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Research article

Bayesian inference of dynamic cumulative residual entropy from Pareto II distribution with application to COVID-19

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Abstract: Dynamic cumulative residual entropy is a recent measure of uncertainty which plays a substantial role in reliability and survival studies. This article comes up with Bayesian estimation of the dynamic cumulative residual entropy of Pareto II distribution in case of non-informative and informative priors. The Bayesian estimator and the corresponding credible interval are obtained under squared error, linear exponential (LINEX) and precautionary loss functions. The Metropolis-Hastings algorithm is employed to generate Markov chain Monte Carlo samples from the posterior distribution. A simulation study is done to implement and compare the accuracy of considered estimates in terms of their relative absolute bias, estimated risk and the width of credible intervals. Regarding the outputs of simulation study, Bayesian estimate of dynamic cumulative residual entropy under LINEX loss function is preferable than the other estimates in most of situations. Further, the estimated risks of dynamic cumulative residual entropy decrease as the value of estimated entropy decreases. Eventually, inferential procedure developed in this paper is illustrated via a real data.

Keywords: Shannon entropy; dynamic cumulative residual entropy; Pareto II distribution; Bayesian estimators; loss functions
Mathematics Subject Classification: 62F15, 94A17, 94A24

1. Introduction

The primary measure of the uncertainty contained in random variable *X* is the Shannon entropy [1]. It plays an undeniable essential role in the field of probability and statistics, financial analysis, engineering, and information theory. Now, there are considerable literatures assigned to the applications, generalizations and properties of Shannon's measure of entropy. Özçam [2] studied an econometric procedure which revises and updates the technical production coefficients of latest Turkish input/output table, as new information about sectorial productions become available. Gençay and Gradojevic [3] provided a comparative analysis of stock market dynamics of the 1987 and 2008 financial crises and discussed the extent to which risk management measures based on entropy which can be successful in predicting aggregate market expectations. Rashidi et al. [4] discussed and simulated the heat transfer flow (using entropy generation) in solar still. Zhang et al. [5] discussed a network entropy method to measure connectivity uncertainty of functional connectivity graphs of the brain sequences. The predictability of Brazilian agricultural commodity prices during the period after food crisis using information theory has been studied by De Araujo et al. [6]. Different types of entropy measures have been discussed by many researchers (see for examples Shakhatreh et al. [7] and Klein and Doll [8]).

Let *X* be a non-negative random variable, the Shannon entropy, say *H*(*X*), of the probability density function density (PDF) is defined by

$$H(X)=-\int_{-\infty}^{\infty} f(x)\log f(x)dx.$$

In recent times, inference problems associated with entropy measures are of interest to several researches. An estimator of the entropy from the generalized half-logistic distribution using upper record value was obtained by Seo et al. [9]. The Bayesian estimators of entropy from Weibull distribution based on generalized progressive hybrid censoring scheme were studied by Cho et al. [10]. Estimation of entropy from generalized exponential distribution via record values was discussed by Chacko and Asha [11]. The maximum likelihood (ML) estimator of Shannon entropy from inverse Weibull distribution was obtained by Hassan and Zaky [12] using multiple censored data. Bayesian estimator of entropy for Lomax distribution was provided by Hassan and Zaky [13] via upper record values. In recent years, measurement of uncertainty for probability distributions became more interested. The entropy for residual lifetime $X_t = (X - t \mid X>t)$ was defined by Ebrahimi [14] as a dynamic form of uncertainty called the residual entropy at time *t* and defined as

$$H(X;t)=-\int_{t}^{\infty}\frac{f(x)}{\bar{F}(t)}\log\frac{f(x)}{\bar{F}(t)}dx,$$

where, $\bar{F}(t)=1-F(t)$ is the survival function. More recently, Rao et al. [15] proposed an alternative measure of uncertainty known as the cumulative residual entropy (CRE), denoted by $\xi(X)$. The CRE of a random variable *X* is defined by

$$\xi(X)=-\int_{-\infty}^{\infty}\bar{F}(x)\log\bar{F}(x)dx.$$

The CRE has the benefits: (*i*) it possess consistent definitions for the continuous and discrete domains, (*ii*) it forever non-negative and (*iii*) it is straightforwardly determined from sample data and these computations asymptotically converge to the true values.

Another measure of uncertainty deals with residual lifetime function is dynamic cumulative residual entropy (DCRE) which can be attractive in many fields like reliability and survival analysis. The entropy for residual lifetime X_t as a dynamic form of uncertainty was defined by Asadi and Zohrev and [16] as follows

$$\xi(X;t) = -\int_t^\infty \frac{\bar{F}(x)}{\bar{F}(t)} \log \frac{\bar{F}(x)}{\bar{F}(t)} dx. \tag{1}$$

It can be noted that, at $t = 0$, the DCRE tends to CRE. The Bayesian estimators of DCRE for the Pareto distribution were studied by Renjini et al. [17] using type II right censored data. Renjini et al. [18] considered the DCRE Bayesian estimators for Pareto distribution via upper record values. The ML and Bayesian estimate of the entropy of inverse Weibull distribution based on generalized progressive hybrid censoring scheme were discussed by Lee [19]. Renjini et al. [20] provided Bayesian estimator of DCRE for Pareto distribution from complete data.

Pareto II (Lomax) distribution was originally developed by Lomax [21] to model business failure data. This distribution has extensive applications in many fields such as income and wealth inequality, firm size and queuing problems, computer science, risk analysis and economics, actuarial science and reliability, the reader can refer to [22–30]. The cumulative distribution function (CDF) and the PDF of Pareto II distribution with shape parameter ϕ and scale parameter δ are defined by

$$f(x;\phi,\delta) = \phi\delta^{\phi}(x+\delta)^{-(\phi+1)} \qquad , x,\phi,\delta > 0. \tag{2}$$

and,

$$F(x;\phi,\delta) = 1-\delta^{\phi}(x+\delta)^{-\phi} \qquad , x,\phi,\delta > 0. \tag{3}$$

The DCRE for Pareto II distribution can be obtained by substituting (3) in (1) as follows:

$$\xi(X;t) = -(t+\delta)^{\phi} \int_t^\infty (x+\delta)^{-\phi} \log\left(\frac{(x+\delta)^{-\phi}}{(t+\delta)^{-\phi}}\right) dx. \tag{4}$$

Using integration by parts, the DCRE of Pareto II distribution will be as follows:

$$\xi(X;t) = \phi\left(\phi-1\right)^{-2}(t+\delta). \tag{5}$$

This is the required expression of the DCRE for Pareto II distribution which it is a function of ϕ and δ.

From the previous literatures, it can see that the Pareto II distribution take attention from theoretical and statisticians basically due to its use in multiple areas. In addition to, the DCRE has found nice interpretations and applications in the fields of reliability and survival analysis. Recently, statistical inference for the DCRE for lifetime distributions attracted appreciable attention. This motivates us to propose the estimation of the DCRE for Pareto II distribution in view of Bayesian procedure. The Bayesian estimator is obtained using non-informative prior (NIP) and informative prior (IP). The considered loss functions are squared error (SE), LINEX and precautionary (PRE). Markov Chain Monte Carlo (MCMC) technique is utilized due to the complicated forms of DCRE Bayesian estimator. Application to COVID 19 data in Egypt appeared that these data contain more information which is useful in mathematical and statistical purposes.

The form of the article is as follows. The next section presents Bayesian estimator of DCRE for Pareto II distribution under NIP for the considered loss functions. Section 3 gives Bayesian estimators of DCRE for Pareto II distribution using proposed loss functions under IP. Section 4 provides simulation issue and application to real data. The paper ends with summary of this work.

