Journal of Industrial and Management Optimization

https://www.aimspress.com/journal/jimo

JIMO, 22(12): 6317–6338.
DOI:10.3934/jimo.2026212
Received: 16 July 2026
Revised: 15 October 2026
Accepted: 26 October 2026
Published: 10 November 2026

Research article

Joint operational decisions for a supply chain considering demand-dependent green investment

Han Zhao[1], Wei Liu[2,*], Yu Liao[1,*], Yuli Zhang[3] and Shiji Song[4]

[1] Joint Logistics Support Force Engineering University, Chongqing 401311, P.R. China
[2] Department of Basic Science, Wuhan Donghu University, Wuhan 430212, P.R. China
[3] School of Management, Beijing Institute of Technology, Beijing 100081, P.R. China; Sustainable Development Research Institute for Economy and Society of Beijing, Beijing 100081, P.R. China
[4] Department of Automation, Tsinghua University, Beijing 100084, P.R. China

* **Correspondence:** Email: Wei-l090@mails.tsinghua.edu.cn; starsea2010_ nk@163.com.

Abstract: This paper investigated operational decisions in a supply chain consisting of a risk-neutral manufacturer and a risk-averse retailer with carbon abatement cost-sharing and call option contracts under carbon tax policy, where market demand is stochastic and dependent on green investment. Specifically, the optimal ordering and green investment strategies were derived for four distinct scenarios: with/without carbon abatement cost-sharing and with/without call option contracts. Further, it was shown that carbon abatement cost-sharing and call option contracts can mutually benefit the manufacturer and the retailer, and enhance the performance of the supply chain. This paper further revealed that the retailer's optimal total order quantity and optimal firm order quantity are monotonically increasing in the manufacturer's green investment. In addition, it was demonstrated that supply-chain coordination requires satisfying two key conditions, which are related to green investment and carbon tax policy. Finally, the numerical experiments in this study serve as illustrative demonstrations to intuitively visualize the theoretical findings and yield practical managerial insights for supply-chain practitioners.

Keywords: supply-chain risk management; supply-chain coordination; green investment; call option contract; carbon tax policy
Mathematics Subject Classification: 90B06, 91A65

1. Introduction

Atmospheric pollution and the greenhouse effect have attracted heightened global concern, rendering carbon abatement an urgent priority and a collective focus for the international community. To accelerate carbon abatement, governments across the globe are actively promoting the transition to a low-carbon

economy and have implemented a portfolio of policy instruments, including carbon cap, carbon cap-and-trade, carbon tax, and energy conservation subsidies [1, 2]. For instance, Portugal raised its carbon tax rate by more than 80% in 2020, while Norway implemented a moderate carbon tax hike of just over 8% relative to the prior year [3].

Extensive literature has investigated the impacts of carbon emission policies on firms' operational decisions. Early studies primarily focused on carbon policy-driven production optimization, operational cost control, and low-carbon model construction. Specifically, the impact of carbon emission trading mechanisms on firms' production and technology investment decisions was explored [4]. Carbon emission costs were quantitatively linked to operational costs throughout logistics processes, including transportation, warehousing, and handling activities [5]. Targeting downstream circulation links, the carbon emission issues arising from product distribution operations were systematically examined [6]. From the perspective of inventory optimization, a carbon-constrained economic order quantity model was developed, and its operational performance was compared with that of the traditional inventory framework to facilitate low-carbon decision-making [7]. A web-based grey analytical approach was proposed to evaluate suppliers' low-carbon participation performance [8]. To achieve the dual goals of reducing emissions and lowering costs, a novel simulation model was designed for carbon abatement cost minimization [9]. Focusing on carbon tax regulation, an enterprise emission abatement decision-making model was constructed [10], while a novel optimization strategy for carbon abatement in the port transportation operation was proposed [11]. Further, a mixed-integer linear programming model was developed to provide managers with cost-effective low-carbon operational strategies [12]. These studies incorporate carbon factors into operational decisions from different views. A research gap remains regarding the systematic application of the conditional value-at-risk (CVaR) to inventory replenishment and green investment decisions.

Driven by governmental environmental regulations and widespread low-carbon advocacy, the low-carbon consciousness of the consumers has been continuously enhanced. Accordingly, the carbon footprint of products and services has become a vital determinant of consumer purchasing decisions. Empirical research indicates that product market demand is negatively correlated with carbon emission levels. This trend has profoundly reshaped corporate competitive landscapes and cost structures, driving firms to revise their operational strategies and actively pursue low-carbon transformation. Carbon emission reduction in modern supply chains is regarded as a systematic and collaborative endeavor that cannot be achieved merely through unilateral efforts of upstream manufacturers. As critical intermediaries connecting manufacturers and end consumers, retailers play an indispensable role in low-carbon promotion. By effectively disseminating corporate decarbonization information to the market, retailers can improve consumer recognition, stimulate green market demand, and ultimately improve the overall economic performance of the supply chain. Practical industrial cases further verify the necessity of supply-chain low-carbon collaboration. For example, Ford Motor Company requires its parts suppliers to obtain third-party certification of environmental management systems, while companies like Coca-Cola and Hewlett-Packard also seek to cooperate with their upstream and downstream partners, and achieve certain results in their low-carbon supply-chain practices [13, 14]. Therefore, to achieve carbon abatement targets, enterprises need to enhance systematic planning and synergistic cooperation among supply-chain members [15–17].

As research on carbon emission reduction deepens, the academic community gradually recognizes the importance of joint carbon abatement efforts across upstream and downstream supply-chain members

to reduce carbon emissions. Existing studies on joint carbon-emission reduction primarily evaluate the effects of various carbon regulatory instruments. The impact of carbon tax policy on enterprises' carbon emission reduction decisions and supply chain performance was explored, and a moderate carbon tax was found to incentivize abatement reduction investment, whereas a excessively high carbon tax suppresses corporate profitability [18]. Under cap-and-trade rules, corporate production and carbon abatement strategies were examined, where the policy's effectiveness in encouraging low-carbon behavior was also evaluated [19]. The carbon emission reduction strategies of the supply chain under carbon quota constraints were discussed, and reasonable quota allocation facilitates are conducive to cooperative carbon abatement [20]. Furthermore, under the carbon emissions trading scheme, operational decisions of the supply chain composed of a carbon emission-dependent manufacturer and a carbon emission permit supplier were examined, and the impact of carbon emission permit factors on the decision results and performance was analyzed [21]. A sustainable supply-chain model was proposed and used to demonstrate the impact of the production, distribution, and disposal stages on the environment [22]. Combined contracts were investigated, and their advantages over single contracts in supply-chain management with a carbon tax policy were discussed [23]. Under the risk-sharing contract, a two-echelon dual-channel supply chain was analyzed, quantifying the impact of the retailer's risk aversion on channels, member profits, and total supply-chain profit [24]. Manufacturer's green investment was explored to increase the selling price of the product while reducing the total carbon emissions generated from the supply-chain [25]. The effects of green investment and circularity index on total profit and carbon emissions were shown [26]. These studies addressed the impact of carbon emission policies on supply-chain decisions, and few studies systematically investigated the integrated design of operational risk control and carbon-emission-reduction coordination mechanisms, with especially lacking research on enhancing the carbon abatement rate of products.

The existing research on the coordination of the supply-chain via option contracts establishes a foundational body of knowledge. Under discrete demand, Contract design and procurement flexibility have been shown to matter for manufacturers and retailers. With a call option contract and a wholesale price contract, a supply chain was investigated, where the hybrid contract outperforms the single one when the manufacturer is risk-averse [27]. Under random spot prices and random demand, procurement flexibility was examined through a optimal combined strategy of a wholesale price contract, a call option contract, and a spot market [28], while firm orders and option orders were jointly investigated, highlighting how the risk-averse retailer's optimal ordering policy depends on its budget [29]. Under the mean-CVaR risk criterion, the inventory replenishment of products with weather-dependent demand was examined, and the retailer's maximum expected utility with the call option contract was derived [30]. With a call option contract, the optimal ordering decisions of a loss-averse retailer were characterized, a solution algorithm was then developed, and the relationship between loss aversion level and the optimal ordering policy was explored [31]. The aforementioned literature addressed demand risk from the perspective of customers, and paid less attention to incorporating supply-side risks. With uncertain demand and yield, the shortages, or the overstock costs were considered, and the optimal production policy of the manufacturer and the optimal ordering policy of the retailer were derived [32]. With random demand and yield, option orders and emergency orders were considered, where the call option contract was shown to mitigate risks from demand and yield uncertainty, create win-win outcomes, and enhanced the performance of the supply-chain [33]. Optimal operational decisions of the risk-neutral supplier and the risk-averse retailer were derived with the conditional value-at-risk and a service requirement,

respectively [34]. Under the carbon tax policy, the ordering strategies of the risk-averse retailer with call option contracts were examined, and the impact of the carbon tax on these policies was strategies [35]. Nevertheless, the extant literature contains prominent research gaps this paper aims to address. Few studies systematically investigated the carbon abatement incentive value of call option contracts within a coordinated low-carbon supply-chain framework. In practical operations, downstream retailers holding call options can adjust actual procurement volumes flexibly in response to real market demand. This mechanism avoids overproduction and redundant inventory, thereby cutting wasteful carbon emissions and corresponding carbon tax liabilities. Further, call options redistribute overproduction risks between manufacturers and retailers, and encourage higher green investment and order volumes.

