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Research article

Boundedness analysis of non-autonomous stochastic differential systems with Lévy noise and mixed delays

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Abstract: The present research studies the boundedness issue of Lévy driven non-autonomous stochastic differential systems with mixed discrete and distributed delays. A set of sufficient conditions of the pth moment globally asymptotical boundedness is obtained by combining the Lyapunov function method with the inequality technique. The proposed results reveal that the convergence rate λ and the coefficients of the estimates for Lyapunov function W and Itô operator $\mathcal{L}W$ can determine the upper bound for the solution. The presented results are demonstrated by an illustrative example.

Keywords: stochastic differential systems; Lévy noise, mixed delays; asymptotical boundedness
Mathematics Subject Classification: 34K50, 34K20, 60G51

1. Introduction

Since the great mathematician Itô initiated and developed his stochastic calculus, the theory of stochastic differential systems has been developed rapidly. At present, stochastic differential systems have been used in many fields, such as mechanics of materials, economic electrical, finance, biology, neural networks, power systems, control engineering and social sciences. A lot of significant results on the theory and application of many kinds of stochastic differential systems have been obtained, for example, the existence-uniqueness, the periodicity, the stability and the boundedness of the solution have been discussed in [1–14], respectively; and the applications of stochastic differential systems in neural networks, epidemic models, chaotic systems and switched systems have been discussed in [15–22], respectively.

Needs to be emphasized that the stochastic differential systems are mainly limited to the case of Gaussian noise in the literature mentioned above. However, many practical system often suffers from sudden environmental perturbations which are unsuitable to be described by Gaussian noise, such as harvesting, earthquakes and hurricanes. Fortunately, as an important non-Gaussian noise, Lévy noise can be used to perfectly describe these phenomena. Recently, some interesting studies have been devoted to stochastic differential systems with Lévy noise [23–27]. These studies mainly focus on the stability of the solution. But there is seldom study focusing on the boundedness of the solution [28].

Based on the above statement, the present article aims to discuss the boundedness issue for non-autonomous stochastic differential systems with Lévy noise and mixed delays. Sufficient conditions of the pth moment globally asymptotical boundedness are obtained by combining the Lyapunov function methods with the inequality techniques. The main contributions of the present research are as follows: (i) both Lévy noises and mixed delays are taken into account for non-autonomous stochastic differential systems; (ii) several sufficient conditions on the asymptotical boundedness are presented for the considered model using the Lyapunov technique; (iii) attracting sets along with the convergence rates of the model are also given.

Notations:

$\Re_+ := [0, \infty)$.

$\Re_{t_0} := [t_0, \infty)$.

$a \wedge b$: the minimum of a and b.

$\lambda_{\min}(A)$: the smallest eigenvalue of a symmetric matrix A.

$\lambda_{\max}(A)$: the largest eigenvalue of a symmetric matrix A.

$|u|$: the Euclidean norm of a vector u.

$(\Omega, \mathscr{F}, \mathbb{P})$: the complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$.

$\omega(t) = (\omega_1(t), ..., \omega_m(t))^T$: the m-dimensional Brownian motion defined on $(\Omega, \mathscr{F}, \mathbb{P})$.

$C[[-\nu, 0], \Re^d]$: the space of continuous $\Re^d$-valued functions ϕ defined on $[-\nu, 0]$ with the norm $|\phi|_\nu = \sup_{-\nu\leq\theta\leq 0} |\phi(\theta)|$.

$\mathbb{E}[\xi]$: the expectation for a stochastic process ξ.

$C^b_{\mathcal{F}_{t_0}}[[-\nu, 0], \Re^d]$: the family of bounded $\mathcal{F}_{t_0}$-measurable, $C[[-\nu, 0], \Re^d]$-valued random variables ϕ such that $\mathbb{E}|\phi|^p_\nu < \infty$.

$C^{2,1}(\Re^d \times \Re_{t_0}, \Re_+)$: the family of all nonnegative functions $W(u, t)$ from $\Re^d \times \Re_{t_0}$ to $\Re_+$, which which are continuously twice differentiable in $u \in \Re^d$ and once differentiable in $t \in \Re_+$.

2. Preliminaries

Consider the non-autonomous stochastic differential systems with Lévy noise and mixed delays

$$\begin{cases} du(t) = \mathcal{X}(u(t), u(t^- - \nu), \int_{t-\nu}^t \alpha(t-\iota)u(\iota)d\iota, t)dt \\ \qquad\qquad +\mathcal{Y}(u(t), u(t^- - \nu), \int_{t-\nu}^t \alpha(t-\iota)u(\iota)d\iota, t)d\omega(t) \\ \qquad\qquad + \int_{|\varsigma|<c} H(u(t^-), u(t^- - \nu), \int_{t-\nu}^t \alpha(t-\iota)u(\iota^-)d\iota, t, \varsigma)\tilde{\Theta}(dt, d\varsigma) \\ \qquad\qquad + \int_{|\varsigma|\geq c} I(u(t^-), u(t^- - \nu), \int_{t-\nu}^t \alpha(t-\iota)u(\iota^-)d\iota, t, \varsigma)\Theta(dt, d\varsigma), t \geq t_0 \geq 0, \\ u(t_0 + \theta) = \phi(\theta), -\nu \leq \theta \leq 0, \end{cases} \tag{2.1}$$

