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Research article

Exponential stability analysis and control design for nonlinear system with time-varying delay

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Abstract: This paper investigates the problem of exponential stability analysis and control design for time delay nonlinear systems with unknown control coefficient. Nussbaum gain function is utilized to solve the problem of unknown control directions at every step. By designing a new Lyapunov-Krasovskii functional, the problem of unknown time-varying delay is solved. Under the frame of adaptive backstepping recursive design, an exponential stabilization control algorithm is developed, which demonstrates that all solutions of controlled system are ultimately uniformly bounded (UUB) and exponential converge to zero. Finally, simulation results are displayed to explain the superiority and effectiveness of the developed control method.

Keywords: exponential stabilization; time-varying delay; Nussbaum gain function backstepping recursive design
Mathematics Subject Classification: 93B52, 93C42

1. Introduction

Since there exists the time-delay phenomenon in the process of signal transmission, the control problems of time delay nonlinear systems have been widely paid attention in the field of industrial engineering, and some works have been received, such as [1–9]. The commonly time-delay nonlinear systems contain input time-delay [1–3] and state time-delay [4–9]. On the one hand, the authors in [1] and [2] investigated the fuzzy adaptive sampling control for nonlinear systems with input time-delay. By adopting Pade approximation method, [3] developed fuzzy adaptive tracking control algorithm for nonlinear system with input delay. On the other hand, when considering the state time-delay, the authors in [4] and [5] investigated the robust adaptive control design problems for nonlinear systems with unknown time-delay. However, in [4] and [5], the considered time-delay belongs to the constant delay, the difficulty of control design process is less than that of time-varying delay system. Thus, the authors in [6–8] developed adaptive tracking control algorithm for time-varying delay nonlinear systems, and [9] studied the adaptive tracking output feedback control issue for time-varying delay system by constructing a state observer.

It is worth noting that the above developed control algorithms are all required the control direction is known. However, in practical engineering systems, the control direction is unknown, it will increase the design difficulty of these systems. Thus, the Nussbaum gain function technique is developed to cope with this issue, and some interesting works have been published, see [10–13]. In [10], the authors studied the adaptive fuzzy output-feedback control problem for nonlinear system with unknown control gain functions. The work [11] developed adaptive robust tracking control method for nonlinear system with unknown control direction by adopting smooth projection operator and Nussbaum gain function. By adopting the approximation property of FLS, the authors in [12] and [13] studied the fuzzy adaptive output feedback control issues for nonlinear systems with unknown control coefficient.

Noted that the convergence rate of the system states has an important influence in practical industry systems. Obviously, compared with the asymptotic stability in the above results, the exponential stability has the better control performance. Thus, the authors in [14] first studied the exponential stability for nonlinear system. Then, inspired by [14], the authors in [15] developed exponential stabilization for uncertain nonholonomic systems, and [16] studied the output feedback exponential stability for nonlinear system. When considering the interconnection of each subsystems and stochastic disturbance, the authors in [17] and [18] developed the exponential stability control for nonlinear systems. The works [19] and [20] developed global exponential stabilization control algorithm for nonlinear systems. It is worth pointing out that there are no available results about the exponential stability analysis and control design for nonlinear systems with unknown control coefficient and time-varying delay.

This paper studies the problem of adaptive exponential stability analysis and control design for time-varying delay nonlinear system with unknown control coefficients. Nussbaum gain functions is adopted in each step to solve the issue of unknown control direction. By designing a Lyapunov-krasovskii functional, the issue of time-varying delay is solved. Compared with the existing results, the major highlights of this paper can be summarized as

1) This paper first studied the adaptive exponential stability analysis and control design problem for SISO nonlinear systems. Under the adaptive backstepping control technique, this paper developed adaptive exponential stabilization control algorithm, which can guarantee all solutions of the controlled system are UUB and exponential converge to zero.

2) Compared with [17], the Lyapunov-Krasovskii functionals are adopted to deal with the problems of unknown time-varying delay, and the considered system is nonlinear systems, instead of linear ones.

The remainder of this paper is organized as follows. In Section II, the problem description and the preliminary knowledge are formulated. Exponential controller design and stability analysis are given in Section III. Simulation studies illustrating the effectiveness of the developed control algorithm are given in Section IV. Finally, we conclude the paper in Section V.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider a class of nonlinear systems as

$$\begin{cases} \dot{x}_i = \theta_i^T(t)\varphi_i(\bar{x}_i) + g_i x_{i+1} + q_i(y(t-\tau_i(t))) \\ \dot{x}_n = \theta_n^T(t)\varphi_n(x) + g_n u + q_n(y(t-\tau_n(t))) \\ y = x_1\ ,\ i = 1,2,\cdots,n-1, \end{cases} \tag{2.1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T$ ($x = [x_1, x_2, \cdots, x_n]^T$) is the state vector, y and u are the output and control input, respectively. $q_i(t)$ are unknown nonlinear functions. $\theta_i(t)$ are vectors of time-varying and uncertain parameters. $\varphi_i(\cdot)$ are known continuous nonlinear functions. $g_i \neq 0 (i = 1, 2, \cdots, n)$ are unknown constants, and they are referred to as virtual control coefficients. $\tau_i(t)$ is the time-varying delay and satisfies $\dot{\tau}_i(t) \le \tau^* \le 1$, $|\tau_i| \le \tau$ with constants τ^* and τ.

