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*Research article*

# A hybrid framework for multi-depot vehicle routing under stochastic demand: integrating adaptive redispatching with joint distribution

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**Abstract:** E-commerce and last-mile growth intensify routing demands, while deterministic demand, independent distribution, and single-mode rescheduling hinder efficiency and realism. To address these issues, we proposed a multi-depot capacity-constrained vehicle routing problem with stochastic demands and joint distribution (MCVRPSD-JD), embedding adaptive skip, reassigning, and redistributing redispatching strategies for failed delivery points. The model integrates chance-constrained programming with cooperative scheduling mechanisms to enhance vehicle utilization and mitigate capacity imbalance under stochastic demand. A hybrid variable neighborhood descent scatter search (HVNDSS) algorithm is developed to balance global search and local refinement across optimization stages. Experimental results demonstrated that the proposed method outperformed baseline algorithms, reducing solution error to 0.17% while achieving notable distribution cost savings. Compared with independent distribution, the joint distribution model lowered costs by an average of 15.91% and reduced failure points. This study contributes a scalable framework for stochastic logistics, offering practical insights for cost-efficient urban distribution systems.

**Keywords:** urban logistics; vehicle routing problem; stochastic demand; joint distribution; hybrid optimization model
**Mathematics Subject Classification:** 90B06, 90C29

## 1. Introduction

With the rapid expansion of e-commerce platforms, instant delivery services, and last-mile urban logistics, urban distribution faces unprecedented pressure and challenges within the logistics system, and has become a key part affecting overall cost optimization and logistics performance [1,2]. The rationality of the vehicle distribution route directly affects the logistics cost to a large extent. Researchers have pointed out that in urban distribution networks, irrational routing can lead to an increase in distribution costs up to 25% [3,4]. Thus, an in-depth investigation of vehicle routing optimization is highly necessary for modern logistics systems.

Vehicle routing problem (VRP), a classical problem in logistics, aims to find the optimal set of routes for a fleet of vehicles delivering goods or services to a given customer base. Some researchers provided a comprehensive classification of VRP studies [5]. Many variants of VRP have been studied extensively, such as VRP with time windows (VRPTW) [6], capacity-constrained VRP (CVRP) [7], green VRP (GVRP) [8,9], VRP with pickups and deliveries (VRPPD) [10], multi-depot VRP (MVRP) [11], VRP with delivery options (VRPDO) [12]. Most models assume that all input demand data are deterministic and known in advance. However, in real-world logistics, different demand setups have been observed in most distribution processes, which are typically dynamic and hardly predictable, posing significant solution challenges compared to deterministic types [13]. Thus, considering demand uncertainty in the distribution process is gradually becoming a core problem faced by logistics companies in their planning and operations [14,15].

To cope with the challenge of fluctuating customer demand, some scholars have proposed VRP with stochastic demands (VRPSD) [16,17]. For this problem, most researchers have used a two-phase solution logic, where a complete prior route is designed in the first phase, and the planned route is implemented in the second phase to find a sequence of routes that minimizes the total distribution cost [18,19]. During the second implementation phase, failures may occur when the vehicle load is insufficient to meet the observed customer demand. In this case, strategies such as preventive return trips [19], chance-constraint modeling [20], and recourse actions [21] are adopted to compensate for the failure points of the designed routes. While these strategies satisfy customer needs as much as possible, they inevitably increase additional distribution costs. To achieve further cost savings, collaborative strategies for multiple logistics facilities are essential in distribution network optimization [22]. Among them, the joint distribution model of multiple centers, which focuses on the integration of vehicle capacity, shows significant potential for reducing duplicate transportation, optimizing distribution routes, and improving vehicle utilization by intelligent scheduling under digital connectivity and the sharing economy [23,24]. However, traditional VRPSD optimization of distribution logistics often ignores this cooperative mechanism among centers and vehicles, making it difficult to mitigate the effects of distribution capacity imbalance.

Based on the above review, studies lack integrated frameworks for multi-depot coordination under stochastic demand that incorporate adaptive dynamic redispatching, as well as hybrid methodological frameworks that effectively balance global search with local optimization. To bridge these gaps, we construct a multi-depot capacity-constrained VRPSD with joint distribution (MCVRPSD-JD) model to achieve cooperative vehicle scheduling optimization among multiple depots under stochastic demand, thereby improving delivery efficiency and reducing distribution costs. The major contributions are shown as follows:

(1) An optimal MCVRPSD-JD model is developed based on stochastic chance-constrained programming considering transportation resource sharing. Unlike prior VRPSD models that treat depots independently or rely on single-mode replenishment, the proposed model embeds an adaptive cross-depot redispatching strategy capable of dynamically determining whether to skip, reassign, or redistribute tasks to accommodate the time-varying random customer demand.

(2) Considering the global search capability of the scatter search (SS) algorithm and the local optimization advantage of the variable neighborhood descent (VND) algorithm, a hybrid variable neighborhood descent scatter search algorithm (HVNDSS) is designed to improve the solution performance of the MCVRPSD-JD model.

(3) Multiple benchmark distribution models and experimental cases are built to validate the proposed method in terms of distribution cost, failure reduction, and vehicle utilization. Policy implications are analyzed through multidimensional parameter sensitivity studies. This method is scalable for various urban cases with different characteristics and provides a new paradigm for stochastic logistics distribution systems.

The organization of this paper is structured as follows: In Section 2, we conduct a comprehensive review of pertinent prior research. In Section 3, the research problem is delineated, succeeded by Section 4, where we elaborate on the methodology proposed. In Section 5, we showcase the experiments conducted and the corresponding results. Final remarks are concluded in Section 6, while Appendices A and B list the main case information and route results from the paper.

## 2. Literature review

### 2.1. Vehicle routing under uncertainty

For the uncertain vehicle routing problem in logistics distribution, the researchers mainly focus on the following three directions: stochastic demand modeling, restocking strategy optimization, and solution algorithm design. In terms of stochastic demand modeling, many researchers share a common goal of improving routing efficiency while accounting for unpredictable factors, such as travel times, demand, and service times. For example, dealing with stochastic demand based on probability distributions or distributional ambiguities, some scholars established routing methods based on demand evaluation metrics (e.g., requirements index or satisfaction level) to assess the performance of route optimization schemes in meeting uncertain travel times, demand, and vehicle capacity [13,25]. When historical data are available to estimate the demand distribution, a probability distribution of stochastic planning uncertainties is applied to optimize the expected outcomes, and the VRPSD problem was formulated as a chance-constrained stochastic optimization problem in distribution systems [20,26]. Additionally, considering the uncertainties of supply and demand, travel time, and service time, some scholars have established a multi-objective stochastic programming model and a multi-objective Green VRP (GVRP) model to trade off from the perspectives of travel cost, energy consumption, and customer satisfaction [27–29]. To further increase the robustness of path planning schemes, robust optimization is widely used to handle uncertain demand problems. Studies differ in how uncertainty is represented and how conservatism is controlled, including hard time windows [14], cross-docking networks [15], distributionally robust models [30], and uncertain travel times [31]. Although these approaches improve route feasibility, they focus on single-system routing and lack mechanisms for cross-depot coordination or adaptive redispatching after demand realization.

In terms of restocking strategy optimization, due to the random nature of demand, many scholars have taken corresponding measures to address the prominent practical issues of unmet demand. Early on, a paired vehicle contingency strategy for single-depot VRPSD was proposed [3]. In this strategy, vehicles are assigned in pairs so that in the event of demand overload, one vehicle can use its remaining capacity to assist another vehicle in completing unfinished tasks. An approximate dynamic programming algorithm was then used to improve the efficiency of redispatching in multi-stage decision-making in VRPSD scenarios [18]. Moreover, an exact procedure was proposed to design the optimal restocking strategy with the goal of minimizing the total expected routing cost [16]. Additionally, some scholars have introduced a rule-based recourse policy where vehicles can engage in proactive preventive replenishment, returning early to restock in specific situations to avoid subsequent failures [19,21,32]. The active restocking strategy is bound to result in cost savings over a prior path strategy, but accurately determining the optimal proactive replenishment point remains challenging due to the random nature of customer demand.

In terms of solution methods, they can be broadly classified into two major categories. The first category includes exact algorithms for solving small-scale problems, such as stochastic planning models with recourse [16,33], Monte Carlo scenario models [18], chance-constrained and dynamic planning models [20], robust optimization models [34], based on decomposition and pricing algorithms such as branch-and-bound pricing [12] and integer L-shape algorithms [19]. However, since VRPSD is an NP-hard combinatorial optimization problem, the exact method is not effective when dealing with practically larger-scale problems [35]. The second category is given by heuristic and metaheuristic methods aiming to find near-optimal solutions, including tabu search algorithms [3,25], genetic algorithms [35,36], large neighborhood search algorithms [14,31,37], the cuckoo optimization algorithm [38], and hybrid metaheuristic algorithms [29,39]. The use of heuristic and metaheuristic methods in VRP variants has also been systematically reviewed [40]. In more complex environments, some scholars have chosen reinforcement learning-based approaches to seek high-quality solutions [41–43]. However, this often requires strategy training based on large amounts of real data. Hence, in uncertain scenarios where obtaining substantial real-world data is challenging, consideration should be given to developing efficient solution algorithms.

### *2.2. Joint distribution logistics*

Logistics companies have typically operated distribution independently and, as a result, a lot of research has been conducted on the single distribution model [15,35,40]. The obvious drawbacks of single distribution are poor full load rates, a very high number of rented vehicles, and high freight costs, which urgently require further development of the logistics industry. Driven by digital connectivity and the sharing economy, the joint distribution model, with capacity integration at its core, is gradually being combined with intelligent scheduling as a new effective way to realize sustainable logistics development [9,23,24]. Researchers have developed joint distribution models from different perspectives, including two-echelon network optimization [44], electric-vehicle routing [45], online crowdsourced delivery [46,47], and blockchain-enabled coordination [48]. For example, linear optimization models for two-echelon logistics joint distribution networks have been established to optimize distribution paths and provide a cooperative mechanism for logistics companies [22,46]. By sharing vehicles and customers, joint distribution of electric vehicles based on charging and battery-swapping in fresh food logistics has also been examined under the constraint of minimum freshness of fresh products, which effectively reduces distribution cost [48]. These studies demonstrate the value of resource sharing and cooperative scheduling, but most of them assume deterministic or scenario-specific settings, with limited treatment of stochastic customer demand and adaptive cross-depot redispatching.

