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Research article

Reversed S-shaped connected component for second-order periodic boundary value problem with sign-changing weight

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Abstract: In this paper, we consider the existence of an reversed S-shaped connected component in the set of positive solutions for second order periodic boundary value problem with a sign-changing weight function. By bifurcation technique, we identify the interval of bifurcation parameter in which the periodic boundary value problem has one or two or three positive solutions according to the asymptotic behavior of f at 0 and ∞.

Keywords: positive solution; periodic boundary value problem; bifurcation
Mathematics Subject Classification: 34B10, 34B18

1. Introduction

In this paper, we show the existence of reversed S-shaped connected component of positive solutions for second order periodic boundary value problem(PBVP)

$$\begin{cases} -u''(t) + q(t)u(t) = \lambda h(t)f(u(t)), \quad t \in (0, 2\pi), \\ u(0) = u(2\pi), \quad u'(0) = u'(2\pi), \end{cases} \tag{1.1}$$

where $q \in C([0, 2\pi], (0, \infty))$, $f \in C(\mathbb{R}, \mathbb{R})$, $\lambda > 0$ is a parameter, and $h \in C[0, 2\pi]$ is a sign-changing function.

In recent years, periodic boundary value problem have been discussed by many authors via topological degree theory, fixed point theorems and bifurcation technique (see, for example, [2, 4–10, 12–14, 18] and the references therein).

In [4], Dai et al. studied a unilateral global bifurcation result for a class of quasilinear PBVP

$$\begin{cases} -(\varphi_p(u'))' + q(t)\varphi_p(u) = \lambda m(t)f(u), \quad t \in (0, T), \\ u(0) = u(T), \quad u'(0) = u'(T), \end{cases} \tag{1.2}$$

where $\varphi_p(s) = |s|^{p-2}s,\ p > 1,\ q \in C([0, T], (0, \infty)),\ f \in C(\mathbb{R}, \mathbb{R}),\ m \in C[0, T]$ change its sign, λ is a parameter. They using the bifurcation technique to derive that the interval of parameter λ in which the above problem (1.2) has one or two positive solutions according to the asymptotic behavior of f at 0 and ∞.

In 2015, Sim and Tanaka [17] studied the global structure of positive solutions set for the p-Laplacian problem

$$\begin{cases} -(\varphi_p(u'))' + \lambda h(x) f(u) = 0, & x \in (0, 1), \\ u(0) = u(1) = 0, \end{cases} \tag{1.3}$$

where $\lambda > 0$ is a parameter, $f \in C[0, \infty)$, $f(0) = 0$, $f > 0$ for all $s > 0$, the weight function $h \in C[0, 1]$ satisfies the condition

(F0) *there exist* $a, b \in [0, 1]$ *such that* $a < b,\ h(x) > 0$ *on* (a, b) *and* $h(x) \le 0$ *on* $[0, 1] \backslash [a, b]$.

By bifurcation technique, they proved that there is an unbounded component which is bifurcating from the trivial solution, grows to the right from the initial point, to the left at some point, and to the right near $\lambda = \infty$. Roughly speaking, there exists an S*-shaped connected component* of positive solutions for problem (1.3). See Figure 1(i). Inspired by [17], in [9], the authors have established a similar result to the problem (1.1) with the weight function h satisfies condition (F0).