Remark 1. Nonlinear system (2.1) is a huger class of nonlinear SISO strict-feedback systems and has been studied extensively in some published results. In [11], the adaptive robust control of the unknown control coefficients was addressed for nonlinear systems. However, [11] are not considered the unknown time-varying delays problems. In fact, when the time-varying delays appears in systems, the control design in [11] will need to be reconstructed. In this paper, the time-varying delays will be handled by designing a Lyapunov-Krasovskii functional.

Assumption 1. ([9]) There exist positive constant ϖ_i and known function $Q_i(\cdot)$, nonlinear function $q_i(\cdot)$ satisfies

$$|q_i(y(t-\tau_i(t)))|^2 \le z_1(t-\tau_i(t))Q_{i1}(z_1(t-\tau_i(t))) + \varpi_i$$

Control Objective: This paper will develop an exponential stabilization control algorithm such that all solutions of the controlled system are UUB and exponentially converge to zero.

To deal with the issue of unknown control coefficient g_i, the Nussbaum gain technique is utilized in this note.

Definition 1. ([11, 12]) Nussbaum-type function $N(\zeta)$ satisfies

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\zeta)d\zeta = \infty \ \text{ and } \ \lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\zeta)d\zeta = -\infty \tag{2.2}$$

The Nussbaum functions that are commonly used are $\exp(\zeta^2)\cos((\pi/2)\zeta)$, $\zeta^2\sin(\zeta)$, $\zeta^2\cos(\zeta)$. In this note, we choose the Nussbaum functions as $N(\zeta) = \zeta^2\cos(\zeta)$.

Lemma 1. ([11,12]) $\zeta(t)$ is a continuous functions defined on $[0, t_f)$, $N(\zeta)$ is called as Nussbaum function. If the positive definite function $V(t)$ satisfies

$$V(t) \le D + \mathrm{e}^{-\lambda t}\int_0^t g(\tau)N(\zeta)\dot{\zeta}\mathrm{e}^{\lambda\tau}d\tau + \mathrm{e}^{-\lambda t}\int_0^t \dot{\zeta}\mathrm{e}^{\lambda\tau}d\tau$$

where $D > 0$ $\lambda > 0$ and $g(\tau)$ is time-varying parameter in $I := [l^-, l^+]$ $(0 \notin I)$, thus $V(t)$, $\zeta(t)$ and $\int_0^t g(\tau)N(\zeta)\dot{\zeta}d\tau$ are bounded on $[0, t_f)$.

3. Exponential controller design

In this section, an exponential stabilization controller needs to be designed.

Define the following cooperation transactions as

$$z_1 = x_1 \tag{3.1}$$

$$z_i = x_i - \alpha_{i-1} \tag{3.2}$$

where z_i are the virtual errors, α_i $(i = 2, \cdots, n)$ is the virtual control function, which will be designed later.

Step 1. According to (2.1) and (3.1), we have

$$\begin{aligned}\dot{z}_1 &= g_1 x_2 + \theta_1^T(t)\varphi_1(x_1) + q_1(y(t-\tau_1(t))) \\ &= g_1(z_2+\alpha_1) + \theta_1^T(t)\varphi_1(x_1) + q_1(y(t-\tau_1(t)))\end{aligned} \tag{3.3}$$

Choose Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + W_1 + \frac{1}{2\gamma_1}\tilde{\theta}_{m,1}^T\tilde{\theta}_{m,1} \tag{3.4}$$

where $\gamma_1 > 0$ is a design constant, $\hat{\theta}_{m,1}$ is the estimation of $\theta_{m,1}$ and $\tilde{\theta}_{m,1} = \theta_{m,1} - \hat{\theta}_{m,1}$. Define $W_1 = \frac{e^{r(\tau-t)}}{2(1-\tau_1^*)}\int_{t-\tau_1(t)}^{t} e^{rs} z_1(s)Q_{1,1}(z_1(s))ds$, thus, $\dot{W}_1 \leq -rW_1 + \frac{e^{r\tau}}{2(1-\tau^*)}z_1(t)Q_{1,1}(z_1(t)) - \frac{1}{2}z_1(t-\tau_1(t))Q_{1,1}(z_1(t-\tau_1(t)))$.

From (3.3) and (3.4), we have

$$\dot{V}_1 = z_1[g_1z_2 + g_1\alpha_1 + \theta_{m,1}^T(t)\varphi_{m,1}(x_1) + q_1(y(t-\tau_1(t)))] + \dot{W}_1 - \frac{1}{\gamma_1}\tilde{\theta}_{m,1}^T\dot{\hat{\theta}}_{m,1} \tag{3.5}$$

where $\theta_{m,1} = \theta_1$ and $\varphi_{m,1} = \varphi_1$.

Utilizing Young's inequality [26,27]

$$c^T d \leq \frac{\epsilon^m}{m}\|c\|^m + \frac{1}{n\epsilon^n}\|d\|^n \tag{3.6}$$

where $\epsilon > 0$, $c, d \in R$, $n, m > 1$ with $(n-1)(m-1) = 1$. One has

$$g_1z_1z_2 \leq \frac{1}{4}z_1^2 + \bar{g}_1^2 z_2^2 \tag{3.7}$$

$$z_1q_1(y(t-\tau_1(t))) \leq \tfrac{1}{2}z_1^2 + \tfrac{1}{2}z_1(t-\tau_1(t))Q_{1,1}(z_1(t-\tau_1(t))) + \tfrac{1}{2}\varpi_1 \tag{3.8}$$

where $\bar{g}_1$ is positive constant and satisfies $|g_1| < \bar{g}_1$.

