



Research article

Dynamic pricing, advertising and inventory optimization for perishable products under trade credit

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Abstract: This paper develops a continuous-time model for expiration-sensitive products to jointly determine optimal dynamic pricing, advertising investment, and inventory decisions under trade-credit financing. Demand depends on both selling price and advertising-generated goodwill, with goodwill following Nerlove–Arrow dynamics, whereas inventory availability is subject to sales depletion and age-dependent deterioration over time. The trade-credit mechanism introduces regime-switching dynamics based on the relationship between the credit period and the replenishment cycle, leading to a hybrid optimal control problem analyzed using Pontryagin’s Maximum Principle. A post-solution approximation of the goodwill trajectory is proposed that preserves the structure of the original optimality conditions while providing semi-closed-form expressions for the optimal policies. Numerical experiments demonstrate that the approximation closely reproduces the exact optimal trajectories and objective values. The results further show that trade credit enhances retailer profitability by relaxing liquidity constraints, thereby enabling lower inventory levels, stronger advertising investment, smoother pricing policies, and higher goodwill, particularly for products with longer expiration periods. The proposed framework provides a decision-support tool for jointly coordinating operational, marketing, and financial decisions in expiration-sensitive supply chains.

Keywords: trade credit; deteriorating products; dynamic pricing; goodwill; advertising investment; optimal control

Mathematics Subject Classification: 90B05

1. Introduction

The management of deteriorating products remains a significant challenge in modern supply chains due to the combined effects of limited shelf life, inventory losses, and uncertain market demand. Products such as packaged food, pharmaceutical products, cosmetics, and seasonal goods experience a finite selling horizon, requiring retailers to coordinate operational and marketing decisions before products lose their commercial value. Inefficient management of these products may result in substantial economic losses caused by product deterioration, disposal costs, and inappropriate pricing decisions. Therefore, developing effective decision-making frameworks for deteriorating products is essential for improving profitability and reducing waste.

Deteriorating products require decisions that evolve dynamically over time because inventory availability changes as products age. In many practical environments, products that fail to meet quality standards are removed from available inventory before reaching customers. Therefore, deterioration does not necessarily imply that consumers continuously observe a decline in product quality; rather, it represents a reduction in the quantity of saleable inventory. This characteristic is particularly relevant for expiration-sensitive products managed through quality inspection and shelf-life control procedures. Retailers increasingly rely on advertising activities to influence consumer demand. Advertising not only generates immediate sales but also contributes to the accumulation of goodwill, which represents consumer awareness, brand recognition, and market reputation. Since goodwill evolves over time, pricing and advertising decisions should be coordinated dynamically with inventory decisions. Moreover, trade credit has become an important financial mechanism in retailer–supplier relationships by allowing delayed payments and improving liquidity. However, credit conditions may significantly influence operational decisions, including pricing, advertising, and inventory management.

Research on inventory control for deteriorating products has progressively expanded to incorporate additional decision dimensions, such as dynamic pricing, advertising decisions, and trade-credit policies. Studies on dynamic pricing and inventory decisions have extensively investigated how retailers coordinate pricing and replenishment decisions for deteriorating products with limited lifetimes. Various deterioration patterns, including constant, non-instantaneous, and time-dependent deterioration, have been incorporated into these models to capture the operational challenges associated with product deterioration. These studies provide important insights into how retailers adapt their decisions in response to product deterioration.

Advertising decisions and goodwill accumulation have been incorporated into deteriorating-product models to capture the interaction between marketing and operational decisions. Existing studies recognize that advertising affects demand through goodwill formation and provide important foundations for integrating marketing efforts with inventory and pricing decisions. Nevertheless, most advertising-based models assume that deterioration influences consumer purchasing behavior through freshness or quality perceptions. Less attention has been given to products in which deteriorated units are removed before reaching customers and deterioration primarily affects inventory availability rather than consumer valuation.

Trade-credit mechanisms have been widely studied in deteriorating inventory systems to investigate the role of financial flexibility in operational decisions. Although existing research has examined payment policies, financing conditions, and inventory decisions, most models do not incorporate the simultaneous effects of dynamic advertising decisions and goodwill evolution. Consequently, the simultaneous integration of dynamic pricing, advertising-generated goodwill, age-dependent deterioration, and trade-credit financing remains a research direction, leading to the following research questions:

1. How do trade-credit conditions affect the joint coordination of dynamic pricing, advertising, and replenishment decisions when demand evolves through price and accumulated goodwill?
2. How does an age-dependent deterioration process influence inventory decisions when deteriorated products are removed before reaching customers?

In this framework, this paper develops a continuous-time optimal control framework for a retailer managing deteriorating products under a trade-credit payment scheme. The retailer jointly determines dynamic pricing, advertising effort, and replenishment cycle decisions. Demand is assumed to depend on price and advertising-generated goodwill, where advertising creates a dynamic market asset representing consumer awareness and brand recognition. The deterioration process is modeled through an age-dependent mechanism representing the increasing likelihood that products become unsuitable for sale as they approach their maximum allowable shelf life. Unlike freshness-dependent demand models, the proposed framework assumes that deteriorated units are removed before reaching consumers; therefore, deterioration affects the quantity of available inventory rather than directly reducing customer willingness to purchase. The proposed model considers trade-credit regimes according to the relationship between the credit period and the replenishment cycle, together with a benchmark case without trade credit. By adopting a continuous-time optimal control approach, the model captures the dynamic interactions among pricing decisions, advertising investment, goodwill evolution, inventory deterioration, and financial conditions. Pontryagin's Maximum Principle is employed to characterize the optimal policies and derive structural properties of the solution. From a methodological perspective, this paper also develops a computationally efficient approximation of the goodwill dynamics that closely reproduces the exact optimal-control solution and leads to semi-closed-form solutions.

The remainder of this paper is organized as follows. Section 2 reviews the literature on deteriorating-product models based on dynamic pricing, advertising and goodwill, and trade-credit decisions. Section 3 presents the notation and assumptions. Section 4 formulates the optimization problem and develops analytical results. Section 5 provides numerical analysis and managerial insights. Section 6 concludes the paper.

2. Literature review

The literature on deteriorating inventory systems has evolved significantly over the past decades, expanding from basic deterministic models to more complex frameworks incorporating pricing, financial policies, and investment decisions. Comprehensive surveys, such as Janssen et al. [1] and Asghari [2], highlight that recent research increasingly focuses on multi-dimensional decision-making environments involving interactions between operational, marketing, and financial factors. This study relates to three primary streams of literature on deteriorating products: (i) joint dynamic pricing and inventory control, (ii) dynamic advertising and goodwill accumulation, (iii) inventory models under trade credit. While each stream has developed substantial insights independently, their integration remains limited, particularly in a continuous-time optimal control framework.

Numerous studies have examined joint pricing and inventory decisions for deteriorating goods. Early studies on dynamic pricing of perishable products mainly focused on coordinating price decisions with inventory evolution. Feng et al. [3] developed a continuous-time optimal control model in which pricing and inventory decisions were jointly optimized using Pontryagin's Maximum Principle. Wang et al. [4] considered non-instantaneously deteriorating products under an optimal control framework, assuming an initial non-deterioration period followed by a deterioration phase and incorporating inventory-dependent demand. Zhang et al. [5] and Tashakkor et al. [6] incorporated more

realistic deterioration patterns by considering non-instantaneous and time-dependent deterioration, respectively, while jointly optimizing pricing and replenishment decisions. Herbon and Khmelnitsky [7] further investigated dynamic pricing policies by considering alternative demand specifications in which product age influences market demand.

Later studies extended this framework by incorporating additional operational factors, such as production decisions [8], age- and price-dependent demand [9], replenishment decisions under behavioral assumptions [10], multi-channel and multi-store environments [11], production and pricing policies under inventory-level-dependent demand [12], pricing and freshness-keeping effort under price-dependent demand [13], and freshness, stock preservation investment, and greening decisions [14].

Another stream of literature integrates advertising and goodwill dynamics into pricing and inventory decisions. Motivated by the seminal work of Nerlove and Arrow [15] who modeled goodwill as a dynamic stock variable, subsequent studies extend this framework by incorporating features, such as negative goodwill and no smooth dynamics [16], cooperative advertising in supply chains [17] and reference price effects [18]. Feng et al. [3] studied a joint dynamic pricing and advertising problem in which demand decreases with price and increases with goodwill. They developed a dynamic optimization model to maximize total profit under advertising capacity constraints. Subsequent studies extended this framework by incorporating inventory-dependent demand [19] and multi-period formulations that account for perishability effects [20]. More recent works also considered supply chain interactions and strategic behavior, such as two-echelon systems [21], distributor entry timing [22] and integrated pricing–advertising–inventory decisions under regulatory constraints [23].

Trade credit has been widely studied as a mechanism for coordinating supply chain financing and operational decisions. The foundational model of Goyal [24] extended in numerous directions, including time-dependent demand [25], deteriorating items [26], partial payment schemes [27], order-linked trade credit [28], two-level trade credit and preservation decisions [29]. Other studies examine joint pricing and replenishment decisions under financing mode selection [30], various payment structures, including hybrid and buy-now-pay-later schemes [31,32], multi-item two-echelon inventory model [33], preservation technology and inflation effects [34].

Table 1 reports a comparison of the most closely related studies, selected according to their relevance to the main features of the present model, namely dynamic pricing, marketing-related decisions, deterioration effects, demand formulation and optimal control methodology. The table highlights the similarities and differences between these studies, and the proposed model.

