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Research article

On quantum aspects of general theory of relativity and its detectors

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Abstract: Starting from limitations of quantum theory and its measurements we have discussed the concept of quantization in general theory of relativity in strong regime as well as quantization of gravity, which leads to gravitons, in weak field limitation. This is based on the fact that general theory of relativity based on strong principle of equivalence which is incompatible with quantum principle. It has shown that this is the complete agreement with the implications following from the measurement analysis. To discuss the physical consequences of the limitations arising for quantum GRT, Compton effect and Euler scattering are discussed.

Keywords: Compton effect; Euler scattering; vacuum polarization effect; quantum gravity; gravitons **Mathematics Subject Classification:** 83C45, 83C47

1. Introduction

In this paper, the relation between GRT and quantum theory has discussed. That means, it is not asked the question "what do you expect a quantum theory of gravity to look like?" but it is posed the query "how does classical GRT (without any modification) relate to quantum theory"? We are therefore dealing with same question for GRT, which was discussed for electrodynamics after the foundation of quantum theory [14, 17, 20].

To discuss this topic, first there will be presented some results from [2,3] and in the literature cited there in Section 2 while in Section 3 some problems concerning Euler scattering will be considered.

2. Limitation of field measurements resulting from background quantization

We can think quantum operator g_{ij} as a classical background operator η_{ij} with some perturbation h_{ij} .

$$g_{ij} = \eta_{ij} + h_{ij} \tag{2.1}$$

This method was developed by Dewitt [4, 5]. Apply Einstein-Hilbert action Σ after using tensor notations ($g^i = \eta^i + h^i$), the following functional series is obtained:

$$\Sigma[g] = \Sigma[\eta + h] = \Sigma_{i_1}[\eta] h^{i_1} + \frac{1}{2} \Sigma_{i_1 i_2}[\eta] h^{i_1} h^{i_2} + \frac{1}{3!} \Sigma_{i_1 i_2 i_3}[\eta] h^{i_1} h^{i_2} h^{i_3} + \dots$$
(2.2)

Since classical operator η_{ij} is a background operator, it satisfied the equation

$$\Sigma_{,i}\left[\eta\right] = 0 \tag{2.3}$$

in other words,

$$R_{ij}\left(\eta_{ij}\right) = 0 \tag{2.4}$$

Ricci curvature tensor vanishing here permits us to use solution of Einstein's equations in vacuum as a background field. One can recover the classical approach to quantum gravity that use particle physics by putting η_{ij} to ζ_{ij} in Eq. (2.2) [15]. Then Eqs. (2.1) and (2.2) reduce to

$$g_{ij} = \zeta_{ij} + h_{ij} \tag{2.5}$$

and

$$\Sigma[g] = \Sigma_{FP}(\eta,\zeta) + \frac{1}{2}F_{;i_1i_2}[\zeta]h^{i_1}h^{i_2} + \frac{1}{3!}F_{;i_1i_2i_3}[\eta]h^{i_1}h^{i_2}h^{i_3} + \dots$$
(2.6)

Here $F_{;i_1}$ is a tensor that comes from $\Sigma_{;i_1i_2}$ when we use gauge condition and Σ_{FP} represents the evaluation of action at zero point corresponds to flat empty spacetime which may describe the supplementation of vector fictitions particles. In this manner, one can recover classical field theory using a very nonlinear (typically higher order terms i.e., polynomial) interaction [19]. This interaction would be between massless spin of two gravitons, which are propagating in a Minkowski space. Here quantum general relativity is explicitly curtailed to level of usual particle physics withal the renormalizability problems known in this model [16].

Another implication of condition (2.3) consists of the fact that, for pure gravity Σ matrix elements without divergences result on the one-loop level. Indeed, it was shown in [1, 14] that the covariant counter terms will be added in the Lagrangian i.e.,

$$\Delta L \sim \sqrt{-g} \left[a R \left[\eta \right]^2 + b R_{ij} \left[\eta \right] R^{ij} \left[\eta \right] \right]$$
(2.7)

From Eq. (2.3) this term vanishes in vacuum.

