



Research article

Optimizing electric vehicle routing for cross-period distribution of multitype scarce resources under hybrid time windows and dynamic priorities

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Abstract: This study investigates the problem of multitype scarce resource distribution using electric vehicles, incorporating practical factors including hybrid time windows, cross-period delivery, and priority-based allocation. We develop a mixed integer programming model to minimize the total distribution costs subject to limits on electric vehicle routing, hybrid time windows, priority, cross-period assignment, load capacity, and battery swapping. A tailored genetic algorithm is designed to solve the proposed model. Numerical experiments demonstrate the model's effectiveness and superiority over benchmark scenarios – including models without priority, demand splitting, battery swapping stations, and region partitioning, as well as intraperiod and soft time windows – in terms of both the distribution cost and demand satisfaction rate. To further balance service fairness across all customers, we introduce an adaptive dynamic priority mechanism based on the cumulative demand satisfaction rate and service start time, and propose a ε -adaptive dynamic priority-based multitype scarce resource distribution model with cross-period hybrid time windows. The results show that the enhanced model significantly improves equity performance while maintaining operational efficiency.

Keywords: multitype scarce resources; electric vehicle routing; hybrid time window; cross-period; adaptive dynamic priority

Mathematics Subject Classification: 90B30, 90B50, 90B06

1. Introduction

In recent years, frequent climate anomalies, natural disasters, and epidemics have caused severe shortages of essential living and medical materials in affected regions. The efficient distribution of scarce resources is critical for sustaining lives, ensuring safety, and maintaining social stability. Concurrently, in response to escalating carbon emissions and the greenhouse effect, many countries are actively implementing carbon neutrality and peak carbon strategies. As a low-carbon and green transportation mode, electric vehicles (EVs) are being widely adopted in the logistics and public transport sectors, and are poised to replace internal combustion engine vehicles. Under this dual background of escalating disaster relief needs and the green transportation transition, this study investigates the approximately optimal distribution of scarce resources using a capacitated EV fleet. We assume that EVs can exchange batteries at swapping stations to overcome range limitations. This work addresses the pressing problem of designing an effective EV-based distribution system to alleviate material shortages under complex real-world operational constraints.

Although existing studies have made substantial improvements in EV-based distribution systems for scarce resources, there remain three fundamental research gaps that need to be filled.

First, there is inadequate modeling of resource criticality and delivery urgency. Existing studies typically classify relief items solely on the basis of their physical characteristics, e.g., consumable vs. nonconsumable [1,2]. Balcik et al. [1] categorized relief items into consumable items, e.g., water and medication, and nonconsumable items, e.g., shelter. Researchers hold different viewpoints on whether the two kinds of items can be delayed in delivery. Balcik et al. [1] considered that nonconsumable items can be backordered, while consumable items cannot; Lin et al. [3], Afshar and Haghani [4], and Huang and Rafiei [2] argued that consumable items and nonconsumable items can be backordered. However, literature on the delivery of multitype scarce resources lacks a classification system directly built on the severity of shortage. Furthermore, while soft time window [5–7] and hard time window [8–10] are widely studied in vehicle routing problems, most of these studies adopt a uniform time window setting for resources, failing to capture the nuanced delivery urgency required for different categories of scarce resources. The application of hybrid time windows specifically tailored to different resource types according to their criticality remains largely unexplored. Therefore, we put emphasis on scarce resource classification by the degree of shortage and match them with different types of time windows.

Second, there is a lack of rigid demand fulfillment paradigms in multiperiod contexts. The majority of multiperiod vehicle routing problem (VRP) research [11–13] operates under the unrealistic assumption that one customer's demand must be fully satisfied within a single period, which proves impractical under persistent resource scarcity and limited vehicle capacity. Although customer demand that cannot be met at one time can be split for complete distribution by multiple batches [14–16], the previous literature on VRPs with demand splitting is almost about static decisions. There are few studies about dynamic vehicle routing decisions [17–19] or multiperiodic vehicle routing decisions [2,20,21] due to difficulties in their solution methodologies. Although priority mechanisms are recognized as crucial in emergency logistics [2,22,23], most models rely on static priorities that cannot adapt to evolving distribution scenarios. Therefore, we will lay stress on cross-period demand satisfaction, which means that one customer's demand can be dynamically split and satisfied across multiple periods, and design a new priority mechanism to reflect the multiperiod

dynamicity, resource scarcity degree, and customers' urgency and satisfaction regarding scarce resources.

Third, the existing literature separately considers EV routing constraints in emergency logistics. While EV routing problem represents a growing research field [24–26], studies that comprehensively integrate EVs' operational constraints (battery capacity, battery swapping) with the complex practical factors of emergency relief logistics remain highly limited. The critical interplay between EV-specific limitations and key emergency logistics considerations – including multitype resource categorization, hybrid time windows, cross-period split delivery, and dynamic prioritization – creates problem complexity that current literature has not adequately addressed.

To fill these research gaps, this study proposes a novel and comprehensive research framework that optimizes scarce resource distribution along four integrated dimensions: Resources, time, space, and capacity. The main contributions are fourfold, which are rarely considered in previous studies:

(1) We introduce a new classification of scarce resources based on the severity of shortage. Crucially, we map each resource type to a specific type of time window (hard or soft), creating a more realistic model that directly links the material's criticality to delivery urgency. Importantly, we pioneer a comprehensive modeling framework that unifies three key advances in scarce resource distribution.

(2) We introduce dynamic cross-period fulfillment, allowing the same kind of scarce resources to be split and satisfied not only by multiple vehicles within a single period but also across multiple periods – a critical capability under prolonged resource scarcity. The combination of cross-period fulfillment and priority can effectively safeguard customers' needs. The cross-period rolling delivery alongside dynamic priority adjustment is designed to further improve the fairness of service among different customers.

(3) We design a novel adaptive dynamic priority mechanism for receiving points, in which priority levels are not fixed but adjustable in response to the evolving distribution process, current material scarcities, and changing urgency of customers' needs.

(4) We develop new holistic and sustainable models by integrating multiple real-world elements of multitype scarce resources, hybrid time windows, multiperiod (cross-period), demand splitting, customer priority (especially the extended adaptive dynamic priority and ε -adaptive dynamic priority constraint), region partition, and en-route battery swapping. These models significantly enhance the flexibility, efficiency, and practicality of emergency relief operations under complex real-world conditions.

The remainder of this paper is organized as follows. Section 2 provides a detailed problem description with the basic assumptions and notations. Section 3 develops the mathematical formulation of the proposed distribution model based on our integrated framework. Section 4 designs advanced solution methods to solve the complex model. Section 5 presents computational experiments and case studies. Section 6 proposes an extended model variant and discusses its performance. Finally, Section 7 concludes with the findings and future research directions.

2. Model setting and basic assumptions

In the distribution system, there are a set of distribution centers SD , a set of receiving points SC , and a set of battery swapping stations SR . Each receiving point demands two kinds of scarce resources, very scarce resources and moderately scarce resources, which are denoted as s and n and are associated with hard time windows and soft time windows, respectively. A distribution center supplies very scarce resources D^{ds} and moderately scarce resources D^{dn} ($d \in SD$). $|K_d|$

EVs are used for delivering scarce resources stored in distribution center $d \in SD$, where K_d is the set of EVs in $d \in SD$. Every distribution center $d \in SD$ belongs to a service subregion, to which some receiving points are assigned indicated by a binary symbol v_i^d , $i \in SC$. Each EV delivers two types of scarce resources. The load capacity of an EV is Q . The battery capacity (also referred to as the maximum travel distance) of an EV is denoted by $f_{\max}$. An EV can travel to battery swapping station $r \in SR$ to swap its battery. Assume that EVs start delivering at time zero and return to the distribution center before time T . The delivery plan horizon $[0, T]$ is divided into a set of periods P . EVs can deliver materials across many periods to meet the demand for the very scarce resource m_i^s and moderately scarce resource m_i^n at the receiving point $i \in SC$ associated with priority. The very scarce and moderately scarce resources at receiving point $i \in SC$ are associated with hard time windows (no delay allowed) (e_i^s, l_i^s) and soft time windows (delay allowed) (e_i^n, l_i^n) , respectively. The notations for the model are listed in Table 1.

The problem addressed in this study involves complicated factors and relations. To reduce the complexity of the model, some assumptions are given as follows:

- (1) The travel speed of EVs is constant.
- (2) The start time and braking time of an EV are not considered.
- (3) The battery swapped for an EV is fully charged, and the time spent swapping batteries is constant at battery swapping stations. In addition, the two swapped batteries are the same except for their state of charge (SOC, i.e., remaining driving distance or remaining power).
- (4) The start time of the service is consistent with the arrival time of an EV at a receiving point.
- (5) The battery swapping and scarce resource unloading begin simultaneously at a receiving point that is also a battery swapping station.

Table 1. Notation definition.