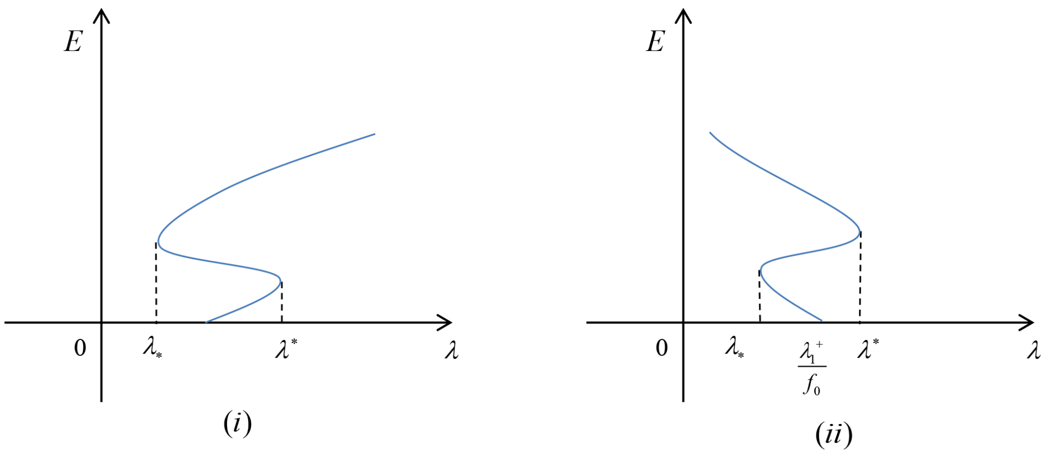

Figure 1. (i) S-shaped connected component. (ii) reversed S-shaped connected component.

We mention that recently existence and multiplicity results for nonlinear nonhomogeneous parametric robin problems were proved by Papageorgiou et al. [15, 16]. In particular, in [16], the authors proved a bifurcation-type theorem for small values of the parameter. More precise, they showed that there exists $\lambda^* > 0$ such that for all $\lambda \in (0, \lambda^*)$, the robin problem has at least two positive solutions, and for all $\lambda = \lambda^*$, the problem has at least one positive solution, and for all $\lambda > \lambda^*$, the problem has no positive solution.

Motivated by these studies, in this paper, we show the existence of reversed S-shaped connected component of positive solutions for (1.1). As a by-product, we determine the interval of λ in which problem (1.1) has one or two or three positive solutions under the suitable conditions on the weight function and nonlinearity.

Consider problem (1.1). Suppose that f satisfies
(H0) $f \in C(\mathbb{R}, \mathbb{R})$ with $f(s)s > 0$ for $s \neq 0$.

Let λ_0^+ be the first eigenvalue for the following linear eigenvalue problem

$$\begin{cases} -u''(t) + q(t)u(t) = \lambda h(t)u(t), \quad t \in (0, 2\pi), \\ u(0) = u(2\pi), \quad u'(0) = u'(2\pi). \end{cases} \tag{1.4}$$

It is well-known that λ_0^+ is simple eigenvalue with positive eigenfunction. See [3].
Remark 1.1. The eigenvalue λ_0^+ is the minimum of the "Rayleigh quotient", that is

$$\lambda_0^+ = \inf\Big\{ \int_0^{2\pi} ((v')^2 + qv^2)dt |\, v(0) = v(2\pi),\ v'(0) = v'(2\pi), \int_0^{2\pi} hv^2 dt = 1\Big\}.$$

Furthermore, we assume that
(H1) there exist constants $\alpha > 0, f_0 > 0$, and $f_1 > 0$ such that

$$\lim_{s\to 0^+} \frac{f(s) - f_0 s}{s^{1+\alpha}} = f_1;$$

(H2) $f_\infty := \lim\limits_{s\to\infty} \frac{f(s)}{s} = \infty$;
(H3) there exists $s_0 > 0$ such that $0 \leq s \leq s_0$ implies that

$$f(s) \leq \frac{f_0}{2\pi\lambda_0^+ \hat{h} M} s_0,$$

where $\hat{q} = \max\limits_{s\in[0,2\pi]} q(s)$, $\hat{h} = \max_{s\in[0,2\pi]} |h(s)|$, M be as in (2.1).

Notice that if (H1) holds, then

$$\lim_{s\to 0^+} \frac{f(s)}{s} = f_0. \tag{1.5}$$

Indeed, under (H0) and (H1), we have an unbounded subcontinuum which is bifurcating from $(\lambda_0^+/f_0, 0)$ and goes leftward. Conditions (H0) and (H3) lead the unbounded sub-continuum to the right at some point, and finally to the left near $\lambda = 0$. Roughly speaking, we shall show that there exists a *reversed S-shaped connected component* of positive solutions of (1.1). See Figure 1(ii).

