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*Research article*

# Nonnegative periodicity on high-order proportional delayed cellular neural networks involving $D$ operator

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**Abstract:** This paper aims to deal with the dynamic behaviors of nonnegative periodic solutions for one kind of high-order proportional delayed cellular neural networks involving $D$ operator. By utilizing Lyapunov functional approach, combined with some dynamic inequalities, we establish a new assertion to guarantee the existence and global exponential stability of nonnegative periodic solutions for the addressed networks. The obtained results supplement and improve some existing ones. In addition, the correctness of the analytical results are verified by numerical simulations.

**Keywords:** high-order cellular neural network; proportional delay; nonnegative periodic solution; global exponential stability; $D$ operator
**Mathematics Subject Classification:** 34C25, 34K13, 34K25

## 1. Introduction

As is known to all, time delay unavoidably exists in the process of signal transmission, which may lead to performance degradation, oscillation and even instability of the system [1–9]. Analyzing the dynamic behaviors of the system under the influence of time delay has become a fundamental problem [10, 11]. Especially, unlike bounded time-varying delay, distributed delay and constant delay, proportional delay is a class of monotonically increasing unbounded time-varying delay, which has the strengths of predictability and controllability [12]. Moreover, in many practical applications of neural networks dynamics, neutral delay with $D$ operator has more realistic significance than one based on non-operator [13–15]. As a result, the global exponential convergence of equilibrium points for the high-order proportional delayed cellular neural networks (HPDCNNs) involving $D$ operator:

$$\begin{aligned}&[x_i(t)-m_i(t)x_i(k_it)]'\\&=-b_i(t)x_i(t)+\sum_{j=1}^{n}\mu_{ij}(t)J_j(x_j(t))+\sum_{j=1}^{n}\nu_{ij}(t)F_j(x_j\left(d_{ij}t\right))\\&\quad+\sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t)R_j(x_j\left(h_{ijl}t\right))R_l(x_l\left(r_{ijl}t\right))+I_i(t),\quad t\geq t_0>0.\end{aligned}\tag{1.1}$$

was investigated in [16–18]. Here $n$ is the units number, $x_i(t)$ denotes the $i$th neuron state, $b_i(t)$ is the decay rate, $m_i(t)$, $\mu_{ij}(t)$, $\nu_{ij}(t)$ and $\theta_{ijl}(t)$ designate the connection weights, $J_j$, $F_j$ and $R_j$ represent the activation functions, the proportional delay factors $k_i, d_{ij}, h_{ijl}, r_{ijl} \in (0,1)$, for all $i, j, l \in Z = \{1, 2, \cdots, n\}$. The detailed biological description of input function $I_i(t)$ can be seen in [17, 18]. The initial condition of system (1.1) can be characterized via:

$$x_i(s)=\varphi_i(s),\ s\in[e_it_0,t_0],\varphi_i\in C([e_it_0,t_0],\mathbb{R}),e_i=\min_{l,j\in Z}\{d_{ij},h_{ijl},r_{ijl},k_i\},i\in Z.\tag{1.2}$$

A noticeable phenomenon is that the relevant state variables are often regarded as the light intensity level, proteins, molecules or electric charge in the process of establishing neural networks, which needs to ensure that they are nonnegative [19–21]. The systems mentioned above are often called as nonnegative systems. In recent years, more attention has been paid to the positivity and stability for the equilibrium points [3, 22–25], periodic solutions [26–28] and almost periodic solutions [29, 30] in many various neural networks systems. However, the aforementioned literature are all based on non-operator neural networks systems, and their methods for positivity cannot be used for neural networks systems involving $D$ operator directly. Besides, the proportional delay is monotonically increasing and obviously does not satisfy the periodicity, which will increase the difficulty of investigating periodic solutions for HPDCNNs. For all we know, there exists no reference on the existence and stability of the nonnegative periodic solution for HPDCNNs (1.1).

In view of the above considerations, we desire to establish a criterion on the existence and stability of the nonnegative periodic solution for HPDCNNs (1.1). The main approaches of this paper are Lyapunov functional methods, as well as employing some dynamic inequalities. It should be pointed out that the results obtained are novel and complement some existing ones in [16–18, 22–31].

The main framework of this paper is furnished as below. A criterion is proposed in Section 2 to insure that all global solutions are exponentially attractive to each other. The existence and global exponential stability for the nonnegative periodic solution are stated and guaranteed in Section 3. A numerical case is presented to prove the efficacy of our method in Section 4. We summarize this paper in Section 5.

## 2. Exponential attractivity of solutions

For the sake of convenience, we first describe some basic notations:

$$E_n=(1)_{n\times n},\ e=(e_1,\ e_2,\cdots,e_n)^T\in\mathbb{R}^n,\ \|e\|=\max_{i\in Z}|e_i|,$$

$$h^+=\sup_{t\in\mathbb{R}}|h(t)|,\ h^-=\inf_{t\in\mathbb{R}}|h(t)|,$$

where $E_n$ designates the identity matrix of order $n$, $h$ is a bounded and continuous function defined on $\mathbb{R}$. Furthermore, let $\Gamma$ and $\bar{\Gamma}$ be two matrices or vectors, $\Gamma \geq \mathbf{0}$ is denoted that each item of $\Gamma$ is greater than or equal to zero, the definition of $\Gamma > \mathbf{0}$ is similar. And $\Gamma \geq \bar{\Gamma}$ (resp. $\Gamma > \bar{\Gamma}$) means that $\Gamma - \bar{\Gamma} \geq \mathbf{0}$ (resp. $\Gamma - \bar{\Gamma} > \mathbf{0}$).

**Lemma 2.1.** (see [3]). If $B \geq 0$ is an $n \times n$ matrix and the spectral radius $\rho(B) < 1$, then $E_n - B$ is an $M$-matrix, and $(E_n - B)^{-1} \geq \mathbf{0}$.

Throughout this paper, we assume that $b_i$, $m_i$, $\mu_{ij}$, $\nu_{ij}$, $\theta_{ijl}$, $I_i : \mathbb{R} \to [0, +\infty)$ are continuous $T$-periodic functions ($T > 0$) with respect to time $t$ and the following assumptions are true for $i, j, l \in Z$.

