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Research article

Exponential stability of stochastic Hopfield neural network with mixed multiple delays

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Abstract: This paper investigates the problem for exponential stability of stochastic Hopfield neural networks involving multiple discrete time-varying delays and multiple distributed time-varying delays. The exponential stability of such neural systems has not been given much attention in the past literature because this type of neural systems cannot be transformed into the vector forms and it is difficult to derive the easily verified stability conditions expressed in terms of the linear matrix inequality. Therefore, this paper tries to establish the easily verified sufficient conditions of the linear matrix inequality forms to ensure the mean-square exponential stability and the almost sure exponential stability for this type of neural systems by constructing a suitable Lyapunov-Krasovskii functional and inequality techniques. Four examples are provided to demonstrate the effectiveness of the proposed theoretical results and compare the established stability conditions to the previous results.

Keywords: stochastic Hopfield neural network; multiple time-varying delays; exponential stability
Mathematics Subject Classification: 34D20

1. Introduction

Since Hopfield neural network was proposed in 1982, many mathematicians, physicists and computer experts have been working on the dynamic behaviors of this network and its applications in pattern recognition, associative memory and optimization [1–4]. The stability analysis of nonlinear nature of neural networks is of great interest when designing neural networks for practical applications because the existence of stable equilibrium points of such neural networks can avoid some suboptimal responses. Therefore, the stability analysis of dynamic neural system has always been a research hotspot. It is also known that it is inevitable to encounter various types of time delay which might cause great damage to the stability in the process of neural network implementation. Among the various types of time delay, time-varying delay and distributed delay are the most common. The time-varying delay must exist due to the finite switching speed of amplifiers and the distributed delay often occur beacuse a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths. Recently, some research papers have analyzed the stability of various delayed neural networks and obtained useful stability results, see, for example, [5–19], and references therein.

On the other hand, it has been well recognized that stochastic perturbations are ubiquitous and inevitable in the real nervous systems [20]. Recently, some valuable stability results of stochastic delayed neural networks can be found in some famous journals related to mathematics, physics and neural network, for example, see [21–38] and references therein. It is noted that most of these literatures have studied the networks which can be expressed in the vector forms and established various stability criteria in the linear matrix inequality forms. Different from them, stochastic neural networks investigated in this paper cannot be transformed into the vector forms because of the existence of the multiple delays, which causes the difficulty of establishing the stability condition expressed in terms of the linear matrix inequality. The existence of the stochastic perturbations, the time-varying delays and the distributed delays in the stochastic neural networks further increase the difficulty. Perhaps, it is the reason that the exponential stability of such neural networks has not been given much attention in the past literature.

In this paper, we mainly consider the mean-square stability and almost sure exponential stability for nonlinear stochastic Hopfield neural networks involving multiple discrete time-varying delays and multiple distributed time-varying delays. The main aim of this paper is to establish the stability conditions of the linear matrix inequality form for such stochastic Hopfield neural networks by constructing a suitable Lyapunov-Krasovskii functional and inequality techniques. Since the systems studied in [5, 24, 33] are some special cases of our proposed system, the stability conditions we established are valid for these systems while their stability conditions are invalid for our proposed system. Four examples are provided to demonstrate the effectiveness of our proposed theoretical results and compare the established stability conditions to the previous results in [5, 24, 33]. These examples show that the established stability conditions are easily verified by MATLAB LMI control toolbox and better than the stability conditions in [5, 24, 33]. Therefore, for the neural networks in [5, 24, 33], our results provide novel sufficient conditions which are easy to verify. Our proposed approach can be applied to study the exponential stability for other types of stochastic (or deterministic) neural networks with multiple delays.

2. Preliminaries

This paper considers the following stochastic Hopfield neural networks with the mixed multiple delays

$$dx_i(t) \;=\; [-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_{ij}(t)))$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\int_{t-\rho_{ij}(t)}^{t} d_{ij}h_j(x_j(s))ds]dt \\
&+\sum_{j=1}^{n}\sigma_{ij}(x_j(t),x_j(t-\tau_{ij}(t)))dw_j(t), i=1,\cdots,n,
\end{aligned} \tag{2.1}
$$

where c_i is the self-feedback connection weight satisfying $c_i > 0$; a_{ij}, b_{ij} and d_{ij} present the connection weight coefficients; $\tau_{ij}(t)$ and $\rho_{ij}(t)$ are multiple delays; $\sigma_{ij}(\cdot,\cdot)$ are the diffusion functions; $f_i(\cdot), g_i(\cdot)$ and $h_i(\cdot)$ denote the nonlinear activation functions; $w(t) = (w_1(t),\cdots,w_n(t))^T$ is n-dimensional Brownian motion defined on a complete probability space $(\Omega,\mathcal{F},\mathbb{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ generated by $\{w(t)\}$, where we associate Ω with the canonical space generated by $w(t)$, and denote by $\mathcal{F}$ the associated σ-algebra generated by $\{w(s): 0\leq s\leq t\}$ with the probability measure $\mathbb{P}$.

