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Theory article

Pricing and green innovation for national and private brands on the e-retail platform

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Abstract: This study examines pricing and green innovation decisions in an e-retail platform supply chain where a national brand (NB) competes directly with a platform-owned private brand (PB). Using a Stackelberg game framework, we analyze four scenarios: No innovation, NB-only innovation, PB-only innovation, and bilateral innovation. The key findings are as follows. Network externalities consistently benefit all supply chain members and enhance green innovation levels across scenarios, but their impact on retail prices is scenario-dependent. The NB manufacturer achieves its highest profit under NB-only innovation, whereas the e-retail platform prefers bilateral innovation, which constitutes the unique Nash equilibrium. This misalignment creates a green innovation prisoner's dilemma: Although bilateral innovation maximizes the total supply chain profit, the manufacturer earns less than under NB-only innovation. Moreover, standalone green innovation by either party yields higher green levels than bilateral innovation. These findings suggest that coordinating bilateral green innovation may require profit-sharing mechanisms, and that innovation strategies should be tailored to the market conditions.

Keywords: platform supply chain; pricing; private brand; green innovation; network externality
Mathematics Subject Classification: 91A65, 91A10, 91A80

1. Introduction

The development of private brands (PBs) has a long-standing history in the traditional retail sector. For instance, Walmart's "Huiyi" and Boxmart's "Boxmax" have successfully enhanced consumer loyalty through differentiated positioning and quality improvements. In recent years, e-retail platforms have similarly embraced PB strategies. Amazon has launched Amazon Basics and Solimo; Alibaba has developed its own branded products through channels such as Tmall Supermarket and Taobao Xinxuan; and JD.com has introduced PBs including J.ZAO and JIA BEST, covering a wide range of product categories. These initiatives not only improve the profitability and user loyalty of e-retail platforms but also enable them to respond more accurately to evolving market demands.

However, the rise of PBs has fundamentally reshaped the relationship between e-retail platforms and NB manufacturers, giving rise to what the literature terms a “coopetitive supply chain” [1]. This relationship is inherently dualistic: On the one hand, e-retail platforms compete directly with NB manufacturers in the product market through their PBs, exerting both price and channel pressure; on the other hand, platforms continue to rely on NB manufacturers for production capacity, technological expertise, and brand equity to maintain category diversity and ensure the supply chain’s stability. This coopetitive dynamic compels both parties to seek equilibrium across multiple dimensions, including product differentiation, pricing strategies, supply chain efficiency, and data sharing.

Among various approaches to product innovation, green innovation is widely regarded as an effective strategy for mitigating brand competition. Empirical evidence supports this view: Studies show that 57% of European consumers and 75% of US consumers are willing to pay a premium for eco-friendly products [2]. Governments are also accelerating the development of green innovation through policy initiatives. China’s “Action Plan for Energy Conservation and Carbon Reduction 2024–2025” [3] emphasizes the synergistic development of green supply chains; the United States incentivizes emissions reductions through its Paris Agreement commitments [4] and carbon market regulations; and the European Union’s “Fit for 55” package [5] establishes stringent emission reduction targets for 2030. Driven by both policy support and growing green market demand, NB manufacturers and e-retail platforms alike have strong incentives to invest in green product innovation as a means of market expansion. Industry practice reflects this strategic shift. For instance, H&M has invested in low-carbon textile technologies [6], while Uniqlo encourages its suppliers to adopt organic cotton and recycled fibers[1]. In the e-retail sector, Amazon has optimized its packaging to reduce plastic usage[2], and JD.com’s PB J.ZAO incorporates eco-friendly materials and streamlined packaging in products such as LED desk lamps, effectively balancing functionality with sustainability. These cases also reveal an underlying conflict: While national brand manufacturers invest in green innovation to differentiate themselves and command higher wholesale prices, e-retail platforms often use their PBs—which may not bear the same innovation costs—to undercut NBs’ products on price, capturing green-sensitive consumers without making comparable investments. This tension directly motivates our modeling of green innovation competition between the NB and the PB.

However, despite the growing importance of green innovation in platform supply chains, existing research suffers from three notable gaps. First, most studies focus on unilateral green innovation by national brand manufacturers, largely neglecting that e-retail platforms—after introducing PB—also have strong incentives to engage in green innovation for their PBs’ products. Second, although network externalities are a defining feature of e-retail platform economies, their

[1] Retrieved October 5, 2024, from https://www.sohu.com/a/618734146_121123735.

[2] Retrieved October 5, 2024, from https://news.sohu.com/a/787445316_121823407.

impact on green innovation decisions under PB and NB competition has yet to be systematically investigated. Third, the existing literature rarely compares multiple innovation scenarios (no innovation, unilateral innovation by either party, and bilateral innovation) within a unified framework, leaving the strategic interactions and equilibrium outcomes unclear. This paper aims to fill these gaps.

Against this backdrop, the rapid development of a PB on e-retail platforms has intensified competition with the NB within the same platform ecosystem. In this increasingly complex market environment, both e-retail platforms and NB manufacturers may enhance their market competitiveness through green product innovation. Consequently, supply chain members must carefully coordinate both pricing and green innovation decisions. This study seeks to explore (i) the factors that influence green innovation levels and pricing decisions for different brands' products, and (ii) how supply chain members can leverage green innovation to enhance competitiveness and achieve profitability growth. Specifically, this paper addresses the following research questions.

(1) Compared with the baseline scenario where neither brand engages in green innovation, how do equilibrium outcomes change when only one party engages in green innovation or when both do so simultaneously?

(2) How do intensity of competition, green awareness, and innovation efficiency moderate these outcomes, particularly in the presence of platform network externalities?

(3) What are the optimal green innovation strategies for the e-retail platform and the NB manufacturer? What is the Nash equilibrium of the game, and under what conditions can a win–win outcome be achieved?

The corresponding theoretical contributions and practical values of answering these questions are threefold. First, by incorporating bilateral green innovation decisions, our study extends the green supply chain literature to the platform context, offering a more complete picture of innovation incentives. Practically, this helps e-retail platforms and NB manufacturers understand when bilateral innovation is mutually beneficial. Second, by explicitly modeling network externalities, we reveal how platform-specific features shape green innovation outcomes, providing managers with thresholds to adjust their strategies according to market conditions. Third, through a comprehensive comparison of four scenarios, we identify a green innovation prisoner's dilemma and propose a green innovation cost-sharing contract to resolve it, offering actionable guidance for coordinating green innovation in e-retail platform supply chains.

To address these questions, we develop a game-theoretic model involving a NB manufacturer (NB) and an e-retail platform that has introduced a PB product (PB). The model is built on a rigorous analytical framework. To ensure the existence and uniqueness of the equilibrium solutions, we impose parameter constraints derived from the negative definiteness of the Hessian matrices for each scenario (e.g., $\eta > \eta_0$, as specified in Section 3.2). All equilibrium solutions are derived via backward induction, and the concavity of the profit functions is verified for each scenario. Within this framework, the products of NB and PB compete directly on the same e-retail platform. We assume that both brands offer functionally similar products within the same category and consider four distinct green innovation scenarios on the basis of which the entities engage in green innovation: (i) neither NB nor PB product engages in green innovation (Scenario N); (ii) both NB and PB products engage in green innovation (Scenario B); (iii) only NB engages in green innovation (Scenario M); and (iv) only the PB product engages in green innovation (Scenario E). In this Stackelberg game, the e-retail platform acts as the leader, while the NB manufacturer serves as the follower. This leader—follower structure reflects the reality that major e-retail platforms (e.g., Amazon, JD.com) possess greater market power and data advantages than individual national brand manufacturers. The rationality assumption applies to both parties, each maximizing its own profit. Both parties are assumed to be rational decision-makers aiming to maximize their own profits.

Our research yields several interesting findings. Collectively, these findings reveal the strategic interactions between green innovation and platform-based competition, and highlight the need for coordination mechanisms to align individual and collective incentives.

First, stronger network externalities consistently benefit manufacturers' profit, wholesale price, and e-retail platform revenue, yet their impact on retail pricing is highly scenario-dependent. Specifically, when network externalities increase, the following occur: In Scenario N, both brands' retail prices decline; in Scenario B, both brands' prices rise only under weak competition and high green efficiency; in Scenario M, the platform consistently reduces PB's retail price but raises NB's retail price only under high green efficiency; in Scenario E, the platform consistently reduces NB's retail price but raises PB's retail price only under high green efficiency. The effects of other parameters also exhibit pronounced scenario dependence. (i) In Scenario N, intensified competition always harms the manufacturer's profit, while the e-retail platform benefits only in larger markets. (ii) In Scenario B, competition consistently erodes the manufacturer's profit, with the platform benefiting only in larger markets. Green preference boosts the platform's profit universally but benefits the manufacturer only within certain thresholds. (iii) In Scenario M, heightened competition continues to undermine the manufacturer's profit, while increased green preference can yield a win–win outcome. (iv) In scenario E, competition still impairs the manufacturer's profit, with the e-retail platform gaining only in larger markets; greater green preference enables the e-retail platform to profit at the manufacturer's expense.

Secondly, the manufacturer consistently favors Scenario M, while the platform prefers Scenario B, revealing an asymmetric equilibrium. Scenario B, in which both NB and PB undertake green innovation, constitutes the game's sole Nash equilibrium, creating a green innovation prisoner's dilemma: Although Scenario B maximizes total supply chain profits, the manufacturer earns less than under Scenario M.

Finally, standalone green innovation by either NB or PB more readily achieves high levels of green innovation. Under Scenarios M or E, green innovation levels significantly surpass those under Scenario B, as differentiated monopoly premiums cover the innovation costs. Conversely, simultaneous innovation (Scenario B) leads to homogenization of green attributes, suppressing investment in both brands' products.

The remainder of this paper is organized as follows. Section 2 reviews the extant literature on platform-based PBs, supply chain pricing, and green supply chain management. Section 3 presents the models' setup and assumptions, and constructs the game framework for the four green innovation scenarios. Section 4 derives the equilibrium solutions for each scenario and analyzes the impacts of key parameters. Section 5 compares the equilibrium prices, demands, and green innovation levels between NB and PB products, and examines the green innovation policy preferences of platform-based supply chain members, and then proposes a green innovation cost-sharing mechanism to resolve the green innovation dilemma. Finally, Section 6 summarizes the research findings and discusses the managerial implications as well as directions for future research. All proofs are provided in the Appendix.

2. Literature review

This study is related to three streams of literature: Platform-based PBs, supply chain pricing, and green innovation.

2.1. Platform-based private brands

With the evolution of e-retail platforms in recent years, PBs have become a research focus in fields such as supply chain management and marketing. The extant literature relevant to this study primarily revolves around three themes: the motivations and consequences of the PB's introduction, NB manufacturer's strategies in response to PB competition from the private brand, and the impact of network externalities on supply chain decisions, which are a defining feature of platform economies.

First, several scholars have examined retailers' motivations for introducing PBs and the subsequent impact on supply chain performance. Mills et al. [7–9] suggested that retailers' incentives to develop PBs stem from improved financial performance (e.g., higher margins), marketing objectives (e.g., capturing greater market share), and operational benefits (e.g., reducing procurement costs for NBs). According to an analysis of the practical conditions for implementing retailers' PB strategies, Wang et al. [10] distinguished between traditional Original Equipment Manufacturer (OEM)-based branding, emerging OEM-based branding, and PBs, offering recommendations for advancing PB strategies. Shan et al. [11] explored the impact of high-end PBs on retailers' competitive capabilities, providing corresponding evaluations and future research directions. Li et al. [12] examined the impact of introducing a PB on the performance of both manufacturers and retailers, revealing that while PB increases retailer profit and its profit share, it harms the manufacturer's profitability. Fan et al. [13] studied the impact of introducing PBs of varying quality levels on supply chain member's performance, concluding that regardless of the quality level, introduction of a PB damages the manufacturer's profits. Collectively, these studies demonstrate that the introduction of platform-based PBs exerts significant market pressure on the upstream manufacturers, fundamentally reshaping the coopetitive relationship between platforms and manufacturers.