By invoking (3.5)–(3.8), we have

$$\begin{aligned}\dot{V}_1 \leq\ & z_1[g_1\alpha_1 + \hat{\theta}_{m,1}^T(t)\varphi_{m,1}(x_1) + \tfrac{3}{4}z_1 + \tfrac{e^{r\tau}}{2(1-\tau_1^*)}Q_{1,1}(z_1(t))] \\ &+ \tfrac{\tilde{\theta}_{m,1}^T}{\gamma_1}[\gamma_1 z_1\varphi_{m,1}(x_1) - \dot{\hat{\theta}}_{m,1}] + \bar{g}_1^2z_2^2 + \tfrac{1}{2}\varpi_1 - rW_1\end{aligned} \tag{3.9}$$

Design the virtual control function α_1 and update law $\dot{\hat{\theta}}_{m,1}$ as

$$\alpha_1 = N(\zeta_1)[c_1 z_1 + \hat{\theta}^T_{m,1}(t)\varphi_{m,1}(x_1) + \tfrac{3}{4}z_1 + \sum_{j=1}^{n} \bar{Q}_j\{z_1(t)\}] \tag{3.10}$$

$$\dot{\hat{\theta}}_{m,1} = \gamma_1 z_1 \varphi_{m,1}(x_1) - \sigma_1 \hat{\theta}_{m,1} \tag{3.11}$$

$$\dot{\zeta}_1 = z_1[c_1 z_1 + \hat{\theta}^T_{m,1}(t)\varphi_{m,1}(x_1) + \tfrac{3}{4}z_1 + \sum_{j=1}^{n} \bar{Q}_j\{z_1(t)\}] \tag{3.12}$$

where $\sigma_1 > 0$ and $c_1 > 0$ are design parameters. $\bar{Q}_j = \sum_{k=1}^{j} \frac{e^{r\tau}}{2(1-\tau_k^*)} Q_{k,1}(z_1(t))$.

Thus, (3.9) can be rewritten as

$$\begin{aligned} \dot{V}_1 \le& -c_1 z_1^2 + (g_1 N(\zeta_1) + 1)\dot{\zeta}_1 + \bar{g}_1^2 z_2^2 + \tfrac{1}{2}\varpi_1 \\ &+ \tfrac{\sigma_1}{\gamma_1}\tilde{\theta}^T_{m,1}\hat{\theta}_{m,1} - \sum_{j=2}^{n} z_1(t)\bar{Q}_j\{z_1(t)\} - rW_1 \end{aligned} \tag{3.13}$$

Step 2. According to (2.1) and (3.2), we have

$$\begin{aligned} \dot{z}_2 &= g_2 x_3 + \theta_2^T(t)\varphi_2(\bar{x}_2) + q_2(y(t-\tau_2(t))) - \dot{\alpha}_1 \\ &= g_2(z_3 + \alpha_2) + \theta^T_{m,2}(t)\varphi_{m,2}(\bar{x}_2) + q_2(y(t-\tau_2(t))) \\ &\quad - \tfrac{\partial\alpha_1}{\partial x_1} q_1(y(t-\tau_1(t))) - H_2 \end{aligned} \tag{3.14}$$

where $H_2 = \frac{\partial\alpha_1}{\partial\hat{\theta}_{m,1}}\dot{\hat{\theta}}_{m,1} + \frac{\partial\alpha_1}{\partial\zeta_1}\dot{\zeta}_1$. $\varphi_{m,2} = [\varphi_2^T(\bar{x}_2), -(\partial\alpha_1/\partial x_1)\varphi_1^T(x_1), -(\partial\alpha_1/\partial x_1)x_2]^T$, $\theta_{m,2} = [\theta_2^T, \theta_1^T, g_1]^T$.

Choose Lyapunov function as

$$V_2 = V_1 + \tfrac{1}{2}z_2^2 + W_2 + \tfrac{1}{2\gamma_2}\tilde{\theta}^T_{m,2}\tilde{\theta}_{m,2} \tag{3.15}$$

where $\gamma_2 > 0$ is a design constant. $\hat{\theta}_{m,2}$ is the estimation of $\theta_{m,2}$ and $\tilde{\theta}_{m,2} = \theta_{m,2} - \hat{\theta}_{m,2}$.

Define $W_2 = \sum_{k=1}^{2} \frac{2^{r(\tau-t)}}{2(1-\tau_k^*)} \int_{t-\tau_k(t)}^{t} e^{rs} z_1(s) Q_{k,1}(z_1(s))ds$, thus, we have $\dot{W}_2 \le -rW_2 + \sum_{k=1}^{2} \frac{e^{r\tau}}{1-\tau_k^*} z_1(t)Q_{k,1}(z_1(t)) - \frac{1}{2}\sum_{k=1}^{2} z_1(t-\tau_k(t))Q_{k,1}(z_1(t-\tau_k(t)))$.