The philosophy behind the Eq. (2.3) says that whenever Eq. (2.3) holds, true external gravitons (associated to η_{ij}) are physical, that is, are on mass shell. This shows that founders of the background method start with the conviction that gravitons should rigorously be attributed to gravitational fields so that even the classical background η_{ij} can be separated into ζ_{ij} and ϕ_{ij} . That is

$$\eta^r = \zeta^r + \phi^r \tag{2.8}$$

and

$$\eta_{ij} = \zeta_{ij} + \phi_{ij} \tag{2.9}$$

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Where ϕ_{ij} is to be interpreted in terms of external gravitons. Because we, however, want to examine the relation of GRT to quantum theory (i.e., the justification of the graviton hypothesis). In a weak-field approximation, spacetime looks like

$$\hat{g}_{ij} = \eta_{ij} + \varepsilon h_{ij} \tag{2.10}$$

 η_{ij} is again the usual background when Ricci curvature vanishes mostly, i.e., $\eta_{ij} \sim 1$. Order of magnitude of h_{ij} is same as of η_{ij} and $\varepsilon > 0$. In same way order of magnitude is $\partial \eta \sim \frac{r}{L}$ and $\partial h \sim \frac{h}{\lambda}$ where L and λ are characteristics lengths. We can express Ricci tensor in terms of Taylor series expansion,

$$R_{ij}\left[\eta_{ij} + \varepsilon h_{ij}\right] = R_{ij}\left(\eta_{ij}\right) + \varepsilon R_{ij}^{(1)}\left(h_{ij}\right) + \varepsilon^2 R_{ij}^{(2)}\left(h_{ij}\right) + \varepsilon^3 R_{ij}^{(3)}\left(h_{ij}\right) + \dots$$
(2.11)

with

$$R_{ij}\left(\eta_{ij}\right) = O\left(L^{-2}\right), \quad \varepsilon R_{ij}^{(1)}\left(h_{ij}\right) = O\left(\varepsilon\lambda^{-2}\right), \quad \varepsilon^2 R_{ij}^{(2)}\left(h_{ij}\right) = O\left(\varepsilon^2\lambda^{-2}\right), \dots \quad (2.12)$$

$$\varepsilon^{r} R_{ij}^{(r)} \left(h_{ij} \right) = O \left(\varepsilon^{r} \lambda^{-2} \right)$$

Let us distinguish now three cases $\frac{\lambda}{L} \gg \varepsilon$, $\frac{\lambda}{L} \ll \varepsilon$ and $\frac{\lambda}{L} \sim \varepsilon$.

1. $\frac{\lambda}{L} \gg \varepsilon$: In this case we have the order of magnitude relation

$$R_{ij}\left(\eta_{ij}\right) \gg \varepsilon^{r} R_{ij}^{(r)}\left(h_{ij}\right) \tag{2.13}$$

so that $R_{ij}(\eta_{ij})$ is the contributing term and all other terms are correction term only. First order correction in Ricci tensor leads to

$$R_{ij}(\eta_{ij})=0$$

and hence, in this case background field method is most suitable one. The non-linearity of the Einstein tensors and Ricci tensors lead to weak field interaction and probability of occurrence of nonlinear graviton-graviton interactions are high. But it is not (and need not) considered a back reaction of the h_{ij} field on the η_{ij} background field $\cong L^{-2}$ [7]. In this approximation, the quantum field may be regarded as propagation upon a fixed background field.

2. $\frac{\lambda}{L} \ll \varepsilon$: Regarding again the relations (2.12), we get from Eq. (2.11)

$$R_{ij}(\eta_{ij}) \ll \varepsilon^2 R_{ij}^{(2)}(h_{ij}) \tag{2.14}$$

In this case background field method does not work. Independent of all questions on quantization, background method opposes the spirit of GRT for such high frequencies.

3. $\frac{\lambda}{L} \sim \varepsilon$: Weak field assumption provides $\varepsilon \ll 1$ thus $\lambda \ll L$ so that the h_{ij} field is of high frequency. But the frequency is not so large here that the decomposition (2.10) loses its sense or, roughly speaking, the η_{ij} background is dissolved into fluctuations. Usual background considerations must however be modified because, instead of Eq. (2.3), now Eq. (2.13) holds, therefore we have to assume

$$R_{ij}\left(\eta_{ij}\right) - \frac{1}{2}\eta_{ij}R\left(\eta_{ij}\right) = -\frac{G}{c^4}T_{ij}^{eff}$$
(2.15)

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where

$$T_{ij}^{eff} \equiv \frac{c^4 \varepsilon^2}{G} \left[R_{ij}^{(2)} \left(h_{ij} \right) - \frac{1}{2} \eta_{ij} R^{(2)} \left(h_{ij} \right) \right]$$
(2.16)

is the effective stress tensor because of the high frequency field.

