{n-k}{n}\right)^2\left(k-1\right)\left(k+1\right)\frac{b^2}{12}<\left(k-1\right)\left(k+1\right)\frac{b^2}{12}$$

on simplification, the condition reduces to

$$\left(\frac{n-k}{n}\right)^2 < 1 \tag{30}$$

which always holds for $n>k$, making the proposed method more efficient than linear-systematic sampling.

3.2. Subramani's modified systematic sampling

The proposed sampling scheme is more efficient than Subramani's modified linear systematic sampling scheme if,

$$Var\left(\overline{y}_{mdsy}\right) < Var\left(\overline{y}_{msy}\right) \tag{31}$$

Using (22) and (26) in (31),

$$\left(\frac{n-k}{n}\right)^2 (k-1)(k+1)\frac{b^2}{12} < \left(\frac{(n-1)^2+1}{n^2}\right)(k-1)(k+1)\frac{b^2}{12}$$

$$\text{Or} \quad \left(\frac{n-k}{n}\right)^2 < \left(\frac{(n-1)^2+1}{n^2}\right)$$

On further simplification it yields,

$$(n-k)^2 < (n-1)^2 + 1 \tag{32}$$

Since $n>k$, so condition (32) always holds. Hence, the proposed sampling scheme is more efficient than Subarmani's modified Systematic Sampling scheme.

3.3. Diagonal systematic sampling

The proposed sampling scheme is more efficient than the diagonal systematic sampling scheme if,

$$Var\left(\overline{y}_{mdsy}\right) < Var\left(\overline{y}_{dsy}\right) \tag{33}$$

Using (20) and (26) in (33)

$$\left(\frac{n-k}{n}\right)^2 (k-1)(k+1)\frac{b^2}{12} < (k-n)\left[n(k-n)+2\right]\frac{b^2}{12n}$$

$$\text{Or,} \quad \frac{(k-n)^2}{n^2}(k-1)(k+1) < (k-n)\left[n(k-n)+2\right]\frac{1}{n}$$

$$\text{Or, } (k-n)(k-1)(k+1) < n^2(k-n) + 2n$$

$$\text{Or, } (k-n)(k^2-1) - n^2(k-n) - 2n < 0$$

Further simplification yields

$$(n-k)\left[k^2 - (n^2+1)\right] + 2n \geq 0 \tag{34}$$

The proposed method will be more efficient provided that the condition in (34) is satisfied.

4. Empirical results

The proposed method experimented with real data (milk yield). The results show that the proposed method is more efficient as compared to the commonly used methods. The milk yield (in liters) of the S-19 brand of Sahiwal cows for 252 days from the date of calving is taken from Pandey and Kumar [19]. The data shows that the milk yield tends to decrease over time, thus creating a high degree of a linear trend. The variances of different sampling schemes based on milk yield data are given in Table 2. The results show that the proposed mixed-mode sampling scheme is more efficient than the existing sampling schemes.

Table 2. Variances of different sampling schemes for milk yield data.

n	k	$Var(\bar{y}_r)$	$Var(\bar{y}_{sy})$	$Var(\bar{y}_{dsy})$	$Var(\bar{y}_{rsy})$	$Var(\bar{y}_{mdsy})$
83	3	1.7329	2.0410	1.6124	0.9154	0.8653
63	4	1.5051	1.1436	1.0070	0.7900	0.5180
49	5	1.0628	0.7286	0.6023	0.5826	0.3496
42	6	0.9291	0.6542	0.5317	0.4604	0.2604
35	7	0.7690	0.5881	0.4781	0.3713	0.2267
31	8	0.7157	0.5017	0.4496	0.3418	0.1807
28	9	0.6143	0.4210	0.3305	0.2501	0.1268
25	10	0.5243	0.3781	0.2851	0.1947	0.1097
22	11	0.5034	0.3535	0.2641	0.1693	0.0988
21	12	0.4632	0.3130	0.2345	0.1471	0.0923
19	13	0.4153	0.3042	0.2160	0.1260	0.0891
18	14	0.3613	0.2896	0.1990	0.1063	0.0856
16	15	0.3650	0.2775	0.1630	0.0993	0.0833

Since the population size is N = 252 units and systematic sampling requires $N = nk$, therefore, for some choices of n and k, some observations from the original population were randomly deleted to reconcile between the values of N, n, and k. For example, for n = 10 and k = 25, two units were randomly removed from the population to make the population size N = 250 instead of N = 252. The efficiency has also been compared in the case of a perfect linear trend for various choices of N, n, and *k*. The variances of the proposed and other sampling methods for various choices of *N*, *n*, and *k* have been presented in Table 3.

