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Research article

The principal-agent model in venture investment based on fairness preference

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Abstract: The fairness preference in the principal-agent relationship is a vital factor that can even determine the success or failure of one program. Under normal circumstances, the capital invested by VC is often several times that of EN, which is one of the reasons for the profit gap between EN and VC. Therefore, when establishing a principal-agent model with fairness preferences, it is necessary to project the utility of VC to the level of EN and compare it with the utility of venture entrepreneurs, which will better reflect the profit gap between the two. On the basis of previous studies, this paper considers the amount of contribution of the participants, builds four principal-agent models to find the optimal distribution of income between the Venture Entrepreneur (EN) and the Venture Capital (VC) in a venture capital investment program, two without fairness preference and others with fairness preference. After the simulation we confirm that the fairness preference coefficient exerts a great impact on the distribution of income in both situations where information is symmetric and asymmetric, and a strong fairness preference will lead to a greater net profit gap between the EN and the VC. Thus, the EN should carefully choose the level of his efforts to realize the maximum return for him. In the case of information asymmetry, EN's optimal effort level decreases as the fairness preference coefficient increases.This will affect project revenue. And then affect the VC income.

Keywords: Venture Capital; principal-agent model; fairness preference
Mathematics Subject Classification: 91B08, 91B43

1. Introduction

Venture capital refers to the business investment activities of professional investment personnel for new creation or market value, also known as venture capital.Venture capital is a form of private equity investment. Different from other investments, venture capital has the characteristics of high failure and high return [1]. The United States is a pioneer of modern venture capital industry, its venture capital has developed rapidly from 1975. At present, it is still the largest venture capital country in the world, with more than 250 billion US dollars of venture capital under active management. The operation of venture capital involves three key subjects, investors, venture capitalists and venture entrepreneurs, among whom there are double problems of moral hazard [2, 3]. VC focuses on the early-stage and high-tech companies with the highest degree of information asymmetry. Venture capital can not only provide funds for start-ups, but also provide a variety of value-added services [4–6]. Kortum and Lerner [7] empirically analyzed the impact of venture capital on the technological innovation of 20 manufacturing industries in the United States from 1965 to 1992. Venture capital plays an important role in promoting the economy based on entrepreneurship, promoting the adjustment and upgrading of national and regional economic structures, maintaining continuous innovation and sustained and stable development of the economy, and increasing the comprehensive competitiveness of countries and regions in the era of knowledge economy with increasingly fierce global competition [8].

Venture capital activities are based on the cooperation between VC and EN. The development of VC and EN cooperation mechanism directly affects the success of venture capital activities and the development of venture capital industry [9]. Therefore, this paper only studies and analyzes the relationship between VC and EN. Due to the high risk and uncertainty of the project, it is of great theoretical and practical significance to ensure the high return of VC and the safety of venture capital. In terms of income, VC expects to maximize its own utility [10]. In the absence of moral hazard, it is ideal for both sides to make joint efforts. However, VC and EN have moral hazard in real life [11, 12]. EN, because of its information superiority, has more information to participate in risk projects. Therefore, the moral degree of EN is the key to the success of projects. Then, VC designing incentive contracts to make EN pay more efforts is the common method, some factors must also be possessed in venture capital investment allocation decision, such as: professional knowledge, solid reputation and status. [13–15], as well as the professional knowledge, solid reputation and status that must be possessed in venture capital investment allocation decision [16]. In the venture capital system, the problem of information asymmetry is more serious than in other industries, and there are huge risks in the formation of principal-agent relationship [17]. The adverse selection mechanism caused by information asymmetry will lead to "lemon market" in which inferior goods drive out superior goods and finally reach zero value equilibrium [18]. EN's efforts are long-term and constantly changing process. At present, it is all about constructing multi-stage principal-agent model to design incentive contracts. The stage is a basic operation form of venture capital [19–21]. Segmented investment can not only alleviate information asymmetry, control risks and reduce moral risks, but also play an effective supplement role to contracts and an effective way to ensure the safety of venture capital and realize dynamic adjustment of control rights of risk enterprises [22, 23].

Some psychologists and behavioral economists have shown that, the hypothesis of "participation and people is purely self-interested" in traditional economics is not consistent with the reality, through a large number of experiments and empirical studies [24]. Fehr and Schmidt [25] built the theoretical model of equity preference, quantified the utility of subjects with equity preference, and provided more ideas for later researchers to study the model of equity preference. This paper uses the equity preference theoretical model of Fehr and Schmidt to establish a multi-period principal-agent model based on equity preference. Furthermore, this paper designs multi-incentive mechanism, and designs the incentive mechanism of venture capital based on fairness preference.

Zhao and Chen [26] reconstructed and expanded the classical principal-agent theory, proposed obo-Endeavor theorem, verified the moral hazard of implied behavior, and found the synergistic effect of effort level on effort effect index through Newton method and computer graphics to calculate the long-term efforts of risk entrepreneurs. Li and Wang [27] designed two kinds of linear incentive contracts consisting of linear screening contract and linear pooling contract to solve the principal-agent problem with asymmetric information and moral hazard. Wang and Song [28] analyzed the relationship among venture entrepreneurs, venture firms and venture capitalists in order to study the basis of cooperative decision-making by stakeholders in venture capital. Then the cooperative game model of venture capital decision-making is established by taking investment amount and management level as decision variables, venture entrepreneurs and venture capitalists as participants. Guan and Ye [29] also considered the principal-agent efficiency and believed that the principal-agent efficiency was the key to the smooth operation of the enterprise. For more research, see [30–34].

In the studies and analysis of the multi-stage process of venture capital, Ulrich and Dirk [35] considered the provision of venture capital from the dynamic agent model. The optimal contract was a time-varying share contract, which allowed the inter-temporal risks sharing between venture capitalists and entrepreneurs. Malcomson et al. [36] pointed out that in the multi-period principal-agent model, the long-term contract is better than the short-term contract, only when the long-term contract makes the principal or agent earn less than the short-term contract in a certain future situation. Chen et al. [37] proposed the project overall concept model, in order to improve phase, project risk investment rate of return, success rate and reduce the investment risk. Hsu [38] in the principal-agent framework research installment decision problem of venture capitalists, found that subsection investment not only gave a waiting for the choice of VC, and alleviates the problem of agency for the enterprise is too conservative. Wei and Yong [39] studied and designed the optimal payment contract based on equity preference by using the behavioral contract theory, believing that equity preference would only lead to a loss of incentive efficiency and a risk of fair compensation. Zheng et al. [40] introduced fair preference into the principal-agent model between venture capitalists and entrepreneurs in venture capital market under the principle of bounded rationality, and conducted research and analysis based on the analysis of venture entrepreneurs' financing decisions. Guo [41] established a principal-agent model based on the equity preference theory, and maked the model parameters specific. She studied incentive contract and incentive efficiency under equity preference, and found that the equity preference has a considerable influence on the structure and efficiency of incentive contract, and has a positive influence on the optimal effort level of the agent. Wang [42] regarded effort variables as multi-stage dynamic variables and constructed a principal-agent model of venture capital with multi-stage efforts for venture entrepreneurs, on which basis he studied the theory of equity preference.

2. Model assumption and model analysis

2.1. Model assumptions

Some assumptions are listed as follows.

(1) An EN wants to start a business and he owns funds C_1, The total funds that needed are T, so a VC would provide the rest funds C_2 to support the EN's business. The share of holdings of the EN is $d = C_1/(C_1 + C_2)$. Apparently, $T = C_1 + C_2$ [43].

(2) Since the firm is managed by the EN, whether the program will be successful or failed is heavily contingent on the EN's ability and effort. We denote his ability as θ, and the efforts that the EN puts out at stage i is e_i. The cost of effort of the EN is $c(e_i) = be_i^2/2\theta$. Then the firm's total revenue in stage i will be $X_i = e_i + \theta + \varepsilon_i$ in the form of cash flow if the program succeeds, ε_i represents the random variable in stage i. The random variables ε_i and ε_j are independent, and $\varepsilon_i \sim N(0, \sigma_i)$.

(3) The VC will give the EN some incentives to unify the financial goals of them. We suppose that the incentive coefficient in stage i is β_i, and the fixed income for the EN is α. Then the total income for the EN is $w_i = \alpha + \beta_i X_i + (1 - \beta_i)dX_i$.