As there was argued in [2], such high frequency considerations show only for $\lambda \ge L$. This means that only for comparatively low frequencies, the usual background method (with Eq. (2.13)) can be applied. The assumption $R_{ij}(\eta) = 0$ for all λ (made in the conventional approach) is according to a strong supplementary condition changing the mathematical and physical contents of GRT.

The high frequency arguments hitherto given in this section show nonlinear nature of Einstein's equation. There are frequencies for which one has to consider the background as dynamically determined. Therefore, the algebraic splitting (2.9) (which is always possible) cannot be maintained on the level of the field equations. Now our point is, this typical feature of GRT leads to limitations on quantum GRT. Indeed, according to Eq. (2.15) the total energy density $\rho = \frac{c^4}{G} \frac{\varepsilon^2}{\lambda^2}$ of the h_{ij} field is to be equal or smaller than $\frac{c^4}{G}L^{-2}$, i.e.,

$$L^{-2} \ge \left(\frac{\varepsilon}{\lambda}\right)^2$$

(The inequality sign is used if, besides gravitational waves h_{ij} , other sources of energy are present) and in accordance to the fact that h_{ij} describes a quantum field $\rho = \frac{h\nu}{L_0^3}$ [18].

If L_0 denotes the linear extension of the volume considered. We obtain

$$\lambda \ge \frac{1}{L_0^3} \left(l_p L \right)^2 \tag{2.17}$$

and for an optimal measurement

$$\lambda \ge l_0 \equiv \left(l_p L\right)^{1/2} \tag{2.18}$$

where $l_p = \left(\frac{hG}{c^3}\right)^{1/2}$ is Planck's length.

The above inequality relations say that these limitations on the quantization procedure signaling an intrinsic incompatibility of classical GRT and quantum theory at high frequencies (high energies) and short distances, respectively.

We could now be tempted to consider the conclusion mentioned above that, it is not so much as a result concerning typical features of quantum GRT but as showing the limits of the background field method used here. There are however some arguments opposing this interpretation. First, we should remember that principle limitation on quantum GRT were deduced by Rosenfeld [23] arguing mainly within the canonical approach. Rosenfeld showed that in quantum GRT there occurs a principle limitation for length measurements given by $L_0 \ge l_p$, which results, by regarding typical features of GRT like Q = GM and $\Delta |g_{ij}| < 1$, from the Bohr-Rosenfeld relation for field measurements,

$$\Delta F.L_0^2 \ge \frac{h}{c} \frac{Q}{M} \tag{2.19}$$

originally derived for electrodynamics. Here L_0 denotes the linear extension of the volume considered, Q the charge of the measurement body, M its mass, and F the field strength to be measured. This shows that the canonical approach does not change the situation, it rather provides the absolute limit on quantum GRT caused by the three universal constants h, c and G lying on the basis of quantum GRT and the covariant approach completes this picture. It shows that due to the nonlinear structure of Einstein's equations and to the identification of metric and field, the cut off arises gradually before (not rapidly) at Planck's length. The latter point is an interesting feature of the situation in principle characterized by the appearance of Planck's cut-off units, because nonlinearity and metric field identification are expressions of the strong principle of equivalence. The gradual cut-off given by equations (2.17) and (2.18) is thus an implication of the basic principle of GRT.

A second argument showing the fundamental meaning of the cut-off length stems from the measurement analysis not referring to a special field quantization method. Following Bohr and Rosenfeld, it was shown that Eq. (2.19) can be derived by pure measurement consideration if one assumes a measurement body whose structure is described by classical physics and whose displacement Δx in the field to be measured obeys Heisenberg's uncertainty relation

$$\Delta p_x \Delta x \ge h \tag{2.20}$$

Due to Q = GM and in the case of gravity, Eq. (2.19) reads

$$\Delta g. L_0^2 \ge \frac{hG}{c^3} \tag{2.21}$$

and

$$\Delta\Gamma.L_0^3 \ge \frac{hG}{c^3} \tag{2.22}$$

(g and Γ denote the components of the metric and the affinity, respectively.)

