AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 6(4): 3455–3464.
DOI: 10.3934/math.2021206
Received: 15 October 2020
Accepted: 13 January 2021
Published: 20 January 2021

Research article

Daily nonparametric ARCH(1) model estimation using intraday high frequency data

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Abstract: In this paper, the intraday high-frequency data are used to estimate the volatility function of daily nonparametric ARCH(1) model. A nonparametric volatility proxy model is proposed to achieve this objective. Under regular assumptions, the asymptotic distribution of the proposed estimator is established. The impact of different proxies on the estimation precision is also discussed. Simulation and empirical studies show that using the intraday high frequency data can significantly improve the estimation accuracy of the considered model. The idea of this article can be easily extended to other nonparametric or semiparametric ARCH/GARCH models.

Keywords: volatility function; volatility proxy; local polynomial estimation
Mathematics Subject Classification: 62G05, 62G20

1. Introduction

Since the autoregressive conditional heteroscedasticity (ARCH) model in Engle [1] and its generalized version, the generalized autoregressive conditional heteroscedasticity (GARCH) model in Bollerslev [2] were proposed, scholars have extended them and applied them to the financial market. See, for example, Nelson [3], Hentschel [4], Klüppelberg et al. [5], Pan et al. [6], Zou et al. [7], Tetsuya [8], Davide [9], Samuel [10], Zhang et al. [11] and Linton et al. [12]. To avoid the misspecification problem for volatility function, nonparametric ARCH or GARCH models have been widely studied since the work of Engle and Ng [13]. See, for example, Härdle and Tsybakov [14], Bühlmann and McNeil [15], Yang [16], Giordano and Parrella [17], Chen et al. [18] and the references therein. With the popularity of electronic trading systems, intraday high frequency data become easily available. Such data are valuable in modeling and parameter estimation. Visser [19] utilized the intraday high frequency data to improve the estimation of daily GARCH model. The framework was further extended to the estimation of GJR-GARCH model and robust estimation of GARCH model (Huang et al. [20], Wang et al. [21]). Deng [22] utilized the intraday high frequency data to study the parameter test of daily GARCH model. Existent results following Visser [19] mainly focus on parametric GARCH type models, which may cause misspecification problem for practical data. An alternative perspective to study the volatility is from a nonparametric way.

The main goal of this paper is to use the intraday high frequency data to improve the estimation of daily nonparametric ARCH model. For simplicity, we focus on the nonparametric ARCH (1) model. Giordano and Parrella [17] showed GARCH (1, 1) model can be well approximated by a nonparametric ARCH (1) model, implying this model could be adequate in many situations. Moreover, the idea of this article can be easily extended to other nonparametric or semiparametric ARCH/GARCH models.

The rest of the paper is organized as follows. Section 2 introduces the models and estimators. Section 3 derives the asymptotic results. Section 4 investigates the estimation performance based on simulation studies. Empirical study is provided in Section 5. A concluding remark is given in Section 6. The appendix shows the proofs of theorems.

2. Model and estimation

2.1. Nonparametric volatility proxy model

Let Y_t be the daily returns of a certain asset. The nonparametric ARCH (1) model, denoted as NARCH(1), has the form

$$Y_t = g^{1/2}(Y_{t-1})\xi_t, \tag{2.1}$$

where $g(Y_{t-1})$ is the unknown volatility function depending on Y_{t-1}. The errors ξ_t are independent and identically distributed random noises such that $E(\xi_t) = 0$ and $E(\xi_t^2) = 1$, and they are independent of Y_s for $s \leq t$. Model (2.1) can be rewritten as

$$Y_t^2 = g(Y_{t-1}) + g(Y_{t-1})(\xi_t^2 - 1). \tag{2.2}$$

It follows that

$$E(Y_t^2|Y_{t-1} = y) = g(y), \;\; \mathrm{Var}(Y_t^2|Y_{t-1} = y) = g^2(y)(m_4 - 1) \; and \;\; m_4 = E\xi_t^4. \tag{2.3}$$

According to the equation $E(Y_t^2|Y_{t-1} = y) = g(y)$ in (2.3), the unknown function $g(y)$ can be estimated by many nonparametric regression techniques such as kernel regression, local polynomial regression and spline method.

