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Research article

Periodic solutions of Cohen-Grossberg-type Bi-directional associative memory neural networks with neutral delays and impulses

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Abstract: This paper considers a class of delayed Cohen-Grossberg-type bi-directonal associative memory neural networks with impulses. By using Mawhin continuation theorem and constructing a new Lyapunov function, some sufficient conditions are presented to guarantee the existence and stability of periodic solutions for the impulsive neural network systems. A simulation example is carried out to illustrate the efficiency of the theoretical results.

Keywords: Cohen-Grossberg; BAM neural networks; impulse; periodic solution; stability; Mawhin coincidence degree; Lyapunov function
Mathematics Subject Classification: 34C25, 92B20

1. Introduction

In a diverse world, the dynamics of neural networks has been studied extensively due to their convergence and stability are important in real applications, such as pattern recognition, image and signal processing, associative memories and so on [1–3]. A lot of literatures have been reported for different types of neural networks based on their various topological structures. To name a few, Cohen-Grossberg neural networks [4], bi-directonal associative memory (BAM) neural networks [5], Hopfield-type neural networks [6] and cellular neural networks [7].

Among them, Cohen-Grossberg neural networks were originally introduced by Cohen and Grossberg in 1983, which can be reduced to Hopfield-type neural networks and cellular neural networks as well in [4]. And the dynamics of such neural networks have received increasing interest, such as stability, synchronization and stabilization [8–12]. In order to make a detailed description between different layers of networks, BAM neural networks have been proposed by Kosko in 1988, and many methods and techniques were developed to discuss the dynamical characteristics of BAM neural networks in [13–16]. Recently, a class of Cohen-Grossberg-type BAM neural networks have drawn significant attention owing to the wide application in practice [17–20].

In fact, time delays occur in neural networks due to the finite speed of signal propagation. Generally, time delays can be divided into several types, such as discrete-type delays [21], distributed-type delays [22] and neutral-type delays [23]. As we all know, it is difficult to verify the dynamics of neutral delayed neural networks since they contain important information about the derivative of the past state. Some authors focused on neutral neural networks and several results have been obtained. For example, Cheng et al. [24] discussed neutral Cohen-Grossberg neural networks by means of Lyapunov stability method. Liu and Zong [25] dealt with a kind of neutral BAM neural networks based on some new integral inequalities and the Lyapunov-Krasovskii functional approach.

On the other hand, many real-world systems often receive sudden external disturbance, which entail systems undergo abrupt changes in very short time. This phenomenon is viewed as impulse [26–28]. The existence of impulse is also one of the key factors leading to the instability of neural networks. Recently impulsive neural networks have aroused a lot of interest [29–31]. Gu et al. [32] established the existence and global exponential stability of BAM-type impulsive neural networks with time-varying delays. They mainly used the method of the continuation theorem of coincidence degree theory and Lyapunov functional analysis. Liao et al. [33] gave the results of global asymptotic stability of periodic solutions for inertial delayed BAM neural networks by combining Mawhin continuation theorem of coincidence degree theory, Lyapunov functional method and inequality techniques. Especially, they seek periodic solutions by means of Lyapunov functional method instead of the prior estimate method. The addition of delays and impulses in neural networks make it more accurate to describe the evolutionary process of the systems.

To the best of our knowledge, Cohen-Grossberg-type BAM neural networks with neutral delays and impulses have not been investigated. The aim of this paper is to establish the existence and asymptotic stability of periodic solutions.

To study the dynamics of neural networks, many methods and techniques are developed, such as the matrix theory, set-valued maps theory and functional differential inclusions. Our results are based on the famous Mawhin coincidence degree theory and Lyapunov functional analysis. To do this, our highlights lie in four aspects:

• Proposing a new model with neutral-type time delays and impulses, some previous considered neural network models can be regarded as the special cases of ours, such as [15, 20, 32].

• Establishing some sufficient conditions to guarantee the existence and asymptotic stability of the periodic solutions by means of Mawhin coincidence degree theory and the contraction of a suitable Lyapunov functional. We not only employ the method of the prior classical estimation in Section 3, but also seek periodic solutions by means of Lyapunov functional method in Appendix.

• The impulse terms in this paper are more relaxing from linear functions as well in [16].

• The theoretical findings play a key role in designing the electric implementation of Cohen-Grossberg-type BAM neural networks and processing its signals transmission.

This paper is organized as follows. In Section 2, the model description and necessary knowledge are provided. In Section 3, by using the continuation theorem of coincidence degree theory, some conditions for the existence of periodic solutions are obtained. In Section 4, the global asymptotic stability of periodic solutions is discussed. In Section 5, an illustrative example is given to show the effectiveness of our criterions.

2. Model description and preliminaries

2.1. Model description

Consider the following Cohen-Grossberg-type BAM neural networks with neutral-type time delays and impulses, i.e.,

$$\begin{cases} \dot{x}_i(t) = -a_i(x_i(t))\Bigg[b_i(x_i(t)) - \sum_{j=1}^{m} a_{ij}(t) f_j(y_j(t-\tau_{ij}(t))) \\ \qquad\qquad - \sum_{j=1}^{m} b_{ij}(t) f_j(\dot{y}_j(t-\tilde{\tau}_{ij}(t))) - I_i(t)\Bigg], \quad t>0, \quad t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k) = I_{ik}(x_i(t_k)), \quad i = 1,2,\cdots,n, \quad k = 1,2,\cdots, \\ \dot{y}_j(t) = -c_j(y_j(t))\Bigg[d_j(y_j(t)) - \sum_{i=1}^{n} c_{ji}(t) g_i(x_i(t-\sigma_{ji}(t))) \\ \qquad\qquad - \sum_{i=1}^{n} d_{ji}(t) g_i(\dot{x}_i(t-\tilde{\sigma}_{ji}(t))) - J_j(t)\Bigg], \quad t>0, \quad t \neq t_k, \\ \Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k) = J_{jk}(y_j(t_k)), \quad j = 1,2,\cdots,m, \quad k = 1,2,\cdots. \end{cases} \tag{2.1}$$

The initial conditions associated with (2.1) are of the form

$$\begin{cases} x_i(t) = \varphi_i(t), \quad t \in (-\tau, 0], \quad \tau = \max_{1\le i\le n, 1\le j\le m}\{\tau_{ij}^*\}, \quad i = 1,2,\cdots,n, \\ y_j(t) = \psi_j(t), \quad t \in (-\sigma, 0], \quad \sigma = \max_{1\le i\le n, 1\le j\le m}\{\sigma_{ji}^*\}, \quad j = 1,2,\cdots,m, \end{cases} \tag{2.2}$$

where

$$\tau_{ij}^* = \max_{0\le t\le\omega}\{\tau_{ij}(t), \tilde{\tau}_{ij}(t)\}, \text{ and } \sigma_{ji}^* = \max_{0\le t\le\omega}\{\sigma_{ji}(t), \tilde{\sigma}_{ji}(t)\}.$$

Obviously $\Delta x_i(t_k)$ and $\Delta y_j(t_k)$ are the impulses at moments t_k and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to+\infty} t_k = +\infty$.

The ecological meaning of parameters are as follows. Among the system (2.1), $x_i(t),\ y_j(t)$ represent the potential (or voltage) of cell $i,\ j$ at time t respectively; $n,\ m$ correspond to the number of neurons in the $X-$layer and $Y-$layer; $a_i(\cdot),\ c_j(\cdot)$ denote amplification functions; $b_i(\cdot),\ d_j(\cdot)$ mean appropriately behaved functions such that the solutions of system (2.1) remain bounded; $a_{ij}(t),\ b_{ij}(t),\ c_{ji}(t),\ d_{ji}(t)$ describe the connection strengths of connectivity between cell i and j at the time t; $f_j(\cdot),\ g_i(\cdot)$ are the activation functions; $I_i(t),\ J_j(t)$ show the external inputs at time t.