where the initial value $\phi(\theta) \in C^b_{\mathcal{F}_{t_0}}[[-\nu, 0], \Re^d]$, $\alpha : \Re \to \Re$ is a continuous function, $u(\iota^-) = \lim_{\theta\uparrow\iota} u(\theta)$, $\mathcal{X} : \Re^d \times \Re^d \times \Re^d \times \Re_{t_0} \to \Re^d$, $\mathcal{Y} : \Re^d \times \Re^d \times \Re^d \times \Re_{t_0} \to \Re^{d\times m}$; $H, I : \Re^d \times \Re^d \times \Re^d \times \Re_{t_0} \times \Re^d \to \Re^d$, the constant $c \in (0, \infty]$ represents the maximum allowable jump size, $\Theta(\cdot, \cdot)$ represents a Poisson random measure defined on $\Re_{t_0} \times (\Re^d - \{0\})$ with compensator $\tilde{\Theta}(\cdot, \cdot)$ and intensity measure v. Suppose that $\Theta(\cdot, \cdot)$ is independent of ω and v represents a lévy measure such that

$$\tilde{\Theta}(dt, d\varsigma) = \Theta(dt, d\varsigma) - v(d\varsigma)dt, \int_{\Re^d \backslash \{0\}} (|\varsigma|^p \wedge 1) v(d\varsigma) < \infty.$$

The notation (ω, Θ) is often referred to as the Lévy noise.

Now let us recall the definition of the operator $\mathcal{L}W$ (one may refer to [24]).
If $W \in C^{2,1}(\Re^d \times \Re_{t_0}, \Re_+)$, define an operator $\mathcal{L}W$ from $\Re^d \times \Re_{t_0}$ to $\Re$ by

$$\begin{aligned} \mathcal{L}W(u,t) =& W_t(u,t) + W_u(u,t)\mathcal{X} + \frac{1}{2} trac(\mathcal{Y}^T W_{uu}(u,t)\mathcal{Y}) \\ &+ \int_{|\varsigma|<c} [W(u+H,t) - W(u,t) - HW_u(u,t)] v(d\varsigma) \\ &+ \int_{|\varsigma|\geq c} [W(u+I,t) - W(u,t)] v(d\varsigma), \end{aligned} \tag{2.2}$$

where $\mathcal{X}$, $\mathcal{Y}$, H and I are the functions in model (2.1), and

$$W_t(u,t) = \frac{\partial W(u,t)}{\partial t}, W_u(u,t) = (\frac{\partial W(u,t)}{\partial u_1}, ..., \frac{\partial W(u,t)}{\partial u_d}), W_{uu}(u,t) = (\frac{\partial^2 W(u,t)}{\partial u_i u_j})_{d\times d}.$$

Lemma 2.1.([29]). *For $a_k \geq 0$, $b_k > 0$ and $\sum_{k=1}^n b_k = 1$,*

$$\prod_{k=1}^n a_k^{b_k} \leq \sum_{k=1}^n b_k a_k. \tag{2.3}$$

Definition 2.2. Model (2.1) is said to be pth moment globally asymptotically bounded (p-GAB) if there exist a positive constant $\mathfrak{z}_0$ such that for $\forall\ \phi \in C^b_{\mathcal{F}_{t_0}}[[-\nu, 0], \Re^d]$,

$$\limsup_{t\to\infty} \mathbb{E}|x(t; t_0, \phi)|^p \leq \mathfrak{z}_0, p \geq 2, t \geq t_0.$$

When $p = 2$, it is usually said to be GAB in mean square.

Definition 2.3. Model (2.1) is said to be pth moment globally exponentially ultimately bounded (p-GEUB) if there exist positive constants λ, $\mathfrak{z}_1$ and $\mathfrak{z}_2$ such that for $\forall\ \phi \in C^b_{\mathcal{F}_{t_0}}[[-\nu, 0], \Re^d]$,

$$\mathbb{E}|x(t; t_0, \phi)|^p \leq \mathfrak{z}_1 \mathbb{E}|\phi|^p_\nu e^{-\lambda(t-t_0)} + \mathfrak{z}_2, p \geq 2, t \geq t_0.$$

When $p = 2$, it is usually said to be GEUB in mean square.

Remark 2.4. The above definitions are very important to stochastic systems. For more detail on these definitions, one may refer to [30, 31].

3. Asymptotical boundedness

In this section, several sufficient conditions on the asymptotical boundedness will be presented for the model (2.1) using the Lyapunov technique.

Theorem 3.1. *Let $\eta(s)$ be a continuous function and $W(u,t) \in C^{2,1}(\Re^d \times \Re_{t_0}, \Re_+)$. If there exist constants $p \geq 2$, $\hat{\varpi} > 0$, $\varrho > 0$, $\gamma_i > 0 (i = 1, 2, ..., 5)$ and $\gamma_6 \geq 0$ such that for all $(u,t) \in \Re^d \times \Re_{t_0}$,*

(i)

$$\gamma_1 |u|^p \leq W(u,t) \leq \gamma_2 |u|^p; \tag{3.1}$$

(ii)

$$\mathcal{L}W(u,t) \leq \varpi(t)[-\gamma_3 W(u(t),t) + \gamma_4 W(u(t-\nu),t) + \gamma_5 \int_0^\nu \eta(s) W(u(t-s),t) ds + \gamma_6], \tag{3.2}$$

where $\varpi(t)$ is a positive integrable function satisfying $\varpi(t+\nu) \leq \varrho\varpi(t)\varpi(\nu)$, $\sup_{t \geq t_0} \int_{t-\tau}^t \varpi(s) ds \leq \hat{\varpi}$ and $\lim_{t\to\infty} \int_{t_0}^t \varpi(s) ds = \infty$;