2. Bayesian estimation of DCRE under NIP

The Bayesian estimator of $\xi(X;t)$ under SEL, LINEX and PRE loss functions in case of NIP is obtained. To compute the Bayesian estimator of $\xi(X;t)$, we firstly obtain the Bayesian estimators of ϕ and δ. Assuming the prior of parameters ϕ and δ has a uniform distribution, then the Bayesian estimator of DCRE is obtained under symmetric and asymmetric loss functions. Additionally, the Bayesian credible interval (BCI) estimators are constructed. Consider a random sample of size n from PDF (2) and CDF (3), where ϕ and δ are unknown. Then, given the sample $\underline{x}=(x_1,x_2,...,x_n)$, the likelihood function of Pareto II distribution is

$$L(\phi,\delta\,|\,\underline{x})=\phi^n\delta^{n\phi}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)},\quad i=1,2,...,n$$

Considering that the prior of parameters ϕ and δ, denoted by $\pi_1(\phi)$ and $\pi_2(\delta)$, has the following uniform distribution

$$\pi_1(\phi)=\phi^{-1},\,\pi_2(\delta)=\delta^{-1}.$$

So, the joint posterior for parameters, denoted by $\pi_1^*(\phi,\delta)$, is

$$\pi_1^*(\phi,\delta\,|\,\underline{x})=K^{-1}\,\phi^{n-1}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)},$$

where,

$$K=\int_0^\infty\int_0^\infty\phi^{n-1}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}d\phi\, d\delta,$$

So, the marginal posterior PDF of parameters ϕ and δ are given, respectively, as follows

$$\pi_1^{**}(\phi\,|\,\underline{x})=K^{-1}\int_0^{\infty}\phi^{n-1}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\,\delta,$$

and (6)

$$\pi_2^{**}(\delta\,|\,\underline{x})=K^{-1}\int_0^{\infty}\phi^{n-1}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\,\phi.$$

Based on SE loss function, the Bayesian estimators of ϕ and δ denoted by $\hat{\phi}_{(SE)_1}$, and $\hat{\delta}_{(SE)_1}$ are obtained as posterior mean as follows:

$$\hat{\phi}_{(SE)_1}=\int_0^{\infty}\phi\,\pi_1^{**}(\phi\,|\,\underline{x})d\,\phi=K^{-1}\int_0^{\infty}\phi^{n}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\,\phi\,d\,\delta. \tag{7}$$

Also,

$$\hat{\delta}_{(SE)_1}=\int_0^{\infty}\delta\,\pi_2^{**}(\delta\,|\,\underline{x})\,d\,\delta=K^{-1}\int_0^{\infty}\int_0^{\infty}\phi^{n-1}\,\delta^{n\phi}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\,\phi\,d\,\delta. \tag{8}$$

Under LINEX loss function, the Bayesian estimators of ϕ and δ, say $\hat{\phi}_{(LINEX)_1}$, and $\hat{\delta}_{(LINEX)_1}$ are given, respectively, as follows

$$\hat{\phi}_{(LINEX)_1}=\frac{-1}{\nu}\log E\,(e^{-\phi\nu})=\frac{-1}{\nu}\log\left(K^{-1}\int_0^{\infty}\int_0^{\infty}e^{-\phi\nu}\,\phi^{n-1}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}d\,\phi\,d\,\delta\right),\nu\neq 0 \tag{9}$$

and,

$$\hat{\delta}_{(LINEX)_1}=\frac{-1}{\nu}\log E\,(e^{-\delta\nu})=\frac{-1}{\nu}\log\left(K^{-1}\int_0^{\infty}\int_0^{\infty}e^{-\delta\nu}\,\phi^{n-1}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\,\phi\,d\,\delta\right), \tag{10}$$

where, ν is a real number. Furthermore, the Bayesian estimators of parameters ϕ and δ, under PRE loss function, denoted by $\hat{\phi}_{(PRE)_1}$, and $\hat{\delta}_{(PRE)_1}$ are given as follows

$$\hat{\phi}_{(PRE)_1}=\sqrt{E\,(\phi^2\,|\,\underline{x})}=\left(K^{-1}\int_0^{\infty}\int_0^{\infty}\phi^{n+1}\,\delta^{n\phi-1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\,\phi\,d\,\delta\right)^{\frac{1}{2}}, \tag{11}$$

and,

$$\hat{\delta}_{(PRE)_1}=\sqrt{E\,(\delta^2\,|\underline{x})}==\left(K^{-1}\int_0^{\infty}\int_0^{\infty}\phi^{n-1}\,\delta^{n\phi+1}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\,\phi\,d\,\delta\right)^{\frac{1}{2}}. \tag{12}$$

Integrals (7)–(12) do not have a closed form, therefore Metropolis-Hastings (M-H) and random-walk Metropolis algorithms are employed to generate MCMC samples from posterior density functions (6). After getting MCMC samples from the posterior distribution, we can find the Bayes estimate for the parameters. The M-H algorithm is described as follows:

Step 1: Let g(.) be the density of Pareto II distribution.

Step 2: Initialize a starting value x_0 and determine the number of samples N.

Step 3: For i = 2 to N set $x = x_{i-1}$.

Step 4: Generate u from uniform (0, 1) and generate y from g(.).

Step 5: If $u \le \frac{\pi^{**}{}_{\phi}(y)\,g(x)}{\pi^{**}{}_{\phi}(\text{x})\,g(\text{y})}$ [where, π^{**} is posterior distribution provided in Equation (6)], then

set $x_{\text{i}} = y$ else set $x_{\text{i}} = x$.

Step 6: Set $i = i + 1$ and return to step 2 and repeat the previous steps N times.

Hence, the Bayes estimates (BEs) of ϕ and δ under SE, LINEX and PRE loss functions can be obtained as the mean of the simulated samples from their posteriors. Further, once the BEs of ϕ and δ are obtained, the BE of DCRE of Pareto II distribution is yielded. Therefore, the BEs of the DCRE under SE, LINEX, PRE loss functions, denoted by $\hat{\xi}(X;t)_{(SE)_1}$, $\hat{\xi}(X;t)_{(LINEX)_1}$, and $\hat{\xi}(X;t)_{(PRE)_1}$, are obtained, by using Equation (5). Furthermore, BCI is a useful summary of the posterior distribution which reflects its variation that is used to quantify the statistical uncertainty. The Bayesian analogy of a confidence interval is called a credible interval. A credible interval of entropy is the probability that a real value of entropy will fall between an upper and lower bounds of a probability distribution. Therefore, using the same algorithm introduced by Chen and Shao [31], we obtain an approximate highest posterior density interval for $\xi(X;t)$.

3. Bayesian estimation of DCRE under IP

In this section, we obtain the Bayesian estimator of DCRE under SE, LINEX and PRE loss functions by considering the prior of parameters ϕ and δ has a gamma distribution. Additionally, the BCI estimators are constructed. Following [32], assuming that the prior of ϕ and δ denoted by $\pi_3(\phi)$ and $\pi_4(\delta)$, has a gamma distribution with parameters (a_1, b_1) and (a_2, b_2) respectively.

$$\pi_3(\phi) = \frac{a_1^{b_1}}{\Gamma(b_1)}\phi^{b_1-1}e^{-\phi a_1}, \pi_4(\delta) = \frac{a_2^{b_2}}{\Gamma(b_2)}\delta^{b_2-1}e^{-\delta a_2},$$

where a_{j} and b_{j}, j = 1,2 are known and non-negative, so, the joint posterior for parameters, denoted by $\pi_3^*(\phi,\delta\,|\,\underline{x})$, is

$$\pi_3^*(\phi,\delta\,|\,\underline{x})=C^{-1}\ \phi^{n+b_1-1}\,\delta^{n\phi+b_2-1}e^{-(\phi a_1+\delta a_2)}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)},$$

where, $C=\int_0^\infty\int_0^\infty \phi^{n+b_1-1}\,\delta^{n\phi+b_2-1}e^{-(\phi a_1+\delta a_2)}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\phi\, d\delta.$