In order to establish the existence of periodic solutions of systems (2.1) and (2.2), we assume the following hypotheses:

(H1) $\tau_{ij}(t),\ \tilde{\tau}_{ij}(t),\ \sigma_{ji}(t),\ \tilde{\sigma}_{ji}(t),\ a_{ij}(t),\ b_{ij}(t),\ c_{ji}(t),\ d_{ji}(t)$ are continuous $\omega-$periodic functions and

$$a_{ij}^M = \max_{0\le t\le\omega} a_{ij}(t), \quad b_{ij}^M = \max_{0\le t\le\omega} b_{ij}(t), \quad c_{ji}^M = \max_{0\le t\le\omega} c_{ji}(t), \quad d_{ji}^M = \max_{0\le t\le\omega} d_{ji}(t).$$

(H2) $f_j(x),\ g_i(y)$ are bounded and globally Lipschitz continuous, i.e., there exist positive constants $F_j,\ G_i,\ \tilde{F}_j,\ \tilde{G}_i$, such that

$$|f_j(x) - f_j(y)| \le F_j|x-y|, \qquad |f_j(x)| \le \tilde{F}_j, \quad j = 1,2,\cdots,m,$$

$$|g_i(x) - g_i(y)| \leq G_i|x - y|, \qquad |g_i(y)| \leq \tilde{G}_i, \quad i = 1, 2, \cdots, n.$$

It is easy to see that

$$|f_j(x)| \leq F_j|x| + |f_j(0)|, \qquad |g_i(x)| \leq G_i|x| + |g_i(0)|, \quad \text{for} \quad y = 0.$$

(H3) $a_i(u),\ b_i(u),\ c_j(u),\ d_j(u) \in C(\mathbb{R}, \mathbb{R})$, and there exist positive constants $a_i^L,\ a_i^M,\ c_j^L,\ c_j^M,\ b_i^L,\ b_i^M,\ d_j^L,\ d_j^M$, such that

$$0 < a_i^L \leq a_i(u) \leq a_i^M, \quad 0 < b_i^L|u| \leq b_i(u) \leq b_i^M|u|, \quad i = 1, 2, \cdots, n,$$

$$0 < c_j^L \leq c_j(u) \leq c_j^M, \quad 0 < d_j^L|u| \leq d_j(u) \leq d_j^M|u|, \quad j = 1, 2, \cdots, m.$$

(H4) for all $x, y \in \mathbb{R}$, there exists a positive integer p such that

$$t_{k+p} = t_k + \omega, \quad I_{i(k+p)}(x) = I_{ik}(x), \quad J_{j(k+p)}(y) = J_{jp}(y).$$

(H5) $I_{ik}(\cdot)$ and $J_{jk}(\cdot)$ are bounded and Lipschitz continuous functions, that is, there exist constants $s_i,\ r_j,\ s_{ik}$ and r_{jk} such that

$$|I_{ik}(x) - I_{ik}(y)| \leq s_{ik}|x - y|, \qquad |I_{ik}(\cdot)| < s_i, \quad i = 1, 2, \cdots, n,$$

$$|J_{jk}(x) - J_{jk}(y)| \leq r_{jk}|x - y|, \qquad |J_{jk}(\cdot)| < r_j, \quad j = 1, 2, \cdots, m.$$

It is easy to see that

$$|I_{ik}(x)| \leq s_{ik}|x| + |I_{ik}(0)|, \qquad |J_{jk}(x)| \leq r_{jk}|x| + |J_{jk}(0)|, \quad \text{for} \quad y = 0.$$

2.2. Preliminaries

Before presenting our results on the existence and stability of periodic solutions of systems (2.1) and (2.2), we briefly introduce the Mawhin coincidence degree theorem [34].

Let X and Y be two Banach spaces, $L : \mathrm{Dom}L \cap X \to Y$ be a linear mapping and $N : X \to Y$ be a continuous mapping. The mapping L is called a Fredholm mapping of index zero if $\dim\mathrm{Ker}L = \mathrm{codim}\mathrm{Im}L < +\infty$ and $\mathrm{Im}L$ is closed in Y. If L is a Fredholm mapping of index zero, there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\mathrm{Im}P = \mathrm{Ker}L$, $\mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I - Q)$, then the restriction L_p of L to $\mathrm{Dom}L \cap \mathrm{Ker}P$ is invertible. We denote the inverse of that mapping by K_p. If Ω is an open bounded subset of X, the mapping N is said to be $L-$compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_p(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\mathrm{Im}Q$ is isomorphic to $\mathrm{Ker}L$, there exists an isomorphism $J : \mathrm{Im}Q \to \mathrm{Ker}L$.

Lemma 1. *Let X and Y be two Banach spaces, $L : DomL \cap X \to Y$ be a Fredholm mapping with index zero, $\Omega \subset X$ be an open bounded set and $N : \bar{\Omega} \to Y$ be $L-$compact on $\bar{\Omega}$. Assume that:*

(a) for each $\lambda \in (0, 1)$ and $x \in \partial\Omega \cap DomL$, $Lx \neq \lambda Nx$,

(b) for each $x \in \partial\Omega \cap KerL$, $QNx \neq 0$,

(c) $deg\{JQN, \Omega \cap KerL, 0\} \neq 0$. Then equation $Lx = \lambda Nx$ has at least one solution in $\bar{\Omega} \cap DomL$.

3. Existence of the periodic solution

In this section, we study the existence of periodic solutions of systems (2.1) and (2.2) based on Mawhin coincidence degree theorem.

Let

$$z(t) = (x^T(t), y^T(t))^T = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T,$$

obviously if $z(t)$ is a solution of systems (2.1) and (2.2) defined on $[0, \omega]$ such that $z(0) = z(t)$, then according to the periodicity of systems (2.1) and (2.2) in t, the function $z^*(t)$ defined by

$$z^*(t) = z(t - k\omega), \quad t \in [l\omega, (l+1)\omega] \setminus \{t_k\}, \quad k = 1, 2, \cdots, \ l = 1, 2 \cdots.$$

in which $z^*(t)$ is left continuous at $t = t_k$. Thus $z^*(t)$ is an ω−periodic solution of systems (2.1) and (2.2).

For any non-negative integer q, let $C^q[0, \omega : t_1, \cdots, t_p] = \{z : [0, \omega] \to \mathbb{R}^{n+m} \mid z^{(q)}(t)$ exists for $t \neq t_1, \cdots, t_p; z^{(q)}(t^+)$ and $z^{(q)}(t^-)$exists at $t_1, \cdots, t_p$ and $z^{(j)}(t_k) = z^{(j)}(t_k^-), k = 1, \cdots, p, \ j = 1, \cdots, q\}$.

In order to establish the existence of ω−periodic solutions of systems (2.1) and (2.2), we take

$$X = \{z \in C[0, \omega : t_1, \cdots, t_p] | z(t) = z(t + \omega)\}, \quad Y = X \times \mathbb{R}^{(n+m)\times(p+1)},$$

and

$$\|z\| = \sum_{i=1}^{n+m} \max_{0\le t\le\omega} |z_i(t)| = \sum_{i=1}^{n} \max_{0\le t\le\omega} |x_i(t)| + \sum_{j=1}^{m} \max_{0\le t\le\omega} \left|y_j(t)\right|,$$

then X and Y are both Banach space.

Set

$$L : \mathrm{Dom}L \cap X \to Y, \quad z \to (\dot{z}(t), \Delta z(t_1), \Delta z(t_2), \cdots, \Delta z(t_p), 0), \tag{3.1}$$

where $\mathrm{Dom}L = \{z \in C^1[0, \omega : t_1, \cdots, t_p], z(t) = z(t + \omega)\}$.