In addition to theoretical modeling research, scholars in various countries study different joint distribution methods in relation to practice. For example, a joint distribution GVRP model based on the cold chain logistics industry has been constructed in combination with the carbon trading mechanism to achieve a balance between the generated economic and environmental benefits [45]. With the deepening of supply chain collaboration and the advancement of technological innovation, a digital connectivity-enabled B2C and O2O joint distribution system has been presented, which can significantly improve logistics services and reduce carbon emissions by collecting and scheduling real-time logistics information [24]. Additionally, a hybrid approach combining blockchain technology and a coordination contract for joint distribution in end-to-end logistics has been proposed, which provides a solution to the trust problem in the logistics industry and supply chain management [49]. The use of collaborative strategies between logistics facilities and the formation of distribution alliances constitutes a sustainable approach to VRP-based network optimization. However, there are challenges in coordinating uncertain demands for multi-center joint distribution optimization.

Overall, while research offers valuable insights into vehicle routing under uncertainty and joint distribution logistics, few researchers address the simultaneous incorporation of demand uncertainties in scenarios with multi-depot joint distribution, as well as the adaptive adjustment of vehicle routing strategies. In addition, optimization methods mostly use static or batch-calculation methods, which cannot respond to dynamic demands. Accordingly, we construct an MCVRPSD-JD model embedded with an adaptive redispatching strategy to optimize the real-time distribution plan under uncertain demand environments and design an HVNDSS algorithm for efficient solutions considering global search and local optimization model.

## 3. Problem statement

### 3.1. Problem description

The proposed problem is a complex decision-making process that integrates multiple distribution centers, uncertain customer demand, and vehicle routing optimization. The scenario is shown in Figure 1.

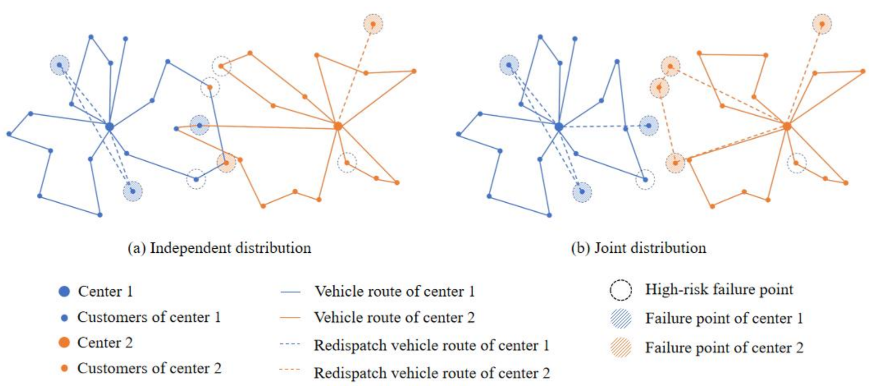


**Figure 1.** Different distribution models.

Figure 1(a) depicts an independent distribution model wherein each center dispatches vehicles according to a demand forecast-based plan during the first stage. However, owing to demand uncertainty, certain customers may not be successfully served, resulting in service failures. Based on historical data, customer locations associated with a high likelihood of service failure are identified as high-risk points. These unserved customers then require a second-stage supplemental delivery, typically executed via rerouting or dedicated backup vehicles, to ensure full demand fulfillment. In contrast, Figure 1(b) illustrates a joint distribution model. In this approach, customer locations may be serviced by vehicles from either distribution center during the pre-optimization phase. By enabling cross-center vehicle assignments, the number of first-stage service failures could be reduced, thereby decreasing the need for costly recourse actions in the second stage. Consequently, the overall distribution cost is minimized across both centers.

A distinctive feature of this problem is that customer demands remain unknown until vehicle arrival, and each customer may potentially be served by a vehicle originating from any distribution center within the distribution network. Accordingly, based on such problem scenarios, the primary objective is to devise efficient vehicle routes and service sequences that accommodate uncertain demands while minimizing the total distribution cost across multi-center logistics networks.

### *3.2. Model assumptions*

For research purposes, it is typically assumed that each distribution center has a limited fleet of vehicles, customer demand follows a known probability distribution, and the scheduling cost between distribution centers can be quantified. The objective is to determine the optimal routing scheme that minimizes distribution cost while satisfying stochastic demand constraints. To clarify the research problem and facilitate model formulation, the following assumptions are made:

(1) Customer demand is modeled as a random variable. For each distribution center, demand follows a known Poisson distribution [20,25]. Actual customer demand remains unknown until the vehicle arrives at the customer location.

(2) The locations of distribution centers and customers are known, and all points in the network are fully interconnected.

(3) Vehicles depart from the distribution centers in a fully loaded state. All vehicles depart from and return to their respective distribution centers.

(4) Each distribution center operates vehicles of identical model and performance. In the pre-optimization stage, each vehicle is assigned to one route.

## 4. Methodologies

Based on the above qualitative problem description and assumptions, in this section, we present the mathematical formulation of the proposed model and algorithm. Most VRPSD studies model demand uncertainty but ignore coordination across depots. Conversely, joint distribution research improves cooperation but relies heavily on deterministic assumptions. Furthermore, rescheduling strategies are either rigid or costly, without adaptive capabilities to respond to real-time demand. These limitations motivate the development of the MCVRPSD-JD model and the HVNDSS algorithm to jointly address stochastic demand, multi-depot coordination, and adaptive redispatching under a unified framework.

### 4.1. MCVRPSD-JD model construction

#### 4.1.1. Stochastic chance-constrained model

The symbols used in the proposed model are listed in Table 1.

**Table 1.** Meaning of symbols.

| Symbols | Meaning |
|---|---|
| $G$ | The logistics distribution network under multiple distribution centers, $G = (D, V, E)$ |
| $N$ | The set of all nodes, $N = D \cup V$ |
| $D$ | The set of distribution centers, and $m$ represents the total number of distribution centers, $D = \{d_1, d_2, \cdots, d_m\}$ |
| $S$ | The set of routes served by vehicle |
| $V$ | The set of customer points for all distribution centers, and $n$ represents the total number of customer points, $V = \{v_1, v_2, \cdots, v_n\}$ |
| $V_d$ | The set of customer points of distribution center $d$ |
| $V_k$ | The set of customer points served by distribution vehicle $k$ after task division, $V_k \subseteq V_d$ |
| $V_f$ | The set of failure-point |
| $E$ | The set of routes between all nodes, $E = \{(i, j) \mid i, j \in N\}$ |
| $K$ | The set of all distribution vehicles, $K = \{k_1, k_2, \cdots, k_t\}$ |
| $k_d$ | The set of vehicles in distribution center $d$ |
| $M_k$ | The expectation of total customer demand for vehicle $k$ |
| $S_k$ | The variance of total customer demand for vehicle $k$ |
| $c_{ij}$ | The cost of paths between point $i$ and point $j$ |
| $Q$ | The capacity of distribution vehicle |
| $\overline{Q}$ | The adjusted artificial capacity |
| $\xi_i$ | The random demand of customer point $i$ |
| $Var[\xi_i]$ | The variance of the customer demand |
| $E[\xi_i]$ | The expectation of the customer demand |
| $q$ | The demand of customer point $i$ |
| $x_{ij}^k$ | Equals 1 if the vehicle $k$ travels directly from point $i$ to point $j$, and 0 otherwise |
| $y_i^k$ | Equals 1 if the customer $i$ is serviced by the vehicle $k$, and 0 otherwise |
| $\lambda$ | The Poisson distribution parameter |
| $\alpha$ | The confidence level |
| $M$ | A penalty cost |
| $\tau$ | The $\alpha$ quantile of the distribution |

The demand $\xi_i$ of customer point $i (i \in V_d)$ is an independent random variable following a discrete probability distribution. Assume that $\xi_i$ follows the Poisson distribution, i.e., $\xi_i \sim \text{Poisson}(\lambda)$. The expectation $E[\xi_i]$ and the variance $Var[\xi_i]$ of the customer demand of distribution center $d$ are both $\lambda$. For example, the specific probability of the demand $q$ of customer point $i$ is $p = \Pr(\xi_i = q)$ and $0 \leq q \leq Q$. The decision variable for route selection is $x_{ij}^k$, which equals 1 if the vehicle $k$ travels directly from point $i$ to point $j$, and 0 otherwise. The decision variable for customer service is $y_i^k$, which equals 1 if the customer $i$ is serviced by the vehicle $k$, and 0 otherwise.

For the classical deterministic VRP (DVRP), customer demand is known in advance, and the optimization objective is typically limited to designing vehicle routes that satisfy this demand. In contrast, for the MCVRPSD-JD proposed in this study, customer demand is uncertain and is only revealed upon the vehicle's arrival at the customer location. This stochasticity increases problem complexity and may render traditional routing schemes infeasible. To address the stochasticity, we adopt a two-stage stochastic chance-constrained programming approach. The stage processes are shown as follows:

- Stage 1: Vehicle routes are optimized based on predicted customer demand, using the probability distribution of demand and subject to vehicle capacity constraints.
- Stage 2: If demand exceeds vehicle capacity during route execution, it may involve adjusting the remaining routes or redispatching vehicles. We employ stochastic chance constraints to ensure that the probability of total demand exceeding vehicle capacity during a route does not surpass a predefined risk tolerance level.

In Stage 2, demand uncertainty may cause vehicle overloading, meaning that the total demand along a route exceeds vehicle capacity $Q$. To control this risk, the chance constraints $\Pr\left(\sum_{i\in V_k}\xi_i \le Q\right) \ge \alpha$ and $\alpha \in [0,1]$ are introduced to denote the risk tolerance level, i.e., the probability of satisfying the capacity constraint. For a given value of $\alpha$, if $\Pr\left(\sum_{i\in V_k}\xi_i \le Q\right) \ge \alpha$, the vehicle can serve all customers on its route, and if $\Pr\left(\sum_{i\in V_k}\xi_i \le Q\right) < \alpha$, the vehicle cannot fulfill all demands. For example, $\alpha = 0.95$ means that there is 95% probability that the vehicle will not be overloaded. The chance constraint guarantees that, with high probability, the vehicle can satisfy customer demand, while permitting a small probability of failure $(1-\alpha)$. Accordingly, the proposed MCVRPSD-JD model based on stochastic chance-constrained programming is formulated as follows:

$$\mathbf{min} \quad \sum_{d\in D}\sum_{i\in N}\sum_{j\in N}\sum_{k\in K_d} c_{ij}x_{ij}^k \tag{1}$$

$$\mathbf{s.t.} \quad \Pr\left(\sum_{i\in N}\sum_{j\in V} x_{ij}^k\xi_j \le Q\right) \ge \alpha, \forall k \in K \tag{2}$$

$$\sum_{i\in N} x_{ij}^k - \sum_{i\in N} x_{ji}^k = 0, \forall j \in V, \forall k \in K \tag{3}$$