The prior research has made significant progress in carbon abatement decisions. However, these studies fail to integrate five core elements (carbon tax regulation, carbon abatement cost-sharing, green investment, CVaR-based retailer risk aversion, and call option contracts) into a unified Stackelberg game optimization model. This paper fills this research vacuum by constructing an integrated coordination framework to explore equilibrium decisions and derive actionable managerial insights for low-carbon supply chains under risk uncertainty. Specifically, building on the work of existing literature [19, 25, 26, 35], this study develops an extended and generalized supply-chain framework that incorporates decision-maker's risk aversion while considering green-investment-dependent stochastic demand. This endogenous coupling mechanism generates new equilibrium properties and coordination insights that differ fundamentally from those in previous studies. From the perspective of joint carbon abatement and low-carbon promotion among upstream and downstream enterprises in the supply-chain, the Stackelberg game approach is employed to investigate the coordination of the supply-chain with carbon tax policy. A call-option-contract model with carbon abatement cost-sharing is proposed to achieve the synergistic optimization of supply-chain collaborative carbon abatement and economic benefits. Specifically, the following questions are addressed.

(1) What is the optimal ordering strategy of the risk-averse retailer and what is the optimal green investment strategy of the manufacturer with carbon abatement cost-sharing and call option contracts?

(2) What effect does the manufacturer's green investment have on the ordering strategy of the risk-averse retailer and the supply-chain system's carbon abatement volume and performance?

(3) How can supply-chain coordination be achieved with carbon abatement cost-sharing and call option contracts?

(4) What effect do the carbon abatement cost-sharing and call option contracts have on the performance of the supply-chain with the carbon tax policy?

The main contributions of this article are summarized as follows:

(1) The optimal ordering and green-investment strategies are derived for four distinct scenarios: with/without carbon abatement cost-sharing and with/without call option contracts.

(2) Carbon abatement cost-sharing and call option contracts can mutually benefit the manufacturer and the retailer, and enhance the performance of the supply-chain.

(3) This paper further reveals that the retailer's optimal total order quantity and optimal firm order quantity are monotonically increasing in the manufacturer's green investment.

(4) The supply-chain coordination requires satisfying two key conditions, which are related to green investment and the carbon tax policy

The remaining sections of this paper are organized as follows. In Section 2, the model formulation and assumptions are provided. The retailer's optimal order policy and the manufacturer's optimal green

investment policy are derived in Section 3. Section 4 analyzes the effects of the carbon abatement cost-sharing and call option contracts on the performance of the supply-chain. Section 5 presents the numerical experiments to intuitively visualize the theoretical findings. Finally, Section 6 draws conclusions and some managerial implications.

2. Model formulation and assumptions

This paper considers the carbon abatement problem in a single-period two-stage supply-chain consisting of one manufacturer and one retailer. The manufacturer is large-scale and capable of risk hedging, is risk-neutral and produces certain types of low-carbon products with short life cycles, and distributes them to consumers via small-scale retailers with weak risk resistance that are risk-averse. It is assumed that consumers have a preference for low-carbon products, and the product market demand is dependent on carbon emission reduction. Further, the products are characterized by a long production lead time, the short selling season, and high demand uncertainty. Once the selling season commences, the retailer has no opportunity for reordering. Consequently, the manufacturer and the retailer must make their decisions before the beginning of the selling season. This paper considers that the carbon tax policy is implemented to curb supply-chain operations' carbon emissions. To comply with the carbon tax policy and boost market demand, the manufacturer is motivated to invest in green production technologies.

Under a carbon tax policy, the government imposes a carbon tax based on the carbon emission levels of products. The retailer is responsible for carbon tax payments generated during procurement, transportation, and storage. Carbon tax costs for production and salvage disposal are incorporated into the manufacturer's cost term c. Let the carbon tax be denoted by α, i.e., the carbon emission cost per unit product. The manufacturer endeavors to enhance the green investment to boost market demand, and induces a carbon abatement cost. It follows that the market demand is dependent on the green investment and the carbon abatement cost is a function of the manufacturer's green investment. Let the green investment of the manufacturer be θ and the carbon abatement cost be $g(\theta)$. Obviously, $g(0) = 0$. The carbon abatement cost increases with the rise in green investment, i.e., $g'(\theta) > 0$. The marginal utility of the carbon abatement cost with respect to the green investment increases monotonically, i.e., $g''(\theta) > 0$. To motivate the retailer to participate in the low-carbon promotion and green supply-chain, the manufacturer negotiates to share a certain proportion of the carbon-emission-reduction cost with the retailer. Let the proportion be δ, $1 > \delta > 0$. The random demand X has a θ-dependent support, $X \geq 0$. For a given green investment θ, let the probability density function of the stochastic demand X be $f(x \mid \theta)$ and the corresponding cumulative distribution function be $F(x \mid \theta)$. $F(x \mid \theta)$ is an increasing, differentiable, non-negative, and invertible function, and $F(0 \mid \theta) = 0$. $F^{-1}(\cdot|\theta)$ stands for the corresponding quantile function, which is well-defined for arguments within probability domain $(0, 1)$. Obviously, the stochastic demand increases as the green investment θ rises, i.e., $\partial F(x \mid \theta)/\partial\theta < 0$. Further, the marginal utility of the stochastic demand with respect to the green investment θ decreases monotonically, i.e., $\partial^2 F(x \mid \theta)/\partial^2\theta > 0$. This demand specification is widely adopted in low-carbon supply-chain research, which can effectively reflect the positive impact of the green investment level on market demand and the negative impact of product price. It is assumed that market demand follows $X = f(\theta)\varepsilon$, where $f(\theta) = \theta^{\frac{1}{\beta}} + \gamma$ represents the scale function capturing demand expansion from green investment, ε is an exogenous noise term, $\varepsilon \sim U[0, \bar{M}]$, β represents the curvature parameter governing the marginal demand return of green investment, $\beta > 1$ implies diminishing marginal demand benefit

from green investment, and γ represents the baseline market demand, representing minimum demand independent of green investment, $\gamma > 0$.

The variables and parameters used to formulate the model are listed in Table 1.

Table 1. Notations.

Notation	Description
p	Retail price
w	Wholesale price through a firm order
c	Production cost
e	Exercise price
o	Option price
v	Salvage value after the selling season
q_1	Firm order quantity
q_2	Call option order quantity
q	Total order quantity, $q = q_1 + q_2$
Q	Production quantity
α	Carbon tax
θ	The manufacturer's green investment
$g(\theta)$	Carbon abatement cost, $g(0) = 0$, $g'(\theta) > 0$, $g''(\theta) > 0$
δ	The retailer's sharing the proportion of $g(\theta)$, $1 > \delta > 0$
η	Risk aversion coefficient of the retailer
X	Stochastic demand, $E(X) = \mu \geq 0$ and $Var(X) = \sigma^2$
$f(x \mid \theta)$	Probability density function of X, $f(0 \mid \theta) = 0$, $f(x \mid \theta) \geq 0$
$F(x \mid \theta)$	Cumulative distribution function of X, $F(0 \mid \theta) = 0$, $\partial F(x \mid \theta) \diagup \partial\theta < 0$, $\partial^2 F(x \mid \theta) \diagup \partial^2\theta > 0$
β	Curvature parameter capturing the marginal demand return to green investment, $\beta > 1$
γ	Baseline market demand, denoting minimum demand independent of green investment, $\gamma > 0$
$f(\theta) = \theta^{1/\beta} + \gamma$	Scale function capturing demand expansion from green investment
ε	Exogenous independent random shock, $\varepsilon \sim U[0, \bar{M}]$

The decision process of the model with carbon abatement cost-sharing and call option contracts can be described as follows. Before the start of the production season, the manufacturer offers the retailer the firm ordering price w, option purchase price o, option exercise price e, green investment θ, and the proportion δ for sharing the carbon abatement cost $g(\theta)$. Before the beginning of the selling season, the retailer engages in low-carbon promotional campaigns to influence consumer purchasing behavior, places the initial firm order quantity q_1 at a wholesale price w, purchases the call option order quantity q_2 at an option price o, and bears the proportion δ of the carbon abatement cost $g(\theta)$. Note that such low-carbon promotional activity is not endogenously optimized within our model. Each call option order grants the retailer the right to purchase one additional unit at the option exercise price e after the actual demand is observed. According to retailer's two types of order quantities, the manufacturer decides the make-to-order production and the production quantity Q of the product. If the manufacturer faces

a stockout, it incurs a substantial stockout cost, which can be interpreted as procuring the alternative product from the spot market for replenishment. At the start of the sales season, the retailer receives q_1 through the firm order. During the sales season, the retailer decides whether and how many call options to exercise based on actual market demand. After the sales season, any remaining products are salvaged by the manufacturer and the retailer at a residual value v.

It is assumed that the manufacturer is risk-neutral and its goal is to maximize the expected profit, while the retailer is risk-averse and aims to maximize the CVaR of its profit. Specifically, when the retailer's profit function is $\pi_r(X; q)$, CVaR is defined as follows [23, 34, 35]:

$$\mathrm{CVaR}_\eta(\pi_r(X;q)) = \max_{\xi \in R} \left\{ \xi - \frac{1}{\eta} E[\xi - \pi_r(X;q)]^+ \right\}, \tag{2.1}$$

where $\xi \in \mathbb{R}$ denotes the target profit level, E denotes the expectation operator, and $\eta \in (0, 1]$ denotes the tail probability for CVaR measurement, which serves as the risk-aversion coefficient of the retailer. Note that the retailer is risk-averse when $0 < \eta < 1$ and the retailer is risk-neutral when $\eta = 1$. Further, a larger value of η corresponds to a lower risk aversion level.