From $N : X \to Y$, we have

$$Nz = \left(\begin{pmatrix} A_1(t) \\ \vdots \\ A_n(t) \\ B_1(t) \\ \vdots \\ B_m(t) \end{pmatrix}, \begin{pmatrix} \Delta x_1(t_1) \\ \vdots \\ \Delta x_n(t_1) \\ \Delta y_1(t_1) \\ \vdots \\ \Delta y_m(t_1) \end{pmatrix}, \begin{pmatrix} \Delta x_1(t_2) \\ \vdots \\ \Delta x_n(t_2) \\ \Delta y_1(t_2) \\ \vdots \\ \Delta y_m(t_2) \end{pmatrix}, \cdots, \begin{pmatrix} \Delta x_1(t_p) \\ \vdots \\ \Delta x_n(t_p) \\ \Delta y_1(t_p) \\ \vdots \\ \Delta y_m(t_p) \end{pmatrix}, \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \right), \tag{3.2}$$

where

$$A_i(t) = -a_i(x_i(t)) \left[b_i(x_i(t)) - \sum_{j=1}^{m} a_{ij}(t) f_j(y_j(t - \tau_{ij}(t))) - \sum_{j=1}^{m} b_{ij}(t) f_j(\dot{y}_j(t - \tilde{\tau}_{ij}(t))) - I_i(t) \right],$$

and

$$B_j(t) = -c_j(y_j(t)) \left[d_j(y_j(t)) - \sum_{i=1}^{n} c_{ji}(t) g_i(x_i(t - \sigma_{ji}(t))) - \sum_{i=1}^{n} d_{ji}(t) g_i(\dot{x}_i(t - \tilde{\sigma}_{ji}(t))) - J_j(t) \right].$$

Obviously,

$$\mathrm{Ker}L = \mathbb{R}^{n+m},$$

and

$$\begin{aligned}\mathrm{Im}L &= \{(h, C_1, C_2, \cdots, C_p, d) \in Y : \int_0^{\omega} h(s)ds + \sum_{k=1}^{p} C_k + d = 0\} \\ &= X \times \mathbb{R}^{(n+m)\times p} \times \{0\},\end{aligned}$$

thus

$$\dim \mathrm{Ker}L = \mathrm{codim}\ \mathrm{Im}L = n + m.$$

It is easy to show that ImL is closed in Y and L is a Fredholm mapping of index zero.

Lemma 2. *Let L and N are two mappings defined by (3.1) and (3.2), then N is L−compact on* $\bar{\Omega}$ *for any open bounded set* $\Omega \subset X$*..*

Proof. Define two projectors:

$$Pz = \frac{1}{\omega}\int_0^{\omega} z(t)dt,$$

and

$$Qz = Q(h, C_1, C_2, \cdots, C_p, d) = \left(\frac{1}{\omega}\left[\int_0^{\omega} h(s)ds + \sum_{k=1}^{p} C_k + d\right], 0, \cdots, 0, 0\right).$$

It is obvious that P and Q are continuous and satisfy

$$\mathrm{Im}P = \mathrm{Ker}L \quad \text{and} \quad \mathrm{Im}L = \mathrm{Ker}Q = \mathrm{Im}(I - Q).$$

Furthermore, the generalized inverse $K_p = L_p^{-1}$ is given by

$$K_p z = \int_0^t h(s)ds + \sum_{t>t_k} C_k - \frac{1}{\omega}\int_0^{\omega}\int_0^t h(s)dsdt - \sum_{k=1}^{p} C_k.$$

Then the expression of QNz is

$$QNz = \left(\begin{matrix}\left(\frac{1}{\omega}\int_0^{\omega} A_i(s)ds + \frac{1}{\omega}\sum_{k=1}^{p} I_{ik}(x_i(t_k))\right)_{n\times 1} \\ \left(\frac{1}{\omega}\int_0^{\omega} B_j(s)ds + \frac{1}{\omega}\sum_{k=1}^{p} J_{jk}(y_j(t_k))\right)_{m\times 1}\end{matrix}, 0, \cdots, 0, 0\right),$$

and

$$K_p(I - Q)Nz = \left(\begin{matrix}\left(\int_0^{t} A_i(s)ds + \sum_{t>t_k} I_{ik}(x_i(t_k))\right)_{n\times 1} \\ \left(\int_0^{t} B_j(s)ds + \sum_{t>t_k} J_{jk}(y_j(t_k))\right)_{m\times 1}\end{matrix}\right)$$

$$-\begin{pmatrix}\left(\frac{1}{\omega}\int_0^\omega\int_0^t A_i(s)dsdt+(\frac{t}{\omega}-\frac{1}{2})\int_0^\omega A_i(s)ds\right)_{n\times1}\\ \left(\frac{1}{\omega}\int_0^\omega\int_0^t B_j(s)dsdt+(\frac{t}{\omega}-\frac{1}{2})\int_0^\omega B_j(s)ds\right)_{m\times1}\end{pmatrix}-\begin{pmatrix}\left(\sum_{k=1}^p I_{ik}(x_i(t_k))\right)_{n\times1}\\ \left(\sum_{k=1}^p J_{jk}(y_j(t_k))\right)_{m\times1}\end{pmatrix}.$$

Thus QN and $K_p(I-Q)N$ are both continuous.

Consider the sequence $\{K_p(I-Q)N\}$, for any open bounded set $\Omega\subset X$ and any $z\in\Omega$, we have

$$\begin{aligned}\|K_p(I-Q)Nz\| &= \max_{0\le t\le\omega}\sum_{i=1}^n\left|\int_0^t A_i(s)ds+\sum_{t>t_k}I_{ik}(x_i(t_k))-\frac{1}{\omega}\int_0^\omega\int_0^t A_i(s)dsdt\right.\\ &\quad\left.-(\frac{t}{\omega}-\frac{1}{2})\int_0^\omega A_i(s)ds-\sum_{k=1}^p I_{ik}(x_i(t_k))\right|\\ &\quad+\max_{0\le t\le\omega}\sum_{j=1}^m\left|\int_0^t B_j(s)ds+\sum_{t>t_k}J_{jk}(y_j(t_k))-\frac{1}{\omega}\int_0^\omega\int_0^t B_j(s)dsdt\right.\\ &\quad\left.-(\frac{t}{\omega}-\frac{1}{2})\int_0^\omega B_j(s)ds-\sum_{k=1}^p J_{jk}(y_j(t_k))\right|\\ &\le \sum_{i=1}^n\frac{5}{2}\int_0^\omega|A_i(s)|ds+2ps_i+\sum_{j=1}^m\frac{5}{2}\int_0^\omega|B_j(s)|ds+2pr_j,\end{aligned}$$

then $K_p(I-Q)N$ is uniformly bounded on Ω. For any $z,\tilde{z}\in\Omega$, we have

$$\begin{aligned}&\|K_p(I-Q)Nz-K_p(I-Q)N\tilde{z}\|\\ &=\max_{0\le t\le\omega}\sum_{i=1}^n\left|\sum_{t>t_k}(I_{ik}(x_i(t_k))-I_{ik}(\tilde{x}_i(t_k)))-\sum_{k=1}^p(I_{ik}(x_i(t_k))-I_{ik}(\tilde{x}_i(t_k)))\right|\\ &+\max_{0\le t\le\omega}\sum_{j=1}^m\left|\sum_{t>t_k}(J_{jk}(y_j(t_k))-J_{jk}(\tilde{y}_j(t_k)))-\sum_{k=1}^p(J_{jk}(y_j(t_k))-J_{jk}(\tilde{y}_j(t_k)))\right|\\ &\le\sum_{i=1}^n\sum_{k=1}^p|(I_{ik}(x_i(t_k))-I_{ik}(\tilde{x}_i(t_k)))|+\sum_{j=1}^m\sum_{k=1}^p\left|(J_{jk}(y_j(t_k))-J_{jk}(\tilde{y}_j(t_k)))\right|\\ &\le\sum_{i=1}^n\sum_{k=1}^p s_{ik}\,|x_i(t_k)-\tilde{x}_i(t_k)|+\sum_{i=1}^m\sum_{k=1}^p r_{jk}\left|y_j(t_k)-\tilde{y}_j(t_k)\right|,\end{aligned}$$

then $K_p(I-Q)N$ is equicontinuous on Ω. By virtue of the Arzela-Ascoli Theorem, $K_p(I-Q)N(\Omega)$ is a sequentially compact set. Therefore, $K_p(I-Q)N(\bar{\Omega})$ is compact. Moreover, $QN(\bar{\Omega})$ is bounded. Thus, N is $L-$compact on $\bar{\Omega}$ for any open bounded set $\Omega\subset X$. □

Now, we need to show that there exists a domain Ω that satisfies all the requirements given in Lemma 1.

