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Research article

Computing μ-values for LTI Systems

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Abstract: In this article we consider certain linear time-varying control systems and investigate their stability using structured singular values (μ-values). We use the low rank ordinary differential equations based methodology to compute the lower bounds for μ-values. The inner-outer algorithm computes the local extremizer of an admissible perturbation and adjusts the desired perturbation level. Further, we present a comparison of our results via the well-known MATLAB routine mussv which is available in MATLAB control toolbox.

Keywords: eigenvalues; singular values; structured singular values; low rank ODE's
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1. Introduction

A time-varying (TV) system is a system whose dynamics changes over time. Consider the example of a rocket traveling through space. As a rocket burns a lot of fuel its mass reduces quickly over a short periods of time. The dynamics of a linear time–varying (LTV) system is usually described by a system of differential equation with variable coefficients. Other examples of such systems includes wireless communication networks, amplitude modulation, aircrafts and human vocal tract. These systems are also extensively used to model the dynamics of a large class of engineering and biological systems. The linear time invariant (LTI) systems are a particular case of the TV system. Therefore TV systems are studied as a generalization of time invariant systems [1, 2].

The most important characteristic of a dynamical system is its stability [3]. One has to ensure the stability for a system to determine its accurate behavior. During the construction, testing and simulation, it is essential to check whether the test object is stable and we need to define the settings for which this system is stable. The stability conditions are a quick and a reliable tool to ensure stability. An important question is whether the state space approach of time invariant systems extends naturally to TV systems. It has been observed [3] that state space transition matrices are almost impossible to compute for most of the TV systems. The most important distinction here is that stability of LTI systems mostly depend on system parameters but in the case of TV systems it does not. For example, it is known that the stability of time invariant systems totally depends on the eigenvalues of its system matrix. In the case of TV systems, however, there is no direct relation can be established between stability and the location of eigenvalues of their system matrix.

Consider the following TV system $\dot{x}(t) = A(t)x(t)$, where $t \in [a, \infty)$ for some finite number a. We take $A(t)$ to be an $n \times n$ matrix of piece wise continuous functions over $[a, \infty)$. We will denote by $\Phi(t, t_0)$, t, $t_0 \in [a, \infty)$, $t \geq t_0$, the state transition matrix for this system. The system is said to be stable if for every $\epsilon > 0$ and for every $t_0 \in [a, \infty)$, there exists a $\delta > 0$ (δ depends on t_0 and ϵ) such that $|x(t_0)| < \delta$ implies $|x(t)| < \epsilon$. It is well known [3] that the above system is sable if and only if $|\Phi(t, t_0)| \leq k(t_0)$ for any $t_0 \in [a, \infty)$ and for some $k(t_0) > 0$. It is worth mentioning here that there are other notions of stability in literature such as uniform stability (δ does not depend on t_0) and exponential stability [4].

Structured Singular Value (SSV) (μ-values) for complex matrices in terms of a family of block diagonal matrices is introduced in [5]. In the μ-theory [6], the number of structured uncertainty embedded in a given system is rather large. We denote by $\Delta_i(jw)$ an uncertain element. Each of $\Delta_i(jw)$ can be consider as LTI value. The uncertainties $\Delta_i(jw)$ can either be real, complex or mixed (both real and complex). If $\Delta_i = 0$ we say that the system is nominal and it is called perturbed if $\Delta_i \neq 0$.

Conceptually, SSV is a straight forward generalization of singular values. SSV provides a tool to analyze and synthesize the robustness and performance of feedback systems in linear control theory. These tools are helpful to study stability analysis of structured eigenvalue perturbation theory and that of uncertain linear control systems. We put our attention to robust stability problem related to standard feedback interconnection of stable matrices $M(s)$ and $\Delta(s)$ for some $s \in \mathbb{C}_+$. Next we write $M(s)$ and $\Delta(s)$ as M and Δ.