Thus,the derivation of V_2 is

$$\begin{aligned} \dot{V}_2 =& \dot{V}_1 + z_2[g_2(z_3 + \alpha_2) + \theta^T_{m,2}(t)\varphi_{m,2}(\bar{x}_2) + q_2(y(t-\tau_2(t))) \\ &- \tfrac{\partial\alpha_1}{\partial x_1} q_1(y(t-\tau_1(t))) - H_2] - \tfrac{1}{\gamma_2}\tilde{\theta}^T_{m,2}\dot{\hat{\theta}}_{m,2} + \dot{W}_2 \end{aligned} \tag{3.16}$$

From (3.6), we can get

$$z_2(g_2 z_3 + q_2) \le \tfrac{3}{4}z_2^2 + \bar{g}_2^2 z_3^2 + \tfrac{1}{2}z_1(t-\tau_2(t))Q_{2,1}(z_1(t-\tau_2(t))) + \tfrac{1}{2}\varpi_2 \tag{3.17}$$

$$-z_2\tfrac{\partial\alpha_1}{\partial x_1}q_1 \le \tfrac{1}{2}z_2^2(\tfrac{\partial\alpha_1}{\partial x_1})^2 + \tfrac{1}{2}z_1(t-\tau_1(t))Q_{1,1}(z_1(t-\tau_1(t))) + \tfrac{1}{2}\varpi_1 \tag{3.18}$$

where $\bar{g}_2$ is positive constant and satisfies $|g_2| < \bar{g}_2$.

By invoking (3.16)–(3.18), we have

$$\begin{aligned} \dot{V}_2 \leq& -c_1 z_1^2 + (g_1 N(\zeta_1) + 1)\dot{\zeta}_1 + \tfrac{\sigma_1}{\gamma_1}\tilde{\theta}_{m,1}^T \hat{\theta}_{m,1} - \sum_{j=3}^{n} z_1(t)\bar{Q}_j\{z_1(t)\} \\ &+ z_2[g_2\alpha_2 + \bar{g}_1^2 z_2 + \hat{\theta}_{m,2}^T(t)\varphi_{m,2}(\bar{x}_2) + \tfrac{3}{4}z_2 + \tfrac{1}{2}z_2(\tfrac{\partial\alpha_1}{\partial x_1})^2 - H_2] \\ &+ \tfrac{1}{\gamma_2}\tilde{\theta}_{m,2}^T[\gamma_2 z_2 \varphi_{m,2}(\bar{x}_2) - \dot{\hat{\theta}}_{m,2}] + \bar{g}_2^2 z_3^2 + \varpi_1 + \tfrac{1}{2}\varpi_2 - \sum_{k=1}^{2} rW_k \end{aligned} \tag{3.19}$$

Design the virtual control function α_2 and update law $\dot{\hat{\theta}}_{m,2}$ as

$$\alpha_2 = N(\zeta_2)[c_2 z_2 + \bar{g}_1^2 z_2 + \hat{\theta}_{m,2}^T(t)\varphi_{m,2}(\bar{x}_2) + \tfrac{3}{4}z_2 + \tfrac{1}{2}z_2(\tfrac{\partial\alpha_1}{\partial x_1})^2 - H_2] \tag{3.20}$$

$$\dot{\hat{\theta}}_{m,2} = \gamma_2 z_2 \varphi_{m,2}(\bar{x}_2) - \sigma_2 \hat{\theta}_{m,2} \tag{3.21}$$

$$\dot{\zeta}_2 = z_2[c_2 z_2 + \bar{g}_1^2 z_2 + \hat{\theta}_{m,2}^T(t)\varphi_{m,2}(\bar{x}_2) + \tfrac{3}{4}z_2 + \tfrac{1}{2}z_2(\tfrac{\partial\alpha_1}{\partial x_1})^2 - H_2] \tag{3.22}$$

where $\sigma_2 > 0$ and $c_2 > 0$ are design parameters.

Thus, rewrite (3.19) as

$$\begin{aligned} \dot{V}_2 \leq& -\sum_{k=1}^{2} c_k z_k^2 + \sum_{k=1}^{2}[(g_k N(\zeta_k) + 1)\dot{\zeta}_k] + \varpi_1 + \tfrac{1}{2}\varpi_2 \\ &+ \sum_{k=1}^{2}\tfrac{\sigma_k}{\gamma_k}\tilde{\theta}_{m,k}^T\hat{\theta}_{m,k} - \sum_{j=3}^{n} z_1(t)\bar{Q}_j\{z_1(t)\} + \bar{g}_2^2 z_3^2 - \sum_{k=1}^{2} rW_k \end{aligned} \tag{3.23}$$

Step i ($3 \leq i \leq n-1$): From (2.1) and (3.2), one has

$$\begin{aligned} \dot{z}_i =& g_i x_{i+1} + \theta_i^T(t)\varphi_i(\bar{x}_i) + q_i(y(t-\tau_i(t))) - \dot{\alpha}_{i-1} \\ =& g_i(z_{i+1} + \alpha_i) + \theta_{m,i}^T(t)\varphi_{m,i}(\bar{x}_i) + q_i(y(t-\tau_i(t))) \\ &- \sum_{l=1}^{i-1}\tfrac{\partial\alpha_{i-1}}{\partial x_l} q_l(y(t-\tau_l(t))) - H_i \end{aligned} \tag{3.24}$$

where $H_i = \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_{m,l}}\dot{\hat{\theta}}_{m,l} + \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\zeta_l}\dot{\zeta}_l$. $\varphi_{m,i} = [\varphi_i^T(\bar{x}_i), -\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}\varphi_{i-1}^T(\bar{x}_{i-1}), \cdots, -\frac{\partial\alpha_{i-1}}{\partial x_1}\varphi_1^T(\bar{x}_1), -\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}x_i, \cdots, -\frac{\partial\alpha_{i-1}}{\partial x_1}x_2]^T$, $\theta_{m,i} = [\theta_i^T, \cdots, \theta_1^T, g_{i-1}, \cdots, g_1]^T$.