Theorem 1. *Assume that (H1)-(H5) hold, systems (2.1) and (2.2) have at least one $\omega-$periodic solution.*

Proof. Corresponding to the operator equation $Lz = \lambda Nz,\ \lambda \in (0,1)$, we have

$$\begin{cases} \dot{x}_i(t) = -\lambda a_i(x_i(t))\Bigg[b_i(x_i(t)) - \sum_{j=1}^{m} a_{ij}(t) f_j(y_j(t-\tau_{ij}(t))) \\ \qquad\qquad - \sum_{j=1}^{m} b_{ij}(t) f_j(\dot{y}_j(t-\tilde{\tau}_{ij}(t))) - I_i(t)\Bigg], \quad t>0, \quad t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k) = \lambda I_{ik}(x_i(t_k)), \quad i=1,2,\cdots,n, \quad k=1,2,\cdots, \\ \dot{y}_j(t) = -\lambda c_j(y_j(t))\Bigg[d_j(y_j(t)) - \sum_{i=1}^{n} c_{ji}(t) g_i(x_i(t-\sigma_{ji}(t))) \\ \qquad\qquad - \sum_{i=1}^{n} d_{ji}(t) g_i(\dot{x}_i(t-\tilde{\sigma}_{ji}(t))) - J_j(t)\Bigg], \quad t>0, \quad t \neq t_k, \\ \Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k) = \lambda J_{jk}(y_j(t_k)), \quad j=1,2,\cdots,m, \quad k=1,2,\cdots. \end{cases} \tag{3.3}$$

Suppose that $(x_1(t),\cdots,x_n(t),y_1(t),\cdots,y_m(t))^T \in X$ is a solution of system (3.3) for some $\lambda \in (0,1)$. Integrating system (3.3) over the interval $[0,\omega]$, we obtain

$$\begin{cases} \int_0^{\omega}\Bigg\{-a_i(x_i(s))\Bigg[b_i(x_i(s)) - \sum_{j=1}^{m} a_{ij}(t) f_j(y_j(s-\tau_{ij}(s))) - \sum_{j=1}^{m} b_{ij}(t) f_j(\dot{y}_j(s-\tilde{\tau}_{ij}(s))) - I_i(s)\Bigg]\Bigg\} ds \\ \quad + \sum_{k=1}^{p} I_{ik}(x_i(t_k)) = 0, \\ \int_0^{\omega}\Bigg\{-c_j(y_j(s))\Bigg[d_j(y_j(s)) - \sum_{i=1}^{n} c_{ji}(t) g_i(x_i(s-\sigma_{ji}(s))) - \sum_{i=1}^{n} d_{ji}(t) g_i(\dot{x}_i(s-\tilde{\sigma}_{ji}(s))) - J_j(s)\Bigg]\Bigg\} ds \\ \quad + \sum_{k=1}^{p} J_{jk}(y_j(t_k)) = 0. \end{cases} \tag{3.4}$$

Let $\xi^-,\ \eta^- \in [0,\omega]$, and

$$x_i(\xi^-) = \inf_{0\le t\le\omega} x_i(t),\ i=1,2,\cdots,n; \quad y_j(\eta^-) = \inf_{0\le t\le\omega} y_j(t),\ j=1,2,\cdots,m.$$

From (3.4), we have

$$\begin{aligned} x_i(\xi^-) a_i^L b_i^L \omega &\le \int_0^{\omega} a_i(x_i(s)) b_i(x_i(s)) ds \\ &= \int_0^{\omega} \{-a_i(x_i(s))[-\sum_{j=1}^{m} a_{ij}(t) f_j(y_j(s-\tau_{ij}(s))) - \sum_{j=1}^{m} b_{ij}(t) f_j(\dot{y}_j(s-\tilde{\tau}_{ij}(s))) \\ &\quad -I_i(s)]\} ds + \sum_{k=1}^{p} I_{ik} x_i(t_k) \\ &\le \int_0^{\omega} |a_i(x_i(s))[-\sum_{j=1}^{m} a_{ij}(t) f_j(y_j(s-\tau_{ij}(s))) - \sum_{j=1}^{m} b_{ij}(t) f_j(\dot{y}_j(s-\tilde{\tau}_{ij}(s))) \end{aligned}$$

$$
\begin{aligned}
&-I_i(s)]|ds + |\sum_{k=1}^{p} I_{ik}x_i(t_k)| \\
\leq\ & a_i^M\omega\sum_{j=1}^{m}(a_{ij}^M + b_{ij}^M)\tilde{F}_j + \sum_{k=1}^{p} I_{ik}x_i(t_k),
\end{aligned}
$$

that is,

$$x_i(\xi^-) \leq \frac{a_i^M\omega\sum_{j=1}^{m}(a_{ij}^M + b_{ij}^M)\tilde{F}_j + \sum_{k=1}^{p} I_{ik}x_i(t_k)}{a_i^L b_i^L\omega} := T_{1i}^-.$$

Similarly we obtain

$$y_j(\eta^-) \leq \frac{c_j^M\omega\sum_{i=1}^{n}(c_{ji}^M + d_{ji}^M)\tilde{G}_i + \sum_{k=1}^{p} J_{jk}y_j(t_k)}{c_j^L d_j^L\omega} := T_{2j}^-.$$

Again set $\xi^+,\ \eta^+ \in [0,\omega]$, and

$$x_i(\xi^+) = \sup_{0\leq t\leq\omega} x_i(t),\ i = 1,2,\cdots,n;\quad y_j(\eta^+) = \sup_{0\leq t\leq\omega} y_j(t),\ j = 1,2,\cdots,m,$$

obviously

$$x_i(\xi^+) \geq -\frac{a_i^M\omega\sum_{j=1}^{m}(a_{ij}^M + b_{ij}^M)\tilde{F}_j + \sum_{k=1}^{p} I_{ik}x_i(t_k)}{a_i^M b_i^M\omega} := T_{1i}^+.$$

$$y_j(\eta^+) \geq -\frac{c_j^M\omega\sum_{i=1}^{n}(c_{ji}^M + d_{ji}^M)\tilde{G}_i + \sum_{k=1}^{p} J_{jk}y_j(t_k)}{c_j^M d_j^M\omega} := T_{2j}^+.$$

Denote $H_i = \max\limits_{0\leq t\leq\omega}|z_i(t)| < \max\{|T_{1i}^+|, |T_{1i}^-|\}$, and

$$H = \sum_{i=1}^{n+m} H_i + E,$$

where E is a sufficiently large positive constant. It is obvious that H is independent of λ. Let

$$\Omega = \{z(t) = (x^T(t), y^T(t))^T \in X : \|z(t)\| < H,\ z(t_k^+) \in \Omega,\ k = 1,2\cdots,p\}.$$

where $x(t) = (x_1(t),\cdots,x_n(t))^T,\ y(t) = (y_1(t),\cdots,y_m(t))^T$.

Next we check the three conditions in Lemma 1.

(a) For each $\lambda \in (0,1),\ z(t) \in \partial\Omega \cap \mathrm{Dom}L$ with the norm $\|z(t)\| = H$, we have $Lz \neq \lambda Nz$.

(b) For any $z \in \partial\Omega \cap \mathbb{R}^{n+m}$, z is a constant vector in $\mathbb{R}^{n+m}$, $\|z\| = H$, then $QNz \neq 0$.

(c) Let $J : \mathrm{Im}Q \to \mathrm{Ker}L$, then

$$JQNz = QNz = \begin{pmatrix} \left(\frac{1}{\omega} \int_0^\omega A_i(s)ds + \frac{1}{\omega} \sum_{k=1}^{p} I_{ik}(x_i(t_k)) \right)_{n\times 1}, 0, \cdots, 0, 0 \\ \left(\frac{1}{\omega} \int_0^\omega B_j(s)ds + \frac{1}{\omega} \sum_{k=1}^{p} J_{jk}(y_j(t_k)) \right)_{m\times 1} \end{pmatrix}.$$

Define $\Psi : \mathrm{Ker}L \times [0, 1] \to X$ by

$$\Psi(z, \mu) = -\mu z + (1 - \mu)QNz.$$

It is easy to verify $\Psi(z, \mu) \neq (0, 0, \cdots, 0)$ for any $z \in \partial\Omega \cap \mathrm{Ker}L$.