Our main result is the following theorem

Theorem 1.1. Assume that (H0)–(H3) hold. Then there exist $\lambda_* \in (0, \frac{\lambda_0^+}{f_0})$ and $\lambda^* > \frac{\lambda_0^+}{f_0}$ such that
(i) (1.1) has at least one positive solution if $0 < \lambda < \lambda_*$;
(ii) (1.1) has at least two positive solutions if $\lambda = \lambda_*$;
(iii) (1.1) has at least three positive solutions if $\lambda_* < \lambda < \lambda_0^+/f_0$;
(iv) (1.1) has at least two positive solutions if $\lambda_0^+/f_0 < \lambda \leq \lambda^*$;
(v) (1.1) has at least one positive solution if $\lambda > \lambda^*$.
Remark 1.1. Let us consider the function

$$f(s) = ms + s\ln(1 + s),\ m > 0,\ s \in [0, \infty).$$

Obviously, f satisfies (H1) and (H2) with

$$\alpha = 1, f_0 = m, f_1 = 1.$$

It is easy to see that if $m > 0$ is sufficiently large, then this function satisfies (H3).

Remark 1.2. If the conditions of f in this paper are compared with those of f in [9, 17], it is easy to see that f is superlinear near ∞ according to (H2), so we cannot find a constant $f^* > 0$ such that $f(s) \leq f^* s$ for all $s \geq 0$. This will bring great difficulty to the study of the global structure of the positive solutions set for (1.1). On the other hand, in [9, 17], the authors used a key condition about the weight function h, see (F0). We note that the concavity and convexity of the positive solutions of (1.1) can be deduced directly from (F0) and the nonlinearity in the equation. However, in this paper, we don't make any additional assumptions about the weight function h.

The paper is organized as follows. In Section 2, we show the existence of bifurcation from simply eigenvalue for the corresponding problem and the leftward direction near the initial point. Section 3 is devoted to showing the change of direction of bifurcation, and complete the proof of Theorem 1.1.

2. Preliminaries and leftward bifurcation

Let $G(t, s)$ is the Green's function of the homogeneous PBVP

$$\begin{cases} -u''(t) + q(t)u(t) = 0, & t \in (0, 2\pi), \\ u(0) = u(2\pi), \quad u'(0) = u'(2\pi). \end{cases}$$

By [1, Theorem 2.5], we know that $G(t, s) > 0, \ \forall t, s \in [0, 2\pi]$.

Denote

$$M = \max_{0 \leq t, s \leq 2\pi} G(t, s). \tag{2.1}$$

Let $Y = C[0, 2\pi]$ with the norm

$$\|u\|_\infty = \max_{t \in [0, 2\pi]} |u(t)|,$$

$E = \{u \in C^1[0, 2\pi] : u(0) = u(2\pi), u'(0) = u'(2\pi)\}$ with the norm

$$\|u\| = \max_{t \in [0, 2\pi]} |u(t)| + \max_{t \in [0, 2\pi]} |u'(t)|,$$

and $P = \{u \in E : u(t) \geq 0\}$ be the positive cone in E.

Lemma 2.1. ([4, Theorem 4.3]) Assume $f_0 \in (0, \infty)$, $f_\infty \in (0, \infty)$, then $(\frac{\lambda_0^+}{f_0}, 0)$ is a bifurcation point of problem (1.1). Moreover, there is an unbounded component C of the set for the solutions of problem (1.1) and

$$C \subset \Big(\{(\mathbb{R}^+ \times \mathrm{int} P)\} \cup \{(\frac{\lambda_0^+}{f_0}, 0)\} \Big).$$

Furthermore, C emanating from $(\frac{\lambda_0^+}{f_0}, 0)$ and joining to $(\frac{\lambda_0^+}{f_\infty}, \infty)$.

Lemma 2.2. ([11]) Let X be a Banach space and let C_n be a family of closed connected subsets of X. Assume that:

(i) there exist $z_n \in C_n$, $n = 1, 2, \cdots$, and $z^* \in X$, such that $z_n \to z^*$;

(ii) $r_n = \sup\{\|x\| | x \in C_n\} = \infty$;

(iii) for every $R > 0$, $(\bigcup_{n=1}^\infty C_n) \cap \bar{B}_R(0)$ is a relatively compact set of X.

Then $D := \limsup_{n \to \infty} C_n$ is unbounded, closed and connected.