$$\sum_{j\in V} x_{dj}^k = \sum_{i\in V} x_{id}^k \le 1, \forall k \in K, \forall d \in D \tag{4}$$

$$x_{ij}^k = 0, \forall i = j, \forall i,j \in V, \forall k \in K \tag{5}$$

$$\sum_{j\in V} x_{ij}^k = y_i^k, \forall i \in V, \forall k \in K \tag{6}$$

$$\sum_{i\in V} x_{ij}^k = y_j^k, \forall j \in V, \forall k \in K \tag{7}$$

$$\sum_{k\in K} y_i^k = 1\,, \forall i \in V \tag{8}$$

$$\sum_{k\in K_d} y_i^k + M\sum_{k\notin K_d} y_i^k = 1,\ \ i \in V, \forall d \in D \tag{9}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ij}^k \le |S| - 1\,, \forall S \subseteq V, \forall k \in K \tag{10}$$

$$x_{ij}^k \in \{0,1\}, y_i^k \in \{0,1\}, \forall i,j \in V, k \in K \tag{11}$$

where Equation (1) means that the objective is to minimize the total distribution cost of vehicles across all distribution centers. Equation (2) ensures that, with a confidence level of $\alpha$, the total demand of customers served along each route does not exceed the vehicle capacity. Equation (3) guarantees flow balance, which means the route entering and leaving each customer point must be identical. Equation (4) ensures that vehicles from each distribution center can have only one departure route and that they must return to their original distribution center. Equation (5) indicates that there is no path connectivity between identical points. Equations (6) and (7) link the decision variables $x_{ij}^k$ and $y_i^k$, ensuring that the customer point is connected to a path whenever it is served by the vehicle, thereby ensuring consistency between route planning and customer assignment. Equation (8) ensures that each customer point can be served by only one vehicle. Equation (9) ensures that the solution prioritizes the use of the original distribution center's vehicles by setting a penalty cost $M$. If the customer is served by a vehicle from the original distribution center, there is no additional cost. However, if a vehicle from another distribution center is used, an additional penalty $M$ is imposed. Equation (10) prevents subroute loops and ensures that each route is a complete path from and back to the distribution center. If some customers are served by a vehicle, the number of paths the vehicle takes between those customers cannot be equal to $|S|$ but must be less than $|S| - 1$, and $S$ represents the set of routes served by the vehicle. Equation (11) shows the decision variables $x_{ij}^k$ and $y_i^k$, which indicate whether to select a route and whether to assign a vehicle to a customer, respectively.

#### 4.1.2. Equivalent handling of chance constraints

The stochastic chance constraints, as shown in Equation (2), can be reformulated as an equivalent deterministic constraint. For distribution vehicle $k$, the demand of served customer points $\xi_j$ is an independent and identically distributed random variable. The aggregate demand across all customers is the sum of their individual demands on the route. Based on the Central Limit Theorem, the total demand of the customers can be approximated by a normal distribution, as shown in Equation (12).

$$\sum_{i,j\in V} x_{ij}^k \xi_j \sim N\left(\sum_{i,j\in V} x_{ij}^k E[\xi_j], \sum_{i,j\in V} \left(x_{ij}^k\right)^2 Var[\xi_j]\right) \tag{12}$$

Then the expectation $M_k$ and the standard deviation $S_k$ of the total customer demand for the vehicle are shown in Equations (13) and (14).

$$M_k = \sum_{i,j\in V} x_{ij}^k E[\xi_j] \tag{13}$$

$$S_k = \sqrt{\sum_{i,j\in V} \left(x_{ij}^k\right)^2 Var[\xi_j]} \tag{14}$$

If there exists a constant $\tau$ that is the $\alpha$ quantile of the distribution and satisfies the condition shown in Equation (15), then the constraint of Equation (2) holds and can be replaced by the deterministic constraint shown in Equation (16).

$$\Pr\left(\left(\sum_{i\in N}\sum_{j\in V} x_{ijk}\cdot\xi_j - M_k\right)/S_k \le \tau\right) = \alpha, \forall k \in K \tag{15}$$

$$M_k + \tau S_k \le Q, \forall k \in K \tag{16}$$

The constraint is usually nonlinear and has an expanded form, as shown in Equation (17).

$$\sum_{i,j\in V} x_{ij}^k E[\xi_j] + \tau\sqrt{\sum_{i,j\in V}\left(x_{ij}^k\right)^2 Var[\xi_j]} \le Q, \forall k \in K \tag{17}$$

Last-mile logistics demand is usually discrete and arrival-based, such as the number of parcels or orders generated within a service period. Since these orders can be regarded as independent customer arrivals, the Poisson distribution is suitable for describing stochastic customer demand [17, 20, 25]. Given that $\xi_j$ is assumed to follow a Poisson distribution, the random component of the original constraint $\sum_{i\in N}\sum_{j\in V} x_{ij}^k \xi_j$ can be represented by its expectation $E[\xi_j]$, thereby linearizing the formulation. Assuming the random variable distribution has no extreme deviations (e.g., outliers far from the average), and given the property of the Poisson distribution that its mean equals its variance, fluctuations can be considered negligible. Thus, the standard deviation $S_k$ can be analytically incorporated into the adjusted artificial capacity, which simplifies the original constraint of Equation (2) to Equation (18).

$$\sum_{i,j\in V} x_{ij}^k E[\xi_j] \le \overline{Q} \tag{18}$$

where $\overline{Q}$ represents the adjusted artificial capacity and $\overline{Q} = \left(2Q + \tau^2 - \sqrt{\tau^4 + 4Q\tau^2}\right)/2$. The artificial capacity $\overline{Q}$ ensures that the vehicle can satisfy probabilistic demand. Although Equation (18) improves computational tractability, it is an approximate deterministic reformulation rather than a strict equivalent transformation. If customer demand is over-dispersed, temporally clustered, or affected by extreme orders, the Poisson property may no longer hold, and this approximation may underestimate route-level demand fluctuation, which is suitable for scenarios characterized by moderate demand levels without obvious outliers. Additionally, the artificial capacity ($\overline{Q}$) and confidence level ($\alpha$) help control the risk from stochastic demand.

*4.2. HVNDSS algorithm design*

To address the stochastic nature of customer demand in MCVRPSD-JD, we propose an HVNDSS algorithm, which integrates Scatter Search (SS) and Variable Neighborhood Descent (VND) methods. HVNDSS leverages the complementary strengths of SS and VND to enhance solution quality and computational efficiency. SS primarily performs global exploration by combining features of solutions to generate new candidates. This improves solution diversity, enabling the algorithm to escape local optima. VND focuses on local search by systematically switching among neighborhood structures. It avoids premature convergence and improves solution quality. The HVNDSS algorithm comprises pre-optimization and failure-point redispatching stages, as illustrated in Figure 2. The overall pseudocode of the HVNDSS algorithm is shown in Algorithm 1.

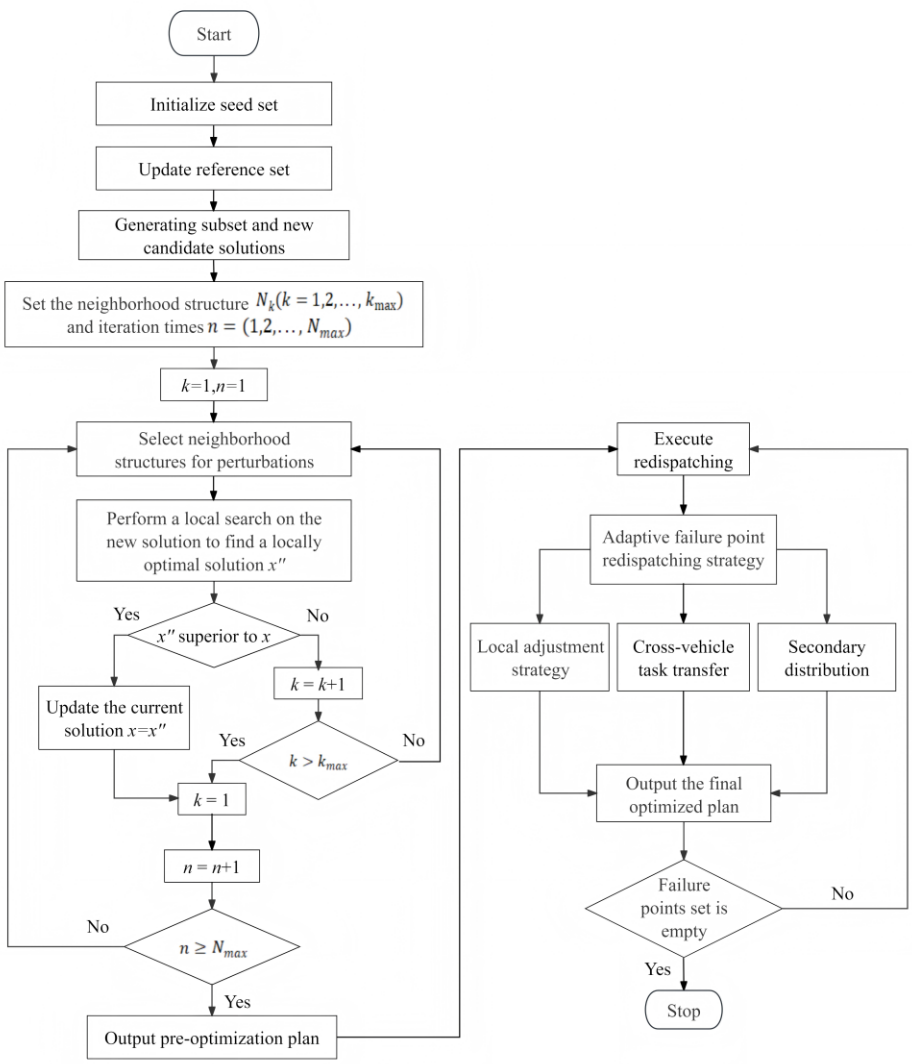


**Figure 2.** The framework of the HVNDSS algorithm.

Algorithm 1 Overall workflow of HVNDSS

Input: Customer set $C$, depot set $D$, vehicle set $K$, vehicle capacity $Q$, stochastic demand $d_i$, confidence level $\alpha$, maximum iterations $N_{max}$

Initialize a feasible solution population based on customer permutations and capacity constraints

Improve the initial solutions using a basic local search strategy

Construct the reference set according to solution quality and diversity

while the stopping criterion is not satisfied ($n < N_{max}$) do

  Generate new candidate solutions through subset combination in SS

  Apply VND to each candidate solution using relocate, exchange, and 2-opt neighborhoods

  Evaluate the improved solutions according to the total distribution cost

  Update the reference set if better or more diverse solutions are obtained

end while

Obtain the pre-optimized vehicle routes

Execute the routes under stochastic customer demand

if failure points occur then

  Evaluate the remaining load and stochastic feasibility of subsequent customers

  Apply the adaptive skip, reassigning, and redistributing redispatching strategy

end if

Output: Final vehicle routes and total distribution cost

In the pre-optimization stage, an initial set of feasible solutions is rapidly constructed to satisfy problem constraints. A reference set updating strategy is then employed to iteratively enhance solution quality. High-quality solutions obtained from local SS optimization are subsequently incorporated into the VND phase, which strengthens global exploration and prevents premature convergence. Within the VND phase, multiple neighborhood structures (relocation, exchange, and 2-opt) are applied sequentially to improve solutions. The search process within each neighborhood is dynamically adjusted according to the quality of the current and neighboring solutions. In the failure-point redispatching stage, unmet customer demands are resolved by adjusting the pre-optimization plan. The final optimal solution is then obtained after redispatching and returned as the algorithm's output.

