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Research article

A Berry-Esséen bound of wavelet estimation for a nonparametric regression model under linear process errors based on LNQD sequence

Xueping Hu* **and Jingya Wang**

School of Mathematics and Physics, Anqing Normal University, Anhui, Anqing 246133, China

* **Correspondence:** Email: hxprob@163.com; Tel: +8618055690595.

Abstract: By using some inequalities for linearly negative quadrant dependent random variables, Berry-Esséen bound of wavelet estimation for a nonparametric regression model is investigated under linear process errors based on linearly negative quadrant dependent sequence. The rate of uniform asymptotic normality is presented and the rate of convergence is near $O(n^{-\frac{1}{6}})$ under mild conditions, which generalizes or extends the corresponding results of Li et al.(2008) under associated random samples in some sense.

Keywords: wavelet estimation; Berry-Esséen bound; linearly negative quadrant dependent sequence
Mathematics Subject Classification: 60G05, 62G20

1. Introduction

Consider the classical nonparametric regression model

$$Y_{ni} = g(x_{ni}) + \varepsilon_{ni}, 1 \le i \le n, \tag{1.1}$$

where $\{\varepsilon_{ni}, 1 \le i \le n\}$ are random errors and $\{x_{ni}, 1 \le i \le n\}$ are known fixed design points, $g(\cdot)$ is an unknown bounded real valued function on $[0, 1]$. It is well known that the model (1.1) has been widely studied by many authors in the literature. The uniformly asymptotic normality for a general weighted regression estimator of $g(\cdot)$, which was proposed by Georgiev [1], had got extensive investigated. One can refer to Georgiev [2] under independent random errors, Roussas et al. [3] under strong mixing random errors, Yang [4] under negatively associated errors, and so on. Under the assumption that the errors case is a weakly stationary linear process based on a martingale difference sequence, Tran et al. [5] studied the asymptotic normality. Liang and Li [6] obtained the Berry-Esséen bound based on linear process errors under negatively associated random variables.

In recent years, wavelets techniques, owing to their ability to adapt to local features of curves, have been widely used in statistics, engineering and technological fields. Many authors have considered employing wavelet methods to estimate nonparametric and semiparametric models. For instance, one can refer to the papers [7–13].

The problem we face here is to derive a Berry-Esséen bound of wavelet estimator of $g(\cdot)$ proposed by Antoniadis et al. [7]

$$\hat{g}_n(t) = \sum_{i=1}^{n} Y_{ni} \int_{A_i} E_m(t, s) ds, \tag{1.2}$$

where the wavelet kernel $E_m(t, s)$ can be defined by $E_m(t, s) = 2^m \sum_{k \in Z} \phi(2^m t - k)\phi(2^m s - k)$, $\phi(\cdot)$ is a scaling function, $m = m(n) > 0$ is an integer depending only on n, $A_i = [s_{i-1}, s_i), i = 1, 2, \cdots, n$ are intervals that partition $[0, 1]$.

We recall the concepts of negative associated (NA, in short), negative quadrant dependent (NQD, in short) and linearly negative quadrant dependent (LNQD, in short) sequences.

Definition 1.1. [14] A finite collection of random variables $\{X_i\}_{1 \le i \le n}$ are said to be NA, if for every disjoint subsets $A, B \subset \{1, 2, \cdots, n\}$

$$\mathrm{Cov}(f(X_i, i \in A), g(X_j, j \in B)) \le 0,$$

where f and g are real coordinate-wise nondecreasing functions such that this covariance exists. An infinite sequence of random variables $\{X_n\}_{n \ge 1}$ are said to be NA, if for every $n \ge 2$, $X_1, X_2, \cdots, X_n$ are NA.

Definition 1.2. [15] Two random variables X, Y are said to be NQD, if for any $x, y \in R$,

$$P(X < x, Y < y) \le P(X < x)P(Y < y).$$

A sequence of random variables $\{X_n\}_{n \ge 1}$ are said to be pairwise negative quadrant dependent (PNQD, in short), if every pair of random variables in the sequence are NQD.

Definition 1.3. [16] A sequence $\{X_i\}_{1 \le i \le n}$ of random variables are said to be LNQD, if for any disjoint subsets $A, B \subset Z^+$ and positive $r_j's$, $\sum_{i \in A} r_i X_i$ and $\sum_{j \in B} r_j X_j$ are NQD.

