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Research article

Quantifying some distance topological properties of the non-zero component graph

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Abstract: Several bioactivities of chemical compounds in a molecular graph can be expected by using many topological descriptors. A topological descriptor is a numeric quantity which quantify the topology of a graph. By defining the metric on a graph related with a vector space, we consider this graph in the context of few topological descriptors, and quantify the Wiener index, hyper Wiener index, Reciprocal complimentary Wiener index, Schultz molecular topological index and Harary index. We also provide the graphical comparison of our results to describe the relationship and dependence of these descriptors on the involved parameters.

Keywords: distance; distance-based topological descriptors; non-zero component graph
Mathematics Subject Classification: 20E45, 05C25, 05C12, 05C07, 05C69, 15A27

1. Preliminaries

In this article, we always consider simple, non-empty and connected graph G with vertex set $V(G)$ and edge set $E(G)$. The quantity $d(u, v)$ represents the *distance* between u and v for any $u, v \in V(G)$. Note, that the distance between any two vertices u and v of G is the length of a shortest path between u

and v. The quantity $ecc(v) = \max_{u \in V(G)} d(u,v)$ is the *eccentricity* of v in G. The *diameter* $D(G)$ of G is the maximum eccentricity among all the eccentricities of the vertices of G. The quantity d_v represents the degree of $v \in V(G)$, which is the number of edges incident with v.

Cheminformatics is a new subject defined by the combination of chemistry, mathematics and information. The biological activities and properties of chemical compounds have been predicted by cheminformatics, by studying relationships between Quantitative structure-activity and Quantitative structure-property (QSAR/QSPR). In the study of QSAR/QSPR, the bioactivity of a chemical compounds has been predicted by using physico-chemical properties and topological descriptors such as Wiener index, hyper-Wiener index, Harary index and Schultz index. A topological descriptor is, in a numeric quantity, related to the chemical composition, aiming for correlation between chemical structure and physico-chemical properties. They are, in fact, designed on the ground of transformation of a molecular graph into a number which signifies the topology of that graph. Significant research on topological descriptors of various families of graphs has been done because of their chemical importance [5, 6, 15, 16].

The first non-trivial distance-based topological descriptor was introduced by Wiener, in 1947, to study the boiling points of paraffins. At first, this descriptor was called *path number* by him and was later on named as *Wiener index* [22]. There has been considerable research interest in Wiener index and related indices (see the bibliography and therein [17, 20]). This index was stated in terms of edge weights which was defined on trees in the beginning. Traditionally, its generalization on general graphs G is defined as:

$$W(G) = \sum_{u,v \in V(G)} d(u,v).$$

Several mathematical chemists have studied and introduced a large number of generalizations, and extensions of the Wiener index during the period of last two decades. There is an extensive bibliography on this topic can by viewed in [4, 11]. One of these extensions was proposed by Randić for trees [19] called, *hyper-Wiener index*. It was extended by Klein *et al.* [9] to all connected graphs. This index significantly used as a structure descriptor for prediction of physicochemical properties of chemical compounds, which are important for pharmacology, agriculture and environment protection [3, 9, 19]. This index is defined as:

$$WW(G) = \frac{1}{2} \sum_{u,v \in V(G)} d(u,v)(1 + d(u,v)).$$

In 2000, Ivancivc *et al.* [7] introduced reciprocal complementary Wiener index, and is defined as follows:

$$RCW(G) = \sum_{u,v \in V(G)} \frac{1}{D(G) + 1 - d(u,v)}.$$

The generalization of the Wiener index is the *Schultz molecular topological index*, which was introduced in 1989 [21], and is defined as:

$$MTI(G) = \sum_{u,v \in V(G)} (d_u + d_v)\, d(u,v).$$

In 1993, Plavšić *et al.* [18] and Ivancivc *et al.* [8] proposed, independently, a novel topological descriptor and named it as the Harary index in honor of Frank Harary on the occasion of his 70^{th}

birthday. Actually, a version of the Harary index was first defined by Mihalić and Trinajstić [14] in 1992 as follows:

$$H_{old}(G) = \sum_{u \neq v \in V(G)} \left(\frac{1}{d(u,v)} \right)^2.$$

However, the Harary index introduced in [8, 18] is one of the very much studied topological descriptors and is defined as follows:

$$H(G) = \sum_{u \neq v \in V(G)} \frac{1}{d(u,v)}.$$

2. Non-zero component graph

Let $\mathbb{V}$ be a finite dimensional vector space over $\mathbb{F}$ (where $\mathbb{F}$ is a field) with basis set $\{\beta_1, \beta_2, \ldots, \beta_n\}$ and θ as a null vector space. Then any vector v of $\mathbb{V}$ can be represented uniquely as a linear combination of the form

$$v = b_1\beta_1 + b_2\beta_2 + \ldots + b_n\beta_n.$$

The aforementioned representation of v is the basic representation of v with respect to $\{\beta_1, \beta_2, \ldots, \beta_n\}$. Non-zero component graph of a finite dimensional vector space V is represented by $\Gamma(\mathbb{V}) = (V, E)$ and is defined with respect to a basis $\{\beta_1, \beta_2, \ldots, \beta_n\}$ as follows: $V = \mathbb{V} - \theta$ and for $v_1, v_2 \in V$, $(v_1, v_2) \in E$ if v_1 and v_2 shares at least one β_i with non-zero coefficient in their basic representation, *i.e.*, their exists at least one β_i along which both v_1 and v_2 have non-zero components. Unless otherwise mentioned, we take the basis on which the graph is build as $\{\beta_1, \beta_2, \ldots, \beta_n\}$ [1, 2]. The following are some important results about $\Gamma(\mathbb{V})$.

Theorem 1. [1] *Let $\mathbb{V}$ be a vector space with dimension n, over a field $\mathbb{F}$ having q elements. Then the order of $\Gamma(\mathbb{V})$ is $q^n - 1$ and the size of $\Gamma(\mathbb{V})$ is*

$$\frac{q^{2n} - q^n + 1 - (2q-1)^n}{2}.$$

Theorem 2. [1] *Suppose $\mathbb{V}$ is a vector space over $\mathbb{F}$ ($\mathbb{F}$ is a finite field) having q elements and Γ be its related graph corresponding to basis $\{\beta_1, \beta_2, \ldots, \beta_n\}$. Then the degree of the vertex $d_1\beta_{i_1} + d_2\beta_{i_2} + \ldots + d_n\beta_{i_n}$, where $d_1, d_2, \ldots, d_k \neq 0$, is $(q^k - 1)q^{n-k} - 1$.*

Theorem 3. [1] *$\Gamma(\mathbb{V})$ is connected and $D(\Gamma(\mathbb{V})) = 2$.*

Theorem 4. [1] *$\Gamma(\mathbb{V})$ is complete if and only if $\mathbb{V}$ is $1-$dimensional.*

Suppose $\mathbb{V}$ is a vector space with dimension $n \geq 1$ over a field $\mathbb{F}$ of order $q \geq 2$. Then there are $\binom{n}{k}(q-1)^k$ unique vectors $\beta_1 v_1 + \beta_2 v_2 + \ldots + \beta_k v_k$ of length k in $\Gamma(\mathbb{V})$ for each $1 \leq k \leq n$. Let us denote the vertex corresponds to the ith vector of length k by v_i^k, $1 \leq i \leq \binom{n}{k}(q-1)^k$ *and* $1 \leq k \leq n$. So the vertex of $\Gamma(\mathbb{V})$ is $V = \{v_i^k;\ 1 \leq k \leq n, 1 \leq i \leq \binom{n}{k}(q-1)^k\}$.
Let us denote the degree of a vertex $v_i^k, 1 \leq i \leq \binom{n}{k}(q-1)^k$, by d_k which is then

$$d_k = (q^k - 1)q^{n-k} - 1 \text{ for } k \in [1, n].$$

For n exactly 1 and q at least 2, $\Gamma(\mathbb{V})$ is a complete graph by Theorem 4. In this case, all the topological descriptors can be obtained easily. In the next section, we consider $\Gamma(\mathbb{V})$ for more than 1-dimensional vector space in the context of the topological descriptors mentioned in the previous section.