In terms of computational complexity, the proposed HVNDSS algorithm has an approximate time complexity of $O(N_{max} \cdot b^2 \cdot n^2)$, where $N_{max}$ is the maximum number of SS iterations, $b$ is the reference set size, and $n$ is the number of customer points. This complexity is dominated by the VND local search, which evaluates up to three neighborhood structures (relocate, exchange, 2-opt) each with $O(n^2)$ operations per iteration. The theoretical complexity indicates that the algorithm scales quadratically with problem size, which is acceptable for offline planning in multi-depot stochastic routing problems.

### 4.2.1. Optimization of the SS algorithm

The SS algorithm begins with seed set initialization, where candidate solutions are generated using integer encoding. In this scheme, 0 denotes the distribution center, while positive integers represent customer points. Feasible solutions are constructed by randomly permuting all customer points, with each permutation representing a potential visiting sequence. This randomization ensures broad coverage of the solution space and maintains diversity. To further enhance the quality of the initial population, local search is applied to refine these randomly generated solutions. For instance, a 2-opt strategy is employed to remove redundant or inefficient path segments.

Next, the reference set updating process is performed. Given an initial solution set $P$, solution quality is evaluated to extract a high-quality subset $refset1$ of size $b_1$. In parallel, a diversity subset $refset2$ of size $b_2$ is constructed based on geometric distance measures. Specifically, for each solution in $P$ (except $refset1$) and $refset1$, the solution with the maximum of calculated distances is selected as the current diversity solution. The screening process is repeated $b_2$ times until $refset2$ is formed. Through iterative updating, the seed set is continually enriched with new solutions, thereby preserving quality and diversity.

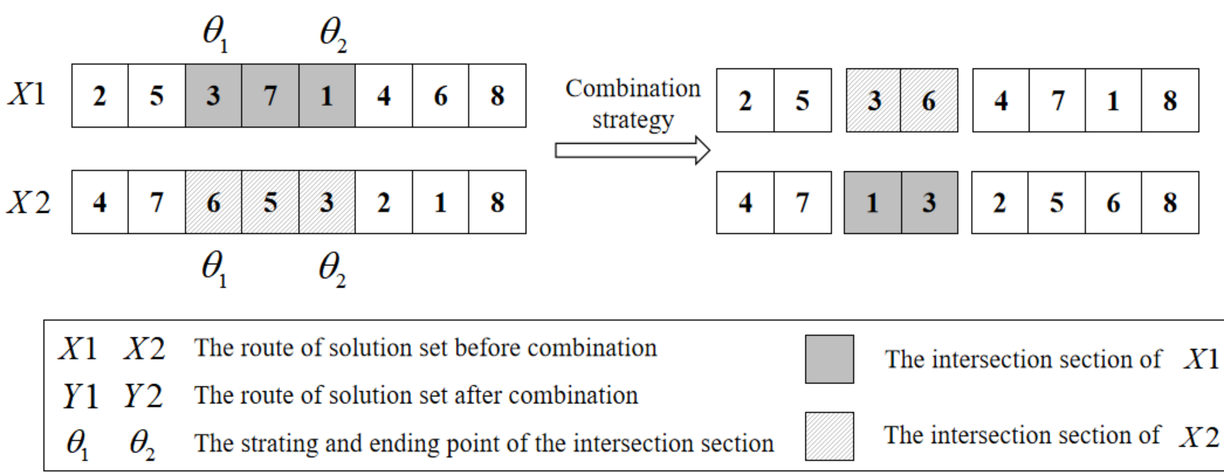


**Figure 3.** Example of a subset combination to generate a new solution.

Finally, a solution combination is carried out to expand the search space. Representative solutions are selected from the reference set based on quality and diversity, and subsets of size 2–5 are constructed to balance exploration with computational efficiency. New solutions are then generated using a crossover-based recombination strategy. Specifically, given two reference solutions $X1$ and $X2$, random crossover segments are selected by positions $\theta_1$ and $\theta_2$, where $\theta_1 < \theta_2$. In constructing $Y1$, the path from the start point of $X1$ to $\theta_1$ is retained, while the segment from $\theta_1$ to $\theta_2$ in $X2$ is inserted in reverse order. Duplicate customer points are removed to ensure feasibility, and unvisited customers are reinserted in their original order. A symmetric operation is performed to generate $Y2$, with $X1$ and $X2$ interchanged. This combination strategy enhances structural diversity, avoids premature convergence, and produces solutions with improved path characteristics. An illustrative example of this procedure is presented in Figure 3.

### 4.2.2. Optimization of the VND algorithm

Unlike traditional local search, the VND algorithm dynamically switches among multiple neighborhood structures during the search process, thereby enhancing its ability to escape local optima. Based on the literature [14], three neighborhood structure schemes are incorporated into the VND mechanism: relocate, exchange, and 2-opt operation.

The relocate operation removes a point from its current position and reinserts it elsewhere in the path. Specific operations are shown as follows: Select an element $s_i$ in the solution $S$ , select a target location $j(j \neq i)$, move $s_i$ to location $j$, and generate a new solution $S'$. The location $j$ can be a relocation within the same path (internal relocate), as shown in Figure 4(a), or a relocation across paths (external relocate), as shown in Figure 4(b). This operation is particularly effective in resolving load imbalance or excessively long routes, as it quickly adjusts inefficient paths and is especially suitable for correcting failure points caused by fluctuations in customer demand.

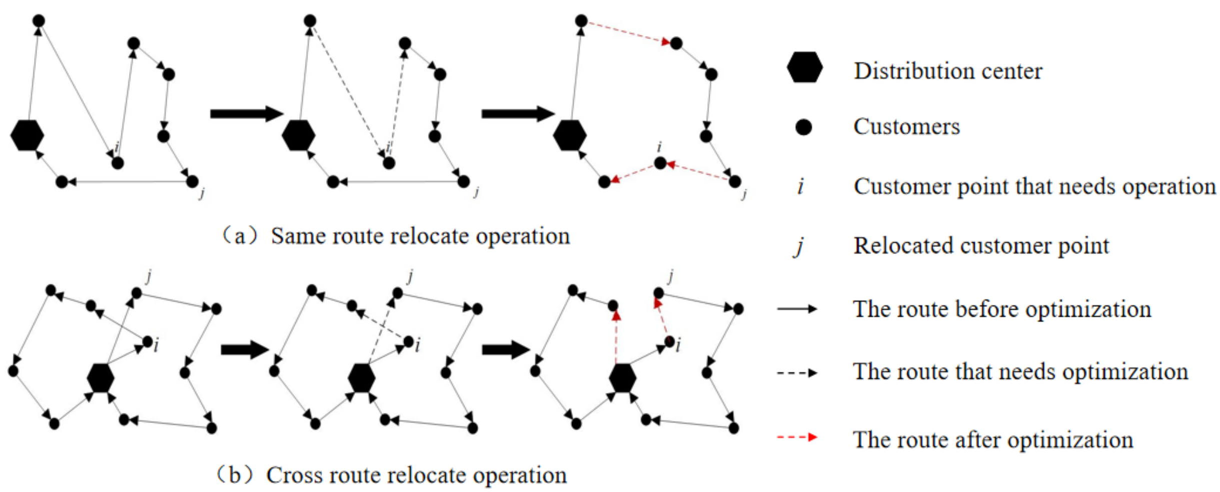


**Figure 4.** The relocate operation.

The exchange operation generates a new candidate solution by swapping two elements, thereby exploring a better solution. The specific operation is to select two elements, $s_i$ and $s_j$, in the solution $S$, and exchange the positions of $s_i$ and $s_j$ to generate a new solution $S'$. This operation can be performed within the same route (internal exchange), as illustrated in Figure 5(a), or across routes (external exchange), as shown in Figure 5(b). Compared to the relocate operation, the exchange operation exerts a smaller impact on the overall path structure, making it particularly suitable for detailed optimization.

The 2-opt operation explores improved solutions by removing two edges from the current solution and reconnecting them to generate a new candidate route. When $i$ and $j$ are in the same path, the operation is performed by selecting two edges $(s_i, s_{i+1})$ and $(s_j, s_{j+1})$ $(i < j)$ in the solution $S$, deleting the selected two edges, reconnecting the paths to $(s_i, s_j)$ and $(s_{i+1}, s_{j+1})$, and reversing the sub-paths from $s_{i+1}$ to $s_j$ to generate a new solution $S'$, which is shown in Figure 6(a). When $i$ and $j$ are not in the same line, there are four major exchange methods, depending on how the paths are connected and whether the paths are reversed, including sequential exchange, reverse exchange, sequential cross-exchange, and reverse cross-exchange. The paths in which $i$ and $j$ are located are noted as $P_1$ and $P_2$, respectively, as shown in Figure 6(b). The 2-opt operation is particularly effective in reducing path crossings and shortening total travel distance. Deleting and reconnecting edges enables significant structural adjustments to the solution, thereby achieving strong optimization effects.