Second, manufacturer's response strategies to retailers' PB competition constitute a central focus of academic inquiry. Szymanowski et al. [14] considered that retailers may sell their PBs through competing outlets, found that consumers may not be concerned about which retailer is the origin a particular PB. Manzur et al. [15] suggested that manufacturers can use advertising strategies to counter the impact of retailers introducing a PB. Nasser et al. [16] proposed three response strategies for manufacturers facing retailers' PBs: nonresponse, becoming a PB manufacturer, and introducing new products. Lu et al. [17] studied the effects of manufacturer's advertising investment and advertising cost-related information sharing on retailers' PBs. Amrouche et al. [18] examined how NB manufacturers should respond to PBs and found that revenue sharing may benefit the overall supply chain and the manufacturer, but may harm the retailer. Shen et al. [19] discussed whether manufacturer advertising can deter a retailer from introducing a PB and found that advertising is a dominant strategy for manufacturers and may simultaneously increase retailers' profits.

Third, as a core feature of platform economies, network externalities exert profound influence on decision-making throughout the platform supply chain. Guo et al. [20] examined how network externalities drive the adoption of blockchain technology on secondhand goods trading platforms. Shen et al. [21] confirmed that enhanced network externalities positively impact market expansion and value creation for e-commerce enterprises. In the digital product domain, Li et al. [22] explored the critical influence of online platform network externalities on digital content providers' channel strategies (direct sales or platform distribution). Shen et al. [23] further examined the impact of network externalities on advertising strategies and members' profits within platform-based supply chains, noting that manufacturers' fairness concerns may mitigate the positive effects of network externalities. Zheng et al. [1] used evolutionary game theory to model the strategic interactions and evolutionary paths between platforms and suppliers under varying market uncertainties. Numerical simulations confirmed the significant impact of factors such as network externalities on equilibrium outcomes. Cui [24] demonstrated that network externalities indirectly influence pricing and service decisions in e-retail platform supply chains by affecting sales volume. Furthermore, he found that enhancing sales service levels effectively attracts consumers, thereby boosting the system's profitability. These studies indicate that network externalities not only affect platforms' user bases and market scales but also profoundly shape supply chain members' pricing strategies, channel choices and innovation incentives.

2.2. Supply chain pricing

Supply chain pricing serves as a core reflection of e-retail platform operations and brand competition. In the context of e-retail platforms introducing PBs, the allocation of pricing power and the complexity of pricing strategies have increased substantially. Several studies examine the relationship between the timing of PB introduction and pricing strategies. Li et al. [25] systematically compared the differential impacts of uniform pricing versus price discrimination strategies on market competition, brand positioning, and supply chain members' profits following the introduction of private labels by retail platforms, using analytical modeling. Another stream of research delves into the interplay between specific PB introduction models and the dynamics of pricing power. Lyu et al. [26] analyzed e-retail platforms' decisions to adopt the consumer-to-manufacturer (C2M) model for private brand launches, finding its advantages in precisely matching consumer demand and reducing brand mismatch risks. These advantages directly translate into enhanced market pricing power. Huang et al. [27] demonstrated that the potential market size of the platform's PB, order fulfillment costs, and the intensity of competition between NBs and PBs collectively form a complex decision-making environment. This environment ultimately shapes the pricing structure across the entire supply chain by influencing the choice of sales model, which determines the allocation of pricing power. Zhao et al. [28] provided an insightful analysis within reselling and agency models, examining the strategic interaction between platforms' adoption of small-to-medium brand strategies and the innovation strategies of these brands' manufacturers. Their work profoundly reveals how the allocation of pricing power under different models affects firms' innovation incentives and profit margins.

2.3. Green supply chain management

With the growing emphasis on low-carbon and sustainable development, green supply chain management has attracted increasing attention from both academia and industry. The literature relevant to this study can be broadly classified into two streams: The first stream examines internal green cooperation mechanisms among supply chain members, while the second stream focuses on how external environmental factors influence green innovation decisions.

A substantial body of research investigates how green cooperation mechanisms, including carbon emission reduction agreements, cost-sharing strategies, and revenue-sharing contracts, affect supply chain performance. Xia et al. [29] examined how manufacturers can design contracts to ensure profitability and environmental compliance when retailers possess private information about their green marketing efforts and actual market size, and proposed carbon emission reduction contracts for both decentralized and centralized supply chains. Peng et al. [30] investigated the green innovation decisions of supply chain enterprises when different entities undertake corporate social responsibility (CSR) under random demand for green products, Hafezi et al. [31] analyzed two distinct cooperation strategies, specifically innovation sharing and investment sharing, alongside a non-cooperative approach of independent development, and explored how coopetition influences pricing and green innovation investment for green products. Motivated by the practical challenge of stalled green transformation in retailer-led supply chains where the suppliers bear the green innovation costs independently, Yenipazarli et al. [32] systematically investigated the roles of cost-sharing and revenue-sharing mechanisms in coordinating upstream and downstream innovation investments. Hong et al. [33] studied a two-echelon supply chain comprising manufacturer-associated green production and retailer green marketing, comparing three cooperation modes: Price contracts, green marketing cost-sharing contracts, and two-part tariff contracts. Wei and Wang [34] examined the interaction between supply chain enterprises' carbon reduction technology innovation and government intervention under decentralized decisions without cost sharing, decentralized decisions with cost sharing, and centralized decisions respectively. Liu et al. [35] introduced behavior-based pricing strategies based on purchase history into green supply chains, examining pricing mechanisms under centralized and decentralized decision-making. Yang et al. [36] investigated green R&D cooperation models among supply chain members, and found that revenue-sharing contracts offer greater flexibility in supply chain coordination.

Another stream of literature focuses on how external environmental factors, such as supply chain structure, capital constraints, and information sharing, influence supply chains' green innovation decisions. Shi et al. [37] examined the choice of green technology investment entity in a competitive supply chain comprising one manufacturer and two retailers. Du et al. [38] investigated pricing and environmental innovation decisions when none, one, or both manufacturers implement green product innovation. Xia et al. [39] studied optimal financing strategies and green innovation decisions under capital constraints in a low-carbon supply chain with chain-to-chain competition. Zhang et al. [40] explored how competing manufacturers' green innovation strategies affect retailers' preferences for demand information sharing, identifying the number of manufacturers engaging in green innovation, competition intensity, market uncertainty, and innovation efficiency as key determinants of retailers' information-sharing preferences. Furthermore, Liu et al. [41] examined the impact of consumers' environmental awareness on supply chain members under the dual effects of product competition and retail competition. Zhu et al. [42] investigated how the supply chain structure (centralized vs. decentralized), product type (R&D-intensive vs. marginal cost-intensive), and competition type (price competition vs. green competition) influence product-related greenness decisions in competitive environments. These studies indicate that network externalities not only affect e-retail platforms' user bases and market scales but also profoundly shape supply chain members' pricing strategies, channel choices, and innovation incentives.

In summary, while the abovementioned streams of literature have provided valuable insights, they share three common limitations.

First, although the literature on platform-based PBs has extensively studied the motivations for PB introduction and NB manufacturers' response strategies (Section 2.1), and the literature on green supply chain management has examined various green cooperation mechanisms and external environmental factors (Section 2.3), these two streams have largely developed in parallel. Specifically, green innovation is predominantly treated as a unilateral strategy of NB manufacturers, while the potential for e-retail platforms to engage in green innovation for their own PBs is rarely considered. This omission is significant because, as documented in our Introduction (e.g., JD.com's J.ZAO using eco-friendly materials), platforms invest in green innovation in practice. By neglecting platform-led green innovation, existing studies fail to capture the full spectrum of strategic interactions between NB and PB in the green context.

Second, network externalities—despite being a defining feature of platform economies and having been shown to significantly impact pricing, channel selection, and user acquisition (see Section 2.1)—are rarely incorporated into the analysis of green innovation competition. Most existing green supply chain models (e.g., [38,40,42]) are developed in traditional retail settings where network externalities are absent or irrelevant. When platforms introduce PBs and compete with NB manufacturers on green innovation, however, network externalities may fundamentally alter the returns to green investment by amplifying market expansion effects or shifting consumer demand. Ignoring this platform-specific factor thus limits the applicability of existing green innovation findings to e-retail platform contexts.

Third, while several studies have compared green innovation strategies under different supply chain structures (e.g., [38,42]), they typically focus on whether manufacturers should innovate or not. A systematic comparison of multiple innovation scenarios—no innovation, unilateral innovation by either party (manufacturer-only or platform-only), and bilateral innovation—within a unified game-theoretic framework is lacking in the existing literature. As a result, the strategic interactions between the NB manufacturer and the e-retail platform regarding green innovation, the equilibrium outcomes under different scenarios, and the conditions under which bilateral innovation is mutually beneficial remain unclear. Without such a unified framework, it is difficult to determine whether and when both parties should invest in green innovation simultaneously, or whether unilateral innovation by one party might dominate.

These three limitations collectively define the research gap that this paper addresses. The first gap establishes that both parties—not just the manufacturer—can potentially engage in green innovation. The second gap identifies network externalities as a platform-specific factor that must be explicitly modeled to obtain valid insights for e-retail settings. The third gap highlights the need for a unified framework that compares all relevant innovation scenarios to understand strategic interactions and equilibrium outcomes. By addressing these gaps simultaneously, our study provides a more complete and realistic analysis of green innovation in e-retail platform supply chains than existing literature.

The innovations and contributions of our study are as follows. First, we develop a game-theoretic framework that captures bilateral green innovation decisions in a platform supply chain in which an e-retail platform with a PB competes directly with a NB manufacturer. Unlike prior studies that treat green innovation as a manufacturer-only strategy, our framework allows both the NB manufacturer and the e-retail platform to decide whether to invest in green innovation for their respective products. Second, we explicitly incorporate network externalities into the analysis of green innovation competition, examining how this platform-specific feature influences green innovation incentives and equilibrium outcomes. This distinguishes our work from existing green supply chain models that operate in traditional retail settings without network effects. Third, we investigate the joint determination of pricing and green innovation strategies under four distinct innovation scenarios (N, B, M, E), providing insights into the strategic interactions between the NB manufacturer and the e-retail platform in the context of green innovation. To the best of our knowledge, this study is the first to systematically compare all four scenarios within a single coherent Stackelberg framework for an e-retail platform supply chain with PB competition.

3. Model

3.1. Problem description

The e-commerce supply chain system described in this paper involves products from two brands (NB and PB) being sold on the same e-retail platform. Similar supply chain structures are described in the literature of Huang et al. [43] and Liu et al. [44]. In reality, the power structure in a supply chain is not fixed; it can be influenced by multiple factors such as market power, brand influence, and negotiation ability. In some cases, downstream retailers may become the supply chain leader, while the upstream supplier brands may also become the leader. Moreover, channel power affects the final profit allocation. Given the rapid development of information networks and e-commerce platforms, third-party e-retail platforms have easier access to market demand and end consumers. Therefore, the power structure considered in this study is that the downstream e-retail platform is the leader, and the upstream traditional brand manufacturer is the follower.

In this paper, we consider four distinct scenarios based on which party (or parties) engages in green innovation. These scenarios are designed to capture the full range of strategic possibilities in the platform-based supply chain and to isolate the effects of green innovation from different sources. Below, we describe each scenario in detail.

(1) Scenario N (no innovation): Neither the NB manufacturer nor the e-retail platform invests in green innovation. Both NB and PB products are conventional (non-green) products. This scenario serves as a benchmark for comparing the effects of green innovation, as it represents the baseline case where no party bears any green innovation cost, and consumer green preference does not affect demand.

(2) Scenario B (bilateral innovation): Both the NB manufacturer and the e-retail platform invest in green innovation for their respective products. Both NB and PB products are green, but their green attributes are differentiated. Each party bears its own innovation cost. This scenario represents a green competition situation where both brands seek to capture green-sensitive consumers through their own innovation efforts.

(3) Scenario M (manufacturer-only innovation): Only the NB manufacturer invests in green innovation for its NB product. The e-retail platform does not invest in green innovation for its PB product. In this scenario, the NB product gains a green differentiation advantage over the PB product. The manufacturer bears the full innovation cost, while the platform free-rides on the green attribute of the NB product indirectly through its pricing strategy.

(4) Scenario E (platform-only innovation): Only the e-retail platform invests in green innovation for its PB product. The NB manufacturer does not invest in green innovation for its NB product. In this scenario, the PB product gains a green differentiation advantage over the NB product. The platform bears the full innovation cost, while the manufacturer competes with a green PB without any green investment of its own.

In the wholesale model, the game proceeds as follows.

Stage 1: The platform and manufacturer simultaneously decide whether to pursue green innovation.