3. Methodology

Graph theoretical tools (path, length, distance, eccentricity, diameter etc.), shortest path algorithm, vertex partitioning method, and combinatorial computing are use to build few helpful parameters and to examine our main descriptors. Moreover, we use Matlab (MathWorks, Natick, MA, USA) for mathematical calculations and verifications (see https://en.wikipedia.org/wiki/MATLAB)[13] to provide a numeric comparison of the examined descriptors, which is shown in Table 1. We also use the maple software (Maplesoft, McKinney, TX, USA) for plotting our mathematical results (see https://en.wikipedia.org/wiki/Maplesoftware)[12].

4. Construction

In this part, we define and build few effective parameters, which are helpful in the assessment of predefined topological discriptors. These are defined as follows:
The distance number of a vertex v in G

$$D(v|G) = \sum_{u\in V(G)} d(u, v);$$

the double distance number of a vertex v in G

$$DD(v|G) = \frac{1}{2}\left(D(v|G) + \sum_{u\in V(G)} (d(u, v))^2\right);$$

the sum distance number of a vertex v in G

$$D_s(v|G) = \sum_{u\in V(G)-\{v\}} \frac{1}{(D(G) + 1 - d(u, v)};$$

and the reciprocal distance number of a vertex v in G

$$D_r(v|G) = \sum_{u\in V(G)} \frac{1}{d(u, v)}.$$

According to these parameters, the distance based topological indices, defined in Section 1, become:

$$W(G) = \frac{1}{2}\sum_{v\in V(G)} D(v|G),$$

$$WW(G) = \frac{1}{2}\sum_{v\in V(G)} DD(v|G),$$

$$RCW(G) = \frac{|G|}{D(G) + 1}+\sum_{v\in V(G)} D_s(v|G),$$

$$MTI(G) = \sum_{v\in V(G)} (d(v))^2+\sum_{v\in V(G)} d(v)D(v|G),$$

$$H(G) = \frac{1}{2}\sum_{v\in V(G)} D_r(v|G).$$

Remark 5. *If dim*($\mathbb{V}$) *is n where n is at least 2 and* $q = o(\mathbb{F}) \geq 2$*, then for all* $k \in [1, n]$ *and for each* $1 \leq i \leq \binom{n}{k}(q-1)^k$*,* $d(v_i^k, v) = 1$ *or* 2 *by Theorem 3 for any* $v \in V - \{v_i^k\}$*. Since the number of vertices lying at distance* 1 *from* v_i^k *in* $\Gamma(\mathbb{V})$ *is the degree* d_k *of* v_i^k*, so the number of vertices lying at distance* 2 *from* v_i^k *is*

$$|\Gamma(\mathbb{V})| - d_k - 1 = (q^n - 1) - \left(q^{n-k}(q^k - 1) - 1\right) - 1 = q^{n-k} - 1.$$

Now, we construct the above defined parameters for $\Gamma(\mathbb{V})$ in the following proposition:

Proposition 6. *Suppose dim*($\mathbb{V}$) $= n \geq 2$ *and* $q = o(\mathbb{F}) \geq 2$*, then for any* $k \in [1, n]$ *and for each* $1 \leq i \leq \binom{n}{k}(q-1)^k$*, we have*

$$D(v_i^k|\Gamma(\mathbb{V})) = q^{n-k}(q^k+1)-3,$$

$$DD(v_i^k|\Gamma(\mathbb{V})) = q^{n-k}(q^k+2)-4,$$

$$D_s(v_i^k|\Gamma(\mathbb{V})) = \frac{q^{n-k} + q^n - 3}{2},$$

$$D_r(v_i^k|\Gamma(\mathbb{V})) = \frac{q^{n-k}(2q^k - 1) - 3}{2}.$$

Proof. By Remark 5, the distance number of any vertex of $\Gamma(\mathbb{V})$ is:

$$D(v_i^k|\Gamma(\mathbb{V})) = \left[q^{n-k}(q^k - 1) - 1\right](1) + [q^{n-k} - 1](2) = q^{n-k}(q^k + 1) - 3.$$