$(S_1)$ $J_j, F_j, R_j : \mathbb{R} \to \mathbb{R}$ are non-decreasing functions. Moreover, there are constants $L_j^J, L_j^F, L_j^R, Q_j^R \in [0, +\infty)$ such that

$$J_j(0) = F_j(0) = R_j(0) = 0, |J_j(a) - J_j(b)| \leq L_j^J |a - b|, |F_j(a) - F_j(b)| \leq L_j^F |a - b|,$$

and

$$|R_j(a) - R_j(b)| \leq L_j^R |a - b|, \ \ |R_j(a)| \leq Q_j^R, \ \text{ for all } \ a, b \in \mathbb{R}.$$

$(S_2)$ $\Lambda_i < 0$, $m_i^+ e^{\ln \frac{1}{k_i}} < 1$, $\rho(V) < 1$, $I_i^- > m_i^+ b_i^+ \chi_i$, $N_i + m_i^+ < 1$, $i \in Z$, where

$$\begin{cases} \Lambda_i = \sup\limits_{t \in \mathbb{R}} \Big[ 1 - b_i(t) + \frac{b_i(t) m_i(t)}{1 - m_i^+ e^{\ln \frac{1}{k_i}}} e^{\ln \frac{1}{k_i}} + \sum\limits_{j=1}^{n} \frac{L_j^J \mu_{ij}(t)}{1 - m_j^+ e^{\ln \frac{1}{k_j}}} + \sum\limits_{j=1}^{n} \frac{L_j^F \nu_{ij}(t)}{1 - m_j^+ e^{\ln \frac{1}{k_j}}} e^{\ln \frac{1}{d_{ij}}} \\ \qquad + \sum\limits_{j=1}^{n} \sum\limits_{l=1}^{n} \theta_{ijl}(t) \big( \frac{Q_l^R L_j^R}{1 - m_j^+ e^{\ln \frac{1}{k_j}}} e^{\ln \frac{1}{h_{ijl}}} + \frac{Q_j^R L_l^R}{1 - m_l^+ e^{\ln \frac{1}{k_l}}} e^{\ln \frac{1}{r_{ijl}}} \big) \Big], \\ V = (v_{ij})_{n \times n} = \Big( \frac{1}{b_i^- (1 - m_i^+)} \big[ \mu_{ij}^+ L_j^J + \nu_{ij}^+ L_j^F + \sum\limits_{l=1}^{n} \theta_{ijl}^+ L_j^R Q_l^R \big] \Big)_{n \times n}, \\ \beta = \{\beta_i\} = \Big\{ \frac{I_i^+}{b_i^- (1 - m_i^+)} \Big\}, \mathscr{X} = \{\chi_i\} = (E_n - V)^{-1} \beta, \\ N_i = \sup\limits_{t \in \mathbb{R}} \frac{1}{b_i(t)} \Big[ b_i(t) m_i(t) + \sum\limits_{j=1}^{n} L_j^J \mu_{ij}(t) + \sum\limits_{j=1}^{n} L_j^F \nu_{ij}(t) \\ \qquad + \sum\limits_{j=1}^{n} \sum\limits_{l=1}^{n} \theta_{ijl}(t) (Q_l^R L_j^R + Q_j^R L_l^R) \Big]. \end{cases}$$

**Remark 2.1.** From $(S_2)$ and Lemma 2.1, it is easy to see that $(E_n - V)$ is an $M$-matrix, $(E_n - V)^{-1} \geq \mathbf{0}$, and then there is a positive vector $\xi^*$ such that $\xi = (E_n - V)\xi^* > \mathbf{0}$.

**Lemma 2.2. (see [17], Lemma 2.1).** Under the above assumptions, HPDCNNs (1.1) involving initial value (1.2) has unique solution $x(t)$ on $[t_0, +\infty)$.

**Lemma 2.3.** Suppose $(S_1)$ and $(S_2)$ be satisfied, and let

$$\gamma(t) = (\gamma_1(t), \gamma_2(t), \cdots, \gamma_n(t))^T \text{and } \zeta(t) = (\zeta_1(t), \zeta_2(t), \cdots, \zeta_n(t))^T$$

be two arbitrary solutions of HPDCNNs (1.1) obeying the following initial conditions

$$\gamma_i(s) = \varphi_i^\gamma(s), \ \zeta_i(s) = \varphi_i^\zeta(s), \ s \in [e_i t_0, t_0], \ \varphi_i^\gamma, \varphi_i^\zeta \in C([e_i t_0, t_0], \mathbb{R}), \ i \in Z, \tag{2.1}$$

then there are two positive constants $P = P(\varphi^\gamma, \varphi^\zeta)$ and $\kappa > 1$ satisfying that

$$|\gamma_i(t) - \zeta_i(t)| \leq P \left( \frac{1 + t_0}{1 + t} \right)^\kappa, \ \text{ for all } \ t \geq t_0, \ i \in Z.$$

**Proof**. Let $\mathscr{H}_i(s) = m_i^+ e^{s\ln\frac{1}{k_i}}$ and

$$
\begin{aligned}
&\mathscr{G}_i(s)\\
&= \sup_{t\in\mathbb{R}}\Big[s - b_i(t) + \frac{b_i(t)m_i(t)}{1-m_i^+e^{s\ln\frac{1}{k_i}}}e^{s\ln\frac{1}{k_i}} + \sum_{j=1}^{n}\frac{L_j^J\mu_{ij}(t)}{1-m_j^+e^{s\ln\frac{1}{k_j}}} + \sum_{j=1}^{n}\frac{L_j^F\nu_{ij}(t)}{1-m_j^+e^{s\ln\frac{1}{k_j}}}\\
&\quad\times e^{s\ln\frac{1}{d_{ij}}} + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t)\Big(\frac{Q_l^RL_j^R}{1-m_j^+e^{s\ln\frac{1}{k_j}}}e^{s\ln\frac{1}{h_{ijl}}} + \frac{Q_j^RL_l^R}{1-m_l^+e^{s\ln\frac{1}{k_l}}}e^{s\ln\frac{1}{r_{ijl}}}\Big)\Big].
\end{aligned}
\tag{2.2}
$$