Table 1. Comparison of studies integrating dynamic pricing with advertising, goodwill or related demand-enhancing decisions.

Study	Decision variables	State variables	Deterioration rate / product life-cycle assumption	Demand structure	Advertising / goodwill / quality effect	Trade credit
Feng et al. [3]	Dynamic price, advertising effort	Inventory	Perishable products with deterioration during the selling horizon	Price, and advertising-dependent demand	Advertising directly increases demand; no separate goodwill state variable	No
Feng [35]	Dynamic price, quality investment, replenishment decisions	Inventory quality state	Physical deterioration combined with quality deterioration	Price, and quality-dependent demand	Quality investment improves product condition and affects demand; no advertising goodwill	No

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Study	Decision variables	State variables	Deterioration rate / product life-cycle assumption	Demand structure	Advertising / goodwill / quality effect	Trade credit
Dye [20]	Dynamic price, advertising effort, replenishment decisions	Inventory goodwill, freshness level	Freshness deterioration; product quality decreases over time	Price, goodwill, inventory, and freshness-dependent demand	Advertising accumulates goodwill, while freshness directly affects customer demand	No
Liu et al. [23]	Dynamic price, advertising effort, replenishment decisions	Inventory and goodwill	Perishable inventory	Price, goodwill, and time-dependent demand	Advertising generates goodwill that affects future demand	No
This study	Dynamic price, advertising, effort, replenishment decisions	Inventory goodwill	Age-dependent deterioration	Price- and goodwill-dependent demand	Advertising evolves through goodwill dynamics, deterioration affects inventory availability rather than customer valuation	Yes

As shown in Table 1, previous studies have investigated either the impact of advertising and goodwill on demand or the impact of deterioration and product quality on inventory decisions. However, existing advertising-based models generally assume that deterioration influences customers through freshness or quality perception. The present study considers expiration-sensitive products in which deteriorated units are removed before reaching customers; therefore, deterioration affects the quantity of saleable inventory rather than demand perception. Furthermore, unlike previous studies, the proposed model integrates advertising-generated goodwill, age-dependent deterioration, and trade-credit financing within a continuous-time optimal control framework.

3. Notation and assumptions

In this section the notation and assumptions are introduced.

3.1. Notation

δ	Goodwill decay rate.
M	Credit period offered by the supplier.
β_0	Potential market demand.
β_1	Price sensitivity coefficient.
β_2	Coefficient representing demand sensitivity to product goodwill.
c	Unit procurement cost.
m	Product's expiration date.
K	Ordering cost per order.
a	Advertising cost coefficient.
h	Holding cost (excluding interest paid) per unit per unit time.
η_1	Interest charged per unit per unit time.
η_2	Interest earned per unit per unit time.
φ	Approximation parameter in goodwill effect, $0 < \varphi < 1$.

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δ	Goodwill decay rate.
$D(t)$	The demand rate at time t .
$I_i(t)$	Inventory level at time t under scenario $i \in \{cd, nd\}$, where $i = cd$: supplier offers trade credit $i = nd$: no trade credit.
$I_i(0)$	Replenishment quantity, $i \in \{cd, nd\}$
J_i	Total profit per unit time of retailer under scenario $i \in \{cd, nd\}$.
<i>Decision variables</i>	
T	The replenishment cycle time.
$u(t)$	Advertising investment at time t .
$p(t)$	Selling price charged per unit product at time t , $p(t) > c$.

Superscripts $k \in \{1, 2\}$ represent different scenarios under trade credit, where $k = 1: M > T$, $k = 2: M \leq T$.

3.2. Assumptions

(1) Demand is assumed to depend on selling price and advertising-generated goodwill and is expressed as:

$$D(p(t), G(t)) = \beta_0 - \beta_1 p(t) + \beta_2 G(t), 0 \leq t \leq T$$

where β_0 represents the basic market potential, β_1 is the price sensitivity coefficient, and β_2 measures the impact of goodwill on demand. This formulation reflects the conventional economic relationships that demand decreases with price and increases with accumulated goodwill generated by advertising investment. The admissible price range $p(t) \in \left(0, \frac{\beta_0 + \beta_2 G(t)}{\beta_1}\right)$ ensures non-negative demand. Similar linear demand specifications combining price and advertising effects have been widely adopted in dynamic pricing and advertising models [23,35].

(2) The product is assumed to be subject to continuous deterioration during storage. The deterioration rate is age-dependent and defined as:

$$f(t) = \frac{t}{m}, 0 \leq t \leq T \leq m$$

where m represents the maximum allowable shelf-life of the product [36–38]. This formulation captures the increasing likelihood that products become unsuitable for sale as storage time increases. In particular, deterioration is interpreted as a loss of saleable inventory, rather than a direct reduction in consumers' willingness to purchase. This assumption is appropriate for expiration-sensitive products subject to quality inspection and withdrawal procedures, such as food and other regulated products with shelf-life limits. Accordingly, products that fail to satisfy the required quality standards are removed from inventory before reaching customers. Therefore, demand depends on price and goodwill rather than freshness, while deterioration affects inventory availability through the inventory balance equation.

(3) Advertising expenditure is assumed to be a convex quadratic function of advertising effort:

$$C(u(t)) = \frac{a}{2} u^2(t), a > 0.$$

This assumption reflects increasing marginal costs associated with higher advertising intensity and ensures convexity of the control cost. Quadratic advertising costs are widely used in optimal control models involving dynamic goodwill accumulation [3,17,23].

(4) The supplier provides a fixed credit period M . The financial structure depends on the relationship between the credit period and the replenishment cycle T :

- If $T \geq M$, the retailer pays the supplier at time M and incurs interest charges on the remaining inventory investment during the period $(M, T]$ at rate η_1 .
- If $T < M$, the retailer does not make payment during the selling cycle and earns interest on sales revenue accumulated during $[0, T]$ at rate η_2 .

This trade-credit structure represents common supplier–retailer financing arrangements in inventory systems and follows the classical trade-credit framework [25].

(5) Shortages are not permitted and replenishment is assumed to be instantaneous with zero lead time. The inventory cycle ends when all saleable inventory has either been sold or removed due to deterioration so $I(T) = 0$. This assumption is appropriate for expiration-sensitive products with a finite selling horizon, where inventory cannot be carried beyond the maximum allowable shelf-life. It allows the model to focus on the interaction among pricing, advertising, deterioration and financial decisions without additional complexity arising from shortages or delayed replenishment.

4. Model formulation and optimization

In this section, we develop optimal control models to characterize the retailer's joint dynamic pricing, advertising investment, and inventory decisions for perishable products. The objective is to maximize the retailer's total profit per unit time over a replenishment cycle. The model incorporates two key state variables: inventory level and goodwill. Inventory evolves according to demand and product deterioration, while goodwill captures the cumulative effect of advertising and influences market demand. The goodwill dynamics follow the classical Nerlove–Arrow framework and its subsequent applications, including those proposed by Feng et al. [3], Dye [20], and Liu et al. [23]. To evaluate the effect of trade-credit financing on the retailer's optimal decisions, we formulate two scenarios: one with a credit payment scheme and one without trade credit. Under the credit-payment scenario, the relative length of the credit period and the replenishment cycle determines the applicable financing regime. Therefore, two cases are considered depending on whether the credit period exceeds the replenishment cycle.

4.1. Dynamic system

Over the interval $[0, T]$, the inventory level, $I(t)$, decreases due to both market demand and product deterioration. The inventory dynamics are governed by:

$$\frac{dI(t)}{dt} = -f(t)I(t) - D(p(t), G(t)), I(T) = 0. \quad (1)$$

The goodwill dynamics follow the Nerlove–Arrow [15], model:

$$\frac{dG(t)}{dt} = u(t) - \delta G(t), G(0) = G_0.$$

The exact solution is:

$$G(t) = e^{-\delta t} \left(G_0 + \int_0^t e^{\delta x} u(x) dx \right), \quad (2)$$

which shows that goodwill depends on the entire past history of advertising effort.

4.2. Dynamic decision-making under trade credit

We now consider the retailer's dynamic decision-making problem under a supplier-provided trade credit arrangement. Under this payment scheme, the retailer is allowed to delay payment for a fixed credit period M . During the credit period, sales revenue may generate interest income, whereas inventory remaining after the expiration of the credit period incurs financing costs. The retailer's optimal pricing, advertising, and replenishment decisions therefore depend critically on the relationship between the credit period M and the replenishment cycle T . Accordingly, we distinguish between the following two cases:

- **Case 1:** $M > T$, where the replenishment cycle is shorter than the credit period;
- **Case 2:** $M \leq T$, where the replenishment cycle extends beyond the credit period.