So, the marginal posterior PDF of ϕ and δ are given respectively by:

$$\pi_3^{**}(\phi\,|\,\underline{x})=C^{-1}\int_0^\infty \phi^{n+b_1-1}\,\delta^{n\phi+b_2-1}e^{-(\phi a_1+\delta a_2)}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\delta,$$

and, $\pi_4^{**}(\delta\,|\,\underline{x})=C^{-1}\int_0^\infty \phi^{n+b_1-1}\,\delta^{n\phi+b_2-1}e^{-(\phi a_1+\delta a_2)}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\phi.$

Therefore, the Bayesian estimators of ϕ under SE, LINEX and PRE loss function, say $\hat{\phi}_{(SE)_2}$, $\hat{\phi}_{(LINEX)_2}$ and $\hat{\phi}_{(PRE)_2}$ are obtained as follows:

$$\hat{\phi}_{(SE)_2}=C^{-1}\int_0^\infty\int_0^\infty \phi^{n+b_1}\,\delta^{n\phi+b_2-1}e^{-(\phi a_1+\delta a_2)}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\phi\, d\delta,$$

$$\hat{\phi}_{(LINEX)_2}=\frac{-1}{\varpi}\log\left(C^{-1}\int_0^\infty\int_0^\infty \phi^{n+b_1-1}\,\delta^{n\phi+b_2-1}e^{-(\varpi\phi+\phi a_1+\delta a_2)}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}d\phi\, d\delta\right)\ \varpi\neq 0,$$

and (13)

$$\hat{\phi}_{(PRE)_2}=\left(C^{-1}\int_0^\infty\int_0^\infty \phi^{n+b_1+1}\,\delta^{n\phi+b_2-1}e^{-(\phi a_1+\delta a_2)}\prod_{i=1}^{n}(x_i+\delta)^{-(\phi+1)}\,d\phi\, d\delta\right)^{\frac{1}{2}},$$

Similarly, the Bayesian estimators of δ under, SE, LINEX and PRE loss functions are obtained. As mentioned in previous section, MCMC technique is used to approximate the integral Equations (13). The M-H algorithm will be implemented to compute the BE as well as BCI width under proposed loss functions. Hence, the BE of $\xi(X;t)$, under different loss functions is obtained based on Equation (5).

4. Simulation and application

Here, performance of the different estimates is examined and a real data set is provided to illustrate the theoretical results.

4.1. Simulation illustration

A numerical study is performed in order to study the behavior of the BE for DCRE of the Pareto II distribution. MCMC simulations are performed for different sample sizes (n) under proposed loss functions. The true values of DCRE measure are selected as $\xi(X;t)=1.1111$, 2 and 6 at t = 0.5 and $\xi(X;t)=$ 2.2222, 4, and 8 at t = 1.5 [the parameter values are selected as (ϕ,δ) = (2.5, 0.5), (0.5, 0.5) and (0.5, 2.5)]. The hyper-parameters for gamma prior are selected as a_1=a_2=1 and b_1=b_2=4. Take (ν,ϖ) = (−2, 2) for LINEX loss function. 5000 random samples of sizes, n = 10, 30, 50, 70 and 100 are generated from Pareto II distribution. The relative absolute biases (RABs), estimated risks (ERs) and the width of BCI are computed to evaluate the behavior of the BEs.

4.2.Numerical results in Case of NIP

Tables 1–6 give simulation results of the DCRE in case of the NIP. Also, numerical outcomes are represented in Figures 1–4. So, we conclude the following about the behavior of the DCRE estimates.

- The estimated value of DCRE decreases as the value of δ decreases for same value of ϕ. The estimated value of DCRE decreases as the value of ϕ increases for same value of δ. ERs for DCRE estimates get the smallest value at $\xi(X;t)=1.1111$ and $\xi(X;t)=2.2222$.
- As the true value of $\xi(X;t)$ decreases, the ER of $\xi(X;t)$ decreases under different loss functions. The ER of DCRE under LINEX at $\nu=-2$ takes the smallest values at n = 10, 30 and 100, while the ER of DCRE under LINEX at ν = 2 takes the smallest values at n = 50 and 70 (see Table 1).
- The ERs for DCRE estimates get the smallest values at $\xi(X;t)=1.1111$ for all n (see for example Figures 1 and 2).

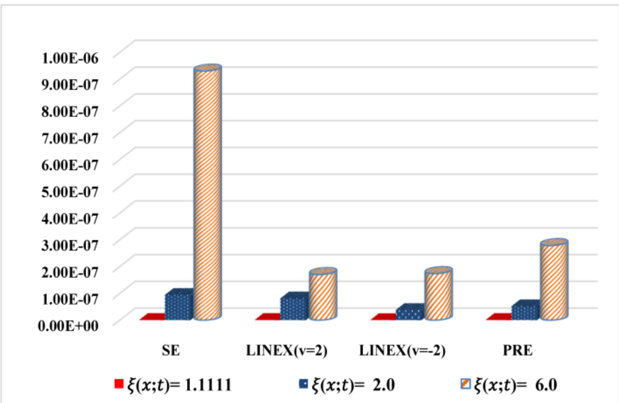

Figure 1. ERs of $\hat{\xi}(X;t)$ under different loss functions for NIP at n = 10 and t = 0.5.

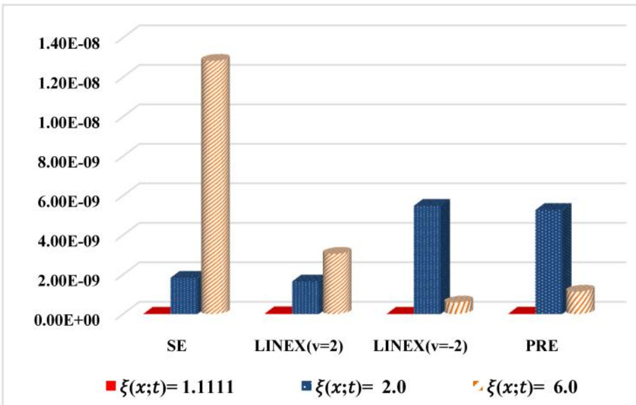

Figure 2. ERs of $\hat{\xi}(X;t)$ under different loss functions for NIP at n = 100 and t = 0.5.

- As the value of t increases the ER of $\xi(X;t)$ increases under all loss functions. The ER of DCRE under LINEX at $\nu = 2$ takes the smallest values at n = 30 and 100, while the ER of DCRE under SE takes the smallest values at n = 50 and 70. The width of BCI for DCRE under LINEX at $\nu = 2$ is the shortest compared to the width of BCI in case of PRE and SE loss functions for most values of n (see Table 2).
- The ER of DCRE under LINEX at $\nu = 2$ gets the smallest values at n = 10, 30 and 70. For n = 50, 70 and 100, the width of BCI for DCRE under SE is the shortest compared to the width of BCI in case of PRE and LINEX loss functions (see Table 3). The ERs for DCRE estimates get the smallest values at $\xi(X;t) = 2.2222.$ for all n (see for example Figures 3 and 4).

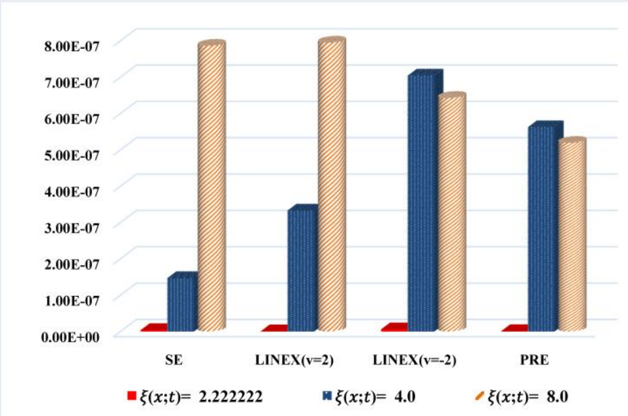

Figure 3. ERs of $\hat{\xi}(X;t)$ under different loss functions for NIP at n = 10 and t = 1.5.