By Remark 5, the double distance number of any vertex of $\Gamma(\mathbb{V})$ is:

$$\begin{aligned} DD(v_i^k|\Gamma(\mathbb{V})) &= \left[[q^{n-k}(q^k + 1) - 3] + [q^{n-k}(q^k - 1)](1^2) + [q^{n-k} - 1](2)^2\right] \\ &= q^{n-k}(q^k + 2) - 4. \end{aligned}$$

Using Theorem 3 and Remark 5, the sum distance number of any vertex of $\Gamma(\mathbb{V})$ is:

$$\begin{aligned} D_s(v_i^k|\Gamma(\mathbb{V})) &= \sum_{u \in V(G) - v_i^k} \frac{1}{2 + 1 - d(u, v_i^k)} \\ &= [q^{n-k}(q^k - 1) - 1](\frac{1}{3-1}) + [q^{n-k} - 1](\frac{1}{3-2}) = \frac{q^{n-k} + q^n - 3}{2}. \end{aligned}$$

By Remark 5, the reciprocal distance number of any vertex of $\Gamma(\mathbb{V})$ is:

$$D_r(v_i^k|\Gamma(\mathbb{V})) = [q^{n-k}(q^k - 1) - 1](1) + [q^{n-k} - 1](2) = \frac{q^{n-k}(2q^k - 1) - 3}{2}.$$

□

5. Results

The following result is about computation of Wiener index of $\Gamma(\mathbb{V})$.

Theorem 7. *Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ over a field $\mathbb{F}$ with $dim(\mathbb{V}) = n \geq 2$ and $o(\mathbb{F}) = q \geq 2$. Then*

$$W(G) = \frac{1}{2}(q^{2n} - 5q^n + (2q-1)^n + 3).$$

Proof. According to the vertex set of G, the formula for the Wiener index is:

$$W(G) = \frac{1}{2}\sum_{k=1}^{n}\sum_{v_i^k \in V(G)} D(v_i^k|G).$$

Using the distance number, given in Proposition 6, for each vertex v_i^k, we have

$$\begin{aligned}
W(G) &= \frac{1}{2}\sum_{k=1}^{n}\sum_{v_i^k \in V(G)} q^{n-k}(q^k+1) - 3 \\
&= \frac{1}{2}\left[\binom{n}{1}(q-1)(q^{n-1}(q^1+1)-3)\right] + \frac{1}{2}\left[\binom{n}{2}(q-1)^2(q^{n-2}(q^2+1)-3)\right] \\
&+ \ldots + \frac{1}{2}\left[\binom{n}{n}(q-1)^n((q^n+1)-3)\right] \\
&= \frac{1}{2}\left[\binom{n}{1}(q-1)(q^n+q^{n-1}-3)\right] + \frac{1}{2}\left[\binom{n}{2}(q-1)^2(q^n+q^{n-2}-3)\right] \\
&+ \ldots + \frac{1}{2}\left[\binom{n}{n}(q-1)^n(q^n+q^{n-n}-3)\right] \\
&= \frac{1}{2}\left[\binom{n}{1}(q-1)(q^n-3) + \binom{n}{1}(q-1)q^{n-1}\right] + \frac{1}{2}\left[\binom{n}{2}(q-1)^2(q^n-3) + \binom{n}{2}(q-1)^2q^{n-2}\right] \\
&+ \ldots + \frac{1}{2}\left[\binom{n}{n}(q-1)^n(q^n-3) + \binom{n}{n}(q-1)^nq^{n-n}\right] \\
&= \frac{1}{2}\left[(q^n-3)[\binom{n}{1}(q-1) + \binom{n}{2}(q-1)^2 + \ldots + \binom{n}{n}(q-1)^n]\right] \\
&+ \frac{1}{2}\left[\binom{n}{1}(q-1)q^{n-1} + \binom{n}{2}(q-1)^2q^{n-2} + \ldots + \binom{n}{n}(q-1)^nq^{n-n}\right].
\end{aligned}$$

By simplifications using binomial expansion, we obtain the required index. □

The following theorem give us the formula for the hyper-Wiener index of $\Gamma(\mathbb{V})$.