Notation	Definition
Sets:	
SS	Set of all points
SC	Set of receiving points
SD	Set of distribution centers
SR	Set of battery swapping stations; $SR \cap SC$ may be empty or not
P	Set of time periods
K_d	Set of EVs belonging to distribution center $d \in SD$
K	Set of EVs, $K = \bigcup_{d \in SD} K_d$
Variables:	
s_r^k	$s_r^k = t_r$ if EV k swaps battery at battery swapping station r and zero otherwise
m_i^s, m_i^n	Demand for very scarce and moderately scarce resources at receiving point i , respectively; unit, ton
$m_i'^{skp}, m_i'^{nkp}$	Quantity of very scarce and moderately scarce resources delivered by EV k at receiving point i in period p , respectively; unit, ton
g_i^{sp}, g_i^{np}	Cumulative demand satisfaction of very scarce and moderately scarce resources for receiving point i at period p , which can be expressed as $\sum_{k \in K_d} \sum_{p'=1, \dots, p, p \in P} m_i'^{skp'} / m_i^s$ and $\sum_{k \in K_d} \sum_{p'=1, \dots, p, p \in P} m_i'^{nkp'} / m_i^n$, respectively

Continued on next page

ST_i^{sk}, ST_i^{nk}	Service start time of EV k at point i for very scarce and moderately scarce resources, respectively; unit, h
u_i^p	The periodic priority of receiving point i , with a greater priority value indicating a lower priority
f_i^k	SOC of an EV k at receiving point i ; unit, km
f_r^k	Incremental electricity of battery of EV k after swapping at battery swapping station r compared with the previous battery; unit, km
v_i^d	A binary notation indicating whether receiving point i belongs to the delivery area of distribution center d ($v_i^d=1$) or not ($v_i^d=0$)
η_r^k	A binary variable indicating whether EV k swaps battery at battery swapping station r ($\eta_r^k=1$) or not ($\eta_r^k=0$)
x_{ij}^{kp}	A binary variable indicating whether EV k travels from receiving point i to receiving point j in period p ($x_{ij}^{kp}=1$) or not ($x_{ij}^{kp}=0$)
y_{id}^{kp}	A binary variable indicating whether EV k arrives at distribution center d from distribution center i in period p ($y_{id}^{kp}=1$) or not ($y_{id}^{kp}=0$)
z_i	A binary notation indicating whether i is a battery swapping station ($z_i=1$) or not ($z_i=0$)
Parameters:	
Q	Load capacity of each EV; unit, ton
γ	Transportation cost per unit distance for an EV; unit, RMB/km
β	Unit usage cost one trip of an EV; unit, RMB
α	Penalty cost for an EV arriving at a receiving point a unit time after the latest (or before the earliest) service time of a time window; unit, RMB/h
δ	Cost induced by the demand not satisfied at a receiving point; unit, RMB/ton
T	The latest time for an EV to return to a distribution center; unit, h
$f_{\max}$	Battery capacity (i.e., the maximum driving distance for a fully charged EV)
ω	A sufficiently large positive value
Q^s, Q^n	The capacity of an EV for loading very scarce and moderately scarce resources, respectively; unit, ton
D^{ds}, D^{dn}	Actual supply of very scarce and moderately scarce resources at distribution center $d \in SD$, respectively; unit, ton
$[e_i^s, l_i^s]$	Service time window for very scarce resources at receiving point i ; unit, h
$[e_i^n, l_i^n]$	Service time window for moderately scarce resources at receiving point i ; unit, h
u_i	The initial priority of receiving point i with a greater priority value indicating a lower priority
c_{ij}	Travel distance from $i \in SS$ to $j \in SS$; unit, km
t_{ij}	The EV travel time from $i \in SS$ to $j \in SS$; unit, h
s_i	Service duration time at receiving point i ; unit, h
τ_r	Unit cost for battery swapping at battery swapping station r ; unit, RMB/km
t_r	Time spent for swapping a battery at battery swapping station r ; unit, h

3. Formulation of model - M1

This section proposes a model for the cross-period electric VRP for delivering multitype scarce resources, denoted as M1. The objective function is the total distribution cost. The constraints

include typical VRP constraints, dynamic split and cross-period assignment constraints, capacity constraints, distribution constraints of multitype scarce resources, priority constraints, and EV battery swapping constraints.

3.1. Objective function

The aim of the optimization model, M1, is to minimize the total distribution cost, which includes transportation cost, EV usage cost, loss cost, penalty cost, and battery swapping cost of EVs. The objective is in essence a multi-objective problem, e.g., Yin and Xiang [27]. The loss cost refers to the cost induced by the shortage of materials that fail to fully satisfy the receivers' demand. The penalty cost covers two parts: Penalties from insufficient electricity for delivery travel, and penalties from violation of the soft time window for moderately scarce resources. The objective function is formulated as (1).

$$\begin{aligned}
 \min Z &= Z_1 + Z_2 + Z_3 + Z_4 + Z_5 + Z_6 \\
 Z_1 &= \sum_{k \in K} \gamma \left(\sum_{i \in SC} \sum_{p \in P} \sum_{j \in SC} x_{ij}^{kp} c_{ij} + \sum_{d \in SD} \sum_{p \in P} \sum_{j \in SC} y_{dj}^{kp} c_{dj} + \sum_{d \in SD} \sum_{p \in P} \sum_{j \in SC} y_{jd}^{kp} c_{jd} \right), \\
 Z_2 &= \beta \sum_{k \in K} \sum_{j \in SC} \sum_{p \in P} \sum_{d \in SD} x_{dj}^{kp}, \\
 Z_3 &= \sum_{k \in K} \sum_{i \in SC} S(ST_i^{nk}), \\
 Z_4 &= \sum_{i \in SS} \sum_{k \in K} P(f_i^k), \\
 Z_5 &= \tau_r \sum_{k \in K} \sum_{r \in SR} \eta_r^k f_r^k, \\
 Z_6 &= \sum_{p \in P} \sum_{k \in K_d} \sum_{i \in SC} \delta \left[(m_i^s - m_i'^{skp}) + (m_i^n - m_i'^{nkp}) \right]
 \end{aligned} \tag{1}$$

In Formula (1), Z_1 represents the total transportation cost for traveling between receiving points, traveling from distribution centers to receiving points, and traveling from receiving points to distribution centers. Z_2 represents the total operation cost due to the use of EVs. Z_3 and Z_4 represent the penalty costs triggered by delivery time window violations and electricity shortages, respectively. Z_5 is the cost for swapping batteries at distribution centers and battery swapping stations. Z_6 is the loss cost.

The penalty imposed on EV k for violating the soft time windows of moderately scarce resources at receiving point i is given in (2).

$$S(ST_i^{nk}) = \begin{cases} \alpha(e_i - ST_i^{nk}), & ST_i^{nk} \leq e_i \\ 0, & e_i^n < ST_i^{nk} \leq l_i^n \\ \alpha(ST_i^{nk} - l_i), & l_i \leq ST_i^{nk} \end{cases}, \quad \forall i \in SC, k \in K \tag{2}$$

where the nonzero value of $S(ST_i^{nk})$ represents the penalty on EV k if the EV reaches receiving point i outside the soft time window required by the receiving point. α is the penalty coefficient.

$$P(f_i^k) = \begin{cases} \omega, (f_i^k < c_{ij}) \& (f_i^k < c_{id}), i \neq j \neq d \\ \omega, (f_i^k < c_{ij}) \& (f_i^k < c_{ir}), i \neq j \neq r \\ \omega, (f_i^k = c_{ij}, z_j = 0) \& (f_i^k < c_{id}), i \neq j \neq d \\ \omega, (f_i^k = c_{ij}, z_j = 0) \& (f_i^k < c_{ir}), i \neq j \neq r, \forall i \in SS, j \in SC, k \in K_d, d \in SD, r \in SR \\ 0, (f_i^k > c_{ij}) \vee (f_i^k \geq c_{id}), i \neq j \neq d \\ 0, c_{ir} \leq f_i^k \leq c_{ij}, z_j = 0, \\ 0, f_i^k = c_{ij}, z_j = 1, i \neq j \end{cases} \quad (3)$$

The penalty for electricity shortage incurred by EV k at receiving point i is formulated in (3) by judging whether the SOC of EV k can reach the next receiving point, the distribution center, or a battery swapping station ($P(f_i^k) = \omega$) or not ($P(f_i^k) = 0$).

3.2. Routing and demand assignment constraints

This subsection presents the vehicle routing constraints, priority constraints, EV load capacity constraints, dynamic demand splitting and cross-period assignment constraints, distribution center storage constraints, and definition constraints.

A typical kind of fixed priority constraint is given by (4).

$$u_j^p - \omega(1 - x_{ij}^{kp}) > u_i^p, \forall i, j \in SC, k \in K, p \in P \quad (4)$$

Formula (4) guarantees that the priorities (values of priority) of any two adjacent receiving points on a path must be in decreasing (increasing) order.

The constraints for dynamic split and cross-period demand satisfaction are formulated as follows:

$$\sum_{k \in K_d} \sum_{i \in SC} \sum_{p \in P} \left[v_i^d m_i^{tskp} \left(\sum_{d \in SD} y_{di}^{kp} + \sum_{j \in SC} x_{ij}^{kp} \right) \right] \leq D^{ds}, \forall d \in SD \quad (5)$$

$$\sum_{k \in K_d} \sum_{i \in SC} \sum_{p \in P} \left[v_i^d m_i^{nkp} \left(\sum_{d \in SD} y_{di}^{kp} + \sum_{j \in SC} x_{ij}^{kp} \right) \right] \leq D^{dn}, \forall d \in SD \quad (6)$$

$$\sum_{k \in K_d} \sum_{p \in P} m_i^{tskp} \leq m_i^s, \forall i \in SC, d \in SD \quad (7)$$

$$\sum_{k \in K_d} \sum_{p \in P} m_i^{nkp} \leq m_i^n, \forall i \in SC, d \in SD \quad (8)$$

Equations (5) and (6) mean that the actual supply of each type of scarce resources delivered from a distribution center cannot exceed the available supply of the distribution center. Although (5) and (6) are used, the demand of the receiving points can be met to the maximum extent because the loss cost induced by the shortage of materials is minimized in the objective function. Equations (7) and (8) are the dynamic demand splitting constraints for each receiving point. They permit cross-period delivery across the entire distribution planning horizon. In other words, a receiving point can be served across multiple periods if the service time falls in the time window of the receiving point. These constraints also make sure that the delivered quantity of each type of scarce resource does not exceed the

demand of very or moderately scarce resources at a receiving point. Because the loss cost induced by material shortage is already embedded within the objective function for all receiving points, the delivered quantity is not required to be more than the demand of any receiving point.