The instability of feedback system is directly related with the measure of quantity $(I - M\,\Delta)$ to be singular where I is the identity matrix with dimension similar to M and Δ. This restrict the choice of Δ in the sense of $\|\Delta\|_\infty$ which cause instability to closed loop system. For stability analysis of closed loop system $\|\Delta\|_\infty < \alpha$, $\alpha \in \mathbb{R}$, $\alpha > 0$.

The increase in α up to α_{max} allows feedback system to be unstable. The quantity α_{max} gives robust stability margin of feedback system. Since stability analysis of a feedback system depends upon the fact that quantity $(I - M\,\Delta)$ remains non-singular for the values of M and Δ.

In [7], Zhou gives a detailed account of problems and progress on the stability of TV system. He then gives some results on the asymptotic stability of LTV systems. He also considers a class of upper triangular TV systems and establishes their stability. We consider LTV systems with the matrices $A(t)$ given as follows [1, 7].

$$A(t) = \begin{pmatrix} -1 & e^{\alpha_{12} t} \\ 0 & -3 \end{pmatrix}, \quad \forall t \in [0, \infty), \tag{1.1}$$

where α_{12} is a constant scalar.

$$A(t) = \begin{pmatrix} 0 & 1 \\ -\frac{1}{t+1} & -10 \end{pmatrix}, \quad \forall t \in [0, \infty). \tag{1.2}$$

$$A(t) = \begin{pmatrix} \frac{e^{-t}-3}{e^{-t}-2} & 0 & \frac{2e^{-t}-1}{e^{-t}-2} \\ 0 & 1 & 0 \\ \frac{1}{2-e^{-t}} & 0 & \frac{3(e^{-t}-1)}{e^{-t}-2} \end{pmatrix}. \tag{1.3}$$

$$A(t) = \begin{pmatrix} 2+\cos(t^2) & te^{-t} & 0 \\ -1 & 1-|\sin(t)| & 1 \\ -1 & 0 & \arctan(t)e^{-t^2} \end{pmatrix}. \tag{1.4}$$

The aim of this paper is to study stability analysis of linear time-varying systems in control. Furthermore, we also discuss the pseudo-spectrum with the help of EigTool of coefficient matrices $A(t)$ describing such systems. Our approach is based on low-rank ordinary differential equations which involves the computation of admissible perturbations and perturbation level to approximate the bounds of μ-values. The bounds of μ-values describes the stability of linear time-varying systems.

2. Preliminaries

Definition 2.1. The spectrum of a square complex valued matrix $M \in \mathbb{C}^{n,n}$ is defined as

$$\Lambda(M) = \{\lambda \in \mathbb{C} : \ |(\lambda I - M)| = 0\}.$$

Definition 2.2. The pseudospectrum of a complex matrix $M \in \mathbb{C}^{n,n}$ with a small positive real parameter $\epsilon > 0$ is defined as

$$\Lambda_\epsilon(M) = \{\lambda \in \mathbb{C} : \ |(\lambda I - M)^{-1}| \geq \frac{1}{\epsilon}\}.$$

Definition 2.3. Unstructured uncertainty $\mathbb{B}$ or structured uncertainty $\mathbb{B}$ is stable transfer matrix or structured stable transfer matrix having the form

$$\mathbb{B} = \{diag(\delta_1 I_1, \delta_2 I_2, ..., \delta_s I_S; \Delta_1, \Delta_2, ..., \Delta_F) : \delta_i \in \mathbb{C}, \Delta_j \in \mathbb{C}^{m_j,m_j}, \forall i = 1 : S, j = 1 : F\}.$$

Definition 2.4. For n-dimensional matrix $M \in \mathbb{C}^{n,n}$ and a perturbation set $\mathbb{B}$, the μ-value is defined as following:

$$\mu_{\mathbb{B}}(M) := \frac{1}{min\{\|\Delta\|_2 : \Delta \in \mathbb{B}, det(I - M\Delta) = 0\}},$$

with no such Δ that cause the perturbed matrix $(I - M\Delta)$ to be singular such that $\mu_{\mathbb{B}}(M) = 0$.