Remark 1.1. It easily follows that if $\{X_n\}_{n \ge 1}$ is a sequence LNQD random variables, then $\{aX_n + b\}_{n \ge 1}$ is still a sequence of LNQD, where a and b are real numbers. Furthermore, NA implies LNQD from the definitions, LNQD random variables are NQD random variables, but the converse is not true.

The concept of LNQD sequence was introduced by Newman [16], some applications can be found in many monographs. For example, Newman investigated the central limit theorem for a strictly stationary LNQD process. Wang et al. [17] established the exponential inequalities and complete convergence for a LNQD sequence. Li et al. [18] obtained some inequalities and gave some applications for a fixed-design regression model. Ding et al. [19] derived the Berry-Esséen bound of weighted kernel estimator for model (1.1) based on linear process errors under a LNQD sequence.

In this paper, we shall consider the above wavelet estimator of nonparametric regression problem with linear process errors generated by a LNQD sequence. Our main purpose is to derive the Berry-Esséen bound of the wavelet estimator (1.2).

The layout of the rest is as follows. In Section 2, we present some basic assumptions and main results. In Section 3, some preliminary lemmas are stated and proof of Theorem 2.1 is provided. In Section 4, proofs of some preliminary lemmas are given. The Appendix contains some known results.

Throughout the paper, $C, C_1, C_2, \cdots$ denote some positive constants not depending on n, which may be different in various places. $\lfloor x \rfloor$ denotes the largest integer not exceeding x. All limits are taken as the sample size n tends to ∞, unless specified otherwise.

2. Some basic assumptions and main results

To obtain our results, the following basic assumptions are sufficient:
Assumption (A1) For each n, $\{\varepsilon_{ni}, 1 \le i \le n\}$ have the same joint distribution as $\{\xi_1, \cdots, \xi_n\}$, where $\xi_t = \sum_{j=-\infty}^{\infty} |a_j| e_{t-j}$ and $\{a_j\}$ is a sequence of real numbers with $\sum_{j=-\infty}^{\infty} |a_j| < \infty$. Here $\{e_j\}$ are identically distribution, LNQD random variables with $Ee_0 = 0, E|e_0|^{2+\delta} < \infty, 0 < \delta \le 1$.
Assumption (A2) The spectral density function $f(\omega)$ of $\{\xi_i\}$ is bounded away from zero and infinity, i.e., for $\omega \in (-\pi, \pi], 0 < C_1 \le f(\omega) \le C_2 < \infty$.
Assumption (A3) (i)$\phi(\cdot)$ is $r-$ regular (r is a positive integer), and satisfies the Lipschitz condition with order 1 with a compact support. Furthermore, $|\hat{\phi}(\xi) - 1| = O(\xi)$ as $\xi \to \infty$, where $\hat{\phi}$ denotes the Fourier transform of ϕ. (ii) $\max_{1\le i\le n} |s_i - s_{i-1}| = O(n^{-1})$.
Assumption (A4) (i) $g(\cdot) \in H^v, v > 1/2$, where H^v presents Sobolev space of order v, i.e., if $h \in H^v$ then $\int |\hat{h}(\omega)|^2 (1+\omega^2)^v d\omega < \infty$ with $\hat{h}$ denoting the Fourier transform of h. (ii) $g(\cdot)$ satisfies the Lipschitz condition of order 1.
Assumption (A5) There exist positive integers $p := p_n, q := q_n$ and $k := k_n = \lfloor \frac{3n}{p+q} \rfloor$, such that, for $p + q \le 3n, qp^{-1} \to 0$, and let $\gamma_{in} \to 0, i = 1, 2, 3$, where $\gamma_{1n} = qp^{-1}2^m, \gamma_{2n} = pn^{-1}2^m, \gamma_{3n} = n(\sum_{|j|>n} |a_j|)^2$.
Remark 2.1. Assumption (A1) is the general condition of the LNQD sequence, Assumptions (A2–A4) are mild regularity conditions for wavelet estimate in the recent literature, such as Sun and Chai [10], Li et al. [11,12], Liang and Qi [8]. In Assumption (A5) $\gamma_{in} \to 0, i = 1, 2, 3$ are easily satisfied if p, q, m are chosen reasonable, see e.g., Li et al. [11,12] and Liang et al. [6,8].