Other common constraints are presented as follows:

$$\sum_{k \in K_d} \sum_{j \in SC} \sum_{p \in P} y_{dj}^{kp} = \sum_{k \in K_d} \sum_{j \in SC} \sum_{p \in P} y_{jd}^{kp}, \quad \forall d \in SD \quad (9)$$

$$x_{ij}^{kp} \cdot y_{dj}^{kp} = 0, \quad \forall i, j \in SC, k \in K_d, d \in SD, p \in P \quad (10)$$

$$\sum_{k \in K_d} \sum_{p \in P} \left(\sum_{i \in SC} x_{ij}^{kp} + \sum_{d \in SD} y_{dj}^{kp} \right) = v_j^d, \quad \forall j \in SC, d \in SD \quad (11)$$

$$\sum_{i \in SC} \sum_{p \in P} m_i^{nkp} \left(\sum_{d \in SD} y_{di}^{kp} + \sum_{j \in SC} x_{ij}^{kp} \right) \leq Q^n, \quad \forall k \in K_d, d \in SD \quad (12)$$

$$\sum_{i \in SC} \sum_{p \in P} m_i^{skp} \left(\sum_{d \in SD} y_{di}^{kp} + \sum_{j \in SC} x_{ij}^{kp} \right) \leq Q^s, \quad \forall k \in K_d, d \in SD \quad (13)$$

$$Q^n + Q^s = Q \quad (14)$$

$$\sum_{k \in K} \sum_{i \in SC} \sum_{p \in P} x_{ij}^{kp} + \sum_{k \in K} \sum_{d \in SD} \sum_{p \in P} y_{dj}^{kp} = \sum_{k \in K} \sum_{i \in SC} \sum_{p \in P} x_{ji}^{kp} + \sum_{k \in K} \sum_{d \in SD} \sum_{p \in P} y_{jd}^{kp}, \quad \forall j \in SC \quad (15)$$

$$x_{ij}^{kp} \in \{0, 1\}, \quad \forall k \in K_d, d \in SD, i, j \in SC, p \in P \quad (16)$$

$$y_{di}^{kp} \in \{0, 1\}, \quad \forall k \in K_d, d \in SD, i \in SC, p \in P \quad (17)$$

$$m_i^{skp} \geq 0, m_i^{nkp} \geq 0, \quad \forall i \in SC, k \in K_d, p \in P \quad (18)$$

Formula (9) ensures that the origin and destination distribution center of an EV are consistent. Formula (10) ensures that an EV should not start from a distribution center and a receiving point at the same time. Formula (11) means that each receiving point is only served once in each period to ensure that all receiving points can obtain scarce resources as soon as possible. Formulae (12), (13), and (14) ensure that all types of scarce resources delivered by an EV do not exceed the EV's load capacity. Formula (15) is the vehicle flow balance constraints. Formulae (16), (17), and (18) are definition constraints for y_{ik} , x_{ijk} , m_i^{skp} , and m_i^{nkp} .

3.3. Hybrid time window constraints

The hybrid time window constraints contain soft time window constraints for general scarce resources and hard time window constraints for very scarce resources.

First, we formulate soft time window constraints for general scarce resources delivered by EVs to each receiving point as follows:

$$ST_j^{nk} + \omega(1 - x_{ij}^{kp}) \geq \begin{cases} ST_i^{nk} + \max[s_i, s_i^k \cdot z_i] + t_{ij}, & \text{if } x_{ij}^{kp} = 1, \forall k \in K, j, i \in SC, j \neq i, p \in P \\ ST_r^{nk} + s_r^k + t_{rj}, & \text{if } x_{rj}^{kp} = 1, \forall k \in K, r \in SR, r \notin SC, j \in SC, r \neq j, p \in P \end{cases} \quad (19)$$

$$T \geq \begin{cases} ST_i^{nk} + \max[s_i, s_i^k \cdot z_i] + t_{id}, & \text{if } y_{id}^{kp} = 1, \forall k \in K_d, d \in SD, i \in SC, i \neq d, p \in P \\ ST_r^{nk} + s_r^k + t_{rd}, & \text{if } y_{rd}^{kp} = 1, \forall k \in K_d, d \in SD, r \in SR, r \notin SC, r \neq d, p \in P \end{cases} \quad (20)$$

$$ST_i^{nk} = \begin{cases} t_{di}, & \text{if } y_{di}^{kp} = 1, \forall k \in K_d, d \in SD, i \in SC, p \in P \\ ST_j^{nk} + \max[s_j, s_j^k \cdot z_j] + t_{ji}, & \text{if } x_{ji}^{kp} = 1, \forall k \in K_d, d \in SD, j \in SC, i \in SC, p \in P, j \neq i \neq d \\ ST_r^{nk} + s_r^k + t_{ri}, & \text{if } x_{ri}^{kp} = 1, \forall k \in K_d, d \in SD, r \in SR, r \notin SC, i \in SC, p \in P, r \neq i \neq d \end{cases} \quad (21)$$

The constraints listed in (19) define the relationship between the service start times at two adjacent receiving points for an EV that delivers moderately scarce resources. Equation (20) indicates that an EV must return to its distribution center prior to the latest return time. Equation (21) calculates the service start time of EV k delivering moderately scarce resources to receiving point i . The first formula of (21) sets the service start time at receiving point i to which EV k travels from a distribution center. The battery swapping time of an EV at the distribution center is omitted in the formula, which is reasonable, because there is enough time for the preparation of an EV. The second formula of (21) calculates the service start time at receiving point i to which an EV travels from another receiving point j . The third formula of (21) calculates the service start time at receiving point i to which an EV travels from a battery swapping station r .

Second, we formulate the hard time window constraints for very scarce resources delivered by EVs to each receiving point as follows:

$$ST_j^{sk} + \omega(1 - x_{ij}^{kp}) \geq \begin{cases} ST_i^{sk} + \max[s_i, s_i^k \cdot z_i] + t_{ij}, & \text{if } x_{ij}^{kp} = 1, \forall k \in K, j, i \in SC, j \neq i, p \in P \\ ST_r^{sk} + s_r^k + t_{rj}, & \text{if } x_{rj}^{kp} = 1, \forall k \in K, r \in SR, r \notin SC, j \in SC, r \neq j, p \in P \end{cases} \quad (22)$$

$$T \geq \begin{cases} ST_i^{sk} + \max[s_i, s_i^k \cdot z_i] + t_{id}, & \text{if } y_{id}^{kp} = 1, \forall k \in K_d, d \in SD, i \in SC, i \neq d, p \in P \\ ST_r^{sk} + s_r^k + t_{rd}, & \text{if } y_{rd}^{kp} = 1, \forall k \in K_d, d \in SD, r \in SR, r \notin SC, r \neq d, p \in P \end{cases} \quad (23)$$

$$e_i^s \leq ST_i^{sk} \leq l_i^s, \quad \forall i \in SC, k \in K \quad (24)$$

$$ST_i^{sk} = \begin{cases} t_{di}, & \text{if } y_{di}^{kp} = 1, \forall k \in K_d, d \in SD, i \in SC, p \in P \\ ST_j^{sk} + \max[s_j, s_j^k \cdot z_j] + t_{ji}, & \text{if } x_{ji}^{kp} = 1, \forall k \in K_d, d \in SD, j \in SC, i \in SC, p \in P, j \neq i \neq d \\ ST_r^{sk} + s_r^k + t_{ri}, & \text{if } x_{ri}^{kp} = 1, \forall k \in K_d, d \in SD, r \in SR, r \notin SC, i \in SC, p \in P, r \neq i \neq d \end{cases} \quad (25)$$

The meanings of (22), (23), and (25) are similar to those of (19), (20), and (21), respectively. Equation (24) indicates that the service start time of an EV does not exceed the hard time window of very scarce resources required by receiving point i .

3.4. Battery swapping constraints

The battery swapping constraints for EVs include mainly the battery swapping condition, the battery swapping time, and the SOC of battery of an EV at a receiving point. The constraints are formulated as follows:

$$f_j^k = \begin{cases} f_i^k + z_i \eta_i^k (f_{\max} - f_i^k) - c_{ij}, & \text{if } x_{ij}^{kp} = 1, \forall k \in K_d, d \in SD, i, j \in SC, p \in P \\ f_r^k + \eta_r^k (f_{\max} - f_r^k) - c_{rj}, & \text{if } x_{rj}^{kp} = 1, \forall k \in K_d, d \in SD, r \in SR, r \notin SC, j \in SC, p \in P \\ f_{\max} - c_{dj}, & \text{if } y_{dj}^{kp} = 1, \forall k \in K_d, d \in SD, j \in SC, p \in P \end{cases} \quad (26)$$

$$s_r^k = \eta_r^k t_r, \forall r \in SR, k \in K \quad (27)$$

$$0 \leq f_i^k \leq f_{\max}, \forall i \in SC, k \in K \quad (28)$$

$$\eta_r^k = \begin{cases} 1, & \text{if } (c_{ir} \leq f_i^k < c_{ij}) \vee (c_{ir} \leq f_i^k < c_{id}), \forall k \in K_d, d \in SD, r \in SR, i \in SC, j \in SC, r \neq i \neq j \\ 0, & \text{otherwise} \end{cases} \quad (29)$$

Equation (26) determines the SOC of an EV upon arrival at its immediate downstream receiving point. The first expression applies to the case where an EV departs from a receiving point that may be also a battery swapping station. The second expression applies to the case where an EV starts from a battery swapping station. The third expression calculates the SOC if the immediate upstream point of an EV is a distribution center. Equation (27) determines the swapping duration spent at battery swapping station r . Equation (28) ensures that the SOC of EV k is not greater than the battery capacity. Equation (29) defines the battery swapping condition. If an EV can reach a battery swapping station yet lacks sufficient charge to travel onward to the next receiving point or distribution center, then a battery swap is performed for the EV; otherwise, no swap occurs.

In summation, the fixed priority-based cross-period hybrid time window region-partitioned distribution model is built up as follows:

M1: (1)~(29).

4. Solution methods

The VRP is a well-known NP-hard problem [28,29]. The genetic algorithm (GA), the tabu search algorithm, and the simulated annealing algorithm demonstrate strong performance in solving such problems [30,31]. The GA has a strong adaptive ability and global search ability for approximately optimal solutions [32], so we develop a solution method based on the framework of the GA to solve the proposed model.

4.1. Main operations

The main operations in the GA include chromosome encoding, chromosome selection, chromosome crossover, and chromosome mutation.