Theorem 2.5. (Small Gain Theorem [8]). The control system is well-posed and stable for an admissible perturbation Δ and bounded above by 1 iff

$$\|M\|_\infty : Sup(\|M(jw)\|) < 1,$$

for some $w \in \mathbb{R}^+$, the frequency.

Theorem 2.6. [9]. For two structured uncertainties $\mathbb{B}_1 \subset \mathbb{B}_2$, then

$$\mu_{\mathbb{B}_1}(\|M(jw)\|) < \mu_{\mathbb{B}_2}(\|M(jw)\|).$$

The control system is well-posed and internally stable for a perturbation $\Delta \in \mathbb{B}$ with $\|\Delta\|_2 \leq 1$ iff $Sup(M(jw)) < 1$ for some $w \in \mathbb{R}^+$.

3. Reformulation of μ-values

This section is devoted to reformulation of μ-values. The key idea for the reformulation of the structured singular values is to shift the largest eigenvalue of $I - M\Delta(t)$ such that for $\lambda_{max} = 1$ the new eigenvalue $\eta = 0$ as $\eta = 1 - \lambda_{max}$ and it achieve the maximum value to be 1 when $\lambda_{max} = 0$. On the basis of this mathematical construction, the reformulation of structured singular values is given as below.

Definition 3.1. For $M \in \mathbb{C}^{n,n}$ and an admissible perturbation level $\epsilon > 0$, structured spectral value set is denoted by $\Lambda_\epsilon^{\mathbb{B}}(M)$ and is defined as follows:

$$\Lambda_\epsilon^{\mathbb{B}}(M) = \{\lambda \in \Lambda(\epsilon M\Delta), \Delta \in \mathbb{B}, \|\Delta\|_2 \leq 1\},$$

where $\Lambda(\epsilon M\Delta)$ denotes the spectrum of $(\epsilon M\Delta)$ and is simply a disk centered at origin 0.

Definition 3.2. The structured epsilon spectral value set for $M \in \mathbb{C}^{n,n}$ and $\epsilon \geq 0$ is defined as

$$\Sigma_\epsilon^{\mathbb{B}}(M) = \{\eta : 1 - \lambda : \lambda \in \Lambda_\epsilon^{\mathbb{B}}(M)\}.$$

Definition 3.3. For a given $M \in \mathbb{C}^{n,n}$ and an underlying perturbation set $\mathbb{B}$ the μ-value is defined as

$$\mu_{\mathbb{B}}(M) = \frac{1}{arg\ min_{\epsilon > 0}\{max|\lambda| = 1, \lambda \in \Lambda_\epsilon^{\mathbb{B}}(M)\}}.$$

4. Pseudo-spectra

In this section we present the pseudospectra for matrices under consideration to whom the goal is to approximate structured singular values. For this purpose we make use of the software package EigTool [10]. EigTool is routinely used for plotting unstructured pseudospectra of the matrices under consideration. In Figures 1 and 2, we show the computation of the pseudospectra of a different matrices as taken in examples 1–4.