Next, we try to measure the constraints on background metric. Uncertainty arises due to measurement of metric g near the origin in Riemannian coordinates must be added to the uncertainties $\Delta g(0)$ given by Eq. (2.21);

$$\Delta g\left(x\right) = \Delta g\left(0\right) + \frac{\alpha}{L^2} x^2 \tag{2.23}$$

where g(0) regarded as the average volume of g over a domain V of order $V \sim L_0^3$. Here we assume that background is flat and average curvature has been measured which is g(x). The Riemannian background curvature is calculated by L^{-1} , and $\alpha > 0$ is a numerical constant of order 1. Using Eq. (2.21) we obtain that the least value of $\Delta g(x)$ is accessed for $l_0 \sim (l_p L)^{1/2}$.

Thus all limitations deducible in field quantum formalism can be reproduced by a measurement analysis. This supports the fact that the limitations arising are not limits of the used quantization procedure, which one can overcome by another one, but limits of quantum GRT itself.

3. Implications of the limits arising in quantum GRT

As these were shown earlier, the limitation on quantum GRT discussed above imply a cut-off for the high frequency pars of such quantum effects as Bremsstrahlung, gravitational Compton effect, Lamb shift and pair creation [2,21]. In difference to quantum electrodynamics, these effects cannot be used to test quantum GRT (and the corresponding graviton conception) over the whole frequency scale [11,24].

This is especially easy to see in the case of the gravitational Compton Effect given by the formula:

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$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{mc^2} \left(1 - \cos\theta\right)$$
(3.1)

where v is the frequency of the incoming waves, v' of the scattered waves, θ is the scattering angle and *m* denotes the mass of the scattering particle. According to Heisenberg, Eq. (3.1) is valid upto frequencies $\lambda \Lambda \gg r_0^2$ where Λ , the Compton wavelength of the scattering particle, λ is the length of the scattered waves and r_0 is cut-off length. If we assume

$$r_0 = l_0 \equiv \left(l_p L \right)^{1/2}$$
(3.2)

then we obtain $\lambda \gg l_p$ for $\Lambda = L$, while $\lambda \Lambda \gg r_0^2$ is not satisfied for $\Lambda \approx \lambda$. The occurrence of the wavelength given by Eq. (2.18) thus cuts-off high frequency Compton effect.

In the remainder of this section, we will discuss some aspects of gravitational vacuum polarization, namely Euler scattering [25]. It is interesting because pure vacuum effects are especially essential in order to decide the interpretation of quantization of field. Bohr and Rosenfeld [23] addressed field quantization when they were trying to answer the question whether quantum electrodynamics is physically meaningful or whether it is a purely mathematical formalism. A similar position was taken by Planck when he coined a quest regarding physical reality of light quanta.

We should start from the effective Lagrangian derived from quantum GRT to consider gravitational vacuum polarization in details [25]. Because there might not so viable quantum GRT (or quantum theory of gravity). For this reason, many considerations start with the so called semi classical version of quantum GRT to arrive at Leff [13]. This is however physically and mathematically inconsistent approach as shown by Borzeszkowski [2]. Therefore, we prefer here to presuppose the existence of an effective Lagrangian describing in analogy to Euler scattering in quantum electrodynamics. This analogy helps to determine the frequency region for which such a Lagrangian exists and, by this, to find the limits within such effects as gravitational Euler scattering can be considered as test of quantum GRT [8, 10, 12].

These two conditions must be satisfied in order to search a Lagrangian responsible for scattering of light quantas:

1. We must choose such regions where the electric field |E| must be small so that it must not create particles from vacuum. In other words we can say that energy change ΔE of a charge particle *e* shifted towards a distance $\Lambda = \frac{h}{mc}$ must be smaller than mc^2 i.e.,

$$\Delta E \sim \frac{\partial E}{\partial x} \Lambda \sim \frac{e^2}{x^2} \frac{\hbar}{mc} \ll mc^2$$
(3.3)

So that we obtain

$$|E| \ll |E_k| \equiv \frac{m^2 c^3}{e\hbar} \tag{3.4}$$

or

$$x \gg x_k \sim (\lambda d)^{1/2} \sim \alpha^{-1/2} d \tag{3.5}$$

where $\alpha = e^2/hc$ is the Sommerfeld constant and $d = e^2/mc^2$ is the classical radius of the electron. This implies, electric field strength must be smaller than the critical field E_k . That is, Eq. (3.5) gives the distance x when pure vacuum effects happening. Such effects do not contribute in the matter coupling. 2. We must make an assumption that

$$h\omega \ll mc^2$$
 and $c|k| \ll mc^2$ (3.6)

i.e., the change in electromagnetic field must be happened but in a very long and slow process otherwise Lagrangian may contain some extra term other than invariants of the field. Rapidly changing field must require extra term other than the Larmor Lagrangian. Lagrangian of such field must also contain derivatives of the field [6].