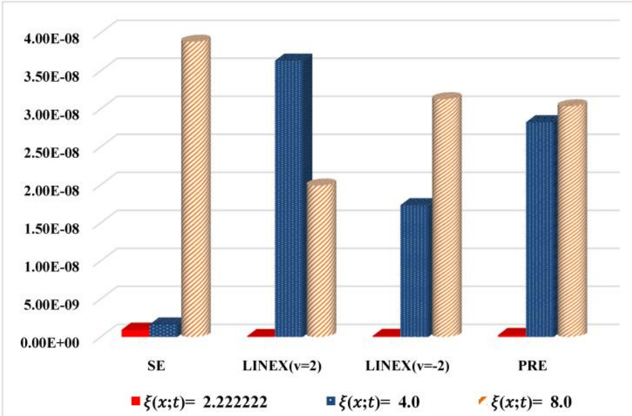

Figure 4. ERs of $\hat{\xi}(X;t)$ under different loss functions for NIP at n = 100 and t = 1.5.

- The ER of $\hat{\xi}(X;t)_{(LINEX)_1}$ at $\nu = 2$ takes the smallest values at n = 10, 30 and 100, while the ER of $\hat{\xi}(X;t)_{(LINEX)_1}$ at $\nu = -2$ takes the smallest values at n = 50 and 70. At n = 70 and 100, the width of BCI for $\hat{\xi}(X;t)_{(LINEX)_1}$ at $\nu = 2$ is the shortest compared to the width of BCI in case of PRE and SE loss functions (see Table 4).
- The ER of DCRE under SE takes the smallest values for all n. The width of BCI of DCRE under LINEX at $\nu = 2$ is the shortest compared to the width of BCI in case of PRE and SE loss functions for all n except at n = 100 (see Table 5).
- The ER of DCRE under PRE takes the smallest values for all n except at n = 100. The width of BCI for DCRE under LINEX at $\nu = -2$ is the shortest compared to the width of BCI in case of PRE and SE loss functions for all n except at n = 10 (see Table 6).

Table 1. BE, RAB, ER and width of DCRE at (ϕ,δ) = (2.5, 0.5) and t = 0.5 under NIP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		1.1111				
SE	BE	1.1129	1.1119	1.1118	1.1110	1.1113
	RAB	0.0016	0.0007	0.0006	0.0001	0.0002
	ER	6.07E-10	1.18E-10	9.23E-11	7.01E-11	5.78E-12
	width	0.0031	0.0020	0.0020	0.0019	0.0019
LINEX (v = 2)	BE	1.1127	1.1096	1.1117	1.1104	1.1099
	RAB	0.0014	0.0014	0.0005	0.0006	0.0011
	ER	5.06E-10	4.86E-10	6.35E-11	4.02E-11	2.90E-11
	width	0.0033	0.0031	0.0022	0.0022	0.0021
LINEX (v = −2)	BE	1.1114	1.1105	1.1120	1.1118	1.1112
	RAB	0.0002	0.0006	0.0008	0.0006	0.0001
	ER	9.27E-11	7.93E-11	6.73E-11	5.35E-11	3.58E-12
	width	0.0030	0.0026	0.0022	0.0019	0.0015
PRE	BE	1.1131	1.1105	1.1109	1.1113	1.1104
	RAB	0.0018	0.0005	0.0002	0.0002	0.0007
	ER	7.58E-10	6.70E-10	5.58E-11	4.53E-11	1.08E-11
	width	0.0026	0.0025	0.0025	0.0024	0.0020

Note: E-a: stands for 10^-a.

Table 2. BE, RAB, ER and width of DCRE at (ϕ,δ) = (0.5, 0.5) and t = 0.5 under NIP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		2.0				
SE	BE	1.99532	2.02025	2.00360	1.99691	1.99045
	RAB	0.00234	0.01013	0.00180	0.00155	0.00478
	ER	9.379E-08	8.202E-08	2.588E-09	1.913E-09	1.825E-09
	width	0.03882	0.02937	0.01338	0.02417	0.01952
LINEX (v = 2)	BE	1.99799	1.99765	2.00641	1.99598	2.00905
	RAB	0.00101	0.00117	0.00320	0.00201	0.00453
	ER	8.095E-08	9.102E-09	8.206E-09	3.240E-09	1.639E-09
	width	0.03135	0.02967	0.01261	0.01187	0.01003
LINEX (v = −2)	BE	2.01375	2.01198	1.98447	1.99197	2.01654
	RAB	0.00687	0.00599	0.00776	0.00402	0.00827
	ER	3.780E-08	2.872E-08	2.823E-08	1.291E-08	5.471E-09
	width	0.02987	0.02717	0.02698	0.02463	0.02178
PRE	BE	2.00760	1.98742	2.00603	2.00281	1.99487
	RAB	0.00380	0.00629	0.00302	0.00141	0.00256
	ER	5.155E-08	3.168E-08	7.276E-09	5.581E-09	5.259E-09
	width	0.02891	0.02675	0.02363	0.02181	0.01953

Note: E-a: stands for 10^-a.

Table 3. BE, RAB, ER and width of DCRE at (ϕ,δ) = (0.5, 2.5) and t = 0.5 under NIP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		6.0				
SE	BE	6.02546	6.068974	6.009779	6.01213	5.974681
	RAB	0.004243	0.011496	0.00163	0.002022	0.00422
	ER	9.30E-07	6.51E-07	4.91E-08	2.94E-08	1.28E-08
	width	0.149819	0.097667	0.044076	0.043662	0.034056
LINEX (v = 2)	BE	5.970834	6.024524	6.01785	5.996316	6.012336
	RAB	0.004861	0.004087	0.002975	0.000614	0.002056
	ER	1.70E-07	1.20E-07	6.37E-08	2.71E-08	3.04E-09
	width	0.072616	0.068967	0.052788	0.047875	0.04169
LINEX (v = −2)	BE	6.029378	6.029448	6.01762	6.047852	5.998299
	RAB	0.004896	0.004908	0.002937	0.007975	0.000283
	ER	1.74E-07	1.73E-07	6.21E-08	4.58E-08	5.79E-10
	width	0.058549	0.056825	0.053469	0.047115	0.046906
PRE	BE	6.011797	5.964629	6.018637	5.989266	6.002373
	RAB	0.001966	0.005895	0.003106	0.001789	0.000396
	ER	2.78E-07	2.50E-07	6.95E-08	2.80E-08	1.13E-09
	width	0.068916	0.066612	0.062829	0.054263	0.053632

Note: E-a: stands for 10^-a.

Table 4. BE, RAB, ER and width of DCRE at (ϕ,δ) = (2.5, 0.5) and t = 1.5 under NIP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		2.2222				
SE	BE	2.226397	2.225899	2.218533	2.218556	2.220049
	RAB	0.001879	0.001654	0.00166	0.00165	0.000978
	ER	3.49E-09	2.74E-09	2.72E-09	2.69E-09	9.44E-10
	width	0.007484	0.006809	0.006664	0.005188	0.005093
LINEX (v = 2)	BE	2.220678	2.221263	2.223683	2.22065	2.222941
	RAB	0.000695	0.000432	0.000657	0.000707	0.000323
	ER	4.87E-10	4.84E-10	4.27E-10	4.14E-10	1.03E-10
	width	0.00569	0.005229	0.00466	0.002028	0.00195
LINEX (v = −2)	BE	2.220546	2.219501	2.222949	2.219954	2.225297
	RAB	0.000754	0.001225	0.000327	0.001021	0.001383
	ER	5.62E-09	1.48E-09	1.36E-10	1.23E-10	1.19E-10
	width	0.005723	0.00508	0.004397	0.003961	0.003478
PRE	BE	2.220297	2.221733	2.222725	2.221037	2.221118
	RAB	0.000866	0.00022	0.000226	0.000533	0.000497
	ER	7.41E-10	5.78E-10	5.06E-10	2.81E-10	2.44E-10
	width	0.004525	0.003979	0.00375	0.003014	0.002158

Note: E-a: stands for 10^-a.