Stage 2: The platform sets retail prices for NB and PB products; if the platform is innovating, it also decides the PB product's green innovation level.

Stage 3: The manufacturer sets NB product's wholesale price, and if the manufacturer is innovating, it must also decide on the level of green innovation for the NB product.

Stage 4: Consumers purchase according to the price and green innovation level; demand functions determine sales.

Key assumptions include the following: The e-retail platform fully controls the PB products' supply chain; NB and PB products are functionally comparable; green innovation refers to environmental improvements via technology, following Zhang et al. [40], Liu et al. [44] and Zhang et al. [45]; the wholesale model is dominant, with the e-retail platform as the Stackelberg leader and the NB manufacturer as follower, which is consistent with platforms like Amazon and JD.com [40,42]; and despite market competition, the two parties cooperate upstream/downstream, thus considering competition intensity, consumer green preference, green innovation efficiency, and network externalities in their innovation and pricing decisions.

In the supply chain ecosystem considered in this paper, the e-retail platform, as the Stackelberg leader, actively maintains the equilibrium between NB and PB products through the following three mechanisms.

The first is price coordination. The platform simultaneously sets the retail prices of both NB and PB products. As shown in our equilibrium analysis (Section 4), the platform adjusts the relative prices of the two brands to balance their market shares and mitigate excessive price competition. For example, in Scenario M, the platform deliberately reduces the PB retail price to make it appear more affordable, thereby protecting NB demand while maintaining PB sales. The second is green innovation investment control. The platform not only decides the green innovation level of its own PB product but also influences the manufacturer's green innovation incentives through its pricing and wholesale price strategies. In Scenario B, where both brands innovate, the platform's pricing power allows it to capture a larger share of the green premium, which, in turn, affects how much the manufacturer is willing to invest in green innovation. The third involves coordination mechanisms. When bilateral innovation (Scenario B) leads to a prisoner's dilemma—where the manufacturer earns less than under Scenario M—the platform can offer a cost-sharing contract (as detailed in Section 5.5) to compensate for part of the manufacturer's green innovation cost. By doing so, the platform aligns the manufacturer's incentive with the ecosystem's collective interest, achieving a stable equilibrium where both parties invest in green innovation and total supply chain profit is maximized.

Through this combination of price coordination, innovation control, and contractual coordination, the e-retail platform sustains a dynamic equilibrium of the supply chain ecosystem, balancing its own profit, the manufacturer's profit, and the competitive dynamics between NB and PB products.

3.2. Market structure and notations

The wholesale and retail prices of the NB product are denoted as w_m and p_m, respectively, while the retail price of the PB product is denoted p_r. If the NB or PB product implements green innovation, the level of green innovation of the product is denoted η_i, where $i \in \{m, r\}$, m represents the NB manufacturer, and r represents the e-retail platform's PB. Product-related green innovation incurs a certain innovation cost that is generally related to innovation efficiency and the level of green innovation. We assume that the green innovation cost coefficient for NB and PB products is η_m and η_r, respectively, and the innovation cost is $\frac{1}{2}\eta_i\alpha^2$. A larger η_i thus implies a higher innovation cost for a given green innovation level α. Referring to the relevant literature of Zhang et al [40], in order to simplify the analysis, it is assumed that the two brands of products adopt the same method of green innovation; therefore, the efficiency is also the same, i.e., $\eta_m = \eta_r = \eta > 0$. We also acknowledge that assuming identical green innovation efficiency η for both the NB manufacturer and the e-retail platform is a simplification. In practice, their cost structures may differ. However, our primary focus is on the strategic behavior of green innovation rather than efficiency-related heterogeneity. A detailed analysis of asymmetric efficiency is left for future research. In order to ensure that the equilibrium solution of the model for the four scenarios under the wholesale sales model has practical significance, combined with the constraint condition for the negative definiteness of the Hessian matrix in Section 4, the parameter conditions are as follows: $\eta > \eta_0$, where $\eta_0 = \max\{\eta^{1*}, \ \eta^{2*}\}$, $\eta^{1*} = \frac{3\theta^2}{4}$ and $\eta^{2*} = \frac{\theta^2(1+\lambda)}{2}$. Our key findings are also derived from the range of $\eta > \eta_0$. Taking Scenarios B and M as examples, the market structure and decision variables of supply chain members are shown in Figure 1 and Figure 2 below.

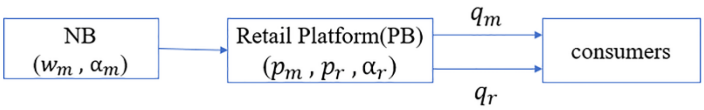

Figure 1. Diagram of the supply chain structure in Scenario B.

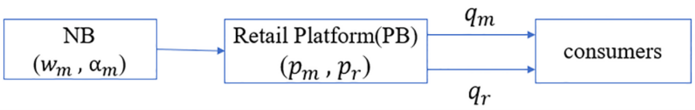

Figure 2. Diagram of the supply chain structure in Scenario M.

In the context of green innovation, the distinctiveness of a PB or NB product is manifested primarily in two ways: Its functionality and the extent of its green innovation. Specifically, the greater the functionality-related distinctiveness of the products, the less intense the price competition. The parameter λ can be regarded as a measure of the competitive intensity between the two brands. As λ increases, the substitutability between the two brands' products increases, and the competition intensifies. The substitutability between two brands and the impact of green innovation levels can be quantified using the same parameter λ. In addition, consumers in the market exhibit specific environmental preferences and are sensitive to the products' prices and green innovation levels. The parameter θ is used to measure consumers' environmental preferences, with higher θ values indicating greater consumer sensitivity to green innovation levels. Assuming that both brands have implemented green innovation, the consumer demand for NB and PB products is as follows:

$$q_m = d - (p_m - \lambda p_r) + \theta(\alpha_m - \lambda\alpha_r), \tag{1}$$

$$q_r = d - (p_r - \lambda p_m) + \theta(\alpha_r - \lambda\alpha_m). \tag{2}$$

The linear demand functions in (1) and (2) follow the standard formulation in the literature on brand competition and green supply chain management (Liu et al. [2], Zhang et al. [40], and Zhou et al. [46]). This specification can be derived from a quadratic consumer utility function. The linear form ensures analytical tractability and yields closed-form thresholds that clearly reveal how competition intensity, green preference, and cross-effects influence the equilibrium outcomes. We acknowledge that actual e-retail demand often exhibits stochastic fluctuations (e.g., due to promotions, seasonality, or unforeseen events) and may involve nonlinear network externalities that cannot be captured by the linear deterministic structure. The current model deliberately simplifies these aspects to establish a baseline. If stochastic demand were introduced, or if network externalities took a nonlinear form, the concavity of the objective functions could be lost, potentially leading to multiple equilibria or requiring numerical solutions. We leave these extensions for future research, noting that the linear demand model adopted in this paper provides a clear benchmark for such generalizations.

The variable d denotes the potential market demand for a given product. The demand function for brand i, $i \in \{NB, PB\}$, is influenced not only by the brand's own e-retail price and the level of green innovation but also by the retail price and green innovation level of the competing brand. Furthermore, the impact of the retail price and green innovation level of brand i on its own demand is greater than that of the competing brand. All the elements of the model, such as potential market demand (d), the intensity of competition (λ), the degree of consumer preference for green innovation (θ), and the strength of the platform's network externalities (k), are stable and certain over a period of time. It is difficult for supply chain participants to gain a competitive advantage from these factors. This study explores the selection of innovative strategies for competing brands in a setting where information is perfect. The profitability functions of the manufacturer and the e-retail platform are as follows:

$$\pi_r = q_m(p_m - w_m) + q_r p_r - \frac{1}{2}\eta\alpha_r^2 + k(q_m + q_r), \tag{3}$$

$$\pi_m = q_m w_m - \frac{1}{2}\eta\alpha_m^2. \tag{4}$$

The parameter k captures the strength of network externalities on the e-retail platform, which generally grows with platform scale: A larger user base corresponds to stronger network effects [1]. Unlike traditional supply chains, e-retail platforms enable consumers to readily access product sales, quality, and review information, allowing network externalities to shape perceived value. As more users purchase similar or complementary products, individual consumer satisfaction rises. This study focuses on positive network externalities, whereby rising adoption attracts additional consumers and generates incremental revenue for the platform. The other symbols used in this study are explained in Table 1.

Table 1. Symbols.

Symbols	Explanation
$i \in \{m, r\}$	m : NB manufacturer; r : e-retail platform
$j \in \{N, B, M, E\}$	Scenario: N (no innovation), B (bilateral innovation), M (manufacturer-only innovation), E (platform-only innovation)
α_i^j	Green innovation level of brand i in scenario j (decision variables), $\alpha_i^j > 0$; higher value indicates greater environmental improvement
p_i^j	Retail price of brand i 's product in scenario j
w_m^j	Wholesale price of the NB product in scenario j (decision variable for manufacturer)
q_m, q_r	Market demand function of NB manufacturer and e-retail platform's PB
λ	Competition intensity between NB and PB; higher λ means stronger substitutability and more intense price competition, $\lambda \in (0,1)$
θ	Consumer green preference; a higher θ indicates greater willingness to pay for green products, $\theta \in (0,1)$
k	Strength of network externalities on the e-retail platform; a large k means more value from user scale, $k \in (0,1)$
d	Product's potential market demand
π_i^j	Equilibrium profitability of supply chain member i in scenario j
π_s^j	Equilibrium profitability of the entire supply chain in scenario j , $\pi_s^j = \pi_m^j + \pi_r^j$
*	Denotes equilibrium value.

4. Game equilibrium solution

Given the choice of product-related green innovation strategies for the e-retail platform and the manufacturer, there are four scenarios: N, B, M, and E. In this section, we will derive and analyze the equilibrium decisions of the two brands in each scenario.

4.1. Scenario N

In Scenario N, neither the e-retail platform's PB nor the NB manufacturer's products implement green innovation. The demand functions for the two brands' products are given, respectively, by

$$q_m = d - (p_m - \lambda p_r) , \tag{5}$$

$$q_r = d - (p_r - \lambda p_m) . \tag{6}$$

The profit functions for NB manufacturer and the e-retail platform are

$$\pi_m^N = q_m w_m^N , \tag{7}$$

$$\pi_r^N = q_m(p_m^N - w_m^N) + q_r p_r^N + k(q_m + q_r). \tag{8}$$

The equilibrium result under Scenario N can be obtained by backward induction as follows: $w_m^{N*} = \frac{1}{4}(d + k - k\lambda)$, $p_m^{N*} = \frac{k\lambda^2 - k + d(3-\lambda)}{4(1-\lambda)}$, $p_r^{N*} = \frac{d - k(1-\lambda)}{2-2\lambda}$, $\pi_r^{N*} = \frac{(3+\lambda)(d+k-k\lambda)^2}{8(1-\lambda)}$, and $\pi_m^{N*} = \frac{1}{16}(d + k - k\lambda)^2$. An analysis of the impact of the main parameters on the equilibrium decisions of the e-retail platform and the NB manufacturer in Scenario N leads to the conclusions of Corollary 4.1 and Corollary 4.2.

Corollary 4.1 (1) $\frac{\partial w_m^{N*}}{\partial \lambda} < 0$, $\frac{\partial \pi_m^{N*}}{\partial \lambda} < 0$, $\frac{\partial p_r^{N*}}{\partial \lambda} > 0$; (2) if $d > \frac{k(1-\lambda)^2}{2}$, then $\frac{\partial p_m^{N*}}{\partial \lambda} > 0$, $\frac{\partial \pi_r^{N*}}{\partial \lambda} > 0$, otherwise $\frac{\partial p_m^{N*}}{\partial \lambda} < 0$, $\frac{\partial \pi_r^{N*}}{\partial \lambda} < 0$.

Corollary 4.1 (1) indicates that as competition intensifies, the NB and PB products become more substitutable. The platform leverages this substitutability and its dominance to pressure the manufacturer into lowering the NB's wholesale prices, thereby reducing the manufacturer's margins. Meanwhile, the platform raises the PB's retail prices—both to boost the PB's margin and to make the NB appear more affordable, thus maximizing total profit while avoiding direct retail competition. According to Corollary 4.1 (2), potential market size critically moderates the effect of competition. Only in large markets does the NB's retail price and the platform's profit rise with competition, as sales losses are relatively minor. Here, the platform converts competitive pressure into lower wholesale prices and higher retail prices, turning competition into profit—an outcome unattainable in small markets.