Throughout this paper, the following assumptions are required for system (2.1):

(A_1) : There exist constants $\tau > 0, \rho > 0$ and μ such that for $t \geq 0$,

$$
0 \leq \tau_{ij}(t) \leq \tau, 0 \leq \rho_{ij}(t) \leq \rho, \dot{\tau}_{ij}(t) \leq \mu < 1.
$$

(A_2) : The diffusion functions $\sigma_{ij}(\cdot,\cdot)$ satisfy $\sigma_{ij}(0,0) = 0$ and that there exist nonnegative constants L_{ij} and M_{ij} such that for all $x, y \in \mathbb{R}$,

$$
|\sigma_{ij}(x,y)| \leq L_{ij}|x| + M_{ij}|y|.
$$

(A_3) : $f_i(\cdot), g_i(\cdot)$ and $h_i(\cdot)$ satisfy $f_i(0) = g_i(0) = h_i(0) = 0$ and that there exist some constants $\alpha_i^-, \alpha_i^+, \beta_i^-, \beta_i^+, \gamma_i^-$ and γ_i^+ such that for all $x, y \in \mathbb{R}(x \neq y)$,

$$
\alpha_i^- \leq \frac{f_i(x)-f_i(y)}{x-y} \leq \alpha_i^+, \beta_i^- \leq \frac{g_i(x)-g_i(y)}{x-y} \leq \beta_i^+, \gamma_i^- \leq \frac{h_i(x)-h_i(y)}{x-y} \leq \gamma_i^+.
$$

The initial condition $x_i(s) = \xi_i(s), s \in [-\max\{\tau,\rho\},0]$, and $\xi = \{(\xi_1(s),\cdots,\xi_1(s))^T : -\max\{\tau,\rho\} \leq s \leq 0\}$ is $C([-\max\{\tau,\rho\},0];\mathbb{R}^n)$-valued function and $\mathcal{F}_0$-measurable $\mathbb{R}^n$-valued random variable satisfying

$$
\|\xi\|^2 = \sup_{-\max\{\tau,\,\rho\}\leq t\leq 0} \mathbb{E}\|\xi(t)\|^2 < \infty,
$$

where $\|\cdot\|$ denotes the Euclidean norm and $C([-\max\{\tau,\rho\},0];\mathbb{R}^n)$ denotes the space of all continuous $\mathbb{R}^n$-valued functions defined on $[-\max\{\tau,\rho\},0]$.

Remark 1. It is noted that assumption (A_3) is less conservative than the Lipschitz conditions satisfied by $f_i(\cdot)$ and $g_i(\cdot)$ in [15, 22–24, 29, 38] since $\alpha_i^-, \alpha_i^+, \beta_i^-$ and β_i^+ ($\alpha_i^- < \alpha_i^+, \beta_i^- < \beta_i^+$) in (A_3) can be any real numbers.

System (2.1) is a more general mathematical expression and can be described in different mathematical forms by changing the system parameters and functions. When $\tau_{ij}(t) = \rho_{ij}(t) = \tau_j(t), f_j = g_j, w_1(t) = \cdots = w_j(t) = w(t)$ and $\sigma_{ij}(x_j(t), x_j(t-\tau_{ij}(t))) = L_{ij}x_j(t) + M_{ij}x_j(t-\tau_j(t))$, system (2.1) transforms into the following equation studied in [33]:

$$
dx_i(t) = [-c_ix_i(t) + \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j(t)))
$$

$$+\sum_{j=1}^{n}\int_{t-\tau_j(t)}^{t} d_{ij}h_j(x_j(s))ds]dt$$
$$+\sum_{j=1}^{n}[L_{ij}x_j(t)+M_{ij}x_j(t-\tau_j(t))]dw(t), i=1,\cdots,n. \tag{2.2}$$

When $\tau_{ij}(t) = \tau_j, d_{ij} = 0$ and $\sigma_{ij}(x_j(t), x_j(t-\tau_{ij}(t))) = \sigma_{ij}(x_j(t))$, system (2.1) transforms into the following equation studied in [24]

$$\begin{aligned} dx_i(t) &= [-c_ix_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}g_j(x_j(t-\tau_j))]dt \\ &\quad+\sum_{j=1}^{n}\sigma_{ij}(x_j(t))dw_j(t), i=1,\cdots,n, \end{aligned} \tag{2.3}$$

When $f_j = g_j, d_{ij} = 0$ and $\sigma_{ij}(x_j(t), x_j(t-\tau_{ij}(t))) = 0$, system (2.1) transforms into the following deterministic system studied in [5]:

$$dx_i(t) = -c_ix_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_{ij}(t))), i=1,\cdots,n. \tag{2.4}$$

3. Stability results and examples

In this section, novel sufficient conditions of exponential stability of zero solution of system (2.1) are presented. Four examples are given to demonstrate the effectiveness of our theoretical results and compare the stability conditions to the previous results in [5, 24, 33].