Lemma 2.3. If $f_0 \in (0, \infty)$ and $f_\infty = \infty$, then the unbounded subcontinuum C of positive solutions for (1.1) joins $(\frac{\lambda_0^+}{f_0}, 0)$ to $(0, \infty)$.

Proof. Define the cut-off function of f as the following

$$f_n(s) = \begin{cases} f(s), & s \in [0, n], \\ \dfrac{2n^2 - f(n)}{n}(s - n) + f(n), & s \in (n, 2n), \\ ns, & s \in [2n, \infty). \end{cases}$$

Consider the following second order periodic boundary value problem

$$\begin{cases} -u''(t) + q(t)u(t) = \lambda h(t) f_n(u(t)), \quad t \in (0, 2\pi), \\ u(0) = u(2\pi), \quad u'(0) = u'(2\pi). \end{cases} \tag{2.2}$$

Clearly, $\lim_{n\to\infty} f_n(s) = f(s), (f_n)_0 = f_0$ and $(f_n)_\infty = n$. Lemma 2.1 implies that there exists a sequence of unbounded continua $(C)_n$ of solutions to problem (2.2) emanating from $(\frac{\lambda_0^+}{f_0}, 0)$ and joining to $(\frac{\lambda_0^+}{n}, \infty)$.

By Lemma 2.2, there exists an unbounded component C of $\lim_{n\to\infty} C_n$ such that $(\frac{\lambda_0^+}{f_0}, 0) \in C$ and $(0, \infty) \in C$. This completes the proof. □

Lemma 2.4. [9, Lemma 2.2] Assume (H1). Let $\{(\lambda_n, u_n)\}$ is a sequence of positive solutions to (1.1) which satisfies $\lambda_n \to \frac{\lambda_0^+}{f_0}$ and $\|u_n\| \to 0$. Let ϕ is the positive eigenfunction corresponding to λ_0^+, which satisfies $\|\phi\| = 1$. Then there exists a subsequence of $\{u_n\}$, again denoted by $\{u_n\}$, such that $\frac{u_n}{\|u_n\|}$ converges uniformly to ϕ on $[0, 2\pi]$.

Lemma 2.5. Let $\alpha \geq 0$ and let ϕ be the positive eigenfunction corresponding to λ_0^+. Then

$$\int_0^{2\pi} h(t)[\phi(t)]^{2+\alpha} dt > 0.$$

Proof. Multiplying the equation of (1.4) by $\phi^{\alpha+1}$ and integrating it over $[0, 2\pi]$, we obtain

$$\begin{aligned} \mu_1 \int_0^{2\pi} h(t)[\phi(t)]^{\alpha+2} dt &= -\int_0^{2\pi} \phi''(t)[\phi(t)]^{\alpha+1} dt + \int_0^{2\pi} q(t)[\phi(t)]^{\alpha+2} dt \\ &= (\alpha + 1)\int_0^{2\pi} [\phi'(t)]^2[\phi(t)]^\alpha dt + \int_0^{2\pi} q(t)[\phi(t)]^{\alpha+2} dt > 0. \end{aligned}$$

□

Lemma 2.6. Assume that (H0)–(H2) hold. Let C be as in Lemma 2.3. Then there exists $\delta > 0$ such that $(\lambda, u) \in C$ and $|\lambda - \frac{\lambda_0^+}{f_0}| + \|u\| \leq \delta$ imply $\lambda < \frac{\lambda_0^+}{f_0}$.

Proof. Assume to the contrary that there exists a sequence $\{(\lambda_n, u_n)\} \subset C$ such that $\lambda_n \to \frac{\lambda_0^+}{f_0}$, $\|u_n\| \to 0$ and $\lambda_n \geq \frac{\lambda_0^+}{f_0}$. By Lemma 2.4 there exists a subsequence of $\{u_n\}$, again denoted by $\{u_n\}$, such that $\frac{u_n}{\|u_n\|_\infty}$ converges uniformly to ϕ on $[0, 2\pi]$. Multiplying the equation of (1.1) with $(\lambda, u) = (\lambda_n, u_n)$ by ϕ and integrating it over $[0, 2\pi]$, we have

$$\int_0^{2\pi} \phi(x)(-u_n''(x) + q(x)u_n(x))dx = \lambda_n \int_0^{2\pi} h(x) f(u_n(x))\phi(x) dx. \tag{2.3}$$