(4) The interest rate in every stage is r, and we do not take tax into account [44].

(5) The probability that the program will succeed in stage i is p_i, and the reserve utility of the EN is u_0.

(6) The EN is risk-averse and the VC is risk-neutral.

2.2. Model analysis

To simplify our model, we only consider the situation where the venture capital investment program consists of two stages.

In the first stage, the expected income for EN is

$$\pi_1 = p_1[\alpha + \beta_1 X_1 + (1 - \beta_1)dX_1] + (1 - p_1)\alpha. \tag{2.1}$$

Because the contract would terminate in stage $i + 1$ if the program failed in stage i, the success or failure in the second stage is strictly based on the success in the first stage. Then the probability that the program will succeed in the second stage is $p_1 p_2$, otherwise the probability is $p_1(1 - p_2)$. So, the expected income for the EN in the second stage is

$$\pi_2 = p_1 p_2[\alpha + \beta_2 X_2 + (1 - \beta_2)dX_2] + p_1(1 - p_2)\alpha. \tag{2.2}$$

We have assumed that the interest rate in every stage is r, then the discount factor is $\delta = 1/(1 + r)$. The net profit of the EN is

$$R_{en} = \pi_1 + \delta\pi_2 - c(e_1) - c(e_2) - C_1. \tag{2.3}$$

Substituting (2.1), (2.2) into (2.3), then,

$$\begin{aligned} R_{en} =& (1 + \delta p_1)\alpha + p_1[\beta_1 + (1 - \beta_1)d](e_1 + \theta + \varepsilon_1) \\ &+ \delta p_1 p_2[\beta_2 + (1 - \beta_2)d](e_2 + \theta + \varepsilon_2) \\ &- \frac{1}{2\theta}be_1^2 - \frac{1}{2\theta}be_2^2 - C_1. \end{aligned} \tag{2.4}$$

Since we have assumed that the EN is risk-adverse in the context, and his utility function is shown as $u(R_{en}) = -e^{-\rho R_{en}}$, where $\rho > 0$. According to the definition of certainty equivalent, a consideration of risk premium is needed to obtain the utility of the EN. More specifically,

$$\begin{aligned} Uen =&(1+\delta p_1)\alpha + p_1[\beta_1 + (1-\beta_1)d](e_1+\theta) \\ &+ \delta p_1 p_2[\beta_2 + (1-\beta_2)d](e_2+\theta) \\ &- \frac{1}{2}\rho p_1^2[\beta_1 + (1-\beta_1)d]^2\sigma_1^2 \\ &- \frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2 + (1-\beta_2)d]^2\sigma_2^2 \\ &- \frac{1}{2\theta}be_1^2 - \frac{1}{2\theta}be_2^2 - C_1. \end{aligned} \tag{2.5}$$

In this function, $-\frac{1}{2}\rho p_1^2[\beta_1+(1-\beta_1)d]^2\sigma_1^2 - \frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2+(1-\beta_2)d]^2\sigma_2^2$ represents the risk premium. The net profit of the VC would be

$$R_{vc} = X_1 + \delta X_2 - \pi_1 - \delta\pi_2 - C_2. \tag{2.6}$$

Similarly, substituting (2.1)-(2.2) into (2.6), then,

$$\begin{aligned} R_{vc} =& -(1+\delta p_1)\alpha + \{1 - p_1[\beta_1 + (1-\beta_1)d]\}(e_1+\theta+\varepsilon_1) \\ &+ \{\delta - \delta p_1 p_2[\beta_2 + (1-\beta_2)d]\}(e_2+\theta+\varepsilon_2) - C_2. \end{aligned} \tag{2.7}$$

Because the VC is a risk-neutral subject, his expected utility equals his expected net profit, that is

$$\begin{aligned} U_{vc} =& -(1+\delta p_1)\alpha + \{1 - p_1[\beta_1 + (1-\beta_1)d]\}(e_1+\theta) \\ &+ \{\delta - \delta p_1 p_2[\beta_2 + (1-\beta_2)d]\}(e_2+\theta) - C_2. \end{aligned} \tag{2.8}$$

EN will only accept the contract if its actual profit is not less than its reserved utility before the contract is signed by EN [45]. And of course, VC wants to pay EN as little as possible until it equals u_0, so EN's personal rational constraint is

$$\begin{aligned} (IR)U_{en} =&(1+\delta p_1)\alpha + p_1[\beta_1 + (1-\beta_1)d](e_1+\theta) + \delta p_1 p_2[\beta_2 \\ &+ (1-\beta_2)d](e_2+\theta) - \frac{1}{2}\rho p_1^2[\beta_1 + (1-\beta_1)d]^2\sigma_1^2 \\ &- \frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2 + (1-\beta_2)d]^2\sigma_2^2 \\ &- \frac{1}{2\theta}be_1^2 - \frac{1}{2\theta}be_2^2 - C_1 = u_0. \end{aligned}$$

The incentive compatibility constraint (IC) should be considered in some situations, and these situations will be demonstrated later in this paper [46]. The incentive compatibility constraint (IC) is

$$\begin{aligned} (IC)\ \max_{\alpha,\beta_1,\beta_2} &(1+\delta p_1)\alpha + p_1[\beta_1 + (1-\beta_1)d](e_1+\theta) \\ &+ \delta p_1 p_2[\beta_2 + (1-\beta_2)d](e_2+\theta) \end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{2}\rho p_1^2[\beta_1+(1-\beta_1)d]^2\sigma_1^2\\
&-\frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2+(1-\beta_2)d]^2\sigma_2^2\\
&-\frac{1}{2\theta}be_1^2-\frac{1}{2\theta}be_2^2-C_1.
\end{aligned}
$$

3. Model

3.1. Multistage principal-agent model based on complete rationality

3.1.1. The model under symmetric information

Under the situation where information is symmetric, there is no obstruction between the EN and the VC to get the information from the other party, which means that the VC can observe the degree of effort of the EN directly [47]. Then the mathematical model(I) is built as follows

$$
\begin{aligned}
\max_{\alpha,\beta_1,\beta_2} U_{vc} =&-(1+\delta p_1)\alpha+\{1-p_1[\beta_1+(1-\beta_1)d]\}(e_1+\theta)\\
&+\{\delta-\delta p_1 p_2[\beta_2+(1-\beta_2)d]\}(e_2+\theta)-C_2,
\end{aligned}
$$

$$
\begin{aligned}
s.t. \qquad (IR)U_{en}=&(1+\delta p_1)\alpha+p_1[\beta_1+(1-\beta_1)d](e_1+\theta)\\
&+\delta p_1 p_2[\beta_2+(1-\beta_2)d](e_2+\theta)\\
&-\frac{1}{2}\rho p_1^2[\beta_1+(1-\beta_1)d]^2\sigma_1^2\\
&-\frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2+(1-\beta_2)d]^2\sigma_2^2\\
&-\frac{1}{2\theta}be_1^2-\frac{1}{2\theta}be_2^2-C_1=u_0.
\end{aligned}
$$

From the Kuhn-Tuck condition, the participation constraint is equal.The second equation of model (I) is solved to obtain

$$
\begin{aligned}
\alpha=&-\frac{1}{1+\delta p_1}\{-u_0+p_1\left[\beta_1+(1-\beta_1)d\right](e_1+\theta)+\delta p_1 p_2\left[\beta_2+(1-\beta_2)d\right](e_2+\theta)\\
&-\frac{1}{2\theta}be_1^2-\frac{1}{2\theta}be_2^2-C_1-\frac{1}{2}\rho p_1^2\left[\beta_1+(1-\beta_1)d\right]^2\sigma_1^2-\frac{1}{2}\rho\delta^2 p_1^2 p_2^2\left[\beta_2+(1-\beta_2)d\right]^2\sigma_2^2\}
\end{aligned}
\tag{3.1}
$$

Substitute (3.1) into the first equation of model (I) to get

$$
\begin{aligned}
\max_{e_1,e_2,\beta_1,\beta_2} U_{vc}=&(e_1+\theta)+\delta(e_2+\theta)-\frac{1}{2\theta}be_1^2-\frac{1}{2\theta}be_2^2\\
&-\frac{1}{2}\rho p_1^2[\beta_1+(1-\beta_1)d]^2\sigma_1^2\\
&-\frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2+(1-\beta_2)d]^2\sigma_2^2-C_1-C_2-u_0.
\end{aligned}
\tag{3.2}
$$