From $(S_2)$, we gain

$$
\mathscr{H}_i(1) < 1 \text{ and } \mathscr{G}_i(1) < 0,\ i \in Z, \tag{2.3}
$$

which, together with the continuities of $\mathscr{H}_i(s)$ and $\mathscr{G}_i(s)$, results that there is a positive constant $\kappa \in (1, \min_{i\in Z} b_i^-)$ such that

$$
m_i^+e^{\kappa\ln\frac{1}{k_i}} < 1, \quad i \in Z, \tag{2.4}
$$

and

$$
\begin{aligned}
&\sup_{t\in\mathbb{R}}\Big[\kappa - b_i(t) + \frac{b_i(t)m_i(t)}{1-m_i^+e^{\kappa\ln\frac{1}{k_i}}}e^{\kappa\ln\frac{1}{k_i}} + \sum_{j=1}^{n}\frac{L_j^J\mu_{ij}(t)}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}} + \sum_{j=1}^{n}\frac{L_j^F\nu_{ij}(t)}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}}e^{\kappa\ln\frac{1}{d_{ij}}}\\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t)\Big(\frac{Q_l^RL_j^R}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}}e^{\kappa\ln\frac{1}{h_{ijl}}} + \frac{Q_j^RL_l^R}{1-m_l^+e^{\kappa\ln\frac{1}{k_l}}}e^{\kappa\ln\frac{1}{r_{ijl}}}\Big)\Big] < 0,\ \ i \in Z.
\end{aligned}
\tag{2.5}
$$

According to (2.5) and the fact that

$$
\frac{\kappa}{1+t} \le \kappa,\ \ln\Big(\frac{1+t}{1+\alpha t}\Big) \le \ln\frac{1}{\alpha},\ \text{for all}\ \ t \ge 0,\ 0 < \alpha < 1,
$$

we obtain

$$
\begin{aligned}
&\sup_{t\in\mathbb{R}}\Big[\frac{\kappa}{1+t} - b_i(t) + \frac{b_i(t)m_i(t)}{1-m_i^+e^{\kappa\ln\frac{1}{k_i}}}e^{\kappa\ln\frac{1+t}{1+k_it}}\\
&\quad + \sum_{j=1}^{n}\frac{L_j^J\mu_{ij}(t)}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}} + \sum_{j=1}^{n}\frac{L_j^F\nu_{ij}(t)}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}}e^{\kappa\ln\frac{1+t}{1+d_{ij}t}} + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t)\\
&\quad\times\Big(\frac{Q_l^RL_j^R}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}}e^{\kappa\ln\frac{1+t}{1+h_{ijl}t}} + \frac{Q_j^RL_l^R}{1-m_l^+e^{\kappa\ln\frac{1}{k_l}}}e^{\kappa\ln\frac{1+t}{1+r_{ijl}t}}\Big)\Big]\\
\le\ &\sup_{t\in\mathbb{R}}\Big[\kappa - b_i(t) + \frac{b_i(t)m_i(t)}{1-m_i^+e^{\kappa\ln\frac{1}{k_i}}}e^{\kappa\ln\frac{1}{k_i}} + \sum_{j=1}^{n}\frac{L_j^J\mu_{ij}(t)}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}}\\
&\quad + \sum_{j=1}^{n}\frac{L_j^F\nu_{ij}(t)}{1-m_j^+e^{\kappa\ln\frac{1}{k_j}}}e^{\kappa\ln\frac{1}{d_{ij}}} + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t)
\end{aligned}
$$

$$\times\Big(\frac{Q_l^R L_j^R}{1-m_j^+ e^{\kappa \ln \frac{1}{k_j}}} e^{\kappa \ln \frac{1}{h_{ijl}}} + \frac{Q_j^R L_l^R}{1-m_l^+ e^{\kappa \ln \frac{1}{k_l}}} e^{\kappa \ln \frac{1}{r_{ijl}}}\Big)\Big] < 0,\ i \in Z. \tag{2.6}$$

Set

$$\gamma_i(t) = \varphi_i^\gamma(t) = \varphi_i^\gamma(e_i t_0), \quad \zeta_i(t) = \varphi_i^\zeta(t) = \varphi_i^\zeta(e_i t_0), \text{ for all } t \in [k_i e_i t_0, e_i t_0], \tag{2.7}$$

and

$$\varrho_i(t) = \gamma_i(t) - \zeta_i(t), \quad X_i(t) = \varrho_i(t) - m_i(t)\varrho_i(k_i t),\ i \in Z, \tag{2.8}$$

it follows from (1.1) that

$$\begin{aligned} X_i'(t) &= -b_i(t)X_i(t) - b_i(t)m_i(t)\varrho_i(k_i t) + \sum_{j=1}^n \mu_{ij}(t)(J_j(\gamma_j(t)) - J_j(\zeta_j(t))) \\ &\quad + \sum_{j=1}^n \nu_{ij}(t)(F_j(\gamma_j(d_{ij}t)) - F_j(\zeta_j(d_{ij}t))) + \sum_{j=1}^n \sum_{l=1}^n \theta_{ijl}(t) \\ &\quad \times [R_j(\gamma_j(h_{ijl}t))R_l(\gamma_l(r_{ijl}t)) - R_j(\zeta_j(h_{ijl}t))R_l(\zeta_l(r_{ijl}t))],\ \ i \in Z. \end{aligned} \tag{2.9}$$

Label

$$\|\varphi\|_0 = \max_{i\in Z} \sup_{t\in[e_i t_0, t_0]} \Big|(\varphi_i^\gamma(t) - m_i(t)\varphi_i^\gamma(k_i t)) - (\varphi_i^\zeta(t) - m_i(t)\varphi_i^\zeta(k_i t))\Big|, \tag{2.10}$$

and suppose $\|\varphi\|_0 > 0$, then, for any $\varepsilon > 0$, one can choose a constant $M > n+1$ such that

$$\|X(t)\| < (\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+t}{1+t_0}} < M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+t}{1+t_0}}, \ \text{ for all } t \in [e_i t_0,\ t_0],\ i \in Z. \tag{2.11}$$

Now, we will reveal that

$$\|X(t)\| < M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+t}{1+t_0}}, \ \text{ for all } t > t_0. \tag{2.12}$$