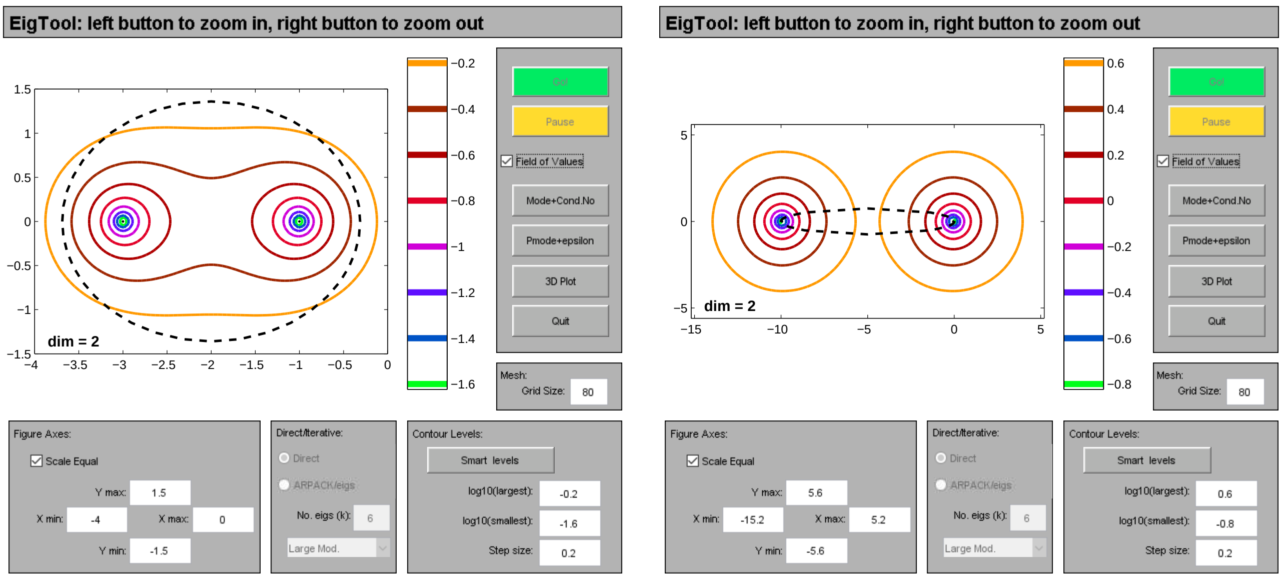

Figure 1. Matlab interface for computing pseudo-spectrum of Examples 1 and 2.

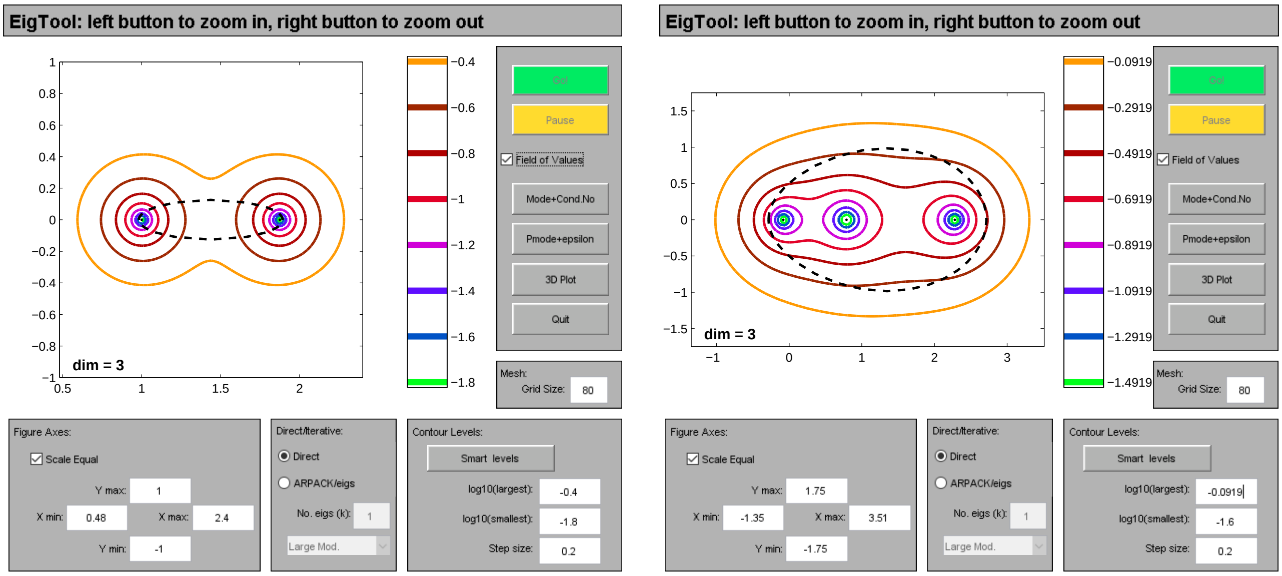

Figure 2. Matlab interface for computing pseudo-spectrum of Examples 3 and 4.