Choose the Lyapunov function as

$$V_i = V_{i-1} + \tfrac{1}{2}z_i^2 + W_i + \tfrac{1}{2\gamma_2}\tilde{\theta}_{m,i}^T\tilde{\theta}_{m,i} \tag{3.25}$$

where $\gamma_i > 0$ is a design constant. $\hat{\theta}_{m,i}$ is the estimation of $\theta_{m,i}$ and $\tilde{\theta}_{m,i} = \theta_{m,i} - \hat{\theta}_{m,i}$.

Define $W_i = \sum_{k=1}^{i}\frac{2^{r(\tau-t)}}{2(1-\tau_k^*)}\int_{t-\tau_k(t)}^{t} e^{rs} z_1(s)Q_{k,1}(z_1(s))ds$, thus, we have $\dot{W}_i \leq -rW_i + \sum_{k=1}^{i}\frac{e^{r\tau}}{1-\tau_k^*}z_1(t)Q_{k,1}(z_1(t)) - \frac{1}{2}\sum_{k=1}^{i} z_1(t-\tau_k(t))Q_{k,1}(z_1(t-\tau_k(t)))$.

Thus, the derivation of V_i is

$$\begin{aligned}\dot{V}_i = {} & \dot{V}_{i-1} + z_i[g_i(z_{i+1}+\alpha_i) + \theta_{m,i}^T(t)\varphi_{m,i}(\bar{x}_i) + q_i(y(t-\tau_i(t))) \\ & - \sum_{l=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_l} q_l(y(t-\tau_l(t))) - H_i] - \tfrac{1}{\gamma_i}\tilde{\theta}_{m,i}^T \dot{\hat{\theta}}_{m,i} + \dot{W}_i\end{aligned} \tag{3.26}$$

From (3.6), we can get

$$z_i(g_i z_{i+1} + q_i) \le \tfrac{3}{4} z_i^2 + \bar{g}_i^2 z_{i+1}^2 + \tfrac{1}{2} z_1(t-\tau_i(t)) Q_{i,1}(z_1(t-\tau_i(t))) + \tfrac{1}{2}\varpi_i \tag{3.27}$$

$$-z_i \sum_{l=1}^{i-1} \tfrac{\partial \alpha_{i-1}}{\partial x_l} q_l \le \tfrac{1}{2} z_i^2 (\tfrac{\partial \alpha_{i-1}}{\partial x_l})^2 + \tfrac{1}{2}\sum_{l=1}^{i-1} z_1(t-\tau_l(t)) Q_{l,1}(z_1(t-\tau_l(t))) + \tfrac{1}{2}\sum_{l=1}^{i-1}\varpi_l \tag{3.28}$$

where $\bar{g}_i$ is positive constant and satisfies $|g_i| < \bar{g}_i$.

By invoking (3.26)–(3.28) yields

$$\begin{aligned}\dot{V}_i \le {} & -\sum_{k=1}^{i-1} c_k z_k^2 + \sum_{k=1}^{i-1}[(g_k N(\zeta_k)+1)\dot{\zeta}_k] + \sum_{k=1}^{i-1}\tfrac{\sigma_k}{\gamma_k}\tilde{\theta}_{m,k}^T \hat{\theta}_{m,k} + \tfrac{1}{2}\sum_{k=1}^{i}\sum_{j=1}^{k}\varpi_j \\ & - \sum_{j=i+1}^{n} z_1(t)\bar{Q}_j\{z_1(t)\} + \bar{g}_i^2 z_{i+1}^2 + z_i[g_i\alpha_i + \hat{\theta}_{m,i}^T(t)\varphi_{m,i}(\bar{x}_i) + \bar{g}_{i-1}^2 z_i \\ & + \tfrac{3}{4} z_i + \tfrac{1}{2} z_i \sum_{l=1}^{i-1}(\tfrac{\partial \alpha_{i-1}}{\partial x_l})^2 - H_i] - \sum_{k=1}^{i} r W_k + \tfrac{1}{\gamma_i}\tilde{\theta}_{m,i}^T[\gamma_i z_i \varphi_{m,i}(\bar{x}_i) - \dot{\hat{\theta}}_{m,i}]\end{aligned} \tag{3.29}$$

Design the virtual control function α_i and update law $\dot{\hat{\theta}}_{m,i}$ as

$$\alpha_i = N(\zeta_i)[c_i z_i + \hat{\theta}_{m,i}^T(t)\varphi_{m,i}(\bar{x}_i) + \bar{g}_{i-1}^2 z_i + \tfrac{3}{4} z_i + \tfrac{1}{2} z_i \sum_{l=1}^{i-1}(\tfrac{\partial \alpha_{i-1}}{\partial x_l})^2 - H_i] \tag{3.30}$$

$$\dot{\hat{\theta}}_{m,i} = \gamma_i z_i \varphi_{m,i}(\bar{x}_i) - \sigma_i \hat{\theta}_{m,i} \tag{3.31}$$

$$\dot{\zeta}_i = z_i[c_i z_i + \bar{g}_{i-1}^2 z_i + \hat{\theta}_{m,i}^T(t)\varphi_{m,i}(\bar{x}_i) + \tfrac{3}{4} z_i + \tfrac{1}{2} z_i \sum_{l=1}^{i-1}(\tfrac{\partial \alpha_{i-1}}{\partial x_l})^2 - H_i] \tag{3.32}$$

where $\sigma_i > 0$ and $c_i > 0$ are design parameters.