In order to formulate our main results, let $\sigma_n^2 := \sigma_n^2(t) = \mathrm{Var}(\hat{g}_n(t)) > 0$, $S_n := S_n(t) = \sigma_n^{-1}\{\hat{g}_n(t) - E\hat{g}_n(t)\}$, $u(q) = \sup_{j\ge1} \sum_{j:|j-i|\ge q} |\mathrm{Cov}(e_i, e_j)|$.
Theorem 2.1. Assume that Assumptions (A1)–(A5) are satisfied, then for each $t \in [0, 1]$, we have

$$\sup_x |P(S_n(t) \le x) - \Phi(x)| \le C(\gamma_{1n}^{1/3} + \gamma_{2n}^{1/3} + \gamma_{2n}^{\delta/2} + \gamma_{3n}^{1/3} + u^{1/3}(q)), \tag{2.1}$$

where $\Phi(x)$ is the distribution function of $N(0, 1)$.
Remark 2.2. Theorem 2.1 extends the results of Li et al. [11] from associated samples to linear process errors generated by a LNQD sequence.
Corollary 2.1. Assume that conditions of Theorem 2.1 hold and $u(1) < \infty$, for each $t \in [0, 1]$, then

$$\sup_x |P(S_n(t) \le x) - \Phi(x)| = o(1). \tag{2.2}$$

Corollary 2.2. Under the conditions of Theorem 2.1, let $\delta = \frac{2}{3}$, $n^{-1}2^m = O(n^{-\theta}), u(n) = O(n^{-\frac{\theta-\rho}{2\rho-1}})$ and $\sup_{n\ge1} \left(n^{\frac{\theta-\rho+1}{2}}\right) \sum_{|j|>n} |a_j| < \infty$, where $\frac{1}{2} < \rho \le \theta < 1$, for each $t \in [0, 1]$, then

$$\sup_x |P(S_n(t) \le x) - \Phi(x)| = O(n^{-\frac{\theta-\rho}{3}}). \tag{2.3}$$

Remark 2.3. From Corollary 2.2, taking $\theta \approx 1$ and $\rho \approx \frac{1}{2}$, the rate of convergence is near $O(n^{-1/6})$.

3. Some preliminary lemmas

In order to prove our main results we introduce the following preliminary lemmas. At first, some notations are introduced for the sake of convenience and brevity.

According to the Eq (1.1) and (1.2), we have

$$\begin{aligned}S_n &= \sigma_n^{-1}\sum_{i=1}^{n}\xi_i\int_{A_i}E_m(t,s)ds\\&=\sigma_n^{-1}\sum_{i=1}^{n}\int_{A_i}E_m(t,s)ds\sum_{j=-n}^{n}|a_j|e_{i-j}+\sigma_n^{-1}\sum_{i=1}^{n}\int_{A_i}E_m(t,s)ds\sum_{|j|>n}|a_j|e_{i-j}\\&:=S_{1n}+S_{2n}.\end{aligned}$$

Note that

$$S_{1n}=\sum_{l=1-n}^{2n}\sigma_n^{-1}\left(\sum_{i=\max\{1,l-n\}}^{\min\{n,l+n\}}|a_{i-l}|\int_{A_i}E_m(t,s)ds\right)e_l:=\sum_{l=1-n}^{2n}Z_{nl}.$$

Set $S_{1n}=S'_{1n}+S''_{1n}+S'''_{1n}$, where $S'_{1n}=\sum_{m=1}^{k}y_{nm}, S''_{1n}=\sum_{m=1}^{k}y'_{nm}, S'''_{1n}=y'_{nk+1}, y_{nm}=\sum_{i=k_m}^{k_m+p-1}Z_{ni}, y'_{nm}=\sum_{i=l_m}^{l_m+q-1}Z_{ni}, y'_{nk+1}=\sum_{i=k(p+q)-n+1}^{2n}Z_{ni}, k_m=(m-1)(p+q)+1-n, l_m=(m-1)(p+q)+p+1-n, m=1,2,\cdots,k$. Then we have $S_n=S'_{1n}+S''_{1n}+S'''_{1n}+S_{2n}$.