Theorem 1. Suppose that there exist some positive real numbers $p_1,\cdots,p_n, u_{i1},\cdots,u_{in}$ $(i = 1,2,3)$ such that

$$\Gamma = \begin{pmatrix} \Delta & PA+U_1\Sigma_4 & U_2\Sigma_6 & U_3\Sigma_8 \\ * & -2U_1 & 0 & 0 \\ * & * & -2U_2+\frac{1}{1-\mu}B_2 & 0 \\ * & * & * & -2U_3+\rho^2D_2 \end{pmatrix} < 0,$$

where $*$ means the symmetric terms, $\Gamma < 0$ means that matrix Γ is symmetric negative definite,

$$\Delta = -2PC+PB_1+PD_1+\Sigma_1+\frac{1}{1-\mu}\Sigma_2-2U_1\Sigma_3-2U_2\Sigma_5-2U_3\Sigma_7,$$
$$A = (a_{ij})_{n\times n}, C = diag\{c_1,\cdots,c_n\}, P = diag\{p_1,\cdots,p_n\},$$
$$U_1 = diag\{u_{11},\cdots,u_{1n}\}, U_2 = diag\{u_{21},\cdots,u_{2n}\}, U_3 = diag\{u_{31},\cdots,u_{3n}\},$$
$$B_1 = diag\{\sum_{j=1}^{n}|b_{1j}|,\cdots,\sum_{j=1}^{n}|b_{nj}|\}, B_2 = diag\{\sum_{j=1}^{n}p_j|b_{j1}|,\cdots,\sum_{j=1}^{n}p_j|b_{jn}|\},$$
$$D_1 = diag\{\sum_{j=1}^{n}|d_{1j}|,\cdots,\sum_{j=1}^{n}|d_{nj}|\}, D_2 = diag\{\sum_{j=1}^{n}p_j|d_{j1}|,\cdots,\sum_{j=1}^{n}p_j|d_{jn}|\},$$

$$\Sigma_1 = 2\ diag\{\sum_{j=1}^{n} p_j L_{j1}^2, \cdots, \sum_{j=1}^{n} p_j L_{jn}^2\}, \Sigma_2 = 2\ diag\{\sum_{j=1}^{n} p_j M_{j1}^2, \cdots, \sum_{j=1}^{n} p_j M_{jn}^2\},$$
$$\Sigma_3 = diag\{\alpha_1^- \alpha_1^+, \cdots, \alpha_n^- \alpha_n^+\}, \Sigma_4 = diag\{\alpha_1^- + \alpha_1^+, \cdots, \alpha_n^- + \alpha_n^+\},$$
$$\Sigma_5 = diag\{\beta_1^- \beta_1^+, \cdots, \beta_n^- \beta_n^+\}, \Sigma_6 = diag\{\beta_1^- + \beta_1^+, \cdots, \beta_n^- + \beta_n^+\},$$
$$\Sigma_7 = diag\{\gamma_1^- \gamma_1^+, \cdots, \gamma_n^- \gamma_n^+\}, \Sigma_8 = diag\{\gamma_1^- + \gamma_1^+, \cdots, \gamma_n^- + \gamma_n^+\}.$$

Then zero solution of system (2.1) is almost surely exponentially stable and exponentially stable in mean square.

Proof. $\Gamma < 0$ implies that there exists a sufficient small real number $\lambda > 0$ such that

$$\bar{\Gamma} = \begin{pmatrix} \bar{\Delta} & PA + U_1\Sigma_4 & U_2\Sigma_6 & U_3\Sigma_8 \\ * & -2U_1 & 0 & 0 \\ * & * & -2U_2 + \frac{1}{1-\mu} e^{\lambda\tau} B_2 & 0 \\ * & * & * & -2U_3 + \rho^2 e^{\lambda\rho} D_2 \end{pmatrix} < 0,$$

in which

$$\bar{\Delta} = \lambda P - 2PC + PB_1 + PD_1 + \Sigma_1 + \frac{e^{\lambda\tau}}{1-\mu}\Sigma_2 - 2U_1\Sigma_3 - 2U_2\Sigma_5 - 2U_3\Sigma_7.$$

Constructing the following Lyapunov-Krasovskii functional

$$\begin{aligned} V(t) &= e^{\lambda t}\sum_{i=1}^{n} p_i x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{t-\tau_{ij}(t)}^{t} e^{\lambda(s+\tau)} p_i \frac{|b_{ij}| g_j^2(x_j(s)) + 2M_{ij}^2 x_j^2(s)}{1-\mu} ds \\ &\quad + \int_{-\rho}^{0}\int_{t+s}^{t}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i |d_{ij}| \rho e^{\lambda(\theta+\rho)} h_j^2(x_j(\theta)) d\theta ds. \end{aligned} \tag{3.1}$$