Theorem 8. *Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ over a field $\mathbb{F}$ with $dim(\mathbb{V}) = n \geq 2$ and $o(\mathbb{F}) = q \geq 2$. Then*

$$WW(G) = \frac{1}{2}(q^{2n} - 7q^n + 2(2q-1)^n + 4).$$

Proof. According to the vertex set of G, the formula for the hyper-Wiener index is:

$$WW(G) = \frac{1}{2}\sum_{v_i^k \in V(G)} DD(v_i^k|G).$$

Using the double distance number, given in Proposition 6, for each vertex v_i^k, we have

$$\begin{aligned}
WW(G) &= \frac{1}{2}\sum_{k=1}^{n}\sum_{v_i^k\in V(G)}(q^{n-k}(q^k+2)-4)\\
&= \frac{1}{2}\left[\binom{n}{1}(q-1)(q^{n-1}(q^1+2)-4)\right]+\frac{1}{2}\left[\binom{n}{2}(q-1)^2(q^{n-2}(q^2+2)-4)\right]\\
&+ \ldots+\frac{1}{2}\left[\binom{n}{n}(q-1)^n(q^{n-n}(q^n+2)-4)\right]\\
&= \frac{1}{2}\left[\binom{n}{1}(q-1)(q^n+2q^{n-1}-4)\right]+\frac{1}{2}\left[\binom{n}{2}(q-1)^2(q^n+2q^{n-2}-4)\right]\\
&+ \ldots+\frac{1}{2}\left[\binom{n}{n}(q-1)^n(q^n+2q^{n-n}(q^n-4)\right]\\
&= \frac{1}{2}\left[\binom{n}{1}(q-1)(q^n-4)+\binom{n}{1}(q-1)2q^{n-1}\right]\\
&+ \frac{1}{2}\left[\binom{n}{2}(q-1)^2(q^n-4)+\binom{n}{2}(q-1)^2 2q^{n-2}\right]\\
&+ \ldots+\frac{1}{2}\left[\binom{n}{n}(q-1)^n(q^n-4)+\binom{n}{n}(q-1)^n 2q^{n-n}\right]\\
&= \frac{1}{2}\left[(q^n-4)[\binom{n}{1}(q-1)+\binom{n}{2}(q-1)^2+\ldots+\binom{n}{n}(q-1)^n]\right]\\
&+ \frac{1}{2}\left[2[\binom{n}{1}(q-1)q^{n-1}+\binom{n}{2}(q-1)^2q^{n-2}+\ldots+\binom{n}{n}(q-1)^nq^{n-n}]\right].
\end{aligned}$$

Using binomial expansion, it is quite simple to obtain the required index. □

The reciprocal complementary Wiener index of $\Gamma(\mathbb{V})$ is computed in the next result.

Theorem 9. *Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ over a field $\mathbb{F}$ with $dim(\mathbb{V}) = n \geq 2$ and $o(\mathbb{F}) = q \geq 2$. Then*

$$RCW(G) = \frac{1}{12}(3q^{2n} - 11q^n + 3(2q-1)^n + 5).$$

Proof. According to the vertex set of G, the formula for the reciprocal complementary Wiener index is:

$$RCW(G) = \frac{|G|}{D(G)+1} + \sum_{v_i^k\in V(G)} D_s(v_i^k|G).$$

Using Theorems 1, 3 and the sum distance number given in Proposition 6, for each vertex v_i^k, we have

$$RCW(G) = \frac{q^n-1}{3} + \sum_{k=1}^{n}\sum_{v_i^k\in V(G)}\frac{q^{n-k}+q^n-3}{2}$$