(1) Chromosome encoding

The chromosomes are generated using natural number encoding. A natural number at a gene locus represents the identification number of a receiving point. For example, $\{(5, 7, 9, 10, 12), (20, 17, 19, 1), (2, 11, 15, 18), (3, 4, 6, 13, 8), (14, 16)\}$ is a chromosome in which there are five subpaths (a subpath is a sequence of some receiving points), i.e., 5–7–9–10–12, 20–17–19–1, 2–11–15–18, 3–4–6–13–8, and 14–16.

Note that any two genes in a chromosome cannot take the same value during initialization because demand splitting is not executed at this step. However, during iterative optimization, certain receiving points require multiple deliveries due to demand splitting; hence, there may be repeated values across several different gene loci.

The detailed encoding method is presented here. First, following the rule of the minimum Euclidean distance between each receiving point and the distribution centers, all receiving points are divided into $|SD|$ parts. Part d contains the sequence of receiving points assigned to distribution center d , where $d=1,\dots,|SD|$. Sort the identity numbers of receiving points in each part in increasing order. Second, for Part d , execute the following operations (i)-(iv).

(i) Generate a sequence of $|SC|$ different random integers based on the total number of receiving points.

(ii) Establish a one-to-one mapping from the random number sequence to the sequence of identity numbers of the receiving points.

(iii) Obtain the sequence of identity numbers of the receiving points by sorting the random integer sequence.

(iv) Sort the sequence just obtained in increasing order according to the priority value of the receiving points; thus the substring of a chromosome is obtained. Third, after all substrings are obtained for all $|SD|$ parts, an initial chromosome is generated.

(2) Selection operator

The roulette function for chromosome selection is given below [33]:

$$q_{\bar{p}} = \sum_{\bar{p}=1}^N Q(\bar{p}) \quad (30)$$

where $q_{\bar{p}}$ is the cumulative probability of chromosome $\bar{p}$, and $Q(\bar{p})$ is the actual selection probability of chromosome $\bar{p}$, which is mainly determined by the fitness values of chromosomes. Generate a random number via the C language's built-in function `rand()`; if this value falls in the interval $[q_{\bar{p}-1}, q_{\bar{p}}]$, chromosome $\bar{p}$ is selected.

(3) Crossover operator

We adopt the partial matching crossover method to perform crossover operations. Let us take an example. Two chromosomes, $\{2, 7, 9, 15, 4, 1, 19, 16, 8, 17\}$ and $\{14, 5, 3, 10, 12, 20, 6, 11, 18, 13\}$, are selected according to the crossover probability. First, Genes 1 to 9 in Chromosome 1 are selected randomly, and Genes 1 to 3 and Genes 5 to 10 in Chromosome 2 are selected randomly. Second, the first selected gene (i.e., Gene 1) in Chromosome 1 is crossed with a gene randomly selected from Genes 1 to 3 and 5 to 10 in Chromosome 2. This process repeats until the last selected gene (i.e., Gene 9) in Chromosome 1 is matched and crossed with a gene that is randomly selected in the remaining genes of Genes 1 to 3 and 5 to 10 in Chromosome 2. Finally, two new chromosomes are generated: $\{12, 14, 18, 6, 5, 3, 13, 20, 11, 17\}$ and $\{7, 4, 1, 10, 2, 16, 15, 8, 9, 19\}$.

(4) Mutation operator

According to a predefined mutation probability, we perform the mutation operation on the randomly selected chromosome using the gene interchange mutation method. An example is given. In the selected chromosome $\{2, 7, 9, 15, 4, 1, 19, 16, 8, 17\}$, Genes 2 and 4 are randomly selected, and then the two genes are swapped to generate a new chromosome sequence $\{2, 15, 9, 7, 4, 1, 19, 16, 8, 17\}$.

(5) Fitness function

We define the fitness function using the objective function value corresponding to chromosome $\bar{p}$ as follows:

$$fit_{\bar{p}} = \frac{1}{Z_{\bar{p}}}, \quad \bar{p} = 1, 2, \dots, N \quad (31)$$

4.2. Procedure of the solution algorithm

According to the primal GA and the main operations presented above, we propose the solution procedure as follows.

Step 1: Initialization. Initialize all parameters including the population size N , the maximum evolution generation $Ite_{\max}$, the crossover probability, the mutation probability, and the initial EV SOC. Set the iteration counter $tt=1$. Generate the chromosomes using natural number encoding. All different chromosomes form the initial population of size N .

Execute the initial allocation of demands at each receiving point according to the greedy criterion. According to the sequence of receiving points in a chromosome, one EV in a trip meets the maximum demand of two types of the scarce resources at each receiving point in turn until the EV cannot meet the demand of any type of scarce resources of a receiving point. Then, using the same method, another EV delivers scarce resources to the remaining receiving points whose demand has not been fully satisfied. This process repeats until both types of demands of all receiving points are met.

Step 2: Partial feasibility verification for a chromosome.

Step 2.1: For a distribution center, ensure that the total delivered quantity of scarce resources does not exceed the actual supply of the corresponding distribution center; that is, Constraints (5) and (6) hold true.

Step 2.2: For an EV, calculate the delivered quantity of each type of scarce resources, and ensure that this quantity does not exceed the load capacity of the EV; that is, Constraints (12), (13), and (14) hold true.

Step 2.3: For a receiving point, calculate the delivered quantity of each type of scarce resources, and ensure that this quantity is not greater than the demand of the receiving point; that is, Constraints (7) and (8) hold true.

If the chromosomes in the initial population satisfy these constraints, proceed to Step 3; otherwise, return to Step 1.

Step 3: Calculate the fitness value for each chromosome according to (31).

Step 4: Selection. The roulette wheel method is used to select chromosomes according to their fitness values. In (30), $Q(\bar{p})$ is given as $Q(\bar{p}) = fit_{\bar{p}} / \sum_{\bar{p}'=1}^N fit_{\bar{p}'}$, where $\bar{p}=1,2,\dots,N$. Finally, this step will generate a new population.

Step 5: Crossover. We use the partial matching crossover method to perform crossover operations.

Step 6: Mutation. The gene interchange mutation method is adopted to perform mutation operations.

Step 7: Partial feasibility verification and repair of a chromosome.

In this step, we conduct partial feasibility verification for a chromosome to eliminate the following circumstances (i)–(iii) caused by the crossover and mutation operators.

(i) The priority order of all receiving points in each chromosome fails to meet the requirements.

(ii) For very scarce resources, the arrival time of an EV at a receiving point does not fall in its hard time window.

(iii) An EV has insufficient electricity because its battery is not swapped in time (caused by position shifts of battery swapping stations in a chromosome).

Partial feasibility verification for a chromosome are presented as follows.

Step 7.1: Identify all paths in a chromosome.

If the following (i)–(ii) are satisfied for an EV at gene locus i , then insert the code of the corresponding distribution center; otherwise, discard the chromosome.

(i) The SOC of an EV at gene locus i is only sufficient to return to its distribution center within the required time.

(ii) The quantity of each type of scarce resources and the total quantity of the two types delivered by the EV satisfy the EV's load capacity constraints.

Step 7.2: We compare the priority of each gene (receiving point) in a chromosome and determine a gene sequence such that priority rule in Formula (4) holds.

Step 7.3: For all routes in a chromosome, execute the following operations.

First, determine which EV is associated with a certain path.

Second, under the assumption that each EV departs at time zero from its corresponding distribution center, calculate the arrival time of an EV at a receiving point in a path.

Finally, at each receiving point i , following the gene sequence order of the chromosome, the service start time of EV k is checked to guarantee that it falls within the hard time window for the very scarce resources required by receiving point i , i.e., (24) holds, involving (22), (23), (25), and (27).

Step 7.4: Evaluate the SOC of an EV at every point.

First, the EV is fully charged at a distribution center.

Second, calculate the EV's current SOC and assess whether its battery can be swapped when it arrives at a receiving point as following (1)–(3).

(1) Calculate the distance between the EV position and the next receiving point, and the distance between the EV's position and the nearest battery swapping station. If the current SOC of the EV is insufficient (or just sufficient) to reach the next receiving point (that is not a battery swapping station) and is enough to reach the nearest battery swapping station, i.e.

$$c_{ir} \leq f_i^k \leq c_{ij}, z_j = 0, \forall r \in SR, i \in SS, j \in SC, r \neq i \neq j, k \in K, \quad (32)$$

or the EV has just enough charge to reach the next receiving point that is also a battery swapping station, then the EV travels to the nearest battery swapping station for swapping.

(2) If the current SOC of the EV is insufficient to reach the next receiving point and cannot return to its distribution center, i.e.

$$(f_i^k < c_{ij}) \& (f_i^k < c_{id}), i \neq j \neq d, \quad (33)$$

or if the current SOC of the EV is insufficient to reach the next receiving point and the nearest battery swapping station, i.e.

$$(f_i^k < c_{ij}) \& (f_i^k < c_{ir}), i \neq j \neq r, \quad (34)$$

then the objective value of the corresponding chromosome is set as an infinitely positive number ω .

(3) If the current SOC of the EV is not enough to reach the next receiving point (that is not a battery swapping station) but cannot return to its distribution center, i.e.

$$(f_i^k = c_{ij}, z_j = 0) \& (f_i^k < c_{id}), i \neq j \neq d, \quad (35)$$

or the current SOC of the EV can just right to reach the next receiving point (that is not a battery swapping station) and is not enough to reach the nearest battery swapping station, i.e.

$$(f_i^k = c_{ij}, z_j = 0) \& (f_i^k < c_{ir}), i \neq j \neq r, \quad (36)$$

then the objective value of the corresponding chromosome is set to an sufficiently large positive value ω .

If an EV at a certain receiving point has sufficient electricity to travel to a battery swapping station, i.e., (28) holds, then insert the code of the battery swapping station into the corresponding chromosome and update the travel time from the receiving point to the battery swapping station.

Step 7.5: Possible demand splitting at receiving points. For a receiving point in the gene sequence of each chromosome, judge whether demand splitting is needed and assign the demand for the receiving point.

First, assess whether the SOC f_{i-1}^k of EV k is sufficient to travel to receiving point i , i.e., (28) holds.