4.2.1. Case 1: $M > T$

When the credit period exceeds the replenishment cycle, all inventory is depleted before payment becomes due. Consequently, the retailer incurs no financing costs and earns interest income on accumulated sales revenue during the credit period. At the beginning of the cycle, the retailer replenishes inventory to the level $I_{cd}(0)$. Over the interval $[0, T]$, inventory decreases due to market demand and product deterioration according to the dynamic system introduced in Section 5.1. The retailer's profit over a replenishment cycle consists of:

Sales revenue:

$$\int_0^T p(t)D(p(t), G(t)) dt$$

Advertising cost:

$$\int_0^T \frac{a}{2} u^2(t) dt$$

Inventory holding cost:

$$\int_0^T hI_{cd}(t) dt$$

Interest income:

$$\eta_2 \int_0^T (M - t) p(t)D(p(t), G(t)) dt$$

Fixed costs:

$$cI_{cd}(0) + K$$

The total profit per unit time is therefore:

$$\begin{aligned} J_{cd}^1(p(t), u(t), T) \\ = \frac{1}{T} \left\{ \int_0^T \left[p(t)D(p(t), G(t))(1 + \eta_2(M - t)) - \frac{a}{2} u^2(t) - hI_{cd}(t) \right] dt - cI_{cd}(0) - K \right\} \end{aligned} \quad (3)$$

The retailer seeks to solve the following problem (P1):

$$\max_{p(t), u(t), T} J_{cd}^1(p(t), u(t), T)$$

subject to:

$$\begin{aligned}\frac{dI(t)}{dt} &= -f(t)I_{cd}(t) - D(p(t), G(t)), I_{cd}(T) = 0, \\ \frac{dG(t)}{dt} &= u(t) - \delta G(t), G(0) = G_0, \\ p(t) &\in \left(0, \frac{\beta_0 + \beta_2 G(t)}{\beta_1}\right), u(t) \geq 0, 0 \leq T \leq M.\end{aligned}$$

For a given replenishment cycle T , we denote the objective function as $J_{cd}^1(T)$. The optimal dynamic pricing and advertising policies are then obtained by applying Pontryagin's Maximum Principle. By introducing adjoint variables $\lambda_{cd}^{I1}(t)$ and $\lambda_{cd}^{G1}(t)$ associated with the state equations, the Hamiltonian function is formulated as follows:

$$\begin{aligned}H_{cd}^1(p, u, I_{cd}, G, T, \lambda_{cd}^{I1}, \lambda_{cd}^{G1}, t) &= p(t)(\beta_0 - \beta_1 p(t) + \beta_2 G(t))[1 + \eta_2(M - t)] - \frac{au^2(t)}{2} \\ &\quad - hI_{cd}(t) + \lambda_{cd}^{I1}(t)[-f(t)I_{cd}(t) - (\beta_0 - \beta_1 p(t) + \beta_2 G(t))] + \lambda_{cd}^{G1}(t)[u(t) - G(t)]\end{aligned}\quad (4)$$

The optimal dynamic decisions for the case $M > T$, given T , are provided in the following proposition.

Proposition 1.

For a fixed replenishment cycle T satisfying $T < M$, the optimal dynamic pricing, advertising, and goodwill policies are characterized by

$$\begin{aligned}p_{cd}^{1*}(t) &= \frac{1}{2} \left[\frac{\beta_0 + \beta_2 G_{cd}^{1*}(t)}{\beta_1} + \frac{\lambda_{cd}^{I1*}(t)}{1 + \eta_2(M - t)} \right], \\ u_{cd}^{1*}(t) &= \max \left\{ 0, \frac{\lambda_{cd}^{G1*}(t)}{a} \right\}, \\ G_{cd}^{1*}(t) &= e^{-\delta t} [G_0 + \varphi t e^{\delta \varphi t} u_{cd}^{1*}(t)].\end{aligned}$$

The adjoint variables are given by

$$\lambda_{cd}^{I1*}(t) = (c + h) \int_0^t e^{-x^2/(2m)} dx e^{t^2/(2m)}$$

and

$$\lambda_{cd}^{G1*}(t) = e^{\delta t} \int_t^T e^{-\delta x} \beta_2 \{ \lambda_{cd}^{I1*}(x) - [1 + \eta_2(M - x)] p_{cd}^{1*}(x) \} dx.$$

Proof. See Appendix A.

When $T < M$, the replenishment cycle ends before the payment due date. Therefore, the retailer can completely deplete inventory before settling the payment with the supplier and consequently avoids financing costs. At the same time, sales revenue accumulated during the permissible delay period generates interest income, thereby increasing the effective profitability of demand. This financial flexibility allows the retailer to adopt more flexible pricing and advertising strategies.

The optimal pricing policy depends on both goodwill and the marginal value of inventory. A higher goodwill level increases demand potential and allows the retailer to maintain a higher selling price. In addition, the inventory adjoint variable, $\lambda_{cd}^{I1*}(t)$, captures the marginal value of inventory over time. Since financing costs are absent in this case, inventory-related pressure remains relatively moderate, allowing the retailer to manage inventory more smoothly throughout the replenishment cycle.

The optimal advertising policy is determined by the goodwill adjoint variable, $\lambda_{cd}^{G1*}(t)$, which measures the future marginal benefit of goodwill accumulation. Advertising investment becomes profitable when the effective revenue generated by demand exceeds the marginal value of inventory, namely when $(1 + \eta_2(M - x))p_1^*(x) > \lambda_{cd}^{G1*}(x)$. Under this condition, additional goodwill accumulation increases future sales profitability more than the associated inventory burden. Consequently, the retailer is encouraged to invest in advertising, which strengthens goodwill and indirectly supports higher selling prices through increased demand.

Therefore, advertising and pricing decisions are jointly connected through the goodwill accumulation mechanism: $\lambda_{cd}^{G1*}(t) \uparrow \Rightarrow u_{cd}^{1*}(t) \uparrow \Rightarrow G_{cd}^{1*}(t) \uparrow \Rightarrow p_{cd}^{1*}(t) \uparrow$. Conversely, when the marginal value of inventory becomes sufficiently large relative to the effective demand benefit, the future value of goodwill accumulation declines. In this situation, advertising investment becomes less attractive, and the retailer prioritizes inventory reduction rather than further demand expansion.

Overall, Proposition 1 shows that when the credit period exceeds the replenishment cycle, trade credit supports goodwill-building and demand-expansion strategies without exposing the retailer to financing pressure. As a result, the retailer can simultaneously maintain relatively stable pricing, stronger advertising investment, and smoother inventory management. The result also highlights the importance of optimizing the replenishment cycle T . Although Proposition 1 characterizes the optimal pricing and advertising policies for a fixed $T < M$, the length of the cycle determines how long the retailer can exploit goodwill-induced demand before the terminal condition is reached. A longer cycle provides more time to benefit from advertising and goodwill accumulation, but it also increases exposure to holding costs and product deterioration. Therefore, even in the absence of financing costs, the replenishment cycle must be optimized to balance demand expansion, inventory deterioration, and ordering costs.

Having characterized the optimal pricing and advertising policies, we next determine the corresponding inventory trajectory, which is necessary for evaluating the optimal replenishment cycle and the retailer's profit per unit time. Since the optimal replenishment cycle depends on the inventory evolution, we first characterize the inventory dynamics. The inventory level evolves over time due to both product deterioration and market demand and satisfies the terminal condition $I(T) = 0$. Accordingly, the inventory trajectory for $T < M$ is governed by:

$$\frac{\partial I_{cd}(t)}{\partial t} = -\frac{t}{m}I_{cd}(t) - (\beta_0 - \beta_1 p_{cd}^1(t) + \beta_2 G_{cd}^1(t)), \quad 0 \leq t \leq T, \quad I(T) = 0 \quad (5)$$

Solving the differential equation (5) yields the inventory level of the perishable product retailer at time t for Case 1 ($M > T$).

$$I_{cd}(t) = e^{-t^2/(2m)} \int_t^T e^{x^2/(2m)} (\beta_0 - \beta_1 p_{cd}^1(x) + \beta_2 G_{cd}^1(x)) dx \quad (6)$$

Further, the initial inventory level can be determined as follows:

$$I_{cd}^1(0) = I_{cd}(0) = \int_0^T e^{\frac{x^2}{2m}} (\beta_0 - \beta_1 p_{cd}^1(x) + \beta_2 G_{cd}^1(x)) dx \quad (7)$$

Substituting the optimal dynamic selling price, advertising investment, and goodwill (see Proposition 1) and inventory level (Eqs. (6) and (7)) into Eq. (3), yields:

$$J_{cd}^1(T) = \frac{1}{T} \left\{ \int_0^T \left(p_{cd}^{1*}(t) D(p_{cd}^{1*}(t), G_{cd}^{1*}(t)) [1 + \eta_2(M - t)] - \frac{a(u_{cd}^{1*}(t))^2}{2} - hI_{cd}^1(t) \right) dt - cI_{cd}^1(0) - K \right\}$$

So, the optimal replenishment cycle time is then determined by solving

$$\max_{0 < T < M} J_{cd}^1(T).$$

Due to the complexity of the function $J_{cd}^1(T)$ and the difficulty of obtaining an analytical solution for T , we use a discrete search procedure over the interval $[0, M]$. The procedure involves evaluating the objective function $J_{cd}^1(T)$ at discrete points T_k with a fixed step size ΔT and selecting the value of T that maximizes $J_{cd}^1(T)$.