It is further assumed that the information held by the retailer and the manufacturer is symmetric. Such an assumption is reasonable because the two supply-chain parties establish stable long-term cooperative interactions and follow standardized and consistent transaction rules throughout the cooperation process. The carbon emissions tax is borne by the retailer, which can be interpreted as a tax on carbon emissions generated in the procurement, inventory, and transportation processes. Further, the tax generated in the production process can be understood as being incorporated into the production cost c. To avoid unreasonable cases, the parameters are required to satisfy conditions: (1) $c + \alpha < w < e + o < p - \alpha$ and $v < e$ ensure a profit for both the manufacturer and retailer. (2) $e < w + o$ and $o + e > w > o$ prevent the retailer from engaging in arbitrage using the call option. (3) $c > o + v$ prevents the manufacturer from engaging in arbitrage using the call option. (4) $c - v < p - \alpha - c$ ensures that the total revenue of the supply-chain is greater than the total cost. (5) $h > c > v$ ensures that the production capacity aligns with market demand. The superscript $*$ denote optimality, and the subscripts s and r denotes the manufacturer and the retailer, respectively. The notation $x^+ = \max\{x, 0\}$.

3. Optimal decisions with carbon abatement cost-sharing and call option contracts

This section formulates the supply-chain model with carbon abatement cost-sharing and call option contracts. The game interaction within the supply-chain is modeled as a Stackelberg game, where the manufacturer, as the leader, determines its optimal green investment strategy to maximize its expectation profit. After observing the manufacturer's decision, the retailer, as the follower, determines its order quantity to maximize the CVaR of its profit. To solve the equilibrium of the Stackelberg game, the backward induction method is employed. Specifically, the retailer first optimizes its order quantity based on the observed green investment level of the manufacturer; subsequently, the manufacturer, by rationally predicting the retailer's reaction function, optimizes and determines the optimal green investment strategy that maximizes its expectation profit. Finally, the supply-chain is coordinated through adjustment of the contract parameters.

3.1. Optimal ordering strategy with carbon abatement cost-sharing and call option contracts

In this context, the retailer employs a two-part procurement strategy: an initial firm order (via a wholesale price contract) and call option orders (via call option contracts). Note that the initial firm order is a binding commitment, and the call option orders provide the right for the retailer to adjust initial firm orders before the start of the selling period. In this case, the retailer faces the problem of optimizing two decision variables: the initial firm order quantity q_1 and the call option order quantity q_2.

With carbon abatement cost-sharing and call option contracts, the risk-averse retailer's random profit is

$$\begin{aligned}\pi_r(X;q_1,q_2) =& p\min(q_1+q_2,X)+v(q_1-X)^+-wq_1-oq_2-e\min(q_2,(X-q_1)^+)\\ &-\alpha[q_1+\min((X-q_1)^+,q_2)]-\delta g(\theta).\end{aligned} \tag{3.1}$$

Equation (3.1) can be reformulated as

$$\pi_r(X;q_1,q_2)=\begin{cases}(p-v)X-(w-v+\alpha)q_1-oq_2-\delta g(\theta), & 0\le X<q_1,\\ (p-e-\alpha)X-(w-e)q_1-oq_2-\delta g(\theta), & q_1\le X<q_1+q_2,\\ (p-\alpha)(q_1+q_2)-wq_1-(o+e)q_2-\delta g(\theta), & q_1+q_2\le X.\end{cases} \tag{3.2}$$

From Equations (2.1) and (3.2), it is derived that $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q_2))$ is

$$\mathrm{CVaR}_\eta(\pi_r(X;q_1,q_2))=\max_{\xi\in R}\left\{\xi-\frac{1}{\eta}\mathrm{E}[\xi-\pi_r(X;q_1,q_2)]^+\right\}. \tag{3.3}$$

.

With carbon abatement cost-sharing and call option contracts, the risk-averse retailer aims to maximize $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q_2))$, and the corresponding optimization problem can be formulated as P_1.

$$\mathrm{P}_1:\quad \max \mathrm{CVaR}_\eta(\pi_r(X;q_1,q_2)) \quad s.t.\begin{cases}q_1\ge 0,\\ q_2\ge 0.\end{cases} \tag{3.4}$$

Let $g(\xi,q_1,q_2)=\xi-\frac{1}{\eta}\mathrm{E}[\xi-\pi_r(X;q_1,q_2)]^+$, and from Equation (3.2),

$$\begin{aligned}g(\xi,q_1,q_2)=&\xi-\frac{1}{\eta}\int_0^{q_1}[\xi+(w+\alpha-v)q_1+oq_2+\delta g(\theta)-x(p-v)]^+f(x\mid\theta)\mathrm{d}x\\ &-\frac{1}{\eta}\int_{q_1}^{q_1+q_2}[\xi+(w-e)q_1+oq_2+\delta g(\theta)-x(p-\alpha-e)]^+f(x\mid\theta)\mathrm{d}x\\ &-\frac{1}{\eta}\int_{q_1+q_2}^{\infty}[\xi+\delta g(\theta)-(p-\alpha-w)q_1-(p-\alpha-o-e)q_2]^+f(x\mid\theta)\mathrm{d}x.\end{aligned}$$

From Equation (3.3), $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q_2))=\max_{\xi\in R}g(\xi,q_1,q_2)$. To derive the closed-form expression of $\mathrm{CVaR}_\eta(\pi_r(X;q_1,q_2))$, the following four scenarios are examined.

Case 1. When $\xi\le-(w+\alpha-v)q_1-oq_2-\delta g(\theta)$, $g(\xi,q_1,q_2)=\xi$ and $\frac{\partial g(\xi,q_1,q_2)}{\partial\xi}=1>0$.

Case 2. When $-(w+\alpha-v)q_1-oq_2-\delta g(\theta)<\xi\le(p-\alpha-w)q_1-oq_2-\delta g(\theta)$, $g(\xi,q_1,q_2)=\xi-\frac{1}{\eta}\int_0^{\frac{\xi+(w+\alpha-v)q_1+oq_2+\delta g(\theta)}{p-v}}[\xi+(w+\alpha-v)q_1+oq_2+\delta g(\theta)-x(p-v)]f(x\mid\theta)\mathrm{d}x$. Since $\frac{\partial g(\xi,q_1,q_2)}{\partial\xi}=1-$

$\frac{1}{\eta}F(\frac{\xi+(w+\alpha-v)q_1+oq_2+\delta g(\theta)}{p-v} \mid \theta)$, then $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} \mid_{\xi=-(w+\alpha-v)q_1-oq_2-\delta g(\theta)} = 1 > 0$ and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} \mid_{\xi=(p-w-\alpha)q_1-oq_2-\delta g(\theta)} = 1 - \frac{1}{\eta}F(q_1 \mid \theta)$.

Case 3. When $(p-\alpha-w)q_1 - oq_2 - \delta g(\theta) < \xi \leq (p-\alpha-w)q_1 + (p-\alpha-o-e)q_2 - \delta g(\theta)$, $g(\xi, q_1, q_2) = \xi - \frac{1}{\eta}\int_0^{q_1}[\xi + (w+\alpha-v)q_1 + oq_2 + \delta g(\theta) - x(p-v)]f(x \mid \theta)\mathrm{d}x - \frac{1}{\eta}\int_{q_1}^{\frac{\xi+(w-e)q_1+oq_2+\delta g(\theta)}{p-\alpha-e}}[\xi + (w-e)q_1 + oq_2 + \delta g(\theta) - x(p-\alpha-e)]f(x \mid \theta)\mathrm{d}x$. Since $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 1 - \frac{1}{\eta}F(\frac{\xi+(w-e)q_1+oq_2+\delta g(\theta)}{p-\alpha-e} \mid \theta)$, then $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} \mid_{\xi=(p-\alpha-w)q_1-oq_2-\delta g(\theta)} = 1 - \frac{1}{\eta}F(q_1 \mid \theta)$ and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} \mid_{\xi=(p-\alpha-w)q_1+(p-\alpha-o-e)q_2-\delta g(\theta)} = 1 - \frac{1}{\eta}F(q_1+q_2 \mid \theta)$.

Case 4. When $(p-\alpha-w)q_1 + (p-\alpha-o-e)q_2 - \delta g(\theta) < \xi$, $g(\xi, q_1, q) = \xi - \frac{1}{\eta}\int_0^{q_1}[\xi + (w+\alpha-v)q_1 + oq_2 + \delta g(\theta) - x(p-v)]f(x \mid \theta)\mathrm{d}x - \frac{1}{\eta}\int_{q_1}^{q_1+q_2}[\xi + wq_1 + oq_2 - eq_1 + \delta g(\theta) - x(p-e-\alpha)]f(x \mid \theta)\mathrm{d}x - \frac{1}{\eta}\int_{q_1+q_2}^{\infty}[\xi + \delta g(\theta) - (p-w-\alpha)q_1 - (p-o-e-\alpha)q_2]f(x \mid \theta)\mathrm{d}x$ and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 1 - \frac{1}{\eta} < 0$.