Second, assess whether the total quantity of scarce resources delivered by EV k at the receiving point i do not exceed the EV's load capacity and whether those delivered by all EVs belonging to a distribution center is not beyond the actual supply of the distribution center; that is, (5), (6), (12), (13), and (14) hold true.

Third, judge whether the arriving time of EV k at the receiving point i does not exceed the hard time window of the receiving point i ; that is, (22), (23), (24), (25), and (27) hold true.

If the three conditions above are satisfied, proceed to execute demand splitting and update the demand for each type of scarce resource for the receiving point i based on (7), (8), (12), and (13).

(i) If the total quantity of very scarce resources delivered by EV k has reached the load capacity of the EV for this type of resource, yet the total quantity of moderately scarce resources has not reached the load capacity of the EV for this resource type, then execute demand splitting and update the delivered quantity for moderately scarce resources at the receiving point i .

(ii) If the total quantity of moderately scarce resources delivered by EV k has reached the load capacity of the EV for this type of resource, yet the total quantity of very scarce resources has not reached the load capacity of the EV for this resource type, then demand splitting is executed and update the delivered quantity for very scarce resources at the receiving point i .

(iii) If the total quantity of very and moderately scarce resources delivered by EV k has not reached the load capacity of the EV for the corresponding resource type, then execute demand splitting and update the delivered quantity for the two resource types at the receiving point i .

Step 7.6: Validation of routing constraints.

Step 7.6.1: For a route, perform the following operations.

If the ending time of the route falls in a time period, then record the occurrence count of the distribution center associated with the path. If the sum of such occurrence numbers for all time periods is divisible by 2, then (9) and (15) are satisfied, and we can proceed to Step 7.6.2; otherwise, discard the corresponding chromosome.

Step 7.6.2: For a route, perform the following operations:

For a receiving point, if it appears multiple times in the route, and the preceding gene represents a distribution center, then discard the corresponding chromosome; otherwise, proceed to Step 7.6.3.

Step 7.6.3: For a receiving point in a route, calculate the total number of services it has received and its total number of incoming arcs. If the two values are equal, then proceed to Step 8; otherwise, discard the corresponding chromosome.

Step 8: Identify the best chromosome of the current generation. Calculate the fitness value of each chromosome in the current population and record the best fitness value and the corresponding chromosome. If the best fitness value in the population outperforms the best fitness value of the previous generation's population, then update the best fitness value and the best-found chromosome.

Step 9: Convergence check. Assess whether the termination condition $tt \geq It_{\max}$ is satisfied. If yes, terminate the iteration and output the best-found solution; otherwise, set $tt = tt + 1$, and return to Step 2.

5. Example study for model M1

The distribution network consists of 2 distribution centers, 20 receiving points, and 5 battery swapping stations, of which receiving points Nos. 14 and 18 function also as battery swapping stations. Each distribution center is equipped with four EVs of the same type. Table 2 lists the coordinates, demand, hybrid time windows, service time, and the fixed priority of each receiving point. The coordinates of the other battery swapping stations are (55, 55), (85, 45), and (31, 50), with unit in km. The coordinates of the two distribution centers are (40, 43) and (80, 15), with the unit in km. The distribution center's supplies are given as $D^{1s} = 2.4$ ton, $D^{1n} = 2.8$ ton, $D^{2s} = 2.6$ ton, and $D^{2n} = 3.1$ ton. The planning horizon spans $[0, 16]$ h, and the duration of each period is 8 hours.

Other parameters are set as follows: $Q^s = 2$ ton; $Q^n = 2$ ton; $f_{\max} = 200$ km; $\beta = 400$ RMB per EV; $t_r = 1/6$ h; $\tau_r = 1.5$ RMB $\cdot$ km $^{-1}$; $\gamma = 1.5$ RMB $\cdot$ km $^{-1}$; $\alpha = 0.5$ RMB $\cdot$ km $^{-1}$; $\delta = 50$ RMB $\cdot$ ton $^{-1}$. The EV's travel speed is set to 60 km $\cdot$ h $^{-1}$. The crossover probability is set to 0.9. The mutation probability is set to 0.001. The population size is set to 60. The maximum iteration count is set to 5000.

Before numerical experiments, we execute region partition. If the Euclidean distance from a receiving point to a distribution center is smaller than that to the other distribution center, then the receiving point is assigned to the former center. According to this rule, we find that receiving points Nos. 1, 2, 3, 4, 6, 7, 10, 13, and 18 are affiliated with Distribution Center 1, and Nos. 5, 8, 9, 11, 12, 14, 15, 16, 17, 19, and 20 belong to Distribution Center 2.

Table 2. Receiving points' information.

$i \in SC$	X axis /km	Y axis /km	m_i^s /ton	m_i^n /ton	e_i^s /h	l_i^s /h	e_i^n /h	l_i^n /h	s_i /h	u_i
1	22	67	0.5	0.3	4	8	1	5	0.5	2
2	72	77	0.2	0.4	2	7	4	8	0.5	2
3	90	77	0.2	0.4	0	4	4	8	0.5	2
4	50	78	0.3	0.2	4	7	0	4	0.5	3
5	90	68	0.3	0.3	2	7	6	10	0.5	3
6	24	52	0.6	0.6	0	4	0	4	0.5	4
7	68	42	0.5	0.4	0	2	6	10	0.5	1
8	68	40	0.4	0.7	2	6	0	4	0.5	5
9	80	40	0.1	0.1	1	5	0	4	0.5	5
10	55	31	0.2	0.8	1	5	5	7	0.5	5
11	65	31	0.3	0.2	0	5	5	9	0.5	2
12	91	10	0.0	1.5	0	ω	0	4	0.5	5
13	34	15	0.1	0.5	8	10	0	4	0.5	3
14	56	17	0.3	0.1	8	10	0	4	0.5	4
15	71	17	0.5	0.2	0	2	3	7	0.5	1
16	87	17	0.3	0.2	3	5	8	12	0.5	2
17	96	17	0.1	0.1	2	7	6	10	0.5	2

Continued on next page

18	34	20	0.7	0.2	0	5	2	6	0.5	3
19	71	10	0.1	0.2	3	9	0	4	0.5	4
20	99	31	0.8	0.2	0	4	4	8	0.5	4

5.1. Comparison between fixed priority and no-priority scenarios

We compare distribution schemes obtained for scenarios with a fixed priority and without a fixed priority of the receiving points by executing 10 independent runs of C++ programs for each scenario. The fixed priority scenario corresponds to the fixed priority-based cross-period hybrid time window region-partitioned distribution model (M1). We record the total distribution costs for the 10 runs. The distribution scheme exhibiting the best performance among the 10 runs is defined as the best-found scheme. The average cost is calculated using the total distribution costs of the 10 runs. Table 3 summarizes the total distribution cost (Z) and cost components (Z_1 , Z_2 , Z_3 , Z_5 , and Z_6) for both the best-found scheme and the average cost. Note that Z_4 is excluded from the table because it equals zero for all feasible distribution schemes.

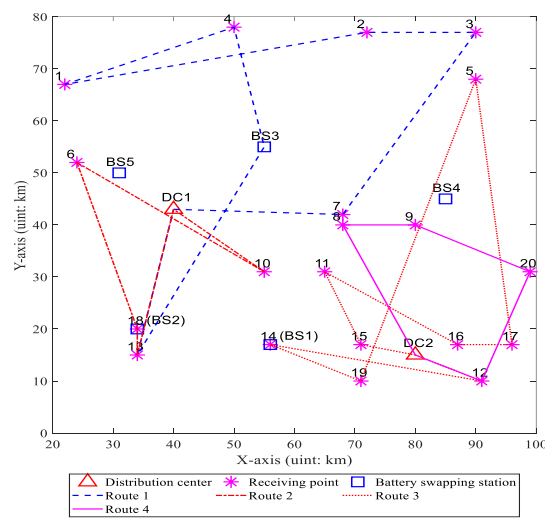
Table 3. Costs of distribution plans with and without fixed priority.

M1	Z_1	Z_2	Z_3	Z_5	Z_6	Z
Best-found	1263.045	1600	17.040	1870.345	156.118	4906.548
Average	1293.822	2320	16.965	1952.156	180.130	5713.074
Without priority	Z_1	Z_2	Z_3	Z_5	Z_6	Z
Best-found	1227.153	2000	16.944	1870.206	194.448	5308.751
Average	1227.999	2600	15.339	2055.241	211.303	6079.882

From Table 3, we can observe the main differences between distribution schemes with and without fixed priority. First, the priority order may lead to detours for EVs and yield longer travel distances for a distribution scheme, as reflected by the value of Z_1 . To further compare the priority and no-priority scenarios, we plot their best-found EV paths, as shown in Figures 1 and 2. These figures validate the abovementioned observation. Second, the EV utilization cost and battery swapping cost of the fixed priority scenario are significantly lower than those of the non-priority scenario. Third, the loss cost Z_6 of the no-priority scenario is significantly higher than that of the priority scenario. For further comparison, we report the satisfaction rates (SRs) of each kind of scarce resource for the two scenarios in the second and third rows of Table 4. The average (best-found) SR here refers to the ratio of total distribution volume to total demand at receiving points for the distribution scheme corresponding to the average value of 10 runs (the best run among the 10 runs). It is evident that the average (best-found) SR of the priority scenario is higher than that of the no-priority scenario.

Table 4. Satisfaction rates of distribution schemes of fixed priority and no-priority scenarios.