Otherwise, there must exist $i \in Z$ and $\theta > t_0$ satisfying that

$$\begin{cases} |X_i(\theta)| = M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+\theta}{1+t_0}}, \\ \|X(t)\| < M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+t}{1+t_0}}, \ \text{ for all } t \in [e_i t_0,\ \theta), \end{cases} \tag{2.13}$$

which, together with (2.7), implies that

$$\begin{aligned} e^{\kappa \ln \frac{1+\nu}{1+t_0}}|\varrho_j(\nu)| &\le e^{\kappa \ln \frac{1+\nu}{1+t_0}}|\varrho_j(\nu) - m_j(\nu)\varrho_j(k_j\nu)| + e^{\kappa \ln \frac{1+\nu}{1+t_0}}|m_j(\nu)\varrho_j(k_j\nu)| \\ &\le e^{\kappa \ln \frac{1+\nu}{1+t_0}}|X_j(\nu)| + m_j^+ e^{\kappa \ln \frac{1+\nu}{1+k_j\nu}} e^{\kappa \ln \frac{1+k_j\nu}{1+t_0}}|\varrho_j(k_j\nu)| \\ &\le M(\|\varphi\|_0 + \varepsilon) + m_j^+ e^{\kappa \ln \frac{1}{k_j}} \sup_{s\in[k_j e_j t_0,\ k_j t]} e^{\kappa \ln \frac{1+s}{1+t_0}}|\varrho_j(s)| \\ &\le M(\|\varphi\|_0 + \varepsilon) + m_j^+ e^{\kappa \ln \frac{1}{k_j}} \sup_{s\in[e_j t_0,\ t]} e^{\kappa \ln \frac{1+s}{1+t_0}}|\varrho_j(s)|, \end{aligned} \tag{2.14}$$

for all $\nu \in [e_j t_0,\ t],\ t \in [t_0,\ \theta),\ j \in Z$, and then

$$e^{\kappa \ln \frac{1+t}{1+t_0}}|\varrho_j(t)| \le \sup_{s\in[e_j t_0,\ t]} e^{\kappa \ln \frac{1+s}{1+t_0}}|\varrho_j(s)| \le \frac{M(\|\varphi\|_0 + \varepsilon)}{1 - m_j^+ e^{\kappa \ln \frac{1}{k_j}}}, \tag{2.15}$$

for all $t \in [e_j t_0,\ \theta),\ j \in Z$.

In views of (2.9), we get

$$\begin{aligned}
X_i(t) &= X_i(t_0)e^{-\int_{t_0}^{t} b_i(u)\mathrm{d}u} + \int_{t_0}^{t} e^{-\int_s^t b_i(u)\mathrm{d}u}\Big( - b_i(s)m_i(s)\varrho_i(k_i s) \\
&\quad + \sum_{j=1}^{n} \mu_{ij}(s)\Big(J_j(\gamma_j(s)) - J_j(\zeta_j(s))\Big) + \sum_{j=1}^{n} \nu_{ij}(s)\Big(F_j(\gamma_j(d_{ij}s)) - F_j(\zeta_j(d_{ij}s))\Big) \\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} \theta_{ijl}(s)\Big[R_j(\gamma_j(h_{ijl}s))R_l(\gamma_l(r_{ijl}s)) - R_j(\zeta_j(h_{ijl}s))R_l(\gamma_l(r_{ijl}s)) \\
&\quad + R_j(\zeta_j(h_{ijl}s))R_l(\gamma_l(r_{ijl}s)) - R_j(\zeta_j(h_{ijl}s))R_l(\zeta_l(r_{ijl}s))\Big]\Big)\mathrm{d}s,\ t \in [t_0, \theta],
\end{aligned}$$

which follows from (2.6), (2.13) and (2.15) that

$$\begin{aligned}
|X_i(\theta)| &= \Big| X_i(t_0)e^{-\int_{t_0}^{\theta} b_i(u)\mathrm{d}u} + \int_{t_0}^{\theta} e^{-\int_s^{\theta} b_i(u)\mathrm{d}u}\Big( - b_i(s)m_i(s)\varrho_i(k_i s) \\
&\quad + \sum_{j=1}^{n} \mu_{ij}(s)\Big(J_j(\gamma_j(s)) - J_j(\zeta_j(s))\Big) + \sum_{j=1}^{n} \nu_{ij}(s)\Big(F_j(\gamma_j(d_{ij}s)) - F_j(\zeta_j(d_{ij}s))\Big) \\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} \theta_{ijl}(s)\Big[R_j(\gamma_j(h_{ijl}s))R_l(\gamma_l(r_{ijl}s)) - R_j(\zeta_j(h_{ijl}s))R_l(\gamma_l(r_{ijl}s)) \\
&\quad + R_j(\zeta_j(h_{ijl}s))R_l(\gamma_l(r_{ijl}s)) - R_j(\zeta_j(h_{ijl}s))R_l(\zeta_l(r_{ijl}s))\Big]\Big)\mathrm{d}s\Big| \\
&\le (\|\varphi\|_0 + \varepsilon)e^{-\int_{t_0}^{\theta} b_i(u)\mathrm{d}u} + \int_{t_0}^{\theta} e^{-\int_s^{\theta} b_i(u)\mathrm{d}u}\Big[| - b_i(s)m_i(s)\varrho_i(k_i s)| \\
&\quad + \sum_{j=1}^{n} L_j^J \mu_{ij}(s)|\varrho_j(s)| + \sum_{j=1}^{n} L_j^F \nu_{ij}(s)|\varrho_j(d_{ij}s)| \\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} \theta_{ijl}(s)(Q_l^R L_j^R|\varrho_j(h_{ijl}s)| + Q_j^R L_l^R|\varrho_l(r_{ijl}s)|)\Big]\mathrm{d}s \\
&\le (\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+\theta}{1+t_0}} e^{-\int_{t_0}^{\theta}(b_i(u) - \frac{\kappa}{1+u})\mathrm{d}u} + M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+\theta}{1+t_0}} \\
&\quad \times \int_{t_0}^{\theta} e^{-\int_s^{\theta}(b_i(u) - \frac{\kappa}{1+u})\mathrm{d}u}\Big[\frac{b_i(s)m_i(s)}{1 - m_i^+ e^{\kappa \ln \frac{1}{k_i}}} e^{\kappa \ln \frac{1+s}{1+k_i s}} \\
&\quad + \sum_{j=1}^{n} \frac{L_j^J \mu_{ij}(s)}{1 - m_j^+ e^{\kappa \ln \frac{1}{k_j}}} + \sum_{j=1}^{n} \frac{L_j^F \nu_{ij}(s)}{1 - m_j^+ e^{\kappa \ln \frac{1}{k_j}}} e^{\kappa \ln \frac{1+s}{1+d_{ij}s}} + \sum_{j=1}^{n}\sum_{l=1}^{n} \theta_{ijl}(s) \\
&\quad \times \Big(\frac{Q_l^R L_j^R}{1 - m_j^+ e^{\kappa \ln \frac{1}{k_j}}} e^{\kappa \ln \frac{1+s}{1+h_{ijl}s}} + \frac{Q_j^R L_l^R}{1 - m_l^+ e^{\kappa \ln \frac{1}{k_l}}} e^{\kappa \ln \frac{1+s}{1+r_{ijl}s}}\Big)\Big]\mathrm{d}s \\
&< M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+\theta}{1+t_0}}\Big[1 - (1 - \frac{1}{M})e^{-\int_{t_0}^{\theta}(b_i(u) - \frac{\kappa}{1+u})\mathrm{d}u}\Big] \\
&< M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+\theta}{1+t_0}}.
\end{aligned}$$