(iii)

$$\gamma_1\gamma_3 > \varrho\gamma_2\gamma_4\varpi(\nu) + \gamma_2\gamma_5 \int_0^\nu \eta(s)\varpi(s) ds. \tag{3.3}$$

Then model (2.1) is p-GAB, and every solution of model (2.1) will eventually converge to the compact set defined by

$$\mathfrak{S} = \left\{ \xi \in C^b_{\mathcal{F}_{t_0}}[[-\nu, 0], \Re^d] | \mathbb{E}|\xi|^p_\nu \leq \frac{\gamma_6}{\gamma_1 \lambda} \right\}, \tag{3.4}$$

where the positive constant λ is defined as

$$\gamma_2\lambda - \gamma_1\gamma_3 + \varrho\gamma_2\gamma_4\varpi(\nu)e^{\lambda\hat{\varpi}} + \varrho\gamma_2\gamma_5 \int_0^\nu \eta(s)\varpi(s)e^{\lambda s} ds < 0. \tag{3.5}$$

Proof. Let m be a positive number and define the stopping time $\mu_m = \inf\{t > t_0 : |u(t)| \geq m\}$. Applying the generalized Itô formula to $W(u,t)$ yields

$$\begin{aligned}
& e^{\lambda \int_{t_0}^{t\wedge\mu_m} \varpi(\sigma)d\sigma} W(u(t\wedge\mu_m), t\wedge\mu_m) - W(u(t_0), t_0) \\
&\quad = \int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma} [\lambda\varpi(\zeta)W(u(\zeta),\zeta) + \mathcal{L}W(u(\zeta),\zeta)] d\zeta \\
&\quad + \int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma} W_u(u(\zeta),\zeta)\mathcal{Y}(u(\zeta), u(\zeta-\nu), \int_{t-\nu}^t \alpha(\zeta-s)u(s)ds, \zeta) d\omega(\zeta) \\
&\quad + \Lambda(t) + \Upsilon(t),
\end{aligned} \tag{3.6}$$

where

$$\Lambda(t) = \int_{t_0}^{t\wedge\mu_m} \int_{|\varsigma|<c} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma} [W(u(\zeta^-)$$

$$+ H(u(\zeta^-), u(\zeta^- - \nu), \int_{\zeta-\nu}^{\zeta} \alpha(\zeta - s)u(s^-)ds, \zeta, \varsigma), \zeta^-)$$
$$- W(u(\zeta^-), \zeta^-)]\tilde{\Theta}(d\zeta, d\varsigma) \tag{3.7}$$

and

$$\Upsilon(t) = \int_{t_0}^{t\wedge\mu_m} \int_{|\varsigma|\geq c} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}[W(u(\zeta^-)$$
$$+ I(u(\zeta^-), u(\zeta^- - \nu), \int_{\zeta-\nu}^{\zeta} \alpha(\zeta - s)u(s^-)ds, \zeta, \varsigma), \zeta^-)$$
$$- W(u(\zeta^-), \zeta^-)]\tilde{\Theta}(d\zeta, d\varsigma) \tag{3.8}$$

are two martingales satisfying $\Lambda(t_0) = \Upsilon(t_0) = 0$. One therefore has that

$$\mathbb{E}\left(e^{\lambda \int_{t_0}^{t\wedge\mu_m} \varpi(\sigma)d\sigma} W(u(t \wedge \mu_m), t \wedge \mu_m) - W(u(t_0), t_0)\right)$$
$$= \mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}[\lambda\varpi(\zeta)W(u(\zeta), \zeta) + \mathcal{L}W(u(\zeta), \zeta)]d\zeta\right). \tag{3.9}$$

This together with the conditions (i) and (ii) yields that

$$\mathbb{E}\left(e^{\lambda \int_{t_0}^{t\wedge\mu_m} \varpi(\sigma)d\sigma} W(u(t \wedge \mu_m), t \wedge \mu_m)\right)$$
$$\leq \gamma_2\mathbb{E}|\phi|^p + \mathbb{E}\bigg(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}[\gamma_2\lambda\varpi(\zeta)|u(\zeta)|^p + \varpi(\zeta)(-\gamma_1\gamma_3|u(\zeta)|^p + \gamma_2\gamma_4|u(\zeta - \nu)|^p$$
$$+ \int_0^{\nu} \gamma_2\gamma_5\eta(s)|u(\zeta - s)|^p ds + \gamma_6)]d\zeta\bigg)$$
$$\leq \gamma_2\mathbb{E}|\phi|^p + (\gamma_2\lambda - \gamma_1\gamma_3)\mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}\varpi(\zeta)|u(\zeta)|^p d\zeta\right)$$
$$+ \gamma_2\gamma_4\mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}\varpi(\zeta)|u(\zeta - s)|^p d\zeta\right)$$
$$+ \mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}\varpi(\zeta) \int_0^{\nu} \gamma_2\gamma_5\eta(s)|u(\zeta - s)|^p dsd\zeta\right) + \frac{\gamma_6}{\lambda}(e^{\lambda \int_{t_0}^{t\wedge\mu_m} \varpi(\sigma)d\sigma} - 1) \tag{3.10}$$