Therefore

$$\begin{aligned} deg\{JQN, \Omega \cap kerL, (0, 0, ..., 0)\} &= deg\{QNz, \Omega \cap kerL, (0, 0, ..., 0)\} \\ &= deg\{-z, \Omega \cap kerL, (0, 0, ..., 0)\} \\ &\neq 0. \end{aligned}$$

All the conditions in Lemma 1 have been verified. We conclude that $Lz = Nz$ has at least one $\omega-$periodic solution. This implies that systems (2.1) and (2.2) have at least one $\omega-$periodic solution. □

Remark 1. *The method of the estimation for Ω is classical and effective. In fact, a new study method is cited in [33], utilizing Lyapunov method to study periodic solutions for neural networks. Hence, we also established Lemma 3 and gave a detailed proof in Appendix.*

4. Global stability of periodic solution

In this section, we will construct a new Lyapunov functional to study the global asymptotic stability of periodic solutions of systems (2.1) and (2.2).

Theorem 2. *Assume that (H1)–(H5) hold, and*

(H6) $a_i(u)$ and $c_j(u)$ are globally Lipschitz continuous, that is, for any $(u, v) \in \mathbb{R}$, there exist constants h_i^a and h_j^c such that

$$|a_i(u) - a_i(v)| \leq h_i^a |u - v|, \quad i = 1, 2, \cdots, n,$$

$$|c_j(u) - c_j(v)| \leq h_j^c |u - v|, \quad j = 1, 2, \cdots, m.$$

(H7) for all i, j $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$, there exist constants L_i^{ab} and L_j^{cd} such that

$$|a_i(u)b_i(u) - a_i(v)b_i(v)| \geq L_i^{ab}|u - v|, \quad |c_j(u)d_j(u) - c_j(v)d_j(v)| \geq L_j^{cd}|u - v|, \quad \forall (u, v) \in \mathbb{R}.$$

(H8) there exist positive constants $\theta_i,\ \theta_{n+j}$ and $\gamma \in (0, \min(\Lambda_{1i}, \Lambda_{2j}))$ such that

$$-\theta_i(\Lambda_{1i} - \gamma) + \sum_{j=1}^{m} \theta_{n+j} c_j^M \frac{c_{ji}^M G_i}{1 - \sigma'^M_{ji}} e^{\gamma \sigma'^M_{ji}} < 0,$$

and

$$-\theta_{n+j}(\Lambda_{2j} - \gamma) + \sum_{i=1}^{n} \theta_i a_i^M \frac{a_{ij}^M F_j}{1 - \tau'^M_{ij}} e^{\gamma \tau'^M_{ij}} < 0,$$

where

$$\Lambda_{1i} = L_i^{ab} - h_i^a \sum_{j=1}^{m} (a_{ij}^M + b_{ij}^M) \tilde{F}_j - h_i^a I_i^M,$$

$$\Lambda_{2j} = L_j^{cd} - h_j^c \sum_{i=1}^{n} (c_{ji}^M + d_{ji}^M) \tilde{G}_i - h_j^c J_j^M,$$

in which

$$I_i^M = \max_{0 \le t \le \omega} I_i(t), \quad J_j^M = \max_{0 \le t \le \omega} J_j(t), \quad \tau'^M_{ij} = \max_{0 \le t \le \omega} \tau'_{ij}(t),$$

$$\sigma'^M_{ji} = \max_{0 \le t \le \omega} \sigma'_{ji}(t), \quad \tilde{\tau}'^M_{ij} = \max_{0 \le t \le \omega} \tau'_{ij}(t), \quad \tilde{\sigma}'^M_{ji} = \max_{0 \le t \le \omega} \sigma'_{ji}(t), .$$

and $1 - \tau'^M_{ij} > 0,\ 1 - \sigma'^M_{ji} > 0,\ 1 - \tilde{\tau}'^M_{ij} > 0,\ 1 - \tilde{\sigma}'^M_{ji} > 0.$

(H9) $-2 < I_{ik} < 0,\ -2 < J_{jk} < 0,\ i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, m,\ k = 1, 2, \cdots, p$. Then the $\omega-$periodic solution $(x^{*T}(t), y^{*T}(t))^T$ of systems (2.1) and (2.2) is global asymptotic stable.

Proof. From Theorem 1, we find that systems (2.1) and (2.2) has at least one $\omega-$periodic solution under assumptions (H1)-(H5). Let $(x^{*T}(t), y^{*T}(t))^T$ be one $\omega-$periodic solution of systems (2.1) and (2.2). For any solution $(x(t)^T, y(t)^T)^T$ of system (2.1), we let

$$u_i(t) = x_i(t) - x_i^*(t), \quad v_j(t) = y_j(t) - y_j^*(t),$$

then

$$\dot{u}_i(t) = \dot{x}_i(t) - \dot{x}_i^*(t), \quad \dot{v}_j(t) = \dot{y}_j(t) - \dot{y}_j^*(t).$$

It is easy to see that system (2.1) can be reduced to the following system

$$\begin{cases} \dfrac{du_i(t)}{dt} = -[a_i(u_i(t)+x_i^*(t))b_i(u_i(t)+x_i^*(t)) - a_i(x_i^*(t))b_i(x_i^*(t))] \\ \qquad +a_i(u_i(t)+x_i^*(t))\displaystyle\sum_{j=1}^{m} a_{ij}(t)f_j(y_j(t-\tau_{ij}(t))) - a_i(u_i(t)+x_i^*(t))\displaystyle\sum_{j=1}^{m} a_{ij}(t)f_j(y_j^*(t-\tau_{ij}(t))) \\ \qquad +a_i(u_i(t)+x_i^*(t))\displaystyle\sum_{j=1}^{m} b_{ij}(t)f_j(\dot{y}_j(t-\tilde{\tau}_{ij}(t))) - a_i(u_i(t)+x_i^*(t))\displaystyle\sum_{j=1}^{m} b_{ij}(t)f_j(\dot{y}_j^*(t-\tilde{\tau}_{ij}(t))) \\ \qquad +(a_i(u_i(t)+x_i^*(t)) - a_i(x_i^*(t)))\displaystyle\sum_{j=1}^{m} a_{ij}(t)f_j(y_j^*(t-\tau_{ij}(t))) \\ \qquad +(a_i(u_i(t)+x_i^*(t)) - a_i(x_i^*(t)))\displaystyle\sum_{j=1}^{m} b_{ij}(t)f_j(\dot{y}_j^*(t-\tilde{\tau}_{ij}(t))) \\ \qquad +(a_i(u_i(t)+x_i^*(t)) - a_i(x_i^*(t)))I_i(t), \quad t\in[o,\omega], \quad t\neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k) = I_{ik}(x_i(t_k)), \quad i=1,2,\cdots,n, \quad k=1,2,\cdots, \\ \dfrac{dv_j(t)}{dt} = -[c_j(v_j(t)+y_j^*(t))d_j(v_j(t)+y_j^*(t)) - c_j(y_j^*(t))d_j(y_j^*(t))] \\ \qquad +c_j(v_j(t)+y_j^*(t))\displaystyle\sum_{i=1}^{n} c_{ji}(t)g_i(x_i(t-\sigma_{ji}(t))) - c_j(v_j(t)+y_j^*(t))\displaystyle\sum_{i=1}^{n} c_{ji}(t)g_i(x_i^*(t-\sigma_{ji}(t))) \\ \qquad +c_j(v_j(t)+y_j^*(t))\displaystyle\sum_{i=1}^{n} d_{ji}(t)g_i(\dot{x}_i(t-\tilde{\sigma}_{ji}(t))) - c_j(v_j(t)+y_j^*(t))\displaystyle\sum_{i=1}^{n} d_{ji}(t)g_i(\dot{x}_i^*(t-\tilde{\sigma}_{ji}(t))) \\ \qquad +(c_j(v_j(t)+y_j^*(t)) - c_j(y_j^*(t)))\displaystyle\sum_{i=1}^{n} c_{ji}(t)g_i(x_i^*(t-\sigma_{ji}(t))) \\ \qquad +(c_j(v_j(t)+y_j^*(t)) - c_j(y_j^*(t)))\displaystyle\sum_{i=1}^{n} d_{ji}(t)g_i(\dot{x}_i^*(t-\tilde{\sigma}_{ji}(t))) \\ \qquad +(c_j(v_j(t)+y_j^*(t)) - c_j(y_j^*(t)))J_j(t), \quad t\in[o,\omega], \quad t\neq t_k, \\ \Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k) = J_{jk}(y_j(t_k)), \quad j=1,2,\cdots,m, \quad k=1,2,\cdots. \end{cases} \tag{4.1}$$