To introduce intraday high frequency data, following Visser [19], we provide the following *nonparametric scaling model* by normalizing the trading day into the unit time interval:

$$Y_t(u) = g^{1/2}(Y_{t-1})\xi_t(u), \; 0 \leq u \leq 1, \tag{2.4}$$

where $Y_t(u)$ denotes the continuous-time intraday log-return process. On different days, the standard noise processes $\xi_k(u)$ and $\xi_l(u)$, $k \neq l$, are assumed to be independent and to follow the same probability distribution. Through standardization, when $u = 1$, we have

$$E\xi_t^2(1) = 1, \; Y_t = Y_t(1), \; \xi_t = \xi_t(1). \tag{2.5}$$

Let $H_t \equiv H(Y_t(u))$ be a function of $Y_t(u)$ with the property of positive homogeneity, given by

$$H_t = H(\alpha Y_t(u)) = \alpha H(Y_t(u)) > 0, \;\; for \;\; \alpha > 0. \tag{2.6}$$

In this paper a proxy is a positive statistic that satisfies the conditions in (2.6), which can be easily satisfied. An example is the daily realized volatility of the form

$$H_t = RV = \sqrt{\sum_k (r_{t,k} - r_{t,k-1})^2}, \tag{2.7}$$

where $r_{t,k}$ denotes the return over the k-th intraday interval in day t. According to (2.4), homogeneity implies $H(Y_t(u)) = g^{1/2}(Y_{t-1})H(\xi_t(u))$. Denote

$$\begin{aligned} &H_t \equiv H(Y_t(u)),\ Z_{Ht} \equiv H(\xi_t(u)),\ \mu_{2H} = EZ_{Ht}^2, \\ &\xi_{Ht} = Z_{Ht}/\sqrt{\mu_{2H}},\ g_H(y) = g(y)\mu_{2H}, \end{aligned} \tag{2.8}$$

then we have the following *nonparametric volatility proxy model*:

$$H_t = g_H^{1/2}(Y_{t-1})\xi_{Ht}, \tag{2.9}$$

where $E(H_t^2|Y_{t-1}) = g_H(Y_{t-1})$ and $E\xi_{Ht}^2 = 1$. Note that the above equation can not only use the information of high frequency data, but also retain the volatility function of NARCH(1) model except for a certain constant, $g_H(y) = g(y)\mu_{2H}$. When $H_t = |Y_t(1)| = |Y_t|$, $E(H_t^2|Y_{t-1}) = g_H(Y_{t-1})$ reduces to $E(Y_t^2|Y_{t-1}) = g(Y_{t-1})$, which means that only daily information Y_t is adopted for estimation. Consequently, (2.9) includes the traditional daily model (2.1) as a special case.

2.2. Volatility function estimation

In practice, $\{Y_t\}_{t=1}^n$ is observable and $\{H_t\}_{t=1}^n$ can be calculated based on discrete intraday high frequency sequence. Let $K(.)$ be a given kernel function, and h be the bandwidth. We firstly define certain symbols as follows:

$$V = \begin{pmatrix} V_2 \\ V_3 \\ \cdots \\ V_n \end{pmatrix} = \begin{pmatrix} H_2^2 \\ H_3^2 \\ \cdots \\ H_n^2 \end{pmatrix},\ Z = \begin{pmatrix} 1 & V_1 - y \\ 1 & V_2 - y \\ \cdots & \cdots \\ 1 & V_{n-1} - y \end{pmatrix},$$

$W = \mathrm{diag}\{\frac{1}{n}K_h(V_1 - y), \cdots, \frac{1}{n}K_h(V_{n-1} - y)\}$, $K_h(u) = \frac{1}{h}K(\frac{u}{h})$ and $E_1 = (1, 0)$. Recall $g_H(y) = E(H_t^2|Y_{t-1} = y)$ and $g(y) = E(Y_t^2|Y_{t-1} = y)$. Then, according to Yang[16], the local linear estimator of $g_H(y)$ is given by

$$\hat{g}_H(y) = E_1(Z^\tau WZ)^{-1}Z^\tau WV, \tag{2.10}$$

and the local linear estimator of $g(y)$, denoted as $\hat{g}(y)$, can be obtained by setting $H_t = |Y_t(1)| = |Y_t|$ in (2.10). Note $\mu_{Z_H} = g_H(y)/g(y)$ is an unknown parameter depending on H_t. Based on (2.10), an estimator for μ_{Z_H} is

$$\hat{\mu}_{Z_H} = \frac{1}{n-1}\sum_{t=2}^{n}\frac{\hat{g}_H(Y_{t-1})}{\hat{g}(Y_{t-1})}. \tag{2.11}$$