Table 5. BE, RAB, ER and width of DCRE at (ϕ,δ) = (0.5, 0.5) and t = 1.5 under NIP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		4.0				
SE	BE	4.027065	3.975195	3.968497	3.999098	4.004808
	RAB	0.006766	0.006201	0.007876	0.000226	0.001202
	ER	1.46E-07	1.23E-07	1.98E-08	1.63E-08	1.62E-09
	width	0.063364	0.053895	0.052946	0.039839	0.024612
LINEX (v = 2)	BE	4.012894	4.039052	3.985898	4.014544	3.978055
	RAB	0.003223	0.009763	0.003525	0.003636	0.005486
	ER	3.32E-07	3.05E-07	6.98E-08	4.23E-08	3.63E-08
	width	0.049488	0.044984	0.042183	0.030235	0.028456
LINEX (v = −2)	BE	3.968082	4.01742	4.024985	4.011722	4.009305
	RAB	0.00798	0.004355	0.006246	0.002931	0.002326
	ER	7.04E-07	6.07E-07	6.25E-08	2.75E-08	1.73E-08
	width	0.066015	0.054591	0.052009	0.034509	0.031416
PRE	BE	4.0053	4.044663	3.992488	3.992203	4.021002
	RAB	0.001325	0.011166	0.001878	0.001949	0.00525
	ER	5.62E-07	3.99E-07	5.13E-08	3.22E-08	2.82E-08
	width	0.083891	0.060246	0.059701	0.043115	0.036387

Note: E-a: stands for 10^-a.

Table 6. BE, RAB, ER and width of DCRE at (ϕ,δ) = (0.5, 2.5) and t = 1.5 under NIP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		8.0				
SE	BE	7.956073	8.05718	8.042204	7.958777	8.01393
	RAB	0.005491	0.007148	0.005276	0.005153	0.001741
	ER	7.86E-07	6.54E-07	3.56E-07	3.40E-07	3.88E-08
	width	0.198678	0.163581	0.097325	0.093346	0.072912
LINEX (v = 2)	BE	7.939398	8.001991	7.978639	7.993182	7.96846
	RAB	0.007575	0.000249	0.00267	0.000852	0.003942
	ER	7.95E-07	7.93E-07	9.43E-08	9.30E-08	1.99E-08
	width	0.171195	0.165343	0.12249	0.099383	0.091496
LINEX (v = −2)	BE	7.991566	7.984006	7.953482	8.08102	8.039532
	RAB	0.001054	0.001999	0.005815	0.010128	0.004942
	ER	6.42E-07	5.12E-07	4.33E-07	7.31E-08	3.13E-08
	width	0.168866	0.075301	0.064749	0.061597	0.012624
PRE	BE	8.024379	7.991098	7.962983	8.014555	7.945081
	RAB	0.003047	0.001113	0.004627	0.001819	0.006865
	ER	5.19E-07	3.58E-07	2.74E-07	4.24E-08	3.03E-08
	width	0.089696	0.088987	0.084625	0.083814	0.045059

Note: E-a: stands for 10^-a.

4.3.Numerical results in case of IP

Tables 7–12 give simulation results of DCRE under IP. Also, numerical outcomes are illustrated through Figures 5–8. So, we conclude the following observations about the behavior of the entropy estimates.

- The ERs for DCRE estimates get the largest value at $\xi(X;t)=6,$ where, t = 0.5 and $\xi(X;t)=8,$ where, t = 1.5. The ERs for DCRE estimates get the largest values at $\xi(X;t)=6,$ for all n (see for example Figures 5 and 6).

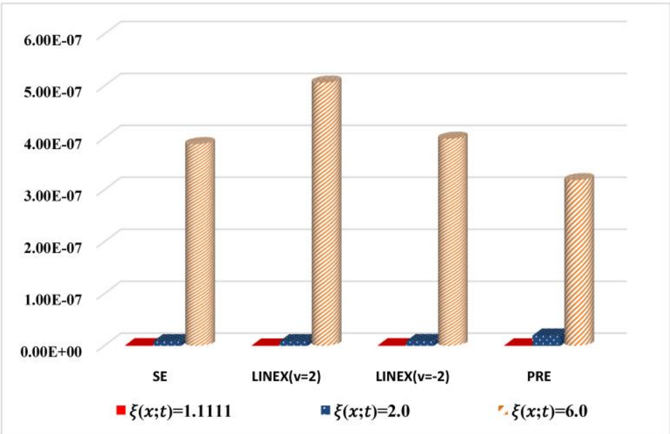

Figure 5. ERs of $\hat{\xi}(X;t)$ under different loss functions for IP at n = 10 and t = 0.5.

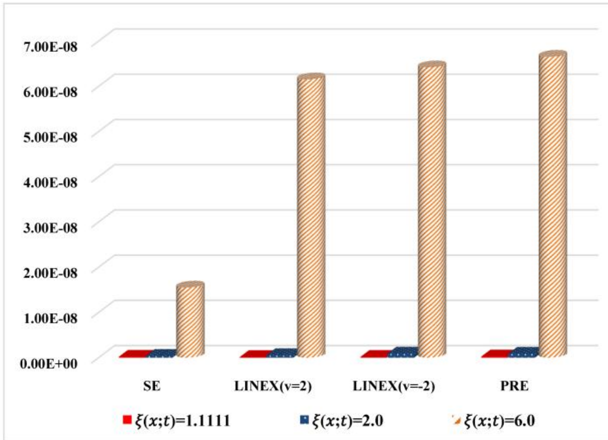

Figure 6. ERs of $\hat{\xi}(X;t)$ under different loss functions for IP at n = 100 and t = 0.5.

- The ER of DCRE under LINEX at ϖ = 2 takes the smallest values for all n. The width of BCI for DCRE under LINEX at ϖ = 2 is the shortest compared to the width of BCI in case of PRE and SE loss functions for all n (see Table 7). The ER of DCRE under SE gets the smallest values for most n. The width of BCI for DCRE under PRE is the shortest compared to the width of BCI in case of LINEX and SE loss functions for all n (see Table 8).
- Under IP, the ER of DCRE under SE takes the smallest values for n greater than 30. The width of BCI for DCRE under LINEX at ϖ = −2 is the shortest compared to the width of BCI in case of PRE and SE loss functions for all n except n = 50 (see Table 9). The ERs for DCRE estimates get the largest values at $\xi(X;t)=8,$ for all n (see for example Figures 7 and 8).

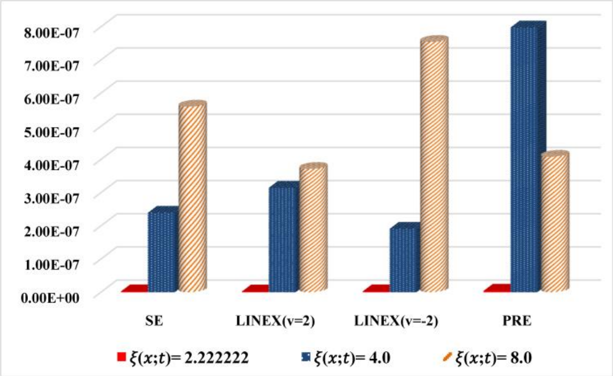

Figure 7. ERs of $\hat{\xi}(X;t)$ under different loss functions for IP at n = 10 and t = 1.5.