Thus, rewrite (3.29) as

$$\begin{aligned}\dot{V}_i \le {} & -\sum_{k=1}^{i} c_k z_k^2 + \sum_{k=1}^{i}[(g_k N(\zeta_k)+1)\dot{\zeta}_k] + \tfrac{1}{2}\sum_{k=1}^{i}\sum_{j=1}^{k}\varpi_j \\ & + \sum_{k=1}^{i}\tfrac{\sigma_k}{\gamma_k}\tilde{\theta}_{m,k}^T\hat{\theta}_{m,k} + \bar{g}_i^2 z_{i+1}^2 - \sum_{k=1}^{i} r W_k - \sum_{j=i+1}^{n} z_1(t)\bar{Q}_j\{z_1(t)\}\end{aligned} \tag{3.33}$$

Step n: According to (2.1) and (3.2), we have

$$\begin{aligned}\dot{z}_n &= g_n u + \theta_n^T(t)\varphi_n(\bar{x}_n) + q_n(y(t-\tau_n(t))) - \dot{\alpha}_{n-1} \\ &= g_n u + \theta_{m,n}^T(t)\varphi_{m,n}(\bar{x}_n) + q_n(y(t-\tau_n(t))) \\ &\quad - \sum_{l=1}^{n-1}\tfrac{\partial \alpha_{n-1}}{\partial x_l} q_l(y(t-\tau_l(t))) - H_n\end{aligned} \tag{3.34}$$

where $H_n = \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_{m,l}} \dot{\hat{\theta}}_{m,l} + \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \zeta_l} \dot{\zeta}_l$. $\theta_{m,n} = [\theta_n^T, \cdots, \theta_1^T, g_{n-1}, \cdots, g_1]^T$, $\varphi_{m,n} = [\varphi_n^T(\bar{x}_n), -\frac{\partial \alpha_{n-1}}{\partial x_{n-1}} \varphi_{n-1}^T(\bar{x}_{n-1}), \cdots, -\frac{\partial \alpha_{n-1}}{\partial x_1} \varphi_1^T(x_1), -\frac{\partial \alpha_{n-1}}{\partial x_{n-1}} x_n, \cdots, -\frac{\partial \alpha_{n-1}}{\partial x_1} x_2]^T$.

Choose the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + W_n + \frac{1}{2\gamma_n} \tilde{\theta}_{m,n}^T \tilde{\theta}_{m,n} \tag{3.35}$$

where $\gamma_n > 0$ is a design constant. $\hat{\theta}_{m,n}$ is the estimate of $\theta_{m,n}$ and $\tilde{\theta}_{m,n} = \theta_{m,n} - \hat{\theta}_{m,n}$.

Define $W_n = \sum_{k=1}^{n} \frac{2^{r(\tau - t)}}{2(1-\tau_k^*)} \int_{t-\tau_k(t)}^{t} e^{rs} z_1(s) Q_{k,1}(z_1(s)) ds$, thus, we have $\dot{W}_n \leq -rW_n + \sum_{k=1}^{n} \frac{e^{r\tau}}{1-\tau_k^*} z_1(t) Q_{k,1}(z_1(t)) - \frac{1}{2} \sum_{k=1}^{n} z_1(t - \tau_k(t)) Q_{k,1}(z_1(t - \tau_k(t)))$.

Thus, the derivation of V_n is

$$\begin{aligned}
\dot{V}_n \leq & - \sum_{k=1}^{n-1} c_k z_k^2 + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{k} \varpi_j + \sum_{k=1}^{n-1} \frac{\sigma_k}{\gamma_k} \tilde{\theta}_{m,k}^T \hat{\theta}_{m,k} + \sum_{k=1}^{n-1} [(g_k N(\zeta_k) + 1)\dot{\zeta}_k] \\
& + z_n [g_n u + \hat{\theta}_{m,n}^T(t) \varphi_{m,n}(\bar{x}_n) + \bar{g}_{n-1}^2 z_n + \frac{1}{2} z_n + \frac{1}{2} z_n \sum_{l=1}^{n-1} (\frac{\partial \alpha_{n-1}}{\partial x_l})^2 \\
& - H_n] + \frac{1}{\gamma_i} \tilde{\theta}_{m,n}^T [\gamma_n z_n \varphi_{m,n}(\bar{x}_n) - \dot{\hat{\theta}}_{m,n}] - \sum_{k=1}^{N} r W_k
\end{aligned} \tag{3.36}$$

Design the controller u and update law $\dot{\hat{\theta}}_{m,n}$ as

$$u = N(\zeta_n)[c_n z_n + \bar{g}_{n-1}^2 z_n + \hat{\theta}_{m,n}^T(t) \varphi_{m,n}(\bar{x}_n) + \frac{1}{2} z_n + \frac{1}{2} z_n \sum_{l=1}^{n-1} (\frac{\partial \alpha_{n-1}}{\partial x_l})^2 - H_n] \tag{3.37}$$

$$\dot{\hat{\theta}}_{m,n} = \gamma_n z_n \varphi_{m,n}(\bar{x}_n) - \sigma_n \hat{\theta}_{m,n} \tag{3.38}$$

$$\dot{\zeta}_n = z_n [c_n z_n + \bar{g}_{n-1}^2 z_n + \hat{\theta}_{m,n}^T(t) \varphi_{m,n}(\bar{x}_n) + \frac{1}{2} z_n + \frac{1}{2} z_n \sum_{l=1}^{n-1} (\frac{\partial \alpha_{n-1}}{\partial x_l})^2 - H_n] \tag{3.39}$$

where $\sigma_n > 0$ and $c_n > 0$ are design parameters.