This contradicts with the fact of $|X_i(\theta)| = M(\|\varphi\|_0 + \varepsilon)e^{-\kappa \ln \frac{1+\theta}{1+t_0}}$. Hence, (2.12) holds. Applying a similar proof to (2.15), from (2.12), we gain

$$|\varrho_j(t)| \leq \sup_{s\in[e_j t_0,\ t]} |\varrho_j(s)| \leq \frac{M(\|\varphi\|_0 + \varepsilon)}{1 - m_j^+ e^{\kappa \ln \frac{1}{k_j}}} e^{-\kappa \ln \frac{1+t}{1+t_0}}, \ \text{ for all } \ t > t_0, j \in Z. \tag{2.16}$$

Let $\varepsilon \to 0^+$, then

$$|\varrho_i(t)| = |\gamma_i(t) - \zeta_i(t)| \leq P\left(\frac{1+t_0}{1+t}\right)^{\kappa}, \ \text{ for all } \ t \geq t_0, i \in Z,$$

where $P = \frac{M\|\varphi\|_0}{1-m_i^+ e^{\kappa \ln \frac{1}{k_i}}}$. The proof is completed.

**Remark 2.2.** According to Lemma 2.3, if $\omega(t)$ is an equilibrium point or a periodic solution for HPDCNNs (1.1), all solutions of HPDCNNs (1.1) will exponentially converge to $\omega(t)$, which indicates that $\omega(t)$ is globally generalized exponentially stable.

## 3. The existence and stability on the nonnegative periodic solution

Based on the above preparations, we now reveal the existence and global exponential stability of the nonnegative periodic solutions for HPDCNNs (1.1).

**Theorem 3.1.** If the assumptions in Section 2 hold, then HPDCNNs (1.1) has a globally exponentially stable nonnegative periodic solution.

**Proof**. Set $\vartheta_i(t) = x_i(t) - m_i(t)x_i(k_i t), i \in Z$, it follows from (1.1) that

$$\begin{aligned}\vartheta_i'(t) =& \big[x_i(t) - m_i(t)x_i(k_i t)\big]' \\ =& - b_i(t)\vartheta_i(t) - b_i(t)m_i(t)x_i(k_i t) + \sum_{j=1}^{n} \mu_{ij}(t)J_j(x_j(t)) + \sum_{j=1}^{n} \nu_{ij}(t) \\ & \times F_j(x_j(d_{ij}t)) + \sum_{j=1}^{n}\sum_{l=1}^{n} \theta_{ijl}(t)R_j(x_j(h_{ijl}t))R_l(x_l(r_{ijl}t)) + I_i(t), \ i \in Z.\end{aligned} \tag{3.1}$$

We define

$$\begin{aligned}\vartheta_i^{\varphi}(t) =& \int_{-\infty}^{t} e^{-\int_s^t b_i(u)\mathrm{d}u}\Big[ - b_i(s)m_i(s)\varphi_i(k_i s) + \sum_{j=1}^{n} \mu_{ij}(s)J_j(\varphi_j(s)) \\ & + \sum_{j=1}^{n} \nu_{ij}(s)F_j(\varphi_j(d_{ij}s)) + \sum_{j=1}^{n}\sum_{l=1}^{n} \theta_{ijl}(s) \\ & \times R_j(\varphi_j(h_{ijl}s))R_l(\varphi_l(r_{ijl}s)) + I_i(s)\Big]\mathrm{d}s,\end{aligned} \tag{3.2}$$

and the nonlinear operator $\mathcal{P}$ by setting

$$(\mathcal{P}\varphi)_i(t) = m_i(t)\varphi_i(k_i t) + \vartheta_i^{\varphi}(t), \ \ t \in R.$$

Take a large enough number $\rho > 0$ such that $\beta > \frac{1}{\rho}\xi$, one has

$$\mathscr{X} = (E_n - V)^{-1}\beta > \frac{1}{\rho}(E_n - V)^{-1}\xi > \mathbf{0}.$$

Let $B^* = \Big\{\varphi(t) = (\varphi_1(t), \varphi_2(t), \cdots, \varphi_n(t))^T \in C(\mathbb{R}, \mathbb{R}^n) : 0 \leq \varphi_i(t) \leq \chi_i, \forall t \in \mathbb{R},\ i \in Z\Big\}$, then $(B^*, \|\cdot\|_\infty)$ is a Banach space, where $\|\varphi\|_\infty = \max\limits_{i\in Z} \sup\limits_{t\in\mathbb{R}} |\varphi_i(t)|$. From $(S_1)$ and $(S_2)$, we gain