Lemma 3.1. Assume that (A1)–(A5) are satisfied, then
$(i)E(S''_{1n})^2\le C\gamma_{1n},\ E(S'''_{1n})^2\le C\gamma_{2n},\ E(S_{2n})^2\le C\gamma_{3n}$;
$(ii)P(|S''_{1n}|\ge\gamma_{1n}^{1/3})\le C\gamma_{1n}^{1/3},\ P(|S'''_{1n}|\ge\gamma_{2n}^{1/3})\le C\gamma_{2n}^{1/3},\ P(|S_{2n}|\ge\gamma_{3n}^{1/3})\le C\gamma_{3n}^{1/3}$.

Lemma 3.2. Suppose that (A1)–(A5) hold, let $s_n^2=\sum_{m=1}^{k}\mathrm{Var}(y_{nm})$, then

$$|s_n^2-1|\le C(\gamma_{1n}^{1/2}+\gamma_{2n}^{1/2}+\gamma_{3n}^{1/2}+u(q)).$$

Let $\{\eta_{nm}:m=1,2,\cdots,k\}$ be independent random variables and η_{nm} have the same distribution as $y_{nm}, m=1,2,\cdots,k$. Set $H_n=\sum_{m=1}^{k}\eta_{nm}$.

Lemma 3.3. Under Assumptions (A1)–(A5), we have

$$\sup_x|P(H_n/s_n\le x)-\Phi(x)|\le C\gamma_{2n}^{\delta/2}.$$

Lemma 3.4. Under the Assumptions of Theorem 2.1, we have

$$\sup_x|P(S'_{1n}\le x)-P(H_n\le x)|\le C(\gamma_{2n}^{\delta/2}+u^{1/3}(q)).$$

Lemma 3.5. Assume that (A1)–(A5) are true, then

$$\sigma_n^2(t)\ge C2^m n^{-1},\sigma_n^{-2}(t)\left|\int_{A_i}E_m(t,s)ds\right|\le C.$$

Proof of Theorem 2.1. Similar to the proof of Theorem 2.1 in [6], it is easily seen that

$$\begin{aligned}\sup_t|P(S'_{1n}\le t)-\Phi(t)|&\le\sup_t|P(S'_{1n}\le t)-P(H_n\le t)|\\&\quad+\sup_t|P(H_n\le t)-\Phi(t/s_n)|+\sup_t|\Phi(t/s_n)-\Phi(t)|\\&:=D_1+D_2+D_3.\end{aligned}$$

From Lemma 3.2 and Lemma 5.2 in Petrov [20], it follows that

$$\begin{aligned} D_3 \le& (2\pi e)^{-1/2}(s_n - 1)I(s_n \ge 1) + (2\pi e)^{-1/2}(s_n^{-1} - 1)I(0 < s_n < 1) \\ \le& C|s_n^2 - 1| \le C[\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2} + u(q)]. \end{aligned}$$

Consequently, by means of Lemmas 3.2–3.4, we can get

$$\sup_x |P(S'_{1n} \le x) - \Phi(x)| \le C(\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{2n}^{\delta/2} + \gamma_{3n}^{1/2} + u^{1/3}(q)). \tag{3.1}$$

According to Lemma A.1 in the Appendix, Lemma 3.1(ii) and the Eq (3.1), we obtain

$$\begin{aligned} &\sup_x |P(S_n \le x) - \Phi(x)| \\ &\le C\left(\sup_x |P(S'_{1n} \le x) - \Phi(x)| + \sum_{i=1}^{3} \gamma_{in}^{1/3} + P(|S''_{1n}| \ge \gamma_{1n}^{1/3}) + P(|S'''_{1n}| \ge \gamma_{2n}^{1/3}) + P(|S_{2n}| \ge \gamma_{3n}^{1/3})\right) \\ &\le C(\gamma_{1n}^{1/3} + \gamma_{2n}^{1/3} + \gamma_{2n}^{\delta/2} + \gamma_{3n}^{1/3} + u^{1/3}(q)). \end{aligned}$$

This completes the proof of Theorem 2.1.