Then the final estimator for volatility function $g(y)$ that takes the intraday high frequency information into account is given by

$$\tilde{g}(y) = \frac{\hat{g}_H(y)}{\hat{\mu}_{Z_H}}. \tag{2.12}$$

3. Asymptotic theory

Before the statements of the limiting theory, we need to define some symbols and list certain assumptions. Let $\|K\|_2^2 = \int K^2(u)du$, $\mu_r(K) = \int \mu^r K(u)du$,

$$S = \begin{pmatrix} \mu_0(K) & \mu_1(K) \\ \mu_1(K) & \mu_2(K) \end{pmatrix},\ S^{-1} = \begin{pmatrix} S_{00} & S_{01} \\ S_{10} & S_{11} \end{pmatrix},$$

$$K_\lambda^*(u) = \sum_{\lambda'=0}^{1} S_{\lambda\lambda'} \mu^{\lambda'} K(u),\ \Lambda_{\lambda,2} = \int K_\lambda^*(u) u^2 du,\ \lambda = 0, 1. \tag{3.1}$$

Throughout this section, we assume $(Y_1, V_2), \cdots, (Y_{n-1}, V_n)$ is a stationary α mixing sequence with mixing coefficient $\alpha(k)$. The following assumptions are needed for our theoretical development, which have been used in Fan and Yao [23] and Yang [16].

Assumptions:

- A1 The kernel function K is bounded with a bounded support.
- A2 The conditional density $f_{Y_0,Y_l|V_1,V_{l+1}}(y_0, y_l|v_1, v_{l+1}) \le A_1 < \infty, \forall l \ge 1$.
- A3 For α mixing sequences, we assume that for some $\delta > 2$ and $a > 1 - 2/\delta$, $\sum_l l^a[\alpha(l)]^{1-2/\delta} < \infty$, $E|V_0|^\delta < \infty$, $f_{Y_0|V_1}(y_0|v_1) \le A_2 < \infty$.
- A4 For α mixing sequences, there exists a sequence of positive integers satisfying $s_n \to \infty$ and $s_n = o\{(nh_n)^{1/2}\}$ such that $(n/h_n)^{1/2}\alpha(s_n) \to 0$ as $n \to \infty$.
- A5 The random variable ξ_{Ht} has a continuous density function, which is positive everywhere and $m_4^H = E\xi_{Ht}^4 < \infty$.
- A6 The random variable Y_t has a stationary density $\varphi(y)$. $g_H(y)$ and $\varphi(y)$ have Lipschitz continuous 2th derivatives. Further, $\inf_{y\in A}\varphi(y) > 0$, where A is a compact subset of R with nonempty interior.

Theorem 1. *Under assumptions A1–A6, for any fixed $y \in A$, as $nh \to \infty, nh^5 = O(1)$,*

$$\sqrt{nh}(\hat{g}_H(y) - g_H(y) - h^2 b_H(y)) \xrightarrow{D} N(0, V_H(y))$$

where $b_H(y) = \Lambda_{0,2} g_H^{(2)}(y)/2!$ and $V_H(y) = \|K_0^*\|_2^2(m_4^H - 1)g_H^2(y)\varphi^{-1}(y)$.

Theorem 2. *Under assumptions A1–A6, if $h \sim n^{-r}$ for some $r \in (1/4, 1)$, then as $n \to \infty$, $(\hat{\mu}_{Z_H} - \mu_{Z_H}) = o_p(n^{-\frac{1}{2}})$ and*

$$\sqrt{nh}(\tilde{g}(y) - g(y) - h^2\tilde{b}(y)) \xrightarrow{D} N(0, \tilde{V}(x)),$$

where $\tilde{b}(y) = \Lambda_{0,2} g^2(y)/2!$ and $\tilde{V}(y) = (m_4^H - 1)\|K_0^*\|_2^2 g^2(y)\varphi^{-1}(y)$.