All the arguments here are non-negative. $0 < p_1 < 1$, $0 < p_2 < 1$, $0 < \beta_1 < 1$, $0 < \beta_2 < 1$. $\rho > 0$ is the absolute risk aversion coefficient of EN, $\theta > 0$ is the individual ability of EN, and $b > 0$ is the effort cost coefficient of EN.So let's compute the first order condition.

$$\begin{aligned} \frac{\partial U_{vc}}{\partial \beta_1} &= -\rho p_1^2 \sigma_1^2 [\beta_1 + (1-\beta_1)d](1-d) < 0, \\ \frac{\partial U_{vc}}{\partial \beta_2} &= -\rho \delta^2 p_1^2 p_2^2 \sigma_2^2 [\beta_2 + (1-\beta_2)d](1-d) < 0. \end{aligned} \tag{3.3}$$

The utility of VC is negatively correlated with β_1. The utility of VC is negatively correlated with β_2. The smaller β_1 and β_2 are, the greater the utility of VC. So when β_1 and β_1 are minimized, the utility function of VC is maximized. Thus,

$$\beta_1^* = 0, \beta_2^* = 0. \tag{3.4}$$

This proves that VC does not have to excite EN in the case of information symmetry.

Substitute $\beta_1^* = 0$ and $\beta_2^* = 0$ into (3.2) the derivative of e_1 and e_2 is equal to 0.

$$\frac{\partial U_{vc}}{\partial e_1} = e_1 - \frac{1}{\theta} b e_1 = 0, \frac{\partial U_{vc}}{\partial e_2} = \delta e_2 - \frac{1}{\theta} b e_2 = 0.$$

So

$$e_1^* = \frac{\theta}{b}, e_2^* = \frac{\delta\theta}{b}. \tag{3.5}$$

Substitute (3.4) and (3.5) into (3.1) to get the optimal fixed remuneration of EN.

$$\alpha^* = -\frac{1}{1+\delta p_1}\left\{-u_0 + p_1 d(\frac{\theta}{b} + \theta) + \delta p_1 p_2 d(\frac{\delta\theta}{b} + \theta) - \frac{\theta}{2b} - \frac{\delta^2\theta}{2b} - \frac{1}{2}\rho p_1^2 d^2 \sigma_1^2 - \frac{1}{2}\rho\delta^2 p_1^2 p_2^2 d^2 \sigma_2^2 - C_1\right\} \tag{3.6}$$

3.1.2. The model under asymmetric information

In this situation, the VC will not receive the information from the EN. Then the incentive compatibility constraint (IC) stands because the VC should give some incentives to the EN to unify their goals [48, 49].

The incentive compatibility constraint (IC) is

$$\begin{aligned} (IC) \max_{e_1, e_2} (1+\delta p_1)\alpha &+ p_1[\beta_1 + (1-\beta_1)d](e_1 + \theta) \\ &+ \delta p_1 p_2 [\beta_2 + (1-\beta_2)d](e_2 + \theta) \\ &- \frac{1}{2}\rho p_1^2 [\beta_1 + (1-\beta_1)d]^2 \sigma_1^2 \\ &- \frac{1}{2}\rho\delta^2 p_1^2 p_2^2 [\beta_2 + (1-\beta_2)d]^2 \sigma_2^2 \\ &- \frac{1}{2\theta} b e_1^2 - \frac{1}{2\theta} b e_2^2 - C_1. \end{aligned}$$

Then, the mathematical model(II) is

$$\begin{aligned}\max_{\alpha,\beta_1,\beta_2} U_{vc} =& -(1+\delta p_1)\alpha + \{1 - p_1[\beta_1 + (1-\beta_1)d]\}(e_1+\theta)\\ &+ \{\delta - \delta p_1 p_2[\beta_2 + (1-\beta_2)d]\}(e_2+\theta) - C_2,\end{aligned}$$

$$\begin{aligned}s.t. \qquad (IR)U_{en} =&(1+\delta p_1)\alpha + p_1[\beta_1 + (1-\beta_1)d](e_1+\theta)\\ &+ \delta p_1 p_2[\beta_2 + (1-\beta_2)d](e_2+\theta)\\ &- \frac{1}{2}\rho p_1^2[\beta_1 + (1-\beta_1)d]^2\sigma_1^2\\ &- \frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2 + (1-\beta_2)d]^2\sigma_2^2\\ &- \frac{1}{2\theta}be_1^2 - \frac{1}{2\theta}be_2^2 - C_1 \geq u_0,\end{aligned}$$

$$\begin{aligned}(IC)\max_{e_1,e_2}&(1+\delta p_1)\alpha + p_1[\beta_1 + (1-\beta_1)d](e_1+\theta)\\ &+ \delta p_1 p_2[\beta_2 + (1-\beta_2)d](e_2+\theta)\\ &- \frac{1}{2}\rho p_1^2[\beta_1 + (1-\beta_1)d]^2\sigma_1^2\\ &- \frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2 + (1-\beta_2)d]^2\sigma_2^2\\ &- \frac{1}{2\theta}be_1^2 - \frac{1}{2\theta}be_2^2 - C_1.\end{aligned}$$

Since EN can choose the level of effort to maximize its own benefits, it takes partial derivatives of e_1 and e_2 in incentive compatibility constraints. And set them equal to 0, and you get

$$dp_1(1-\beta_1) + p_1\beta_1 - \frac{be_1}{\theta} = 0 \Rightarrow e_1 = \frac{(d+\beta_1 - d\beta_1)p_1\theta}{b}, \tag{3.7}$$

$$\delta dp_1p_2(1-\beta_2) + \delta p_1p_2\beta_2 - \frac{be_2}{\theta} = 0 \Rightarrow e_2 = \frac{(d+\beta_2 - d\beta_2)\delta p_1 p_2\theta}{b}. \tag{3.8}$$

From the Kuhn-Tuck condition, the participation constraint is equal.

$$\begin{aligned}\alpha =& -\frac{1}{1+\delta p_1}\{-u_0 + p_1\left[\beta_1 + (1-\beta_1)d\right](e_1+\theta) + \delta p_1 p_2\left[\beta_2 + (1-\beta_2)d\right](e_2+\theta)\\ &-\frac{1}{2\theta}be_1^2 - \frac{1}{2\theta}be_2^2 - C_1 - \frac{1}{2}\rho p_1^2\left[\beta_1 + (1-\beta_1)d\right]^2\sigma_1^2 - \frac{1}{2}\rho\delta^2 p_1^2 p_2^2\left[\beta_2 + (1-\beta_2)d\right]^2\sigma_2^2\}\end{aligned} \tag{3.9}$$

By substituting (3.7)–(3.9) into the first formula of model (II),

$$\begin{aligned}\max_{\beta_1,\beta_2} U_{vc} =& \left[\frac{(d+\beta_1 - d\beta_1)p_1\theta}{b} + \theta\right] + \delta\left[\frac{(d+\beta_2 - d\beta_2)\delta p_1 p_2\theta}{b} + \theta\right]\\ &- \frac{p_1^2[\beta_1 + (1-\beta_1)d]^2\theta}{2b} - \frac{\delta^2 p_1^2 p_2^2[\beta_2 + (1-\beta_2)d]^2\theta}{2b}\\ &- \frac{1}{2}\rho p_1^2[\beta_1 + (1-\beta_1)d]^2\sigma_1^2 - \frac{1}{2}\rho\delta^2 p_1^2 p_2^2[\beta_2 + (1-\beta_2)d]^2\sigma_2^2 - C_1 - C_2 - u_0\end{aligned} \tag{3.10}$$

can be obtained.