Corollary 4.2 $\frac{\partial w_m^{N*}}{\partial k} > 0$, $\frac{\partial p_m^{N*}}{\partial k} < 0$, $\frac{\partial p_r^{N*}}{\partial k} < 0$, $\frac{\partial \pi_r^{N*}}{\partial k} > 0$, $\frac{\partial \pi_m^{N*}}{\partial k} > 0$.

Corollary 4.2 shows that stronger network externalities generally benefit both supply chain members. The NB manufacturer directly gains by raising the wholesale price of NB products, thereby boosting its profit. For the platform, although retail prices for both NB and PB products fall in Scenario N, network-driven market expansion still enables profit growth, as higher sales volume compensates for lower unit margins, resulting in increased total profit for both parties.

4.2. Scenario B

In Scenario B, the e-retail platform and the NB manufacturer both implement green innovation. The demand functions for the two brands' products are given, respectively, by

$$q_m = d - (p_m - \lambda p_r) + \theta(\alpha_m - \lambda \alpha_r), \tag{9}$$

$$q_r = d - (p_r - \lambda p_m) + \theta(\alpha_r - \lambda \alpha_m). \tag{10}$$

The profit functions for NB manufacturer and the e-retail platform are

$$\pi_m^B = q_m w_m^B - \frac{1}{2}\eta\left(\alpha_m^B\right)^2, \tag{11}$$

$$\pi_r^B = q_m(p_m^B - w_m^B) + q_r p_r^B - \frac{1}{2}\eta\left(\alpha_r^B\right)^2 + k(q_m + q_r). \tag{12}$$

By backward induction, the equilibrium results under Scenario B can be obtained as follows: $w_m^{B*} = \frac{\eta(d+k-k\lambda)\left(2\eta-\theta^2(1+\lambda)\right)}{2\left(\left(2\eta-\theta^2\right)^2+\theta^2\left(\eta-\theta^2\right)\lambda^2\right)}$, $\alpha_m^{B*} = \frac{\theta(d+k-k\lambda)\left(2\eta-\theta^2(1+\lambda)\right)}{2\left(\left(2\eta-\theta^2\right)^2+\theta^2\left(\eta-\theta^2\right)\lambda^2\right)}$,

$p_m^{B*} = \frac{k(1-\lambda)\left(3\eta\theta^2-2\eta^2(1+\lambda)+\theta^4\left(-1+\lambda^2\right)\right)+d\left(2\eta^2(3-\lambda)-\eta\theta^2\left(5-2\lambda^2\right)+\theta^4\left(1-\lambda^2\right)\right)}{2(1-\lambda)\left(\left(2\eta-\theta^2\right)^2+\theta^2\left(\eta-\theta^2\right)\lambda^2\right)}$,

$p_r^{B*} = \frac{d\eta\left(4\eta-\theta^2(2+\lambda)\right)-k(1-\lambda)\left(4\eta^2-2\theta^4\left(-1+\lambda^2\right)+\eta\theta^2\left(-6+\lambda+2\lambda^2\right)\right)}{2(1-\lambda)\left(\left(-2\eta+\theta^2\right)^2+\theta^2\left(\eta-\theta^2\right)\lambda^2\right)}$,

$a_r^{B*} = \frac{\theta(d+k-k\lambda)\left(\eta(2+\lambda)-\theta^2(1+\lambda)\right)}{\left(2\eta-\theta^2\right)^2+\theta^2\left(\eta-\theta^2\right)\lambda^2}$, $\pi_r^{B*} = \frac{\eta(d+k-k\lambda)^2\left(2\eta(3+\lambda)-3\theta^2(1+\lambda)\right)}{4(1-\lambda)\left(\left(2\eta-\theta^2\right)^2+\theta^2\left(\eta-\theta^2\right)\lambda^2\right)}$ and

$\pi_m^{B*} = \frac{\eta\left(2\eta-\theta^2\right)(d+k-k\lambda)^2\left(2\eta-\theta^2(1+\lambda)\right)^2}{8\left(\left(2\eta-\theta^2\right)^2+\theta^2\left(\eta-\theta^2\right)\lambda^2\right)^2}$. It is worth noting that, to ensure that the profits of the e-retail platform and the traditional brand manufacturer have a maximum in Scenario B, it is necessary to ensure that the Hessian matrix

$$H_1 = \begin{pmatrix} \frac{2\eta\left(-2\eta+\theta^2\right)}{\left(\eta-\theta^2\right)^2} & \frac{2\eta\left(2\eta-\theta^2\right)\lambda}{\left(\eta-\theta^2\right)^2} & \frac{\eta\theta\left(-3\eta+\theta^2\right)\lambda}{\left(\eta-\theta^2\right)^2} \\ \frac{2\eta\left(2\eta-\theta^2\right)\lambda}{\left(\eta-\theta^2\right)^2} & -2-\frac{2\left(\eta^2+\eta\theta^2-\theta^4\right)\lambda^2}{\left(\eta-\theta^2\right)^2} & \theta-\frac{\theta\left(-2\eta+\theta^2\right)\left(\eta+\theta^2\right)\lambda^2}{\left(\eta-\theta^2\right)^2} \\ \frac{\eta\theta\left(-3\eta+\theta^2\right)\lambda}{\left(\eta-\theta^2\right)^2} & \theta-\frac{\theta\left(-2\eta+\theta^2\right)\left(\eta+\theta^2\right)\lambda^2}{\left(\eta-\theta^2\right)^2} & \eta\left(-1-\frac{2\eta\theta^2\lambda^2}{\left(\eta-\theta^2\right)^2}\right) \end{pmatrix}$$

is negative definite. The constraint condition for the existence of the profit maximum can then be obtained as follows: $\frac{3\theta^2}{4} < \eta$. An analysis of the impact of the main parameters on the equilibrium decisions of e-retail platform and NB manufacturer in Scenario B leads to the conclusions of Corollary 4.3 to Corollary 4.6.

Corollary 4.3 (1) $\frac{\partial w_m^{B*}}{\partial\lambda} < 0$, $\frac{\partial \alpha_m^{B*}}{\partial\lambda} < 0$, $\frac{\partial \pi_m^{B*}}{\partial\lambda} < 0$, $\frac{\partial p_r^{B*}}{\partial\lambda} > 0$; (2) when $d_1 < d$, if $\eta_0 < \eta < \theta^2$ and $\lambda_1 < \lambda < 1$ or $\theta^2 \le \eta$, then $\frac{\partial p_m^{B*}}{\partial\lambda} > 0$; otherwise $\frac{\partial p_m^{B*}}{\partial\lambda} < 0$; (3)

if $\eta_0 < \eta < \theta^2$ and $0 < \lambda \le 2-\sqrt{3}$, or $\eta_1 < \eta < \theta^2$ and $2-\sqrt{3} < \lambda \le \frac{1}{2}$, or $\theta^2 < \eta < \eta_1$ and $\frac{1}{2} < \lambda < 1$, then $\frac{\partial \alpha_r^{B*}}{\partial \lambda} < 0$; otherwise $\frac{\partial \alpha_r^{B*}}{\partial \lambda} > 0$; (4) if $d_2 < d$, then $\frac{\partial \pi_r^{B*}}{\partial \lambda} > 0$; otherwise $\frac{\partial \pi_r^{B*}}{\partial \lambda} < 0$.

Here, $d_1 = \frac{2k\eta^2(-1+\lambda)^2 A_1}{B_1+C_1}$, $d_2 = \frac{k\eta(1-\lambda)\left(4\eta-\theta^2(2+\lambda)\right)A_2}{B_2+C_2}$, $A_1 = \left(4\eta^2+\theta^4\left(1+\lambda+\lambda^2\right)-\eta\theta^2\left(4+\lambda(2+\lambda)\right)\right)$, $A_2 = 2\eta(1+\lambda)-\theta^2(1+2\lambda)$, $B_1 = 16\eta^4+\theta^8\left(-1+\lambda^2\right)^2+4\eta^3\theta^2\left(-9+\lambda+3\lambda^2-\lambda^3\right)$,

$B_2 = 16\eta^3+3\theta^6(-1+\lambda)(1+\lambda)^2$, $C_1 = -3\eta\theta^6\left(3-4\lambda^2+\lambda^4\right)+\eta^2\theta^4\left(28+\lambda\left(-2+\lambda\left(-23+2\lambda(2+\lambda)\right)\right)\right)$,

$C_2 = 2\eta^2\theta^2(2+\lambda)\left(-7+\lambda(2+\lambda)\right)+\eta\theta^4\left(16+\lambda\left(9-\lambda(11+5\lambda)\right)\right)$, $\eta_1 = \frac{1}{8}\left(\theta^2(2+\lambda)^2+\sqrt{\theta^4\lambda^2\left(8+\lambda(8+\lambda)\right)}\right)$,

λ_1 is the smallest real root of the quartic equation in x given by

$$16\eta^4-36\eta^3\theta^2+28\eta^2\theta^4-9\eta\theta^6+\theta^8+\left(4\eta^3\theta^2-2\eta^2\theta^4\right)x +\left(12\eta^3\theta^2-23\eta^2\theta^4+12\eta\theta^6-2\theta^8\right)x^2+\left(-4\eta^3\theta^2+4\eta^2\theta^4\right)x^3+\left(2\eta^2\theta^4-3\eta\theta^6+\theta^8\right)x^4=0.$$

Corollary 4.3 (2) reveals that within certain thresholds, a decline in NB wholesale price may lead to a corresponding reduction in the retail price. In a limited market ($0<d<d_1$), the platform lacks the flexibility to offset competition through volume, making price reduction inevitable. In a large market ($d_1<d$), intense competition diminishes product differentiation, and low green innovation efficiency makes sustaining a green premium difficult, reducing the platform's incentive to maintain a high price for the NB products.

Corollary 4.3 (3) shows that the effect of competitive intensity on PB's green innovation level is nonmonotonic, reflecting the platform's trade-off between competition and innovation efficiency. When green innovation efficiency is low ($\eta_1<\eta$), heightened competition increases product substitutability, making PB-related green innovation costly and further weakening the NB's market position. Conversely, when innovation efficiency is high and competition moderate ($\eta_0<\eta<\eta_1$ and $2-\sqrt{3}<\lambda\le\frac{1}{2}$), the platform can leverage low innovation costs to expand the green premium, allowing the PB to gain greater profit from demand growth than the cost of its innovation investment.

Corollary 4.3 (4) reveals that the impact of competitive intensity on the platform's profit depends on the market size. In a small market ($0<d<d_2$), intensified competition forces the platform to reduce prices for both the NB and PB, and demand expansion is insufficient to compensate, leading to lower profit. In a large market ($d_2<d$), intense competition strengthens the platform's bargaining power, enabling it to shift demand by adjusting price differentials between the two brands, balancing pricing and competitive consumption to increase profit.

Corollary 4.4 (1) $\frac{\partial p_r^{B*}}{\partial \theta} > 0$, $\frac{\partial \alpha_r^{B*}}{\partial \theta} > 0$, $\frac{\partial \pi_r^{B*}}{\partial \theta} > 0$; (2) if $0<\lambda<0.227$, or $0.227<\lambda<\sqrt{3}-1$ and $\eta_2<\eta$, then $\frac{\partial w_m^{B*}}{\partial \theta} > 0$, $\frac{\partial p_m^{B*}}{\partial \theta} > 0$; otherwise $\frac{\partial w_m^{B*}}{\partial \theta} < 0$, $\frac{\partial p_m^{B*}}{\partial \theta} < 0$; (3) if $0<\lambda\le 0.255$, or $0.255<\lambda<1$ and $\eta_3<\eta$, then $\frac{\partial \alpha_m^{B*}}{\partial \theta} > 0$; otherwise $\frac{\partial \alpha_m^{B*}}{\partial \theta} < 0$; (4) if $0 < \lambda \le 0.141$, or $0.141 < \lambda < \sqrt{2} - 1$ and $\eta_4 < \eta$, then $\frac{\partial \pi_m^{B*}}{\partial \theta} > 0$; otherwise $\frac{\partial \pi_m^{B*}}{\partial \theta} < 0$.