Let $\xi^* = \arg\max_{\xi \in R} g(\xi, q_1, q_2)$. Since $g(\xi, q_1, q_2)$ is a concave function of ξ, examining Cases 1–4, it is derived that $\xi^* \in (-oq_2 - (w+\alpha-v)q_1 - \delta g(\theta), (p-\alpha-w)q_1 + (p-\alpha-o-e)q_2 - \delta g(\theta)]$. It is assumed that $1 - \frac{1}{\eta}F(q_1+q_2 \mid \theta) < 0$ and ξ^* is the solution of $\frac{\partial g(\xi,q_1,q_1)}{\partial \xi} = 1 - \frac{1}{\eta}F(\frac{\xi+(w-e)q_1+oq_2+\delta g(\theta)}{p-\alpha-e} \mid \theta) = 0$, i.e., $\xi^* = F^{-1}(\eta \mid \theta)(p-\alpha-e) - oq_2 - (w-e)q_1 - \delta g(\theta)$. Then $g(\xi^*, q_1, q) = \xi^* - \frac{1}{\eta}\int_0^{q_1}[\xi^* + (w+\alpha-v)q_1 + oq_2 + \delta g(\theta) - x(p-v)]f(x \mid \theta)\mathrm{d}x - \frac{1}{\eta}\int_{q_1}^{\frac{\xi^*+(w-e)q_1+oq_2+\delta g(\theta)}{p-\alpha-e}}[\xi^* + (w-e)q_1 + oq_2 + \delta g(\theta) - x(p-\alpha-e)]f(x \mid \theta)\mathrm{d}x$. It is calculated that the first-order partial derivative of $g(\xi^*, q_1, q_2)$ with respect to q_1 and q_2 satisfy $\frac{\partial g(\xi^*,q_1,q_2)}{\partial q_1} = e - w - \frac{1}{\eta}(e - v + \alpha) = -\frac{1}{\eta}[w\eta + \alpha\eta - v + (1-\eta)(e+\alpha)] < 0$ and $\frac{\partial g(\xi^*,q_1,q)}{\partial q} < 0$, respectively, which is inconsistent with the theoretical premises. Thus, it is derived that $1 - \frac{1}{\eta}F(q_1+q_2 \mid \theta) > 0$.

$$\xi^* = arg\max_{\xi \in R} g(\xi, q_1, q_2) = (p-\alpha-w)q_1 + (p-\alpha-o-e)q_2 - \delta g(\theta). \tag{3.5}$$

From Equations (3.3) and (3.5), it follows that

$$\begin{aligned}\mathrm{CVaR}_\eta(\pi_r(X; q_1, q_2)) &= g(\xi^*, q_1, q_2)\\ &= (p-\alpha-w)q_1 + (p-\alpha-o-e)q_2 - \delta g(\theta)\\ &- \frac{1}{\eta}\int_0^{q_1}[(p-v)q_1 + (p-\alpha-e)q_2 - x(p-v)]f(x \mid \theta)\mathrm{d}x\\ &- \frac{1}{\eta}\int_{q_1}^{q_1+q_2}(p-\alpha-e)(q_1+q_2-x)f(x \mid \theta)\mathrm{d}x.\end{aligned} \tag{3.6}$$

Since $q = q_1 + q_2$, the retailer's decision problem with respect to q_1 and q_2 is equivalent to the decision problem with respect to q_1 and q. Equation (3.6) can be reformulated as

$$\begin{aligned}\mathrm{CVaR}_\eta(\pi_r(X; q_1, q)) &= (o+e)q_1 + (p-\alpha-o-e)q - wq_1 - \delta g(\theta)\\ &- \frac{1}{\eta}\int_0^{q_1}[(e+\alpha-v)q_1 + (p-\alpha-e)q - x(p-v)]f(x \mid \theta)\mathrm{d}x\\ &- \frac{1}{\eta}\int_{q_1}^{q}(p-\alpha-e)(q-x)f(x \mid \theta)\mathrm{d}x.\end{aligned} \tag{3.7}$$

Lemma 1. *With carbon abatement cost-sharing and call option contracts, the retailer's* $\mathrm{CVaR}_\eta(\pi_r(X; q_1, q))$ *is a jointly concave function with respect to the decision variables* q_1 *and* q.

Proof. See the Appendix. □

From this lemma, the optimal solutions to the optimization problem P_1 can be derived. Let q_1^* and q_2^* denote these optimal solutions. The expressions of q_1^* and q_2^* are presented in the following proposition.

Proposition 1. *With carbon abatement cost-sharing and call option contracts, the optimal firm order quantity q_1^* and the optimal option order quantity q_2^* of the risk-averse retailer are respectively expressed as follows: $q_1^* = F^{-1}(A \mid \theta)$ and $q_2^* = F^{-1}(B \mid \theta) - F^{-1}(A \mid \theta)$, where $A = \dfrac{(e+o-w)}{e+\alpha-v}\eta$ and $B = \dfrac{(p-\alpha-o-e)}{p-\alpha-e}\eta$.*

Proof. See the Appendix. □

Since $e > v$, $o + e > w$, and $p > \alpha + o + e$ hold, it can be deduced that $A > 0$ and $B > 0$. From Proposition 1, it is shown that $q_1^* > 0$ and $q^* > 0$. From $q_2^* > 0$, it is derived that $B > A$, which in turn leads to $o < \overline{o}$, where $\overline{o} = \frac{(w-v+\alpha)(p-e-\alpha)}{p-v}$. The parameter $\overline{o}$ represents the threshold of the option price at which carbon abatement cost-sharing and call option contracts are activated. Further, if $o \geq \overline{o}$, then it is derived that $q_2^* = 0$, which implies that the retailer solely places the initial firm order under the wholesale price contract. This scenario will be examined in Section 4. The above findings reveal a stable ordering strategy rule for risk-averse retailers. Specifically, when the option price o is sufficiently low ($o < \overline{o}$), the retailer is always willing to adopt a hybrid ordering strategy that combines fixed firm orders and flexible option orders. In other words, a low option price effectively reduces the retailer's procurement risk and capital pressure, thereby motivating the retailer to reserve partial production capacity through call options while maintaining basic fixed order quantities. Accordingly, this study sets a reasonable threshold for the option price to sustain the effectiveness of the hybrid ordering mechanism.

If the carbon tax policy is not considered (i.e., $\alpha = 0$), then $\overline{o}$ changes to $\frac{(w-v)(p-e)}{p-v}$. It is shown that $\frac{(w-v)(p-e)}{p-v} > \frac{(w-v)(p-e-\alpha)}{p-v-\alpha}$. Further, our equilibrium analysis demonstrates that carbon tax regulation lowers the critical threshold of the feasible option price. This finding yields clear managerial implications. On the one hand, increased carbon tax costs amplify the manufacturer's low-carbon operational burden, which indirectly squeezes the overall supply-chain profit margin. On the other hand, to stimulate the retailer's ordering enthusiasm, the manufacturer is required to transfer partial profit margins or undertake corresponding risk-sharing responsibilities. Such a profit incentive mechanism encourages the retailer to raise its total order quantity, thereby improving the overall operational efficiency and carbon abatement performance of the low-carbon supply-chain.

From Proposition 1, the optimal total order quantity of the retailer can be expressed as $q^* = q_1^* + q_2^* = F^{-1}(B)$. Further, it can be derived that both q_1^* and q^* are monotonically increasing with respect to η and monotonically decreasing with respect to α. The following proposition examines how the manufacturer's green investment θ affects the retailer's optimal ordering policy.

Proposition 2. *With carbon abatement cost-sharing and call option contracts, as the green investment of the manufacturer θ increases, both the optimal firm order quantity q_1^* and the optimal total order quantity q^* are monotonically increasing, respectively, i.e., $\frac{dq_1^*}{d\theta} > 0$ and $\frac{dq^*}{d\theta} > 0$.*

Proof. See the Appendix. □

Proposition 2 shows that q_1^* and q^* are monotonically increasing in θ, respectively. This effect is attributed to the increase in green investment by the manufacturer, which reduces carbon emissions,

lowers carbon-related costs (e.g., carbon tax), and simultaneously stimulates market demand. This finding underscores the strategic value of the manufacturer's green investments in driving operational decisions, creating the win-win outcome that improves environmental performance and supply-chain efficiency.

3.2. Optimal green investment strategy with carbon abatement cost-sharing and call option contracts

This context considers the manufacturer's optimal green investment strategy. To stimulate the retailer to increase order quantities, the manufacturer endeavors to enhance the green investment to expand product influence, which results in a carbon abatement cost. Thus, according to the retailer's optimal firm order quantity (q_1^*) and optimal option order quantity (q_2^*), the manufacturer needs to determine the green investment (θ) and the production quantity (Q) before the beginning of the selling season. Since the manufacturer adopts the make-to-order production pattern, it is shown that with carbon abatement cost-sharing and call option contracts, the production quantity Q of the manufacturer satisfies $Q = q^*$.

Under the Stackelberg game framework, the optimal firm order quantity $q_1^*(\theta)$ and the optimal total order quantity $q^*(\theta)$ of the retailer are endogenous responses to the manufacturer's green investment θ. Therefore, the manufacturer's expected profit is a composite function of θ. With carbon abatement cost-sharing and call option contracts, the manufacturer's profit function $\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)$ is

$$\begin{aligned}\pi_s(X; q_1^*(\theta), q^*(\theta), \theta) =& wq_1^*(\theta) + oq_2^*(\theta) + e(\min(q_2^*(\theta), (X - q_1^*(\theta))^+) + v(q_2^*(\theta) \\ &- \min(q_2^*(\theta), (X - q_1^*(\theta))^+) - cq^*(\theta) - (1-\delta)g(\theta).\end{aligned} \tag{3.8}$$

Equation (3.8) can be reformulated as

$$\begin{aligned}&\pi_s(X; q_1^*(\theta), q^*(\theta), \theta) \\ &= \begin{cases} wq_1^*(\theta) + oq_2^*(\theta) + vq_2^*(\theta) - cq^*(\theta) - (1-\delta)g(\theta), & 0 \le X < q_1^*(\theta), \\ (w-e)q_1^*(\theta) + oq_2^*(\theta) + (e-v)X + (v-c)q^*(\theta) - (1-\delta)g(\theta), & q_1^*(\theta) \le X < q^*(\theta), \\ wq_1^*(\theta) + (o+e)q_2^*(\theta) - cq^*(\theta) - (1-\delta)g(\theta), & q^*(\theta) \le X. \end{cases}\end{aligned} \tag{3.9}$$

Let $\mathrm{E}[\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)]$ be the expectation function of $\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)$. With carbon emissions reduction cost-sharing and call option contracts, the manufacturer aims to maximize $\mathrm{E}[\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)]$, and the corresponding optimization problem can be formulated as P_2:

$$\mathrm{P}_2: \quad \max_{0 \le \theta \le 1} \mathrm{E}[\pi_s(X; q_1^*(\theta), q^*(\theta), \theta)] \tag{3.10}$$

Let θ^* be the optimal solution to optimization problem P_2. The expression of θ^* is given in the following proposition.