All the arguments here are non-negative. $0 < p_1 < 1, 0 < p_2 < 1, 0 < \beta_1 < 1, 0 < \beta_2 < 1$. So the first derivative of β_1 is

$$\frac{\partial U_{vc}}{\partial \beta_1} = \frac{\theta p_1(1-d)[1-p_1(d(1-\beta_1)+\beta_1)]}{b} - \rho p_1^2\sigma_1^2(1-d)(d(1-\beta_1)+\beta_1). \tag{3.11}$$

The first derivative of β_2 is

$$\frac{\partial U_{vc}}{\partial \beta_2} = \frac{\theta\delta^2 p_1 p_2(1-d)[1-p_1p_2(d(1-\beta_2)+\beta_2)]}{b} - \rho p_1^2 p_2^2\sigma_2^2(1-d)(d(1-\beta_2)+\beta_2). \tag{3.12}$$

Let $\frac{\partial U_{vc}}{\partial \beta_1} = 0$ and $\frac{\partial U_{vc}}{\partial \beta_2} = 0$, we can get

$$\beta_1^{**} = \frac{\theta - dp_1(\theta + b\rho\sigma_1^2)}{p_1(1-d)(\theta + b\rho\sigma_1^2)}, \tag{3.13}$$

$$\beta_2^{**} = \frac{\theta - dp_1p_2(\theta + b\rho\sigma_2^2)}{p_1p_2(1-d)(\theta + b\rho\sigma_2^2)}. \tag{3.14}$$

Substitute (3.13) and (3.14) for (3.7) and (3.8). They can be drawn that

$$e_1^{**} = \frac{\theta^2}{b(\theta + b\rho\sigma_1^2)}, \tag{3.15}$$

$$e_2^{**} = \frac{\delta\theta^2}{b(\theta + b\rho\sigma_2^2)}. \tag{3.16}$$

Substitute (3.13)–(3.16) into (3.9) to get the optimal fixed remuneration of EN.

$$\begin{aligned}\alpha^{**} = &-\frac{1}{1+\delta p_1}\{-u_0 + p_1\left[d+(1-d)\beta_1^{**}\right](e_1^{**}+\theta) + \delta p_1p_2\left[d+(1-d)\beta_2^{**}\right](e_2^{**}+\theta)\\ &-\frac{1}{2\theta}be_1^{**2} - \frac{1}{2\theta}be_2^{**2} - C_1 - \frac{1}{2}\rho p_1^2\left[d+(1-d)\beta_1^{**}\right]^2\sigma_1^2 - \frac{1}{2}\rho\delta^2p_1^2p_2^2\left[d+(1-d)\beta_2^{**}\right]^2\sigma_2^2\}\end{aligned} \tag{3.17}$$

3.2. Multistage principal-agent model based on fairness preference

The last section demonstrates two models based on the complete rationality. In this paper, we will draw upon the fairness preference theory of Fehr and Schmidt. Fehr and Schmidt maintain that in addition to self-interest preference, people also hold fairness preference. Specifically, suppose there are n participants in a game. Let x_i be the material payoff for the ith participant. Then the utility function for the ith participant is given by

$$\begin{aligned}U_i(x) =& x_i - u_i\frac{1}{n-1}\sum_{j\neq i} max\{x_j - x_i, 0\}\\ &- v_i\frac{1}{n-1}\sum_{j\neq i} max\{x_i - x_j, 0\}.\end{aligned} \tag{3.18}$$

Among that,u_i measures the jealousy inclination and v_i measures the sympathy inclination. Fehr and Schmidt assume that $u_i \geq v_i$ and the value of is in the interval [0 ,1].

The utility function splits the fairness preference into two parts. People tend to accept that they earn more than others. That is to say, they feel unequal when they have less money than others. Further studies also prove that people's jealousy preference is stronger than sympathy preference, then the value of v_i is supposed to be 0.

In effect, there are two participants in this research, namely, the VC and the EN. Given that the VC invests more than the EN, and the EN is subordinate to the VC in terms of their relationship, it is unreasonable that the EN will gain more profit than the VC. More importantly, the profit gap between the EN and the VC is supposed to be measured in the same investment level. Concretely, the EN will not jealous of the VC because of the tiny gap between them, since the VC always provides funds as many times as the EN has. The analysis in the context illustrates that the EN's utility under fairness preference is

$$U_{en}^{'} = U_{en} - u\,max\{\frac{C_1}{C_2}U_{vc} - U_{en}, 0\}.$$

In this function, U_{vc} is multiplied by $\frac{C_1}{C_2}$. The reason is that the investment gap between them should be eliminated and thus we project the profit of the VC to that of the EN to better evaluate the utility losses of the EN caused by fairness preference.

So,

$$\begin{aligned} U_{en}^{'} =&(1 + u + u\frac{C_1}{C_2})(1 + \delta p_1)\alpha - (1 + u)\frac{1}{2\theta}be_1^2 - (1 + u)\frac{1}{2\theta}be_2^2 - C_1 \\ &+ \left\{-u\frac{C_1}{C_2} + \left(1 + u + u\frac{C_1}{C_2}\right)p_1\left[\beta_1 + (1-\beta_1)d\right]\right\}(e_1 + \theta) \\ &+ \left\{-u\delta\frac{C_1}{C_2} + \left(1 + u + u\frac{C_1}{C_2}\right)\delta p_1 p_2\left[\beta_2 + (1-\beta_2)d\right]\right\}(e_2 + \theta) \\ &- (1 + u)\frac{1}{2}\rho p_1^2\left[\beta_1 + (1-\beta_1)d\right]^2\sigma_1^2 - (1 + u)\frac{1}{2}\rho\delta^2 p_1^2 p_2^2\left[\beta_2 + (1-\beta_2)d\right]^2\sigma_2^2. \end{aligned} \tag{3.19}$$

3.2.1. The model under symmetric information

Similarly, in the case of information symmetry, we do not need to consider incentive compatibility constraint (IC), but only the personal rationality constraint (IR) of EN. Therefore, the principal-agent model (III) can be obtained when EN has fair preference in the case of information symmetry.

$$\begin{aligned} \max_{\alpha,\beta_1,\beta_2} U_{vc} =& -(1 + \delta p_1)\alpha + \{1 - p_1[\beta_1 + (1-\beta_1)d]\}(e_1 + \theta) \\ &+ \{\delta - \delta p_1 p_2[\beta_2 + (1-\beta_2)d]\}(e_2 + \theta) - C_2, \end{aligned}$$

$$\begin{aligned} s.t. \qquad (IR)U_{en}^{'} =&(1 + u + u\frac{C_1}{C_2})(1 + \delta p_1)\alpha - (1 + u)\frac{1}{2\theta}be_1^2 - (1 + u)\frac{1}{2\theta}be_2^2 - C_1 \\ &+ \left\{-u\frac{C_1}{C_2} + \left(1 + u + u\frac{C_1}{C_2}\right)p_1\left[\beta_1 + (1-\beta_1)d\right]\right\}(e_1 + \theta) \\ &+ \left\{-u\delta\frac{C_1}{C_2} + \left(1 + u + u\frac{C_1}{C_2}\right)\delta p_1 p_2\left[\beta_2 + (1-\beta_2)d\right]\right\}(e_2 + \theta) \end{aligned}$$

$$- (1+u)\frac{1}{2}\rho p_1^2 \left[\beta_1 + (1-\beta_1)d\right]^2 \sigma_1^2 - (1+u)\frac{1}{2}\rho\delta^2 p_1^2 p_2^2 \left[\beta_2 + (1-\beta_2)d\right]^2 \sigma_2^2 \geq u_0.$$

From the Kuhn-Tuck condition, you just have to participate in the constraint and take the equal sign, which is

$$\begin{aligned}
U_{en}^{'} =& (1+u+u\frac{C_1}{C_2})(1+\delta p_1)\alpha - (1+u)\frac{1}{2\theta}be_1^2 - (1+u)\frac{1}{2\theta}be_2^2 - C_1 \\
&+ \left\{-u\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)p_1\left[\beta_1 + (1-\beta_1)d\right]\right\}(e_1+\theta) \\
&+ \left\{-u\delta\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)\delta p_1 p_2\left[\beta_2 + (1-\beta_2)d\right]\right\}(e_2+\theta) \\
&- (1+u)\frac{1}{2}\rho p_1^2\left[\beta_1 + (1-\beta_1)d\right]^2\sigma_1^2 - (1+u)\frac{1}{2}\rho\delta^2 p_1^2 p_2^2\left[\beta_2 + (1-\beta_2)d\right]^2\sigma_2^2 = u_0,
\end{aligned}$$

and you get

$$\begin{aligned}
\alpha =& -\frac{1}{(1+u+u\frac{C_1}{C_2})(1+\delta p_1)}\left\{-(1+u)\frac{1}{2\theta}be_1^2 - (1+u)\frac{1}{2\theta}be_2^2 - C_1\right. \\
&+ \left[-u\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)p_1\left(\beta_1 + (1-\beta_1)d\right)\right](e_1+\theta) \\
&+ \left[-u\delta\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)\delta p_1 p_2\left(\beta_2 + (1-\beta_2)d\right)\right](e_2+\theta) \\
&\left.-(1+u)\frac{1}{2}\rho p_1^2\left[\beta_1 + (1-\beta_1)d\right]^2\sigma_1^2 - (1+u)\frac{1}{2}\rho\delta^2 p_1^2 p_2^2\left[\beta_2 + (1-\beta_2)d\right]^2\sigma_2^2 - u_0\right\}.
\end{aligned} \tag{3.20}$$