Thus, rewrite (3.36) as

$$\dot{V}_n \leq - \sum_{k=1}^{n} c_k z_k^2 + \sum_{k=1}^{n} [(g_k N(\zeta_k) + 1)\dot{\zeta}_k] + \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{k} \varpi_j + \sum_{k=1}^{n} \frac{\sigma_k}{\gamma_k} \tilde{\theta}_{m,k}^T \hat{\theta}_{m,k} - \sum_{k=1}^{n} r W_k \tag{3.40}$$

3.1. Exponential stability analysis

The property of the developed exponential controller can be summarized as the following Theorem.

Theorem 1. Consider nonlinear system (2.1), under the Assumption 1, the designed exponential controller (3.37), virtual control functions (3.10), (3.20) and (3.30), update laws (3.11), (3.21), (3.31) and (3.37), can guarantee that all signal of controlled system are UUB and exponential converge to origin.

Proof. Choose Lyapunov function as $V = \sum_{i=1}^{n} \{\frac{1}{2} z_i^2 + W_i + \frac{1}{2\gamma_i} \tilde{\theta}_{m,i}^T \tilde{\theta}_{m,i}\}$, from (3.40), one has

$$\dot{V} \leq - \sum_{i=1}^{n} c_i z_i^2 + \sum_{i=1}^{n} [(g_i N(\zeta_i) + 1)\dot{\zeta}_i] + \sum_{i=1}^{n} \frac{\sigma_i}{\gamma_i} \tilde{\theta}_{m,i}^T \hat{\theta}_{m,i} - \sum_{i=1}^{n} r W_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{i} \varpi_j \tag{3.41}$$

By utilizing (3.6), we have

$$\frac{\sigma_i}{\gamma_i}\tilde{\theta}^T_{m,i}\hat{\theta}_{m,i} \leq -\frac{\sigma_i}{2\gamma_i}\tilde{\theta}^T_{m,i}\tilde{\theta}_{m,i} + \frac{\sigma_i}{2\gamma_i}\theta^T_{m,i}\theta_{m,i} \tag{3.42}$$

Thus, let $\lambda = \min\{2c_i, \sigma_i, r\}$ $(i = 1, \cdots, n)$, rewrite (3.41) as

$$\dot{V} \leq -\lambda V + \sum_{i=1}^{n}[(g_i N(\zeta_i) + 1)\dot{\zeta}_i] + \bar{D} \tag{3.43}$$

where $\bar{D} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\varpi_j + \sum_{i=1}^{n}\frac{\sigma_i}{2\gamma_i}\theta^T_{m,i}\theta_{m,i}$.

According to Lemma 1, $\sum_{i=1}^{n}[(g_i N(\zeta_i) + 1)\dot{\zeta}_i]$ is bounded on $[0, t_f]$. Define $D' = \sum_{i=1}^{n}[(g_i N(\zeta_i) + 1)\dot{\zeta}_i]$, $D = \bar{D} + D'$, (3.43) is finally expressed as

$$\dot{V} \leq -\lambda V + D \tag{3.44}$$

Integrating (3.44) over $[0, t]$ yields

$$0 \leq V(t) \leq \frac{D}{\lambda} + e^{-\lambda t}V(0) \tag{3.45}$$

Thus, from (3.45), we can get $|z_i(t)| \leq \sqrt{2(\frac{D}{\lambda} + e^{-\lambda t}V(0))}$, x_i, z_i, $\hat{\theta}_{m,i}$ are UUB and exponential converge to zero. Furthermore, we also can obtain exponential decay rate can be determined by λ, by increasing c_i, σ_i or decreasing γ_i to get good transient performance of controlled system. This completes the proof of Theorem 1. ■

Remark 2. From the above analysis, we know that the size of $|z_i(t)| \leq \sqrt{2(\frac{D}{\lambda} + e^{-\lambda t}V(0))}$ lies the design parameters c_i, γ_i and σ_i. By increasing the design parameters c_i, γ_i or decreasing the design parameters σ_i can make error z_1 be smaller.

4. Simulation example

In this section, a numerical example is provided to display the feasibility of the designed controller.

Example. Consider the nonlinear system as

$$\begin{cases} \dot{x}_1 = g_1 x_2 + \theta^T_1(t)\varphi_1(x_1) + q_1(y(t - \tau_1(t))) \\ \dot{x}_2 = g_2 u + \theta^T_2(t)\varphi_2(\bar{x}_2) + q_2(y(t - \tau_2(t))) \\ y = x_1 \end{cases} \tag{4.1}$$

where $\theta_1 = 0.6$, $\varphi_1(x_1) = x_1^2$, $\theta_2 = [0.8, 0.2]^T$, $\varphi_2(\bar{x}_2) = [x_2\sin(x_1), x_1 x_2]^T$, $g_1 = 2$, $g_2 = 3$, $q_1(y(t - \tau_1(t))) = \frac{x_1(t-\tau_1(t))}{1+x_1^2(t-\tau_1(t))}$, $q_2(y(t - \tau_2(t))) = \frac{\sin(x_1(t))x_1^2(t-\tau_2(t))}{1+x_1^2(t-\tau_2(t))}$.