Proposition 3. *With carbon abatement cost-sharing and call option contracts, the optimal green investment strategy of the manufacturer is expressed as follows:*

$$\theta^* = \begin{cases} 0, & \overline{\theta} \le 0, \\ \overline{\theta}, & 0 < \overline{\theta} < 1, \\ 1, & 1 \le \overline{\theta}, \end{cases}$$

where

$$\bar{M} f'(\overline{\theta}) K - (1-\delta) g'(\overline{\theta}) = 0, \tag{3.11}$$

$M_1 = w - o - e + (e - v)A$, $M_2 = o + e - c - (e - v)B$, and $K = \left[AM_1 + BM_2 + \frac{e - v}{2}(B^2 - A^2)\right]$.

Proof. See the Appendix. □

According to Proposition 3, with carbon abatement cost-sharing and call option contracts, the manufacturer's optimal green investment θ^* is affected by the price parameter v, option contract parameters e and o, carbon abatement cost $g(\theta)$, sharing ratio δ, and wholesale price w. Note that the first-order conditions characterize interior optima. For the complete optimization problem subject to $\theta \in [0, 1]$, boundary solutions $\theta = 0$ or $\theta = 1$ will arise if the unconstrained stationary point lies outside this feasible domain. When $0 < \bar{\theta} < 1$, let $L(\bar{\theta}, \delta) = -(1 - \delta)g'(\bar{\theta}) + \bar{M}f'(\bar{\theta})K$, and it is derived that $\frac{\partial L(\bar{\theta},\delta)}{\partial \delta} = g'(\bar{\theta}) > 0$ and $\frac{\partial L(\bar{\theta},\delta)}{\partial \bar{\theta}} = -(1 - \delta)g''(\bar{\theta}) + \bar{M}f''(\bar{\theta})K < 0$. Further, it is demonstrated that $\frac{d\bar{\theta}}{d\delta} = -\frac{\partial L(\bar{\theta},\delta)}{\partial \delta} / \frac{\partial L(\bar{\theta},\delta)}{\partial \bar{\theta}} > 0$ and $\frac{d\theta^*}{d\delta} \geq 0$. This analytical result reveals a positive monotonic relationship between the manufacturer's equilibrium green investment and the retailer's carbon abatement cost-sharing ratio. Concretely, a larger sharing ratio δ means the retailer covers more abatement expenses and reduces the manufacturer's marginal green investment cost, which in turn drives the manufacturer to raise its optimal green investment intensity. For supply-chain practitioners, designing a well-matched cost-sharing mechanism enables retailers to motivate manufacturers to boost decarbonization efforts under carbon tax policies.

3.3. Supply-chain coordination with carbon abatement cost-sharing and call option contracts

The previous content has analyzed the decision-making strategies of the retailer and the manufacturer. Under the decentralized supply-chain, different members may have disagreements on cost-sharing and profit distribution, and it tends to trigger interest conflicts among members. To improve the performance of the supply-chain, this section focuses on a centralized supply-chain and discusses the issue of supply-chain coordination.

Under a centralized supply-chain, this paper regards supply-chain members as an integrated firm, and examines the centralized system's optimal production policy and optimal green investment strategy. Let Q be the production quantity of the centralized system, Θ be the green investment of the centralized system, and $\pi_c(X; Q, \Theta)$ be the profit function of the centralized system. Then $\pi_c(X; Q, \Theta)$ can be expressed as follows:

$$\pi_c(X; Q, \Theta) = \begin{cases} pX + vQ - vX - \alpha X - g(\Theta) - cQ, & X < Q, \\ pQ - \alpha Q - g(\Theta) - cQ, & X \geq Q. \end{cases} \tag{3.12}$$

From Equation (3.12), the corresponding expected profit function of the centralized system can be formulated as follows:

$$\mathrm{E}[\pi_c(X; Q, \Theta)] = (p - \alpha)\mathrm{E}[\min(Q, X)] + v\mathrm{E}(Q - \min(X, Q))^+ - g(\Theta) - cQ. \tag{3.13}$$

The first term $p\mathrm{E}[\min(Q, X)]$ represents the retail profit, and $\alpha\mathrm{E}[\min(Q, X)]$ represents the carbon tax, the second term $v\mathrm{E}(Q - \min(X, Q))^+$ represents the salvage value of remaining products, the third term $g(\Theta)$ represents the carbon abatement cost, and the fourth term cQ represents the production cost (including the carbon tax incurred in the production process).

With a carbon tax policy, the optimization problem of the centralized system is denoted as P_3, which is expressed as follows:

$$\mathrm{P}_3: \quad \max \quad \mathrm{E}[\pi_c(X; Q, \Theta)] \qquad s.t. \quad \begin{cases} 1 \geq \Theta \geq 0, \\ Q \geq 0. \end{cases} \tag{3.14}$$

Let Q^* and Θ^* be the optimal solutions to the optimization problem P_3. The expressions of Q^* and Θ^* are given in the following proposition.

Proposition 4. *With a carbon tax policy, the centralized system's optimal production quantity and optimal green investment satisfy the following relationships:*

$$Q^* = F^{-1}(C \mid \Theta^*), \tag{3.15}$$

where $C = \dfrac{p - \alpha - c}{p - \alpha - v}$ *and*

$$\Theta^* = \begin{cases} 0, & \overline{\Theta} \leq 0, \\ \overline{\Theta}, & 0 < \overline{\Theta} < 1, \\ 1, & 1 \leq \overline{\Theta}, \end{cases}$$

where

$$g'(\overline{\Theta}) - \frac{(p - \alpha - v)}{2} C^2 \bar{M} f'(\overline{\Theta}) = 0. \tag{3.16}$$

Proof. See the Appendix. □

From Proposition 4, Q^* and Θ^* are affected by multiple factors, including the retail price p, carbon abatement cost $g(\theta)$, production cost c, carbon tax α, salvage value of remaining products v, and market demand uncertainty. Note that the first-order conditions characterize interior optima. For the complete optimization problem subject to $\Theta \in [0, 1]$, boundary solutions $\Theta = 0$ or $\Theta = 1$ will arise if the unconstrained stationary point lies outside this feasible domain. The interdependent transformation logic of these decision variables is intricate, which mirrors the dynamic equilibrium between marginal revenue and marginal abatement cost. Each decision variable endogenously interacts with others and reshapes participants' marginal revenue and cost structures. From a practical perspective, managers should avoid isolated single-variable optimization and coordinate multiple operational parameters to balance economic returns.

The complexity of Q^* and Θ^* is also reflected in the fact that the green investment affects production decisions, while the production decisions also affect green investment. This is caused by the dependence of market demand on the green investment. If it is assumed that the market demand is independent of green investment (i.e., $F(x \mid \theta) = F(x)$), then $\frac{\partial \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial Q} = p - \alpha - c - (p - \alpha - v)F(Q^*)$ and $\frac{\partial \mathrm{E}[\pi_c(X;Q,\Theta)]}{\partial \Theta} = -\frac{dg(\Theta)}{d\Theta}$. It is deduced that the centralized system's optimal green investment θ^* is not affected by random market demand or production quantity, and the optimal production quantity Q^* is not affected by the green investment.

Taking the centralized supply-chain decision as a benchmark, the following proposition explores the conditions for joint carbon abatement and coordination in the supply-chain with carbon abatement cost-sharing and call option contracts.

Proposition 5. *With carbon abatement cost-sharing and call option contracts, the necessary decision-matching conditions for aligning decentralized decisions with the centralized supply-chain optimum are as follows:*

$$(p - c - \alpha)(p - \alpha - e) = (p - \alpha - v)(p - \alpha - o - e)\eta \tag{3.17}$$

and

$$2K = (p - \alpha - v)(1 - \delta)B^2. \tag{3.18}$$

Proof. See the Appendix. □

In the presence of carbon abatement cost-sharing and call option contracts, the necessary decision-matching conditions serve to align decentralized equilibrium outcomes with the optimal solution of the centralized supply chain. According to Proposition 5, with carbon abatement cost-sharing and call option contracts, the conditions of supply-chain coordination require satisfying two important equation relationships, which are affected by the carbon tax α, call option contract parameters e and o, carbon abatement cost $g(\theta)$, sharing ratio δ, and risk aversion coefficient.

From Equation (3.18), the sharing ratio δ of the carbon abatement cost $g(\theta)$ is affected by the interaction of multi-dimensional parameters. The retail price p is significantly positively correlated with the sharing ratio δ (i.e., $\frac{\partial \delta}{\partial p} > 0$), which indicates that the increase in the retail price enables the retailer to cover the carbon abatement cost through higher profits, thereby being willing to bear a larger carbon abatement cost. Further, the call option exercise price e is negatively related to δ (i.e., $\frac{\partial \delta}{\partial e} < 0$). This analytical outcome can be economically interpreted as follows: an excessively high option exercise price squeezes the retailer's profit margin and suppresses its incentive to bear carbon abatement costs. Furthermore, the call option contract parameter o indirectly reshapes equilibrium cost-sharing arrangements by shifting the capacity reservation discount threshold. This result confirms the synergistic interplay between price signals and contractual design, and provides theoretical foundations for dynamically calibrating carbon abatement cost-sharing ratios. In practice, manufacturers ought to avoid setting excessive exercise prices, which would dampen retailers' enthusiasm for cost-sharing and impair supply-chain decarbonization. Instead, managers can take advantage of the interactive linkage between option pricing and cost-sharing mechanisms to formulate flexible adaptive coordination strategies that reconcile economic profits and carbon-reduction obligations.