5. Proposed methodology

For use low-rank ordinary differential equaton technique for solving optimization problem discussed in Definition 7. The numerical technique is composed of two-level algorithm, that is, inner-outer algorithm. In inner-algorithm the main goal is to first construct and then solve a gradient system of ordinary differential equations. In Outer-algorithm we change the perturbation level $\epsilon > 0$ by using Newton's method. The outer-algorithm approximate an exact derivative of extremizer $\Delta(\epsilon)$ for $\Delta \in \mathbb{B}$ and $\epsilon > 0$. A complete detail of numerical method under consideration is given in [11].

6. Numerical experimentation

In this section we compute the structured singular values of the systems given in Eqs (1.1)–(1.4) for increasing values of t. We also demonstrate the stability of given systems in graphs.

Example 1. First we take $\alpha = 1$ and $t = 1$ in (1.1), to get

$$M = \begin{bmatrix} -1 & 2.7182 \\ 0 & -3 \end{bmatrix}.$$

We choose the perturbation set $\mathcal{B} = \{diag(\delta_i I_1) : \delta_i \in \mathbb{R}, \forall\, i = 1 : 2\}$. The MATLAB function **mussv** computes an admissible perturbation $VDelta$ given as

$$VDelta = \begin{bmatrix} -0.0286 & 0.0296 \\ 0.1665 & -0.1729 \end{bmatrix},$$

norm($VDelta$)= 0.2436.

The largest singular value (LSV) of matrix $VDelta$ is 0.2436. The approximate lower and upper bounds given by **mussv** are 4.1054 and 4.1054 respectively. Moreover by using the algorithm [11], the perturbation matrix E is obtained as

$$E = \begin{bmatrix} -0.1172 & 0.1217 \\ 0.6838 & -0.7099 \end{bmatrix}.$$

The LSV of matrix E is computed as 1. The lower bound approximation is 4.1054. For $t = 1.5$ in (1.1), we have

$$M = \begin{bmatrix} -1 & 4.4816 \\ 0 & -3 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is 0.1832. The approximate lower and upper bounds given by **mussv** are 5.4573 and 5.4573 respectively. Moreover by using the algorithm [11], the LSV corresponding to E is computed as 1. The lower bound approximation is 5.4573. For $t = 2.0$ in (1.1), we have

$$M = \begin{bmatrix} -1 & 7.3890 \\ 0 & -3 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is 0.1246. The approximate lower and upper bounds given by **mussv** are 8.0286 and 8.0286 respectively. Moreover by using the algorithm [11], the LSV corresponding to E is computed as 1. The lower bound approximation is 8.0286. For $t = 2.5$ in (1.1), we have

$$M = \begin{bmatrix} -1 & 12.1824 \\ 0 & -3 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is 0.0795. The approximate lower and upper bounds given by **mussv** are 12.5839 and 12.5839 respectively. Moreover by using the algorithm [11], the LSV of matrix E is computed as 1. The lower bound approximation is 12.5839.