Proof of Corollary 2.1. According to $u(1) < \infty$ it easily follows that $u(q) \to 0$, hence Corollary 2.1 holds by Theorem 2.1.

Proof of Corollary 2.2. Taking $p = \lfloor n^{\rho} \rfloor, q = \lfloor n^{2\rho-1} \rfloor$ in Theorem 2.1, for $\delta = 2/3, 1/2 < \rho \le \theta < 1$, we obtain

$$\gamma_{1n}^{1/3} = \gamma_{2n}^{1/3} = O(n^{-\frac{\theta-\rho}{3}}), u^{1/3}(q) = O\left(q^{-\frac{\theta-\rho}{2\rho-1}}\right)^{1/3} = O(n^{-\frac{\theta-\rho}{3}}),$$

$$\gamma_{3n}^{1/3} = n^{-\frac{\theta-\rho}{3}} \left(n^{\frac{\theta-\rho+1}{2}} \sum_{|j|>n} |a_j|\right)^{2/3} = O(n^{-\frac{\theta-\rho}{3}}).$$

Therefore, the conclusion follows from Theorem 2.1.

4. Proofs of some preliminary lemmas

Proof of Lemma 3.1. According to Lemma 3.5, Lemmas A.2, A.4 in the Appendix and Assumptions (A1), (A5), we have

$$\begin{aligned} E(S''_{1n})^2 &= E\left[\sum_{m=1}^{k} \sum_{i=l_m}^{l_m+q-1} \sigma_n^{-1} \left(\sum_{j=\max\{1,i-n\}}^{\min\{n,i+n\}} |a_{j-i}| \int_{A_j} E_m(t,s)ds\right) e_i\right]^2 \\ &\le Ckq\frac{2^m}{n}\left(\sum_{j=\max\{1,i-n\}}^{\min\{n,i+n\}} |a_{j-i}|\right)^2 \le Ckq\frac{2^m}{n}\left(\sum_{j=-\infty}^{\infty} |a_j|\right)^2 \\ &\le Cqp^{-1}2^m = C\gamma_{1n}. \end{aligned}$$

$$
\begin{aligned}
E(S'''_{1n})^2 &= E\left[\sum_{i=k(p+q)+1-n}^{2n} \sigma_n^{-1}\left(\sum_{j=\max\{1,i-n\}}^{\min\{n,i+n\}} |a_{j-i}| \int_{A_j} E_m(t,s)ds\right) e_i\right]^2 \\
&\le C[3n-k(p+q)]\frac{2^m}{n}\left(\sum_{j=\max\{1,i-n\}}^{\min\{n,i+n\}} |a_{j-i}|\right)^2 \\
&\le Cp\frac{2^m}{n}\left(\sum_{j=-\infty}^{\infty} |a_j|\right)^2 \\
&\le Cp(2^m/n) = C\gamma_{2n}.
\end{aligned}
$$

As to S_{2n}, by Lemma A.4 in the Appendix

$$
\begin{aligned}
E(S_{2n})^2 &= E\left(\sigma_n^{-1}\sum_{i=1}^{n}\int_{A_i} E_m(t,s)ds \sum_{|j|>n}|a_j|e_{i-j}\right)^2 \\
&= E\left|\sigma_n^{-1}\sum_{i_1=1}^{n}\int_{A_{i_1}} E_m(t,s)ds \sum_{|j_1|>n}|a_{j_1}|e_{i_1-j_1}\right|\left|\sigma_n^{-1}\sum_{i_2=1}^{n}\int_{A_{i_2}} E_m(t,s)ds \sum_{|j_2|>n}|a_{j_2}|e_{i_2-j_2}\right| \\
&\le CE\left\{\sum_{i_1=1}^{n}\left|\int_{A_{i_1}} E_m(t,s)ds\right|\sum_{i_2=1}^{n}\left|\sum_{|j_1|>n}|a_{j_1}|e_{i_1-j_1}\right|\left|\sum_{|j_2|>n}|a_{j_2}|e_{i_2-j_2}\right|\right\} \\
&\le Cn\left(\sum_{|j|>n}|a_j|\right)^2 = C\gamma_{3n}.
\end{aligned}
$$

Therefore the proof of Lemma 3.1(i) is completed. In addition, by Markov inequality and Lemma 3.1(i) it easily follows that Lemma 3.1(ii) is true.