Applying It$\hat{o}$ formula in [21] to $V(t)$ along the trajectory of system (2.1), we obtain

$$dV(t) = \bar{V}(t)dt + 2e^{\lambda t}\sum_{i=1}^{n} p_i x_i(t)\sum_{j=1}^{n}\sigma_{ij}(x_j(t), x_j(t-\tau_{ij}(t)))dw_j(t), \tag{3.2}$$

where

$$\begin{aligned} \bar{V}(t) &= \lambda e^{\lambda t}\sum_{i=1}^{n} p_i x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n}\Big\{ e^{\lambda(t+\tau)} p_i \frac{|b_{ij}| g_j^2(x_j(t)) + 2M_{ij}^2 x_j^2(t)}{1-\mu} \\ &\quad -(1-\dot{\tau}_{ij}(t)) e^{\lambda(t-\tau_{ij}(t)+\tau)} p_i \frac{|b_{ij}| g_j^2(x_j(t-\tau_{ij}(t))) + 2M_{ij}^2 x_j^2(t-\tau_{ij}(t))}{1-\mu}\Big\} \\ &\quad + \sum_{i=1}^{n}\sum_{j=1}^{n} p_i |d_{ij}| \rho \Big\{\rho e^{\lambda(t+\rho)} h_j^2(x_j(t)) - \int_{-\rho}^{0} e^{\lambda(t+s+\rho)} h_j^2(x_j(t+s)) ds\Big\} \\ &\quad +2e^{\lambda t}\sum_{i=1}^{n} p_i x_i(t)\Big\{ - c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_{ij}(t))) \end{aligned}$$

$$+\sum_{j=1}^{n}\int_{t-\rho_{ij}(t)}^{t} d_{ij}h_j(x_j(s))ds\Big\} + e^{\lambda t}\sum_{i=1}^{n} p_i \sum_{j=1}^{n}\sigma_{ij}^2(x_j(t), x_j(t-\tau_{ij}(t))).$$

From (A_1) and (A_2), we derive

$$\begin{aligned}
\bar{V}(t) \leq\ & \lambda e^{\lambda t}\sum_{i=1}^{n} p_i x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n}\Big\{e^{\lambda(t+\tau)}p_i\frac{|b_{ij}|g_j^2(x_j(t)) + 2M_{ij}^2x_j^2(t)}{1-\mu} \\
& -e^{\lambda t}p_i(|b_{ij}|g_j^2(x_j(t-\tau_{ij}(t))) + 2M_{ij}^2x_j^2(t-\tau_{ij}(t)))\Big\} \\
& +\sum_{i=1}^{n}\sum_{j=1}^{n} p_i|d_{ij}|\rho\Big\{\rho e^{\lambda(t+\rho)}h_j^2(x_j(t)) - \int_{t-\rho}^{t} e^{\lambda(s+\rho)}h_j^2(x_j(s))ds\Big\} \\
& +e^{\lambda t}\sum_{i=1}^{n}\Big\{-2p_ic_ix_i^2(t) + \sum_{j=1}^{n}2p_ia_{ij}x_i(t)f_j(x_j(t)) \\
& +\sum_{j=1}^{n} p_i|b_{ij}|(x_i^2(t) + g_j^2(x_j(t-\tau_{ij}(t)))) \\
& +\sum_{j=1}^{n} p_i|d_{ij}|[x_i^2(t) + (\int_{t-\rho_{ij}(t)}^{t}|h_j(x_j(s))|ds)^2] \\
& +2\sum_{j=1}^{n} p_iL_{ij}^2x_j^2(t) + 2\sum_{j=1}^{n} p_iM_{ij}^2x_j^2(t-\tau_{ij}(t))\Big\} \\
\leq\ & \lambda e^{\lambda t}\sum_{i=1}^{n} p_i x_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda(t+\tau)}p_i\frac{|b_{ij}|g_j^2(x_j(t)) + 2M_{ij}^2x_j^2(t)}{1-\mu} \\
& +\sum_{i=1}^{n}\sum_{j=1}^{n} p_i|d_{ij}|\rho\Big\{\rho e^{\lambda(t+\rho)}h_j^2(x_j(t)) - e^{\lambda t}\int_{t-\rho}^{t} h_j^2(x_j(s))ds\Big\} \\
& +e^{\lambda t}\sum_{i=1}^{n}\Big\{-2p_ic_ix_i^2(t) + \sum_{j=1}^{n}2p_ia_{ij}x_i(t)f_j(x_j(t)) \\
& +\sum_{j=1}^{n} p_i|b_{ij}|x_i^2(t) + \sum_{j=1}^{n} p_i|d_{ij}|x_i^2(t) + 2\sum_{j=1}^{n} p_iL_{ij}^2x_j^2(t)\Big\} \\
& +e^{\lambda t}\sum_{i=1}^{n}\sum_{j=1}^{n} p_i|d_{ij}|\rho\int_{t-\rho}^{t} h_j^2(x_j(s))ds \\
\leq\ & e^{\lambda t}\Big\{x^T(t)\Big(\lambda P - 2PC + PB_1 + PD_1 + \Sigma_1 + \frac{e^{\lambda\tau}\Sigma_2}{1-\mu}\Big)x(t) \\
& +\frac{e^{\lambda\tau}}{1-\mu}g^T(x(t))B_2g(x(t)) + 2x^T(t)PAf(x(t)) \\
& +\rho^2e^{\lambda\rho}h^T(x(t))D_2h(x(t))\Big\},
\end{aligned} \tag{3.3}$$

where

$$x(t) = (x_1(t), \cdots, x_n(t))^T, f(x(t)) = (f_1(x_1(t)), \cdots, f_n(x_n(t)))^T,$$

$$g(x(t)) = (g_1(x_1(t)), \cdots, g_n(x_n(t)))^T, h(x(t)) = (h_1(x_1(t)), \cdots, h_n(x_n(t)))^T.$$