On the other hand,

$$\mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}\varpi(\zeta)|u(\zeta - s)|^p d\zeta\right) \leq \varrho\varpi(\nu)e^{\lambda\hat{\varpi}}\mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}\varpi(\zeta)|u(\zeta)|^p d\zeta\right)$$
$$+ \frac{1}{\lambda}(e^{\lambda\hat{\varpi}} - 1)\mathbb{E}|\phi|^p \tag{3.11}$$

and

$$\mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m} e^{\lambda \int_{t_0}^{\zeta} \varpi(\sigma)d\sigma}\varpi(\zeta) \int_0^{\nu} \gamma_2\gamma_5\eta(s)|u(\zeta - s)|^p dsd\zeta\right)$$

$$
\begin{aligned}
=&\mathbb{E}\left(\int_0^{\nu}\gamma_2\gamma_5\eta(s)\int_{t_0}^{t\wedge\mu_m}e^{\lambda\int_{t_0}^{\zeta}\varpi(\sigma)d\sigma}\varpi(\zeta)|u(\zeta-s)|^p d\zeta ds\right)\\
=&\mathbb{E}\left(\int_0^{\nu}\gamma_2\gamma_5\eta(s)(\int_{t_0-s}^{t\wedge\mu_m-s}e^{\lambda\int_{t_0}^{\zeta+s}\varpi(\sigma)d\sigma}\varpi(\zeta+s)|u(\zeta)|^p d\zeta)ds)\right)\\
\le&\mathbb{E}\left(\int_0^{\nu}\gamma_2\gamma_5\eta(s)e^{\lambda\hat{\varpi}}(\int_{t_0}^{t\wedge\mu_m-s}e^{\lambda\int_{t_0}^{\zeta}\varpi(\sigma)d\sigma}\varpi(\zeta+s)|u(\zeta)|^p d\zeta)ds)\right)\\
&+\mathbb{E}\left(\int_0^{\nu}\gamma_2\gamma_5\eta(s)(\int_{t_0-s}^{t_0}e^{\lambda\int_{t_0}^{\zeta+s}\varpi(\sigma)d\sigma}\varpi(\zeta+s)|u(\zeta)|^p d\zeta)ds)\right)\\
\le&\mathbb{E}\left(\int_0^{\nu}\gamma_2\gamma_5\eta(s)\varpi(s)e^{\lambda\hat{\varpi}}(\int_{t_0}^{t\wedge\mu_m-s}e^{\lambda\int_{t_0}^{\zeta}\varpi(\sigma)d\sigma}\varrho\varpi(\zeta)|u(\zeta)|^p d\zeta)ds)\right)\\
&+\mathbb{E}\left(\int_0^{\nu}\gamma_2\gamma_5\eta(s)(\int_{t_0-s}^{t_0}e^{\lambda\int_{t_0}^{\zeta+s}\varpi(\sigma)d\sigma}\varpi(\zeta+s)|u(\zeta)|^p d\zeta)ds)\right)\\
\le&(\varrho\gamma_2\gamma_5\int_0^{\nu}\eta(s)\varpi(s)e^{\lambda\hat{\varpi}}ds)\mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m}e^{\lambda\int_{t_0}^{\zeta}\varpi(\sigma)d\sigma}\varpi(\zeta)|u(\zeta)|^p d\zeta\right)\\
&+\int_0^{\nu}\gamma_2\gamma_5\eta(s)(\frac{1}{\lambda}(e^{\lambda\hat{\varpi}}-1))ds)\mathbb{E}|\phi|^p
\end{aligned}
\tag{3.12}
$$

Substituting (3.11) and (3.12) into (3.10) yields

$$
\begin{aligned}
&\mathbb{E}\left(e^{\lambda\int_{t_0}^{t\wedge\mu_m}\varpi(\sigma)d\sigma}W(t\wedge\mu_m,u(t\wedge\mu_m))\right)\\
\le&\gamma_2\mathbb{E}|\phi|^p+(\gamma_2\lambda-\gamma_1\gamma_3+\varrho\gamma_2\gamma_4e^{\lambda\hat{\varpi}}+\varrho\gamma_2\gamma_5\int_0^{\nu}\eta(s)\varpi(s)e^{\lambda\hat{\varpi}}ds)\mathbb{E}\left(\int_{t_0}^{t\wedge\mu_m}e^{\lambda\int_{t_0}^{\zeta}\varpi(\sigma)d\sigma}\varpi(\zeta)|u(\zeta)|^p d\zeta\right)\\
&+(\gamma_4+\gamma_5\int_0^{\nu}\eta(s)ds)\frac{\gamma_2}{\lambda}(e^{\lambda\hat{\varpi}}-1)\mathbb{E}|\phi|^p+\frac{\gamma_6}{\lambda}(e^{\lambda\int_{t_0}^{t\wedge\mu_m}\varpi(\sigma)d\sigma}-1)
\end{aligned}
\tag{3.13}
$$

Noting $\gamma_1\gamma_3>\varrho\gamma_2\gamma_4\varpi(\nu)+\varrho\gamma_2\gamma_5\int_0^{\nu}\eta(s)\varpi(s)ds$, there is a positive scalar λ satisfying the inequality (3.5). Letting $n\to\infty$ yields

$$
\mathbb{E}\left(e^{\lambda\int_{t_0}^{t}\varpi(\sigma)d\sigma}W(u(t),t)\right)\le\gamma_2\mathbb{E}|\phi|^p+(\gamma_4+\gamma_5\int_0^{\nu}\eta(s)ds)\frac{\gamma_2}{\lambda}(e^{\lambda\hat{\varpi}}-1)\mathbb{E}|\phi|^p+\frac{\gamma_6}{\lambda}(e^{\lambda\int_{t_0}^{t\wedge\mu_m}\varpi(\sigma)d\sigma}-1)
\tag{3.14}
$$