$$\begin{aligned}
(\mathcal{P}\varphi)_i(t) &= m_i(t)\varphi_i(k_it) + \vartheta_i^\varphi(t) \\
&= m_i(t)\varphi_i(k_it) + \int_{-\infty}^t e^{-\int_s^t b_i(u)\mathrm{d}u}\Big[ - b_i(s)m_i(s)\varphi_i(k_is) \\
&\quad + \sum_{j=1}^n \mu_{ij}(s)J_j(\varphi_j(s)) + \sum_{j=1}^n \nu_{ij}(s)F_j(\varphi_j(d_{ij}s)) \\
&\quad + \sum_{j=1}^n \sum_{l=1}^n \theta_{ijl}(s)R_j(\varphi_j(h_{ijl}s))R_l(\varphi_l(r_{ijl}s)) + I_i(s)\Big]\mathrm{d}s \\
&\leq m_i^+\chi_i + \int_{-\infty}^t e^{-\int_s^t b_i^-\mathrm{d}u}\Big[ \sum_{j=1}^n \mu_{ij}^+L_j^J\chi_j + \sum_{j=1}^n \nu_{ij}^+L_j^F\chi_j \\
&\quad + \sum_{j=1}^n \sum_{l=1}^n \theta_{ijl}^+L_j^RQ_l^R\chi_j + I_i^+\Big]\mathrm{d}s \\
&\leq m_i^+\chi_i + (1-m_i^+)\sum_{j=1}^n \nu_{ij}\chi_j + (1-m_i^+)\beta_i \\
&= \chi_i, \quad \text{for all } \ t \in \mathbb{R},\ i \in Z,
\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}
(\mathcal{P}\varphi)_i(t) &= m_i(t)\varphi_i(k_it) + \vartheta_i^\varphi(t) \\
&= m_i(t)\varphi_i(k_it) + \int_{-\infty}^t e^{-\int_s^t b_i(u)\mathrm{d}u}\Big[ - b_i(s)m_i(s)\varphi_i(k_is) \\
&\quad + \sum_{j=1}^n \mu_{ij}(s)J_j(\varphi_j(s)) + \sum_{j=1}^n \nu_{ij}(s)F_j(\varphi_j(d_{ij}s)) \\
&\quad + \sum_{j=1}^n \sum_{l=1}^n \theta_{ijl}(s)R_j(\varphi_j(h_{ijl}s))R_l(\varphi_l(r_{ijl}s)) + I_i(s)\Big]\mathrm{d}s \\
&\geq m_i^-\varphi_i(k_it) + \int_{-\infty}^t e^{-\int_s^t b_i(u)\mathrm{d}u}\Big[ - b_i(s)m_i(s)\varphi_i(k_is) \\
&\quad + \sum_{j=1}^n \mu_{ij}^-J_j(\varphi_j(s)) + \sum_{j=1}^n \nu_{ij}^-F_j(\varphi_j(d_{ij}s)) \\
&\quad + \sum_{j=1}^n \sum_{l=1}^n \theta_{ijl}^-R_j(\varphi_j(h_{ijl}s))R_l(\varphi_l(r_{ijl}s)) + I_i^-\Big]\mathrm{d}s \\
&\geq \int_{-\infty}^t e^{-\int_s^t b_i(u)\mathrm{d}u}[ - b_i(s)m_i(s)\varphi_i(k_is) + I_i^-]\mathrm{d}s \\
&\geq \int_{-\infty}^t e^{-\int_s^t b_i(u)\mathrm{d}u}[ - m_i^+\chi_ib_i(s)]\mathrm{d}s + \int_{-\infty}^t [e^{-\int_s^t b_i^+\mathrm{d}u}I_i^-]\mathrm{d}s
\end{aligned}$$

$$
\begin{aligned}
&\geq \quad -m_i^+\chi_i + \frac{I_i^-}{b_i^+} \\
&\geq \quad 0, \quad \text{for all } \ t \in \mathbb{R}, \ i \in Z,
\end{aligned} \tag{3.4}
$$

which entail that $\mathcal{P}$ is a continuous mapping from $B^*$ to $B^*$. Now, we prove $\mathcal{P}$ is a contraction mapping of $B^*$. With the help of $(S_1)$ and $(S_2)$, we obtain

$$
\begin{aligned}
& \left|(\mathcal{P}\varphi)_i(t) - (\mathcal{P}\psi)_i(t)\right| \\
= \ & \left|m_i(t)\varphi_i(k_i t) + \vartheta_i^{\varphi}(t) - \left(m_i(t)\psi_i(k_i t) + \vartheta_i^{\psi}(t)\right)\right| \\
\leq \ & \left|m_i(t)(\varphi_i(k_i t) - \psi_i(k_i t))\right| + \left|\vartheta_i^{\varphi}(t) - \vartheta_i^{\psi}(t)\right| \\
\leq \ & \left|m_i^+(\varphi_i(k_i t) - \psi_i(k_i t))\right| + \int_{-\infty}^{t} e^{-\int_s^t b_i(u)\mathrm{d}u}\Big[\left| - b_i(s)m_i(s)(\varphi_i(k_i s) - \psi_i(k_i s))\right| \\
& + \sum_{j=1}^{n}\mu_{ij}(s)\left|J_j(\varphi_j(s)) - J_j(\psi_j(s))\right| + \sum_{j=1}^{n}\nu_{ij}(s)\Big|F_j(\varphi_j(d_{ij}s)) \\
& - F_j(\psi_j(d_{ij}s))\Big| + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(s)\Big|R_j(\varphi_j(h_{ijl}s))R_l(\varphi_l(r_{ijl}s)) \\
& - R_j(\psi_j(h_{ijl}s))R_l(\varphi_l(r_{ijl}s)) + R_j(\psi_j(h_{ijl}s))R_l(\varphi_l(r_{ijl}s)) \\
& - R_j(\psi_j(h_{ijl}s))R_l(\psi_l(r_{ijl}s))\Big|\Big]\mathrm{d}s \\
\leq \ & \Big(m_i^+ + \int_{-\infty}^{t} e^{-\int_s^t b_i(u)\mathrm{d}u}\Big[b_i(s)m_i(s) + \sum_{j=1}^{n}L_j^J\mu_{ij}(s) \\
& + \sum_{j=1}^{n}L_j^F\nu_{ij}(s) + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(s)(Q_l^R L_j^R + Q_j^R L_l^R)\Big]\mathrm{d}s\Big)\|\varphi - \psi\|_\infty \\
\leq \ & (m_i^+ + N_i)\|\varphi - \psi\|_\infty, \quad \text{for all } \ t \in \mathbb{R}, \ \varphi, \psi \in B^*, \ i \in Z,
\end{aligned}
$$

and then

$$
\|(\mathcal{P}\varphi) - (\mathcal{P}\psi)\|_\infty \leq (m_i^+ + N_i)\|\varphi - \psi\|_\infty, \ \ m_i^+ + N_i < 1, \tag{3.5}
$$

which follows from (3.3) and (3.4) that $\mathcal{P}$ is a contraction mapping from $B^*$ to $B^*$. Consequently, the mapping $\mathcal{P}$ exists unique fixed point $x^*(t) = (\mathcal{P}x^*)(t) \in B^*$ satisfying that