4. Discussion

This section investigates the impact of carbon abatement cost-sharing and call option contracts on the supply chain.

4.1. The effect of the call option contract

This subsection considers the case without a call option contract. Under this case, there is no flexibility in supply and ordering. Further, this subsection formulates the model with carbon abatement cost-sharing and wholesale price contracts, and we take this model as a useful benchmark to examine the impact of the call option contract on the joint carbon abatement and coordination of the supply-chain. It is assumed that under this model, the retailer's order quantity is q_0, and all other parameters remain

unchanged. The corresponding profit function of the retailer $\pi_r(X; q_0)$ is

$$\pi_r(X; q_0) = \begin{cases} pX - \alpha q_0 - wq_0 + v(q_0 - X) - \delta g(\theta), & X < q_0, \\ pq_0 - \alpha q_0 - wq_0 - \delta g(\theta), & X \geq q_0. \end{cases} \tag{4.1}$$

Based on the model assumptions, from Equations (2.1) and (4.1), $\mathrm{CVaR}_\eta(\pi_r(X; q_0))$ can be expressed as follows:

$$\mathrm{CVaR}_\eta(\pi_r(X; q_0)) = \max_{\xi \in R} \left\{ \xi - \frac{1}{\eta} \mathrm{E}[\xi - \pi_r(X; q_0)]^+ \right\}. \tag{4.2}$$

With carbon abatement cost-sharing and wholesale price contracts, the risk-averse retailer aims to maximize $\mathrm{CVaR}_\eta(\pi_r(X; q_0))$, and the corresponding optimization problem can be formulated as $\overline{\mathrm{P}_0}$:

$$\begin{aligned} \overline{\mathrm{P}_0}: \quad & \max \quad \mathrm{CVaR}_\eta(\pi_r(X; q_0)) \\ & s.t. \quad q_0 \geq 0. \end{aligned} \tag{4.3}$$

Let q_0^* denote the ordering quantity that maximizes the CVaR of the risk-averse retailer, i.e., the optimal solution to the optimization problem $\overline{\mathrm{P}_0}$. The expression of q_0^* is presented in the following proposition.

Proposition 6. *With carbon abatement cost-sharing and wholesale price contracts, the risk-averse retailer's optimal order quantity q_0^* is expressed as follows: $q_0^* = F^{-1}(D \mid \theta)$, where $D = \frac{(p-w-\alpha)}{p-v}\eta$.*

Proof. The proof is similar to that of Proposition 1 and is omitted. □

According to Proposition 6, without the call option contract, the risk-averse retailer's optimal order quantity is affected by the carbon tax α, wholesale price w, the risk aversion coefficient η, and market demand uncertainty. Without call option contracts, risk-averse retailers can only adopt rigid fixed ordering strategies, which results in a suboptimal equilibrium order quantity. Deprived of flexible capacity reservation instruments, retailers cannot dynamically adjust procurement volumes to respond to uncertain market demand and thus tend to place conservative orders to evade losses from stockouts or overstock.

Let θ_0^* denote the manufacturer's optimal green investment under this scenario, from Proposition 5 and 6, and the following proposition is derived.

Proposition 7. *Under the carbon tax policy, the carbon abatement cost-sharing and wholesale price contracts cannot achieve joint carbon abatement and coordination of the supply-chain. Further, when $\theta_0^* = \Theta^*$, it is shown that $q_0^* < Q^*$.*

Proof. See the Appendix. □

The above research indicates that without the call option contract, the joint carbon abatement and coordination of the supply-chain cannot be achieved. This is because the introduction of the call option contract endows the retailer with greater flexibility in product procurement, eliminates costs associated with returns, reduces carbon emissions, increases purchase quantities, achieves supply-chain coordination and enhances supply-chain performance. From a managerial perspective, pure fixed wholesale contracts fail to hedge demand risks for risk-averse retailers. Manufacturers should integrate call option clauses into coordination agreements to offer downstream firms flexible capacity rights, which encourages retailers to raise their total order quantities, boosts upstream low-carbon investment, and ultimately balances operational risk control and supply-chain decarbonization under carbon tax regulation.

4.2. The effect of the carbon abatement cost-sharing contract

This subsection focuses on the effect of the carbon abatement cost-sharing contract on supply-chain, and considers the case without a carbon abatement cost-sharing contract. In this case, this subsection formulates the model with the call option contract, and we take this model as a useful benchmark to examine the impact of the carbon abatement cost-sharing contract on the supply-chain.

In practice, the analysis process of this subsection is analogous to the previous research. It is assumed that all the parameters remain unchanged except that $\delta \equiv 0$.

Let q_1^* be the retailer's optimal order quantity under this scenario, and under the decentralized supply-chain, from Proposition 1, it is derived that the retailer's optimal total order quantity satisfies the equation

$$\eta(p - e - \alpha - o) - (p - e - \alpha)F(q_1^* \mid \theta) = 0. \tag{4.4}$$

From Proposition 3, the manufacturer's optimal green investment under this scenario is expressed as follows:

$$\theta_1^* = \begin{cases} 0, & \overline{\theta_1} \le 0, \\ \overline{\theta_1}, & 0 < \overline{\theta_1} < 1, \\ 1, & 1 \le \overline{\theta_1}, \end{cases}$$

where

$$\bar{M} f'(\overline{\theta_1})K - g'(\overline{\theta_1}) = 0, \tag{4.5}$$

where $M_1 = w - o - e + (e - v)A$, $M_2 = o + e - c - (e - v)B$, and $K = \left[AM_1 + BM_2 + \dfrac{e - v}{2}(B^2 - A^2)\right]$

Under the centralized supply-chain, the centralized system's optimal production decision strategy and optimal green investment level remain unchanged, and still satisfy Equations (3.15) and (3.16). By the decision-making strategies of decision-makers under the decentralized and centralized supply-chain, the following proposition is derived.

Proposition 8. *Under the carbon tax policy, the supply-chain fails to achieve coordination in the absence of the carbon abatement cost-sharing contract. Further, when $q_1^* = Q^*$, it is deduced that $\theta_1^* \le \Theta^*$.*

Proof. See the Appendix. □

Proposition 8 demonstrates that without the green investment cost-sharing contract, the manufacturer's optimal green investment under the decentralized supply-chain is lower than that under the centralized supply-chain. This can be explained as follows: Under the decentralized supply-chain, the manufacturer bears green investment costs but cannot capture the benefits generated by green investment. Further, the manufacturer's marginal benefit from carbon abatement is lower than that under the centralized supply-chain, and the manufacturer tends to reduce its green investment. Under the centralized supply-chain, the manufacturer and the retailer are a single entity, requiring an integrated consideration of benefits and costs. Green investment brings additional benefits such as increased market share, as it can stimulate higher green investment.

The above research indicates that with carbon abatement cost-sharing alone, the issue of market demand fluctuation risk remains unresolved. This causes the retailer's ordering decision to deviate from the optimal order quantity, and the manufacturer may reduce green investment due to profit uncertainty,

making supply-chain coordination difficult. A standalone call option contract can hedge demand risk, but it fails to address collaborative carbon emission reduction. This results in a lack of motivation for carbon abatement across the entire supply-chain, leading to a gap between the actual green investment and the optimal green investment. Thus, the combination of carbon emission cost-sharing and call option contracts is required to achieve joint carbon abatement and the coordination of the supply-chain.

5. Numerical experiments

This section conducts numerical experiments to further examine the impacts of green investment and the risk aversion coefficient on the optimal operational decisions, and verifies the theoretical findings.

The experimental parameters are set as follows: $p = 50$, $o = 6$, $e = 35$, $w = 33$, $v = 1$, $c = 8$, and $\delta = 0.7$. It is assumed that market demand follows $X = f(\theta)\varepsilon$, where $f(\theta) = \theta^{\frac{1}{\beta}} + \gamma$ represents the demand function dependent on green investment θ, $\beta = 5$, and $\gamma = 1$. Without loss of generality, $\varepsilon \sim U[0, 100]$ is a random demand factor following a uniform distribution. The carbon abatement cost follows $g(\theta) = k\theta^2$, where $k = 50$.

To analyze the impact of the manufacturer's green investment on the optimal order quantity q_1^*, q^*, and q_0^*, the risk aversion coefficient is fixed at $\eta = 0.7$ and the carbon tax is fixed at $\alpha = 5$. Further, the manufacturer's green investment θ increases from 0 to 0.95 with a step size of 0.05. Figure 1 shows that the top curve corresponds to q^* , which maintains the highest order quantity across the entire range of θ. The middle curve represents q_0^* , with values consistently lower than q^*, which is consistent with Proposition 7. Further, from Figure 1, it is shown that q_1^*, q^*, and q_0^* exhibit a monotonically increasing trend with the increase in θ, which illustrates the positive correlation between the manufacturer's green investment and the retailer's optimal ordering quantity.