Using the condition (i) and the relation (3.14), we then have

$$
\mathbb{E}|u(t)|^p\le\frac{\gamma_2}{\gamma_1}[1+\frac{1}{\lambda}(\gamma_4+\gamma_5\int_0^{\nu}\eta(s)ds)(e^{\lambda\hat{\varpi}}-1)]\mathbb{E}|\phi|^pe^{-\lambda\int_{t_0}^{t}\varpi(\sigma)d\sigma}+\frac{\gamma_6}{\gamma_1\lambda}.
\tag{3.15}
$$

The proof is therefore completed. □

Assumption 3.1. There exist functions $\epsilon_i(t)(i=1,2,...,12)$, constants $p\ge 2$, $\hat{\varrho}>0$, $\hat{\delta}>0$, $\hat{\epsilon}_i(i=1,2,...,12)$ and a symmetric positive definite matrix Q such that

$$
(i)\qquad u^TQ\mathcal{X}+\frac{1}{2}trac(\mathcal{Y}^TQ\mathcal{Y})\le\epsilon_1(t)u^TQu+\epsilon_2(t)u^T(t-\nu)Qu(t-\nu)
$$

$$+\epsilon_3(t)\int_0^{\nu}\alpha(s)u^T(t-s)Qu(t-s)ds+\epsilon_4(t); \tag{3.16}$$

$$(ii)\quad |u^TQ\mathcal{Y}|^2 \le \epsilon_5(t)(u^TQu)^2+\epsilon_6(t)(u^T(t-\nu)Qu(t-\nu))^2$$
$$+\epsilon_7(t)\int_0^{\nu}\alpha(s)((u^T(t-s)Qu(t-s)))^2ds+\epsilon_8(t); \tag{3.17}$$

$$(iii)\quad \int_{|\varsigma|<c}[((u+H)^TQ(u+H))^{\frac{p}{2}}-(u^TQu)^{\frac{p}{2}}-p(u^TQu)^{\frac{p}{2}-1}u^TQH]\nu du$$
$$\le \epsilon_9(t)(u^TQu)^{\frac{p}{2}}+\epsilon_{10}(t)(u^T(t-\nu)Qu(t-\nu))^{\frac{p}{2}}$$
$$+\epsilon_{11}(t)\int_0^{\nu}\alpha(s)(u^T(t-s)Qu(t-s))^{\frac{p}{2}}ds+\epsilon_{12}(t); \tag{3.18}$$

$$(iv)\quad \int_{|\varsigma|\ge c}[((u+I)^TQ(u+I))^{\frac{p}{2}}-(u^TQu)^{\frac{p}{2}}]\nu du$$
$$\le \epsilon_{13}(t)(u^TQu)^{\frac{p}{2}}+\epsilon_{14}(t)(u^T(t-\nu)Qu(t-\nu))^{\frac{p}{2}}$$
$$+\epsilon_{15}(t)\int_0^{\nu}\alpha(s)(u^T(t-s)Qu(t-s))^{\frac{p}{2}}ds+\epsilon_{16}(t); \tag{3.19}$$

$$(v)\quad (\lambda_{min}(Q))^{\frac{p}{2}}\hat{\gamma}_3 > \hat{\varrho}\,(\lambda_{max}(Q))^{\frac{p}{2}}\hat{\gamma}_4\delta(\nu)+(\lambda_{max}(Q))^{\frac{p}{2}}\hat{\gamma}_5\int_0^{\nu}\alpha(s)\delta(s)ds; \tag{3.20}$$

$$(vi)\quad \hat{\gamma}_6 = 2\hat{\epsilon}_4+2(p-2)\hat{\epsilon}_8+\hat{\epsilon}_{12}+\hat{\epsilon}_{16}\ge 0; \tag{3.21}$$

$$(vii)\quad \epsilon_i(t)\le \hat{\epsilon}_i\delta(t). \tag{3.22}$$

where $\delta(t)$ is a positive integrable function satisfying $\delta(t+\nu)\le \hat{\varrho}\delta(t)\delta(\nu)$,$\sup_{t\ge t_0}\int_{t-\nu}^{t}\delta(s)ds \le \hat{\delta}$ and $\lim_{t\to\infty}\int_{t_0}^{t}\delta(s)ds=\infty$,

$$\hat{\gamma}_3 = -[p\hat{\epsilon}_1+(\hat{\epsilon}_2+\hat{\epsilon}_4)(p-2)+p(\frac{p}{2}-1)\hat{\epsilon}_5+(p-2)(\frac{p}{2}-2)(\hat{\epsilon}_6+\hat{\epsilon}_8)+\hat{\epsilon}_9+\hat{\epsilon}_{13}$$
$$+(\hat{\epsilon}_3(p-2)+(p-2)(\frac{p}{2}-2)\hat{\epsilon}_7)\int_0^{\nu}\alpha(s)ds)]>0,$$
$$\hat{\gamma}_4 = 2\hat{\epsilon}_2+2(p-2)\hat{\epsilon}_6+\hat{\epsilon}_{10}+\hat{\epsilon}_{14}>0 \tag{3.23}$$

and

$$\hat{\gamma}_5 = 2\hat{\epsilon}_3+2(p-2)\hat{\epsilon}_7+\hat{\epsilon}_{11}+\hat{\epsilon}_{15}>0. \tag{3.24}$$