$$
\begin{aligned}
&= \frac{q^n-1}{3} + \frac{1}{2}\sum_{k=1}^{n}\left[\binom{n}{k}(q-1)^k\frac{q^{n-k}+q^n-3}{2}\right]\\
&= \frac{q^n-1}{3} + \frac{1}{4}\left[\binom{n}{1}(q-1)(q^{n-1}+q^n-3)\right] + \frac{1}{4}\left[\binom{n}{2}(q-1)^2(q^{n-2}+q^n-3)\right]\\
&+ \ldots + \frac{1}{4}\left[\binom{n}{n}(q-1)^n(q^{n-n}+q^n-3)\right]\\
&= \frac{q^n-1}{3} + \frac{1}{4}\left[\binom{n}{1}(q-1)(q^n-3) + \binom{n}{1}(q-1)q^{n-1}\right]\\
&+ \frac{1}{4}\left[\binom{n}{2}(q-1)^2(q^n-3) + \binom{n}{2}(q-1)^2q^{n-2}\right]\\
&+ \ldots + \frac{1}{4}\left[\binom{n}{n}(q-1)^n(q^n-3) + \binom{n}{n}(q-1)^nq^{n-n}\right]\\
&= \frac{q^n-1}{3} + \frac{1}{4}\left[(q^n-3)[\binom{n}{1}(q-1) + \binom{n}{2}(q-1)^2 + \ldots + \binom{n}{n}(q-1)^n]\right]\\
&+ \frac{1}{4}\left[\binom{n}{1}(q-1)q^{n-1} + \binom{n}{2}(q-1)^2q^{n-2} + \ldots + \binom{n}{n}(q-1)^nq^{n-n}\right]
\end{aligned}
$$

After some easy calculations by using binomial expansion, we get the required index. □

The investigation of the Schultz molecular topological index of $\Gamma(\mathbb{V})$ is given in the following result:

Theorem 10. *Let $G = \Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ over a field $\mathbb{F}$ with $dim(\mathbb{V}) = n \geq 2$ and $o(\mathbb{F}) = q \geq 2$. Then*

$$MTI(G) = 2(q^n-2)(q^{2n}-3q^n+(2q-1)^n+1).$$

Proof. According to the vertex set of G, the formula for the Schultz molecular topological index is:

$$MTI(G) = \sum_{v_i^k\in V(G)}(d(v_i^k))^2 + \sum_{v_i^k\in V(G)} d(v_i^k)D(v_i^k|G)$$

Using the degree $d(v_i^k) = d_k$ and distance number, given in Proposition 6, for each vertex v_i^k, we have

$$
\begin{aligned}
MIT(G) &= \sum_{k=1}^{n}\sum_{v_i^k\in V(G)}\left[(q^k-1)q^{n-k}-1\right]^2 + \sum_{k=1}^{n}\sum_{v_i^k\in V(G)}\left[[(q^k-1)q^{n-k}-1]^2[q^{n-k}(q^k+1)-3]\right]\\
&= \sum_{k=1}^{n}\binom{n}{k}(q-1)^k\left((q^k-1)q^{n-k}-1\right)^2\\
&+ \sum_{k=1}^{n}\binom{n}{k}(q-1)^k\left((q^k-1)q^{n-k}-1\right)^2\left(q^{n-k}(q^k+1)-3\right)\\
&= \binom{n}{1}(q-1)^1\left((q^1-1)q^{n-1}-1\right)^2 + \ldots + \binom{n}{n}(q-1)^n\left((q^n-1)q^{n-n}-1\right)^2
\end{aligned}
$$

$$
\begin{aligned}
&+ \binom{n}{1}(q-1)^1\left((q^1-1)q^{n-1}-1\right)^2\left(q^{n-1}(q^1+1)-3\right)\\
&+ \ldots+\binom{n}{n}(q-1)^n\left((q^n-1)q^{n-n}-1\right)^2\left(q^{n-n}(q^k+1)-3\right)\\
&= \binom{n}{1}(q-1)^1((q^1-1)q^{n-1}-1)\left(((q-1)^1q^{n-1}-1)+q^{n-1}(q^1+1)-3\right)\\
&+ \binom{n}{2}(q-1)^2((q^2-1)q^{n-2}-1)\left(((q-1)^2q^{n-2}-1)+q^{n-2}(q^2+1)-3\right)\\
&+ \ldots+\binom{n}{n}(q-1)^n((q^n-1)q^{n-n}-1)\left(((q^n-1)q^{n-n}-1)+q^{n-n}(q^n+1)-3\right).
\end{aligned}
$$

It can necessarily be obtained the required result by performing some simplifications using binomial expansion. □

Theorem 11. *Let $G=\Gamma(\mathbb{V})$ be a non-zero component graph of a vector space $\mathbb{V}$ over a field $\mathbb{F}$ with $dim(\mathbb{V})=n\geq 2$ and $o(\mathbb{F})=q\geq 2$. Then $H(G)=\frac{1}{4}(2q^{2n}-4q^n-(2q-1)^n+3)$.*