	Average SR of very scarce resource	Average SR of moderately scarce resource	Best-found SR of very scarce resource	Best-found SR of moderately scarce resource
M1	0.8005	0.8103	0.8229	0.9351
Without	0.7661	0.7773	0.7844	0.7955
Intraperiod	0.7841	0.7951	0.7984	0.8080
Soft time	0.7811	0.7916	0.7941	0.8024
Without	0.7910	0.8011	0.8031	0.8165

**Figure 1.** The best-found EV path of the distribution scheme with priority.

In Figure 1, four EVs are used to deliver scarce resources to receiving points, with one best-found path assigned to each EV. EVs traveling along Routes 1 and 2 depart from Distribution Center 1 (DC1). Route 1 follows the sequence DC1–7–3–2–1–4–BS3–13–DC1 (the priority values of receiving point sequence are 1, 2, 2, 2, 3, and 3, excluding the battery swapping station BS3); Route 2 follows DC1–13–18–6–10–DC1 (the priority values of receiving point sequence are 3, 3, 4, and 5). Very scarce resources at Receiving Point 13 are exclusively distributed by the EV on path 1. The demand of moderately scarce resources at Receiving Point 13 is split into two batches: The first batch weighs 0.2 ton, and the second batch weighs 0.3 ton. The first batch is fulfilled by the EV on Route 2 at 0.47 h in the first period, and the second batch is fulfilled by the EV on Route 1 at 0.85 h in the second period.

The EV on Route 1 swaps its battery at battery swapping station BS3. The SOC of the EV is 59.59 km upon arriving Receiving Point 4. The EV cannot travel to the subsequent Receiving Point 13, as the energy required to travel from Receiving Point 4 to receiving point 13 corresponds to 65 km of driving range. Thus, the EV detours to BS3, the nearest battery swapping station, for battery replacement.

EVs traveling on routes 3 and 4 depart from Distribution Center 2 (DC2). Route 3 follows DC2–15–11–16–17–5–19–14 (BS1)–12–DC2, and Route 4 follows DC2–12–20–9–8–DC2. The demand for moderately scarce resources at Receiving Point 12 is split into 0.7 ton and 0.8 ton, fulfilled by the EVs on Routes 3 and 4, respectively. The EV on Route 4 arrives at Receiving Point 12 at time 0.20 h in the first period. The EV on Route 3 arrives at Receiving Point 12 at time 1.09 h

in the second period. The SOC of the EV on Path 3 at Receiving Point 14 is 20.754 km, a value lower than the energy consumption (45.22 km) between Receiving Point 14 and the subsequent receiving point 12. Accordingly, the EV performs a battery swap at this location.

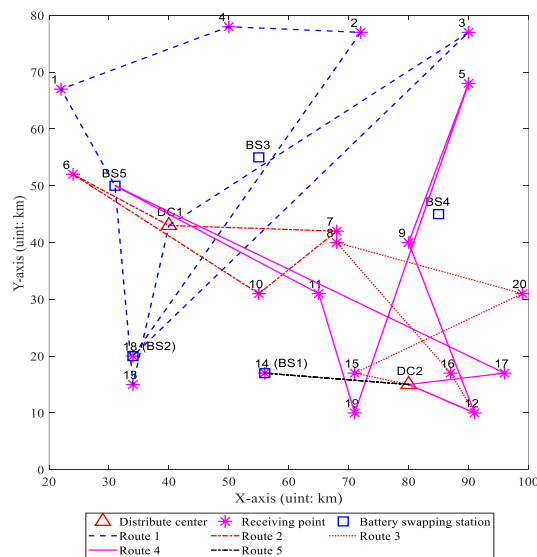


Figure 2. The best-found EV path of the distribution scheme without priority.

In the no-priority scenario, five EVs are deployed to distribute scarce resources through five routes (see Figure 2). The EVs on Paths 1 and 2 belong to DC1. Path 1 is DC1–3–18–2–4–1–BS5–13–DC1, and Path 2 is DC1–7–10–6–DC1. The EV on Route 1 swaps its battery at the battery swapping station BS5. The SOC of the EV is 19.61 km when it arrives at Receiving Point 1. The EV cannot travel to the next receiving point (13) because the energy required to travel from Receiving Point 1 to Receiving Point 13 is 53.37 km. Thus, the EV travels to the nearest battery swapping station, BS5, to swap its battery.

The EVs on paths 3, 4, and 5 belong to DC2. Path 3 follows DC2–15–20–8–16–12–DC2, Path 4 follows DC2–12–9–5–19–11–BS1–17–DC2, and Path 5 follows DC2–14–DC2. The moderately scarce resource demand of Receiving Point 12 is divided into two batches. The first batch of 0.7 ton is satisfied by the EV on Path 4 arriving at 0.20 h in the first period, and the second batch of 0.8 ton is satisfied by the EV on Path 3 arriving at 3.38 h in the same period. The SOC of the EV on Path 3 after reaching Receiving Point 11 is 26.72 km. Since the distance from Receiving Point 11 to Receiving Point 17 is 34.01 km, the EV cannot reach the subsequent receiving point (17). Thus, the EV must travel to battery swapping station BS1 to swap its battery.

Overall, the fixed priority mechanism effectively balances the vehicles' travel cost, battery swapping cost and the resource loss cost, and achieves a higher resource satisfaction rate compared with the no-priority scenario.

5.2. Comparison between cross-period and intraperiod scenarios

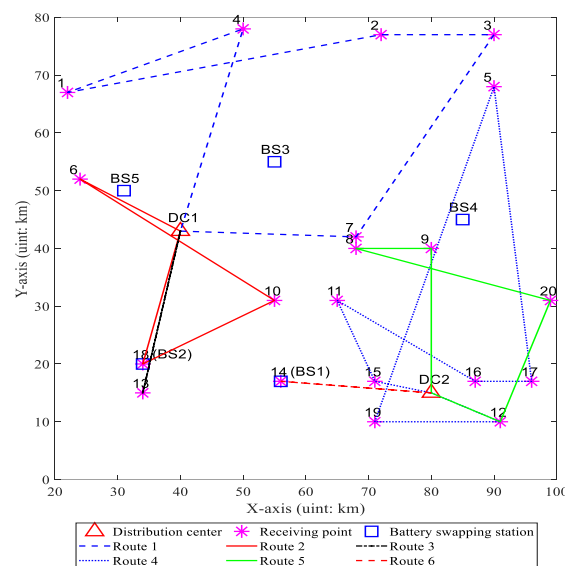
For the intraperiod distribution scenario in which the scarce resource demand of a receiving point can only be satisfied in one period, we run the program and list the cost items of distribution schemes in Table 5. Given the cost items of the cross-period scenario (i.e., M1) shown in Table 3, we can compare the distribution schemes of cross-period and intraperiod scenarios.

Table 5. Costs of distribution schemes for the intraperiod scenario.

	Z_1	Z_2	Z_3	Z_5	Z_6	Z
Best-found	1800	2400	17.882	1219.395	183.071	5620.348
Average	2010	2640	17.512	1266.203	196.193	6129.908

The main differences between the cross-period and intraperiod distribution schemes lie in the EV utilization cost and the battery swapping cost. First, in the intraperiod scenario, when the scarce resource demands of a receiving point cannot be met by one EV in one delivery, it is necessary to divide and meet the demand through multiple trips by one EV or multiple EVs in one period. For example, from the best-found EV travel path for the intraperiod scenario shown in Figure 3, we observe that the general scarce resource demand of Receiving Point 12 is divided into two batches. The first batch is distributed by the EV on Path 5 arriving at 0.20 h in the first period, and the second batch is distributed by the EV on Path 4 arriving at 7.43 h in the first period. Second, the EV usage times in the intraperiod scenario are probably larger than those in the cross-period scenario, which probably leads to greater swapping cost in the intraperiod scenario. For further comparison, we present the average and best-found SR of each kind of scarce resource obtained by the models under cross-period and intraperiod scenarios (see the second and fourth rows of Table 4). The average SRs of very and general scarce resources of the best-found distribution scheme in the cross-period scenario are higher than those of the best-found distribution scheme under the intraperiod scenario. Therefore, the cross-period distribution is better than the intraperiod distribution.

The cross-period distribution strategy breaks the one-period service restriction and optimizes EVs' usage frequency, thus obtaining better comprehensive performance than the intraperiod mode.

**Figure 3.** The best-found EV path of the intraperiod distribution scheme.

5.3 Comparison between hybrid time window and soft time window scenarios

In Model M1, hard and soft time windows are used for very and moderately scarce resources, respectively. The numerical results of cost items can be found in Table 3. We substitute the hard time window constraints of very scarce resources by the soft time window constraints to generate a new

model for the scenario of soft time window and obtain numerical results for the new model as shown in Table 6.

Table 6. Costs of the soft time window distribution scheme.

	Z_1	Z_2	Z_3	Z_5	Z_6	Z
Best-found	1219.467	2000	30.347	1866.425	186.699	5302.938
Average	1283.756	2360	30.931	1951.693	197.785	5824.165

The main differences between the two scenarios that have different time windows are listed as follows. First, for the soft time window scenario, EVs can violate the time windows and can choose the receiving point closest to the current receiving point. Therefore, the EV transportation cost of the soft time window distribution scheme is less than that of the hybrid time window distribution scheme (obtained by M1). Second, the loss cost of soft time window distribution scheme is significantly higher than that of hybrid time window distribution scheme. This can also be reflected by the SRs shown in the second and fifth rows of Table 4, which indicate that the 10 run-based average SRs and the best-found satisfaction rates of both very scarce resources and general scarce resources under the hybrid time window scenario are higher than those under the soft time window scenario.

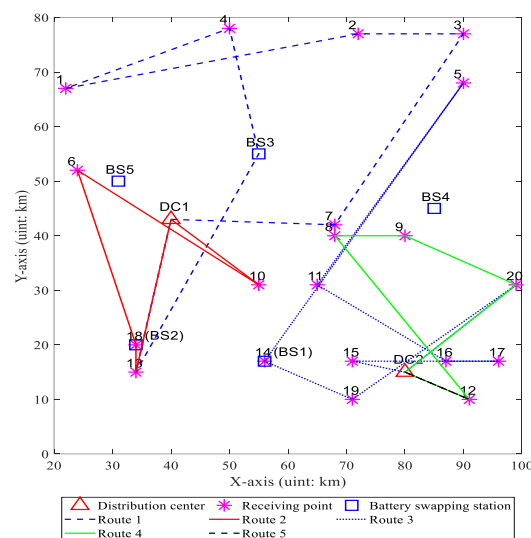


Figure 4. The best-found EV path of the soft time window distribution scheme.