Theorem 3.2. *If Assumption 3.1 holds, then model (2.1) is p-GAB, and every solution of model (2.1) will eventually converge to the compact set defined by*

$$\mathfrak{S}=\left\{\xi\in C^b_{\mathcal{F}_{t_0}}[[-\nu,0],\mathfrak{R}^d]|\mathbb{E}|\xi|^p_{\nu}\le \frac{\hat{\gamma}_6}{(\lambda_{min}(Q))^{\frac{p}{2}}\lambda}\right\}, \tag{3.25}$$

where the positive constant λ is defined as

$$(\lambda_{max}(Q))^{\frac{p}{2}}\lambda-(\lambda_{min}(Q))^{\frac{p}{2}}\hat{\gamma}_3+\hat{\varrho}\,(\lambda_{max}(Q))^{\frac{p}{2}}\hat{\gamma}_4\delta(\nu)e^{\lambda\hat{\delta}}$$
$$+(\lambda_{max}(Q))^{\frac{p}{2}}\hat{\varrho}\hat{\gamma}_5\int_0^{\nu}\alpha(s)\delta(t)e^{\lambda s}ds<0. \tag{3.26}$$

Proof. Defined the function $W(u,t) \in C^{2,1}(\Re^d \times \Re_{t_0}, \Re_+)$ by

$$W(u(t),t) = (u^T(t)Qu(t))^{\frac{p}{2}}. \tag{3.27}$$

Clearly, one has

$$(\lambda_{min}(Q))^{\frac{p}{2}}\, \mathbb{E}|u|^p \leq \mathbb{E}W(u(t),t) \leq (\lambda_{max}(Q))^{\frac{p}{2}}\, \mathbb{E}|u|^p. \tag{3.28}$$

Computing $\mathcal{L}W(u,t)$ by the conditions (3.16)–(3.19) yields

$$\begin{aligned}
\mathcal{L}W(u,t) =& p(u^TQu)^{\frac{p}{2}-1}[u^TQ\mathcal{X} + \frac{1}{2}trac(\mathcal{Y}^TQ\mathcal{Y})] + p(\frac{p}{2}-1)(u^TQu)^{\frac{p}{2}-2}|u^TQ\mathcal{Y}|^2 \\
&+ \int_{|\varsigma|<c}[((u+H)^TQ(u+H))^{\frac{p}{2}} - (u^TQu)^{\frac{p}{2}} - p(u^TQu)^{\frac{p}{2}-1}u^TQH]vdu \\
&+ \int_{|\varsigma|\geq c}[((u+I)^TQ(u+I))^{\frac{p}{2}} - (u^TQu)^{\frac{p}{2}}]vdu \\
\leq& (p\epsilon_1(t) + p(\frac{p}{2}-1)\epsilon_5(t))(u^TQu)^{\frac{p}{2}} + p\epsilon_2(t)(u^TQu)^{\frac{p}{2}-1}u^T(t-\nu)Qu(t-\nu) \\
&+ p\epsilon_3(t)\int_0^{\nu}\alpha(s)(u^TQu)^{\frac{p}{2}-1}u^T(t-s)Qu(t-s)ds \\
&+ \epsilon_4(t)p(u^TQu)^{\frac{p}{2}-1} + p(\frac{p}{2}-1)\epsilon_6(t)(u^TQu)^{\frac{p}{2}-2}(u^T(t-\nu)Qu(t-\nu))^2 \\
&+ p(\frac{p}{2}-1)\epsilon_7(t)\int_0^{\nu}\alpha(s)(u^TQu)^{\frac{p}{2}-2}((u^T(t-s)Qu(t-s)))^2ds \\
&+ \epsilon_8(t)p(\frac{p}{2}-1)(u^TQu)^{\frac{p}{2}-2} + (\epsilon_9(t)+\epsilon_{13}(t))(u^TQu)^{\frac{p}{2}} \\
&+ (\epsilon_{10}(t)+\epsilon_{14}(t))(u^T(t-\nu)Qu(t-\nu))^{\frac{p}{2}} \\
&+ (\epsilon_{11}(t)+\epsilon_{15}(t))\int_0^{\nu}\alpha(s)(u^T(t-s)Qu(t-s))^{\frac{p}{2}}ds + (\epsilon_{12}(t)+\epsilon_{16}(t))
\end{aligned} \tag{3.29}$$

Using Lemma 2.1 and (3.29) produce

$$\begin{aligned}
\mathcal{L}W(u,t) \leq& (p\hat{\epsilon}_1 + p(\frac{p}{2}-1)\hat{\epsilon}_5)\delta(t)(u^TQu)^{\frac{p}{2}} + \hat{\epsilon}_2(p-2)\delta(t)(u^TQu)^{\frac{p}{2}} \\
&+ 2\hat{\epsilon}_2\delta(t)(u^T(t-\nu)Qu(t-\nu))^{\frac{p}{2}} + \hat{\epsilon}_3(p-2)\delta(t)\int_0^{\nu}\alpha(s)(u^TQu)^{\frac{p}{2}}ds \\
&+ 2\hat{\epsilon}_3\delta(t)\int_0^{\nu}\alpha(s)(u^T(t-s)Qu(t-s))^{\frac{p}{2}}ds + \hat{\epsilon}_4(p-2)\delta(t)(u^TQu)^{\frac{p}{2}} + 2\hat{\epsilon}_4\delta(t) \\
&+ (\frac{p}{2}-1)(p-4)\hat{\epsilon}_6\delta(t)(u^TQu)^{\frac{p}{2}} + 2(p-2)\hat{\epsilon}_6\delta(t)((u^T(t-\nu)Qu(t-\nu)))^{\frac{p}{2}} \\
&+ (\frac{p}{2}-1)(p-4)\hat{\epsilon}_7\delta(t)\int_0^{\nu}\alpha(s)(u^TQu)^{\frac{p}{2}}ds \\
&+ 2(p-2)\hat{\epsilon}_7\delta(t)\int_0^{\nu}\alpha(s)((u^T(t-s)Qu(t-s)))^{\frac{p}{2}}ds \\
&+ \hat{\epsilon}_8(\frac{p}{2}-1)(p-4)\delta(t)(u^TQu)^{\frac{p}{2}} + \hat{\epsilon}_8(2p-4)\delta(t) + (\hat{\epsilon}_9+\hat{\epsilon}_{13})\delta(t)(u^TQu)^{\frac{p}{2}}
\end{aligned}$$