In the purview of quantum gravity, the gravitational radius d_g and "gravitational Sommerfeld constant" α_g must be given as

$$\alpha_g = \frac{\left(G^{1/2}m\right)^2}{hc} \tag{3.7}$$

and

$$d_g = \frac{Gm}{c^2} \tag{3.8}$$

So that $x_k = (hc)^{1/2} d/e$ goes over into

$$(x_k)_g = \left(\frac{hG}{c^3}\right)^{1/2} = l_p$$
 (3.9)

Here, we can observe that vacuum polarization effects must occur at $x \gg l_p$ in absolute Planck length regime. Eq. (3.9) contains three fundamental constants namely, h, c and G, those come into quantum gravity picture. Further consideration of quantum gravity may reveal the real range of vacuum polarization.

If we consider $\lambda \sim \varepsilon/L$ then Eq. (2.15) reflects effective energy under a back reaction of the quantum correction εh_{ij} or η_{ij} . Which will be

$$E_g \sim \frac{c^4}{G} \frac{\varepsilon^2}{\lambda^2} L_0^3 \tag{3.10}$$

where L_0 is the length of extended region. Change of energy ΔE_g must be less than mc^2 because of energy described in Eq. (3.10) contributes as like as the energy-momentum tensor of matter. Hence from this point of view, we can say that

$$\frac{c^4}{G}\frac{\varepsilon^2}{\lambda^2}L_0^3\Lambda \ll mc^2 \tag{3.11}$$

i.e.,

$$\frac{1}{d_g} \frac{\varepsilon^2}{\lambda} L_0^3 \Lambda \ll \lambda \tag{3.12}$$

In this regard, h_{ij} is quantized in all components and

$$\varepsilon^2 \sim \frac{\lambda}{L_0^3} l_p^2 \tag{3.13}$$

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holds true then we get

$$\lambda \gg \left(\frac{\Lambda}{d_g}\right)^{\frac{1}{2}} l_p = \Lambda \tag{3.14}$$

From Eq. (3.14) we can observe that wavelength from Eqs. (2.17) and (2.18) would smoothly move in Planck's magnitude regime rather than any drastic change. In the presence of electromagnetic field around a particle, particle–antiparticle pairs reposition themselves such that they partially counteracting the field. These short lived fields therefore will be weaker than would be expected if the vacuum were completely empty. And in this limit the heaviest elementary particles, namely Planckions will be created [9].

$$\Lambda = \frac{\hbar}{mc} \to l_p \text{ for } m \to m_p \tag{3.15}$$

i.e., for $m = m_p$ equation (3.9) will be reproduced.

Thus, if we assume Eq. (3.14), condition (3.11) automatically satisfied and we are awaiting this because Lagrangian in quantum gravity contains quadratic invariants unlike to quantum electrodynamics.

4. Conclusion

To summarize, we can observe that there are regions like weak-field and low frequency regions, where GRT, similar to electrodynamics, can and must be quantized. This quantization overcome mathematical and physical inconsistencies of Einstein's equations describing the interaction of gravity and quantized (or atomistic) non-gravitational matter. In the regions where the non-linearity of Einstein's equations becomes effective, i.e., for strong and high frequency gravitational fields, quantum field effects of gravity are cut off. The effects one can consider in these regions are typical quantum GRT effects because here GRT lies beyond the difference between classical and quantum GRT accordingly, although the conception of gravitons is only a limited one. This gets especially clear in the case of gravitational Euler scattering. The classical non-linearity takes the part played in linear theories by vacuum polarization caused by coupled matter fields. This produces a large cut-off length for effects like Euler scattering.

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Conflict of interest

Author states that there is no conflict of interest.

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