4.2.2. Case 2: $M \leq T$

When $M \leq T$, the retailer earns interest income on sales revenue during $[0, M]$, but pays interest on the unsold balance during $[M, T]$. In this case, using a similar reasoning as in the previous case, the retailer's total profit per unit time is

$$\begin{aligned} J_{cd}^2(p(t), u(t), T) &= \frac{1}{T} \left\{ \int_0^T \left(p(t) D(p(t), G(t)) - \frac{au^2(t)}{2} - hI(t) \right) dt - \eta_1 c \int_M^T I_{cd}(t) dt \right. \\ &\quad \left. + \eta_2 \int_0^M (M - t) p(t) D(p(t), G(t)) dt - cI_{cd}(0) - K \right\}, \end{aligned}$$

or equivalently,

$$\begin{aligned} J_{cd}^2(p(t), u(t), T) &= \frac{1}{T} \left\{ \int_0^M \left[(1 + \eta_2(M - t)) p(t) D(p(t), G(t)) - \frac{au^2(t)}{2} - hI(t) \right] dt \right. \\ &\quad \left. + \int_M^T \left[p(t) D(p(t), G(t)) - \frac{au^2(t)}{2} - (h + \eta_1 c) I(t) \right] dt - cI_{cd}(0) - K \right\}. \end{aligned} \quad (8)$$

The retailer seeks to solve the following problem (P2):

$$\max_{p(t), u(t), T} J_{cd}^2(p(t), u(t), T)$$

subject to:

$$\begin{aligned} \frac{dI_{cd}(t)}{dt} &= -f(t)I(t) - D(p(t), G(t)), I_{cd}(T) = 0, \\ \frac{dG(t)}{dt} &= u(t) - \delta G(t), G(0) = G_0, \\ p(t) &\in \left(0, \frac{\beta_0 + \beta_2 G(t)}{\beta_1} \right), u(t) \geq 0, M \leq T \leq m. \end{aligned}$$

Pontryagin's Maximum Principle is applied to solve the optimal control problem (P2). By introducing adjoint variables $\lambda_{cd}^{I2}(t)$ and $\lambda_{cd}^{G2}(t)$ associated with the state equations, the Hamiltonian function is formulated as follows:

$$H_{cd}^2(p, u, I_{cd}, G, T, \lambda_{cd}^{I2}, \lambda_{cd}^{G2}, t) = \begin{cases} H_{cd}^{21}(p, u, I_{cd}, G, T, \lambda_{cd}^{I2}, \lambda_{cd}^{G2}, t), & t \in [0, M] \\ H_{cd}^{22}(p, u, I_{cd}, G, T, \lambda_{cd}^{I2}, \lambda_{cd}^{G2}, t), & t \in [M, T] \end{cases}$$

where

$$\begin{aligned} H_{cd}^{21}(p, u, I_{cd}, G, T, \lambda_{cd}^{I2}, \lambda_{cd}^{G2}, t) &= [1 + \eta_2(M - t)]p(t)D(p(t), G(t)) - \frac{au^2(t)}{2} - hI_{cd}(t) + \\ &\lambda_{cd}^{G2}(t)[u(t) - \delta G(t)] + \lambda_{cd}^{I2}(t)[-f(t)I_{cd}(t) - D(p(t), G(t))], \\ H_{cd}^{22}(p, u, I_{cd}, G, T, \lambda_{cd}^{I2}, \lambda_{cd}^{G2}, t) &= p(t)D(p(t), G(t)) - \frac{au^2(t)}{2} - (h + \eta_1 c)I_{cd}(t) + \lambda_{cd}^{G2}(t)[u(t) - \\ &\delta G(t)] + \lambda_{cd}^{I2}(t)[-f(t)I_{cd}(t) - D(p(t), G(t))], \end{aligned}$$

For a given replenishment cycle T , the retailer solves the optimal control problem (P2). The resulting optimal dynamic decisions for $M \leq T$ are characterized in Proposition 2.

Proposition 2.

For given T , with $M \leq T \leq m$, the optimal advertising investment, goodwill, and selling price for the perishable product retailer are given by

$$\begin{aligned} u_{cd}^{2*}(t) &= \begin{cases} u_{cd}^{21*}(t) = \max \left\{ 0, \frac{\lambda_{cd}^{G21*}(t)}{a} \right\}, & 0 \leq t \leq M, \\ u_{cd}^{22*}(t) = \max \left\{ 0, \frac{\lambda_{cd}^{G22*}(t)}{a} \right\}, & M \leq t \leq T, \end{cases} \\ G_{cd}^{2*}(t) &= \begin{cases} G_{cd}^{21*}(t) = e^{-\delta t} [G_0 + \varphi t e^{\delta \varphi t} u_{cd}^{21*}(t)], & 0 \leq t \leq M, \\ G_{cd}^{22*}(t) = e^{-\delta t} [G_0 + \varphi t e^{\delta \varphi t} u_{cd}^{22*}(t)], & M \leq t \leq T, \end{cases} \end{aligned}$$

and

$$p_{cd}^{2*}(t) = \begin{cases} p_{cd}^{21*}(t) = \frac{1}{2} \left[\frac{\beta_0 + \beta_2 G_{cd}^{21*}(t)}{\beta_1} + \frac{\lambda_{cd}^{I2*}(t)}{1 + \eta_2(M - t)} \right], & 0 \leq t \leq M, \\ p_{cd}^{22*}(t) = \frac{\beta_0 + \beta_2 G_{cd}^{22*}(t) + \beta_1 \lambda_{cd}^{I2*}(t)}{2\beta_1}, & M \leq t \leq T. \end{cases}$$

The inventory adjoint variable is

$$\lambda_{cd}^{I2*}(t) = \begin{cases} (c + h \int_0^t e^{-x^2/(2m)} dx) e^{\frac{t^2}{2m}}, & 0 \leq t \leq M, \\ e^{(t^2 - M^2)/(2m)} (h + \eta_1 c) \int_M^t e^{-(x^2 - M^2)/(2m)} dx, & M \leq t \leq T. \end{cases}$$

The goodwill adjoint variables are characterized by

$$\lambda_{cd}^{G2*}(t) = \begin{cases} \lambda_{cd}^{G21*}(t), & 0 \leq t \leq M, \\ \lambda_{cd}^{G22*}(t), & M \leq t \leq T, \end{cases}$$

where

$$\lambda_{cd}^{G21*}(t) = e^{\delta(M-t)} \int_t^M e^{-\delta(M-x)} \beta_2 [\lambda_{cd}^{I2*}(x) - (1 + \eta_2(M - x))p_{cd}^{21*}(x)] dx + e^{\delta(M-t)} \lambda_{cd}^{G22*}(M),$$

and

$$\lambda_{cd}^{G22*}(t) = e^{\delta(t-T)} \int_t^T e^{-\delta(x-T)} \beta_2 [\lambda_{cd}^{I2*}(x) - p_{cd}^{22*}(x)] dx.$$

Proof. See Appendix B.

In this case, the retailer faces a change in financial conditions at time $t = M$. Before the expiration of the credit period, the retailer benefits from interest income generated during the permissible delay period, which increases the effective marginal value of demand and supports relatively expansionary pricing and advertising strategies. During this phase, the retailer is encouraged to stimulate demand and accumulate goodwill, since both inventory and goodwill possess relatively high marginal value.

It is worth noting that the optimal policies over the interval $[0, M]$ coincide structurally with those obtained in Proposition 1. During this phase, the retailer operates entirely under the trade-credit benefit regime and earns interest income from delayed payment. Consequently, the pricing, advertising, goodwill, and adjoint structures before $t = M$ are analogous to the case $T < M$. The key structural difference arises after the expiration of the credit period, when financing costs become active and modify the optimal inventory and pricing dynamics.

The optimal pricing policy is influenced by both goodwill and the marginal value of inventory. A higher goodwill level increases demand potential and allows the retailer to sustain a higher selling price. Hence, advertising affects pricing indirectly through goodwill accumulation. In addition, the inventory adjoint variable $\lambda_{cd}^{I2*}(t)$ captures the marginal value of inventory. Before $t = M$, its effect on price is moderated by the trade-credit term $1 + \eta_2(M - t)$, reflecting the financial benefit generated by interest earnings during the permissible delay period. Consequently, the retailer is willing to maintain relatively higher prices while simultaneously stimulating demand through advertising investment.

The optimal advertising policy is directly related to the goodwill adjoint variables $\lambda_{cd}^{G21*}(t)$ and $\lambda_{cd}^{G22*}(t)$, which measure the future marginal value of goodwill accumulation. A higher value of these adjoint variables leads to stronger advertising investment, thereby increasing goodwill and supporting future demand. Therefore, advertising and pricing decisions are jointly connected through the goodwill accumulation process.

After $t = M$, however, unsold inventory incurs financing costs, increasing the effective holding cost and consequently the marginal value of inventory represented by $\lambda_{cd}^{I2*}(t)$. This creates stronger incentives for inventory liquidation and modifies the retailer's pricing behavior. As financing pressure increases, the retailer becomes more concerned with reducing inventory levels and accelerating sales. At the same time, the remaining planning horizon becomes shorter, reducing the future benefit generated by goodwill accumulation. Consequently, the goodwill adjoint variable declines, leading to lower advertising intensity during the later stage of the cycle.

In the post-credit regime, the goodwill adjoint variable $\lambda_{cd}^{G22*}(t)$ remains nonnegative when $\lambda_{cd}^{I2*}(x) - p_{cd}^{22*}(x) > 0$, which indicates that the marginal value of inventory dominates the marginal revenue effect generated by demand expansion. In this situation, additional goodwill accumulation still contributes positively to inventory reduction and sales acceleration. Before $t = M$, the stronger condition $\lambda_{cd}^{I2*}(x) - (1 + \eta_2(M - x))p_{cd}^{21*}(x) > 0$ is sufficient, though not necessary, for $\lambda_{cd}^{G21*}(t)$ to remain nonnegative. This condition naturally holds in the case $T < M$, where the retailer operates entirely under the trade-credit benefit regime. Therefore, the optimal pricing and advertising policies exhibit a clear piecewise structure driven by the transition from interest earnings before $t = M$ to financing costs after the expiration of the trade credit period.