From system (4.1), we have

$$\begin{aligned} D^+u_i(t) &\leq -[L_i^{ab} - h_i^a\sum_{j=1}^{m}(a_{ij}^M + b_{ij}^M)\tilde{F}_j - h_i^a I_i^M]|u_i(t)| + a_i^M\sum_{j=1}^{m} a_{ij}^M F_j(v_j(t-\tau_{ij}(t))) \\ &\quad + a_i^M\sum_{j=1}^{m} b_{ij}^M F_j(\dot{v}_j(t-\tilde{\tau}_{ij}(t))), \end{aligned} \tag{4.2}$$

and

$$\begin{aligned} D^+v_j(t) &\leq -[L_j^{cd} - h_j^c\sum_{i=1}^{n}(c_{ji}^M + d_{ji}^M)\tilde{G}_i - h_j^c J_j^M]|v_j(t)| + c_j^M\sum_{i=1}^{n} c_{ji}^M G_i(u_i(t-\sigma_{ji}(t))) \\ &\quad + c_j^M\sum_{i=1}^{n} d_{ji}^M G_i(\dot{u}_i(t-\tilde{\sigma}_{ji}(t))), \end{aligned} \tag{4.3}$$

for all $i=1,2,\cdots,n$ and $j=1,2,\cdots,m$.

Define $V(t) = V_1(t) + V_2(t)$, where

$$\begin{aligned} V_1(t) &= \sum_{i=1}^{n}\Bigg[\theta_i e^{\gamma t}|u_i(t)| + a_i^M \sum_{j=1}^{m}\theta_i \frac{a_{ij}^M F_j}{1-\tau_{ij}'^M}\int_{t-\tau_{ij}(t)}^{t}|v_j(s)|e^{\gamma(s+\tau_{ij}'^M)}ds \\ &\quad + a_i^M \sum_{j=1}^{m}\theta_i \frac{b_{ij}^M F_j}{1-\tilde{\tau}_{ij}'^M}\int_{t-\tilde{\tau}_{ij}(t)}^{+\infty}|\dot{v}_j(s)|e^{\gamma(s+\tilde{\tau}_{ij}'^M)}ds\Bigg], \end{aligned} \tag{4.4}$$

and

$$\begin{aligned} V_2(t) &= \sum_{j=1}^{m}\Bigg[\theta_{n+j} e^{\gamma t}|v_j(t)| + c_j^M \sum_{i=1}^{n}\theta_{n+j} \frac{c_{ji}^M G_i}{1-\sigma_{ji}'^M}\int_{t-\sigma_{ji}(t)}^{t}|u_i(s)|e^{\gamma(s+\sigma_{ji}'^M)}ds \\ &\quad + c_j^M \sum_{i=1}^{n}\theta_{n+j} \frac{d_{ji}^M G_i}{1-\tilde{\sigma}_{ji}'^M}\int_{t-\tilde{\sigma}_{ji}(t)}^{+\infty}|\dot{u}_i(s)|e^{\gamma(s+\tilde{\sigma}_{ji}'^M)}ds\Bigg], \end{aligned} \tag{4.5}$$

From (4.2) and (4.3), we have

$$\begin{aligned} D^+V_1(t) &\le \sum_{i=1}^{n}\theta_i\Big(-(L_i^{ab} - h_i^a\sum_{j=1}^{m}(a_{ij}^M + b_{ij}^M)\tilde{F}_j - h_i^a I_i^M - \gamma)\Big)|u_i(t)|e^{\gamma t} \\ &+\theta_i e^{\gamma t} a_i^M \sum_{j=1}^{m} a_{ij}^M F_j(v_j(t-\tau_{ij}(t))) + \theta_i e^{\gamma t} a_i^M \sum_{j=1}^{m} b_{ij}^M F_j(\dot{v}_j(t-\tilde{\tau}_{ij}(t))) \\ &+a_i^M\sum_{j=1}^{m}\theta_i\frac{a_{ij}^M F_j}{1-\tau_{ij}'^M}\big[|v_j(t)|e^{\gamma(t+\tau_{ij}'^M)} - |v_j(t-\tau_{ij}(t))|e^{\gamma(t-\tau_{ij}(t)+\tau_{ij}'^M)}(1-\tau_{ij}'(t))\big] \\ &+a_i^M\sum_{j=1}^{m}\theta_i\frac{b_{ij}^M F_j}{1-\tilde{\tau}_{ij}'^M}\big[-|\dot{v}_j(t-\tilde{\tau}_{ij}(t))|e^{\gamma(t-\tilde{\tau}_{ij}(t)+\tilde{\tau}_{ij}'^M)}(1-\tilde{\tau}_{ij}'(t))\big]. \end{aligned}$$

According to

$$\begin{aligned} 1-\tau_{ij}'(t) \ge 1-\tau_{ij}'^M, \quad e^{\gamma(t-\tau_{ij}(t)+\tau_{ij}'^M)} \ge e^{\gamma t}, \\ 1-\tilde{\tau}_{ij}'(t) \ge 1-\tilde{\tau}_{ij}'^M, \quad e^{\gamma(t-\tilde{\tau}_{ij}(t)+\tilde{\tau}_{ij}'^M)} \ge e^{\gamma t}, \end{aligned}$$

we obtain

$$D^+V_1(t) \le \sum_{i=1}^{n}\Bigg[-\theta_i(\Lambda_{1i}-\gamma)|u_i(t)|e^{\gamma t} + \sum_{j=1}^{m}\theta_i a_i^M\frac{a_{ij}^M F_j}{1-\tau_{ij}'^M}|v_j(t)|e^{\gamma(t+\tau_{ij}'^M)}\Bigg],$$

where

$$\Lambda_{1i} = L_i^{ab} - h_i^a\sum_{j=1}^{m}(a_{ij}^M + b_{ij}^M)\tilde{F}_j - h_i^a I_i^M.$$

Similarly to the calculation of $D^+V_1(t)$, we have

$$D^+V_2(t) \le \sum_{j=1}^{m}\Bigg[-\theta_{n+j}(\Lambda_{2i}-\gamma)|v_j(t)|e^{\gamma t} + \sum_{i=1}^{n}\theta_{n+j} c_j^M\frac{c_{ji}^M G_i}{1-\sigma_{ji}'^M}|u_i(t)|e^{\gamma(t+\sigma_{ji}'^M)}\Bigg],$$

Hence

$$\begin{aligned} D^+V(t) &\leq e^{\gamma t}\sum_{i=1}^{n}\left[-\theta_i(\Lambda_{1i}-\gamma)+\sum_{j=1}^{m}\theta_{n+j}c_j^M\frac{c_{ji}^M G_i}{1-\sigma'^M_{ji}}e^{\gamma\sigma'^M_{ji}}\right]|u_i(t)| \\ &\quad +e^{\gamma t}\sum_{j=1}^{m}\left[-\theta_{n+j}(\Lambda_{2i}-\gamma)+\sum_{i=1}^{n}\theta_i a_i^M\frac{a_{ij}^M F_j}{1-\tau'^M_{ij}}e^{\gamma\tau'^M_{ij}}\right]|v_j(t)| \\ &< 0. \end{aligned}$$