$$
\begin{aligned}
&+ (\hat{\epsilon}_{10} + \hat{\epsilon}_{14})\delta(t)(u^T(t-\nu)Qu(t-\nu))^{\frac{p}{2}} \\
&+ (\hat{\epsilon}_{11} + \hat{\epsilon}_{15})\delta(t)\int_0^{\nu} \alpha(s)(u^T(t-s)Qu(t-s))^{\frac{p}{2}}ds + (\hat{\epsilon}_{12} + \hat{\epsilon}_{16})\delta(t) \\
=&\delta(t)[-\hat{\gamma}_3 W(u(t),t) + \hat{\gamma}_4 W(u(t-\nu),t) + \hat{\gamma}_5 \int_0^{\nu} \alpha(s)W(u(t-s),t)ds + \hat{\gamma}_6]. \qquad (3.30)
\end{aligned}
$$

By the continuity and the condition (v), there exists a positive scalar λ satisfying (3.26). Therefore, it follows from (3.28), (3.30) and Theorem 3.1 that

$$
\begin{aligned}
\mathbb{E}|u(t)|^p \leq &\frac{(\lambda_{max}(Q))^{\frac{p}{2}}}{(\lambda_{min}(Q))^{\frac{p}{2}}}[1 + \frac{1}{\lambda}(\hat{\gamma}_4 + \hat{\gamma}_5 \int_0^{\nu} \alpha(s)ds)(e^{\lambda\hat{\delta}} - 1)]\mathbb{E}|\phi|^p e^{-\lambda\int_{t_0}^{t}\delta(\sigma)d\sigma} \\
&+ \frac{\hat{\gamma}_6}{(\lambda_{min}(Q))^{\frac{p}{2}}\lambda}, \qquad (3.31)
\end{aligned}
$$

where the positive scalar λ is determined by (3.26). The proof is therefore completed. □

From the results obtained above, we have the following corollaries immediately.

Corollary 3.3. *Under assumptions of Theorem 3.1. If* $\varpi(t) = 1$*, then model (2.1) is p-GEUB.*

Corollary 3.4. *Under Assumption 3.1. If* $\delta(t) = 1$*, then model (2.1) is p-GEUB.*

Corollary 3.5. *Under assumptions of Theorem 3.1. If* $\gamma_6 = 0$*, then model (2.1) is pth moment globally asymptotically stable (p-GAS).*

Corollary 3.6. *Under Assumption 3.1. if* $\hat{\epsilon}_4 = \hat{\epsilon}_8 = \hat{\epsilon}_{12} = \hat{\epsilon}_{16} = 0$*, then model (2.1) is p-GAS.*

Remark 3.7. *The boundedness of Levy driven non-autonomous stochastic differential systems with infinite distributed delays have been discussed in [28]. One can find that the results in [28] are invalid for model (2.1) since model (2.1) is a mixed delayed system. Even for the case where only distributed delays are considered, our conditions are looser than those in [28] since* $\varpi(t) \not\equiv 1$ *and* $\delta(t) \not\equiv 1$ *in our conditions.*

Remark 3.8. *Compared with ordinary differential systems, partial differential systems have more wide application. Up to now, various partial differential systems have been extensively discussed [32]. Recently, Lévy driven partial differential systems have also aroused many researchers' great interest [33]. But the boundedness issue of Lévy driven partial differential systems is still a challenge. We will discuss it in the future work.*

Remark 3.9. Although the condition (3.2) is relaxed enough for model (2.1), it is harsh on certain types of systems such as the Cohen-Grossberg neural networks since $\varpi(t)$ is dependent of $x(t)$ in Cohen-Grossberg neural networks. How to improve the condition (3.2) so that it is effective for Cohen-Grossberg neural networks is still a challenge.

Remark 3.10. The obtained results can be applied to the boundedness analysis for some real world systems such as capital asset pricing models, DC motor models and population systems. Such applications will be addressed in the future work.

Remark 3.11. It is well-known that, impulsive effects are unavoidable in many real systems, which can affect the boundedness of the systems. In recent years, various impulsive systems, such as impulsive complex-valued systems [34], impulsive fractional systems [35], impulsive stochastic systems [11], have been studied. More recently, impulsive effects have been considered in Lévy driven stochastic differential systems [36]. Therefore, it is necessary to extend the obtained results to the impulsive case. Further research is needed for such extension which will be discussed in the future work.