Based on Proposition 2, the inventory dynamics are also piecewise:

$$\frac{\partial I_{cd}(t)}{\partial t} = \begin{cases} -\frac{t}{m}I_{cd}(t) - (\beta_0 - \beta_1 p_{cd}^{21}(t) + \beta_2 G_{cd}^{21}(t)), & t \in [0, M], \\ -\frac{t}{m}I_{cd}(t) - (\beta_0 - \beta_1 p_{cd}^{22}(t) + \beta_2 G_{cd}^{22}(t)), & t \in [M, T], \end{cases} \quad \text{with } I_{cd}(T) = 0. \quad (9)$$

From Eq. (9), the expression for the inventory level is obtained as:

$$I_{cd}(t) = \begin{cases} e^{\frac{-t^2}{2m}} \left\{ I_{cd}^{22}(M) e^{\frac{M^2}{2m}} + \int_t^M e^{\frac{x^2}{2m}} (\beta_0 - \beta_1 p_{cd}^{21}(t) + \beta_2 G_{cd}^{21}(t)) dx \right\}, & t \in [0, M], \\ e^{\frac{-t^2}{2m}} \int_t^T e^{\frac{x^2}{2m}} (\beta_0 - \beta_1 p_{cd}^{22}(t) + \beta_2 G_{cd}^{22}(t)) dx, & t \in [M, m] \end{cases}$$

Substituting the optimal controls into $J_{cd}^2(p(t), u(t), T)$, the profit function becomes

$$J_{cd}^2(T) = \frac{1}{T} \left\{ \int_0^M \left[(1 + \eta_2(M - t)) p_{cd}^{21}(t) D(p_{cd}^{21}(t), G_{cd}^{21}(t)) - \frac{a(u_{cd}^{21}(t))^2}{2} - h I_{cd}^{21}(t) \right] dt \right. \\ \left. + \int_M^T \left[p_{cd}^{22}(t) D(p_{cd}^{22}(t), G_{cd}^{22}(t)) - \frac{a(u_{cd}^{22}(t))^2}{2} - (h + \eta_1 c) I_{cd}^{22}(t) \right] dt - c I_{cd}^{22}(0) - K \right\}.$$

So, the optimal replenishment cycle time is then determined by solving

$$\max_{M < T < m} J_{cd}^2(T).$$

Due to the complexity of the function $J_{cd}^2(T)$ and the difficulty of obtaining an analytical solution for T , we follow the same discrete search procedure over the interval $[M, m]$ and select the value of T , T_{cd}^{2*} , that maximizes $J_{cd}^{2*}(T)$.

The overall optimum under trade credit scenario is

$$J_{cd}^*(p_{cd}^*(t), u_{cd}^*(t), T_{cd}^*) = \max \{ J_{cd}^{1*}(p_{cd}^{1*}(t), u_{cd}^{1*}(t), T_{cd}^{1*}), J_{cd}^{2*}(p_{cd}^{2*}(t), u_{cd}^{2*}(t), T_{cd}^{2*}) \}$$

4.3. Benchmark model without trade credit

In the absence of trade credit, the retailer neither earns interest on sales revenue nor incurs interest charges on inventory. The demand function and goodwill dynamics remain the same as previously defined. The retailer seeks to determine the optimal dynamic pricing $p(t)$, advertising investment $u(t)$, and replenishment cycle time T that maximize total profit per unit time:

$$J_{nd}(p(t), u(t), T) = \frac{1}{T} \left\{ \int_0^T (p(t) D(p(t), G(t)) - \frac{a}{2} u(t)^2 - h I_{nd}(t)) dt - c I_{nd}(0) - K \right\}$$

The resulting problem, (P3), can then be formulated as follows:

$$\max_{p(t), u(t), T} J_{nd}(p(t), u(t), T)$$

subject to:

$$\begin{aligned} \frac{dI_{nd}(t)}{dt} &= -f(t)I_{nd}(t) - D(p(t), G(t)), I_{nd}(T) = 0, \\ \frac{dG(t)}{dt} &= u(t) - \delta G(t), G(0) = G_0, \\ p(t) &\in \left(0, \frac{\beta_0 + \beta_2 G(t)}{\beta_1} \right), u(t) \geq 0, 0 \leq T \leq M. \end{aligned}$$

Applying Pontryagin's Maximum Principle to solve the optimal control problem (P3), the Hamiltonian function is formulated as follows:

$$H_{nd}(p, u, I_{nd}, G, T, \lambda_{nd}^I, \lambda_{nd}^G, t) = p(t)D(p(t), G(t)) - \frac{au^2(t)}{2} - hI_{nd}(t) + \lambda_{nd}^G(t)[u(t) - \delta G(t)] \\ + \lambda_{nd}^I(t)[-f(t)I_{nd}(t) - D(p(t), G(t))]$$

The optimal dynamic decisions for given T are presented in the next proposition.

Proposition 3.

For benchmark model, the retailer's optimal dynamic advertising investment, goodwill and selling price are given respectively by:

$$u_{nd}^*(t) = \frac{\lambda_{nd}^{G*}(t)}{a} \\ G_{nd}^*(t) = e^{-\delta t}[G_0 + \varphi t e^{\delta \varphi t} u_{nd}(t)] \\ p_{nd}^*(t) = \frac{\beta_0 + \beta_2 G_{nd}^*(t) + \beta_1 \lambda_{nd}^{I*}(t)}{2\beta_1}$$

where

$$\lambda_{nd}^{I*}(t) = (c + h \int_0^t e^{-x^2/(2m)} dx) e^{\frac{t^2}{2m}}. \\ \lambda_{nd}^{G*}(t) = e^{\delta t} \int_t^T e^{-\delta x} \beta_2 [\lambda_{nd}^{I*}(x) - p_{nd}^*(x)] dx.$$

Proof. See Appendix C.

In the benchmark model without trade credit, the retailer's decisions are driven solely by operational factors such as inventory deterioration, holding costs, pricing, and goodwill accumulation. Since no permissible delay in payment is provided, the retailer neither benefits from interest earnings nor incurs financing costs associated with delayed settlement. Consequently, the system evolves under a single operational regime throughout the planning horizon. The optimal pricing policy depends on both goodwill and the marginal value of inventory. A higher goodwill level increases demand potential and allows the retailer to sustain a higher selling price. At the same time, the inventory adjoint variable $\lambda_{nd}^{I*}(t)$ captures the marginal value of inventory over time. As inventory-related pressure increases, the retailer adjusts pricing decisions to balance profitability and inventory depletion.

The optimal advertising policy is determined by the goodwill adjoint variable, $\lambda_{nd}^{G*}(t)$, which measures the future marginal value of goodwill accumulation. Higher values of $\lambda_{nd}^{G*}(t)$ lead to stronger advertising investment, thereby increasing goodwill and stimulating future demand. Consequently, advertising and pricing decisions remain interconnected through the goodwill accumulation process. In particular, the goodwill adjoint variable remains nonnegative when $\lambda_{nd}^{I*}(x) - p_{nd}^*(x) > 0$, indicating that the marginal value of inventory exceeds the unit revenue obtained from current sales. Under this condition, goodwill accumulation still contributes positively to inventory reduction and demand stimulation, supporting continued advertising investment.

The marginal value of goodwill gradually decreases as the planning horizon approaches its terminal point. This follows directly from the integral representation of the goodwill adjoint variable, together with the transversality condition $\lambda_{nd}^{G*}(T) = 0$. As t approaches T , the remaining integration horizon becomes shorter, reducing the future benefit generated by goodwill accumulation. Consequently, the optimal advertising effort declines over time. Simultaneously, inventory-related

considerations become relatively more important, encouraging the retailer to focus increasingly on inventory depletion during the later stages of the cycle. Overall, Proposition 3 characterizes the benchmark operational behavior of the retailer in the absence of trade credit. Unlike the trade-credit cases, the retailer's decisions are determined exclusively by operational trade-offs rather than financial incentives, thereby isolating the effects of goodwill accumulation, deterioration, and inventory management on the optimal pricing and advertising policies.

Turning now to replenishment time, the differential equation governing the inventory level over time in the scenario without credit payment scheme is given by:

$$\frac{\partial I_{nd}(t)}{\partial t} = -\frac{t}{m}I_{nd}(t) - (\beta_0 - \beta_1 p_{nd}^*(t) + \beta_2 G_{nd}^*(t)), 0 \leq t \leq T \text{ with } I_{nd}(T) = 0 \quad (10)$$

Consequently, the inventory level is given by:

$$I_{nd}(t) = e^{-\frac{t^2}{2m}} \int_t^T e^{\frac{x^2}{2m}} (\beta_0 - \beta_1 p_{nd}^*(x) + \beta_2 G_{nd}^*(x)) dx \quad (11)$$

Furthermore, substituting the optimal dynamic strategies in Proposition 3 into $J_{nd}(p(t), u(t), T)$ yields:

$$J_{nd}(T) = \frac{1}{T} \left\{ \int_0^T (p_{nd}^*(t) D(p_{nd}^*(t), G_{nd}^*(t)) - \frac{a(u_{nd}^*(t))^2}{2} - hI_{nd}^*(t)) dt - cI_{nd}(0) - K \right\} \quad (12)$$

So, the optimal replenishment cycle time is then determined by solving

$$\max_{0 < T < M} J_{nd}(T).$$

Due to the complexity of the function $J_{nd}(T)$ and the difficulty of obtaining an analytical solution for T , we follow the same discrete search procedure over the interval $[0, m]$ and select the value of T , T_{nd} , that maximizes $J_{nd}(T)$.