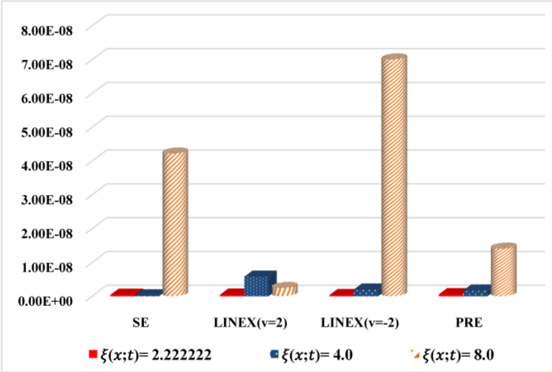

Figure 8. ERs of $\hat{\xi}(X;t)$ under different loss functions for IP at n = 100 and t = 1.5.

- The ER of DCRE under LINEX at ϖ = 2 takes the smallest values at n = 10 and 30, while the ER of DCRE under SE takes the smallest values at n = 50 and 70. The width of BCI for DCRE under LINEX at ϖ = −2 is the shortest compared to the width of BCI in case of PRE and SE loss functions at n = 10, 30 and 100 (see Table 10).
- Under IP, the ER of DCRE under LINEX at ϖ = −2 takes the smallest values at n = 10 and 30, while the ER of DCRE under SE takes the smallest values at n = 100. The width of BCI of DCRE under SE is the shortest compared to the width of BCI in case of PRE and LINEX loss functions for all values of n (see Table 11). From Table 12, the ER of DCRE under LINEX at ϖ = 2 takes the smallest values for all n. The width of BCI for DCRE under SE is the shortest compared to the width of BCI in case of PRE and LINEX loss functions for most values of n.

Table 7. BE, RAB, ER and width of DCRE for (ϕ,δ) = (2.5, 0.5) and t = 0.5 under IP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		1.111				
SE	BE	1.110118	1.113165	1.111933	1.11279	1.110128
	RAB	0.000894	0.001848	0.00074	0.001511	0.000885
	ER	8.97E-10	8.43E-10	6.35E-10	5.64E-10	1.93E-10
	width	0.003617	0.003566	0.002962	0.002683	0.002562
LINEX (ϖ = 2)	BE	1.109951	1.110168	1.110209	1.109762	1.112137
	RAB	0.001044	0.000849	0.000812	0.001214	0.000923
	ER	2.69E-10	1.78E-10	1.63E-10	1.62E-10	1.10E-10
	width	0.003432	0.00309	0.002341	0.002149	0.001411
LINEX (ϖ = −2)	BE	1.109703	1.108662	1.110331	1.110916	1.110214
	RAB	0.001267	0.002204	0.000702	0.000175	0.000808
	ER	7.96E-10	6.20E-09	4.22E-10	2.58E-10	1.61E-10
	width	0.004626	0.004364	0.003503	0.002992	0.002773
PRE	BE	1.110771	1.112018	1.11276	1.112896	1.110762
	RAB	0.000306	0.000816	0.001484	0.001606	0.000314
	ER	6.31E-10	4.64E-10	4.44E-10	3.37E-10	2.44E-10
	width	0.004068	0.003849	0.003775	0.003336	0.002173

Table 8. BE, RAB, ER and width of DCRE for (ϕ,δ) = (0.5, 0.5) and t = 0.5 under IP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		2.0				
SE	BE	2.0042	1.9939	1.9938	1.9966	2.0014
	RAB	0.0021	0.0031	0.0031	0.0017	0.0007
	ER	8.45E-09	7.48E-09	5.61E-09	2.31E-09	3.65E-10
	width	0.04314	0.03013	0.02	0.0181	0.01612
LINEX (ϖ = 2)	BE	1.99639	1.99716	1.99464	1.98086	1.99948
	RAB	0.0018	0.00142	0.00268	0.00957	0.00026
	ER	8.60E-09	6.61E-09	5.76E-09	4.32E-09	5.37E-10
	width	0.0278	0.0277	0.0252	0.0222	0.0197
LINEX (ϖ = −2)	BE	1.9947	2.0059	2.0023	1.9811	1.9979
	RAB	0.0027	0.003	0.0012	0.0094	0.001
	ER	8.69E-09	6.97E-09	5.08E-09	3.12E-09	8.80E-10
	width	0.0294	0.0221	0.0161	0.0157	0.0125
PRE	BE	1.9902	2.004	2.0045	2.0013	1.9794
	RAB	0.0049	0.002	0.0023	0.0006	0.0103
	ER	1.94E-08	7.20E-09	4.10E-09	3.22E-09	8.50E-10
	width	0.0269	0.019	0.0115	0.0107	0.0103

Table 9. BE, RAB, ER and width of DCRE for (ϕ,δ) = (0.5, 2.5) and t = 0.5 under IP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		6.0				
SE	BE	6.044	6.0299	5.9889	6.0252	5.9867
	RAB	0.0073	0.005	0.0018	0.0042	0.0022
	ER	3.88E-07	1.78E-07	2.45E-08	1.87E-08	1.55E-08
	width	0.0966	0.0884	0.0499	0.0452	0.0346
LINEX (ϖ = 2)	BE	6.0503	5.9613	6.0062	6.0254	6.0175
	RAB	0.0084	0.0065	0.001	0.0042	0.0029
	ER	5.06E-07	3.00E-07	1.59E-07	1.29E-07	6.15E-08
	width	0.1342	0.0866	0.0803	0.0737	0.05
LINEX (ϖ = −2)	BE	6.0446	5.9986	5.9668	5.9914	5.9821
	RAB	0.0074	0.0002	0.0055	0.0014	0.003
	ER	3.98E-07	3.07E-07	2.21E-07	6.49E-08	6.42E-08
	width	0.0616	0.0548	0.041	0.0368	0.0317
PRE	BE	6.0399	6.0271	5.9943	6.0191	5.9792
	RAB	0.0067	0.0045	0.0009	0.0032	0.0035
	ER	3.19E-07	1.47E-07	9.44E-08	7.33E-08	6.65E-08
	width	0.0941	0.0447	0.0429	0.0416	0.0384

Table 10. BE, RAB, ER and width for DCRE for (ϕ,δ) = (2.5, 0.5) and t = 1.5 under IP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		2.222				
SE	BE	2.2186	2.223	2.2235	2.2225	2.2237
	RAB	0.0016	0.0004	0.0006	0.0001	0.0006
	ER	2.57E-09	1.35E-09	8.27E-10	5.03E-10	4.17E-10
	width	0.00588	0.00358	0.0033	0.00327	0.00306
LINEX (ϖ = 2)	BE	2.21906	2.22057	2.22103	2.22061	2.22176
	RAB	0.00142	0.00074	0.00054	0.00073	0.00021
	ER	2.00E-09	6.46E-10	5.86E-10	5.20E-10	4.33E-10
	width	0.00719	0.00538	0.0051	0.00371	0.00327
LINEX (ϖ = −2)	BE	2.21868	2.21936	2.21977	2.2239	2.22345
	RAB	0.0016	0.00129	0.0011	0.00076	0.00055
	ER	2.52E-09	1.63E-09	1.20E-09	5.65E-10	3.03E-10
	width	0.00478	0.00424	0.00418	0.00322	0.00289
PRE	BE	2.22079	2.22435	2.21944	2.22345	2.22056
	RAB	0.00065	0.00096	0.00125	0.00055	0.00075
	ER	4.13E-09	2.08E-09	1.55E-09	5.99E-10	5.50E-10
	width	0.00494	0.00485	0.00385	0.00356	0.00318