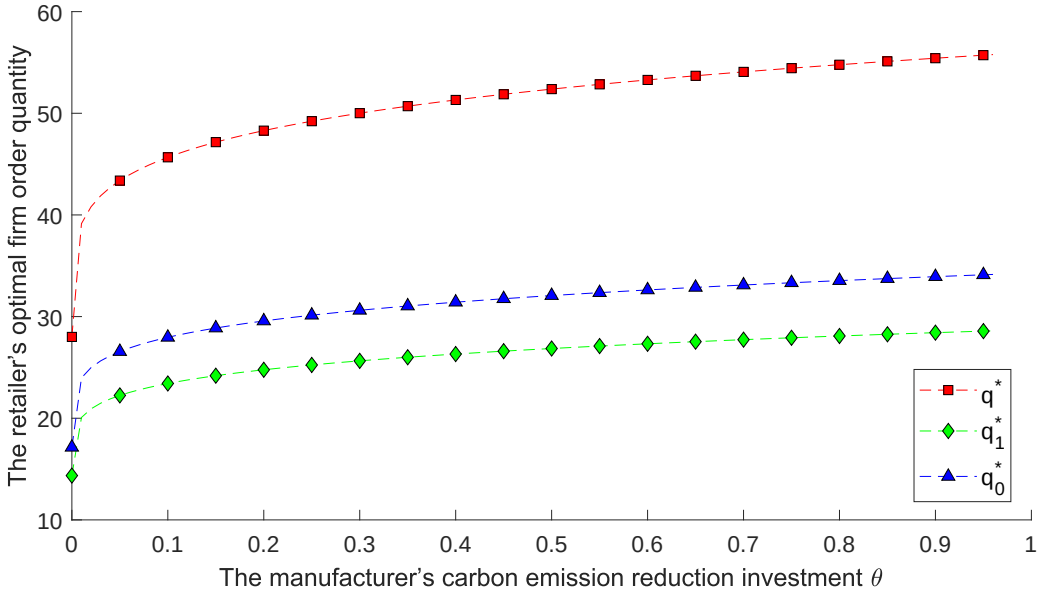


Figure 1. The effect of green investment on the optimal order quantity.

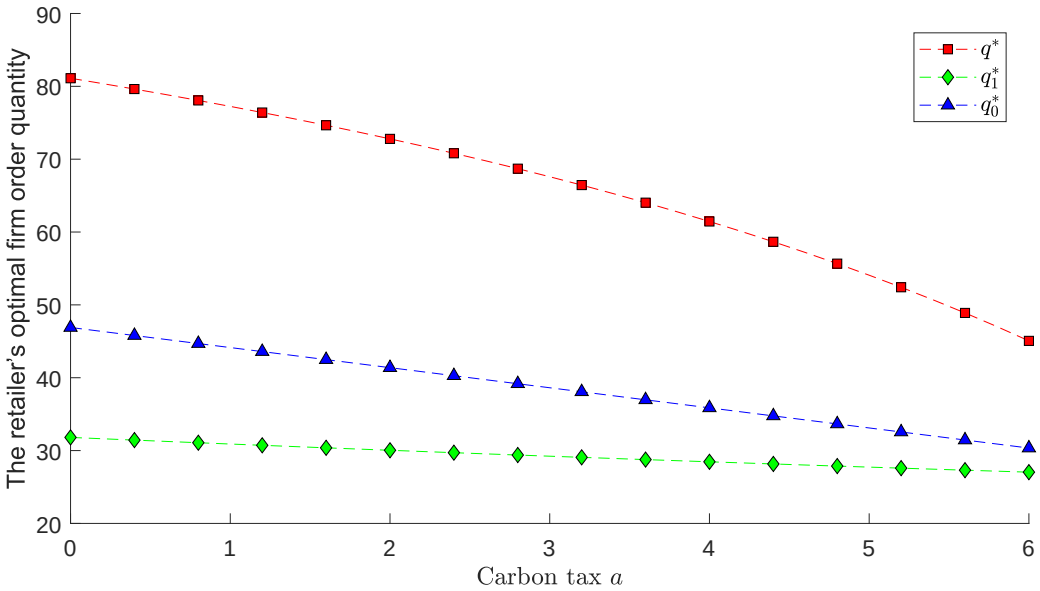


Figure 2. The effect of carbon tax on the optimal order quantity.

To evaluate the effect of carbon tax on the optimal total order quantity q_1^*, q^*, and q_0^*, we vary the carbon tax α from 0 to 6 in steps of 0.5 with fixed values of $\eta = 0.7$. The results of this numerical experiment are presented in Figure 2. The upper curve represents the optimal order quantity q^*, which consistently maintains the highest ordering level across the entire range of the carbon tax α. The middle curve corresponds to q_0^*, whose values are persistently lower than q^*. Overall, q^*, q_1^*, and q_0^* all exhibit a monotonically decreasing trend as α increases. This outcome demonstrates a significant negative correlation between the carbon tax and the retailer's equilibrium ordering quantity.

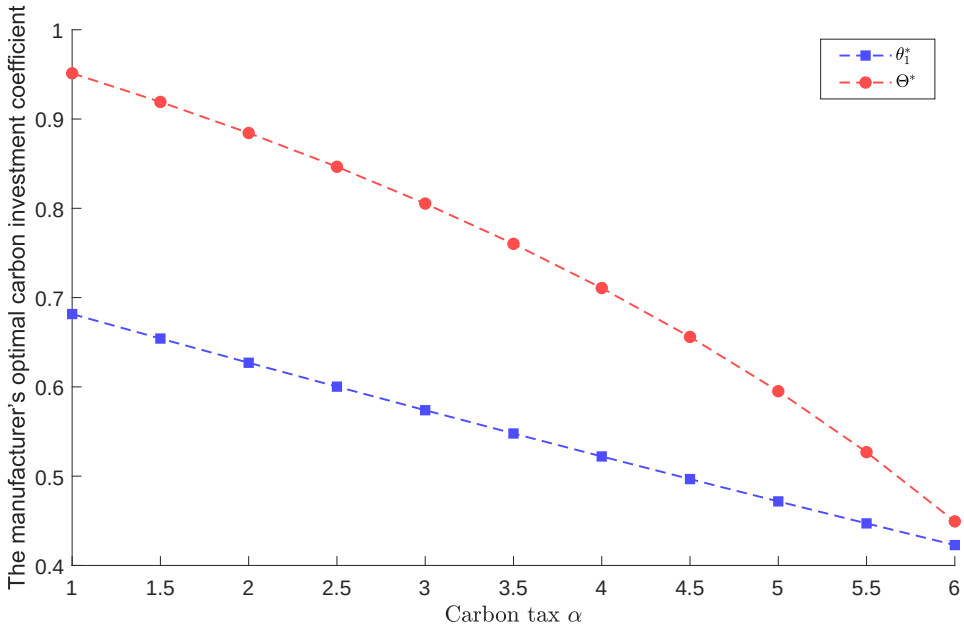


Figure 3. The effect of carbon tax on the manufacturer's optimal green investment.

To illustrate the effect of the carbon tax on the manufacturer's optimal green investment θ_1^* and Θ^* without and with a carbon abatement cost-sharing contract, we fix $\eta = 0.7$ and vary α from 1 to 6 in steps of 0.5. As illustrated in Figure 3, the lower curve corresponds to θ_1^* in the benchmark case lacking a carbon abatement cost-sharing contract, while the upper curve Θ^* captures optimal green investment with a carbon abatement cost-sharing contract. Throughout the considered range of the carbon tax α, θ_1^* is uniformly lower than Θ^*, which is consistent with Proposition 8. Further, Figure 3 also shows that both θ_1^* and Θ^* follow a strictly monotonic decreasing trend with respect to the carbon tax α. As α rises, the vertical gap between θ_1^* and Θ^* gradually shrinks.

6. Conclusions and future research directions

This paper investigates the optimal operational decisions in a supply-chain consisting of a risk-averse retailer and a risk-neutral manufacturer with a carbon tax policy, where the risk aversion coefficient of the retailer is measured by CVaR. The supply-chain incorporates both a carbon abatement cost-sharing contract and the call option contract, where market demand is stochastic and dependent on green investment. Different from existing research, this is the first stochastic supply-chain model that connects inventory replenishment operation and green investment. Further, this paper examines how the carbon tax policy, risk aversion coefficient, carbon abatement cost-sharing ratio, and call option contract jointly affect the supply-chain.

Specifically, the optimal ordering and production policies are derived for four distinct scenarios: with/without a carbon abatement cost-sharing contract and with/without a call option contract. It is shown that carbon abatement cost-sharing and call option contracts can mutually benefit the manufacturer and the retailer, and enhance the performance of the supply-chain. Further, the retailer's optimal firm order quantity and optimal total order quantity are monotonically increasing with the manufacturer's green investment. This paper further reveals that supply-chain coordination requires satisfying two key conditions, which are related to green investment and the carbon tax policy. In addition, it is demonstrated that two contract configurations fail to achieve supply-chain coordination. Finally, the numerical experiments in this study serve as illustrative demonstrations to intuitively visualize the theoretical findings, and yield practical managerial insights for supply-chain practitioners.

This study has several inherent limitations that offer promising directions for future research. First, the current model only considers the manufacturer's green investment as the core low-carbon measure responding to uncertain market demand, while ignoring the potential influences of retailers' low-carbon promotion efforts on supply-chain operational performance and demand volatility. Second, this paper only incorporates demand uncertainty induced by green investment levels and fails to account for supply-side uncertainty factors, which are commonly observed in practical low-carbon supply-chain operations. Third, the complete information symmetry assumption adopted in this study simplifies the interactive decision-making context of supply-chain members. Relaxing this restrictive premise to explore decentralized decision-making under information asymmetry can further enhance the model's practical applicability. Furthermore, this research is conducted based on a single supply-chain framework; future studies can extend the proposed model to competitive supply-chain scenarios to enrich the theoretical boundary of low-carbon coordination mechanisms.

Author contributions

Han Zhao: Conceptualization, Methodology, Formal analysis, Software, Investigation, Writing – original draft, Visualization. Wei Liu: Conceptualization, Supervision, Validation, Writing–review and editing, Funding acquisition. Yu Liao: Investigation, Formal analysis, Validation. Yuli Zhang: Software, Visualization, Validation. Shiji Song: Investigation, Writing–review and editing.