Proof of Lemma 3.2. Set $\Gamma_n = \sum_{1\le i<j\le n} \mathrm{Cov}(y_{ni}, y_{nj})$, then $s_n^2 = E(S'_{1n})^2 - 2\Gamma_n$. Note that $ES_n^2 = 1$ and applying Lemma 3.1(i), we can get

$$
|E(S'_{1n})^2 - 1| = |E(S''_{1n} + S'''_{1n} + S_{2n})^2 - 2E[S_n(S''_{1n} + S'''_{1n} + S_{2n})]| \le C(\gamma_{1n}^{1/2} + \gamma_{2n}^{1/2} + \gamma_{3n}^{1/2}). \tag{4.1}
$$

On the other hand, by Lemma 3.1, Lemma 3.5, Assumptions (A1), (A5), and Lemma A.4 in the Appendix, it follows that

$$
\begin{aligned}
|\Gamma_n| &= |\sum_{1\le i<j\le k} \mathrm{Cov}(y_{ni}, y_{nj})| \\
&\le \sum_{1\le i<j\le k}\sum_{s_1=k_i}^{k_i+p-1}\sum_{t_1=k_j}^{k_j+p-1} |\mathrm{Cov}(Z_{ns_1}, Z_{nt_1})| \\
&\le \sum_{1\le i<j\le k}\sum_{s_1=k_i}^{k_i+p-1}\sum_{t_1=k_j}^{k_j+p-1}\sum_{u=\max\{1,s_1-n\}}^{\min\{n,s_1+n\}}\sum_{v=\max\{1,t_1-n\}}^{\min\{n,t_1+n\}} \sigma_n^{-2}|a_{u-s_1}a_{v-t_1}|
\end{aligned}
$$

$$
\begin{aligned}
&\times\left|\int_{A_u}E_m(t,s)ds\int_{A_v}E_m(t,s)ds\right||\mathrm{Cov}(e_{s_1},e_{t_1})|\\
\le& C\sum_{i=1}^{k-1}\sum_{s_1=k_i}^{k_i+p-1}\sum_{u=\max\{1,s_1-n\}}^{\min\{n,s_1+n\}}|a_{u-s_1}|\left|\int_{A_u}E_m(t,s)ds\right|\\
&\times\sum_{j=i+1}^{k}\sum_{t_1=k_j}^{k_j+p-1}|\mathrm{Cov}(e_{s_1},e_{t_1})|\sum_{v=\max\{1,t_1-n\}}^{\min\{n,t_1+n\}}|a_{v-t_1}| \qquad (4.2)\\
\le& C\sum_{i=1}^{k-1}\sum_{s_1=k_i}^{k_i+p-1}\sum_{u=\max\{1,s_1-n\}}^{\min\{n,s_1+n\}}|a_{u-s_1}|\left|\int_{A_u}E_m(t,s)ds\right|\sup_{t_1\ge1}\sum_{t_1:|t_1-s_1|\ge q}|\mathrm{Cov}(e_{s_1},e_{t_1})|\\
\le& Cu(q)\sum_{u=1}^{n}\left|\int_{A_u}E_m(t,s)ds\right|\left(\sum_{i=1}^{k-1}\sum_{s_1=k_i}^{k_i+p-1}|a_{u-s_1}|\right)\le Cu(q).
\end{aligned}
$$

Thus, (4.1) and (4.2) follow that $|s_n^2-1|\le C(\gamma_{1n}^{1/2}+\gamma_{2n}^{1/2}+\gamma_{3n}^{1/2}+u(q))$.