Remark. *In Theorem 1, when $H_t = |Y_t|$, $\hat{g}_H(y)$ and $g_H(y)$ become $\hat{g}(y)$ and $g(y)$ respectively. Based on Theorem 2, the revised estimator $\tilde{g}(y)$ retains the same bias term with $\hat{g}(y)$ while the asymptotic variance terms are different. The impact of H_t lies in the quantity $(m_4^H - 1)$, and smaller m_4^H will cause lower asymptotic variance of $\tilde{g}(y)$ and hence will lead to a more precise estimator.*

In practice, $m_4^H = c \cdot EH_t^4/(EH_t^2)^2$ and $c = [E(g(Y_{t-1}))]^2/E[g^2(Y_{t-1})]$. Let

$$m_4^{H*} = EH_t^4/(EH_t^2)^2. \tag{3.2}$$

Then we can choose the optimal proxy H_t according to the value of m_4^{H*}.

4. Simulation

In this section, we assess the finite-sample performance of the proposed estimator $\tilde{g}(y)$. To simulate Y_t and $Y_t(u)$, we first need to simulate the intraday noise process $\xi_t(u)$. Following Visser [19], $\xi_t(u)$ was simulated by the following two processes:

$$d\gamma_t(u) = -\delta(\gamma_t(u) - \mu)du + \sigma_\gamma dB_t^{(2)}(u), \tag{4.1}$$

$$d\xi_t(u) = e^{\gamma_t(u)} dB_t^{(1)}(u), \ u \in [0, 1]. \tag{4.2}$$

The Brownian motions $B_t^{(1)}$ and $B_t^{(2)}$ are uncorrelated, $\xi_t(0) = 0$ and $\gamma(0)$ is sampled from $N(\mu, \sigma_r^2)$. We divide the unit time interval $[0, 1]$ into 240 small intervals and set $\delta = 1/2$, $\sigma_\gamma = 1/4$, $\mu = -1/16$, $g(y) = 0.1 + 0.5y^2$. For each day, we consider three settings for the proxy H_t in (2.9): 5-minute realized volatility (denoted as $H5_t$), 30-minute realized volatility (denoted as $H30_t$), and $|Y_t|$. Here, the realized volatility is computed according to (2.7).

To get the estimator $\tilde{g}(y)$ based on (2.12), the bandwidth is simply set as $1.06 * \text{std}(Y_t) * n^{-1/5}$, the kernel function is $K(x) = 0.75(1 - x^2)_+$. The sample sizes of $n = 200$, 400, 800 are considered, and the replication time is set to be 100. For each proxy $H5_t$, $H30_t$ and $|Y_t|$, according to the 20% and 80% percentiles of the simulated Y_{t-1}, the subset A in Theorem 1 is given as $[-0.45, 0.45]$, and the grid point vector is $U = [-0.45 : 0.025 : 0.45]$. For each sample size and each proxy, the 100 replicated estimated curves $\tilde{g}(U_i)$ are plotted in subplot of Figure 1 (green curve), together with the true function curve in bold black line for comparisons. From left to right, the three columns correspond to the cases with $n = 200$, 400 and 800, respectively. In the first column, subplots of $(a_i)(i = 1, 2, 3)$ are the estimation results corresponding the proxy $H5_t$, $H30_t$ and $|Y_t|$ respectively. Similarly, subplots of (b_i) and $(c_i)(i = 1, 2, 3)$ are the estimation results for $n = 400$ and 800 respectively. Subplots $(a_4), (b_4)$ and (c_4) are the box plots of m_4^{H*} in (3.2) for $H5_t$, $H30_t$ and $|Y_t|$ under different sample sizes.

It is found that the estimator under the proxy $H5_t$ performs best among the three proxies considered for each sample size, especially for the case with a small sample size. This is consistent with the box plots of m_4^{H*} in subplots $(a_4), (b_4)$ and (c_4), where the estimated values under $H5_t$ are generally smaller than the others. The estimator under the proxy $H30_t$ shows more precise estimation than that of proxy $|Y_t|$. When the sample size increases, each proxy shows better fitting performance, justifying the asymptotic normality in Theorem 2. According to the simulation results, it is shown that introducing the intraday high frequency data can significantly improve the estimation accuracy of NARCH(1) model.