In the next section, we provide several numerical examples to illustrate the above results, explore the impact of perishable product's expiration date on the optimal inventory level, dynamic selling price, advertising investment and goodwill.

5. Numerical analysis

To illustrate the theoretical results and evaluate the effectiveness of the proposed solution approach, this section presents a series of numerical experiments based on a retailer managing expiration-sensitive products. The experiments examine the effects of product lifetime, trade-credit financing, and key model parameters on the retailer's optimal pricing, advertising, inventory, and replenishment decisions. Particular emphasis is placed on assessing the accuracy of the proposed approximation by comparing it with the exact solution of the original optimal control problem.

Consider a medium-sized supermarket selling premium apples. Inventory is measured in kilograms, time in weeks, and monetary values in euros. At the beginning of each replenishment cycle, the retailer receives a heterogeneous shipment of apples. Although all products arrive simultaneously, individual items differ in maturity, handling conditions, and remaining shelf life. Products that no longer satisfy the retailer's quality standards are removed before reaching consumers. Consequently, the inventory state represents the quantity of saleable products, which decreases because of both customer demand and product expiration. Since expired products are withdrawn before sale, customer demand depends on the selling price and the retailer's accumulated goodwill rather than on product freshness. The supplier offers a fixed trade-credit period during which payment for the replenishment order may be postponed. The retailer therefore jointly determines the selling price, advertising investment, replenishment quantity, and replenishment cycle to maximize the average profit per unit

time. Unless otherwise stated, the parameters are set as follows (consistent with Feng [35]; Chang et al. [38]); baseline potential demand: $\beta_0 = 80\text{kg/week}$, demand reduction per €/kg increase in price: $\beta_1 = 5\text{kg/week}$, demand increase per goodwill unit: $\beta_2 = 1\text{ kg/week}$, goodwill decay rate: $\delta = 0.25$, $h = 0.1\text{ €/kg/week}$, advertising-cost coefficient: $a = 10$, initial goodwill index: $G_0 = 5$, nominal shelf-life horizon: $m = 2.5\text{ weeks}$, $c = 3\text{ €/kg}$, inventory financing rate after M : $\eta_1 = 0.1$, interest-earning rate before payment: $\eta_2 = 0.05$, supplier's trade-credit period: $M = 2\text{ weeks}$, $K = 45\text{ €/order}$, midpoint approximation parameter: $\varphi = 0.5$. For clarity, superscripts e and $*$ denote the exact and approximate solutions, respectively. The exact solution is obtained by numerically solving the complete Pontryagin Maximum Principle system, whereas the approximate solution is computed using the proposed approximation of the goodwill dynamics (see Appendices). The comparison between these two solutions serves to evaluate the accuracy and computational efficiency of the proposed approximation.

5.1. Dynamic decision-making under trade credit

This subsection investigates the retailer's optimal policies under the trade-credit financing scheme. The numerical results first validate the proposed approximation by comparing the exact and approximate objective functions. Subsequently, the dynamic effects of the expiration date on inventory, pricing, advertising, and goodwill are examined through the corresponding optimal trajectories.

Figure 1 compares the exact objective function, $J_{cd}^e(T)$, with its approximate counterpart, $J_{cd}^*(T)$, under the trade-credit policy. Both objective functions exhibit the same unimodal behaviour, indicating that the proposed approximation preserves the overall structure of the optimization problem. The exact solution attains its maximum at $T_{cd}^e = 1.18$, yielding a profit of $J_{cd}^e = 214.99$, whereas the approximate solution reaches its maximum at $T_{cd}^* = 1.16$ with $J_{cd}^* = 213.57$. Thus, the approximation produces only a slight underestimation of both the optimal replenishment cycle and the corresponding profit.

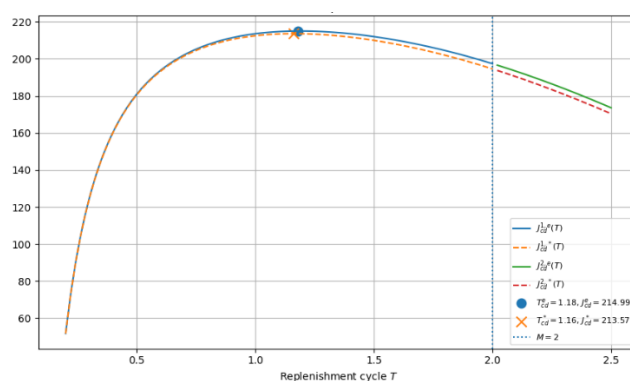


Figure 1. $J_{cd}(T)$ with respect to T .

More importantly, the two objective functions remain nearly indistinguishable throughout the entire range of replenishment cycles. This close agreement indicates that the proposed approximation accurately reproduces the economic behavior of the original optimal control problem while substantially reducing the computational effort required to determine the optimal replenishment policy.

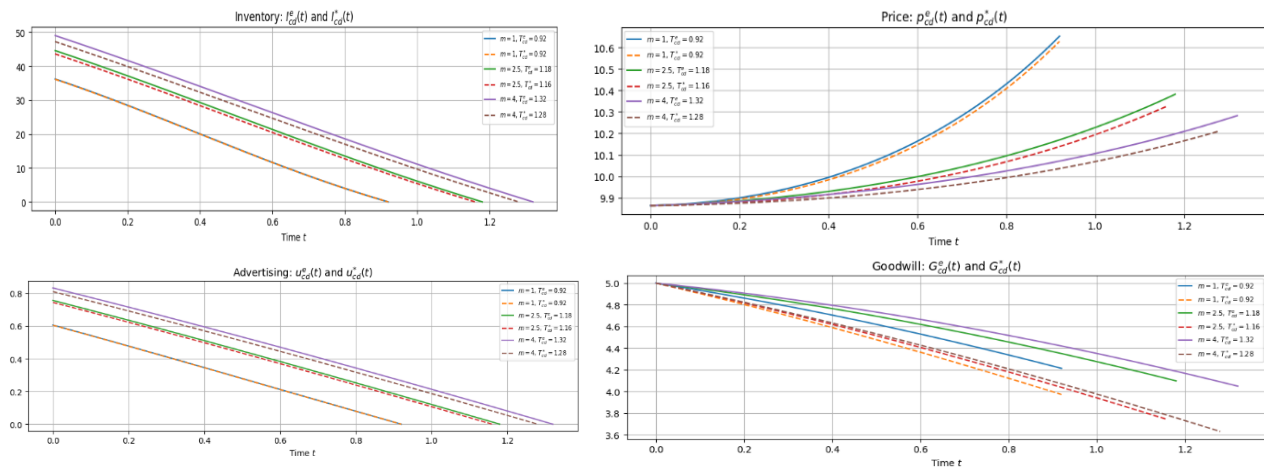


Figure 2. Optimal trajectories of the inventory level, $I_{cd}(t)$, selling price, $p_{cd}(t)$, advertising investment, $u_{cd}(t)$, and goodwill level, $G_{cd}(t)$ under the trade-credit policy for different expiration dates ($m = 1, 2.5, 4$).

To investigate the influence of product lifetime on the retailer's dynamic decisions, Figure 2 compares the exact and approximate trajectories of the inventory level, $I_{cd}(t)$, selling price, $p_{cd}(t)$, advertising investment, $u_{cd}(t)$, and goodwill level, $G_{cd}(t)$, under the trade-credit policy for different expiration dates. The figure demonstrates not only how the expiration date affects the retailer's optimal operational and marketing decisions but also how accurately the proposed approximation reproduces the exact optimal trajectories.

The inventory level decreases monotonically throughout the replenishment cycle as inventory is depleted by customer demand and product expiration. Products with longer expiration dates experience lower deterioration rates and therefore remain saleable for a longer period. Consequently, inventory is depleted more gradually, allowing the retailer to maintain larger inventory levels and adopt longer replenishment cycles. Economically, the slower inventory depletion reflects a more gradual evolution of the inventory shadow value, which reduces the urgency of inventory liquidation. Across all expiration dates, the approximate inventory trajectories are almost indistinguishable from the exact solutions, demonstrating that the proposed approximation accurately captures the inventory dynamics.

The optimal selling price increases continuously over the replenishment cycle. Since deteriorated products are removed before reaching customers, this increase is not associated with declining product quality but rather with the increasing scarcity of saleable inventory. At the beginning of the cycle, inventory availability is high, encouraging relatively low prices that stimulate demand while the retailer benefits from delayed payment under the trade-credit arrangement. As inventory becomes progressively scarcer, the marginal value of the remaining inventory increases, leading to higher prices. Products with shorter expiration dates become scarce more rapidly, producing steeper price increases, whereas longer product lifetimes allow smoother pricing adjustments. The approximate pricing trajectories closely coincide with the exact solutions, indicating that the approximation accurately preserves the retailer's optimal pricing policy.

Advertising investment is highest at the beginning of the replenishment cycle and gradually decreases over time. Early advertising is particularly valuable because the resulting goodwill influences demand over a longer remaining selling period. The availability of trade credit further enhances this effect by postponing payment obligations and improving the retailer's financial flexibility, thereby supporting stronger investment in demand-generation activities. As the replenishment cycle progresses, the remaining period over which advertising can generate future sales

becomes shorter, reducing its marginal benefit and leading to lower advertising expenditures. Longer expiration dates further increase the return on advertising by extending the period during which accumulated goodwill can stimulate demand. Consequently, higher values of m lead to greater advertising investment throughout the replenishment cycle. Once again, the approximate trajectories remain extremely close to the exact solutions, confirming the ability of the proposed approximation to reproduce the optimal advertising policy.