Here, $\eta_2 = \frac{\sqrt{2}}{2}\sqrt{\frac{\theta^4(1-\lambda)(\lambda+\lambda^2)^2}{(2-\lambda(2+\lambda))^2} + \frac{\theta^2(1-\lambda^2)}{2-\lambda(2+\lambda)}}$, η_3 is the third smallest real root of the cubic equation in x given by $\begin{aligned}&\theta^6+\theta^6\lambda-\theta^6\lambda^2-\theta^6\lambda^3+\left(-2\theta^4+4\theta^4\lambda+5\theta^4\lambda^2-\theta^4\lambda^3\right)x\\&+\left(-4\theta^2-12\theta^2\lambda-2\theta^2\lambda^2\right)x^2+8x^3=0\end{aligned}$ and η_4 is the smallest real root of the cubic equation in x given by $\begin{aligned}&\theta^6+\theta^6\lambda-\theta^6\lambda^2-\theta^6\lambda^3+\left(-6\theta^4+9\theta^4\lambda^2+3\theta^4\lambda^3\right)x\\&+\left(12\theta^2-12\theta^2\lambda-18\theta^2\lambda^2\right)x^2+\left(-8+16\lambda+8\lambda^2\right)x^3=0\end{aligned}$.

Corollary 4.4 (1) shows that the e-retail platform raises the PB's prices to transfer green innovation costs to willing consumers, thereby fully capturing consumers' green preference as profit growth. Corollary 4.4 (2) reveals that increased green preference does not always raise the NB's wholesale or retail prices; under high competition ($\sqrt{3}-1 < \lambda < 1$), the platform lowers the NB's retail prices to shift demand toward the higher-margin NB line, forcing the manufacturer to also reduce wholesale prices. When competition is moderate and innovation efficiency is high ($0.255 < \lambda < \sqrt{3}-1$ and $\eta_0 < \eta < \eta_2$), the platform exploits its first-mover advantage and low-cost green edge to further depress the NB's prices. Corollary 4.4 (3) indicates that higher green preference does not necessarily enhance the NB's or PB's green innovation. It boosts the NB's innovation only when competition is low ($0 < \lambda \le 0.255$) or when both competition and innovation efficiency are high ($0.255 < \lambda < 1$ and $\eta_3 < \eta$). Under medium-high λ and low η, however, a rise in θ reduces the NB's innovation because the PB fully harvests the green premium, making NB innovation unprofitable. As evidenced by Corollary 4.4 (4), the manufacturer profits from its own green innovation when competition is low ($0 < \lambda \le 0.141$). Moderate competition combined with low innovation efficiency ($0.141 < \lambda < \sqrt{2}-1$ and $\eta_4 < \eta$) is especially beneficial: faced with high innovation costs, the platform strategically tolerates the NB's green innovation to prevent the overall green market from shrinking, which ultimately benefits the NB manufacturer.

Corollary 4.5 (1) $\frac{\partial p_r^{B*}}{\partial \eta} < 0$, $\frac{\partial \alpha_r^{B*}}{\partial \eta} < 0$, $\frac{\partial \pi_r^{B*}}{\partial \eta} < 0$; (2) if $0 < \lambda \le 0.227$, or $0.227 < \lambda < \sqrt{3} - 1$ and $\eta_1 < \eta$, then $\frac{\partial w_m^{B*}}{\partial \eta} < 0$, $\frac{\partial p_m^{B*}}{\partial \eta} < 0$; otherwise $\frac{\partial w_m^{B*}}{\partial \eta} > 0$, $\frac{\partial p_m^{B*}}{\partial \eta} > 0$; (3) if $\eta_0 < \eta \le \frac{3\theta^2}{2}$ and $0 < \lambda < \lambda_2$, or $\frac{3\theta^2}{2} < \eta$, then $\frac{\partial \alpha_m^{B*}}{\partial \eta} < 0$; otherwise $\frac{\partial \alpha_m^{B*}}{\partial \eta} > 0$; (4) if $0 < \lambda \le 0.141$, or $0.141 < \lambda < \sqrt{2} - 1$ and $\eta_2 < \eta$, then $\frac{\partial \pi_m^{B*}}{\partial \eta} < 0$; otherwise $\frac{\partial \pi_m^{B*}}{\partial \eta} > 0$.

Here, λ_2 is the smallest real root of the cubic equation in x given by $-8\eta^2 + 8\eta\theta^2 - 2\theta^4 + \left(8\eta\theta^2 - 4\theta^4\right)x - \theta^4 x^2 + \theta^4 x^3 = 0$.

Corollary 4.5 (1) shows that higher green innovation costs reduce the price competitiveness of the PB product. To stabilize demand, the e-retail platform lowers both the PB's retail price and its green innovation level, ultimately decreasing the platform's profitability due to joint cost and price pressures. Corollary 4.5 (2) indicates that under high competition intensity or moderate competition with high green innovation efficiency, a rise in green innovation costs induces the NB manufacturer to increase the NB's wholesale price. The platform then raises the NB's retail price accordingly, making the PB products more attractive to price-sensitive consumers and shifting demand toward the higher-margin PB channel. Corollary 4.5 (3) shows that under high competition intensity ($\lambda_2 < \lambda < 1$), the NB manufacturer engages in defensive green innovation to avoid market erosion, increasing investment despite higher unit costs. In low-competition, high-efficiency scenarios ($0 < \lambda < \lambda_2$ and $\eta_0 < \eta \le \frac{3\theta^2}{2}$), as the efficiency advantage decays, the manufacturer further boosts innovation to maintain technological leadership. Corollary 4.5 (4) indicates that the NB manufacturer's profitability increases as green innovation efficiency decreases, either under high competition or under high initial efficiency paired with low competition. This occurs because lower efficiency encourages the platform to ease pricing pressure on the NB product, allowing the manufacturer to secure higher margins. The outcome reflects a strategic equilibrium in which the platform balances competition with cooperative gains within the supply chain.

Corollary 4.6 (1) $\frac{\partial w_m^{B*}}{\partial k} > 0$, $\frac{\partial \pi_r^{B*}}{\partial k} > 0$, $\frac{\partial \pi_m^{B*}}{\partial k} > 0$, $\frac{\partial \alpha_m^{B*}}{\partial k} > 0$, $\frac{\partial \alpha_r^{B*}}{\partial k} > 0$; (2) if $0 < \lambda < \frac{1}{8}$ and $\eta_0 < \eta < \eta_5$, then $\frac{\partial p_m^{B*}}{\partial k} > 0$; otherwise $\frac{\partial p_m^{B*}}{\partial k} < 0$; (3) if $0 < \lambda < \frac{1}{2}$ and $\eta_0 < \eta < \eta_6$, then $\frac{\partial p_r^{B*}}{\partial k} > 0$; otherwise $\frac{\partial p_r^{B*}}{\partial k} < 0$.

Here, $\eta_5 = \frac{1}{4}\left(\frac{3\theta^2}{1+\lambda} + \sqrt{\frac{\theta^4\left(1+8\lambda\left(-1+\lambda+\lambda^2\right)\right)}{(1+\lambda)^2}}\right)$, $\eta_6 = \frac{1}{8}\left(-\theta^2\left(-6+\lambda+2\lambda^2\right)+\theta^2\sqrt{4+\lambda\left(-12+\lambda\left(9+4\lambda(1+\lambda)\right)\right)}\right)$.

From Corollary 4.6 shows that stronger network externalities generally benefit both supply chain parties: The NB manufacturer raises both the wholesale price and the green innovation level of its product, increasing its profit, while the e-retail platform also achieves higher profitability. The effect on the retail prices of NB and PB products is non-monotonic. Under a low competition intensity and high green innovation efficiency, enhanced network externalities raise prices for both brands. The expanded market and increased willingness to pay allow the platform to leverage high efficiency and low interbrand competition to simultaneously increase prices for the NB and PB, thereby boosting overall supply chain profitability.

4.3. Scenario M

In Scenario M, only the NB manufacturer implements green innovation. The demand functions for the two brands are given, respectively, by

$$q_m = d - (p_m - \lambda p_r) + \theta\alpha_m, \tag{13}$$

$$q_r = d - (p_r - \lambda p_m) - \theta\lambda\alpha_m. \tag{14}$$

The profit functions for NB manufacturer and e-retail platform are

$$\pi_m^M = q_m w_m^M - \frac{1}{2}\eta\left(\alpha_m^M\right)^2, \tag{15}$$

$$\pi_r^M = q_m\left(p_m^M - w_m^M\right) + q_r p_r^M + k(q_m + q_r). \tag{16}$$

By backward induction, the equilibrium results under Scenario M can be obtained as follows: $w_m^{M*} = \frac{\eta(d+k-k\lambda)}{4\eta-2\theta^2}$, $p_m^{M*} = \frac{d\left(\eta(3-\lambda)-\theta^2\right)-k(1-\lambda)\left(\eta-\theta^2+\eta\lambda\right)}{2\left(2\eta-\theta^2\right)(1-\lambda)}$, $\alpha_m^{M*} = \frac{\theta(d+k-k\lambda)}{4\eta-2\theta^2}$, $p_r^{M*} = \frac{d-k(1-\lambda)}{2-2\lambda}$, $\pi_r^{M*} = \frac{(d+k-k\lambda)^2\left(\eta(3+\lambda)-\theta^2(1+\lambda)\right)}{4\left(2\eta-\theta^2\right)(1-\lambda)}$ and $\pi_m^{M*} = \frac{\eta(d+k-k\lambda)^2}{16\eta-8\theta^2}$. To ensure that the profits of the e-retail platform and the traditional brand manufacturer have a maximum in Scenario M, it is necessary to ensure that the Hessian matrix $H_2 = \begin{pmatrix} \frac{2\eta\left(-2\eta+\theta^2\right)}{\left(\eta-\theta^2\right)^2} & \frac{2\eta\left(2\eta-\theta^2\right)\lambda}{\left(\eta-\theta^2\right)^2} \\ \frac{2\eta\left(2\eta-\theta^2\right)\lambda}{\left(\eta-\theta^2\right)^2} & -2-\frac{2\left(\eta^2+\eta\theta^2-\theta^4\right)\lambda^2}{\left(\eta-\theta^2\right)^2} \end{pmatrix}$ is negative definite. The constraint condition for the existence of the profit maximum can then be obtained as follows: $\frac{\theta^2}{2} < \eta$. An analysis of the impact of the main parameters on the equilibrium decisions of the e-retail platform and NB manufacturer in Scenario M leads to the conclusions of Corollary 4.7 to Corollary 4.10.

Corollary 4.7 (1) $\frac{\partial w_m^{M*}}{\partial\lambda} < 0$, $\frac{\partial\alpha_m^{M*}}{\partial\lambda} = 0$, $\frac{\partial\pi_m^{M*}}{\partial\lambda} < 0$, $\frac{\partial p_r^{M*}}{\partial\lambda} > 0$; (2) if $\frac{k\eta(1-\lambda)^2}{2\eta-\theta^2} < d$, then $\frac{\partial p_m^{M*}}{\partial\lambda} > 0$; otherwise $\frac{\partial p_m^{M*}}{\partial\lambda} < 0$; (3) if $\frac{k(1-\lambda)\left(\eta+\eta\lambda-\theta^2\lambda\right)}{2\eta-\theta^2} < d$, then $\frac{\partial\pi_r^{M*}}{\partial\lambda} > 0$; otherwise $\frac{\partial\pi_r^{M*}}{\partial\lambda} < 0$.

Corollary 4.7 (1) shows that in Scenario M, the NB's green innovation level remains unaffected by competition. Heightened competition increases the PB's substitution threat, forcing the manufacturer to lower wholesale prices to boost demand. The platform raises the PB's price to make the NB appear more affordable. However, increased demand does not offset the profit losses from price reductions, reducing the manufacturer's overall profitability. Corollary 4.7 (2) and Corollary 4.7 (3) indicate that market size is the main factor affecting the retail pricing of the NB product and the profitability of the e-retail platform. In a small market ($0 < d < \frac{k\eta(1-\lambda)^2}{2\eta-\theta^2}$), increased competition makes the retail price of the NB product lower. For the e-retail platform, only when the market size is large ($\frac{k(1-\lambda)\left(\eta+\eta\lambda-\theta^2\lambda\right)}{2\eta-\theta^2} < d$), as λ increases, the e-retail platform can earn higher profitability through economies of scale.