Substitute (3.20) into the first formula of (III), and get

$$\begin{aligned}
\max_{e_1,e_2,\beta_1,\beta_2} U_{vc} =& \frac{1}{1+u+u\frac{C_1}{C_2}}\left\{-u_0 + \left[1+u-u\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)p_1(\beta_1 + (1-\beta_1)d)\right](e_1+\theta)\right. \\
&+ \left[\left(1+u-u\frac{C_1}{C_2}\right)\delta + \left(1+u+u\frac{C_1}{C_2}\right)\delta p_1 p_2(\beta_2 + (1-\beta_2)d)\right](e_2+\theta) \\
&- \frac{1}{2}\rho\left[-u\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)p_1(\beta_1 + (1-\beta_1)d)\right]^2\sigma_1^2 \\
&\left.-\frac{1}{2}\rho\left[-u\delta\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)\delta p_1 p_2(\beta_2 + (1-\beta_2)d)\right]^2\sigma_2^2 - \left(1+u+u\frac{C_1}{C_2}\right)C_2\right\}
\end{aligned} \tag{3.21}$$

Take the first derivative of β_1 and β_2.

$$\frac{\partial U_{vc}}{\partial \beta_1} = -\frac{C_2(1-d)p_1^2(1+u)[d(1-\beta_1)+\beta_1]\rho\sigma_1^2}{C_2 + C_1 u + C_2 u} < 0, \tag{3.22}$$

$$\frac{\partial U_{vc}}{\partial \beta_2} = -\frac{C_2(1-d)p_1^2 p_2^2(1+u)[d(1-\beta_2)+\beta_2]\delta^2\rho\sigma_1^2}{C_2 + C_1 u + C_2 u} < 0. \tag{3.23}$$

So,

$$\beta_1^{***} = 0, \beta_2^{***} = 0. \tag{3.24}$$

In the case of information symmetry, the behavior of EN is observable, so there is no need to stimulate EN.

Substitute (3.24) in (3.21) and find the first-order conditions for e_1 and e_2 to get

$$\frac{\partial U_{vc}}{\partial e_1} = -\frac{C_2(1+u)(be_1-\theta)}{(C_2+C_1u+C_2u)\theta}, \frac{\partial U_{vc}}{\partial e_2} = -\frac{C_2(1+u)(be_2-\delta\theta)}{(C_2+C_1u+C_2u)\theta}. \tag{3.25}$$

Let $\frac{\partial U_{vc}}{\partial e_1} = 0, \frac{\partial U_{vc}}{\partial e_2} = 0$, we can get

$$e_1^{***} = \frac{\theta}{b}, e_2^{***} = \frac{\delta\theta}{b}. \tag{3.26}$$

This result is the same as (3.5), so in the case of information symmetry, whether EN has fairness preference or not does not affect its own efforts.

Substitute (3.24) and (3.26) into (3.20), and get

$$\begin{aligned}\alpha^{***} = &-\frac{1}{(1+u+u\frac{C_1}{C_2})(1+\delta p_1)}\left\{-(1+u)\frac{\theta}{2b} - (1+u)\frac{\delta^2\theta}{2b} - C_1\right.\\ &+\left[-u\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)p_1 d\right](\frac{\theta}{b}+\theta)\\ &+\left[-u\delta\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)\delta p_1 p_2 d\right](e_2+\theta)\\ &\left.-(1+u)\frac{1}{2}\rho p_1^2 d^2\sigma_1^2 - (1+u)\frac{1}{2}\rho\delta^2 p_1^2 p_2^2 d^2\sigma_2^2 - u_0\right\}.\end{aligned} \tag{3.27}$$

Because $\frac{\partial\alpha^{***}}{\partial u} > 0$, the higher EN's fairness preference, the higher EN's fixed income, which means the lower VC's profit.

3.2.2. The model under asymmetric information

In the same way, it is important to consider the incentive compatibility constraint (IC):

$$\begin{aligned}(IC)\max_{e_1,e_2,}&(1+u+u\frac{C_1}{C_2})(1+\delta p_1)\alpha - (1+u)\frac{1}{2\theta}be_1^2 - (1+u)\frac{1}{2\theta}be_2^2 - C_1\\ &+\left\{-u\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)p_1\left[\beta_1 + (1-\beta_1)d\right]\right\}(e_1+\theta)\\ &+\left\{-u\delta\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right)\delta p_1 p_2\left[\beta_2 + (1-\beta_2)d\right]\right\}(e_2+\theta)\\ &-(1+u)\frac{1}{2}\rho p_1^2\left[\beta_1+(1-\beta_1)d\right]^2\sigma_1^2 - (1+u)\frac{1}{2}\rho\delta^2 p_1^2 p_2^2\left[\beta_2+(1-\beta_2)d\right]^2\sigma_2^2.\end{aligned}$$

Then, the mathematical model(IV) is

$$\max_{\alpha,\beta_1,\beta_2} U_{vc} = -(1+\delta p_1)\alpha + \{1 - p_1[\beta_1 + (1-\beta_1)d]\}(e_1+\theta)$$

$$+\{\delta-\delta p_1p_2[\beta_2+(1-\beta_2)d]\}(e_2+\theta)-C_2,$$

$$\begin{aligned} s.t. \qquad (IR)U_{en}^{'} =&(1+u+u\frac{C_1}{C_2})(1+\delta p_1)\alpha-(1+u)\frac{1}{2\theta}be_1^2-(1+u)\frac{1}{2\theta}be_2^2-C_1\\ &+\left\{-u\frac{C_1}{C_2}+\left(1+u+u\frac{C_1}{C_2}\right)p_1\left[\beta_1+(1-\beta_1)d\right]\right\}(e_1+\theta)\\ &+\left\{-u\delta\frac{C_1}{C_2}+\left(1+u+u\frac{C_1}{C_2}\right)\delta p_1p_2\left[\beta_2+(1-\beta_2)d\right]\right\}(e_2+\theta)\\ &-(1+u)\frac{1}{2}\rho p_1^2\left[\beta_1+(1-\beta_1)d\right]^2\sigma_1^2-(1+u)\frac{1}{2}\rho\delta^2p_1^2p_2^2\left[\beta_2+(1-\beta_2)d\right]^2\sigma_2^2\geq u_0, \end{aligned}$$

$$\begin{aligned} (IC)\max_{e_1,e_2,}&(1+u+u\frac{C_1}{C_2})(1+\delta p_1)\alpha-(1+u)\frac{1}{2\theta}be_1^2-(1+u)\frac{1}{2\theta}be_2^2-C_1\\ &+\left\{-u\frac{C_1}{C_2}+\left(1+u+u\frac{C_1}{C_2}\right)p_1\left[\beta_1+(1-\beta_1)d\right]\right\}(e_1+\theta)\\ &+\left\{-u\delta\frac{C_1}{C_2}+\left(1+u+u\frac{C_1}{C_2}\right)\delta p_1p_2\left[\beta_2+(1-\beta_2)d\right]\right\}(e_2+\theta)\\ &-(1+u)\frac{1}{2}\rho p_1^2\left[\beta_1+(1-\beta_1)d\right]^2\sigma_1^2-(1+u)\frac{1}{2}\rho\delta^2p_1^2p_2^2\left[\beta_2+(1-\beta_2)d\right]^2\sigma_2^2. \end{aligned}$$