In view of assumption (H3), we have

$$\begin{aligned} V(t_k^+) &= \sum_{i=1}^{n}\left[\theta_i e^{\gamma t_k^+}|u_i(t_k^+)| + a_i^M\sum_{j=1}^{m}\theta_i\frac{a_{ij}^M F_j}{1-\tau'^M_{ij}}\int_{t_k^+-\tau_{ij}(t_k^+)}^{t_k^+}|v_j(s)|e^{\gamma(s+\tau'^M_{ij})}ds\right. \\ &\quad \left.+\sum_{j=1}^{m}\theta_i a_i^M\frac{b_{ij}^M F_j}{1-\tilde{\tau}'^M_{ij}}\int_{t_k^+-\tilde{\tau}_{ij}(t_k^+)}^{+\infty}|\dot{v}_j(s)|e^{\gamma(s+\tilde{\tau}'^M_{ij})}ds\right] \\ &\quad +\sum_{j=1}^{m}\left[\theta_{n+j}e^{\gamma t_k^+}|v_j(t_k^+)| + c_j^M\sum_{i=1}^{n}\theta_{n+j}\frac{c_{ji}^M G_i}{1-\sigma'^M_{ji}}\int_{t_k^+-\sigma_{ji}(t_k^+)}^{t_k^+}|u_i(s)|e^{\gamma(s+\sigma'^M_{ji})}ds\right. \\ &\quad \left.+\sum_{j=1}^{m}\sum_{i=1}^{n}\theta_{n+j}c_j^M\frac{d_{ji}^M G_i}{1-\tilde{\sigma}'^M_{ji}}\int_{t_k^+-\tilde{\sigma}_{ji}(t_k^+)}^{+\infty}|\dot{u}_i(s)|e^{\gamma(s+\tilde{\sigma}'^M_{ji})}ds\right] \\ &\leq \sum_{i=1}^{n}\left[\theta_i e^{\gamma t_k}|(1+I_{ik})u_i(t_k)| + a_i^M\sum_{j=1}^{m}\theta_i\frac{a_{ij}^M F_j}{1-\tau'^M_{ij}}\int_{t_k-\tau_{ij}(t_k)}^{t_k}|v_j(s)|e^{\gamma(s+\tau'^M_{ij})}ds\right. \\ &\quad \left.+\sum_{j=1}^{m}\theta_i a_i^M\frac{b_{ij}^M F_j}{1-\tilde{\tau}'^M_{ij}}\int_{t_k-\tilde{\tau}_{ij}(t_k)}^{+\infty}|\dot{v}_j(s)|e^{\gamma(s+\tilde{\tau}'^M_{ij})}ds\right] \\ &\quad +\sum_{j=1}^{m}\left[\theta_{n+j}e^{\gamma t_k}|(1+J_{jk})v_j(t_k)| + c_j^M\sum_{i=1}^{n}\theta_{n+j}\frac{c_{ji}^M G_i}{1-\sigma'^M_{ji}}\int_{t_k-\sigma_{ji}(t_k)}^{t_k}|u_i(s)|e^{\gamma(s+\sigma'^M_{ji})}ds\right. \\ &\quad \left.+\sum_{j=1}^{m}\sum_{i=1}^{n}\theta_{n+j}c_j^M\frac{d_{ji}^M G_i}{1-\tilde{\sigma}'^M_{ji}}\int_{t_k-\tilde{\sigma}_{ji}(t_k)}^{+\infty}|\dot{u}_i(s)|e^{\gamma(s+\tilde{\sigma}'^M_{ji})}ds\right] \\ &\leq V(t_k). \end{aligned}$$

Thus, by the standard Lyapunov functional theory, the periodic solution $(x^{*T}, y^{*T})^T$ is global asymptotic stable. The proof is complete. □

Remark 2. *In [20], the authors discussed a model describing dynamics of delayed Cohen-Grossberg-type BAM neural networks without impulses. In our paper, we not only consider the impact of impulses and construct a new Lyapunov functional to establish the existence and the global asymptotic stability of periodic solutions.*

Remark 3. *In [16], the impulsive function is linear. We discussed a kind of more general bounded and Lipschitz continuous functions in this paper.*

Remark 4. *In [32], the authors discussed a kind of BAM neural networks with time-varying delays and impulses. In this paper, we propose a new kind of Cohen-Grossberg-type BAM neural network systems, which have a more wide application in practice. Obviously Cohen-Grossberg-type BAM neural networks can be reduced to Hopfield-type BAM neural networks and cellular BAM neural networks.*

Corollary 1. *Suppose that the assumptions (H1)–(H3) hold, then the systems (2.1) and (2.2) without impulses has at least one $\omega-$periodic solution.*

Remark 5. *The consideration of neutral delays may affect the stability of systems since the presence of delays may induce complex behaviors for the schemes. The existence of delays plays an increasingly important role in many disciplines like economic, mathematics, science, and engineering, which can also help describe propagation and transport phenomena or population dynamics, etc. For instance, in economic systems, presence delays appear in a natural way since decisions and effects are separated by some time interval.*

5. An illustrative example

As applications, we present an example to illustrate our main results in Theorem 1 and Theorem 2.

Example 1. *Consider the Cohen-Grossberg-type BAM neural networks with $m = n = 1$, which is given as*

$$\begin{cases} \dot{x}_1(t) = -a_1(x_1(t))\left[b_1(x_1(t)) - a_{11}(t)f_1(y_1(t-\tau_{11}(t))) - b_{11}(t)f_1(\dot{y}_1(t-\tilde{\tau}_{11}(t))) - I_1(t)\right], t > 0, t \neq t_k, \\ \Delta x_1(t_k) = I_{1k}(x_1(t_k)), \quad k = 1, 2, \cdots, \\ \dot{y}_1(t) = -c_1(y_1(t))\left[d_1(y_1(t)) - c_{11}(t)g_1(x_1(t-\sigma_{11}(t))) - d_{11}(t)g_1(\dot{x}_1(t-\tilde{\sigma}_{11}(t))) - J_1(t)\right], t > 0, t \neq t_k, \\ \Delta y_1(t_k) = J_{1k}(y_1(t_k)), \quad k = 1, 2, \cdots, \end{cases} \tag{5.1}$$

where

$$a_1(u) = \frac{1}{3} + \frac{1}{3}sinu, \quad b_1(u) = u(\frac{1}{3} - \frac{1}{3}sinu), \quad c_1(u) = \frac{1}{2} + \frac{1}{2}cosu, \quad d_1(u) = u(\frac{1}{2} - \frac{1}{2}cosu),$$

$$f_1(u) = sinu, \quad g_1(u) = \frac{1}{2}(|sinu + 1| - |sinu - 1|), \quad I_1(t) = \frac{1}{15}(-2 + sint),$$

$$J_1(t) = \frac{1}{15}(-2 + cost), \quad a_{11}(t) = b_{11}(t) = c_{11}(t) = d_{11}(t) = \frac{1}{30}sint,$$

$$\tau_{11} = \tilde{\tau}_{11} = \sigma_{11} = \tilde{\sigma}_{11} = \frac{1}{10}sint, \quad I_{1k}(u) = -1 + sinu, \quad J_{1k}(u) = -1 + cosu.$$

By a straightforward calculation, we obtain

$$a_{11}^M = b_{11}^M = c_{11}^M = d_{11}^M = \frac{1}{30}, \quad \tilde{F}_1 = 1, \quad \tilde{G}_1 = 1, \quad I_1^M = J_1^M = -\frac{1}{15},$$

$$h_1^a = \frac{1}{3}, \quad h_1^c = \frac{1}{2}, \quad a_1^L = c_1^L = 0, \quad a_1^M = c_1^M = \frac{2}{3}, \quad b_1^L = d_1^L = 0,$$

$$b_1^M = d_j^M = 1, \quad L_1^{ab} = \frac{1}{9}, \quad L_1^{cd} = \frac{1}{4}, \quad \tau_{11}'^M = \tilde{\tau}_{11}'^M = \sigma_{11}'^M = \tilde{\sigma}_{11}'^M = \frac{1}{10}.$$

Thus

$$\Lambda_{11}=\frac{4}{45}\quad \textit{and}\quad \Lambda_{21}=\frac{13}{60}.$$

Choose $\gamma=\frac{1}{100}\in(0,\min(\Lambda_{11},\Lambda_{21}))$*, there exist positive constants* $\theta_1=100$ *and* $\theta_2=50$*, such that*

$$\begin{aligned}
-\theta_1(\Lambda_{11}-\gamma)+\theta_2 c_1^M\frac{c_{11}^M G_1}{1-\sigma_{ji}'^M}e^{\gamma\sigma_{11}'^M} &= -100\times(\frac{4}{45}-\frac{1}{100})+\frac{100}{81}\times e^{\frac{1}{1000}}\\
&\approx -6.653085802263323<0,
\end{aligned}$$

and

$$\begin{aligned}
-\theta_2(\Lambda_{21}-\gamma)+\theta_1 a_1^M\frac{a_{11}^M F_1}{1-\tau_{11}'^M}e^{\gamma\tau_{11}'^M} &= -50\times(\frac{4}{45}-\frac{1}{100})+\frac{200}{81}\times e^{\frac{1}{1000}}\\
&\approx -1.472838271193313<0.
\end{aligned}$$

By Theorem 1, system (5.1) has a $2\pi-$*periodic solution. From Theorem 2, all other solutions of system (5.1) converges asymptotically to the periodic solution as* $t\to+\infty$*.*

6. Conclusions

We have introduced Cohen-Grossberg-type BAM neural networks (2.1) and (2.2) with neutral-type time delays and impulses, and obtained some results on the existence and stability of periodic solutions. The impulse terms in (2.1) are bounded and Lipschitz continuous functions instead of linear functions. What's more, we utilized both the method of the classical estimation and Lyapunov functional construction to search for the region of periodic solutions.