$$
\begin{aligned}
x_i^*(t) &= m_i(t)x_i^*(k_i t) + \vartheta_i^{x^*}(t) \\
&= m_i(t)x_i^*(k_i t) + \int_{-\infty}^{t} e^{-\int_s^t b_i(u)\mathrm{d}u}\Big[ - b_i(s)m_i(s)x_i^*(k_i s) \\
&+ \sum_{j=1}^{n}\mu_{ij}(s)J_j(x_j^*(s)) + \sum_{j=1}^{n}\nu_{ij}(s)F_j(x_j^*(d_{ij}s)) \\
&+ \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(s)R_j(x_j^*(h_{ijl}s))R_l(x_l^*(r_{ijl}s)) + I_i(s)\Big]\mathrm{d}s,
\end{aligned} \tag{3.6}
$$

and

$$
[x_i^*(t) - m_i(t)x_i^*(k_i t)]'
$$

$$
\begin{aligned}
&= -b_i(t)\vartheta_i^{x^*}(t) - b_i(t)m_i(t)x_i^*(k_it) + \sum_{j=1}^{n}\mu_{ij}(t)J_j\left(x_j^*(t)\right) \\
&\quad + \sum_{j=1}^{n}\nu_{ij}(t)F_j\left(x_j^*\left(d_{ij}t\right)\right) + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t)R_j\left(x_j^*\left(h_{ijl}t\right)\right)R_l(x_l^*\left(r_{ijl}t\right)) + I_i(t) \\
&= -b_i(t)x_i^*(t) + \sum_{j=1}^{n}\mu_{ij}(t)J_j\left(x_j^*(t)\right) + \sum_{j=1}^{n}\nu_{ij}(t)F_j\left(x_j^*\left(d_{ij}t\right)\right) + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t) \\
&\quad \times R_j\left(x_j^*\left(h_{ijl}t\right)\right)R_l(x_l^*\left(r_{ijl}t\right)) + I_i(t), \quad t \geq t_0 > 0,\ i \in Z, \qquad (3.7)
\end{aligned}
$$

which entails that $x^*(t)$ is a nonnegative solution of HPDCNNs (1.1). For any natural number $m$, we get

$$
\begin{aligned}
&\left[x_i^*(t+mT) - m_i(t)x_i^*(k_i \times (t+mT))\right]' \\
&= -b_i(t)x_i^*(t+mT) + \sum_{j=1}^{n}\mu_{ij}(t)J_j\left(x_j^*(t+mT)\right) \\
&\quad + \sum_{j=1}^{n}\nu_{ij}(t)F_j\left(x_j^*(d_{ij} \times (t+mT))\right) + \sum_{j=1}^{n}\sum_{l=1}^{n}\theta_{ijl}(t) \\
&\quad \times R_j\left(x_j^*(h_{ijl} \times (t+mT))\right)R_l\left(x_l^*(r_{ijl} \times (t+mT))\right) + I_i(t),
\end{aligned}
\qquad (3.8)
$$

thus, $x^*(t+mT) \in B^*$ is a solution of HPDCNNs (1.1). Particularly, $v(t) = x^*(t+T)$ is a solution of HPDCNNs (1.1) obeying the initial condition

$$
v_i(t) = \varphi_i^v(t), t \in [e_it_0, t_0], \varphi_i^v \in C([e_it_0, t_0], \mathbb{R}), i \in Z. \qquad (3.9)
$$

According to Lemma 2.3, there is a positive constant $P = P(\varphi^{x^*}, \varphi^v)$ satisfying that

$$
|x_i^*(t) - v_i(t)| \leq P\left(\frac{1+t_0}{1+t}\right)^{\kappa}, \text{ for all } t \geq t_0, i \in Z.
$$

Then, for all $i \in Z$ and any $t + lT \geq 0$,

$$
\left|x_i^*(t+lT) - x_i^*(t+(l+1)T)\right| = \left|x_i^*(t+lT) - v_i(t+lT)\right| \leq P\left(\frac{1+t_0}{1+t+lT}\right)^{\kappa},
$$

which follows from

$$
x_i^*(t+mT) = x_i^*(t) + \sum_{l=0}^{m-1}\left[x_i^*(t+(l+1)T) - x_i^*(t+lT)\right], \ i \in Z,
$$

and $\kappa > 1$ that $\{x^*(t+mT) \in B^*\}_{m\geq 1}$ uniformly converges to a continuous function $\omega(t) \in B^*$ on any compact set of $\mathbb{R}$. Furthermore, it is easy to obtain

$$
\omega(t+T) = \lim_{m\to+\infty} x^*(t+T+mT) = \lim_{(m+1)\to+\infty} x^*(t+(m+1)T) = \omega(t),
$$

which suggests that $\omega(t)$ is $T$-periodic. Letting $m \to +\infty$ in (3.8) yields

$$\begin{aligned}
&\left[\omega_i(t) - m_i(t)\omega_i(k_i t)\right]' \\
=& -b_i(t)\omega_i(t) + \sum_{j=1}^{n} \mu_{ij}(t)J_j(\omega_j(t)) + \sum_{j=1}^{n} \nu_{ij}(t)F_j(\omega_j\left(d_{ij}t\right)) \\
&+ \sum_{j=1}^{n}\sum_{l=1}^{n} \theta_{ijl}(t)R_j(\omega_j\left(h_{ijl}t\right))R_l(\omega_l\left(r_{ijl}t\right)) + I_i(t), \quad t \geq t_0 > 0.
\end{aligned} \tag{3.10}$$

Therefore, $\omega(t)$ is a nonnegative $T$-periodic solution of HPDCNNs (1.1). Again from Remark 2.2, we gain $\omega(t)$ is globally generalized exponentially stable. This ends the proof.