By a simple computation, one has that

$$\int_0^{2\pi}\phi(x)(-u_n''(x)+q(x)u_n(x))dx=\int_0^{2\pi}(-\phi''(x)+q(x)\phi(x))u_n(x)dx \\ =\lambda_0^+\int_0^{2\pi}h(x)\phi(x)u_n(x)dx. \tag{2.4}$$

Combining (2.3) and (2.4), we obtain

$$\int_0^{2\pi}h(x)f(u_n(x))\phi(x)dx=\frac{\lambda_0^+}{\lambda_n}\int_0^{2\pi}h(x)\phi(x)u_n(x)dx,$$

and accordingly,

$$\frac{\int_0^{2\pi}h(x)\phi(x)[f(u_n(x))-f_0u_n(x)]dx}{\|u_n\|_\infty^{1+\alpha}}=\frac{\int_0^{2\pi}h(x)\phi(x)[\frac{\lambda_0^+}{\lambda_n}u_n(x)-f_0u_n(x)]dx}{\|u_n\|_\infty^{1+\alpha}}.$$

Because

$$\frac{\int_0^{2\pi}h(x)\phi(x)[f(u_n(x))-f_0u_n(x)]dx}{\|u_n\|_\infty^{1+\alpha}}=\int_0^{2\pi}h(x)\phi(x)\frac{f(u_n(x))-f_0u_n(x)}{(u_n(x))^{1+\alpha}}\Big[\frac{u_n(x)}{\|u_n\|_\infty}\Big]^{1+\alpha}dx.$$

By Lebesgue's dominated convergence theorem, Lemma 2.5, and condition (H1), we have

$$\frac{\int_0^{2\pi}h(x)\phi(x)[f(u_n(x))-f_0u_n(x)]dx}{\|u_n\|_\infty^{1+\alpha}}\to f_1\int_0^{2\pi}h(x)\phi^{2+\alpha}dx>0.$$

Similarly,

$$\frac{\int_0^{2\pi}h(x)\phi(x)[\frac{\lambda_0^+}{\lambda_n}u_n(x)-f_0u_n(x)]dx}{\|u_n\|_\infty^{1+\alpha}}=\frac{\lambda_0^+-f_0\lambda_n}{\lambda_n\|u_n\|_\infty^{\alpha}}\int_0^{2\pi}h(x)\phi(x)\frac{u_n(x)}{\|u_n\|_\infty}dx\le 0.$$

This is a contradiction. □

3. Proof of Theorem 1.1

In this section, we show that there is a direction turn of the bifurcation under (F3) condition and accordingly we finish the proof of Theorem 1.1.

Lemma 3.1. [9, Lemma 3.1] Let (H1) and (H2) hold. Assume that $\{(\lambda_k,u_k)\}$ is a sequence of positive solutions of (1.1). Assume that $|\lambda_k|<C_0$ for some constant $C_0>0$, and $\lim_{k\to\infty}\|u_k\|\to\infty$, then $\lim_{k\to\infty}\|u_k\|_\infty\to\infty$.

Lemma 3.2. Assume that (H0) and (H3) hold. Let $(\lambda,u)\in\mathcal{C}$ be a positive solution of (1.1) with $\|u\|_\infty=s_0$. Then $\lambda>\frac{\lambda_0^+}{f_0}$.

Proof. Let u be a positive solution of (1.1) with $\|u\|_\infty = s_0$, then

$$\begin{aligned} s_0 = \|u\|_\infty &= \max_{x\in[0,2\pi]} \lambda \Big| \int_0^{2\pi} G(x,s)h(s)f(u(s))ds \Big| \\ &\leq \lambda M \int_0^{2\pi} |h(s)| f(u(s))ds \\ &< \lambda M \cdot 2\pi \hat{h} \frac{f_0}{2\pi\lambda_0^+ \hat{h} M} s_0, \\ &= \lambda \cdot \frac{f_0}{\lambda_0^+} s_0. \end{aligned}$$

So, it is easy to see that $\lambda > \frac{\lambda_0^+}{f_0}$. □

Proof of Theorem 1.1. From Lemma 2.3, there exists an unbounded connected component C in the positive solutions set of (1.1) and C is bifurcating from $(\frac{\lambda_0^+}{f_0}, 0)$ and goes leftward.