Goodwill evolves as a dynamic state variable determined jointly by advertising investment and natural depreciation. As shown in Figure 2, goodwill gradually decreases throughout the replenishment cycle because the depreciation effect dominates the positive contribution of advertising. Although advertising effort is relatively high at the beginning of the cycle, it is insufficient to completely offset goodwill decay. Products with longer expiration dates maintain higher goodwill levels because the extended selling horizon increases the economic return on advertising investment and encourages stronger marketing efforts. In contrast, shorter product lifetimes reduce the effectiveness of advertising, leading to lower advertising expenditure and a faster decline in goodwill. The approximate goodwill trajectories remain nearly indistinguishable from the exact solutions for all expiration dates, confirming that the proposed approximation accurately reproduces the evolution of goodwill despite replacing the original state equation with a simplified analytical representation.

Overall, Figure 2 demonstrates that extending the expiration date improves the retailer's operational and marketing decisions under the trade-credit policy. Longer product lifetimes support larger inventory levels, smoother pricing adjustments, stronger advertising investment, and better preservation of goodwill, all of which contribute to higher profitability. Equally important, the agreement between the exact and approximate trajectories confirms that the proposed approximation preserves both the qualitative behavior and the quantitative characteristics of the optimal policies.

5.2. Benchmark model without credit payment

To isolate the effect of supplier financing on the retailer's decisions, this subsection considers the benchmark model without trade credit. As in the previous subsection, the proposed approximation is first validated through the objective function and is then evaluated by comparing the exact and approximate trajectories of the state and control variables under different expiration dates.

Figure 3 compares the exact objective function, $J_{nd}^e(T)$, with the corresponding approximate objective function, $J_{nd}^*(T)$, for the benchmark model without trade credit. Similar to the trade-credit case, both objective functions exhibit a unimodal shape, indicating that the proposed approximation preserves the fundamental structure of the underlying optimization problem. The exact solution achieves its maximum at $T_{nd}^e = 1.32$, yielding an optimal profit of $J_{nd}^e = 191.30$, whereas the approximate solution attains its maximum at $T_{nd}^* = 1.30$ with $J_{nd}^* = 189.85$. The approximation therefore produces only a slight underestimation of both the optimal replenishment cycle and the corresponding profit.

A comparison with Figure 1 highlights the economic value of trade credit. In the absence of supplier financing, the retailer adopts a longer replenishment cycle while achieving a lower profit. Without delayed payment, inventory investment must be financed entirely at the beginning of the cycle, reducing financial flexibility and limiting the retailer's ability to simultaneously invest in inventory and demand-generation activities. Nevertheless, the close agreement between the exact and approximate objective functions demonstrates that the proposed approximation remains highly accurate even when financing incentives are absent.

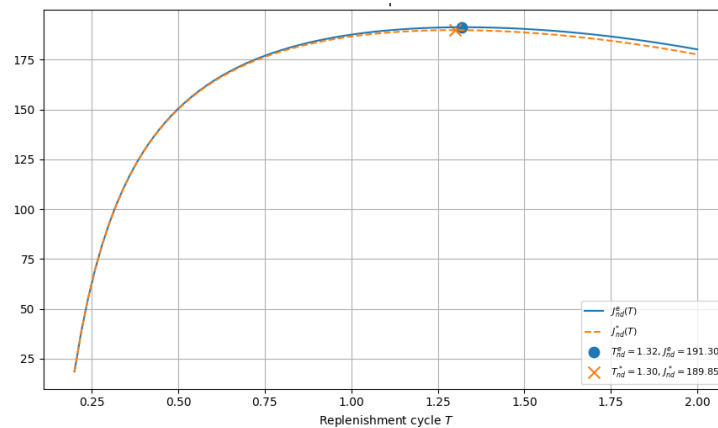


Figure 3. $J_{nd}(T)$ with respect to T .

To further examine the impact of product lifetime in the absence of supplier financing, Figure 4 compares the exact and approximate trajectories of the inventory level, $I_{nd}(t)$, selling price, $p_{nd}(t)$, advertising investment, $u_{nd}(t)$, and goodwill level, $G_{nd}(t)$, for different expiration dates. Besides illustrating the influence of the expiration date on the retailer's optimal decisions, the figure provides an additional validation of the proposed approximation under the benchmark financing environment.

The inventory level decreases monotonically throughout the replenishment cycle because of customer demand and product expiration. As the expiration date increases, deterioration occurs more slowly, allowing products to remain saleable for a longer period and enabling the retailer to maintain higher inventory levels. However, compared with the trade-credit model, the expansion of inventory is noticeably more limited. Since inventory purchases must be fully financed at the beginning of the cycle, tighter liquidity constraints reduce the retailer's willingness to carry large inventories despite the operational advantages associated with longer product lifetimes. The approximate inventory trajectories closely coincide with the exact solutions for all expiration dates, confirming that the proposed approximation accurately reproduces the inventory dynamics.

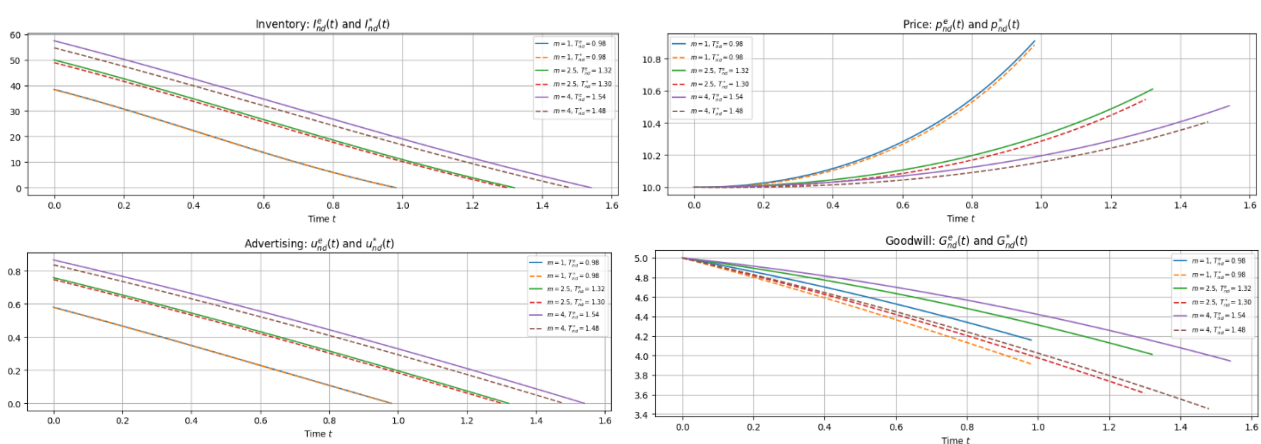


Figure 4. Optimal trajectories of the inventory level, $I_{nd}(t)$, selling price, $p_{nd}(t)$, advertising investment, $u_{nd}(t)$, and goodwill level, $G_{cd}(t)$, under the trade-credit policy for different expiration dates ($m = 1, 2.5, 4$).

The optimal selling price increases steadily throughout the replenishment cycle as the availability of saleable inventory gradually declines. Unlike the trade-credit model, however, the retailer cannot benefit from delayed payment and therefore faces greater pressure to recover inventory-related costs through product sales. Consequently, products with shorter expiration dates exhibit steeper price increases because inventory becomes scarce more rapidly. Longer expiration dates postpone inventory scarcity and allow smoother pricing adjustments, although the retailer still adopts more conservative pricing policies than under trade credit. The approximate pricing trajectories remain almost indistinguishable from the exact solutions, demonstrating that the approximation accurately preserves the optimal pricing behavior.

Advertising investment again declines over the replenishment cycle because the remaining period during which advertising-generated goodwill can influence demand becomes progressively shorter. Although longer expiration dates increase the return on advertising by extending the selling horizon, advertising investment remains consistently lower than under trade credit. Without supplier financing, the retailer must allocate limited financial resources between inventory acquisition and marketing activities, reducing the incentive to invest aggressively in advertising. The approximate advertising trajectories reproduce the exact policies with excellent accuracy, confirming that the approximation effectively captures the retailer's optimal marketing decisions.

Goodwill gradually decreases throughout the replenishment cycle because its natural depreciation exceeds the positive contribution generated by advertising investment. Products with longer expiration dates maintain higher goodwill levels because the longer selling horizon increases the economic value of advertising, encouraging greater investment and slowing the decline of goodwill. Nevertheless, goodwill remains lower than in the trade-credit model, reflecting the retailer's more conservative advertising strategy under tighter liquidity constraints. The approximate goodwill trajectories remain nearly identical to the exact solutions across all expiration dates, further demonstrating the robustness of the proposed approximation.

Overall, the comparison between Figures 2 and 4 clearly illustrates the strategic role of trade credit in dynamic inventory management. Although extending the expiration date improves operational performance under both financing policies by reducing deterioration losses and increasing inventory availability, the presence of trade credit substantially amplifies these benefits. By relaxing liquidity constraints, trade credit enables the retailer to maintain larger inventories, implement smoother pricing policies, invest more heavily in advertising, and preserve higher goodwill throughout the replenishment cycle. Furthermore, the close agreement between the exact and approximate trajectories under both financing environments confirms that the proposed approximation provides a computationally efficient and highly accurate alternative to solving the original optimal control problem.