Since EN can choose the level of effort to maximize its own benefits, it takes partial derivatives of e_1 and e_2 in incentive compatibility constraints. And set them equal to 0, and you get

$$\begin{aligned} &-u\frac{C_1}{C_2}+p_1\left(1+u+u\frac{C_1}{C_2}\right)(d+(1-d)\beta_1)-\frac{be_1(1+u)}{\theta}=0,\\ &-u\delta\frac{C_1}{C_2}+p_1p_2\left(1+u+u\frac{C_1}{C_2}\right)(d+(1-d)\beta_2)\delta-\frac{be_2(1+u)}{\theta}=0. \end{aligned} \tag{3.28}$$

$$\Rightarrow$$

$$\begin{aligned} e_1&=-\frac{\left[u\frac{C_1}{C_2}-p_1\left(1+u+u\frac{C_1}{C_2}\right)(d+(1-d)\beta_1)\right]\theta}{b(1+u)},\\ e_2&=-\frac{\left[u\delta\frac{C_1}{C_2}-p_1p_2\left(1+u+u\frac{C_1}{C_2}\right)(d+(1-d)\beta_2)\delta\right]\theta}{b(1+u)}. \end{aligned} \tag{3.29}$$

From the Kuhn-Tuck condition, you just have to participate in the constraint and take the equal sign, which is

$$\begin{aligned} U_{en}^{'} =&(1+2u)(1+\delta p_1)\alpha-(1+u)\frac{1}{2\theta}be_1^2-(1+u)\frac{1}{2\theta}be_2^2-(1+u)C_1+uC_2\\ &+\left\{-u+(1+2u)p_1\left[\beta_1+(1-\beta_1)d\right]\right\}(e_1+\theta)\\ &+\left\{-\delta u+(1+2u)\delta p_1p_2\left[\beta_2+(1-\beta_2)d\right]\right\}(e_2+\theta)\\ &-(1+u)\frac{1}{2}\rho p_1^2\left[\beta_1+(1-\beta_1)d\right]^2\sigma_1^2-(1+u)\frac{1}{2}\rho\delta^2p_1^2p_2^2\left[\beta_2+(1-\beta_2)d\right]^2\sigma_2^2=u_0, \end{aligned}$$

and you get

$$
\begin{aligned}
\alpha =& - \frac{1}{(1+u+u\frac{C_1}{C_2})(1+\delta p_1)} \left\{ -(1+u)\frac{1}{2\theta} b e_1^2 - (1+u)\frac{1}{2\theta} b e_2^2 - C_1 \right. \\
&+ \left[-u\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right) p_1 \left(\beta_1 + (1-\beta_1)d\right) \right] (e_1+\theta) \\
&+ \left[-u\delta\frac{C_1}{C_2} + \left(1+u+u\frac{C_1}{C_2}\right) \delta p_1 p_2 \left(\beta_2 + (1-\beta_2)d\right) \right] (e_2+\theta) \\
&\left. -(1+u)\frac{1}{2}\rho p_1^2 \left[\beta_1 + (1-\beta_1)d\right]^2 \sigma_1^2 - (1+u)\frac{1}{2}\rho\delta^2 p_1^2 p_2^2 \left[\beta_2 + (1-\beta_2)d\right]^2 \sigma_2^2 - u_0 \right\}.
\end{aligned} \tag{3.30}
$$

Substituting (3.29) and (3.30) into the first formula of (IV), and find the first-order conditions for the β_1 and β_2.

$$
\begin{aligned}
\frac{\partial U_{vc}}{\partial \beta_1} =& \frac{p_1}{(bC_2(1+u)(C_2+C_1u+C_2u)} \left\{ (1-d)(C_2+C_1u+C_2u)^2(1+dp_1(-1+\beta_1)-p_1\beta_1)\theta \right. \\
&\left. - bC_2^2(1-d)p_1(1+u)^2(d+\beta_1-d\beta_1)\rho\sigma_1^2 \right\},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial U_{vc}}{\partial \beta_2} =& \frac{(1-d)p_1p_2\delta}{C_2(1+u)} \left\{ \frac{(C_2+C_1u+C_2u)[\delta+\delta p_1p_2(d(-1+\beta_2)-\beta_2)]\theta}{b} \right. \\
&\left. - \frac{C_2^2 p_1 p_2 (1+u)^2 (d+\beta_2-d\beta_2)\delta\rho\sigma_2^2}{C_2+C_1u+C_2u} \right\}.
\end{aligned}
$$

Let $\frac{\partial U_{vc}}{\partial \beta_1} = 0$ and $\frac{\partial U_{vc}}{\partial \beta_2} = 0$, we can get

$$
\beta_1^{****} = \frac{\left[C_1^2u^2\theta + 2C_1C_2u(1+u)\theta + C_2^2(1+u)^2\theta\right](-1+dp_1) + C_2^2(1+u)^2 bdp_1p_2\rho\sigma_1^2}{(-1+d)p_1\left[C_1^2u^2\theta + 2C_1C_2u(1+u)\theta + C_2^2(1+u)^2(\theta+b\rho\sigma_1^2)\right]}, \tag{3.31}
$$

$$
\beta_2^{****} = \frac{\left[C_1^2u^2\theta + 2C_1C_2u(1+u)\theta + C_2^2(1+u)^2\theta\right](-1+dp_1p_2) + C_2^2(1+u)^2 bdp_1p_2\rho\sigma_2^2}{(-1+d)p_1p_2\left[C_1^2u^2\theta + 2C_1C_2u(1+u)\theta + C_2^2(1+u)^2(\theta+b\rho\sigma_2^2)\right]}. \tag{3.32}
$$

Substitute (3.31) and (3.32) for (3.29). They can be drawn that

$$
e_1^{****} = -\frac{\left[u\frac{C_1}{C_2} - p_1\left(1+u+u\frac{C_1}{C_2}\right)(d+(1-d)\beta_1^{****})\right]\theta}{b(1+u)}, \tag{3.33}
$$

$$
e_2^{****} = -\frac{\left[u\delta\frac{C_1}{C_2} - p_1p_2\left(1+u+u\frac{C_1}{C_2}\right)(d+(1-d)\beta_2^{****})\delta\right]\theta}{b(1+u)}. \tag{3.34}
$$

4. Simulation

4.1. Simulation steps

Since the optimal contracts shown above consist of several variables. Firstly, initiating some parameters. They are shown on Table1.

Table 1. The Values of Parameters.

r	b	C_1	C_2	p_1	p_2	ρ	σ_1^2	σ_2^2	d	θ	u_0
0.07	1.00	0.25	0.75	0.50	0.50	1.00	4.00	4.00	0.25	1.00	0.40

Secondly, the fairness preference coefficients should be considered carefully. The recent studies suppose it follows the uniform distribution in the interval [0,1]. Then we randomly select 30 numbers in it, which are shown in Table 2.

Table 2. The Values of Fairness Preference Coefficients.

0.19500	0.33400	0.00600	0.76700	0.650007	0.04800
0.51900	0.28800	0.27600	0.25300	0.02300	0.01700
0.78700	0.21600	0.81000	0.25900	0.33600	0.85700
0.35500	0.58500	0.69600	0.67100	0.48500	0.62900
0.29900	0.24200	0.84600	0.52400	0.23400	0.11300

Thirdly, we use Microsoft Excel to complete our experiments. The following four tables represent the Numerical conclusion of four models separately (see Tables 3–6).

Table 3. Numerical Example Under Symmetric Information Without Fairness Preference.