Corollary 4.8 $\frac{\partial w_m^{M*}}{\partial\theta} < 0$, $\frac{\partial\alpha_m^{M*}}{\partial\theta} > 0$, $\frac{\partial\pi_m^{M*}}{\partial\theta} > 0$, $\frac{\partial p_m^{M*}}{\partial\theta} > 0$, $\frac{\partial p_r^{M*}}{\partial\theta} = 0$, $\frac{\partial\pi_r^{M*}}{\partial\theta} > 0$.

Corollary 4.8 reveals that the PB's retail price remains unaffected by consumers' green preference. Increased green preference raises both the green innovation level and retail price of NB products. While the platform suppresses the NB's wholesale price, the profitability of both the manufacturer and platform grow, as higher green preference directly boosts NB demand, allowing the manufacturer to offset wholesale price pressure through volume. Simultaneously, the platform compensates for lower PB demand by reselling NB products at higher premiums, creating a symbiotic equilibrium where the manufacturer gains a marginal profit from green innovation, and the platform captures excess returns through channel advantage.

Corollary 4.9 $\frac{\partial w_m^{M*}}{\partial \eta} < 0, \frac{\partial \alpha_m^{M*}}{\partial \eta} < 0, \frac{\partial \pi_m^{M*}}{\partial \eta} < 0, \frac{\partial p_m^{M*}}{\partial \eta} < 0, \frac{\partial p_r^{M*}}{\partial \eta} = 0, \frac{\partial \pi_r^{M*}}{\partial \eta} < 0$.

In contrast to the findings in Scenario B, Corollary 4.9 shows that in Scenario M, reduced green innovation efficiency reduces the profitability of supply chain members. Higher green innovation costs make the NB product less profitable per unit. Consequently, the NB manufacturer also reduces the green innovation level and the wholesale price of the NB product, while the e-retail platform reduces the retail price of the NB product to stimulate demand.

Corollary 4.10 (1) $\frac{\partial w_m^{M*}}{\partial k} > 0, \frac{\partial \alpha_m^{M*}}{\partial k} > 0, \frac{\partial p_r^{M*}}{\partial k} < 0, \frac{\partial \pi_m^{M*}}{\partial k} > 0, \frac{\partial \pi_r^{M*}}{\partial k} > 0$; (2) if $\eta_0 < \eta < \frac{\theta^2}{1+\lambda}$, then $\frac{\partial p_m^{M*}}{\partial k} > 0$; otherwise $\frac{\partial p_m^{M*}}{\partial k} < 0$.

Under Scenario M, enhanced network externalities differentially affect supply chain decisions. Corollary 4.10 shows that as network effects strengthen, the NB manufacturer increases both the green innovation level and wholesale price, thereby raising its profitability. The platform's NB pricing depends on green innovation efficiency: When efficiency is high ($\eta_0 < \eta < \frac{\theta^2}{1+\lambda}$), the platform raises the NB's retail price to capture wider margins; when efficiency is low ($\frac{\theta^2}{1+\lambda} < \eta$), it lowers the NB's price to boost volume. Meanwhile, the platform consistently reduces the PB's price to optimize its product portfolio. Driven by market expansion from network effects, both members achieve synergistic profit growth.

4.4. Scenario E

In Scenario E, only the e-retail platform's PB implements green innovation. The demand functions for the two brands are given, respectively, as follows

$$q_m = d - (p_m - \lambda p_r) - \theta\lambda\alpha_r, \tag{17}$$

$$q_r = d - (p_r - \lambda p_m) + \theta\alpha_r. \tag{18}$$

The profit functions for NB manufacturer and e-retail platform are

$$\pi_m^E = q_m w_m^E, \tag{19}$$

$$\pi_r^E = q_m(p_m^E - w_m^E) + q_r p_r^E - \frac{1}{2}\eta\left(\alpha_r^E\right)^2. \tag{20}$$

By backward induction, the equilibrium results under Scenario E can be obtained as follows: $w_m^{E*}=\frac{(d+k-k\lambda)(2\eta-\theta^2(1+\lambda))}{8\eta-2\theta^2(2-\lambda^2)}$, $p_m^{E*}=\frac{k(1-\lambda)(\theta^2-2\eta(1+\lambda))+d(2\eta(3-\lambda)-\theta^2(3-2\lambda^2))}{2(1-\lambda)(4\eta+\theta^2(-2+\lambda^2))}$, $p_r^{E*}=\frac{d(4\eta-\theta^2\lambda)-k(1-\lambda)(4\eta+\theta^2(-4+\lambda+2\lambda^2))}{2(1-\lambda)(4\eta+\theta^2(-2+\lambda^2))}$, $\alpha_r^{E*}=\frac{\theta(2+\lambda)(d+k-k\lambda)}{4\eta-\theta^2(2-\lambda^2)}$, $\pi_r^{E*}=\frac{(d+k-k\lambda)^2(\theta^2(1+\lambda)-2\eta(3+\lambda))}{4(1-\lambda)(\theta^2(2-\lambda^2)-4\eta)}$ and $\pi_m^{E*}=\frac{(d+k-k\lambda)^2(2\eta-\theta^2(1+\lambda))^2}{4(4\eta-\theta^2(2-\lambda^2))^2}$. To ensure that the profits of the e-retail platform and the traditional brand manufacturer have a maximum in Scenario E, it is necessary to ensure that the Hessian matrix $H_3=\begin{pmatrix}-4 & 4\lambda & -3\theta\lambda\\ 4\lambda & -2(1+\lambda^2) & \theta+2\theta\lambda^2\\ -3\theta\lambda & \theta+2\theta\lambda^2 & -\eta-2\theta^2\lambda^2\end{pmatrix}$ is negative definite. The constraint condition for the existence of the profit maximum can then be obtained as follows: $\frac{\theta^2(1+\lambda)}{2}<\eta$. An analysis of the impact of the main parameters on the equilibrium decisions of the e-retail platform and NB manufacturer in Scenario E leads to the conclusions of Corollary 4.11 to Corollary 4.14.

Corollary 4.11 (1) $\frac{\partial w_m^{E*}}{\partial\lambda}<0$, $\frac{\partial\pi_m^{E*}}{\partial\lambda}<0$; (2) if $0<\lambda\le\sqrt{5}-2$, or $\sqrt{5}-2<\lambda<1$ and $\frac{\theta^2(2+\lambda(4+\lambda))}{4}<\eta$, then $\frac{\partial\alpha_r^{E*}}{\partial\lambda}>0$; otherwise $\frac{\partial\alpha_r^{E*}}{\partial\lambda}<0$; (3) if $d_3<d$, then $\frac{\partial p_m^{E*}}{\partial\lambda}>0$; otherwise $\frac{\partial p_m^{E*}}{\partial\lambda}<0$; (4) if $d_4<d$, then $\frac{\partial p_r^{E*}}{\partial\lambda}>0$; otherwise $\frac{\partial p_r^{E*}}{\partial\lambda}<0$; (5) if $d_5<d$, then $\frac{\partial\pi_r^{E*}}{\partial\lambda}>0$; otherwise $\frac{\partial\pi_r^{E*}}{\partial\lambda}<0$.

Here, $d_3=\frac{2k(1-\lambda)^2(4\eta^2+\theta^4\lambda-\eta\theta^2(2+\lambda(2+\lambda)))}{16\eta^2-4\eta\theta^2(5-\lambda-3\lambda^2+\lambda^3)+\theta^4(6-\lambda(2+5\lambda-2\lambda^3))}$,

$$d_4=\frac{k\theta^2(1-\lambda)^2(\eta(4+8\lambda)-\theta^2(2+\lambda^2))}{16\eta^2+\theta^4(2+\lambda^2-2\lambda^3)-4\eta\theta^2(3+\lambda(2-3\lambda))},$$

$$d_5=\frac{k(1-\lambda)(4\eta-\theta^2\lambda)(2\eta(1+\lambda)-\theta^2)}{16\eta^2-2\eta\theta^2(1+\lambda)(6-\lambda(3+\lambda))+\theta^4(2+\lambda(1-\lambda-\lambda^2))}.$$

In Scenario E, without green innovation advantages, the NB manufacturer can only mitigate profit loss by lowering prices as competition intensifies. Below a certain threshold, rising competition also promotes the PB's green innovation level, as platforms are compelled to invest in green innovation to capture demand from environmentally conscious consumers. Corollary 4.11 (3), (4), and (5) show that the primary factor influencing the NB's and PB's retail prices is market size. When the market size exceeds a certain threshold, platforms raise prices for both brands despite heightened competition, because the overall increase in profitability outweighs demand reduction losses, ultimately boosting overall platform profit.

Corollary 4.12 $\frac{\partial w_m^{E*}}{\partial\theta}>0$, $\frac{\partial\pi_m^{E*}}{\partial\theta}<0$, $\frac{\partial p_m^{E*}}{\partial\theta}<0$, $\frac{\partial\alpha_r^{E*}}{\partial\theta}>0$, $\frac{\partial p_r^{E*}}{\partial\theta}>0$, $\frac{\partial\pi_r^{E*}}{\partial\theta}>0$.

Because the PB's green innovation takes away part of the market demand for the NB product, the e-retail platform lowers the NB's retail price to boost demand when θ increases. Consequently, the profitability of the NB manufacturer also decreases. In contrast, for the e-retail platform, since the PB product is the only product in the market implementing green innovation and the platform has absolute pricing power over the PB, it is always beneficial for the platform to increase consumers' green preference.

Corollary 4.13 $\frac{\partial w_m^{E*}}{\partial \eta} > 0$, $\frac{\partial \pi_m^{E*}}{\partial \eta} > 0$, $\frac{\partial p_m^{E*}}{\partial \eta} > 0$, $\frac{\partial \alpha_r^{E*}}{\partial \eta} < 0$, $\frac{\partial p_r^{E*}}{\partial \eta} < 0$, $\frac{\partial \pi_r^{E*}}{\partial \eta} < 0$.

Reducing green innovation efficiency is favorable to the manufacturer, because higher green innovation costs force the PB to reduce its green innovation level, thereby reducing its encroachment on the NB product's demand market. At the same time, reducing green innovation efficiency is detrimental to the e-retail platform. The platform has to bear higher green innovation costs, which reduce both the green innovation level and the retail price of the PB product, while increasing the retail price of the NB product to balance market demand and sales margins.

Corollary 4.14 (1) $\frac{\partial w_m^{E*}}{\partial k} > 0$, $\frac{\partial \alpha_r^{E*}}{\partial k} > 0$, $\frac{\partial p_m^{E*}}{\partial k} < 0$, $\frac{\partial \pi_m^{E*}}{\partial k} > 0$, $\frac{\partial \pi_r^{E*}}{\partial k} > 0$; (2) if $\eta_0 < \eta < \frac{1}{4}\theta^2\left(4-\lambda-2\lambda^2\right)$, then $\frac{\partial p_r^{E*}}{\partial k} > 0$; otherwise $\frac{\partial p_r^{E*}}{\partial k} < 0$.

Corollary 4.14 demonstrates that under Scenario E, enhanced network externalities strengthen the e-retail platform's strategic dominance while simultaneously creating pricing power for the manufacturer. The platform elevates the green innovation level of its PB's product and adjusts the PB retail price according to its green innovation efficiency, raising the retail price to capture a green premium when efficiency is high, and lowering it to expand market share otherwise. Although the e-retail platform reduces the retail price of the NB product, the manufacturer can still leverage the market expansion driven by network effects to increase profitability by raising its wholesale price. Ultimately, the expansion of market scale resulting from network effects enables both supply chain members to achieve collective profit growth.

5. Analysis of the results

In this section, we conduct a comparative analysis to examine the impact of different green innovation scenarios on the price and demand for NB and PB products. The numerical examples in this section serve solely to visualize the analytical results (propositions and corollaries) derived in Section 5. All parameter values are chosen within the feasible ranges established in Section 3.2 to ensure positive prices, demands, and innovation levels. Specifically, we set $d = 10$, $\theta = 0.8$, $k = 0.5$ and $\eta = 0.8$ as a representative baseline that satisfies the negative definiteness conditions and covers a typical scenario with moderate competition, medium green preference, and efficient innovation. These values do not affect the generality of our analytical conclusions; they merely illustrate the directional relationships already proven mathematically. To further demonstrate robustness, we have also tested alternative parameter combinations (e.g., $\theta = 0.2$ and $\theta = 0.5$) and obtained qualitatively identical patterns. Therefore, the chosen baseline does not distort the broader dynamics.