Example 2. First we take $\alpha = 1$ and $t = 1$ in (1.2), to get

$$M = \begin{bmatrix} 0 & 1 \\ -0.5 & -10 \end{bmatrix}.$$

We choose the perturbation set $\mathcal{B} = \{diag(\delta_i I_1) : \delta_i \in \mathbb{R}, \forall\, i = 1 : 2\}$. The MATLAB function **mussv** computes an admissible perturbation $VDelta$ given as

$$VDelta = \begin{bmatrix} 0.0005 & -0.0049 \\ 0.0099 & -0.0988 \end{bmatrix},$$

norm($VDelta$)= 0.0994. The LSV of matrix $VDelta$ is 0.0994. The approximate lower and upper bounds given by **mussv** are 10.0622 and 10.0622 respectively. Moreover by using the algorithm [11], the perturbation matrix E is obtained as

$$E = \begin{bmatrix} 0.0050 & -0.0499 \\ 0.0999 & -0.9937 \end{bmatrix}.$$

The LSV of matrix E is 1. The lower bound approximation is 10.0622. For $t = 1.5$ in (1.2), we have

$$M = \begin{bmatrix} 0 & 1 \\ -0.4 & -10 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is 0.0994. The approximate lower and upper bounds given by **mussv** are 10.0578 and 10.0578 respectively. Moreover by using the algorithm [11], the LSV corresponding to E is computed as 1. The lower bound approximation is 10.0578. For $t = 2.0$ in (1.2), we have

$$M = \begin{bmatrix} 0 & 1 \\ -1/3 & -10 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is 0.0994. The approximate lower and upper bounds given by **mussv** are 10.0553 and 10.0554 respectively. Moreover by using the algorithm [11], the LSV of matrix E is computed as 1. The lower bound approximation is 10.0332. For $t = 2.5$ in (1.2), we have

$$M = \begin{bmatrix} 0 & 1 \\ -0.2857 & -10 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is 0.0995. The approximate lower and upper bounds given by **mussv** are 10.0539 and 10.0539 respectively. Moreover by using the algorithm [11], the LSV corresponding to E is computed as 1. The lower bound approximation is 10.0539.

Example 3. First we take $\alpha = 1$ and $t = 1$ in (1.3), to get

$$M = \begin{bmatrix} 1.6126 & 0 & 0.1619 \\ 0 & 1 & 0 \\ 0.6126 & 0 & 1.1619 \end{bmatrix}.$$

We choose the perturbation set $\mathcal{B} = \{diag(\delta_i I_1) : \delta_i \in \mathbb{R}, \forall\, i = 1 : 3\}$. The MATLAB function **mussv** computes an admissible perturbation $VDelta$ given as

$$VDelta = \begin{bmatrix} 0.4020 & 0 & 0.2764 \\ 0 & 0 & 0 \\ 0.1899 & 0 & 0.1306 \end{bmatrix},$$

norm($VDelta$)= 0.5395. The LSV of matrix $VDelta$ is 0.5395. The approximate lower and upper bounds given by **mussv** are 1.8535 and 1.8535 respectively. Moreover by using the algorithm [11], the admissible perturbation E is obtained as

$$E = \begin{bmatrix} 0.7450 & 0 & 0.5123 \\ 0 & 0 & 0 \\ 0.3520 & 0 & 0.2420 \end{bmatrix}.$$

The LSV of matrix E is computed as 1. The lower bound approximation is 1.8535. For $t = 1.5$ in (1.3), we have

$$M = \begin{bmatrix} 1.5627 & 0 & 0.3116 \\ 0 & 1 & 0 \\ 0.5627 & 0 & 1.3116 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is 0.5335. The approximate lower and upper bounds given by **mussv** are 1.8743 and 1.8743 respectively. Moreover by using the algorithm [11], the LSV corresponding to E is computed as 1. The lower bound approximation is 1.8743. For $t = 2.0$ in (1.3), we have

$$M = \begin{bmatrix} 1.5362 & 0 & 0.3911 \\ 0 & 1 & 0 \\ 0.5362 & 0 & 1.3911 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is found as 0.5169. The approximate lower and upper bounds given by **mussv** are 1.9347 and 1.9347 respectively. Moreover by using the algorithm [11], the LSV corresponding to E is computed as 1. The lower bound approximation is 1.9347. For $t = 2.5$ in (1.3), we have

$$M = \begin{bmatrix} 1.5213 & 0 & 0.4358 \\ 0 & 1 & 0 \\ 0.5213 & 0 & 1.4358 \end{bmatrix}.$$

The LSV of matrix $VDelta$ is found as 0.5110. The approximate lower and upper bounds given by **mussv** are 1.9571 and 1.9571 respectively. Moreover by using the algorithm given [11], the LSV corresponding to E is computed as 1. The lower bound approximation is 1.9571.