By Lemma 2.3, it follows that there exists $(\lambda_0, u_0) \in C$ such that $\|u_0\|_\infty = s_0$ and Lemma 2.6 implies that $\lambda_0 < \frac{\lambda_0^+}{f_0}$. By Lemmas 2.6, 3.2, C passes through some points $(\frac{\lambda_0^+}{f_0}, v_1)$ and $(\frac{\lambda_0^+}{f_0}, v_2)$ with $\|v_1\|_\infty < s_0 < \|v_2\|_\infty$, and there exist $\underline{\lambda}$ and $\overline{\lambda}$ which satisfy $0 < \underline{\lambda} < \frac{\lambda_0^+}{f_0} < \overline{\lambda}$ and both (i) and (ii):

(i) if $\lambda \in (\frac{\lambda_0^+}{f_0}, \overline{\lambda}]$, then there exist u and v and $(\lambda, u), (\lambda, v) \in C$ satisfy $\|u\|_\infty < s_0 < \|v\|_\infty$;

(ii) if $\lambda \in (\underline{\lambda}, \frac{\lambda_0^+}{f_0}]$, then there exist u and v and $(\lambda, u), (\lambda, v) \in C$ satisfy $\|u\|_\infty < \|v\|_\infty < s_0$.

Let $\lambda^* = \sup\{\overline{\lambda} : \overline{\lambda}$ satisfies (i)$\}$, $\lambda_* = \inf\{\underline{\lambda} : \underline{\lambda}$ satisfies (ii)$\}$. Then second order periodic boundary value problem (1.1) has a positive solution u_{λ_*} at $\lambda = \lambda_*$ and u_{λ^*} at $\lambda = \lambda^*$, respectively.

It is easy to see that C turns to the left at $(\lambda^*, \|u_{\lambda^*}\|_\infty)$ and C turns to the right at $(\lambda_*, \|u_{\lambda_*}\|_\infty)$, finally to the left near $\lambda = 0$. In other words, C is a reversed S-shaped component. This complete the proof of Theorem 1.1. □

Now we strengthen the assumptions on f and h.

(H5) there exists s_1 with $s_1 > 2s_0 > 0$ such that

$$\min_{s\in[s_1,2s_1]} \frac{f(s)}{s} \geq \frac{f_0}{\lambda_0^+ h_0}\Big[(\frac{2\pi}{l})^2 + \hat{q}\Big],$$

where $l = x_2 - x_1$, $\hat{q} = \max_{s\in[0,2\pi]} q(s)$, $h_0 = \{h(t) | t \in [\frac{3x_1+x_2}{4}, \frac{x_1+3x_2}{4}]\}$;

(H6) there exists $\beta > 0$ such that

$$\beta h^+(t) \inf\{\frac{f(s)}{s} : s \in (0, s_1]\} \geq q(t), \quad t \in (x_1, x_2),$$

where $h^+(t) := \max\{h(t), 0\}$.

By an argument similar to proving [9, Lemma 3.5] with obvious changes, we may obtain the following result.

Corollary 3.1. Assume (H0), (H2), (H4)–(H6). Let (λ, u) be a positive solution of (1.1) with $\|u\|_\infty = s_1$. Then $\lambda < \lambda_0^+/f_0$.

Remark 3.1. Suppose that the hypotheses of Theorem 1.1 hold. Moreover, using the method similar to proving Lemma 3.2 and Corollary 3.1 infinitely many times, we can obtain the continuum C is unbounded, join $(\lambda_0^+/f_0, 0)$ to $(0, \infty)$ and oscillates around the axis $\{\lambda = \lambda_0^+/f_0\}$ an infinite number of times. See Figure 2.

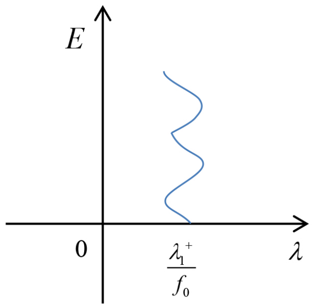

Figure 2. Graph of continuum C.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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