Appendix

The existing proof of periodic solution is very classical and effective approach and the method has been widely applied to studying the periodic solution for more 25 years. The proof of the global stability part has some novel in constructing Laypunov function. Hence, we also apply the new Lyapunov function [33] as in the stability to entail the proof of the existing part.

Lemma 3. *For any* $\lambda\in(0,1)$*, consider the operator equation* $Lz=\lambda Nz$*, if the periodic solutions of system (3.3) exist, then they are bounded and the boundary is independent of the choice of* λ *under assumption (H1)-(H5), Namely, there exists a positive constant* $\bar{H}_0$ *such that when* $\|(x^T(t),y^T(t))^T\|=\|(x_1(t),\cdots,x_n(t),y_1(t),\cdots,y_m(t))^T\|\leq\bar{H}_0$.

Proof. Suppose that $(x_1(t),\cdots,x_n(t),y_1(t),\cdots,y_m(t))^T\in X$ is a solution of system (3.3) for some $\lambda\in(0,1)$. It can be thus obtained from system (3.3) that

$$\begin{aligned}
D^+x_i(t) &\leq -\lambda\left[a_i^L b_i^L|x_i(t)|-\sum_{j=1}^{m}a_i^M(a_{ij}^M+b_{ij}^M)\tilde{F}_j-a_i^M I_i^M\right]\\
&\doteq -\lambda\left[\zeta_{11}|x_i(t)|-\zeta_{12}\right], \qquad (6.1)
\end{aligned}$$

and

$$\begin{aligned} D^+y_j(t) &\leq -\lambda\left[c_j^L d_j^L|y_j(t)| - \sum_{i=1}^{n} c_j^M(c_{ji}^M + d_{ji}^M)\tilde{G}_i - c_j^M J_j^M\right] \\ &\doteq -\lambda\left[\zeta_{21}|x_i(t)| - \zeta_{22}\right], \end{aligned} \tag{6.2}$$

for all $i = 1, 2, \cdots, n$ and $j = 1, 2, \cdots, m$, where

$$\zeta_{11} = a_i^L b_i^L, \quad \zeta_{12} = \sum_{j=1}^{m} a_i^M(a_{ij}^M + b_{ij}^M)\tilde{F}_j + a_i^M I_i^M,$$

$$\zeta_{21} = c_j^L d_j^L, \quad \zeta_{22} = \sum_{i=1}^{n} c_j^M(c_{ji}^M + d_{ji}^M)\tilde{G}_i + c_j^M J_j^M.$$

Define $V(t) = V_1(t) + V_2(t)$, where

$$V_1(t) = \sum_{i=1}^{n} \mu_{1i}|x_i(t)|^2 + \sum_{i=1}^{n} \mu_{2i}|x_i(t)|,$$

and

$$V_2(t) = \sum_{j=1}^{m} \delta_{1j}|y_j(t)|^2 + \sum_{j=1}^{m} \delta_{2j}|y_j(t)|,$$

in which $\mu_{1i}, \mu_{2i}, \delta_{1j}, \delta_{2j} > 0$. Since $(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T$ is a periodic solution of system (3.3), then $V(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))$ is a periodic function.

One may further get

$$\begin{aligned} D^+V_1(t) &\leq -2\lambda\sum_{i=1}^{n} \mu_{1i}|x_i(t)|(\zeta_{11}|x_i(t)| - \zeta_{12}) - \lambda\sum_{i=1}^{n} \mu_{2i}(\zeta_{11}|x_i(t)| - \zeta_{12}) \\ &= \sum_{i=1}^{n}\left[-2\lambda\zeta_{11}\mu_{1i}|x_i(t)|^2 + (2\lambda\mu_{1i}\zeta_{12} - \lambda\mu_{2i}\zeta_{11})|x_i(t)| + \lambda\mu_{2i}\zeta_{12}\right], \end{aligned}$$

and

$$\begin{aligned} D^+V_2(t) &\leq -2\lambda\sum_{j=1}^{m} \delta_{1j}|y_j(t)|(\zeta_{21}|y_j(t)| - \zeta_{22}) - \lambda\sum_{j=1}^{m} \delta_{2j}(\zeta_{21}|y_j(t)| - \zeta_{22}) \\ &= \sum_{j=1}^{m}\left[-2\lambda\zeta_{21}\delta_{1j}|y_j(t)|^2 + (2\lambda\delta_{1j}\zeta_{22} - \lambda\delta_{2j}\zeta_{21})|y_j(t)| + \lambda\delta_{2j}\zeta_{22}\right]. \end{aligned}$$

Thus

$$\begin{aligned} D^+V(t) &\leq \sum_{i=1}^{n}\left[-2\lambda\zeta_{11}\mu_{1i}|x_i(t)|^2 + (2\lambda\mu_{1i}\zeta_{12} - \lambda\mu_{2i}\zeta_{11})|x_i(t)| + \lambda\mu_{2i}\zeta_{12}\right] \\ &+ \sum_{j=1}^{m}\left[-2\lambda\zeta_{21}\delta_{1j}|y_j(t)|^2 + (2\lambda\delta_{1j}\zeta_{22} - \lambda\delta_{2j}\zeta_{21})|y_j(t)| + \lambda\delta_{2j}\zeta_{22}\right]. \end{aligned}$$

From

$$-2\lambda\zeta_{11}\mu_{1i}|x_i(t)|^2 + (2\lambda\mu_{1i}\zeta_{12} - \lambda\mu_{2i}\zeta_{11})|x_i(t)| + \lambda\mu_{2i}\zeta_{12} = 0,$$

and

$$-2\lambda\zeta_{21}\delta_{1j}|y_j(t)|^2 + (2\lambda\delta_{1j}\zeta_{22} - \lambda\delta_{2j}\zeta_{21})|y_j(t)| + \lambda\delta_{2j}\zeta_{22} = 0,$$

there exist positive constants h_1, h_2 such that the solutions satisfy

$$\max_{t\in[0,\omega]} |x_i(t)| > h_1, \qquad \max_{t\in[0,\omega]} |y_j(t)| > h_2.$$

It follows that when $\|x_i(t)\| > h_1, \|y_j(t)\| > h_2$, we can choose a positive constant $\tilde{H} = nh_1 + mh_2$, such that $\|(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T\| > \tilde{H}$, and

$$-2\lambda\zeta_{11}\mu_{1i}|x_i(t)|^2 + (2\lambda\mu_{1i}\zeta_{12} - \lambda\mu_{2i}\zeta_{11})|x_i(t)| + \lambda\mu_{2i}\zeta_{12} < 0,$$

and

$$-2\lambda\zeta_{21}\delta_{1j}|y_j(t)|^2 + (2\lambda\delta_{1j}\zeta_{22} - \lambda\delta_{2j}\zeta_{21})|y_j(t)| + \lambda\delta_{2j}\zeta_{22} < 0.$$

Hence,

$$D^+V(t) < 0.$$

In fact, if $\|(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T\|$ is unbounded, then for any $\bar{H} > \tilde{H}$, we have

$$\|(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T\| > \bar{H} > \tilde{H}.$$

It follows that $D^+V(t) < 0$, which contradicts the fact that $V(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))$ is a periodic function. This implies that $\|(x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_m(t))^T\| \le \bar{H}_0$, where $\bar{H}_0$ is a positive constant. This completes the proof. □

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Conflict of interest

The authors confirm no conflicts of interest in this paper.

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