Example 4. First we take $\alpha = 1$ and $t = 1$ in (1.4), to get

$$M = \begin{bmatrix} 2.5403 & 0.3678 & 0 \\ -1 & 0.1585 & 1 \\ -1 & 0 & 0.2889 \end{bmatrix}.$$

We choose the perturbation set $\mathcal{B} = \{diag(\delta_i I_1) : \delta_i \in \mathbb{R}, \forall\, i = 1 : 3\}$. The MATLAB function **mussv** computes an admissible perturbation $VDelta$ given as

$$VDelta = \begin{bmatrix} 0.2844 & -0.1274 & -0.1159 \\ 0.0249 & -0.0111 & -0.0101 \\ -0.0474 & 0.0212 & 0.0193 \end{bmatrix},$$

norm($VDelta$)= 0.3383. The LSV of matrix $VDelta$ is calculated as 0.3383. The approximate lower and upper bounds given by **mussv** are 2.9556 and 2.9556 respectively. Moreover by using the algorithm [11], the perturbation matrix E is obtained as

$$E = \begin{bmatrix} 0.8406 & -0.3767 & -0.3425 \\ 0.0735 & -0.0329 & -0.0299 \\ -0.1401 & 0.0628 & 0.0571 \end{bmatrix}.$$

The LSV corresponding to perturbation matrix E is computed as 1. The lower bound approximation is 2.9556. For $t = 1.5$ in (1.4), we have

$$M = \begin{bmatrix} 1.3718 & 0.3346 & 0 \\ -1 & 0.0025 & 1 \\ -1 & 0 & 0.1035 \end{bmatrix}.$$

The LSV of matrix *VDelta* is 0.5906. The approximate lower and upper bounds given by **mussv** are 1.6932 and 1.6932 respectively. Moreover by using the algorithm [11], the LSV corresponding to perturbation matrix E is computed as 1. The lower bound approximation is 1.6932. For $t = 2.0$ in (1.4), we have

$$M = \begin{bmatrix} 1.3463 & 0.2706 & 0 \\ -1 & 0.0907 & \\ -1 & 0 & 0.0202 \end{bmatrix}.$$

The LSV of matrix *VDelta* is 0.4838. The approximate lower and upper bounds given by **mussv** are 2.0672 and 2.0672 respectively. Moreover by using the algorithm [11], the LSV corresponding to perturbation matrix E is computed as 1. The lower bound approximation is 2.0672. For $t = 2.5$ in (1.4), we have

$$M = \begin{bmatrix} 2.9994 & 0.2052 & 0 \\ -1 & 0.4015 & 1 \\ -1 & 0 & 0.0022 \end{bmatrix}.$$

The LSV of matrix *VDelta* is 0.3249. The approximate lower and upper bounds given by **mussv** are 3.0776 and 3.0776 respectively. Moreover by using the algorithm [11], the LSV corresponding to admissible perturbation matrix E is computed as 1. The lower bound approximation is 3.0397.

7. Conclusion

In this article we have presented the numerical computations of lower and upper bounds of structured singular values for the matrices arising in LTV systems. The lower bounds of structured singular values discuss the instability of linear system in control. The upper bounds of structured singular values gives stability analysis of control systems. The numerical experiments shows that the obtained bounds (lower and upper) of structured singular values by making using of low rank odes based technique matches with the results obtained by well-known MATLAB function mussv.

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Conflict of interest

The authors declare that they have no competing interests.

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