From (A_3), we derive

$$\begin{aligned} 0 &\leq -2\sum_{i=1}^{n} u_{1i}[f_i(x_i(t)) - \alpha_i^+ x_i(t)][f_i(x_i(t)) - \alpha_i^- x_i(t)] \\ &= -2\sum_{i=1}^{n} u_{1i}[f_i^2(x_i(t)) - (\alpha_i^+ + \alpha_i^-)x_i(t)f_i(x_i(t)) + \alpha_i^+\alpha_i^- x_i^2(t)] \\ &= -2f^T(x(t))U_1 f(x(t)) + 2f^T(x(t))U_1\Sigma_4 x(t) - 2x^T(t)U_1\Sigma_3 x(t), \end{aligned} \tag{3.4}$$

$$\begin{aligned} 0 &\leq -2\sum_{i=1}^{n} u_{2i}[g_i(x_i(t)) - \beta_i^+ x_i(t)][g_i(x_i(t)) - \beta_i^- x_i(t)] \\ &\leq -2g^T(x(t))U_2 g(x(t)) + 2g^T(x(t))U_2\Sigma_6 x(t) - 2x^T(t)U_2\Sigma_5 x(t) \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} 0 &\leq -2\sum_{i=1}^{n} u_{3i}[h_i(x_i(t)) - \gamma_i^+ x_i(t)][h_i(x_i(t)) - \gamma_i^- x_i(t)] \\ &\leq -2h^T(x(t))U_3 h(x(t)) + 2h^T(x(t))U_3\Sigma_8 x(t) - 2x^T(t)U_3\Sigma_7 x(t). \end{aligned} \tag{3.6}$$

Inequalities (3.3)–(3.6) derive

$$\bar{V}(t) \leq e^{\lambda t} y^T(t)\bar{\Gamma} y(t) < 0, \tag{3.7}$$

where $y(t) = (x^T(t), f^T(x(t)), g^T(x(t)), h^T(x(t)))^T$.

Integrating from 0 and t for (3.2) and combining with (3.7), we obtain

$$\begin{aligned} V(t) &= V(0) + \int_0^t \bar{V}(s)ds + \int_0^t 2e^{\lambda s}\sum_{i=1}^{n} p_i x_i(s)\sum_{j=1}^{n}\sigma_{ij}(x_j(s), x_j(s-\tau_{ij}(s)))dw_j(s) \\ &< V(0) + \int_0^t 2e^{\lambda s}\sum_{i=1}^{n} p_i x_i(s)\sum_{j=1}^{n}\sigma_{ij}(x_j(s), x_j(s-\tau_{ij}(s)))dw_j(s). \end{aligned} \tag{3.8}$$

The nonnegative semi-martingale convergence theorem in [21] and (3.8) show that zero solution of system (2.1) is almost surely exponentially stable.

Moreover, (3.1) and (3.8) deduce

$$\begin{aligned} & e^{\lambda t}\min_{1\leq i\leq n}\{p_i\}\mathbb{E}\|x(t)\|^2 \\ \leq\ & \mathbb{E}V(t) < \mathbb{E}V(0) \\ \leq\ & \mathbb{E}\Big\{\max_{1\leq i\leq n}\{p_i\}\|x(0)\|^2 + \sum_{i=1}^{n}\sum_{j=1}^{n}\int_{-\tau}^{0} e^{\lambda(s+\tau)} p_i \frac{|b_{ij}|\beta_j^2 + 2M_{ij}^2}{1-\mu} x_j^2(s)ds \end{aligned}$$

$$+\int_{-\rho}^{0}\int_{s}^{0}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i|d_{ij}|\rho e^{\lambda(\theta+\rho)}\gamma_j^2x_j^2(\theta)d\theta ds\Big\}$$
$$\leq \Big\{\max_{1\leq i\leq n}\{p_i\}+\frac{e^{\lambda\tau}\tau}{1-\mu}\max_{1\leq i\leq n}\{\sum_{j=1}^{n}p_j(|b_{ji}|\beta_i^2+2M_{ji}^2)\}$$
$$+e^{\lambda\rho}\rho^3\max_{1\leq i\leq n}\{\sum_{j=1}^{n}p_j|d_{ji}|\gamma_i^2\}\Big\}||\xi||^2,$$

where $\beta_i = \max\{|\beta_i^-|, |\beta_i^+|\}, \gamma_i = \max\{|\gamma_i^-|, |\gamma_i^+|\}$, which shows that zero solution of system (2.1) is exponentially stable in mean square.

Remark 2. Generally speaking, it is difficult to establish the stability conditions of the linear matrix inequality forms for the system which cannot be transformed into vector-matrix form. For system (2.1), Theorem 1 gives the stability conditions of the linear matrix inequality forms. Unsurprisingly, it is difficult to write an executable Matlab program to solve the matrices P, U_1, U_2 and U_3 by Matlab LMI Control Toolbox because the matrices B_2, D_2, Σ_1 and Σ_2 involve the elements $p_1, \cdots, p_n$ of matrix P.