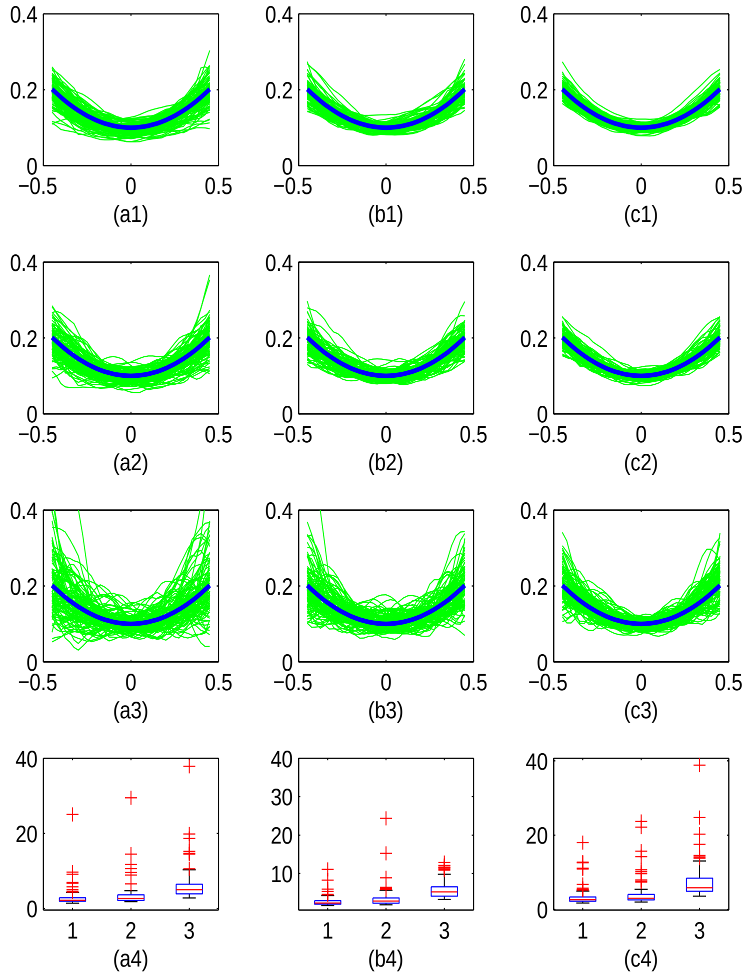

Figure 1. Subplots of $(a_i), (b_i)$ and $(c_i)(i = 1, 2, 3)$ are the estimated curves for $\tilde{g}(U_i)$ (green lines) and true function curve of $g(U_i)$ (bold black line) for different sample size and different proxy. Subplots $(a_4), (b_4)$ and (c_4) are the box plots of m_4^{H*} in (3.2) for $H5_t$, $H30_t$ and $|Y_t|$ (from left to right in each subplot) under different sample size.

5. Empirical study

In this section, based on NARCH(1) model, the proposed method is applied to estimate the volatility function of CSI (China Shanghai-Shenzhen) 300 index, which consist of the 300 largest and most liquid Chinese A-share stocks. The data span the period from 01 Sep 2017 to 12 July 2019, which consist of 466 daily observations. There are 241 price observations in each day based on the intrady sampling frequency of 1 minute. Denote $P_t(u)$ as the t-th intraday price sequence. We can calculate the intraday log-return as

$$Y_t(u) = 100[\log P_t(u) - \log P_{t-1}(1)],\ u \in [0, 1]. \tag{5.1}$$

For each day, we first consider 11 different volatility proxies: 1-minute realized volatility H1 up to 10-minute realized volatility H10, and daily absolute return $|Y_t|$. Here, the 1-minute proxy is computed as

$$H1_t = \sqrt{\Sigma_{k=1}^{240}[Y_t(u_k) - Y_t(u_{k-1})]^2}, \tag{5.2}$$

where $Y_t(u_k)$ denotes the k-th observation of intraday sequence $Y_t(u)$. The formulas of other proxies are similar. According to (3.2), the estimated values of m_4^{H*} for proxies H1-H10 and $|Y_t|$ are: (3.9324, 1.6049, 1.6864, 1.7488, 1.8208, 1.8264, 1.7181, 1.7436, 1.7882, 1.7999, 5.5620), with H2 the smallest. To compare the impact of frequency, we consider H1, H2, H10 and $|Y_t|$ for our studies.