Proof of Lemma 3.3. Applying the Berry-Esséen inequality (cf.Petrov [20], Theorem 5.7), we get

$$
\sup_x|P(H_n/s_n\le x)-\Phi(x)|\le C\sum_{m=1}^{k}(E|y_{nm}|^{2+\delta}/s_n^{2+\delta}). \qquad (4.3)
$$

By Lemma 3.5, from Assumption(A1), (A5) and A.2 in the Appendix it follows that

$$
\begin{aligned}
\sum_{m=1}^{k}E|y_{nm}|^{2+\delta}=&\sum_{m=1}^{k}E\left|\sum_{j=k_m}^{k_m+p-1}\left(\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}}\sigma_n^{-1}|a_{i-j}|\int_{A_i}E_m(t,s)ds\right)e_j\right|^{2+\delta}\\
\le& Cp^{\frac{\delta}{2}}\sum_{m=1}^{k}\sum_{j=k_m}^{k_m+p-1}\left(\sup_j\left|\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}}\sigma_n^{-1}|a_{i-j}|\int_{A_i}E_m(t,s)ds\right|\right)^{2+\delta}\\
\le& Cp^{\frac{\delta}{2}}(\frac{2^m}{n})^{\frac{\delta}{2}}\sum_{m=1}^{k}\sum_{j=k_m}^{k_m+p-1}\left(\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}}|a_{i-j}|\right)^{1+\delta}\sum_{i=\max\{1,j-n\}}^{\min\{n,j+n\}}|a_{i-j}|\left|\int_{A_i}E_m(t,s)ds\right| \qquad (4.4)\\
\le& C\left(\frac{p2^m}{n}\right)^{\frac{\delta}{2}}\left(\sum_{m=1}^{k}\sum_{j=k_m}^{k_m+p-1}|a_{i-j}|\right)\sum_{i=1}^{n}\left|\int_{A_i}E_m(t,s)ds\right|\\
\le& C\left(\frac{p2^m}{n}\right)^{\frac{\delta}{2}}=C\gamma_{2n}^{\delta/2}.
\end{aligned}
$$

Since $s_n\to1$ by Lemma 3.2. From (4.3) and (4.4) we can get Lemma 3.3.

Proof of Lemma 3.4. Let $\psi(t)$ and $\varphi(t)$ be the characteristic functions of S'_{1n} and H_n, respectively. Thus applying the Esséen inequality(Petrov [20], Theorem 5.3), for any $T>0$,

$$
\begin{aligned}
\sup_t|P(S'_{1n}\le t)-P(H_n\le t)|\le&\int_{-T}^{T}\left|\frac{\psi(t)-\varphi(t)}{t}\right|dt\\
&+T\sup_t\int_{|u|\le C/T}|P(H_n\le u+t)-P(H_n\le t)|du \qquad (4.5)\\
:=&D_{1n}+D_{2n}.
\end{aligned}
$$

Similar to (4.2), it follows from Lemma 3.5, A.3 and A.4 in the Appendix that

$$
\begin{aligned}
|\psi(t)-\varphi(t)| &= \left| E\exp\left(\mathrm{i}t\sum_{m=1}^{k} y_{nm}\right) - \prod_{m=1}^{k} E\exp\left(\mathrm{i}ty_{nm}\right)\right| \\
&\le 4t^2 \sum_{1\le i<j\le k}\sum_{s_1=k_i}^{k_i+p-1}\sum_{t_1=k_j}^{k_j+p-1} |\mathrm{Cov}(Z_{ns_1}, Z_{nt_1})| \\
&\le 4Ct^2 u(q),
\end{aligned}
$$

which implies that

$$
D_{1n} = \int_{-T}^{T}\left|\frac{\psi(t)-\varphi(t)}{t}\right| dt \le Cu(q)T^2. \tag{4.6}
$$

Therefore, by Lemma 3.3, we have

$$
\begin{aligned}
&\sup_t |P(H_n \le t+u) - P(H_n \le t)| \le \sup_t \left| P\left(\frac{H_n}{s_n} \le \frac{t+u}{s_n}\right) - \Phi\left(\frac{t+u}{s_n}\right)\right| \\
&+ \sup_t \left| P\left(\frac{H_n}{s_n} \le \frac{t}{s_n}\right) - \Phi\left(\frac{t}{s_n}\right)\right| + \sup_t \left|\Phi\left(\frac{t+u}{s_n}\right) - \Phi\left(\frac{t}{s_n}\right)\right| \\
&\le 2\sup_t \left| P\left(\frac{H_n}{s_n} \le t\right) - \Phi(t)\right| + \sup_t \left|\Phi\left(\frac{t+u}{s_n}\right) - \Phi\left(\frac{t}{s_n}\right)\right| \\
&\le C\left(\gamma_{2n}^{\delta/2} + |\frac{u}{s_n}|\right) \le C\left(\gamma_{2n}^{\delta/2} + |u|\right).
\end{aligned} \tag{4.7}
$$