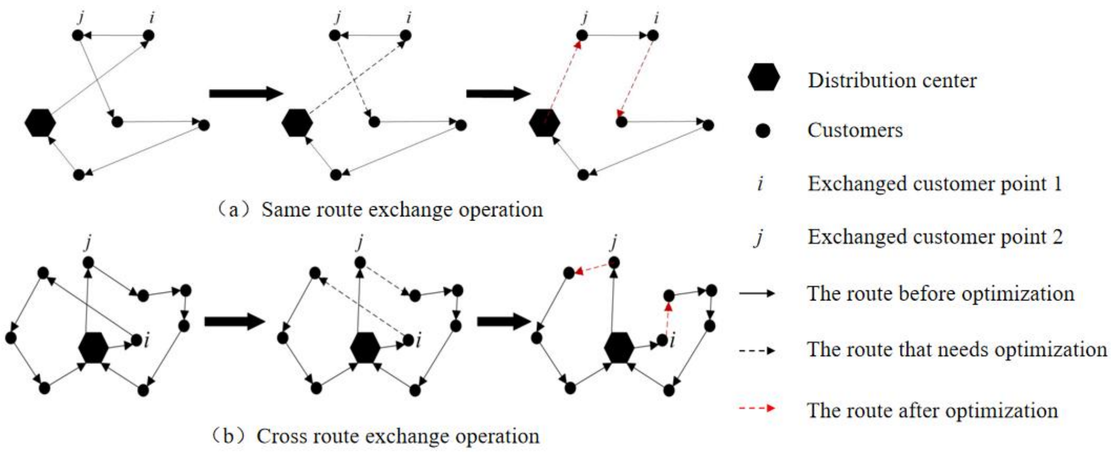


**Figure 5.** The exchange operation.

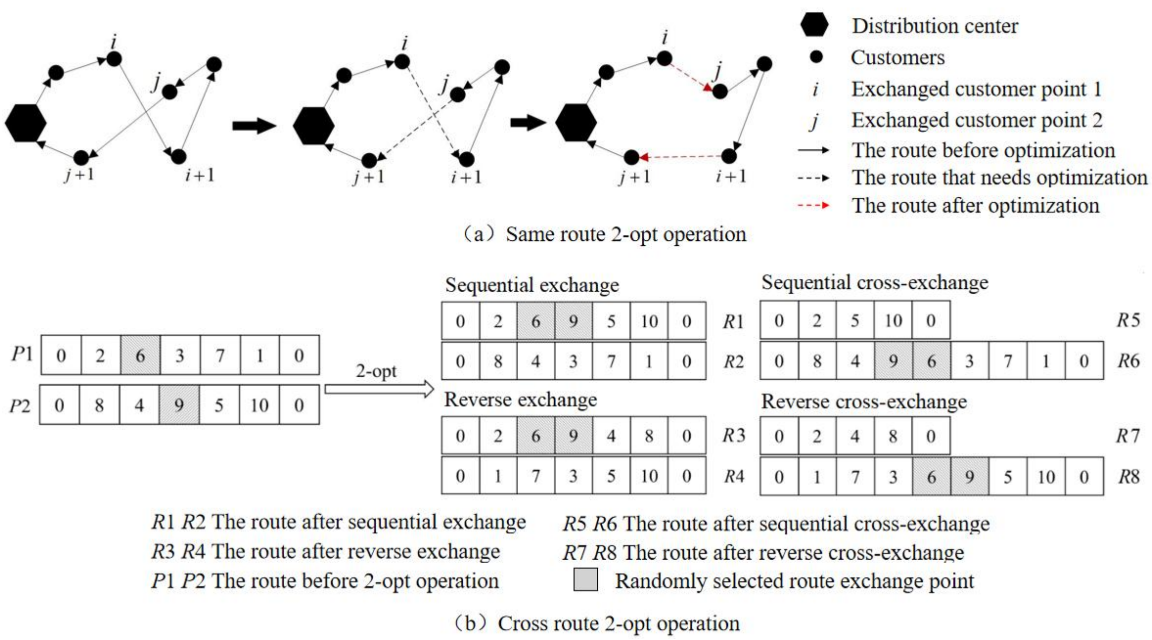


**Figure 6.** The 2-opt operation.

#### 4.2.3. Adaptive redispatching strategy

If the total customer demand along a route exceeds the vehicle's capacity during actual execution, some customers' demands may not be satisfied, thereby generating a failure point. According to the model constraints, failure points arise mostly from two factors, including inappropriate confidence levels in chance constraints and large demand fluctuations. Typically, delivery failures caused by stochastic demand are handled by returning vehicles to depots for replenishment and then re-planning subsequent routes in the second phase. Recourse policies mostly include simple, preventive, and rule-based replenishment strategies [16,21,50]. For example, simple replenishment usually requires the vehicle to return to the depot after a failure occurs, which may significantly increase delivery costs. Preventive replenishment attempts to reduce such costs by considering depot returns in advance during route planning [16,50]. However, due to the random nature of customer demand, accurately determining the optimal replenishment point remains challenging. Most traditional strategies handle failures through depot return, replenishment, or route-level adjustment, with limited consideration of cross-depot cooperation. To overcome these limitations, we propose an adaptive failure-point redispatching strategy. The proposed strategy first evaluates the remaining vehicle load and subsequent demand risk, and then adaptively applies skip, reassign, and redistribute operations across vehicles and depots. The overall process is illustrated in Figure 7.

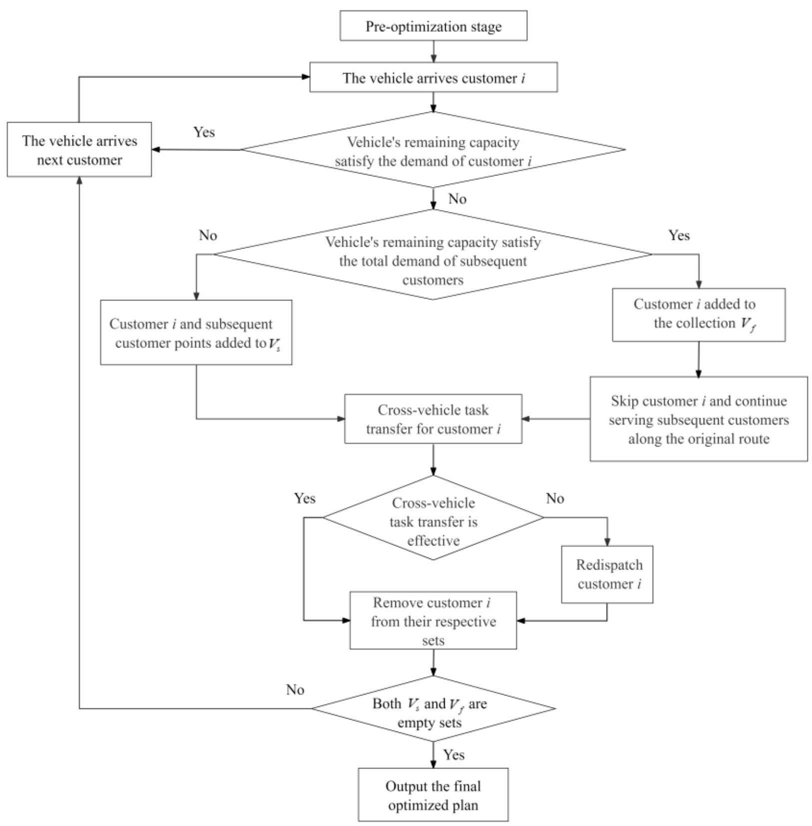


**Figure 7.** The flow of adaptive failure-point redispatching strategy.

Specifically, when a distribution vehicle departs from the distribution center and arrives at a customer point $i$ along the pre-optimization route, its remaining load is first evaluated. If the remaining load is sufficient to meet the demand at point $i$, service is completed as planned. If the load is insufficient, the algorithm then checks whether the remaining capacity can satisfy the demands of subsequent customers on the route.

- If the subsequent demands can be satisfied, customer point $i$ is added to the failure-point set $V_f$, and the vehicle skips the point and continues to serve the remaining customers according to the original route.
- If the subsequent demands cannot be satisfied, customer point $i$ and all remaining customer points on the route are added to the failure-point set $V_s$.

The customers in $V_f$ and $V_s$ are subsequently addressed through inter-vehicle task transfers and redispatching. After each redispatching step, the resolved failure points are removed. When $V_f$ and $V_s$ become empty, all failure points have been successfully handled, and the process terminates with the generation of the final optimized distribution plan. Additionally, to determine whether the remaining load is sufficient for subsequent customers, chance-constrained optimization is applied. Assuming that the vehicle has visited the customer point $i$, the remaining load is $L$, and the path to subsequent customer points $i \in V_R$ with random demand $\xi_i$ follows the Poisson distribution $\xi_i \sim \text{Poisson}(\lambda)$. By the Central Limit Theorem, the Poisson distribution of the cumulative demand approximately follows the normal distribution, i.e., $\sum_{i \in V_R} \xi_i \sim N\left(\sum_{i \in V_R} \lambda_i, \sum_{i \in V_R} \lambda_i\right)$. If $L \geq \sum_{i \in V_R} \lambda_i + z_\alpha \sqrt{\sum_{i \in V_R} \lambda_i}$, then the remaining load is sufficient. Otherwise, rescheduling is required, where $z_\alpha$ is the quantile corresponding to the confidence $\alpha$ found using the standard normal distribution table.

## 5. Experiments

### 5.1. Experimental conditions

In this study, the Christofides and Eilon cases and the Augerat cases are selected to validate the effectiveness of the proposed method. The case experiments are denoted as A-nXX-kY, where A represents the dataset category, XX is the total number of points, and kY indicates the minimum number of vehicles required for the known optimal solution. For example, A-n32-k4 refers to an experiment with 31 customers, one distribution center, and at least four vehicles needed to reach the optimal solution. All the data of the experiment can be obtained from the website (http://vrp.galgos.inf.puc-rio.br). However, these cases all represent definitive independent delivery scenarios. Accordingly, a representative sample of 30 examples is selected for expansion to validate the performance under random customer demands. Specifically, it is assumed that for each customer point $i$, its demand $\xi_i$ is a random variable that satisfies the Poisson distribution, i.e., $\xi_i \sim \text{Poisson}(\lambda_i)$. $\lambda_i$ takes the specified demand values from each example, keeping the customer point and distribution center locations unchanged.

Additionally, to further validate the joint distribution strategy, three examples, A-n33-k6, A-n38-k5, and A-n55-k9, are selected to form a case study. The details of the constructed case are shown in Table A.1–A.5 in Appendix A. The spatial distribution of distribution centers and customer points is illustrated in Figure 8.