Proof. According to the vertex set of G, the formula for the Harary index is:

$$
H(G) = \frac{1}{2}\sum_{v_i^k\in V(G)} D_r(v_i^k)
$$

Using the reciprocal distance number, given in Proposition 6, for each vertex v_i^k, we have

$$
\begin{aligned}
H(G) &= \frac{1}{2}\left[\sum_{k=1}^{n}\sum_{v_i^k\in V(G)}[\frac{q^{n-k}(2q^k-1)-3}{2}]\right]\\
&= \frac{1}{2}\sum_{k=1}^{n}\left[\binom{n}{k}(q-1)^k\right]\left[\frac{q^{n-k}(2q^k-1)-3}{2}\right]\\
&= \frac{1}{2}\left[\binom{n}{1}(q-1)^1\frac{q^{n-1}(2q^1-1)-3}{2}\right]+\frac{1}{2}\left[\binom{n}{2}(q-1)^2\frac{q^{n-2}(2q^2-1)-3}{2}\right]\\
&+ \ldots+\frac{1}{2}\left[\binom{n}{n}(q-1)^n\frac{q^{n-n}(2q^n-1)-3}{2}\right]\\
&= \frac{1}{4}\left[\binom{n}{1}(q-1)(2q^n-3)-\binom{n}{1}(q-1)q^{n-1}\right]+\frac{1}{4}\left[\binom{n}{2}(q-1)^2(2q^n-3)-\binom{n}{2}(q-1)^2q^{n-2}\right]\\
&+ \ldots+\frac{1}{4}\left[\binom{n}{n}(q-1)^n(2q^n-3)-\binom{n}{n}(q-1)^nq^{n-n}\right]\\
&= \frac{1}{4}\left[(2q^n-3)[\binom{n}{1}(q-1)+\binom{n}{2}(q-1)^2+\ldots+\binom{n}{n}(q-1)^n]\right]\\
&- \frac{1}{4}\left[\binom{n}{1}(q-1)q^{n-1}+\binom{n}{2}(q-1)^2q^{n-2}+\ldots+\binom{n}{n}(q-1)^nq^{n-n}\right].
\end{aligned}
$$

Required result can be obtained after some easy simplifications by using the binomial expansion. □

In the following section we compare the aforementioned investigated indices numerically and graphically.

6. Comparisons and plots

We provide a numeric comparison of the indices, which are investigated in the previous section for a graph $\Gamma(\mathbb{V})$ associated with a finite dimensional vector space $\mathbb{V}$ of dimension $n \geq 2$ over a finite filed $\mathbb{F}$ of order $q \geq 2$. We evaluated all the indices for different values of q, n and constructed Table 1. From Table 1, we can see that all the indices are in increasing order as the values of q and n are increasing. The graphical representations of the all indices are depicted in Figures 1–5 for certain values of q and n.

(q,n)	$W(\Gamma(\mathbb{V}))$	$WW(\Gamma(\mathbb{V}))$	$RCW(\Gamma(\mathbb{V}))$	$MTI(\Gamma(\mathbb{V}))$	$H(\Gamma(\mathbb{V}))$
(2,2)	4	5	3	64	2.5
(2,3)	27	33	15.83	840	18
(2,4)	130	155	70	8176	92.5
(2,5)	555	645	287.83	70440	420
(2,6)	2254	2555	1148	574864	1802.5
(3,2)	32	36	18.67	1148	26
(3,3)	361	397	189.17	38800	307
(3,4)	3392	3624	1722.67	1097468	3044
(3,5)	30481	31801	15321.17	29617936	28501
(3,6)	271712	278796	136098.67	792258428	261086
(4,2)	114	123	62	7280	100.5
(4,3)	2061	2169	1051.5	527000	1899
(4,4)	33330	34275	16750	34123376	31912.5
(4,5)	530133	537513	265407.5	2171369816	519063
(4,6)	8437194	8491923	4219962	138234565040	8355100.5
(5,2)	292	308	154	29164	268
(5,3)	7866	8106	3974.33	3931572	7506
(5,4)	197032	199688	98724	492561244	193048
(5,5)	4904526	4930926	2453304.33	61306376292	4864926
(5,6)	122296972	122547068	61153694	7643558811724	121921828
(6,2)	620	645	321.67	89216	582.5
(6,3)	23455	23905	11799.17	20262376	22780
(6,4)	843890	849915	422376.67	4374682736	834852.5
(6,5)	30294175	30366925	15149679.2	942269468296	30185050
(6,6)	1089160310	1089999435	544595707	203263447167056	1087901622.5
(7,2)	1164	1200	598	228044	1110
(7,3)	59067	59823	29647.5	81035240	57933
(7,4)	2890680	2902560	1446140	27762005276	2872860
(7,5)	141381255	141550095	70696229.5	9504777727640	141127995
(7,6)	6922762884	6925058640	3461420658	3257824504263404	6919319250
(7,7)	339140851827	339171402543	169570700427.5	1117188297903540680	339095025753