To estimate the volatility function for the considered data, according to the 20% and 80% percentiles of Y_{t-1}, we set the subset A in Theorem 1 as $[-0.9, 0.9]$ and the grid point vector as $U = [-0.9 : 0.05 : 0.9]$. In terms of (2.8), the ratio $g_H(Y_{t-1})/g(Y_{t-1})$ equals to a constant μ_{2H} and this property is justified by plotted $\{\hat{g}_H(U_i)/\hat{g}(U_i)\}$ in Figure 2, where all the ratios nearly remain constant. This implies that the proposed method is reasonable and adequate for the considered data set. From Figure 3, it is seen the volatility function estimators for $g(Y_{t-1})$ show significant differences among considered proxies. According to estimated values of m_4^{H*}, the proxy H2 is supposed to obtain the most precise estimation. From Figure 3, the volatility function under H2 shows an asymmetric behavior: the function values for $Y_{t-1} < 0$ are larger than those when $Y_{t-1} > 0$. The function shape is also analogous to that of Giordano and Parrella[17] where the volatility of Dow Jones index is studied. Based on the popular view that negative returns usually cause larger volatility, it can be found that the curves estimated by proxy H2 and H10 are similar and more reasonable than other two curves based on H1 and $|Y_t|$. The above results could be explained as follows. H1 may contain much noise causing inefficiency of the proxy, while $|Y_{t-1}|$ makes no use of intraday information and could be inadequate compared to proxies H2 and H10.

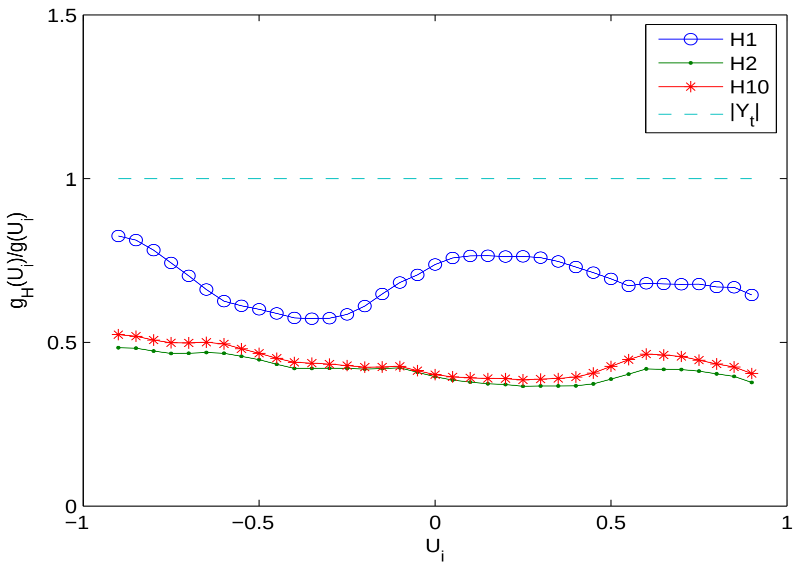

Figure 2. Time series plots of $\{\hat{g}_H(U_i)/\hat{g}(U_i)\}$ for H1, H2, H10 and $|Y_t|$.