In what follows, we express a special case of Theorem 1 for $p_1 = \cdots = p_n = p$, which provides a easily verified sufficient criterion by Matlab LMI Control Toolbox.

Theorem 2. Suppose that there exist positive constants $p, u_{i1}, \cdots, u_{in} (i = 1, 2, 3)$ such that

$$\Gamma=\begin{pmatrix}\Delta & pA+U_1\Sigma_4 & U_2\Sigma_6 & U_3\Sigma_8\\ * & -2U_1 & 0 & 0\\ * & * & -2U_2+\frac{1}{1-\mu}B_2 & 0\\ * & * & * & -2U_3+\rho^2D_2\end{pmatrix}<0,$$

where $*, A, C, B_1, D_1, U_1, U_2, U_3$, and $\Sigma_i (i = 3, 4, 5, 6, 7, 8)$ are defined as in Theorem 1,

$$\Delta = -2pC + pB_1 + pD_1 + \Sigma_1 + \frac{1}{1-\mu}\Sigma_2 - 2U_1\Sigma_3 - 2U_2\Sigma_5 - 2U_3\Sigma_7,$$

$$B_2 = p\ diag\{\sum_{j=1}^{n}|b_{j1}|, \cdots, \sum_{j=1}^{n}|b_{jn}|\}, D_2 = p\ diag\{\sum_{j=1}^{n}|d_{j1}|, \cdots, \sum_{j=1}^{n}|d_{jn}|\},$$

$$\Sigma_1 = 2p\ diag\{\sum_{j=1}^{n}L_{j1}^2, \cdots, \sum_{j=1}^{n}L_{jn}^2\}, \Sigma_2 = 2p\ diag\{\sum_{j=1}^{n}M_{j1}^2, \cdots, \sum_{j=1}^{n}M_{jn}^2\}.$$

Then zero solution of system (2.1) is almost surely exponentially stable and exponentially stable in mean square.

For the systems (2.2)–(2.4), Theorem 2 gives the following results.

Corollary 1. Suppose that there exist positive constants $p, u_{i1}, \cdots, u_{in} (i = 1, 2, 3)$ such that

$$\Gamma=\begin{pmatrix}\Delta & pA+U_1\Sigma_4 & U_2\Sigma_4 & U_3\Sigma_8\\ * & -2U_1 & 0 & 0\\ * & * & -2U_2+\frac{1}{1-\mu}B_2 & 0\\ * & * & * & -2U_3+\tau^2D_2\end{pmatrix}<0,$$

where $\Delta = -2pC + pB_1 + pD_1 + \Sigma_1 + \frac{1}{1-\mu}\Sigma_2 - 2U_1\Sigma_3 - 2U_2\Sigma_3 - 2U_3\Sigma_7$, other symbols are the same as Theorem 2. Then, zero solution of system (2.2) is almost surely exponentially stable and exponentially stable in mean square.

Corollary 2. Suppose that there exist positive constants $p, u_{i1}, \cdots, u_{in}(i = 1, 2, 3)$ such that

$$\Gamma = \begin{pmatrix} \Delta & pA + U_1\Sigma_4 & U_2\Sigma_6 \\ * & -2U_1 & 0 \\ * & * & -2U_2 + B_2 \end{pmatrix} < 0,$$

where $\Delta = -2pC + pB_1 + \Sigma_1 - 2U_1\Sigma_3 - 2U_2\Sigma_5$, other symbols are the same as Theorem 2. Then, zero solution of system (2.3) is almost surely exponentially stable and exponentially stable in mean square.

Corollary 3. Suppose that there exist positive constants $p, u_{i1}, \cdots, u_{in}(i = 1, 2, 3)$ such that

$$\Gamma = \begin{pmatrix} \Delta & pA + U_1\Sigma_4 & U_2\Sigma_4 \\ * & -2U_1 & 0 \\ * & * & -2U_2 + \frac{1}{1-\mu}B_2 \end{pmatrix} < 0,$$

where $\Delta = -2pC + pB_1 - 2U_1\Sigma_3 - 2U_2\Sigma_3$, other symbols are the same as Theorem 2. Then, zero solution of system (2.4) is globally exponentially stable.

Remark 3. Since the networks studied in [5, 24, 33] are some special cases of system (2.1), their stability conditions are invalid for system (2.1). On the contrary, our stability conditions are valid for the systems in [5, 24, 33]. In particular, the deterministic system (2.4) in [5] is a special case of stochastic system (2.1), which leads to that it is easy to transform Theorem 2 into Corollary 3. That is, Theorem 2 for system (2.1) includes Corollary 3 for corresponding system (2.4), which shows the stability result of stochastic system is more general than that of corresponding deterministic system.

Remark 4. Although Theorem 5 in [33] gives the sufficient conditions of the linear matrix inequality forms, the stability conditions of Corollary 1 are more easy to verify. Example 2 demonstrates that the validity of Corollary 1 and the stability conditions of Corollary 1 are better than those of Theorem 5 in [33].