Table 3. Variances of various sampling schemes under the perfect linear trend.

n	k	$Var(\bar{y}_r)$	$Var(\bar{y}_{sy})$	$Var(\bar{y}_{dsy})$	$Var(\bar{y}_{rsy})$	$Var(\bar{y}_{mdsy})$
10	4	10.25	1.25	2.90	1.04	0.45
	6	25.42	2.92	1.27	2.42	0.47
	8	47.25	5.25	0.30	4.34	0.21
30	5	50.33	2.00	51.94	1.87	1.39
	10	225.75	8.25	33.22	7.72	3.67
	15	526.17	18.67	18.67	17.47	4.67
	20	951.58	33.25	8.28	31.12	3.69
	25	1502.00	52.00	2.06	48.66	1.44
50	10	375.75	8.25	133.20	7.93	5.28
	20	1584.92	33.25	74.90	31.95	11.97
	30	3627.42	74.92	33.27	71.98	11.99
	40	6503.25	133.25	8.30	128.03	5.33
100	20	3168.25	33.25	533.20	32.59	21.28
	40	13003.25	133.25	299.90	130.61	47.97
	60	29504.92	299.92	133.27	293.98	47.99
	80	52673.25	533.25	33.30	522.69	21.33
500	100	412508.25	833.25	13333.20	829.92	533.28
	200	1658349.92	3333.25	7499.90	3319.94	1199.97
	300	3737524.92	7499.92	3333.27	7469.98	1199.99
	400	6650033.25	13333.25	833.30	13280.02	533.33

The values of *N*, *n*, and *k* have been chosen in such a way that $N = nk$ and $n > k$. Since the constant b^2 is multiple in the variance of all sampling schemes, $b = 1$ has been used to make the comparison simpler. The results indicate that the proposed modified diagonal systematic sampling scheme is more efficient than the other existing sampling schemes.

5. Conclusions

The proposed method divides the population into two subsets. The method uses diagonal-systematic sampling in the first subset and linear-systematic sampling in the second subset. Thus the proposed method combines the linear, diagonal, and remainder systematic sampling schemes into a new method in such a way as to not only increase the efficiency but is also applicable for any sample and population size. The proposed method's efficiency has also been compared to three basic methods of systematic sampling–linear systematic sampling, diagonal systematic sampling, and remainder systematic sampling. Based on empirical and theoretical studies, the results show that our proposed method is the most efficient in a partial and perfect linear trend.

Acknowledgments

The authors received no funds for this study.

Conflict of interest

The authors have no conflict of interest to declare.

References

1. W. G. Madow, L. H. Madow, On the theory of systematic sampling, I, *Ann. Math. Statist.*, **15** (1944), 1–24.
2. J. Subramani, S. N. Gupta, G. Prabavathy, Circular systematic sampling in the presence of linear trend, *Am. J. Math. Manage. Sci.*, **33** (2014), 1–19.
3. H. J. Chang, K. C. Huang, Remainder linear systematic sampling, *Sankhya: Indian J. Stat. Ser. B*, **62** (2000), 249–256.
4. J. Subramani, Diagonal systematic sampling scheme for finite populations. *J. Indian Soc. Agric. Stat.*, **53** (2000), 187–195.
5. S. Sampath, V. Varalakshmi, Diagonal circular systematic sampling, *Model Assisted Stat. Appl.*, **3** (2008), 345–352.
6. J. Subramani, Further results on diagonal systematic sampling for finite populations, *J. Indian Soc. Agric. Stati.*, **63** (2009), 277–282.
7. J. Subramani, A modification on linear systematic sampling for odd sample size, *Bonfring Int. J. Data Min.*, **2** (2012), 32–36.
8. J. Subramani, S. N. Gupta, Generalized modified linear systematic sampling scheme for finite populations, *Hacet. J. Math. Stat.*, **43** (2014), 529–542.
9. W. G. Madow, On the theory of systematic sampling III-comparison of centered and random start systematic sampling, *Ann. Math. Statist.*, **24** (1953), 101–106.
10. F. Yates, Systematic sampling. *Philos. T. R. Soc A*, **241** (1948), 345–377.
11. D. R. Bellhouse, J. N. K Raom, Systematic sampling in the presence of linear trends, *Biometrika*, **62** (1975), 694–697.
12. D. R. Bellhouse, 6 Systematic sampling, *Handbook Stat.*, **6** (1988), 125–145.
13. R. L. Fountain, P. L. Pathak, Systematic and non-random sampling in the presence of linear trends, *Commun. Stat–Theory M.*, **18** (1989), 2511–2526.
14. S. Sampath, N. Uthayakumaran, Markov systematic sampling, *Biometrical J.*, **40** (1998), 883–895.
15. J. Subramani, A modification on linear systematic sampling, *Model Assisted Stat. Appl.*, **8** (2013), 215–227.
16. D. G. Horvitz, D. J. Thompson, A generalization of sampling without replacement from finite universe, *J. Am. Stat. Assoc.*, **47** (1952), 663–685.
17. V. M. Joshi, Note on the admissibility of the sen-yates-grundy variance estimator and murthy's estimator and its variance estimator for samples of size two, Sankhyā: *Indian J. Stat. Ser. A*, **32** (1970), 431–438.

18. F. Yates, P. M. Grundy, Selection without replacement from within strata with probability proportional to size, *J. R. Stat. Soc. Ser. B*, **15** (1953), 253–261.
19. T. K. Pandey, V. Kumar, Systematic sampling for non-linear trend in milk yield data, *J. Reliab. Stat. Stud.*, **7** (2014), 157–168.