Use of Generative-AI tools declaration

The authors confirm that no generative-AI tools were used in the preparation of this manuscript.

Acknowledgments

This work is supported by the Science and Technology Research Program of the Chongqing Municipal Education Commission (Grant No. KJZD-K202312904) and National Social Science Foundation of China (2025).

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

References

1. X. Xu, X. Xu, H. Ping, Joint production and pricing decisions for multiple products with cap-and-trade and carbon tax regulations, *J. Clean. Prod.*, **112** (2016), 4093–4106. https://doi.org/10.1016/j.jclepro.2015.08.081
2. Y. Zhou, F. Hu, Z. Zhou, Pricing decisions and social welfare in a supply chain with multiple competing retailers and carbon tax policy, *J. Clean. Prod.*, **190** (2018), 752–777. https://doi.org/10.1016/j.jclepro.2018.04.162
3. M. Alegoz, O. Kaya, Z.P. Bayindir, A comparison of pure manufacturing and hybrid manufacturing-remanufacturing systems under carbon tax policy, *Eur. J. Oper. Res.*, **294** (2021), 161–173. https://doi.org/10.1016/j.ejor.2021.01.018
4. H. E. KlingelhöFer, Investments in eop-technologies and emissions trading - results from a linear programming approach and sensitivity analysis, *Eur. J. Oper. Res.*, **196** (2009), 370–383. https://doi.org/10.1016/j.ejor.2008.03.016
5. N. S. Kim, M. Janic, B. V. Wee, Trade-off between carbon dioxide emissions and logistics costs based on multiobjective optimization, *Transp. Res. Rec.*, **2139** (2009), 107–116. https://doi.org/doi:10.3141/2139-13
6. H. Rosič, W. Jammernegg, The economic and environmental performance of dual sourcing: A newsvendor approach, *Int. J. Prod. Econ.*, **143** (2013), 109–119. https://doi.org/10.1016/j.ijpe.2012.12.007

7. D. Battini, A. Persona, F. Sgarbossa, A sustainable EOQ model: Theoretical formulation and applications, *Int. J. Prod. Econ.*, **149** (2014), 145–153. https://doi.org/10.1016/j.ijpe.2013.06.026
8. Y. Dou, Q. Zhu, J. Sarkis, Evaluating green supplier development programs with a grey-analytical network process-based methodology, *Eur. J. Oper. Res.*, **233** (2014), 420–431. https://doi.org/10.1016/j.ejor.2013.03.004
9. I. Harris, M. Naim, A. Palmer, A. Potter, C. Mumford, Assessing the impact of cost optimization based on infrastructure modelling on CO_2 emissions, *Int. J. Prod. Econ.*, **131** (2011), 313–321. https://doi.org/10.1016/j.ijpe.2010.03.005
10. K. Colaneri, R. Frey, and V. Köck, Random carbon tax policy and investment into emission abatement technologies, *Math. Finance*, **36**(2026), 804–825. https://doi.org/10.1111/mafi.70031
11. M. Jin, N. A. Granda-Marulanda, I. Down, The impact of carbon policies on supply chain design and logistics of a major retailer, *J. Clean. Prod.*, **85** (2014), 453–461. https://doi.org/10.1016/j.jclepro.2013.08.042
12. J. Guo, Y. Yang, Z. Ma, Carbon emission reduction for competitive supply chains under the cap-and-trade scheme, *Ind. Eng. J.*, **21** (2018), 36–42. https://doi.org/10.3969/j.issn.1007-7375.e17-1170
13. H. Yang, J. Luo, H. Wang, The role of revenue sharing and first-mover advantage in emission abatement with carbon tax and consumer environmental awareness, *Int. J. Prod. Econ.*, **193** (2017), 691–702. https://doi.org/10.1016/j.ijpe.2017.08.032
14. T. L. Chen, X. Cui, Y. Zou, Carbon emission controls and optimal carbon related regulation in oligopoly: Examination of policy effects on competitiveness and entry deterrence, *J. Clean. Prod.*, **484** (2024), 144292. https://doi.org/10.1016/j.jclepro.2024.144292
15. C. Lu, Q. Tong, X. Liu, The impacts of carbon tax and complementary policies on Chinese economy, *Energy Policy*, **38** (2010), 7278–7285. https://doi.org/10.1016/j.enpol.2010.07.055
16. W. Liu, S. Song, Y. Qiao, H. Zhao, The loss-averse newsvendor problem with random supply capacity, *J. Ind. Manag. Optim.*, **13** (2017), 1417–1429. https://doi.org/10.3934/jimo.2016080
17. W. Liu, S. Song, Y. Qiao, H. Zhao, H. Wang, The loss-averse newsvendor problem with quantity-oriented reference point under CVaR criterion, *J. Ind. Manag. Optim.*, **18** (2022), 2633–2650. https://doi.org/10.3934/jimo.2021085
18. B. M. Beamon, Designing the green supply chain, *Logist. Inf. Manag.*, **12** (1999), 332–342. https://doi.org/10.1108/09576059910284159
19. P. He, Y. Sun, The impacts of retailer's carbon tax on the price and carbon reduction decisions in supply chain, *Comput. Ind. Eng.*, **190** (2024), 110034. https://doi.org/10.1016/j.cie.2024.110034
20. Y. Zhang, T. Zhang, Complex dynamics of a low-carbon supply chain with government green subsidies and carbon cap-and-trade policies, *Int. J. Bifurcat. Chaos*, **32** (2022), 2250090. https://doi.org/10.1142/S0218127422500900
21. L. Yang, Y. Cai, Optimal dynamic pricing for fresh products under the cap-and-trade scheme, *2017 14th Int. Conf. Serv. Syst. Serv. Manag.*, 2017. https://doi.org/10.1109/ICSSSM.2017.7996183
22. G. Ji, A. Gunasekaran, G. Yang, Constructing sustainable supply chain under double environmental medium regulations, *Int. J. Prod. Econ.*, **147** (2014), 211–219. https://doi.org/10.1016/j.ijpe.2013.04.012

23. H. Zhao, S. Song, Y. Zhang, J. N. D. Gupta, A. G. Devlin, R. Chiong, Supply chain coordination with a risk-averse retailer and a combined buy-back and revenue sharing contract, *Asia-Pac. J. Oper. Res.*, **36** (2019), 1950028. https://doi.org/10.1142/S0217595919500283

24. B. Li, P. W. Hou, P. Chen, Q. H. Li, Pricing strategy and coordination in a dual channel supply chain with a risk-averse retailer, *Int. J. Prod. Econ.*, **178** (2016), 154–168. https://doi.org/10.1016/j.ijpe.2016.05.010

25. W. A. Jauhari, S. C. N. Ramadhany, C. N. Rosyidi, U. Mishra, H. Hishamuddin, Pricing and green inventory decisions for a supply chain system with green investment and carbon tax regulation, *J. Clean. Prod.*, **425** (2023), 138897. https://doi.org/10.1016/j.jclepro.2023.138897

26. W. A. Jauhari, C. T. C. Sakti, M. Hisjam, H. Hishamuddin, A sustainable circular economic supply chain model with green production, delays in payment, and carbon tax regulation, *J. Clean. Prod.*, **495** (2025), 145008. https://doi.org/10.1016/j.jclepro.2025.145008

27. K. Jörnsten, S. L. Nonås, L. Sandal, J. Ubøe, Mixed contracts for the newsvendor problem with real options and discrete demand, *Omega*, **41** (2013), 809–819. https://doi.org/10.1016/j.omega.2012.10.011

28. Q. Fu, C. Y. Lee, C. P. Teo, Procurement management using option contracts: random spot price and the portfolio effect, *IIE Trans.*, **42** (2010), 793–811. https://doi.org/10.1080/07408171003670983

29. Y. Feng, Y. Mu, B. Hu, A. Kumar, Commodity options purchasing and credit financing under capital constraint, *Int. J. Prod. Econ.*, **153** (2014), 230–237. https://doi.org/10.1016/j.ijpe.2014.03.003

30. Y. T. Chung, F. Erhun, Designing supply contracts for perishable goods with two periods of shelf life, *IIE Trans.*, **45** (2013), 53–67. https://doi.org/10.1080/0740817X.2012.654847

31. C. Y. Lee, X. Li, M. Yu, The loss-averse newsvendor problem with supply options, *Nav. Res. Logist.*, **62** (2015), 46–59. https://doi.org/10.1002/nav.21613

32. F. Hu, C. C. Lim, Z. Lu, Optimal production and procurement decisions in a supply chain with an option contract and partial backordering under uncertainties, *Appl. Math. Comput.*, **232** (2014), 1225–1234. https://doi.org/10.1016/j.amc.2014.01.149

33. H. Xu, Managing production and procurement through option contracts in supply chains with random yield, *Int. J. Prod. Econ.*, **126** (2010), 306–313. https://doi.org/10.1016/j.ijpe.2010.04.007

34. H. Zhao, B. Sun, H. Wang, S. Song, Y. Zhang, L. Wang, Optimization and coordination in a service-constrained supply chain with the bidirectional option contract under conditional value-at-risk, *J. Ind. Manag. Optim.*, **15** (2022), 3383–3412. https://doi.org/10.3934/dcdss.2022021

35. H. Zhao, S. Song, Y. Zhang, Y. Liao, F. Yue, Optimal decisions in supply chains with a call option contract under the carbon emissions tax regulation, *J. Clean. Prod.*, **271** (2020), 122199. https://doi.org/10.1016/j.jclepro.2020.122199



AIMS Press

© 2026 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0)