Hence, from (4.7) it follows that

$$
D_{2n} = T\sup_t \int_{|u|\le C/T} |P(H_n \le t+u) - P(H_n \le t)|\, du \le C(\gamma_{2n}^{\delta/2} + 1/T) \tag{4.8}
$$

Combining (4.5), (4.6) with (4.8), and choosing $T = u^{-1/3}(q)$, we can easily see that

$$
|P(S'_{1n} \le t) - P(H_n \le t)| \le C(u^{1/3}(q) + \gamma_{2n}^{\delta/2}). \qquad \square
$$

Proof of Lemma 3.5. By Assumption (A1) and Lemma A.5 in the Appendix, it follows that

$$
\sigma_n^2(t) \le C\sum_{i=1}^{n} E\left(\varepsilon_{ni}\int_{A_i} E_m(t,s)ds\right)^2 \le O(2^m/n).
$$

In addition, according to Assumptions (A2)–(A4) and referring to Liang and Qi [8], it is easy to follow Lemma 3.5.

5. Appendix

Lemma A.1 [4] Suppose that $\{\varsigma_n, n \geq 1\}, \{\eta_n, n \geq 1\}$ and $\{\xi_n, n \geq 1\}$ are three random variable sequences,$\{\gamma_n, n \geq 1\}$ is a positive constant sequence, and $\gamma_n \to 0$. If $\sup_x |F_{\varsigma_n}(x) - \Phi(x)| \leq C\gamma_n$, then for any $\varepsilon_1 > 0, \varepsilon_2 > 0$

$$\sup_x |F_{\varsigma_n+\eta_n+\xi_n}(x) - \Phi(x)| \leq C\{\gamma_n + \varepsilon_1 + \varepsilon_2 + P(|\eta_n| \geq \varepsilon_1) + P(|\xi_n| \geq \varepsilon_2)\}.$$

Lemma A.2 [18] Let $\{X_j\}_{j\geq1}$ be a LNQD random variable sequence with zero mean and finite second moment, $\sup_{j\geq1} EX_j^r < \infty$. Assume that $\{a_j\}_{j\geq1}$ be a real constant sequence, $a := \sup_j |a_j| < \infty$. Then for any $r > 1$, there exists a constant C not depending on n such that

$$E\left|\sum_{j=1}^{n} a_j X_j\right|^r \leq Ca^r n^{r/2}.$$

Lemma A.3 [18] If $X_1, \cdots, X_m$ are LNQD random variables with finite second moments, let $\varphi_j(t_j)$ and $\varphi(t_1, \cdots, t_m)$ be c.f.'s of X_j and $(X_1, \cdots, X_m)$, respectively, then for all nonnegative(or non positive) real numbers $t_1, \cdots, t_m$,

$$\left|\varphi(t_1, \cdots, t_m) - \prod_{j=1}^{m} \varphi_j(t_j)\right| \leq 4 \sum_{1\leq l<k\leq m} |t_l t_k||\text{Cov}(X_l, X_k)|.$$

Lemma A.4 [11] Assume that Assumptions (A3) and (A4) hold, then
$(i) \sup_m \int_0^1 |E_m(t, s)|ds \leq C$; $(ii) \sum_{i=1}^n |\int_{A_i} E_m(t, s)ds| \leq C$;
$(iii) |\int_{A_i} E_m(t, s)ds| = O(\frac{2^m}{n}), i = 1, 2, \cdots, n$; $(iv) \sum_{i=1}^n (\int_{A_i} E_m(t, s)ds)^2 = O(\frac{2^m}{n})$.
Lemma A.5 [21] Suppose that $\{X_n; n \geq 1\}$ is a LNQD sequence of random variables with $EX_n = 0$. Then for any $p > 1$, there exists a positive constant C such that

$$E\left|\sum_{i=1}^{n} X_i\right|^p \leq CE\left(\sum_{i=1}^{n} X_i^2\right)^{p/2}, n \geq 1.$$

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Conflict of interest

The authors declare no conflict of interest in this paper.

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