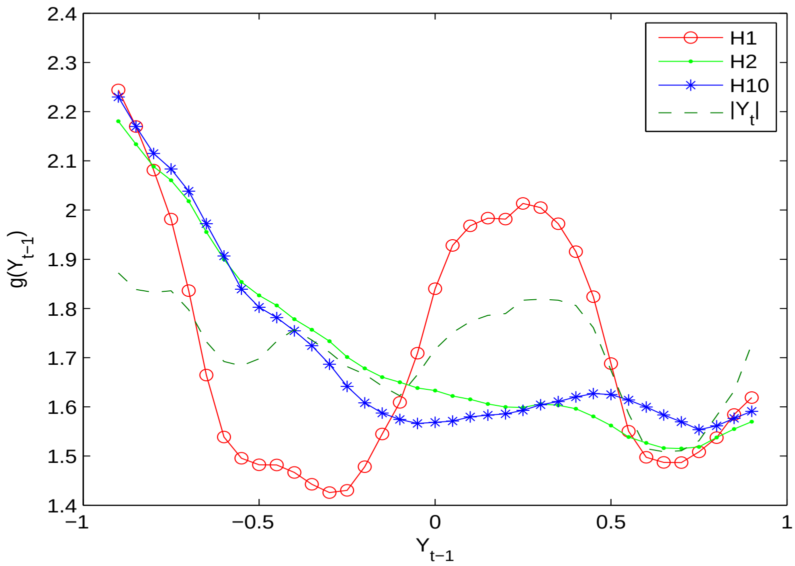

Figure 3. Time series plots of $\{\tilde{g}(Y_{t-1})\}$ for H1, H2, H10 and $|Y_t|$.

6. Conclusions

In this article, an approach is given to utilize the intraday high-frequency data for the estimation of daily nonparametric ARCH(1) model, which has been widely used to forecast the volatility of financial market. The method has potential applications in estimating volatility function of financial asset where the mixed-frequency data are available. Both the theoretical and simulation results show that introducing the intraday high frequency data can significantly improve the estimation precision of daily nonparametric ARCH(1) model, compared to the cases where only daily data is used. The idea of this article is of certain novelty and can provide insights motivating future research on daily nonparametric or semiparametric ARCH/GARCH model estimation by taking the intraday high-frequency data into account.

Appendix

The proof of Theorem 1 is routine and hence omitted. Detailed proof can be found in Yang [16] and Fan and Yao [23] . Next we give simple deduction for Theorem 2. Based on (2.11),

$$\begin{aligned}\hat{\mu}_{Z_H} - \mu_{Z_H} &= \frac{1}{n-1}\sum_{t=2}^{n}\frac{1}{\hat{g}(Y_{t-1})}\{\hat{g}_H(Y_{t-1}) - g_H(Y_{t-1})\} \\ &\quad -\frac{1}{n-1}\sum_{t=2}^{n}\frac{g_H(Y_{t-1})}{\hat{g}(Y_{t-1})g(Y_{t-1})}\{\hat{g}(Y_{t-1}) - g(Y_{t-1})\} \equiv I_1 + I_2.\end{aligned}$$

The above two terms are analogous to the term I_3 in page 383 of Yang [16]. Based on Assumptions A1–A6, when $h \sim n^{-r}$ for some $r \in (1/4, 1)$, following the steps showing $I_3 = o_p(n^{-1/2})$ in Yang [16], we can prove that both I_1 and I_2 are $o_p(n^{-1/2})$ and hence $\hat{\mu}_{Z_H} - \mu_{Z_H} = o_p(n^{-1/2})$. According to (2.10),

$$\begin{aligned}\sqrt{nh}\{\tilde{g}(y) - g(y)\} &= \sqrt{nh}\{\frac{1}{\hat{\mu}_{Z_H}}[\hat{g}_H(y) - g_H(y)]\} \\ &- \sqrt{nh}\{\frac{g_H(y)}{\hat{\mu}_{Z_H}\mu_{Z_H}}[\hat{\mu}_{Z_H} - \mu_{Z_H}]\} \equiv I_3 + I_4.\end{aligned}$$

Further, using the conclusion $\hat{\mu}_{Z_H} - \mu_{Z_H} = o_p(n^{-1/2})$, one can show $I_3 = (1/\mu_{Z_H})\sqrt{nh}\{\hat{g}_H(y) - g_H(y)\} + o_p(1)$ and $I_4 = o_p(1)$. Consequently,

$$\sqrt{nh}\{\tilde{g}(y) - g(y)\} = \frac{1}{\mu_{Z_H}}\sqrt{nh}\{\hat{g}_H(y) - g_H(y)\} + o_p(1),$$

and Theorem 2 holds from the asymptotic normality of $\hat{g}_H(y)$ in Theorem 1.

Acknowledgments

The work is partially supported by National Natural Science Foundation of China 11731015, 11571148, 11701116, Natural Science Foundation of Guangdong Province 192 2018A030310068, Young Innovative Talents Program for Colleges and Universities 191 in Guangdong Province of China 2018KQNCX241 and Guangzhou 193 University Research Funding 69-6209254, 220030401.

Conflict of interest

The authors declare no conflict of interest.

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