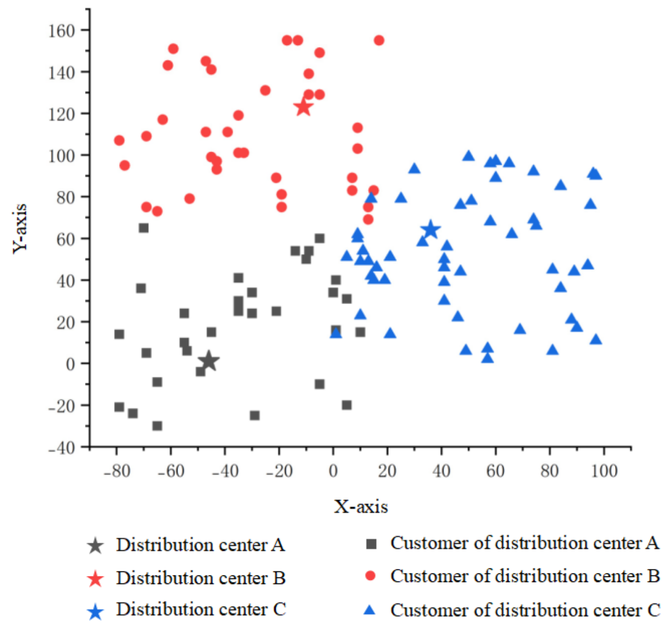


**Figure 8.** Spatial distribution of three distribution centers and customer points.

To ensure comparability, other parameter settings, following the guidelines of Hansen et al. [51], are summarized in Table 2. To focus on algorithmic performance, the experiments adopt abstract units without introducing physical dimensions. Vehicles are assumed to travel at a uniform speed, with identical capacity and performance across the distribution centers. The proposed method is implemented in MATLAB R2022a under the computational environment with a Windows 11 (64-bit) operating system, an Intel i9-14900HX (2.2GHz) CPU, and 64 GB of RAM.

**Table 2.** Other parameter settings.

| Parameter | Value | |
|---|---|---|
| | Joint distribution | Independent distribution |
| Number of iterations for HVNDSS | 50 | 50 |
| Number of iterations for VND | 30 | 20 |
| Initial population size | 90 | 30 |
| Poisson distribution parameters $\lambda_A$, $\lambda_B$, and $\lambda_C$ in distribution centers A, B, and C | 19.28, 13.00, 15.54 | |
| Maximum capacity of distribution vehicles $Q$ | 100 | |
| Route cost $c_{ij}$ (yuan/km) | 1.5 | |

## 5.2. *Comparison of solution algorithms*

We first compare the solution results of the proposed HVNDSS algorithm with those of the Particle Swarm Optimization (PSO) [52] and Simulated Annealing (SA) [53] algorithms under the independent distribution model. In addition, two strategies, the simple replenishment strategy (A) and the adaptive failure-point redispatching strategy (B), are tested under HVNDSS. Each example calculation is repeatedly performed 20 times. The results are presented in Table 3. $\alpha$ represents the probability that the total demand of all customers on a path will not exceed the vehicle capacity $Q$ when the algorithm achieves the optimal value. $Opt$ represents the achieved best solution. $Avg \pm std$ denotes the mean and sample standard deviation over 20 runs. $Dev$ represents the relative deviation of the optimal result achieved by the algorithm from the optimal solution achieved by the three algorithms for the example.

Among the 30 examples, HVNDSS combined with the adaptive failure-point redispatching strategy achieves 20 solutions with a relative deviation ($Dev$) of 0%, and an average deviation of only 0.17% from the optimal solution. Moreover, it achieves an average total cost of 802.99, outperforming PSO (832.40) and SA (900.42), with the smallest average standard deviation (7.40) among all algorithms. This demonstrates that the approach is more stable and consistently capable of producing near-optimal solutions. In a few examples, PSO obtains slightly better results than HVNDSS. A possible reason is that PSO can rapidly exploit promising regions through particle information sharing when the route structure of an instance is favorable to its search mechanism. However, this advantage is case-dependent. PSO is more sensitive to initialization and parameter settings, while HVNDSS combines SS-based diversity preservation and VND-based local refinement, leading to lower average deviation and more stable overall performance. The results also show that even when combined with the simple replenishment strategy, HVNDSS outperforms SA in most examples. Moreover, HVNDSS with the adaptive failure-point redispatching strategy improves upon the simple replenishment strategy by an average of 2.11%, confirming its superiority.

**Table 3.** The solution results of different algorithms.

| Example | HVNDSS | | | | | | PSO | | | SA | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $Opt$ (A) | $Avg \pm std$ (A) | $Opt$ (B) | $Avg \pm std$ (B) | $\alpha$ | $Dev$ (%) | $Opt$ | $Avg \pm std$ | $Dev$ (%) | $Opt$ | $Avg \pm std$ | $Dev$ (%) |
| E-n22-k4 | 390.41 | 399.63±4.97 | 375.85* | 383.54±3.74 | 0.6 | 0.00 | 390.99 | 416.54±10.41 | 4.03 | 411.57 | 452.46±15.17 | 9.50 |
| E-n33-k4 | 851.53 | 871.39±10.31 | 840.12* | 855.72±7.93 | 0.7 | 0.00 | 847.38 | 887.18±18.04 | 0.86 | 850.27 | 925.85±29.69 | 1.21 |
| A-n32-k5 | 826.4 | 850.15±11.17 | 826.32 | 843.45±8.26 | 0.7 | 0.57 | 821.65* | 868.80±18.73 | 0.00 | 853.6 | 933.69±32.59 | 3.89 |
| A-n33-k5 | 689.47 | 706.51±8.40 | 670.82* | 683.07±6.52 | 0.8 | 0.00 | 687.04 | 720.83±15.86 | 2.42 | 704.2 | 772.45±27.96 | 4.95 |
| A-n33-k6 | 760.74 | 778.32±8.96 | 758.18* | 771.95±6.71 | 0.7 | 0.00 | 769.62 | 809.97±16.61 | 1.51 | 793.9 | 863.84±30.11 | 4.71 |
| A-n34-k5 | 790.46 | 809.61±9.50 | 785.64* | 800.04±7.21 | 0.8 | 0.00 | 789.88 | 831.34±17.67 | 0.54 | 826.87 | 912.46±32.46 | 5.25 |
| A-n36-k5 | 847.05 | 869.22±11.18 | 842.32 | 858.48±8.26 | 0.8 | 0.75 | 836.05* | 878.21±17.67 | 0.00 | 858.71 | 940.05±30.76 | 2.71 |
| A-n37-k5 | 691.35 | 709.33±9.08 | 684.6* | 698.69±6.99 | 0.8 | 0.00 | 693.18 | 727.31±14.75 | 1.25 | 708.34 | 769.00±24.50 | 3.47 |
| A-n37-k6 | 1007.35 | 1034.29±11.29 | 1004.45 | 1022.70±9.44 | 0.7 | 0.47 | 999.72* | 1054.21±22.15 | 0.00 | 1030.73 | 1132.86±38.26 | 2.55 |
| A-n38-k5 | 768.63 | 790.07±9.90 | 742.9* | 756.98±7.18 | 0.8 | 0.00 | 756.56 | 799.13±16.96 | 1.84 | 775.14 | 858.04±29.29 | 4.34 |
| A-n39-k5 | 865.92 | 888.61±11.21 | 860.24 | 877.91±8.38 | 0.8 | 0.84 | 853.08* | 894.02±18.55 | 1.53 | 869.18 | 965.79±36.71 | 1.89 |
| A-n39-k6 | 843.16 | 865.85±11.17 | 835.64* | 850.78±7.54 | 0.8 | 0.00 | 847.92 | 892.42±19.20 | 1.47 | 876.6 | 961.84±32.59 | 4.90 |
| A-n44-k6 | 983.73 | 1009.64±13.01 | 984.18 | 1006.35±10.60 | 0.5 | 0.55 | 978.82* | 1034.62±23.14 | 0.00 | 1025.48 | 1133.40±37.60 | 4.90 |
| A-n45-k6 | 996.26 | 1020.84±12.55 | 982.2* | 999.81±9.20 | 0.7 | 0.00 | 997.41 | 1045.26±22.70 | 1.55 | 1026.73 | 1130.97±40.55 | 4.53 |
| A-n46-k7 | 998.45 | 1022.61±12.60 | 979.36* | 995.48±9.05 | 0.8 | 0.00 | 984.98 | 1032.13±21.88 | 0.57 | 1002.22 | 1105.70±41.42 | 2.33 |
| A-n48-k7 | 1149.98 | 1181.48±14.35 | 1134.25 | 1155.97±10.56 | 0.8 | 0.19 | 1132.15* | 1177.74±22.44 | 0.00 | 1187.14 | 1304.05±44.81 | 4.86 |
| E-n51-k5 | 549.25 | 562.47±6.87 | 530.98* | 540.97±4.89 | 0.8 | 0.00 | 544.86 | 573.06±12.27 | 2.61 | 552.26 | 608.24±22.56 | 4.01 |
| P-n50-k7 | 571.39 | 584.75±6.54 | 569.35* | 579.81±5.44 | 0.7 | 0.00 | 570.94 | 597.72±12.11 | 0.28 | 582.37 | 637.23±22.49 | 2.29 |
| P-n50-10 | 740.75 | 758.56±8.88 | 736.63* | 747.44±5.80 | 0.6 | 0.00 | 739.51 | 773.18±14.35 | 0.39 | 760.94 | 837.30±30.59 | 3.3 |
| A-n53-k7 | 1132.15 | 1160.73±14.21 | 1090.74* | 1113.71±10.23 | 0.8 | 0.00 | 1096.6 | 1160.35±24.45 | 0.54 | 1124.27 | 1238.57±42.88 | 3.07 |
| A-n54-k7 | 1269.6 | 1299.43±14.94 | 1228.32 | 1252.19±11.59 | 0.5 | 0.42 | 1223.23* | 1291.06±28.64 | 0.00 | 1287.07 | 1409.19±47.66 | 5.21 |
| A-n55-k9 | 1223.23 | 1253.58±14.58 | 1108.9* | 1128.79±9.72 | 0.8 | 0.00 | 1124.3 | 1183.98±25.86 | 1.39 | 1197.11 | 1314.87±47.60 | 7.95 |
| P-n16-k8 | 465.28 | 478.87±5.48 | 459.52 | 467.84±4.35 | 0.7 | 0.95 | 455.21* | 478.18±9.41 | 0.00 | 512.82 | 551.25±15.69 | 1.27 |
| P-n19-k2 | 212.45 | 217.31±2.61 | 212.34* | 215.90±2.13 | 0.7 | 0.00 | 213.51 | 224.51±4.49 | 1.51 | 224.06 | 245.27±8.30 | 6.60 |
| P-n20-k2 | 225.84 | 230.88±2.81 | 218.63* | 222.14±1.91 | 0.7 | 0.00 | 226.79 | 238.57±5.05 | 3.73 | 233.05 | 255.67±8.44 | 8.58 |