β_1	β_2	e_1	e_2	α	R_{en}	R_{vc}
0.00000	0.00000	1.00000	0.93458	0.85995	0.438077	1.43323

Table 4. Numerical Example Under Symmetric Information With Fairness Preference.

u	β_1	β_2	e_1	e_2	α	R_{en}	R_{vc}
0.195	0	0	1	0.934579439	0.868147	0.445497802	1.42119
0.334	0	0	1	0.934579439	0.872191	0.450789942	1.41526
0.006	0	0	1	0.934579439	0.860263	0.438302524	1.43276
0.767	0	0	1	0.934579439	0.880039	0.467277060	1.40375
0.650	0	0	1	0.934579439	0.878397	0.462821956	1.40615
0.048	0	0	1	0.934579439	0.862338	0.439901886	1.42972
0.519	0	0	1	0.934579439	0.876199	0.457833196	1.40938
0.288	0	0	1	0.934579439	0.870973	0.449039391	1.41705
0.276	0	0	1	0.934579439	0.870637	0.448582126	1.41754
0.253	0	0	1	0.934579439	0.869971	0.447706627	1.41852
0.023	0	0	1	0.934579439	0.861130	0.438949818	1.43149
0.017	0	0	1	0.934579439	0.860828	0.438720779	1.43193
0.787	0	0	1	0.934579439	0.880294	0.468036616	1.40337
0.216	0	0	1	0.934579439	0.868833	0.446298099	1.42019
0.810	0	0	1	0.934579439	0.880580	0.468912649	1.40295
0.259	0	0	1	0.934579439	0.870148	0.447935677	1.41826
0.336	0	0	1	0.934579439	0.872242	0.450866869	1.41519
0.857	0	0	1	0.934579439	0.871648	0.440863998	1.41606
0.355	0	0	1	0.934579439	0.872714	0.451590558	1.41449
0.585	0	0	1	0.934579439	0.877360	0.440863998	1.40768
0.696	0	0	1	0.934579439	0.879074	0.464572769	1.40516
0.671	0	0	1	0.934579439	0.878711	0.463619843	1.40569
0.485	0	0	1	0.934579439	0.875553	0.456540072	1.41033
0.629	0	0	1	0.934579439	0.878073	0.462022892	1.40663
0.299	0	0	1	0.934579439	0.871274	0.449458045	1.41661
0.242	0	0	1	0.934579439	0.869641	0.447286822	1.41900
0.846	0	0	1	0.934579439	0.881011	0.470283506	1.40232
0.524	0	0	1	0.934579439	0.876292	0.458025625	1.40924
0.234	0	0	1	0.934579439	0.869397	0.446982729	1.41936
0.113	0	0	1	0.934579439	0.865151	0.442376832	1.42559

Table 5. Numerical Example Under Asymmetric Information Without Fairness Preference.

β_1	β_2	e_1	e_2	α	R_{en}	R_{vc}
0.20000	0.73333	0.20000	0.18692	0.25591	0.549878	1.22192

Table 6. Numerical Example Under Symmetric Information With Fairness Preference.

u	β_1	β_2	e_1	e_2	α	R_{en}	R_{vc}
0.195	0.246636	0.826605	0.174925156	0.163481392	0.228877	0.611792	0.684439878
0.334	0.271696	0.876726	0.162363307	0.151741535	0.214712	0.657302	0.665769739
0.006	0.201697	0.736727	0.199047184	0.186025348	0.254909	0.551736	0.720491961
0.767	0.324664	0.982661	0.137761462	0.128748963	0.185579	0.803104	0.629330544
0.650	0.313081	0.959496	0.142923352	0.133573358	0.191857	0.763276	0.636969272
0.048	0.213047	0.759428	0.192753593	0.180143664	0.248257	0.564848	0.711056237
0.519	0.298002	0.929338	0.149823832	0.140022403	0.200108	0.719015	0.647183058
0.288	0.263994	0.861321	0.166159131	0.155288845	0.219039	0.642144	0.671405273
0.276	0.261894	0.857122	0.167203522	0.156265099	0.220223	0.638205	0.672956425
0.253	0.257759	0.848852	0.169273374	0.158199540	0.222559	0.630672	0.676033000
0.023	0.206400	0.746133	0.196422554	0.183572422	0.252146	0.557023	0.716554683
0.017	0.204757	0.742848	0.197336264	0.184426533	0.253110	0.555154	0.717925178
0.787	0.326492	0.986318	0.136957265	0.127997578	0.184592	0.809934	0.628141804
0.216	0.250784	0.834901	0.172803230	0.161498284	0.226515	0.618607	0.681282090
0.810	0.328545	0.990423	0.136058178	0.127157109	0.183486	0.817798	0.626811450
0.259	0.258852	0.851038	0.168724571	0.157686639	0.221941	0.632635	0.675216613
0.336	0.272019	0.877372	0.162205392	0.151593951	0.214531	0.657963	0.665535464
0.857	0.332581	0.998496	0.134300000	0.125514000	0.181316	0.833891	0.624211000
0.355	0.275041	0.883415	0.160733267	0.150217943	0.212840	0.664249	0.663351240
0.585	0.305910	0.945153	0.146179857	0.136616623	0.195770	0.741265	0.641788574
0.696	0.317825	0.968984	0.140794605	0.131583876	0.189280	0.778907	0.633818269
0.671	0.315279	0.963892	0.141934510	0.132649207	0.190662	0.770407	0.635505463
0.485	0.293656	0.920645	0.151851428	0.141917157	0.202502	0.707594	0.650186206
0.629	0.310827	0.954987	0.143942322	0.134525468	0.193085	0.756155	0.638476869
0.299	0.265885	0.865103	0.165222037	0.154413056	0.217974	0.645759	0.670014240
0.242	0.255728	0.844790	0.170296149	0.159155404	0.223709	0.627078	0.677552985
0.846	0.331655	0.996644	0.134702547	0.125890365	0.181813	0.830121	0.624806326
0.524	0.298625	0.930584	0.149534728	0.139752212	0.199765	0.720696	0.646755533
0.234	0.254229	0.841791	0.171054075	0.159863560	0.224559	0.624469	0.678679739
0.113	0.228969	0.791271	0.184157071	0.172109352	0.239014	0.585424	0.698202379

4.2. Data analysis

In this section, we utilize the scatter function in MATLAB to draw the corresponding figures so as to make the data showed above clear for people. And also, it is vital to arrive at some conclusions about the figures, especially the trend will be observed between the fairness preference coefficients and other variables. Figure 1 shows the symbol description in the figure.

The first and second columns in Tables 3–6 are shown in Figure 2, and the first and third columns are shown in Figure 3.

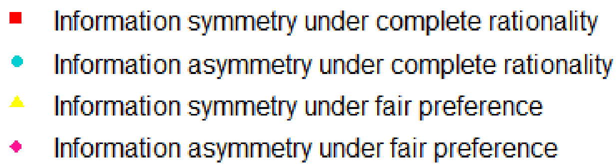

Figure 1. The Symbol Description.

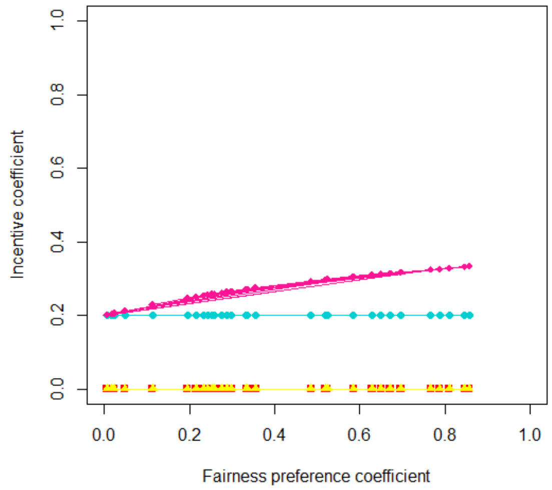

Figure 2. First Stage Optimal Excitation Coefficient.

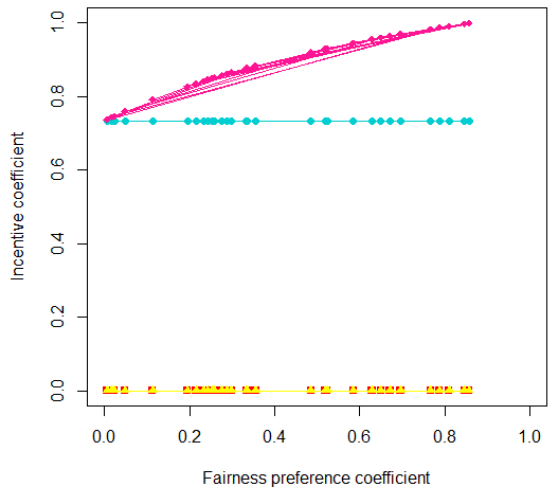

Figure 3. Optimal Excitation Coefficient in the Second Stage.

From Figures 2 and 3, the conclusion 1 could be obtained: Under the situation where information is symmetric, the efforts that the EN puts out will remain the same, no matter what size the fairness preference coefficient is while under the situation where information is asymmetric, the efforts that the EN puts out will go down as the fairness preference coefficient goes up.

According to Tables 3–6, Figure 4 is for the first and fourth columns, and Figure 5 is for the first and fifth columns.