Choose the design parameters in controller, virtual control functions and update laws as: $c_1 = 0.8$, $c_2 = 0.8$, $\tau = 0.6$, $\tau^* = 0.4$, $\gamma_1 = 6$, $\gamma_2 = 4$, $\sigma_1 = 4$, $\sigma_2 = 4$.

Then, the virtual control function α_1, controller u and update laws are:

$$\alpha_1 = N(\zeta_1)[0.8z_1 + \hat{\theta}^T_{m,1}(t)\varphi_{m,1}(x_1) + \frac{3}{4}z_1 + \sum_{j=1}^{2}\bar{Q}_j\{z_1(t)\}]$$

$$u = N(\zeta_2)[0.8z_2 + \bar{g}_1^2 z_2 + \hat{\theta}_{m,2}^T(t)\varphi_{m,2}(\bar{x}_2) + \frac{1}{2}z_2 + \frac{1}{2}z_2(\frac{\partial\alpha_1}{\partial x_1})^2 - H_2]$$

$$\dot{\hat{\theta}}_{m,1} = 6z_1\varphi_{m,1}(x_1) - 4\hat{\theta}_{m,1}$$

$$\dot{\hat{\theta}}_{m,2} = 4z_2\varphi_{m,2}(\bar{x}_2) - 4\hat{\theta}_{m,2}$$

Select the initial conditions od variables as: $x_1(0) = 0.5$, $x_2(0) = 0.5$, $\hat{\theta}_{m,1}(0) = 0.2$, $\hat{\theta}_{m,2}(0) = [0, 0, 0, 0]^T$. Thus, the simulation results are displayed by Figures 1-2. Figure 1 is the curves of states x_1 and x_2; Figure 2 is the controller u.

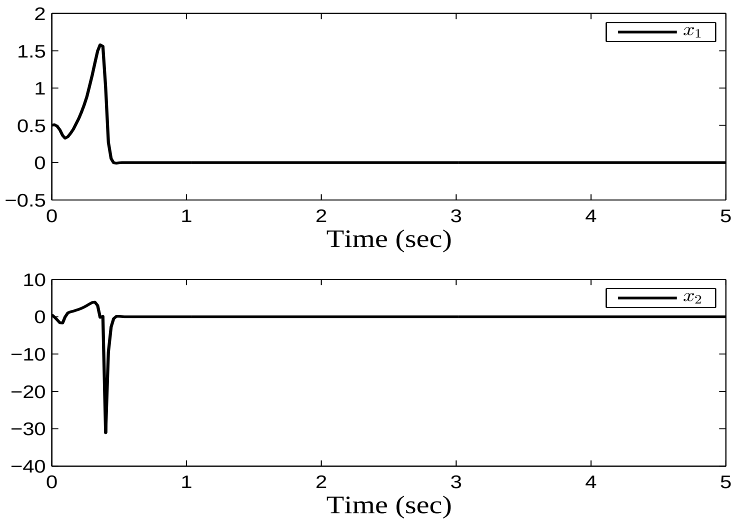

Figure 1. The trajectories of x_i $(i = 1, 2)$.

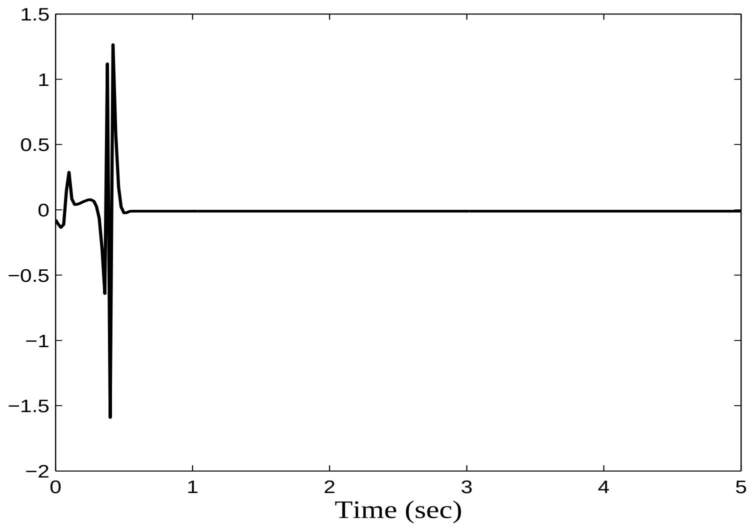

Figure 2. The trajectory of controller u.

From the figures 1 and 2, it means that all the variables of controlled system exponential converge to origin.

5. Conclusion

In this paper, we have studied the exponential stability analysis and controller design issue for time-varying delay nonlinear systems with unknown control direction. In control design, time-varying delay and unknown control directions have been solved. Under the framework of adaptive backstepping recursive design, an exponential stabilization control algorithm has been developed. It is demonstrated that all solutions of controlled system are UUB and exponential converge to origin. The future research directions will focus on the global exponential or fixed-time stabilization control for switched nonlinear systems [21–23]. In addition, the global exponential output-feedback control for nonlinear systems are also our future research topics [24] and [25].

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Conflict of interest

The authors declare no conflict of interest.

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