*Continued on next page*

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P-n21-k2 | 220.38 | 225.08±2.66 | 218.35 | 222.15±2.16 | 0.7 | 0.10 | 218.13* | 228.03±4.65 | 0.00 | 218.96 | 239.58±7.64 | 0.38 |
| P-n55-10 | 739.9 | 756.92±8.79 | 736.35* | 749.93±6.50 | 0.7 | 0.00 | 737.87 | 774.85±15.82 | 0.21 | 745.7 | 812.60±28.53 | 1.27 |
| P-n55-k15 | 1052.34 | 1076.20±11.82 | 1003.52* | 1020.47±8.66 | 0.7 | 0.00 | 1008.6 | 1056.31±21.32 | 0.51 | 1068.05 | 1160.83±37.02 | 6.43 |
| A-n60-k9 | 1490.41 | 1528.91±18.33 | 1456.85 | 1482.83±13.74 | 0.8 | 0.19 | 1454.15* | 1515.10±27.86 | 0.00 | 1529.82 | 1671.13±58.62 | 5.20 |
| P-n60-10 | 802.42 | 821.45±9.20 | 769.53* | 784.56±7.26 | 0.7 | 0.00 | 772.86 | 807.51±16.22 | 0.04 | 804.24 | 868.50±28.92 | 4.51 |
| Average | 805.21 | 825.42±9.91 | 788.24 | 802.99±7.40 | 0.72 | 0.17 | 792.43 | 832.40±17.31 | 0.96 | 821.38 | 900.42±31.05 | 4.20 |

Additionally, the effect of skewness coefficient $S_c$ and variation coefficient $V_c$, which measure the distribution of customer locations relative to the distribution center, on the solution algorithm is analyzed based on the example data. $S_c$ quantifies whether the distance distribution from customer points to the distribution center is skewed toward one side: $S_c > 0$ indicates a right skew, $S_c < 0$ indicates a left skew, and $S_c \approx 0$ indicates a relatively uniform distribution. $V_c$ measures the dispersion of customer points, where $V_c \approx 0$ indicates uniform distribution of all customer points relative to the distribution center, and a smaller or larger $V_c$ signifies greater variation in the distance distribution of customer points.

Figure 9 displays the distributions of $S_c$ and $V_c$ under HVNDSS across examples. For examples where HVNDSS achieves optimal solutions, $S_c$ is primarily concentrated within the (-0.2~0.3) range and $V_c$ predominantly falls within the (0.25~0.5) range, indicating that HVNDSS performs better in scenarios with relatively uniformly distributed customer points and under moderately dispersed customer locations.

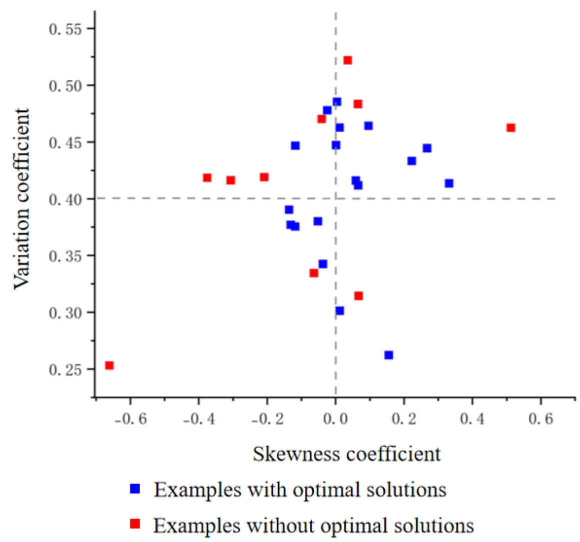


**Figure 9.** The distributions of skewness and variation coefficients.

### *5.3. Comparison of distribution models*

In this subsection, simulation experiments are conducted to compare the independent and joint distribution models. The test operation is repeated 10 times for each experiment. The vehicle routes under the independent distribution network and joint distribution network are shown in Figure 10. The corresponding detailed route information is listed in Table B.1 and B.2 in Appendix B.

Significant adjustments occur in the routes near customer points 20, 19, and 7 in network A. The rescheduling strategy reduces the overall failure rate but results in some vehicles traveling additional distances. Routes near customer points 43, 37, and 39 in network B are reassigned to the redispatching phase due to random chance constraints. Customer points 79 and 74 in network C are taken over by vehicles from other routes, adjusted by the cross-route strategy during the rescheduling phase. Under the joint distribution network, such instances of redispatching are significantly reduced by enabling cross-center scheduling to improve routing efficiency. For instance, customer points 43 and 59 at the boundary of centers B and C, and points 120 and 26 at the boundary of centers A and C, can be reassigned to vehicles from neighboring centers. This reduces additional redispatching costs for all centers. The distribution cost comparison is shown in Table 4.

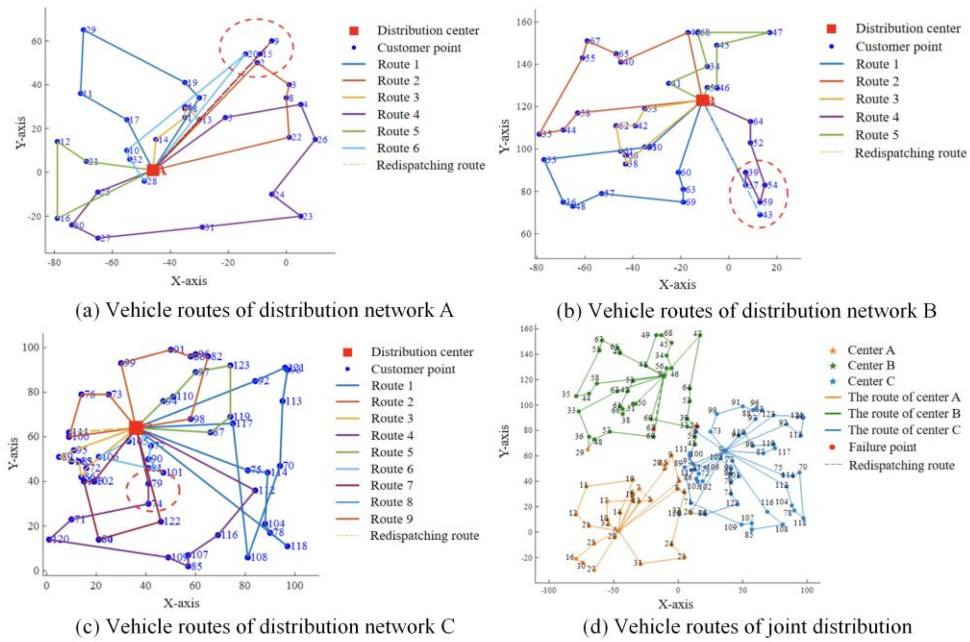

**Figure 10.** The vehicle routes under different distribution networks.

**Table 4.** Comparison of distribution cost under two distribution models.

| Distribution network | Independent distribution cost (yuan) | | Joint distribution cost (yuan) | | Optimization rate（%） |
|---|---|---|---|---|---|
| | Pre-optimization | Redispatching | Pre-optimization | Redispatching | |
| A | 1428.34 | 101.37 | 1106.34 | / | 27.68 |
| B | 1149.54 | 181.28 | 1127.28 | 128.26 | 5.66 |
| C | 1735.61 | 225.96 | 1679.13 | / | 14.40 |
| Total cost | 4822.10 | | 4041.01 | | 15.91 |

Under the joint distribution model, the total distribution cost of the three networks is 4041.01, representing a 15.91% reduction compared to the independent distribution model. In terms of the distribution centers, the joint distribution model reduces costs by 27.68%, 5.66%, and 14.4% in networks A, B, and C, respectively. Center B achieves a smaller optimization rate due to its responsibility for rescheduling tasks, whereas center A achieves the largest cost reduction because its most distant customer (customer point 29) is reassigned to center B. This cost reduction is primarily attributed to the cross-distribution center dispatching strategy under the joint distribution model, which reduces the redispatching cost and achieves a better allocation of resources in the overall route planning.

After 10 rounds of testing, customer points that fail more than twice in the pre-optimization stage are marked as high-risk failure points. The comparison of failure points is shown in Table 5.

**Table 5.** Comparison of failure points under two distribution models.

| Distribution center | Failure points | | Number of high-risk failure points | |
|---|---|---|---|---|
| | Independent distribution | Joint distribution | Independent distribution | Joint distribution |
| A | 20,19,7,9,15,2,30,27,25 | / | 3 | 0 |
| B | 50,43,37,39 | 63 | 2 | 0 |
| C | 74,79,108,111,100,106 | 54 | 2 | 0 |

Note: Customer points highlighted in red are high-risk failure points.

In the independent distribution model, numerous failure points arise. In contrast, in the joint distribution model, failure points can be jointly addressed by vehicles from multiple centers. This increases the diversity of the initial population in the pre-optimization stage and significantly reduces the generation of failure points. There are seven high-risk failure points in the independent distribution model, whereas the number drops to 0 under the joint distribution model. These results demonstrate that the joint distribution model is more effective at handling failure points. Overall, the joint distribution model outperforms the independent distribution model in distribution cost and failure-point handling, achieving path optimization and resource utilization improvement.

### *5.4. Sensitivity analysis*

To further evaluate the adaptability of the proposed independent and joint distribution models under different environments, we perform a sensitivity analysis on key parameters to assess how the models respond to variations in critical variables. The analysis focuses on three aspects: the confidence level, vehicle capacity, and distribution network.

#### 5.4.1. The confidence level

Confidence level $\alpha$ reflects the model's tolerance to failure risk. The optimal solution is obtained by adjusting different values of $\alpha$, typically ranging from 0.5 to 0.8. Figure 11 illustrates the relationship between the number of failure points and the total distribution cost under different values of $\alpha$.

The results show that as $\alpha$ increases, the number of failure points decreases for both distribution models. The distribution cost drops significantly when $\alpha$ increases from 0.5 to 0.7, but rises again when $\alpha$ is 0.8. This U-shaped pattern arises because a lower $\alpha$ permits higher failure risk, leading to frequent redispatching and higher cost. As $\alpha$ increases, this effect diminishes, but when $\alpha$ continues to grow excessively, the model over-prioritizes failure reduction, resulting in overly long pre-optimized routes that increase cost. For all $\alpha$ values, the joint distribution model achieves lower cost and fewer failure points than the independent model, confirming its advantage in balancing cost and reliability. The choice of $\alpha$ should be guided by the service level required in practice. A larger $\alpha$ reduces failure points but may raise cost beyond an optimal point. In our experiments, $\alpha = 0.7$ provides the best trade-off between cost and service reliability. Decision-makers can adjust $\alpha$ based on their tolerance for delivery failures and acceptable cost levels.