5.1. Comparative analysis of price and demand for the NB product

The following key conclusions are derived from a comparative analysis of prices and demand for NB product in the four innovation scenarios.

Proposition 5.1 If $0<\lambda\le 3-\sqrt{7}$, or $3-\sqrt{7}<\lambda\le\sqrt{3}-1$ and $\frac{\theta^2(1-\lambda^2)}{2-\lambda(2+\lambda)}<\eta$, then

$w_m^{M*}>w_m^{B*}>w_m^{N*}>w_m^{E*}$, $p_m^{M*}>p_m^{B*}>p_m^{N*}>p_m^{E*}$, $d_m^{M*}>d_m^{B*}>d_m^{N*}>d_m^{E*}$; otherwise

$w_m^{M*}>w_m^{N*}>w_m^{B*}>w_m^{E*}$, $p_m^{M*}>p_m^{N*}>p_m^{B*}>p_m^{E*}$, $d_m^{M*}>d_m^{N*}>d_m^{B*}>d_m^{E*}$.

Proposition 5.1 can be demonstrated by Figure 3 and 4. The wholesale and retail prices of the NB product are the highest in Scenario M and the lowest in Scenario E. This stems from the market appeal of green products to environmentally conscious consumers. In Scenario M, the NB manufacturer increases the wholesale price of the NB product, leveraging green innovation to enhance product differentiation and market competitiveness. As the supply chain leader, the e-retail platform accepts this higher wholesale price and passes the cost to consumers, raising the NB retail price. In Scenario E, however, the PB's green innovation weakens the NB's market position. To preserve market share, the NB manufacturer lowers the wholesale price, and the platform subsequently reduces the NB's retail price to attract more consumers.

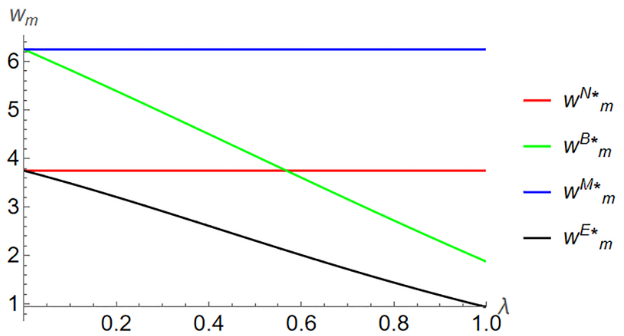

Figure 3. Comparison of equilibrium wholesale prices for the NB product ($d=10$, $\theta=0.8$, $k=0.5$, $\eta=0.8$).

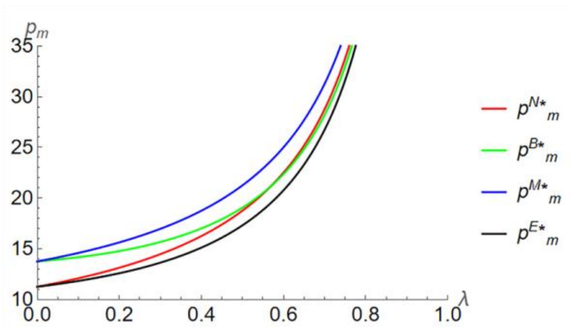

Figure 4. Comparison of equilibrium retail prices of the NB product ($d=10$, $\theta=0.8$, $k=0.5$, $\eta=0.8$).

This also indicates that in Scenario B, the NB's wholesale and retail prices are lower than in Scenario N ($w_m^{N*} > w_m^{B*}, \eta_0 < \eta < \frac{\theta^2(1-\lambda^2)}{2-\lambda(2+\lambda)}$) under high competition and high innovation efficiency ($\sqrt{3}-1 < \lambda < 1$ and $\eta_0 < \eta < \frac{\theta^2(1-\lambda^2)}{2-\lambda(2+\lambda)}$). This stems from two factors. First, product differentiation mitigates price competition under low competition, enabling the NB to raise prices, but under high competition, this effect is insufficient to offset competitive pressure, limiting price increases. The second is the impact of innovation efficiency: When green innovation is inefficient, rising costs drive price increases; when efficiency is high, increased demand can offset innovation costs, and the NB's prices may not necessarily rise.

The trend in NB demand aligns with its price changes, deviating from traditional supply-demand theory where higher prices typically reduce demand. Several factors could explain this. First, the presence of green-preferring consumers allows the NB to sustain demand even at higher prices (analogous to a luxury effect). Second, the price can serve as a signal of green innovation, attracting consumers and creating a positive feedback loop between price and demand. Finally, the competition intensity and market size moderate this relationship. Under low competition, the NB can pass on green innovation costs through higher prices while product differentiation attracts specific consumer segments, yielding both high prices and high demand. In highly competitive settings, where traditional Bertrand competition would drive prices to a marginal cost, green innovation helps the NB overcome homogenization, avoid price wars, and maintain both price and demand at elevated levels.

5.2. Comparative analysis of price and demand for the PB product

The following key conclusions are derived from a comparative analysis of prices and demand for the PB product in the four innovation scenarios.

Proposition 5.2 $p_r^{E*} > p_r^{B*} > p_r^{N*} = p_r^{M*}$.

Proposition 5.2 suggests that the PB attains its highest retail price in Scenario E, driven by its exclusive green attributes and the platform's strategic leadership. Without a green competitor, the PB leverages consumer willingness to pay to sustain a premium price. Meanwhile, the platform directs resources toward the PB in this scenario and captures green innovation gains directly through high pricing, avoiding profit sharing with the NB manufacturer. In Scenarios N and M, the retail price of the PB product is the same and is the lowest, as the platform deliberately suppresses the PB's price to emphasize the NB's differentiation advantage and avoid direct interbrand conflict.

Proposition 5.3 $d_r^{E*} > d_r^{B*} > d_r^{N*} > d_r^{M*}$.

Proposition 5.3 shows that the PB's green innovation raises its market demand, with the highest demand occurring in Scenario E due to its exclusive appeal to green-preferring consumers. Demand ranks second in Scenario B, where the platform balances the PB's and NB's retail profits by strategically limiting PB demand to avoid excessive cannibalization of NB sales. In Scenario M, PB demand is the lowest because the NB's green innovation diverts consumers who might otherwise choose the PB.

5.3. Comparative analysis of green innovation levels in the PB and NB

The following key conclusions are derived from a comparative analysis of green innovation levels for NB and PB products in the four innovation scenarios.

Proposition 5.4 $\alpha_r^{E*} > \alpha_r^{B*} > \alpha_m^{M*} > \alpha_m^{B*}$.

As revealed in Proposition 5.4 and Figure 5, the PB consistently achieves higher green innovation than the NB. As the Stackelberg leader, the e-retail platform shapes the manufacturer's strategic options, limiting the NB's green innovation investment. The platform fully captures the PB's green benefits and recovers innovation costs through higher PB retail prices, whereas the manufacturer must account for the platform's pricing when setting wholesale prices, constraining its cost recovery. By suppressing the NB's green innovation, the platform further protects the PB's profit margins, demonstrating stronger motivation to maximize the PB's green innovation level.

Additionally, both the PB and NB achieve higher green innovation levels when innovating separately (Scenarios E, M) than when innovating simultaneously (Scenario B). Full green differentiation in separate scenarios strengthens consumers' willingness to pay, encouraging higher investment. In Scenario B, however, partial differentiation disperses consumers' choice, forcing a balance between innovation and price competition, thereby reducing green levels for both products.

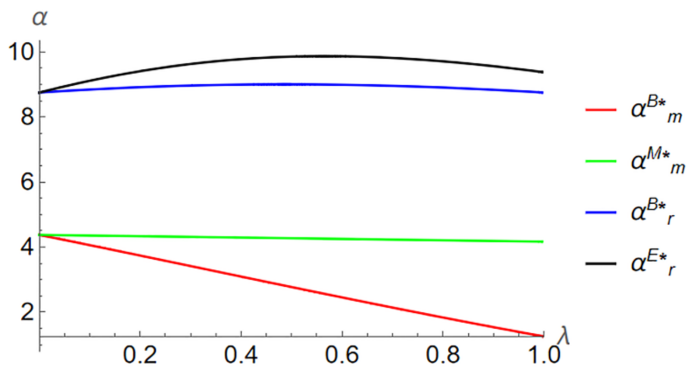

Figure 5. Comparison of green innovation levels for the NB and PB products ($d = 10, \theta = 0.8, k = 0.5, \eta = 0.8$).

5.4. Equilibrium profitability comparison and member preferences

This subsection analyzes the equilibrium profitability of the NB manufacturer and the e-retail platform under four green innovation scenarios to obtain the green innovation scenario preferences of the supply chain members and the whole supply chain. The main conclusions are as follows.

Proposition 5.5 If $\sqrt{3}-\sqrt{4-\sqrt{3}}<\lambda\le\sqrt{2}-1$ and $\eta_0<\eta<\eta_7$, or $\sqrt{2}-1<\lambda<1$,

then $\pi_m^{M*}>\pi_m^{N*}>\pi_m^{B*}>\pi_m^{E*}$; otherwise $\pi_m^{M*}>\pi_m^{B*}>\pi_m^{N*}>\pi_m^{E*}$.

Here, η_7 is the third smallest real root of the cubic equation in x given by

$$\begin{aligned}&\theta^6-2\theta^6\lambda^2+\theta^6\lambda^4+\left(-6\theta^4+4\theta^4\lambda+12\theta^4\lambda^2-2\theta^4\lambda^4\right)x\\&+\left(12\theta^2-16\theta^2\lambda-20\theta^2\lambda^2+\theta^2\lambda^4\right)x^2+\left(-8+16\lambda+8\lambda^2\right)x^3=0\end{aligned}.$$

Proposition 5.5 and Figure 6 show that competitive intensity and green innovation efficiency have the greatest impact on the equilibrium profits of the NB manufacturer. The NB manufacturer always prefers Scenario M, as the profit of the NB manufacturer is maximized in Scenario M and minimized in Scenario E.

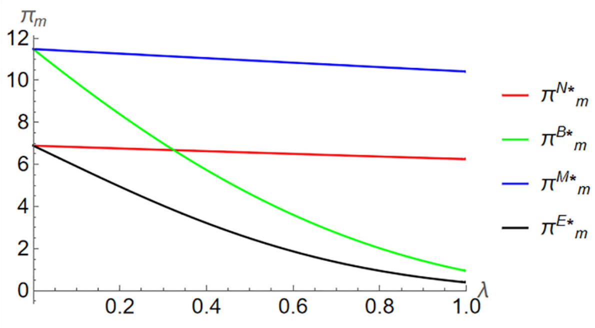

Figure 6. Comparison of the profits of the NB manufacturer ($d=10$, $\theta=0.8$, $k=0.5$, $\eta=0.8$).

Proposition 5.6 $\pi_r^{B*}>\pi_r^{E*}>\pi_r^{M*}>\pi_r^{N*}$.

Proposition 5.6 and Figure 7 show that it is always beneficial for the e-retail platform when either brand implements green innovation. Furthermore, when both brands innovate simultaneously (Scenario B), it is always the optimal choice for the e-retail platform. Therefore, the e-retail platform always has an incentive to promote green innovation for both brands.

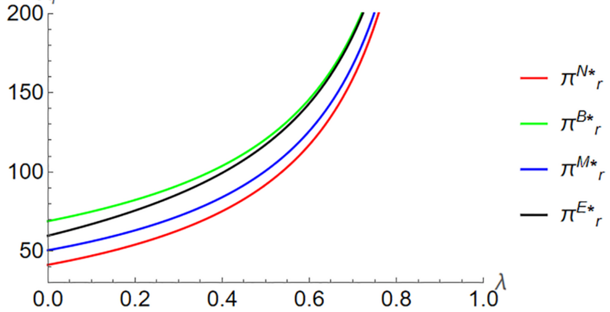

Figure 7. Comparison of the profits of e-retail platform. ($d=10$, $\theta=0.8$, $k=0.5$, $\eta=0.8$)

Combining Proposition 5.5 and Proposition 5.6, regardless of whether the NB manufacturer implements green innovation for the NB product, the PB product will implement green innovation. Given that the PB product will implement green innovation, the NB manufacturer will definitely choose to implement green innovation for the NB product. Therefore, green innovation for both brands (Scenario B) is the pure strategy Nash equilibrium of this game. However, the NB manufacturer and e-retail platform cannot maximize their respective profits through noncooperative games.