Remark 5. Theorem 3.1 in [24] gives the sufficient conditions of the algebraic forms by using Lyapunov function $e^{\lambda t}|x(t)|^2$. This Lyapunov function cannot be applied to study the system with time-varying delays. Example 3 demonstrates that the validity of Corollary 2 and the invalidity of Theorem 3.1 in [24], which shows that the stability conditions of Corollary 2 are better.

Remark 6. In [5], Theorem 2.4 provides the stability condition of the spectral radius form which requires that the absolute values of all eigenvalues of matrix are less than 1. Example 4 demonstrates that the validity of Corollary 3 and the invalidity of Theorem 2.4 in [5], which shows that the stability conditions of Corollary 3 are better.

Example 1. Consider system (2.1) with the following parameters and functions:

$$A = (a_{ij})_{4\times4} = \begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 \end{pmatrix}, B = (b_{ij})_{4\times4} = \begin{pmatrix} -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 1 \end{pmatrix},$$

$$D=(d_{ij})_{4\times4}=\begin{pmatrix}-1&-1&1&-1\\-1&1&-1&-1\\1&-1&1&1\\1&-1&-1&-1\end{pmatrix},C=\begin{pmatrix}6&0&0&0\\0&6&0&0\\0&0&5&0\\0&0&0&6\end{pmatrix},$$

$f_i(x)=0.5tanh(x), g_i(x)=0.4tanh(x), h_i(x)=0.3tanh(x), L_{ij}=M_{ij}=0.1, \tau_{ij}(t)=0.2sint, \rho_{ij}(t)=0.5cost, i=j; \tau_{ij}(t)=0.2cost, \rho_{ij}(t)=0.5sint, i\neq j, i,j=1,2,3,4.$

Then we calculate that $\Sigma_3=\Sigma_5=\Sigma_7=0, B_1=D_1=4I, B_2=D_2=4pI, \Sigma_1=\Sigma_2=0.08pI, \Sigma_4=0.5I, \Sigma_6=0.4I, \Sigma_8=0.3I, \mu=0.2, \rho=0.5$, where I denotes identity matrix.

By using Matlab LMI Control Toolbox, we calculate $P=0.1668I, U_1=diag\{0.5170, 0.5170, 0.5777, 0.5777\}, U_2=0.8794I$ and $U_3=0.6455I$ satisfy the condition of Theorem 2, which demonstrates the effectiveness of our theoretical result.

Example 2. Consider system (2.2) with the following parameters and functions:

$$D=(d_{ij})_{4\times4}=\begin{pmatrix}-1&-1&1&-1\\-1&1&-1&-1\\1&-1&1&1\\1&-1&-1&-1\end{pmatrix},C=\begin{pmatrix}6&0&0&0\\0&6&0&0\\0&0&5&0\\0&0&0&6\end{pmatrix},$$

$f_i(x)=g_i(x)=0.5tanh(x), h_i(x)=0.3tanh(x), L_{ij}=M_{ij}=0.1, \tau_j(t)=0.2sint, i,j=1,2,3,4$, the matrices A and B are the same as in Example 1.

Then we calculate that $\Sigma_3=\Sigma_5=\Sigma_7=0, B_1=D_1=4I, B_2=D_2=4pI, \Sigma_1=\Sigma_2=0.08pI, \Sigma_4=\Sigma_6=0.5I, \Sigma_8=0.3I, \mu=\tau=0.2$. By using Matlab LMI Control Toolbox, we know Corollary 1 holds when $P=0.5791I, U_1=diag\{2.6460, 2.4054, 1.5394, 3.2237\}$, $U_2=diag\{2.8763, 2.8877, 2.8635, 2.8751\}$ and $U_3=diag\{1.6561, 1.6904, 1.3281, 1.8608\}$.

On the other hand, Theorem 5 in [33] shows that zero solution of system (2.2) is almost surely exponentially stable and exponentially stable in mean square provided that there exist some matrices $P>0, U_i=diag\{u_{i1},\cdots,u_{in}\}\geq 0 (i=1,2,3)$ and positive constants $\gamma_1,\gamma_2,\lambda$ such that $\lambda^{-1}\tau\gamma_2^{-1}\in(0,1)$ and

$$\Sigma=\begin{pmatrix}\Delta_1&0&PA+U_1L_2&PB&U_3M_2\\ *&\Delta_2&0&U_2L_2&0\\ *&*&\Delta_3&0&0\\ *&*&*&\Delta_4&0\\ *&*&*&*&\Delta_5\end{pmatrix}<0,$$

where $\sigma_1=(L_{ij})_{n\times n}, \sigma_2=(M_{ij})_{n\times n}, L_1=\Sigma_3, L_2=\Sigma_4, M_1=\Sigma_7, M_2=\Sigma_8$,

$$\Delta_1=(\gamma_1+2\lambda)P-2PC+2\sigma_1^TP\sigma_1+U_1(\lambda I-2L_1)+U_3(\lambda I-2M_1),$$

$$\Delta_2=2\sigma_2^TP\sigma_2+U_2(\lambda I-2L_1), \Delta_3=(2\lambda-2)U_1,$$
$$\Delta_4=(2\lambda-2)U_2, \Delta_5=(2\lambda-2)U_3+\gamma_2D^TPD.$$

It is clear that the above stability condition is more difficult to verify than that of Corollary 1. Moreover, when we choose $\lambda=\gamma_1=\gamma_2=0.5$, we can not find the suitable matrices P, U_1, U_2 and U_3 satisfying the condition of Theorem 5 in [33] by using Matlab LMI Control Toolbox. Therefore, Theorem 5 in [33] is invalid for the system (2.2) in Example 2.