For intensive analysis, we plot the best-found EV travel path for the soft time window scenario in Figure 4. Five EVs are used to distribute scarce resources. The EVs on Paths 1 and 2 belong to DC1. Path 1 is DC1–7–3–2–1–4–BS3–13–DC1, and Path 2 is DC1–13–18–6–10–DC1. The very scarce resource demand of Receiving Point 13 is totally satisfied by the EV of Path 1, and the moderately scarce resources of Receiving Point 13 are divided into two batches. The first batch, 0.3 ton, is delivered by the EV on Path 1 that arrives at Receiving Point 13 at 0.83 h in the second period. The second batch, 0.2 ton, is delivered through Path 2 by the EV arriving at 0.47 h in the first period. In addition, the SOC of the EV on Path 1 when reaching Receiving Point 4 is 25.407 km and the distance from Receiving Point 4 to Receiving Point 13 is 65 km, so the EV is not able to travel to the next receiving point (13) and it travels to battery swapping station BS3 to swap its battery.

The EVs on Paths 3, 4, and 5 belong to DC2. Path 3 is DC2–15–17–16–11–5–14 (BS1)–19–20–DC2, Path 4 is DC2–20–9–8–12–DC1, and Path 5 is DC2–12–DC2. The moderately scarce resource demand of Receiving Point 20 is totally met by the EV on Path 3, and the very scarce resources of Receiving Point 20 are split into 0.1 ton and 0.7 ton that are distributed by the EVs on Paths 3 and 4, respectively. The times the EVs on Paths 4 and 3 arrive at Receiver 20 are 0.41 h in Period 1 and 1.89 h in Period 2, respectively. The moderately scarce resource demand of Receiving Point 12 is also divided into 1.0 ton and 0.5 ton that are delivered by the EVs on Paths 4 and 5, respectively. The EV on path 5 arrives at Receiver 12 at 0.20 h in Period 1, and the EV on Path 4 arrives at Receiver 12 at 4.04 h in Period 1. In addition, the SOC of the EV on Path 3 when reaching Receiving Point 14 is 33.975 km. Although it is enough to travel to the next Receiving Point 19, it cannot travel from Receiving Point 19 to the next receiving point (20) (since the SOC of the EV when reaching Receiving Point 19 is 17.422 km). Therefore, the EV travels to battery swapping station BS1 to swap its battery.

To sum up, although the soft time window scheme cuts down transportation costs by allowing route flexibility, it leads to obvious increases in loss cost and lower resource satisfaction. The hybrid time window setting adopted in our model can well balance operational cost and service quality.

5.4. Comparison between scenarios with and without battery swapping stations

Whether EVs can swap their battery during the distribution process affects not only the receiving point sequence of a distribution path, but also the satisfaction rates of receiving points and the total cost of a distribution scheme. For the distribution scenario without battery swapping stations, we execute 10 runs and list the distribution costs in Table 7.

Table 7. Costs for delivery scheme without battery swapping stations.

	Z_1	Z_2	Z_3	Z_5	Z_6	Z
Best-found	1282.843	2400	17.738	1800	176.114	5676.695
Average	1278.046	2720	17.597	2040	184.684	6230.327

In Tables 3 and 5, we observed the difference between the two distribution scenarios with and without battery swapping stations. It can be seen from Z_2 that the EV utilization cost is greater for the scenario without battery swapping stations than the scenario with battery swapping stations. The reason is that if there is no battery swapping station, more EV trips are needed to meet the demands of all receiving points, since the travel distance of an EV is limited. At the same time, the total swapping cost of the EVs (Z_5) is greater for the scenario without battery swapping stations than the scenario with battery swapping stations, which is also because more use of EVs implies more swapping costs in distribution centers. Although the EV transportation cost under scenario without battery swapping stations is lower than that with battery swapping stations, the average total cost of the former is still higher than that of the latter. Moreover, we give the average SR for the situation of no battery swapping station, as shown in the second and sixth rows of Table 4. For very scarce resources or moderately scarce resources, the average and best-found SRs under the situation with battery swapping stations is higher than those under the situation without battery swapping stations.

We also present the best-found EV paths for the no battery swapping station scenario in Figure 5. EVs are sent out six times to distribute scarce resources. The EVs on Paths 1, 2, and 3 belong to one distribution center, and the EVs on Paths 4, 5, and 6 belong to the other distribution center. We can see that all EVs do not swap their batteries en-route since there is no swapping station. However,

if battery swapping stations are available in the distribution network, EVs can choose to swap their batteries to reduce the system-wide cost. For example, in the fixed priority-based cross-period hybrid time window distribution scheme in Section 5.1, the EV on Path 1 (i.e., DC1–7–3–2–1–4–BS3–13–DC1) enters the battery swapping station BS3 en-route to swap batteries.

The deployment of battery swapping stations effectively eliminates the driving range limitation of EVs. It reduces the overall operation cost and improves the resource SR, demonstrating the practical value of integrating battery swapping stations into the distribution network.

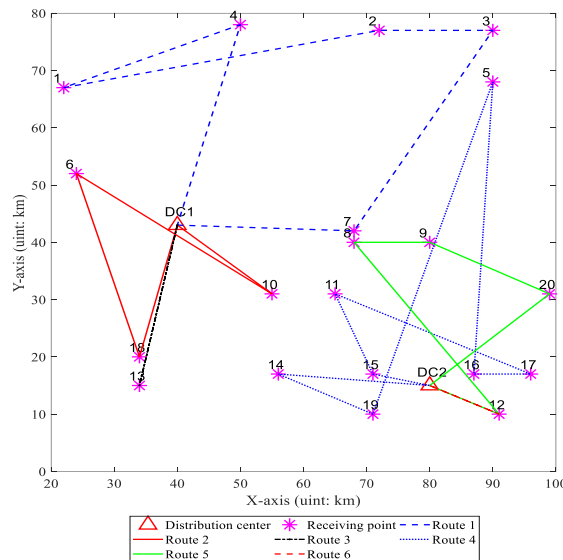


Figure 5. The best-found EV path of the distribution scheme without battery swapping stations.

5.5. Performance comparison of different distribution models

Alongside the above comparative analyses presented above, we also calculate and analyze the fixed priority-based cross-period hybrid time window distribution model with no region partition (the unpartitioned model) and the intraperiod soft time window distribution model without priority, region partition, or demand splitting (a simplified model without practical factors).

To observe the differences among all models involved in these analyses, we obtain the average and best-found total distribution cost and SR through 10 independent runs of C++ programs using the same parameters. The results are shown in Figure 6. The average of total cost of ten runs obtained by the fixed priority-based cross-period hybrid time window region-partitioned distribution model (Model M1) is only 5713.074 RMB. Compared with the Model M1, the total cost of the distribution scheme obtained by the no-priority cross-period hybrid time window region-partitioned distribution model (the no-priority model) is 6.42% higher, the total cost of the distribution scheme obtained by the fixed priority-based intraperiod hybrid time window region-partitioned distribution model (the intraperiod model) is 7.29% higher, the total cost of the distribution scheme obtained by the fixed priority-based cross-period soft time window region-partitioned distribution model (the soft time window model) is 1.94% higher, and the cost of the unpartitioned model is 17.61% higher; while the cost of the simplified model without practical factors is 2.95% higher. We should note that Model M1, the no-priority model, the intraperiod model, and the soft time window model are all based on the assumption that battery swapping stations are available in the distribution network. The total distribution cost obtained by the fixed priority-based cross-period hybrid time window region-partitioned distribution model without battery swapping stations (the no battery swapping

station model) is 9.57% higher than Model M1. Regarding the best-found distribution cost, from Figure 6, we can also observe similar comparative advantages of Model M1.

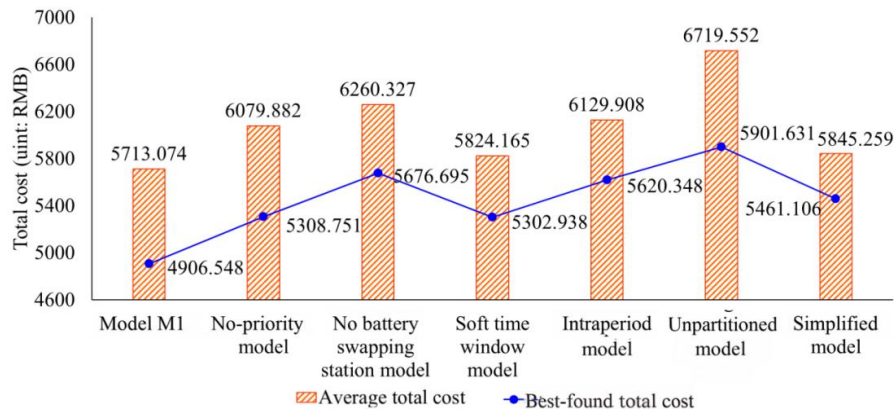


Figure 6. Total distribution costs obtained by different models.

The SRs of the models for all scenarios are presented in Figure 7. The SRs of very scarce resources and moderately scarce resources obtained by Model M1 are 80.05% and 81.03%, respectively, which are higher than those obtained by the no-priority model, the intraperiod model, the soft time window model, the no battery swapping station model, the unpartitioned model, and the simplified model without practical factors. Therefore, Model M1 is superior to the other models in terms of the total distribution cost and SRs of demand.

Comprehensive comparison results fully confirm that our proposed Model M1 outperforms other alternative models regarding total distribution cost and resource SR, which adequately demonstrates the advantages of incorporating multiple practical constraints within this framework.

Beyond the comparative analysis of different operation scenarios, we further discuss the sensitivity of several key parameters including the EV operating parameters, algorithm parameters, and priority threshold based on the existing experimental results. It can be found that the model maintains stable performance within the adopted parameter range, and the variation of a single parameter will not cause a sharp deterioration of the total distribution cost and resource SR, which verifies the robustness of the proposed model.