Proposition 5.7 $\pi_s^{B*} > \pi_s^{E*} > \pi_s^{M*} > \pi_s^{N*}$.

Proposition 5.7 implies that the policy preferences of the entire supply chain are the same as those of the e-retail platform. For the entire supply chain, it is always beneficial for any brand to implement green innovation, and the total supply chain's profit is maximized in Scenario B.

5.5. Resolving the prisoner's dilemma: A green innovation cost-sharing contract

As shown in Proposition 5.5 and Proposition 5.6, Scenario B (both parties innovate) is the unique Nash equilibrium, but the manufacturer's profit is lower than in Scenario M (manufacturer-only innovation). To resolve this prisoner's dilemma, we propose a simple cost-sharing contract between the e-retail platform and the manufacturer. Under Scenario B, let the e-retail platform agree to bear a fraction $s \in (0,1)$ of the manufacturer's green innovation cost. The manufacturer's profit function then becomes:

$$\pi_m^{B,contract} = q_m w_m^B - (1-s)\frac{1}{2}\eta\left(\alpha_m^B\right)^2, \tag{21}$$

$$\pi_r^{B,contract} = q_m(p_m^B - w_m^B) + q_r p_r^B - \frac{1}{2}\eta\left(\alpha_r^B\right)^2 + k(q_m + q_r) - s\frac{1}{2}\eta\left(\alpha_m^B\right)^2. \tag{22}$$

The equilibrium decisions (w_m^{B*}, α_m^{B*}, p_m^{B*}, p_r^{B*}, a_r^{B*}) remain the same as in the original Scenario B because the first-order conditions with respect to these variables are unaffected by the lump-sum transfer of the cost (the cost sharing only changes the profit distribution, not the optimal decisions). The manufacturer's profit after cost-sharing is

$$\pi_m^{B,contract*} = \pi_m^{B*} + s\frac{1}{2}\eta\left(\alpha_m^{B*}\right)^2. \tag{23}$$

To make the manufacturer at least as well off as in Scenario M, we need $\pi_m^{B,contract*} > \pi_m^{M*}$. Solving for s gives $s \ge s_{\min} = \dfrac{\pi_m^{M*} - \pi_m^{B*}}{\frac{1}{2}\eta\left(\alpha_m^{B*}\right)^2} = \dfrac{\lambda\left(2\left(2\eta-\theta^2\right)^3 + \left(2\eta-3\theta^2\right)\left(-2\eta+\theta^2\right)^2\lambda + \left(-\eta\theta+\theta^3\right)^2\lambda^3\right)}{\left(2\eta-\theta^2\right)\left(-2\eta+\theta^2(1+\lambda)\right)^2}$, since $\pi_m^{M*} > \pi_m^{B*}$ in the prisoner's dilemma, $s_{\min} > 0$. The e-retail platform will accept the contract only if its own profit after sharing does not fall below its original profit in Scenario B (or below its profit in Scenario M). Given that the total supply chain's profit under scenario B is higher than under scenario M (i.e., $\pi_s^{B*} > \pi_s^{E*} > \pi_s^{M*} > \pi_s^{N*}$), there is a range of s that makes both parties better off. Setting $s = s_{\min}$ exactly lifts the manufacturer's profit to the Scenario M level, while the e-retail platform retains the extra surplus. Thus, the cost-sharing contract resolves the prisoner's dilemma and aligns incentives with the globally efficient outcome. Practical implementation requires the e-retail platform to monitor the manufacturer's innovation expenditure and reimburse a pre-agreed percentage.

6. Conclusions and managerial insights

6.1. Conclusions

This paper develops a rigorous game-theoretic framework to investigate pricing and green innovation decisions in an e-retail platform supply chain where a NB competes directly with a platform-owned PB. The Stackelberg model, with the e-retail platform as the leader and the NB manufacturer as the follower, is solved under four distinct innovation scenarios (N, B, M, E). Parameter constraints (e.g., $\eta > \eta_0$, as specified in Section 3.2) are imposed throughout to ensure the negative definiteness of the Hessian matrices and thus the existence and uniqueness of the equilibrium solutions. All analytical results are derived via backward induction and are supported by rigorous mathematical proofs provided in the Appendix. Within this framework, we obtain the following core conclusions.

Green innovation in products is considered an effective strategy to mitigate brand competition. Our paper focuses on the platform supply chain composed of a manufacturer and an e-retail platform with a PB, and discusses the product pricing and green innovation policies of the NB and the PB in the context of competition on the same e-retail platform. By constructing a Stackelberg game model, we analyze green innovation in four scenarios under the wholesale model and draw the following core conclusions

Firstly, the analysis of the impact of key parameters on the game's equilibrium shows that enhanced network externalities (k) consistently benefit the manufacturer's profit, the wholesale price, and the platform's revenue. Specifically, when the network externalities increase, the following occur: In Scenario N, retail prices for both brands decrease; in Scenario B, retail prices for both brands increase only when competition is weak and green innovation efficiency is high; in Scenario M, the platform consistently reduces the retail price of the PB product and raises the retail price of the NB product only when green innovation efficiency is high ($\eta_0 < \eta < \frac{\theta^2}{1+\lambda}$); in Scenario E, the platform consistently reduces the retail price of the NB product and raises the retail price of the PB product only when green innovation efficiency is high ($\eta_0 < \eta < \frac{1}{4}\theta^2\left(4-\lambda-2\lambda^2\right)$). The effects of other parameters also demonstrate significant scenario dependence.

(i.) In Scenario N, regardless of the market size, intensified competition consistently harms the manufacturer's interests, while the platform can transform competitive pressure into profit growth only in larger markets (Corollary 4.1).

(ii.) In Scenario B, the impact of competition on retail prices is jointly moderated by the market size, competitive intensity, and green innovation efficiency; competition consistently harms the manufacturer's profit, with the platform benefiting only in larger markets (Corollary 4.3). Increased green preference consistently enhances the platform's profit but improves the manufacturer's profit only within certain thresholds of competition and efficiency (Corollary 4.4). Reduced green innovation efficiency consistently reduces the platform's profit while benefiting the manufacturer only within specific thresholds (Corollary 4.5).

(iii.) In Scenario M, competition (increased λ) continues to harm the manufacturer's profit; the platform's profit fluctuates only with market size, and the NB's green innovation level remains unaffected by competitive intensity (Corollary 4.7). Increased green preference can achieve a win–win outcome (Corollary 4.8), while reducing green innovation efficiency comprehensively affects innovation, prices, and profits (Corollary 4.9).

(iv.) In Scenario E, competition still impairs the manufacturer's profit, with the platform benefiting from intensified competition only in larger markets; the PB's green innovation level is jointly influenced by innovation efficiency and competitive intensity (Corollary 4.11). Increased green preference allows the platform to profit from the PB's green monopoly position at the expense of the manufacturer (Corollary 4.12), while reducing green innovation efficiency protects the manufacturer's profit by raising competitive barriers, though severely diminishing the platform's revenue (Corollary 4.13).

In summary, network externalities generally benefit both parties, while intensified competition consistently exerts pressure on the manufacturer across multiple scenarios, with the impact of each parameter being highly dependent on the specific scenario.

Secondly, the manufacturer consistently favors Scenario M, while the e-retail platform consistently favors Scenario B, indicating an asymmetric equilibrium in the green innovation game studied here. Our analysis indicates that Scenario B (green innovation for both NB and PB products) is the game's sole Nash equilibrium strategy: The e-retail platform consistently pushes for green innovation in PB products, compelling the manufacturer to follow suit with the NB products to avoid profit losses. This equilibrium creates a green innovation prisoner's dilemma. Although Scenario B yields the highest total supply chain profits, the manufacturer earns less than under Scenario M.

Finally, standalone innovation by either the NB or PB product more readily achieves high green levels. Under Scenarios M or E, the green innovation levels of the NB or PB product significantly exceed those under Scenario B. This occurs because differentiated monopoly premiums can cover the innovation costs. Conversely, simultaneous green innovation (Scenario B) leads to homogenization of the green attributes, suppressing green innovation investments by both the NB and PB products.

Returning to the three research questions posed in the Introduction, (1) compared with the no-innovation baseline, unilateral or bilateral innovation significantly alters the equilibrium outcomes, with Scenario M maximizing the manufacturer's profit and Scenario B maximizing the platform's profit and total supply chain profit. (2) Competition intensity, green preference, and innovation efficiency all exhibit scenario-dependent effects, with network externalities consistently amplifying the benefits of green innovation. (3) The unique Nash equilibrium is Scenario B (bilateral innovation), but it generates a prisoner's dilemma; a cost-sharing contract is shown to resolve this dilemma and align individual and collective incentives. These findings collectively confirm the theoretical contributions of this paper, extending the green supply chain literature to the platform context with bilateral innovation decisions, revealing the role of network externalities, and identifying the prisoner's dilemma with a corresponding coordination mechanism.

6.2. Managerial insights

Based on the findings, the following management recommendations are proposed.

(1) The e-retail platform achieves its highest profit in Scenario B (bilateral green innovation). To resolve the prisoner's dilemma, it should offer a cost-sharing contract to compensate the NB manufacturer's innovation cost, thereby incentivizing the manufacturer to participate in green

innovation. By doing so, the platform actively maintains the equilibrium of the supply chain ecosystem—balancing its own profit, the manufacturer's profit, and the competitive dynamics between NB and PB products. Without such a coordination mechanism, the manufacturer would prefer Scenario M (manufacturer-only innovation), leading to a suboptimal equilibrium for the overall system. Thus, a cost-sharing contract serves as an effective tool for the platform to sustain a stable and efficient ecosystem. Additionally, the e-retail platform should cultivate consumers' green preference and strengthen network externalities, as both increase the overall market demand and amplify the benefits of green innovation.

(2) The manufacturer's profit is always highest in Scenario M (manufacturer-only innovation). When competition intensity is high and innovation efficiency is low, increasing green investment can paradoxically raise the manufacturer's profit, because the platform relaxes the price pressure on NB products when innovation is costly. Therefore, manufacturers should not automatically cut green innovation investment when efficiency drops; instead, they should first evaluate the platform's strategic response.

(3) Policy-makers should design incentive-compatible measures, such as including network externality indicators in green supply chain assessments and offering innovation subsidies to NB manufacturers in highly competitive categories to counteract the innovation-suppressing effect of platform-related price pressure.

6.3. Limitations and future research

This paper assumes that the e-retail platform and the NB manufacturer have symmetric information regarding consumers' green preference and market demand. However, in practice, e-retail platforms often possess superior data analytics and digital leadership capabilities, which may create information asymmetry. If the e-retail platform can leverage its data-driven decision-making to better predict consumers' green preferences, it could gain a strategic advantage, particularly in Scenario B. This assumption was deliberately adopted to establish a clean baseline that isolates the strategic effects of green innovation and network externalities without confounding informational asymmetries. Relaxing this assumption would require us to analyze signaling or screening games, which we leave for future research. Second, this study only focuses on the resale mode as a traditional cooperation model. However, in the platform economy, the agency mode is also an important sales model that has not been explored in this research. The resale mode serves as a canonical setting in platform-based supply chain research, and our findings provide a benchmark for future comparisons with agency mode outcomes.

Finally, our study considers only deterministic demand and does not account for various sources of uncertainty that exist in real-world e-retail platform operations, such as stochastic consumer demand, fluctuations in green preference, and market risks. Future research could extend our deterministic Stackelberg game framework to incorporate uncertainty by drawing on the rich literature on cooperative game theory under uncertainty (e.g., [47,48]), stochastic optimal control with regime switching and memory (e.g., [49,50]), and stochastic factor models under model uncertainty [51]. For demand forecasting techniques, future research could also draw on the relevant literature (e.g., [52–54]) to better characterize the impact of dynamic market demand on the decision-making of supply chain members. Moreover, future studies could also consider behavioral uncertainties such as investor sentiment (e.g., [55,56]). These extensions would enable a more robust analysis of green innovation and pricing decisions in e-retail platform supply chains under risk and uncertainty.

Author contributions

Yingying Chen: Methodology and writing – original draft, primarily responsible for model development and computation, as well as writing the original draft.;

Chuanxu Wang: Supervision and writing – review and editing, primarily responsible for teaching, training, moderating and providing professional advice to team members.

All authors have read and agreed to the published version of the manuscript.

Use of Generative –AI tools declaration

The authors declare that no artificial intelligence (AI) tools were used in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest.

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