Table 1. Numeric comparison of investigated topological indices at some values of q and n.

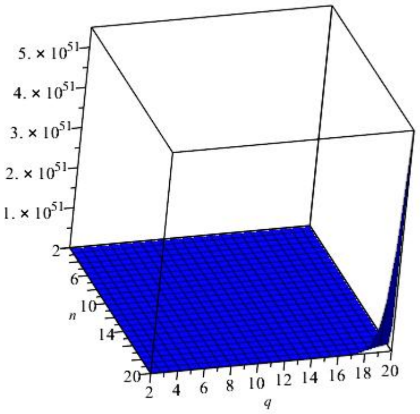

Figure 1. Wiener index of Γ(𝕍).

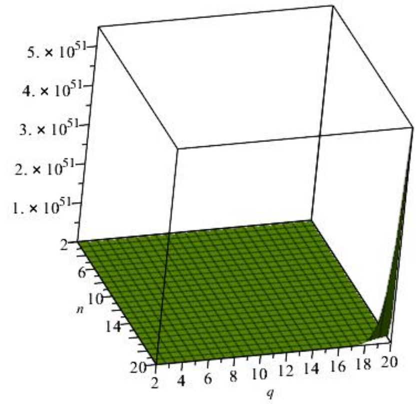

Figure 2. Hyper Wiener index of Γ(𝕍).

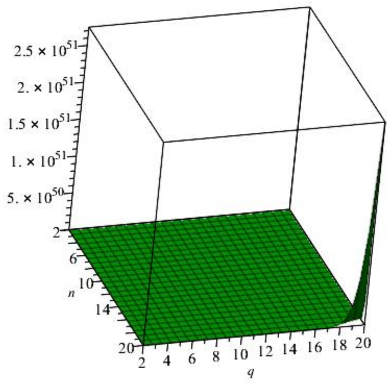

Figure 3. Reciprocal Wiener index of Γ(𝕍).

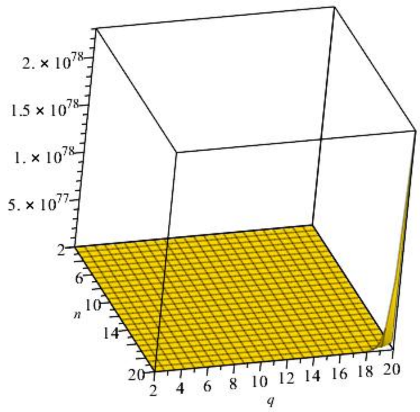

Figure 4. Schultz molecular topological index of Γ(𝕍).

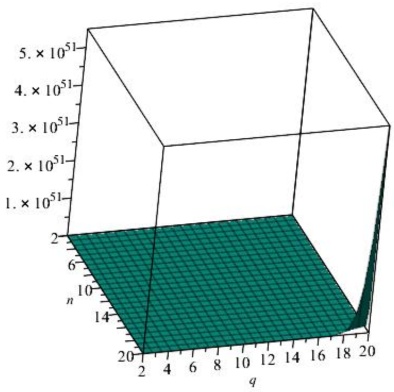

Figure 5. Reciprocal Wiener index of Γ(𝕍).

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Conflict of interest

The authors declare that there is no conflicts of interests regarding the publication of this paper.

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