5.3. Approximation accuracy and sensitivity analysis

This subsection evaluates the accuracy and robustness of the proposed approximation through the numerical results reported in Table 2. The table presents the optimal replenishment cycle and the corresponding objective value obtained under both the trade-credit and no-trade-credit policies for different values of the ordering cost (K), the price sensitivity coefficient (β_1), and the goodwill effectiveness coefficient (β_2). For each parameter setting, the approximate solution (*) is compared with the exact solution (e) obtained by numerically solving the original optimal control problem.

Table 2 shows that the proposed approximation closely reproduces the exact optimal solutions across all parameter settings. Although slight differences are observed in the optimal replenishment cycle, the corresponding objective values remain remarkably close under both financing policies.

These minor discrepancies are expected because the objective function is relatively flat in the neighborhood of its optimum, implying that small changes in the replenishment cycle have only a negligible effect on profitability. Consequently, the proposed approximation preserves the essential economic characteristics of the original optimal control model while substantially reducing the computational effort required to obtain the optimal solution. This makes the approximation particularly suitable for repeated optimization, sensitivity analysis, and managerial decision-support applications.

Table 2. Impact of key parameters on the optimal replenishment cycle and profit.

		T_{cd}^*	J_{cd}^*	T_{nd}^*	J_{nd}^*	T_{cd}^e	J_{cd}^e	T_{nd}^e	J_{nd}^e
K	25	0.92	232.65	1.06	206.75	0.94	233.64	1.08	207.80
	45	1.16	213.57	1.30	189.85	1.18	214.99	1.32	191.30
	65	1.34	197.57	1.46	175.39	1.36	199.32	1.50	177.12
β_1	3	1.02	462.09	1.20	419.82	1.08	466.08	1.28	424.35
	5	1.16	213.57	1.30	189.85	1.18	214.99	1.32	191.30
	7	1.26	110.62	1.38	94.84	1.28	111.28	1.38	95.47
β_2	0.5	1.20	198.23	1.34	176.13	1.22	198.58	1.36	176.48
	1	1.16	213.57	1.30	189.85	1.16	213.57	1.18	214.99
	1.5	1.12	229.31	1.24	203.89	1.12	229.31	1.18	232.58

Table 2 further illustrates the influence of the principal model parameters on the retailer's optimal decisions. Increasing the ordering cost K leads to longer replenishment cycles and lower profitability under both financing policies. As replenishment becomes more expensive, the retailer orders less frequently to distribute the fixed ordering cost over larger inventory quantities. Nevertheless, the trade-credit policy consistently yields higher profits than the benchmark model because delayed payment alleviates liquidity constraints and reduces the financial burden associated with inventory acquisition.

The price sensitivity coefficient β_1 has a pronounced impact on profitability. As β_1 increases, customers become more responsive to price changes, limiting the retailer's ability to charge higher prices without reducing demand. Consequently, the optimal profit decreases substantially under both financing policies. Despite this reduction, the retailer operating under trade credit continues to outperform the benchmark case, indicating that additional financial flexibility partially offsets the adverse effects of highly price-sensitive markets.

The goodwill effectiveness coefficient β_2 has the opposite effect. Higher values of β_2 increase the contribution of advertising-generated goodwill to customer demand, thereby improving the return on advertising investment and increasing the retailer's profitability. The improvement is more pronounced under the trade-credit policy because the retailer can invest more aggressively in advertising while simultaneously maintaining adequate inventory levels. These results highlight the complementary relationship between financial flexibility and demand-enhancing marketing strategies.

Overall, the results reported in Table 2 demonstrate that the proposed approximation remains highly accurate over a broad range of operational and market conditions. At the same time, the sensitivity analysis confirms that lower ordering costs, lower price sensitivity, and stronger goodwill effects improve system performance, while trade credit consistently enhances profitability by increasing the retailer's financial flexibility. These findings further support the proposed

approximation as an efficient and reliable tool for analyzing dynamic pricing, advertising, and inventory decisions for expiration-sensitive products.

5.4. Managerial insights

The numerical analysis provides several managerial insights for retailers and suppliers managing expiration-sensitive products, such as food and fresh agricultural products, in environments characterized by dynamic pricing, advertising, and trade-credit financing. The results demonstrate that trade credit should be viewed as a strategic coordination mechanism rather than merely a financing arrangement. By postponing payment obligations, trade credit relaxes the retailer's liquidity constraints, allowing greater investment in inventory replenishment and advertising while maintaining more flexible pricing policies. Consequently, suppliers can improve the performance of downstream retailers by offering appropriate trade-credit terms, particularly for products with relatively long selling horizons.

The expiration date plays a fundamental role in determining the retailer's optimal operating strategy. Products with longer shelf lives experience slower inventory deterioration, enabling retailers to maintain larger inventories, implement smoother pricing adjustments, and invest more in advertising. These findings suggest that investments in preservation technologies, improved packaging, or cold-chain logistics can generate benefits beyond reducing deterioration losses by increasing the effectiveness of both inventory and marketing decisions. In addition, the results highlight the importance of jointly determining pricing and advertising policies.

Advertising investment enhances goodwill, which stimulates future demand, whereas pricing regulates the rate of inventory depletion and revenue generation. Optimizing these decisions simultaneously enables retailers to balance short-term profitability with long-term market development. This coordination becomes increasingly valuable as product lifetime increases because advertising has a longer period over which it can influence customer demand.

The sensitivity analysis indicates that optimal decisions need to be adapted to market characteristics. When customers become more price-sensitive, retailers may adopt more conservative pricing strategies while maintaining sufficient inventory availability to preserve demand. Conversely, when goodwill has a stronger influence on purchasing behavior, increasing advertising investment becomes more profitable because its contribution to future demand outweighs its additional cost. These results highlight the importance of continuously adjusting operational and marketing policies in response to changing market conditions.

Finally, the proposed approximation offers important practical advantages for managerial decision-making. The numerical results demonstrate that the approximate solution closely reproduces the exact optimal policies and objective values across a wide range of parameter settings while substantially reducing computational effort. As a result, managers can efficiently evaluate alternative replenishment, pricing, advertising, and financing strategies through near closed-form expressions, avoiding the need to repeatedly solve the complete optimal control problem.

Overall, the findings indicate that superior performance in expiration-sensitive supply chains depends on the coordinated management of financing, inventory, pricing, and advertising decisions. Integrating these decisions within a unified dynamic optimization framework enables retailers to improve profitability while responding effectively to product deterioration and evolving market conditions.

6. Conclusion

This paper developed a continuous-time optimal control framework for managing expiration-sensitive products under dynamic pricing, advertising, and trade-credit financing. The model jointly

determines the retailer's optimal replenishment cycle, inventory level, selling price, advertising investment, and goodwill while explicitly accounting for product expiration, goodwill dynamics, and supplier financing. The resulting optimization problem was formulated as a free-terminal-time optimal control problem and solved using Pontryagin's Maximum Principle. An approximation of the goodwill dynamics is proposed that provides semi-closed-form solutions while preserving the essential structure of the problem. Numerical comparisons demonstrate that the approximate solution closely reproduces the exact objective values, optimal replenishment cycles, and dynamic trajectories of the inventory level, selling price, advertising investment, and goodwill across a broad range of parameter settings.

The numerical experiments further demonstrate the significant role of trade-credit financing. By postponing payment obligations, trade credit relaxes liquidity constraints, enabling retailers to maintain higher inventory levels, increase advertising investment, preserve higher goodwill levels, and implement more flexible pricing strategies. In addition, products with longer expiration dates allow slower inventory depletion and increase the effectiveness of advertising, leading to improved profitability under both financing policies. The sensitivity analysis further shows that lower ordering costs, lower price sensitivity, and stronger goodwill effects contribute to better system performance, while the trade-credit policy consistently enhances profitability by increasing financial flexibility.

From a managerial perspective, the results highlight the importance of jointly coordinating operational, marketing, and financial decisions. Rather than treating inventory management, pricing, advertising, and financing decisions independently, retailers can integrate these decisions within a unified dynamic optimization framework to effectively address product deterioration and changing market conditions. The proposed approximation also provides a practical decision-support tool for efficiently evaluating alternative replenishment, pricing, advertising, and financing strategies without repeatedly solving the complete optimal control problem.

There are several directions for future research. Sustainability considerations may be incorporated by jointly optimizing profitability and waste reduction objectives. Since product deterioration directly contributes to food waste and environmental impact, extending the current framework toward bi-objective optimization could provide valuable insights into the interaction between operational efficiency and sustainability goals under trade credit financing arrangements. Related developments in perishable inventory management and waste reduction further motivate such extensions [39]. Extending the framework to multi-product or multi-echelon supply chain environments may provide deeper insights into coordination, competition, and inventory-sharing effects across different retail channels. Alternative goodwill and marketing dynamics, including digital advertising, data-driven demand estimation, and consumer learning behavior, may further enrich the analysis. Finally, future studies may investigate stochastic demand environments, behavioral customer responses, and large-scale computational approaches such as reinforcement learning methods to address more complex and realistic retail systems.

Author contributions

Yan Shi: conceptualization, methodology, investigation, software, data curation, validation, visualization, writing – original draft, funding acquisition. Xiang Liu: review and validation. Konstantina Skouri: supervision, conceptualization, methodology, validation, visualization, formal analysis, writing – review & editing.

Use of Generative-AI tools declaration

The authors declare they have not used Generative-AI tools in the creation of this article.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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