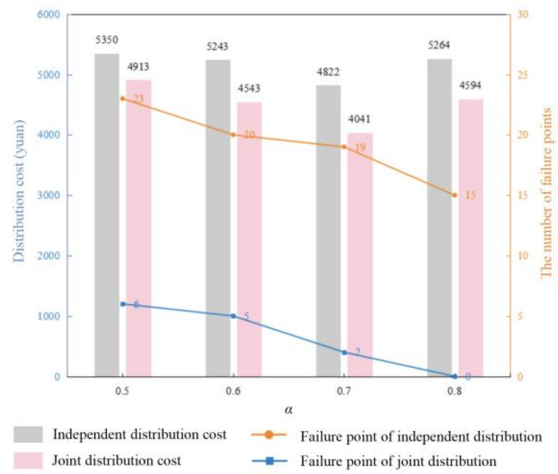


**Figure 11.** The change in the distribution cost and the number of failure points.

5.4.2. Vehicle capacity

Vehicle capacity critically affects distribution efficiency, failure points, and total distribution costs. Too small a capacity increases overloading risk, unmet demand, and extra dispatches. Conversely, too large a capacity reduces failure points but may lead to underutilization and lower the mileage optimization rate of joint distribution. Figure 12 illustrates the distribution mileage and its optimization rate of independent and joint distribution models under different vehicle capacity settings.

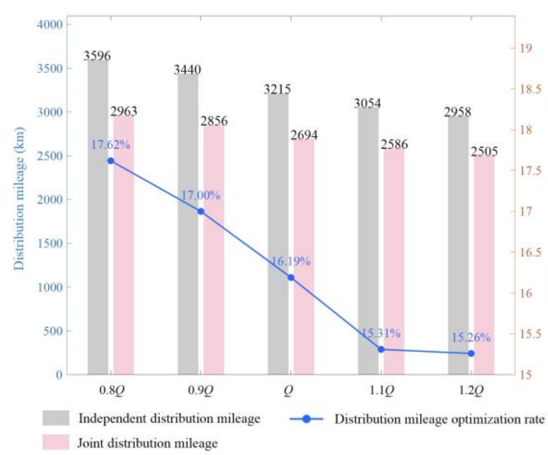


**Figure 12.** The change in the distribution mileage and its optimization rate.

As vehicle capacity increases from $0.8Q$ to $1.2Q$, the distribution mileage decreases significantly. This improvement is mainly due to the higher loading rate of larger vehicles, which reduces delivery frequency and total distribution mileage. Under lower vehicle capacities, the joint distribution model demonstrates a significantly higher optimization rate, reaching 17.62%, indicating that shared dispatching is more effective in resource-constrained environments. As vehicle capacity increases, the optimization rate tends to stabilize while continuously reducing overall distribution mileage. Excessive capacity can dilute fixed-cost savings and may even increase total expenses. Therefore, route optimization should jointly consider vehicle configuration and cost structure to identify the optimal capacity range. Practitioners should avoid oversizing fleets merely to reduce failure points, and instead balance capacity with anticipated demand variability and the benefits of joint dispatching.

#### 5.4.3. Distribution network

In the logistics distribution network, the size of the distribution network and the number of distribution centers directly affect the distribution of customer points, the route optimization process, and the occurrence of failure points. Referring to the benchmark design of Uchoa [7], the size of the distribution network is categorized as follows: n≤50 (small scale), 50<n≤100 (medium scale), and n>100 (large scale). Three sizes of the networks are selected for the analysis. The size of the distribution network and the examples are shown in Table 6.

The example information is obtained from the website (http://vrp.galgos.inf.puc-rio.br/index.php/en/). The proposed method is used to solve for route optimization for three sizes of one-center distribution network with independent distribution, two-center distribution network with joint distribution, and three-center distribution network with joint distribution, respectively. The solution results are shown in Table 7.

**Table 6.** Different distribution network sizes and their examples.

| Size of the distribution network | Number of customer points | Example | Distribution network ID |
|---|---|---|---|
| | | A-n32-k5 | S1 |
| Small | n≤50 | A-n33-k5 | S2 |
| | | A-n33-k6 | S3 |
| | | P-n70-k6 | M1 |
| Medium | 50<n≤100 | P-n76-k4 | M2 |
| | | P-n76-k5 | M3 |
| | | M-n121-k7 | L1 |
| Large | n>100 | M-n151-k12 | L2 |
| | | M-n151-k12 | L3 |

For small-size networks, the one-center independent distribution model produces only a few failure points, indicating that a single depot can generally satisfy demand. Joint distribution brings limited additional improvement, with cost reductions of about 5%~7% for two centers and 7.61% for three centers. In the medium-size network, the independent model yields many failure points. Two-center joint distribution shows inconsistent results and cannot fully eliminate failures, reflecting scheduling constraints. The three-center joint distribution model substantially reduces failure points, underscoring the role of scheduling and routing optimization. Cost optimization is also more evident at this scale. Two centers reduce costs by up to 11.66%, and three centers achieve the highest reduction of 15.64%. Thus, the medium-size network achieves the best balance of optimization benefits.

**Table 7.** Solution results of different distribution models for different distribution networks.

| Network size | Distribution model | Distribution network | Number of failure points | Joint distribution cost (yuan) | Sum of independent distribution cost (yuan) | Distribution cost optimization rate (%) |
|---|---|---|---|---|---|---|
| Small | One-center independent distribution | S1 | 2 | / | 601.53 | 0 |
| | | S2 | 3 | / | 635.72 | 0 |
| | | S3 | 2 | / | 639.00 | 0 |
| | Two-center joint distribution | S1, S2 | 2 | 1170.22 | 1237.25 | 5.42 |
| | | S2, S3 | 1 | 1188.36 | 1274.72 | 6.77 |
| | | S1, S3 | 1 | 1177.29 | 1240.53 | 5.18 |
| | Three-center joint distribution | S1, S2, S3 | 0 | 1733.47 | 1876.25 | 7.61 |
| Medium | One-center independent distribution | M1 | 5 | / | 826.32 | 0 |
| | | M2 | 7 | / | 670.82 | 0 |
| | | M3 | 6 | / | 758.18 | 0 |
| | Two-center joint distribution | M1, M2 | 9 | 1329.15 | 1497.14 | 11.25 |
| | | M2, M3 | 6 | 1294.52 | 1429.00 | 9.41 |
| | | M1, M3 | 8 | 1399.75 | 1584.50 | 11.66 |
| | Three-center joint distribution | M1, M2, M3 | 4 | 1902.59 | 2255.32 | 15.64 |
| Large | One-center independent distribution | L1 | 12 | / | 1075.00 | 0 |
| | | L2 | 13 | / | 1100.53 | 0 |
| | | L3 | 11 | / | 1095.46 | 0 |
| | Two-center joint distribution | L1, L2 | 16 | 2018.67 | 2175.53 | 7.21 |
| | | L2, L3 | 15 | 2017.68 | 2195.99 | 8.12 |
| | | L1, L3 | 14 | 1983.58 | 2170.46 | 8.61 |
| | Three-center joint distribution | L1, L2, L3 | 12 | 2920.01 | 3270.99 | 10.73 |

In the large-size network, the independent model leads to the highest number of total failure points and distribution costs, as the capacity of a single center is insufficient. The two-center model provides only moderate improvements in service level and cost reduction (7%~9%). By contrast, the three-center model reduces failure points by 66.7% and cuts costs by 10.73%. However, in larger networks, the distribution routes are scattered over a larger area and most vehicles serve relatively independent areas, which reduces the relative share of overlapping paths that joint distribution can eliminate. As a result, although the absolute distance savings grow, the marginal effect of adding more distribution centers diminishes, leading to a lower overall cost optimization rate.

Overall, the findings suggest that distribution strategies should be tailored to network scale. For small networks, independent distribution remains efficient, and joint distribution should be applied selectively to improve service reliability. For medium networks, multi-center joint distribution offers the greatest balance between cost savings and operational complexity and should be actively promoted as the preferred strategy. For large networks, where coordination difficulty increases, managers should focus on advanced scheduling technologies, digital coordination platforms, and zonal joint distribution to sustain efficiency.

## 6. Conclusions

In this study, we address the complexity arising from demand uncertainty in modern logistics distribution through theoretical modeling and algorithmic design. Specifically, an MCVRPSD-JD model is first constructed to achieve cooperative vehicle scheduling optimization among depots under stochastic customer demand. Then, a two-stage HVNDSS algorithm with an adaptive failure-point redispatching strategy is designed to improve the solution quality across the stages of pre-optimization and dynamic adjustment. Experimental validation demonstrates that the proposed method outperforms traditional PSO and SA approaches when faced with random customer demands. The error between the solution of the proposed method and the actual optimal solution is only 0.17%, which is about 1/6 of the error of the PSO solution and 1/25 of the error of the SA solution. The HVNDSS algorithm achieves near-optimal performance with outstanding robustness. Additionally, the proposed adaptive redispatching strategy improves results by 2.11% compared with the simple replenishment strategy, thereby achieving global route optimization and effective cost control. The joint distribution model is superior to the independent distribution model, saving an average of 15.91% of distribution costs while significantly reducing the number of failure points. Sensitivity analysis showsthat the confidence level of the model, distribution vehicle capacity, and the size of distribution networks have different degrees of impact on the efficiency of distribution path optimization. In medium-scale distribution networks, the adaptability and cost-optimization advantages of the joint distribution mode become more pronounced. However, vehicle capacity and the allowable risk level of delivery failures must be configured according to actual operational conditions to maintain an appropriate balance between customer demand fulfillment and distribution costs. Overall, the study provides valuable insights for decision-making in logistics distribution systems.

Although this study has achieved some notable results, several limitations remain. The experimental design relies on simplified road networks from standard benchmark datasets, and random customer demand is modeled within a unified framework based on specific probability distributions (e.g., Poisson distribution). In future studies, researchers could construct more realistic environments by incorporating time-varying travel times, considering heterogeneous fleets comprising different types of vehicles, and employing learning-based methods for demand forecasting. Additionally, we do not quantify the individual contributions of the SS and VND components. Conducting ablation experiments would further clarify their respective roles. Nevertheless, we propose a scalable approach that, when applied to large-scale heterogeneous distribution networks in real-world scenarios, would enhance its practical value and provide deeper insights for academic research and industry practice.

## Author contributions

Jingshuai Yang: Conceptualization, Methodology, Funding acquisition, Writing-original draft; Zhengyu Liu: Software, Visualization, Data Curation; Zixin Zhou: Methodology, Software, Resources; Huan Pang: Validation, Writing-Review & Editing; Minghui Xie: Supervision, Funding acquisition, Writing-Review & Editing; Deyin Jiang: Investigation, Formal analysis.

## Use of Generative-AI tools declaration

In the preparation of this work, the authors used Generative AI tools such as ChatGPT to assist in improving the clarity of the language.


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## Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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