## 4. Numerical simulations

We propose the following HPDCNNs:

$$\begin{cases}
[x_1(t) - \frac{\sin^2 2t}{100}x_1(\frac{t}{5})]' = & -\left(2 + \frac{3}{10}|\sin 2t|\right)x_1(t) + \left(\frac{1}{100}|\sin 2t|\right)x_1(t) \\
& + \left(\frac{1}{100}|\sin 2t|\right)x_2(t) + \left(\frac{1}{100}|\sin 2t|\right)x_1(\frac{t}{2}) + \left(\frac{1}{100}|\sin\sqrt{2}t|\right) \\
& \times x_2(\frac{t}{3}) + \frac{1}{100}|\sin 2t|\left[\arctan^2 x_1(\frac{t}{5}) + \arctan^2 x_2(\frac{t}{6})\right. \\
& \left. +2\arctan x_1(\frac{t}{5})\arctan x_2(\frac{t}{6})\right] + 20|\sin 2t| + 3, \\
[x_2(t) - \frac{\cos^2 2t}{100}x_2(\frac{t}{6})]' = & -\left(2 + \frac{3}{10}|\cos 2t|\right)x_2(t) + \left(\frac{1}{100}|\sin 2t|\right)x_1(t) \\
& + \left(\frac{1}{100}|\sin 2t|\right)x_2(t) + \left(\frac{1}{100}|\sin 2t|\right)x_1(\frac{t}{7}) + \left(\frac{1}{100}|\sin 2t|\right) \\
& \times x_2(\frac{t}{9}) + \frac{1}{100}|\sin 2t|\left[\arctan^2 x_1(\frac{t}{8}) + \arctan^2 x_2(\frac{t}{12})\right. \\
& \left. +2\arctan x_1(\frac{t}{8})\arctan x_2(\frac{t}{12})\right] + 30|\cos 2t| + 5,
\end{cases} \tag{4.1}$$

to verify the correctness of the obtained results. Clearly,

$$b_i, m_i, \mu_{ij}, \nu_{ij}, \theta_{ijl}, I_i :\in C(\mathbb{R} \to \mathbb{R}^+) \text{ are } T\text{-periodic functions}(T = \frac{\pi}{2})$$

and

$$J_j(x) = F_j(x) = x,\ R_j(x) = \arctan x,\ \ i, j \in Z = \{1, 2\},$$

which indicates that

$$m_i^+ = \frac{1}{100} < 1, b_i^- = 2 > 0, i = 1, 2.$$

Take

$$L_j^J = L_j^F = L_j^R = 1, Q_j^R = \frac{\pi}{2}, j = 1, 2,$$

then

$$V = \begin{bmatrix} a & a \\ a & a \end{bmatrix}, \beta = \begin{bmatrix} \frac{1150}{99} \\ \frac{1750}{99} \end{bmatrix}, \mathscr{X} = \frac{1}{1-2a}\begin{bmatrix} \frac{1150}{99} + \frac{600}{99}a \\ \frac{1750}{99} - \frac{600}{99}a \end{bmatrix} \text{and} \begin{bmatrix} I_1^- \\ I_2^- \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix},$$

where $a = \frac{\pi+2}{198}$. Using some direct calculations, we gain

$$\Lambda_i < 0,\ m_i^+ e^{\ln\frac{1}{k_i}} < 1,\ \rho(V) < 1,\ I_i^- > m_i^+ b_i^+ \chi_i,\ N_i + m_i^+ < 1, \ \text{ for all } \ i \in Z = \{1, 2\}.$$

Therefore, the HPDCNNs (4.1) satisfies all the assumptions proposed in Section 2. Applying Theorem 3.1, we know that the HPDCNNs (4.1) has a unique nonnegative periodic solution, which is globally exponentially stable. This can be seen in Figure 1.

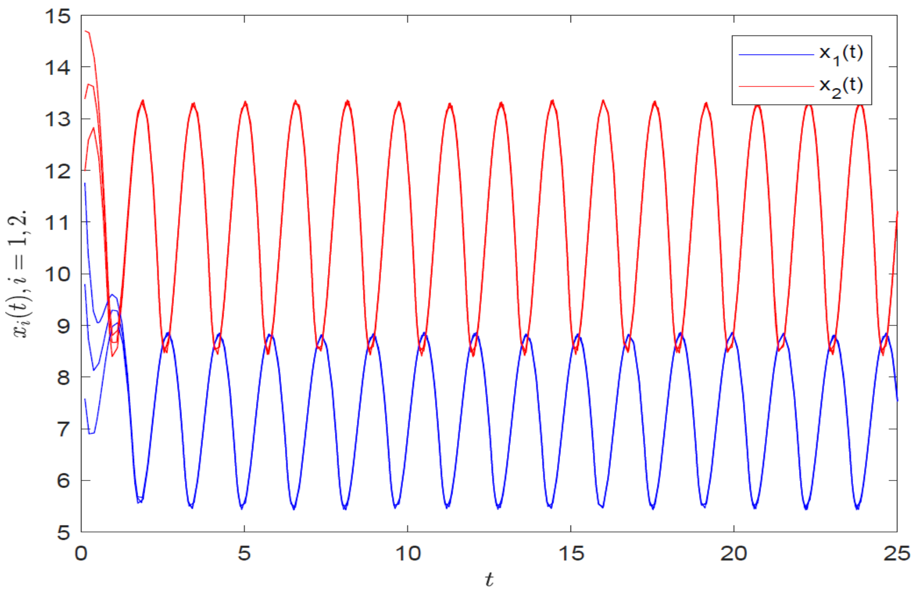


**Figure 1.** Numerical solutions $x(t)$ to system (4.1) with initial values $(\varphi_1(s), \varphi_2(s)) = (10\cos(2s), 10 + 10\sin(2s)), (12\cos(2s), 11 + 12\sin(2s)), (5 + 13\sin(2s), 15\cos(2s))$.

**Remark 4.1.** It should be noted that the existence and global exponential stability of the nonnegative periodic solution for high-order proportional delayed cellular neural networks involving $D$ operator have not been studied in the previous references, and all results proposed in [16–18, 22–61] are invalid for HPDCNNs (4.1).

## 5. Conclusions

Our main aim in this paper is to study the existence and global exponential stability of nonnegative periodic solutions for high-order proportional delayed cellular neural networks involving $D$ operator. The main contributions of this paper are listed as follows.

(1) To the best of our knowledge, this is the first time to study the existence and stability of nonnegative periodic solutions for high-order proportional delayed cellular neural networks involving $D$ operator.

(2) To establish the existence on nonnegative periodic solutions for the addressed neural networks models, the principle of contractive mapping, Lyapunov functional method and new analysis techniques are used in this paper to avoid the difficulties caused by unbounded delays.

(3) A very interesting fact shows that under certain conditions, the HPDCNNs will produce a globally exponentially stable nonnegative periodic solution. And these conditions are easy to check through some basic computations in practice.

Moreover, the method of this paper can also be used to study the periodicity for the other proportional delayed neural networks involving $D$ operator. It is our future work to study the positive periodicity for neutral neural networks involving proportional delays and $D$ operator.

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**Conflict of interests**

We confirm that we have no conflict of interest.

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