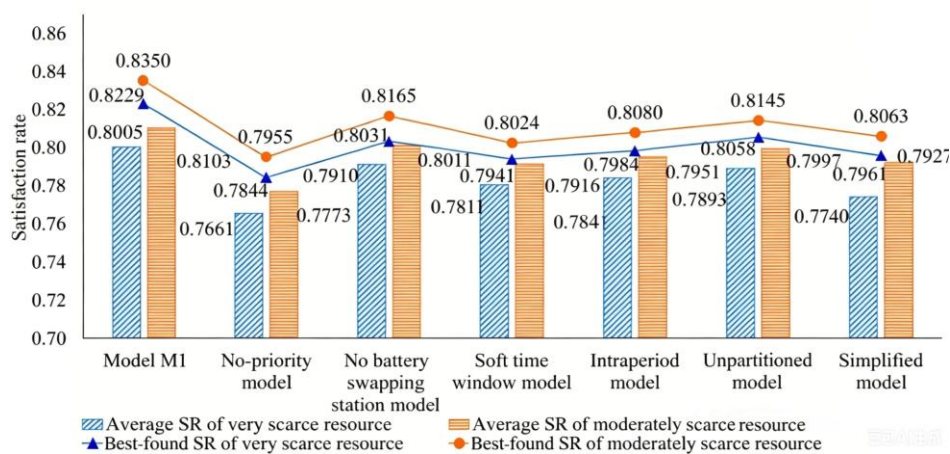


Figure 7. Satisfaction rates obtained by different models.

6. Distribution model based on adaptive dynamic priority

In the previous sections, we have verified the superiority of the proposed Model M1. This section proposes an adaptive dynamic priority and further improves M1 by integrating the proposed adaptive dynamic priority to generate a model (M2).

Different definitions of priority have been proposed, tailored to scenario characteristics or decision-makers' objectives, including region-based priority, item-based priority, customer-based priority, dynamic priority, etc. Generally, one receiving point gains service prior to another if its priority value is lower. However, if the priority is static, then a receiving point with a smaller priority value that has received enough scarce resources will still have superiority over those with greater priority values as time goes on. If the priority value of a receiving point that has received very few or even no scarce resources will remain larger in subsequent periods than those receiving points that have received many materials, then this is unfair for the former receiving point. Therefore, considering that the materials to be delivered is in short supply, all receiving points are eager for these materials, and the delivering process is dynamic, we propose an adaptive dynamic priority based on dynamicity, material scarcity, and receiving customers' requirements. Dynamicity is depicted by the cumulative demand SR and arriving time at a receiving point. Material scarcity and customer requirements are primarily captured by the initial priority and the customer's eagerness to be served as early as possible. Given a priority u_i of receiving point i for the initial period, the adaptive dynamic priority u_i^p is defined as follows:

$$u_i^p = \left\{ u_i \left(1 + \frac{g_i^{sp} + g_i^{np}}{2} \right) \right\}^{\frac{1}{2} \left(\frac{1}{ST_i^{sk}} + \frac{1}{ST_i^{nk}} \right)}, \forall i \in SC, p \in P. \quad (37)$$

Formula (37) indicates that the higher the cumulative SR, the lower the new priority; however, the later the arrival time, the higher the new priority.

To adjust the proposed Model M1, we use typical kind of priority constraint (4) for the first time period. At the same time, we define a kind of ε -adaptive dynamic priority constraint as follows:

$$u_j^p < u_i^p + \omega(1 - x_{ij}^{kp}) + \varepsilon, \text{ if } \frac{g_j^{sp} + g_j^{np}}{2} > \frac{g_i^{sp} + g_i^{np}}{2}, \forall i, j \in SC, k \in K, p \in P, \quad (38)$$

combined with the fixed priority constraint

$$u_i^p > u_j^p + \omega(1 - x_{ji}^{kp}), \text{ if } \frac{g_i^{sp} + g_i^{np}}{2} > \frac{g_j^{sp} + g_j^{np}}{2}, \forall i, j \in SC, k \in K, p \in P \quad (39)$$

for other time periods. In Formula (38), ε is a positive number. Formula (38) indicates that a receiving point will be served in priority if it has a lower cumulative SR and a lower value of adaptive dynamic priority. The difference between the larger adaptive dynamic priority value and the smaller one is less than ε . Formula (39) ensures that a receiving point is served in priority if it has a lower cumulative SR and a lower value of fixed priority.

Since in case of material shortage, all receiving points are eager to obtain scarce resources, we restrict the demand SR of each type of scarce resources in the first period to be not greater than a threshold, e.g., 50%.

$$\sum_{k \in K_d} m_i'^{skp} \leq 50\% m_i^s, \forall i \in SC, p = 1 \quad (40)$$

$$\sum_{k \in K_d} m_i'^{nkp} \leq 50\% m_i^n, \forall i \in SC, p = 1 \quad (41)$$

According to (40) and (41), there will be as many receiving points as possible to obtain materials in the initiate period. Constraints (38)–(41) can guarantee that as many receiving points as possible can be served as early as possible.

Therefore, we propose a new model based on the mixture of ε -adaptive dynamic priority and fixed priority:

M2: (1)–(29), and (37)–(41).

To evaluate the fairness among receivers for the proposed models, we apply an inequity measure [34]. Let $\mathcal{G}_i^s = \sum_{k \in K} \sum_{p \in P} m_i'^{skp} / m_i^s$, $\mathcal{G}_i^n = \sum_{k \in K} \sum_{p \in P} m_i'^{nkp} / m_i^n$ be the demand satisfaction of very scarce and moderately scarce resources for receiving point $i \in SC$. If we let $\mathcal{G}_i = (\mathcal{G}_i^s + \mathcal{G}_i^n)/2$ and $\bar{\mathcal{G}} = \sum_{i \in SC} \mathcal{G}_i / |SC|$, then we define an inequity measure as the standard deviation of demand satisfaction of all receiving points, i.e., $Ine = \sqrt{\sum_{i \in SC} (\mathcal{G}_i - \bar{\mathcal{G}})^2 / |SC|}$.

We consider three models here: M1, M1 without priority, and M2. M1 is the model with fixed priority. M2 is the model with the mixture of ε -adaptive dynamic priority and fixed priority. For numerical tests in this section, we set all parameters to be the same as those in Section 5 except for $\varepsilon = 0.249055$, $T = 24$ h, and some receiving points' hybrid time windows listed in Table 8.

Table 8. Hybrid time windows.

$i \in SC$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
e_i^s/h	1	1	0	2	1	0	0	0	1	0	0	0	4	4	0	1
l_i^s/h	10	14	15	10	17	18	16	19	15	17	17	ω	14	12	18	16
e_i^n/h	1	4	4	0	6	0	6	0	0	5	5	0	0	0	3	8
l_i^n/h	7	8	8	13	10	12	10	10	12	7	9	9	10	10	7	12

The inequities of the best-found distribution schemes obtained for the three models are shown in Table 9. From the perspective of both the value of Ine obtained by each run and the average value of Ine , we observe that the ε -adaptive dynamic priority-based model, M2, is the best regarding equity. Take the average value of Ine for example. The equity of the best-found distribution scheme obtained by M2 is 26.71% greater than that obtained by M1, and is 43.61% greater than that obtained by M1 without priority.

Table 9. Inequity of distribution plans.

Run	M1	M1 without priority	M2
1	0.349	0.441	0.229
2	0.349	0.445	0.232
3	0.280	0.399	0.218
4	0.349	0.377	0.223
5	0.286	0.286	0.218
6	0.349	0.433	0.220
7	0.284	0.445	0.233
8	0.255	0.392	0.230

Continued on next page

9	0.285	0.427	0.222
10	0.282	0.349	0.224
Average	0.307	0.399	0.225

The proposed adaptive dynamic priority mechanism effectively reduces the service inequity among all receiving points. Compared with the fixed priority and priority-free schemes, model M2 achieves a notable improvement in distribution fairness while maintaining favorable operational performance.

7. Conclusion

Amid the increasingly frequent natural disasters and public health emergencies, shortages of food, medicine, and daily supplies commonly occur, making research on scarce material distribution highly valuable. Driven by carbon neutrality and carbon peaking goals, EVs have been widely adopted in modern logistics. This paper investigates the cross-period routing and allocation problem for distributing multitype scarce resources with EVs and establishes two distribution models to address the problem.

The proposed models integrate multiple practical factors, including multitype scarce resources, cross-period distribution, receiving points' priority, hybrid time window, the demand and supply split, region partition, and en-route battery swapping. These models describe the complex interactions among these factors and their influence on the best-found routing and allocation schemes. Since the models are NP-hard, we propose a GA-based solution method to solve the model. We perform numerical experiments and analyses for some situations, and the results verify the superiority of our methods. Experimental outcomes demonstrate that the fixed-priority model achieves lower operational costs and higher demand SRs. Meanwhile, the newly designed adaptive dynamic priority mechanism effectively improves service fairness among all receivers compared with the fixed-priority mechanism.

Regarding applicability, the proposed models are well suited for emergency material distribution under prolonged resource scarcity, especially for logistics networks equipped with battery swapping stations and facing multiperiod delivery demands. As for limitations, the current models do not take uncertain factors into account, such as uncertain supply, dynamic road conditions, and fluctuating on-site demand during actual distribution.

Although we have considered the dynamic split of demand and cross-period scheduling, all parameters and demands are set as deterministic in this study. For future research, we will focus on distribution optimization under fully dynamic and uncertain demand ^[35] and resilient supply ^[36]. In addition, we will carry out large-scale case verification and systematic parameter sensitivity analyses to further enrich and improve the proposed models.

Author contributions

Minghua Zeng: Conceptualization, methodology, supervision, writing – original draft, writing – review and editing, funding acquisition, project administration; Yiwei Cui: Investigation, data curation, software, validation, visualization, writing – original draft; Rong Zhang: Visualization; Yanbing Wang: Resources; Ni Dong: Investigation, writing – review and editing, funding acquisition; Jin Zhang: Formal analysis, project administration.

Use of Generative-AI tools declaration

No generative artificial intelligence tools were used during the preparation of this manuscript. All modeling, simulation, writing, and revision work was completed independently by the authors.

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Conflicts of interest

We declare that we have no conflicts of interest.

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