Table 11. BE, RAB, ER and width of DCRE for (ϕ,δ) = (0.5, 0.5) and t = 1.5 under IP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		4.0				
SE	BE	4.0109	4.0034	4.0181	3.9937	3.9989
	RAB	0.0027	0.0009	0.0045	0.0016	0.0003
	ER	2.39E-07	2.38E-07	6.55E-08	8.01E-09	2.34E-10
	width	0.028868	0.024903	0.023434	0.019349	0.010273
LINEX (ϖ = 2)	BE	4.003966	4.003171	4.011339	4.013368	4.005361
	RAB	0.000992	0.000793	0.002835	0.003342	0.00134
	ER	3.15E-07	2.01E-07	4.57E-08	3.57E-08	5.75E-09
	width	0.065721	0.058115	0.050726	0.048513	0.032623
LINEX (ϖ = −2)	BE	3.990231	3.973805	3.983057	3.993077	4.031082
	RAB	0.002442	0.006549	0.004236	0.001731	0.00777
	ER	1.91E-07	1.37E-07	5.74E-08	9.59E-09	1.93E-09
	width	0.059764	0.040588	0.034511	0.031052	0.01101
PRE	BE	3.98003	3.945267	3.993328	3.994272	3.971136
	RAB	0.004993	0.013683	0.001668	0.001432	0.007216
	ER	7.98E-07	5.99E-07	8.90E-08	6.56E-09	1.67E-09
	width	0.074305	0.071163	0.02797	0.019786	0.01527

Table 12. BE, RAB, ER and width of DCRE for (ϕ,δ) = (0.5, 2.5) and t = 1.5 under IP.

n		**10**	**30**	**50**	**70**	**100**
Exact value of $\xi(X;t)$		8.0				
SE	BE	7.964	7.9542	7.9628	7.982	8.0459
	RAB	0.0045	0.0057	0.0047	0.0023	0.0057
	ER	5.59E-07	4.20E-07	2.77E-07	6.50E-08	4.22E-08
	width	0.13698	0.10276	0.08731	0.04921	0.01093
LINEX (ϖ = 2)	BE	8.01363	7.98158	8.03575	8.00609	8.03478
	RAB	0.0017	0.0023	0.00447	0.00076	0.00435
	ER	3.72E-07	6.79E-08	2.56E-08	7.42E-09	2.42E-09
	width	0.17329	0.12095	0.07442	0.07294	0.01217
LINEX (ϖ = −2)	BE	8.00357	8.05921	7.96138	7.93724	7.99813
	RAB	0.00045	0.0074	0.00483	0.00784	0.00023
	ER	7.54E-07	7.01E-07	2.98E-07	7.88E-08	7.00E-08
	width	0.13923	0.11615	0.06969	0.06458	0.06313
PRE	BE	7.95482	7.95937	7.95312	8.00328	8.02652
	RAB	0.00565	0.00508	0.00586	0.00041	0.00331
	ER	4.08E-07	3.30E-07	2.40E-07	2.15E-08	1.41E-08
	width	0.1405	0.07411	0.07259	0.07079	0.0667

Note: E-a: stands for 10^-a.

4.4.Application to real data

Here, the real data sets can be used to illustrate the method proposed in previous sections.

Data 1: The corona virus, COVID-19, is affecting for most countries all over the world. Confirmed total deaths data of the COVID-19 in Egypt since its start till May, 27, 2020, based on the National Vital Statistics System are recorded as follows.

1	2	2	2	2	2	4	6	6	7	8	10	14	19
21	21	24	30	36	40	41	46	52	52	66	71	78	85
103	118	135	146	159	164	178	183	196	205	224	239	250	264
264	276	294	307	307	337	359	380	392	406	415	429	436	452
469	482	503	514	525	533	544	556	571	592	612	630	645	659
680	696	707	735	764	783	797							

The validity of the fitted model has been checked by Abd-El-Monsef et al. [33]. Regarding this data, the Bayesian estimates of DCRE under SE, LINEX and PRE loss functions are obtained and listed in Table 13.

Table 13. Bayes Estimates of DCRE and their ERs (in brackets) under different loss functions at t = 0.5 and 1.5 for COVID-19 Data.

t	Prior	SE	LINEX (v = 2)	LINEX (v = −2)	PRE
0.5	Uniform	1.155031	1.155022	1.155002	1.155042
		(1.012238 E-10)	(4.292564 E-11)	(1.058868 E-11)	(5.926787 E-11)
1.5		1.166518	1.166519	1.166517	1.16652
		(3.782837 E-10)	(2.791847 E-11)	(1.136111 E-11)	(4.628114 E-11)
t	Prior	SE	LINEX (ϖ = 2)	LINEX (ϖ = −2)	PRE
0.5	Gamma	1.155021	1.155022	1.155027	1.155005
		(7.389608 E-11)	(6.941756 E-11)	(1.40135 E-11)	(1.761893 E-11)
1.5		1.166545	1.166516	1.1665	1.166534
		(3.154903 E-11)	(1.909039 E-11)	(3.952208 E-12)	(1.8328 E-11)

From Table 13, it can be seen that Bayes estimates of DCRE are slightly increasing as time t increases. Based on ER, the Bayesian estimate under LINEX loss function is the most appropriate for DCRE.

The estimated entropy of total deaths is very low;this indicates that these data containing more information that can be useful in mathematical and statistical purposes. So,it's better to study the entropy forthe detailed information about the daily death in Egypt for COVID-19 in future researches.

Data 2: The real data set was obtained from a meteorological study by Simpson [34] which represent the radar-evaluated rainfalls from single Florida cumulus clouds (from 1968 to 1970) from 52 south Florida cumulus clouds, 26 seeded clouds, and 26 control clouds. The validity of the fitted model has been checked by [32]. The Kolmogorov-Smirnov goodness of fit test is employed for real data and its p value indicates that the Pareto II distribution fits the data. The data are recorded as follows

129.6	31.4	2.745.6	489.1	430	302.8	119	4.1	92.4	17.5
200.7	274.7	274.7	7.7	1	656	978	198.6	703.4	1697.8
334.1	118.3	255	115.3	242.5	32.7	40.6	26.1	26.3	87
95	372.4	17.3	24.4	11.5	321.2	68.5	81.2	47.3	28.6
830.1	345.5	1202.6	36.6	4.9	4.9	41.1	29	163	244.3
147.8	21.7								

Regarding this data, the Bayes estimate of DCRE under SE, LINEX and PRE loss functions are obtained and listed in Table 14.

Table 14. Bayes Estimate of DCRE and their ERs (in brackets) under different loss functions at t = 0.5 and 1.5 for radar-evaluated rainfalls data.

t	Prior	SE	LINEX (v = 2)	LINEX (v = −2)	PRE
0.5	Uniform	5.972426	5.998918	5.987861	5.971833
		(5.412273 E-08)	(3.155479 E-08)	(5.129795 E-08)	(4.423818 E-07)
1.5		7.963097	7.982504	7.947595	8.020214
		(3.105256 E-07)	(1.524957 E-08)	(1.019482 E-07)	(2.820544 E-08)
t	**Prior**	**SE**	**LINEX (ϖ = 2)**	**LINEX (ϖ = −2)**	**PRE**
0.5	Gamma	6.022452	6.013078	5.977622	5.981887
		(2.469948 E-08)	(1.275901 E-08)	(9.075968 E-08)	(8.142539 E-08)
1.5		7.991222	8.005705	8.09864	7.958168
		(8.207474 E-07)	(1.13322 E-07)	(1.779665 E-07)	(4.023482 E-07)

As anticipated, from this example that the estimates of DCRE are increasing function on time as the time *t* increases under gamma and uniform priors for the proposed loss functions. Based on ER, the Bayes estimate of DCRE, under LINEX loss function is suitable than the other estimates.

5. Summary and conclusion

The Bayesian estimation of dynamic cumulative residual entropy is considered for Pareto II distribution. The Bayesian estimators of DCRE for Pareto II model are obtained in case of non-informative and informative priorsfor symmetric and asymmetric loss functions. The MCMC procedure is employed to compute the Bayes estimates and the BCIs. The behavior of DCRE estimates for Pareto II distribution is evaluated through their relative absolute bias, estimated risk and the width of credible intervals. Application to real data and simulation issues are provided.