4. Illustrative example

Example 4.1. Consider the following 1-D stochastic differential systems with Lévy noise and mixed delays

$$\begin{aligned} du(t) =&(2+\cos t)(-19u(t)+2u(t-1)+\int_{t-1}^{t} e^{-3(t-s)}u(s)ds+3)dt \\ &+\sqrt{(2+\cos t)}[\sqrt{3}u(t)+u(t-1)]d\omega(t) \\ &+\int_{|\varsigma|<1}\sqrt{2+\cos t}\int_{t-1}^{t} e^{-3(t-s)}u(s^-)ds\tilde{\Theta}(dt,d\varsigma) \\ &+\int_{|\varsigma|\geq 1} 2\sqrt{2+\cos t}\int_{t-1}^{t} e^{-3(t-s)}u(s^-)ds\Theta(dt,d\varsigma),\ t\geq 0, \end{aligned} \tag{4.1}$$

with the Lévy measure v satisfing $v(d\varsigma)=\frac{d\varsigma}{1+|\varsigma|^2}$.

Taking $W(u,t)=u^2$, one has

$$W_u(t,u(t))\mathcal{X} \leq (2+\cos t)[-30u^2(t)+2u^2(t-1)+\int_0^1 e^{-3s}u^2(t-s)ds+3], \tag{4.2}$$

$$\frac{1}{2}W_{uu}(t,u(t))\mathcal{Y}^2 \leq (2+\cos t)[6u^2(t)+u^2(t-1)], \tag{4.3}$$

$$\begin{aligned} &\int_{|\varsigma|<1}[W(u+H,t)-W(u,t)-HW_u(u,t)]v(d\varsigma) \\ &\quad=\int_{|\varsigma|<1}[(u+\sqrt{2+\cos t}\int_{t-1}^{t} e^{-3(t-s)}u(s)ds)^2-u^2 \\ &\quad\quad-2\sqrt{2+\cos t}u\int_{t-1}^{t} e^{-3(t-s)}u(s)ds]\frac{d\varsigma}{1+|\varsigma|^2} \\ &\quad=\int_{|\varsigma|<1}(\sqrt{2+\cos t}\int_{t-1}^{t} e^{-3(t-s)}u(s)ds)^2\frac{d\varsigma}{1+|\varsigma|^2} \end{aligned}$$

$$\leq \frac{\pi}{2}(2+\cos t)\int_0^1 e^{-3s}u^2(t-s)ds, \tag{4.4}$$

$$\begin{aligned}
&\int_{|\varsigma|\geq 1}[W(t,u+I)-W(u,t)]\nu(d\varsigma)\\
&\qquad = \int_{|\varsigma|\geq 1}[(u(t)+2\sqrt{2+\cos t}\int_{t-1}^{t} e^{-3(t-s)}u(s)ds)^2-u^2]\nu(d\varsigma)\\
&\qquad \leq (2+\cos t)[\pi u^2+3\pi\int_0^1 e^{-3s}u^2(t-s)ds].
\end{aligned} \tag{4.5}$$

Hence

$$\begin{aligned}
\mathcal{L}W(t,u(t)) =&(2+\cos t)[-32u^2(t)+2u^2(t-1)+\int_0^1 e^{-3s}u^2(t-s)ds+3]\\
&+6(2+\cos t)u^2(t)+(2+\cos t)u^2(t-1)\\
&+\frac{\pi}{2}\int_0^1 e^{-3s}u^2(t-s)ds+\pi u^2+3\pi\int_0^1 e^{-3s}u^2(t-s)ds\\
\leq&(2+\cos t)[(-26+\pi)u^2(t)+2u^2(t-1)+(1+\frac{7\pi}{2})\int_0^1 e^{-3s}u^2(t-s)ds+3], t\geq t_0.
\end{aligned} \tag{4.6}$$

The conditions (i) and (ii) of Theorem 3.1 can be easily verified by choosing $\gamma_1=\gamma_2=1$, $\gamma_3=26-\pi$, $\gamma_4=2$, $\gamma_5=1+\frac{7\pi}{2}$, $\gamma_6=3$, $\eta(s)=e^{-3s}$, $\varpi(t)=2+\cos t$, $\varrho=1$ and $p=2$. On the other hand, the condition (iii) is also satisfied by

$$\begin{aligned}
\varrho\gamma_2\gamma_4\varpi(\nu)+\gamma_2\gamma_5\int_0^{\nu}\eta(s)\varpi(s)ds &= 2(2+\cos 1)+(1+\frac{7\pi}{2})\int_0^1 3e^{-3s}ds\\
&< 7+\frac{7\pi}{2}<\gamma_1\gamma_3=26-\pi.
\end{aligned}$$

In this example, one can take $\lambda=0.05$ which satisfies the relation (3.5). Therefore, by Theorem 3.1, model (4.1) is GAB in mean square, and every solution of model (4.1) will eventually converge to the compact set defined by

$$\mathfrak{S}=\{\xi\in C^b_{\mathcal{F}_{t_0}}[[-\nu,0],\Re^d]|\mathbb{E}|\xi|^2_\nu\leq\frac{\gamma_6}{\gamma_1\lambda}=60\}. \tag{4.7}$$

5. Conclusions

This article has studied the boundedness issue for non-autonomous stochastic differential systems with Lévy noise and mixed delays. Sufficient conditions of the pth moment globally asymptotical boundedness have been obtained by combining the Lyapunov function approach with the inequality technique. The presented results have been demonstrated by an illustrative example. In the future, we will discuss the problems mentioned in Remarks 3.8–3.11.

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Conflict of interest

No potential conflict of interest was reported by the authors.

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