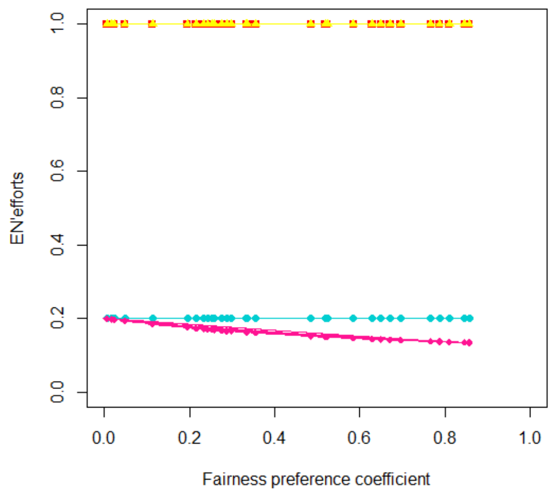

Figure 4. Optimal Effort Level of EN in Stage 1.

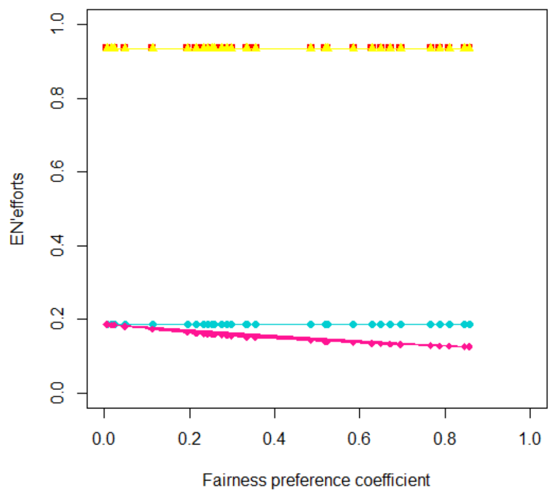

Figure 5. Optimal Effort Level of EN in Stage 2.

From Figures 4 and 5, the conclusion 2 could be obtained: Under the situation where information is symmetric, the EN's incentive coefficients will always be zero, no matter what size the fairness preference coefficient is while under the situation where information is asymmetric, the EN's incentive coefficients will increase as the fairness preference coefficient goes up.

According to Figure 6 in the first and sixth columns of Tables 3–6.

From Figure 6, the conclusion 3 would be obtained: Under the situation where information is symmetric, the EN's fixed income will not change substantially no matter what size the fairness preference coefficient is while under the situation where information is asymmetric, the EN's fixed income will decrease as the fairness preference coefficient goes up. In the case of symmetric information, EN has the largest fixed income.

Figure 7 is shown in the first and seventh columns of Tables 3–6.

From Figure 7, the conclusion 4 would be obtained: Under the situation where information is symmetric, the EN's net profit with fairness preference will lower than that without fairness preference when the fairness preference coefficient is smaller than a specific number. Once the fairness preference coefficient is greater than that specific number, the EN's net profit with fairness preference will greater than that without fairness preference; While under the situation where information is asymmetric, the EN's net profit will increase as the fairness preference coefficient goes up.

Figure 8 is shown in the first and eighth columns of Tables 3–6. From Figure 8, the conclusion 5 would be obtained: VC's optimal net profit under complete rationality is higher than the income under fairness preferences.Under complete rationality and information symmetry VC's optimal net profit is the highest. VC's optimal net profit decreases as the fairness preferences coefficient increases. Therefore, EN's jealousy preference will damage VC's income.

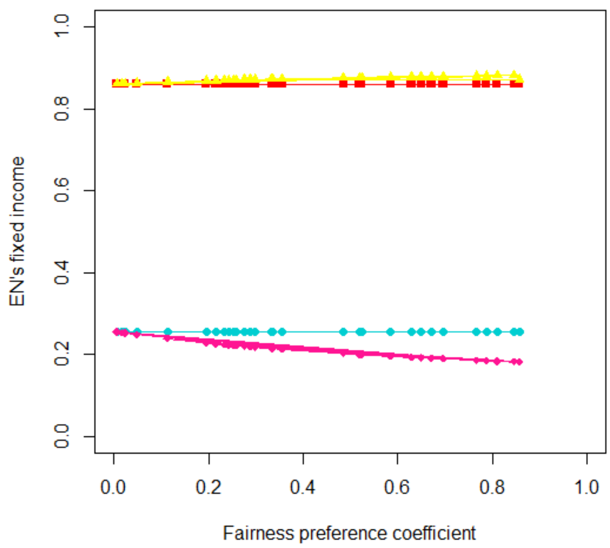

Figure 6. EN's Optimal Fixed Income.

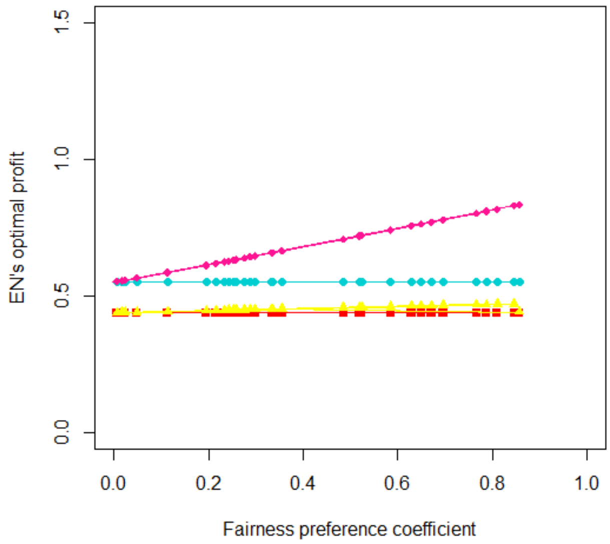

Figure 7. EN's Optimal Net Profit.

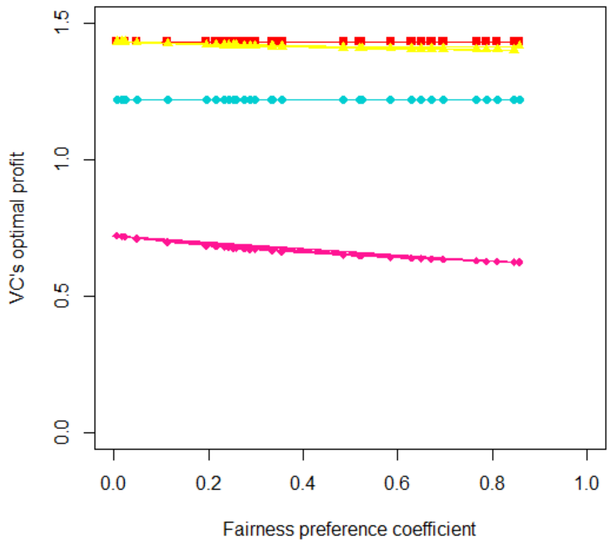

Figure 8. VC's Optimal Net Profit.

5. Conclusions

This article has carried on the preferences of fair entrust a representative model of venture capital research, according to two different dimensions lists the four different models respectively. This paper is focused on the size of the equity preference impact on investment income, with complete rational risk entrepreneurs to do comparison, some conclusions are drawn, the following for the conclusion of this paper are summarized.

This article argues that a large investment gap between VC and EN can lead to a large gap in their net profit, but the same criteria should be used to measure the negative utility of envy. Compared to the returns of EN and VC at the same investment level, then, we multiply the utility of VC by the Fehr-Schmidt fair preference function, so that the investment level of VC can be projected onto the investment level of EN.

Through data analysis, the research shows that the EN will prudent with the decrease in his effort, especially under the situation where information is asymmetric. The VC's purely self-interested preference usually represents that his ultimate goal is to maximize his own profit. When the EN lowers his level of effort because of the fairness preference, the VC might choose to increase the incentive coefficient. We confirm that the fairness preference coefficient exerts a great impact on the distribution of income in both situations where information is symmetric and asymmetric, and a strong fairness preference will lead to a greater net profit gap between the EN and the VC. If the VC pays attention to EN's jealous preference, the VC needs to invest a little more than the income sharing coefficient when it is completely rational. Not only does the project income increase, but it can also achieve mutual benefit harmonious relationship. Since the income of VC is negatively correlated with the coefficient of fairness preference, VC should not choose EN with strong jealousy.

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Conflict of interest

The authors declare no conflict of interest.

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