According to outcomes of study we conclude that, under NIP, for small true values of DCRE the width of BCIs for estimated values of DCRE under LINEX loss function is smaller than the corresponding based on SE and PRE loss functions for large *n* at *t* = 0.5 and 1.5. For large true values of DCRE, at *t* = 0.5, the width of BCIs for estimated values of DCRE under SE loss function is smaller than the corresponding other loss functions for large *n*, but the width of BCIs for estimated values of DCRE under LINEX loss function is smaller than the corresponding based on the other loss functions for large sample size at *t* = 1.5.

Under IP, for small true values of DCRE, the width of BCIs of DCRE under LINEX loss function is smaller than the corresponding based on SE and PRE loss functions for large sample size at *t* = 0.5 and 1.5. For large true values of DCRE, at *t* = 0.5, the width of BCIs for estimated values of entropy under LINEX loss function is smaller than the corresponding estimated values based on SE and PRE loss functions for selected large *n*, but the width of BCIs for estimated values of DCRE under SE loss function is smaller than the corresponding estimated values based on SE and PRE loss functions for large *n* at *t* = 1.5.

Generally, the Bayesian estimate of DCRE approaches the true value as *n* increases. The DCRE values and ERs are directly proportional, that is; if the real value of DCRE decreases, the ERs decrease. As the time increases, the Bayesian estimate of DCRE increases. As *n* increases, the ER and the width of BCIs decrease. Bayesian estimates under LINEX loss function are more suitable than other loss functions for different types of prior functions in most of situations.

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Conflicts of interest

All authors declare no conflicts of interest in this paper.

References

1. C. E. Shannon, A mathematical theory of communication. *Bell Syst. Tech. J*., **27** (1948), 379–432.
2. A. Öz çam, Entropy estimation and interpretation of the inter-sectoral linkages of Turkish economy based on Leontief input/output model, *J. Econ. Stud.*, **36** (2009), 490–507.
3. R. Gen çay, N. Gradojevic, The tale of two financial crises: An entropic perspective, *Entropy*, **19** (2017), 244.
4. S. Rashidi, S. Akar, M. Bovand, R. Ellahi, Volume of fluid model to simulate the nanofluid flow and entropy generation in a single slope solar still, *Renew. Energ.*, **115** (2018), 400–410.
5. C. Zhang, F. Cong, T. Kujala, W. Liu, J. Liu, T. Parviainen, et al. Network entropy for the sequence analysis of functional connectivity graphs of the brain, *Entropy*, **20** (2018), 311.
6. F. H. A. De Araujo, L. Bejan, O. A. Rosso, T. Stosic, Permutation entropy and statistical complexity analysis of Brazilian agricultural commodities, *Entropy*, **21** (2019), 1220.
7. M. K. Shakhatreh, S. Dey, M. T. Alodat, Objective Bayesian analysis for the differential entropy of the Weibull distribution, *Appl. Math. Model.*, **89** (2021), 314–332.
8. I. Klein, M. Doll, (Generalized) Maximum cumulative direct, residual, and paired Φ entropy approach, *Entropy*, **22** (2020), 91.
9. J. I. Seo, H. J. Lee, S. B. Kan, Estimation for generalized half logistic distribution based on records, *J. Korea Inf. Sci. Soc.*, **23** (2012), 1249–1257.
10. Y. Cho, H. Sun, K. Lee, Estimating the entropy of a Weibull distribution under generalized progressive hybrid censoring, *Entropy*, **17** (2015), 101–122.
11. M. Chacko, P. S. Asha, Estimation of entropy for generalized exponential distribution based on record values. *J. Indian Soc. Probab. Stat.*, **19** (2018), 79–96.
12. A. S. Hassan, A. N. Zaky, Estimation of entropy for inverse Weibull distribution under multiple censored data. *J. Taibah Univ. Sci.*, **13** (2019), 331–337.
13. A. S. Hassan, A. N. Zaky, Entropy Bayesian estimation for Lomax distribution based on record. *Thail. Stat.*, (in press).
14. N. Ebrahimi, How to measure uncertainty in the residual lifetime distribution. *Sankhya A.*, **58** (1996), 48–56.
15. M. Rao, Y. Chen, B. C. Vemuri, F. Wang, Cumulative residual entropy: a new measure of information. *IEEE. Trans. Inf. Theory.*, **50** (2004), 1220–1228.
16. M. Asadi, Y. Zohrevand, On the dynamic cumulative residual entropy. *J. Stat. Plan. Infer.*, **137** (2007), 1931–1941.
17. K. R. Renjini, E. I. Abdul-Sathar, G. Rajesh, Bayes estimation of dynamic cumulative residual entropy for Pareto distribution under Type-II right censored data. *Appl. Math. Model.*, **40** (2016), 8424–8434.

18. K. R. Renjini, E. I. Abdul-Sathar, G. Rajesh, A study of the effect of loss functions on the Bayes estimates of dynamic cumulative residual entropy for Pareto distribution under upper record values. *J. Stat. Comput. Sim.*, **86** (2016), 324–339.
19. K. Lee, Estimation of entropy of the inverse Weibull distribution under generalized progressive hybrid censored data. *J. Korea Inf. Sci. Soc.*, **28** (2017), 659–668.
20. K. R. Renjini, E. I. Abdul-Sathar, G. Rajesh, Bayesian estimation of dynamic cumulative residual entropy for classical Pareto distribution. *AM. J. MATH-S.*, **37** (2018), 1–13.
21. K. S. Lomax, Business failures. Another example of the analysis of failure data. *J. Am. Stat. Assoc.*, **49** (1954), 847–852.
22. A. B. Atkinson, A. J. Harrison, *Distribution of Personal Wealth in Britain*. Cambridge: Cambridge Univ. Press: New York, NY, USA, 1978.
23. C. M. Harris, The Pareto distribution as a queue service discipline. *Oper. Res.*, **16** (1968), 307–313.
24. A. S. Hassan, A. Al-Ghamdi, Optimum step stress accelerated life testing for Lomax distribution. *J. Appl. Sci. Res.*, **5** (2009), 2153–2164.
25. A. S. Hassan, S. M. Assar, A. Shelbaia, Optimum step-stress accelerated life test plan for Lomax distribution with an adaptive type-II progressive hybrid censoring. *J. Adv. Math. Comp. Sci.*, **13** (2016), 1–19.
26. A. S. Hassan, R. E. Mohamed, Parameter estimation for inverted exponentiated Lomax distribution with right censored data. *Gazi Univ. J. Sci.*, **32** (2019), 1370–1386.
27. A. S. Hassan, M. Elgarhy, R. E. Mohamed, Statistical properties and estimation of type II half logistic Lomax distribution. *Thail. Stat.*, **18** (2020), 290–305.
28. A. S. Hassan, M. A. Sabry, A. M. Elsehetry, Truncated power Lomax distribution with application to flood data. *J. Stat. Appl. Prob.*, **9** (2020), 347–359.
29. R. Bantan, A. S. Hassan, M. Elsehetry, Zubair Lomax distribution: properties and estimation based on ranked set sampling. *CMC-Comput. Mater. Con.*, **65** (2020), 2169–2187.
30. A. S. Hassan, M. A. Sabry, A. M. Elsehetry, A new family of upper-truncated distributions: properties and estimation. *Thail. Stat.*, **18** (2020), 196–214.
31. M. H. Chen, Q. M. Shao, Monte Carlo estimation of Bayesian credible and HPD Intervals. *J. Comput. Graph. Stat.*, **8** (1999), 69–92.
32. A. Pak, M. R. Mahmoudi, Estimating the parameters of Lomax distribution from imprecise information. *J. Stat. Theory Appl.*, **17** (2018), 122–135.
33. M. M. Z. Abd El-Monsef, N. H. Sweilam, M. A. Sabry, The exponentiated power Lomax distribution and its applications. *Qual. Reliab. Engng. Int.*, (**2020)**, 1–24.
34. J. Simpson, Use of the gamma distribution in single-cloud rainfall analysis, *Mon. Weather Rev.*, **100** (1972), 309–312.

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