Example 3. Consider system (2.3) with $C = diag\{3.5, 5, 5, 5\}$, $f_i(x) = 0.5tanh(x)$,$g_i(x) = 0.4tanh(x)$, $L_{ij} = 0.1, \tau_j = 0.2, i, j = 1, 2, 3, 4$, the matrices A and B are the same as in Example 1.

Then we calculate that $\Sigma_i = 0(i = 2, 3, 5, 7, 8), B_1 = 4I, B_2 = 4pI, \Sigma_1 = 0.08pI, \Sigma_4 = 0.5I, \Sigma_6 = 0.4I, \tau = 0.2, \mu = 0$. By using Matlab LMI Control Toolbox, we know that Corollary 2 holds when $P = 0.1817I, U_1 = diag\{0.6431, 0.6431, 0.7092, 0.7092\}$ and $U_2 = 0.9723I$.

On the other hand, Theorem 3.1 in [24] shows that the following inequalities

$$-2c_i + \sum_{j=1}^{n} |a_{ij}|\alpha_j + \sum_{j=1}^{n} |b_{ij}|\beta_j + \sum_{j=1}^{n} |a_{ji}|\alpha_i + \sum_{j=1}^{n} |b_{ji}|\beta_i + \sum_{j=1}^{n} L_{ji}^2 < 0(i = 1, \cdots, n)$$

are the sufficient conditions of almost sure exponential stability and mean square exponential stability of system (2.3), where α_i and β_i correspond to $\max\{|\alpha_i^-|, |\alpha_i^+|\}$ and $\max\{|\beta_i^-|, |\beta_i^+|\}$ in this paper, respectively.

Then, we calculate that for $i = 1, 2, 3, 4, \alpha_i = 0.5, \beta_i = 0.4$ and

$$-2c_i + \sum_{j=1}^{4} |a_{ij}|\alpha_j + \sum_{j=1}^{4} |b_{ij}|\beta_j + \sum_{j=1}^{4} |a_{ji}|\alpha_i + \sum_{j=1}^{4} |b_{ji}|\beta_i + \sum_{j=1}^{4} L_{ji}^2 = \begin{cases} 0.24, & i = 1; \\ -2.76, & i = 2, 3, 4. \end{cases}$$

Therefore, Theorem 3.1 in [24] is invalid for the system (2.3) in Example 3.

Example 4. Consider system (2.4) with $C = 4I, f_i(x) = 0.5tanh(x), \tau_{ij}(t) = 0.2sint, i = j; \tau_{ij}(t) = 0.2cost, i \neq j, i, j = 1, 2, 3, 4$, the matrices A and B are the same as in Example 1.

Then we calculate that $\Sigma_i = 0(i = 1, 2, 3, 5, 7, 8), B_1 = 4I, B_2 = 4pI, \Sigma_1 = 0.08pI, \Sigma_4 = 0.5I, \Sigma_6 = 0.4I, \tau = \mu = 0.2$. By using Matlab LMI Control Toolbox, we know the matrices $P = 0.1679I, U_1 = diag\{0.5707, 0.5707, 0.6318, 0.6318\}$ and $U_2 = 0.9065I$ satisfy the condition of Corollary 3.

On the other hand, Theorem 2.4 in [5] shows that if $\rho(K) < 1$, then zero solution of system (2.4) is globally exponentially stable, where $\rho(K)$ denotes spectral radius of matrix $K = (k_{ij})_{n\times n}$, $k_{ij} = c_i^{-1}(|a_{ij}| + |b_{ij}|)\alpha_j, \alpha_j$ corresponds to $\max\{|\alpha_j^-|, |\alpha_j^+|\}$ in this paper.

Then, we calculate that for $i = 1, 2, 3, 4, \alpha_i = 0.5$ and $\rho(K) = 1$, where

$$K = (k_{ij})_{4\times 4} = \begin{pmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{pmatrix}.$$

Therefore, the condition of Theorem 2.4 in [5] is not satisfied for the system (2.4) in Example 4.

4. Conclusions

This paper has investigated the problem for exponential stability of stochastic Hopfield neural networks involving multiple discrete time-varying delays and multiple distributed time-varying delays. The exponential stability of such neural systems has not been given much attention because it is difficult to derive the easily verified stability conditions of the linear matrix inequality forms for this type of neural systems that cannot be transformed into the vector forms. This paper has established the easily verified sufficient conditions of the linear matrix inequality forms to ensure the mean-square exponential stability and the almost sure exponential stability by constructing a suitable Lyapunov-Krasovskii functional and inequality techniques. Four examples demonstrate the effectiveness of the proposed theoretical results and show that the established stability conditions are better than the conditions of the previous stability results.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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